Is there an off-shell pion and can it be used at JLab12?

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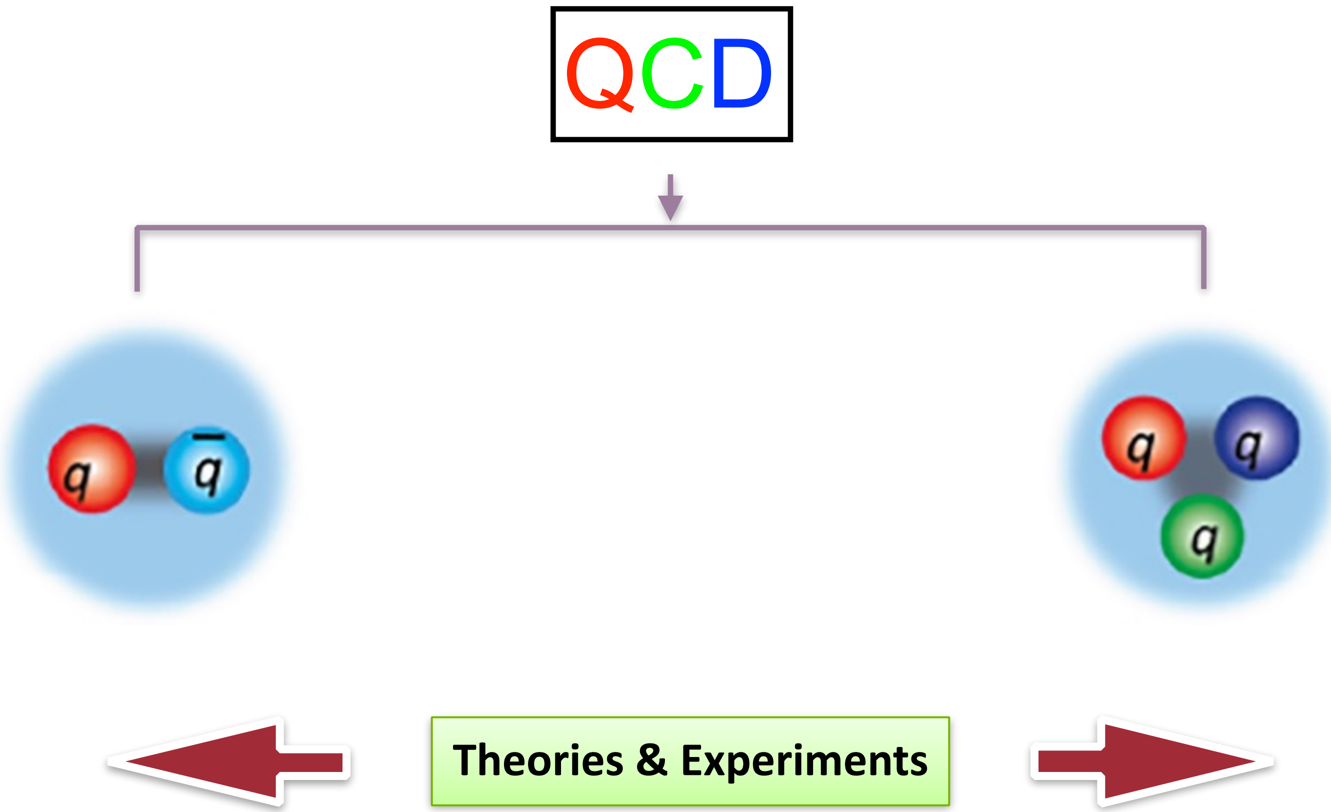
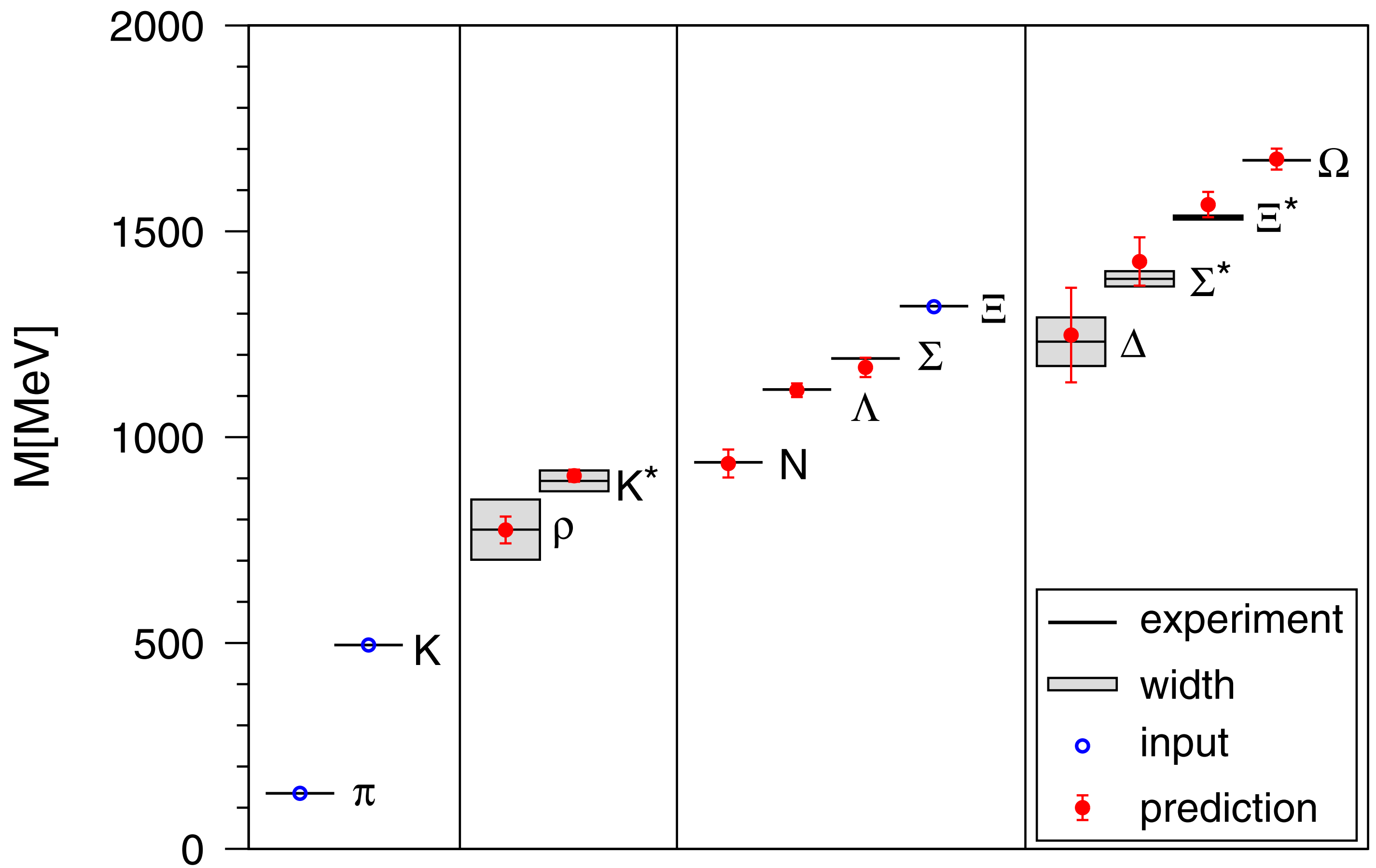
Hadron: Bound-state of QCD

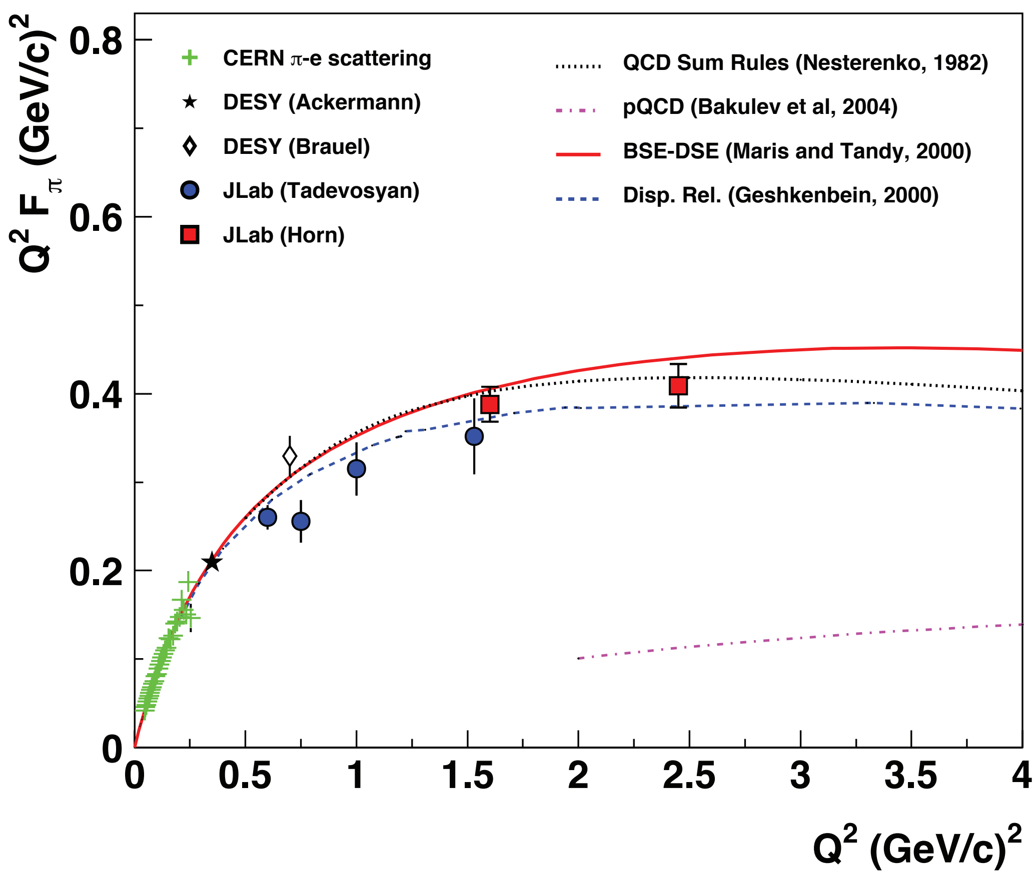


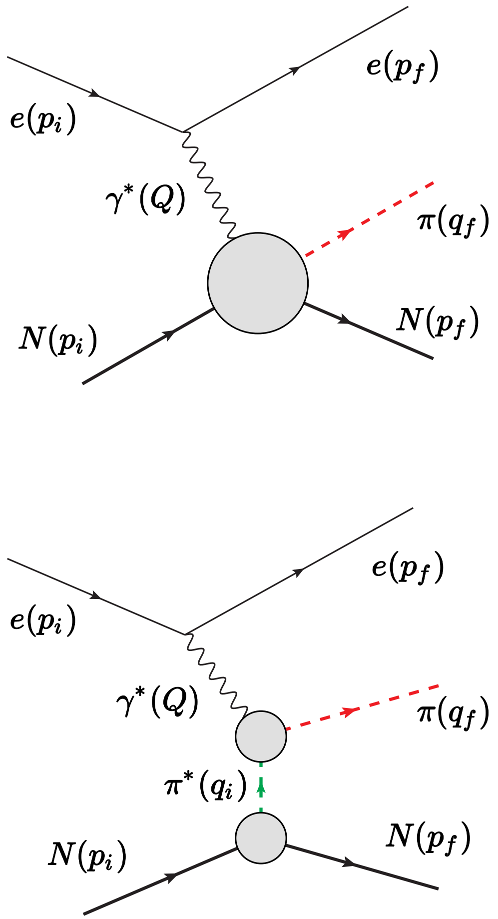
Question I: What matter is possible? — Hadron spectrum



Durr, et al. (Budapest-Marseille-Wuppertal Collaboration), Science 322, 1224 (2008)

Question II: How is it constituted? — Hadron Structure





Horn et al., PRL 97, 192001 (2006)

Solve QCD: Generating functional

$$\mathcal{Z}[\phi, j] = \int [\mathcal{D}\phi] e^{-S(x)+\int_x \phi(x) j(x)}$$

Green functions

$$G^{(n)}(x_1, ..., x_n) := \frac{1}{\mathcal{Z}[0]} \frac{\delta^n \mathcal{Z}[j]}{\delta j(x_1)...j(x_n)}\bigg|_{j=0}$$

Approaches

Lattice QCD

Phenomenological Models

Dyson-Schwinger equations

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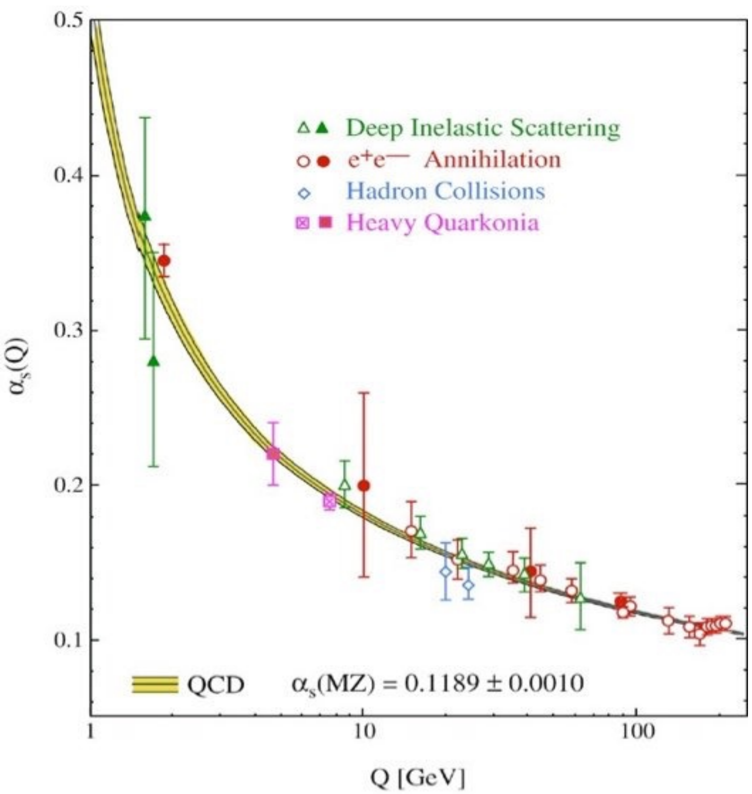
Non-perturbative



QCD running coupling constant

Green functions

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Dyson-Schwinger Equations: Equation of motion of Green functions

Classical Mechanics | Quantum Field Theory

Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0$$

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Equations of Motion (EoM)

Euler-Lagrange Equation | Dyson-Schwinger Equation

Dyson-Schwinger Equations: Equation of motion of Green functions

G. Eichmann, arXiv:0909.0703

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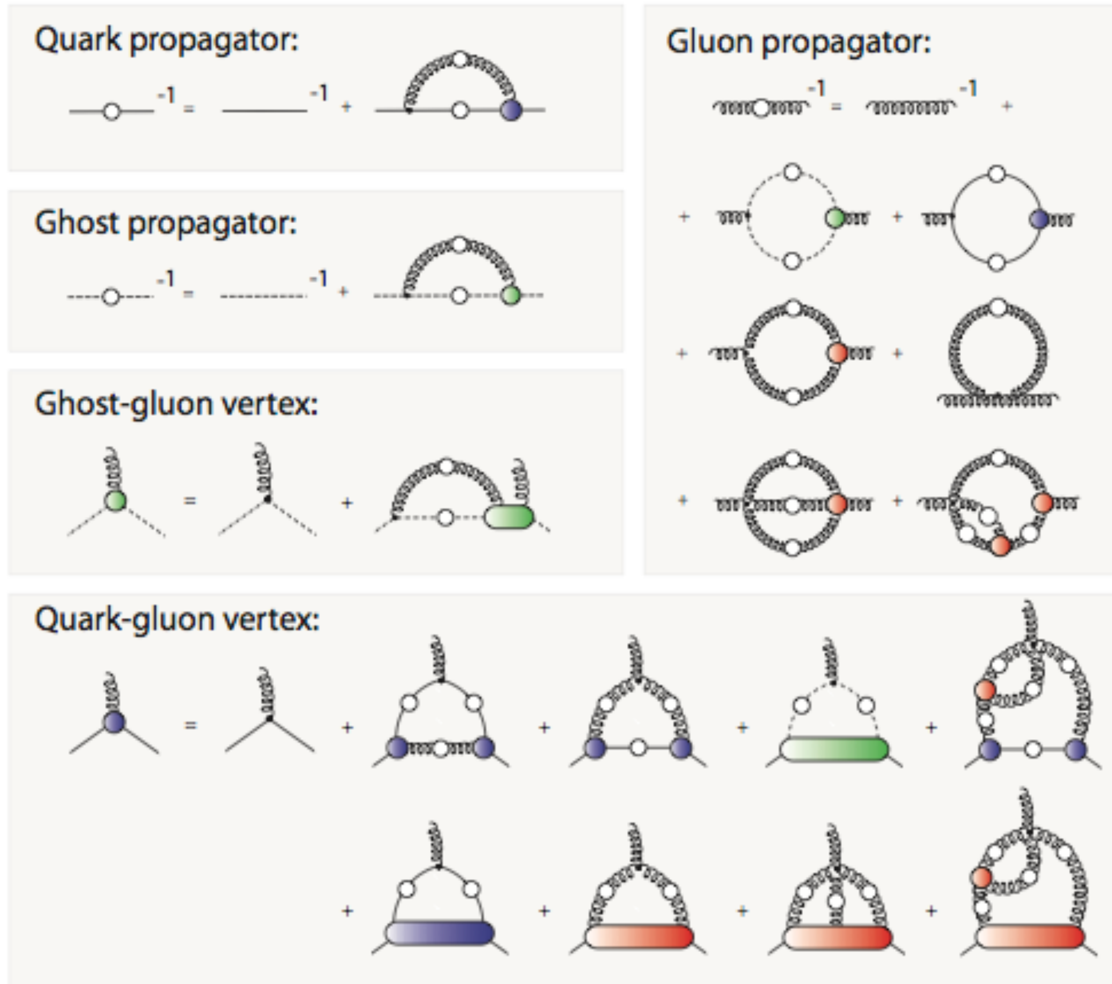
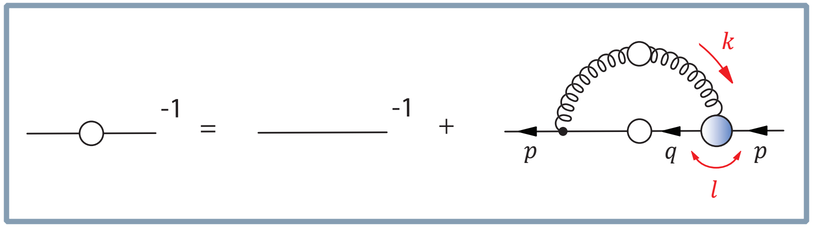
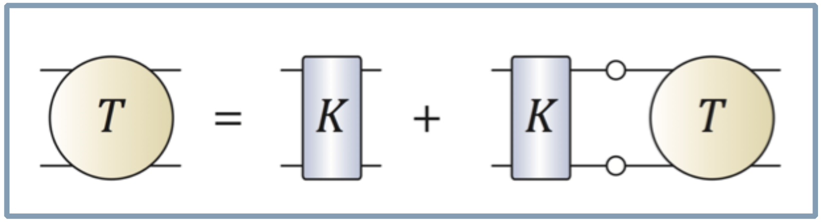
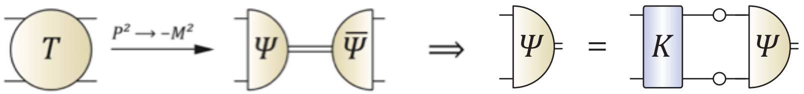


Dyson-Schwinger Equations: Equations for mesons







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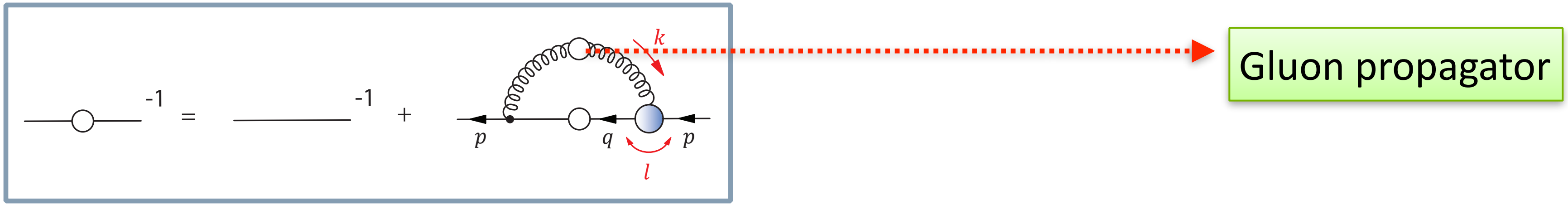

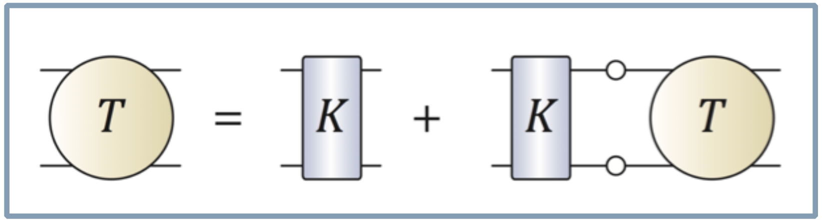

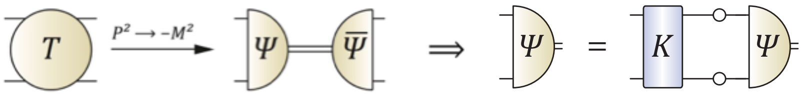

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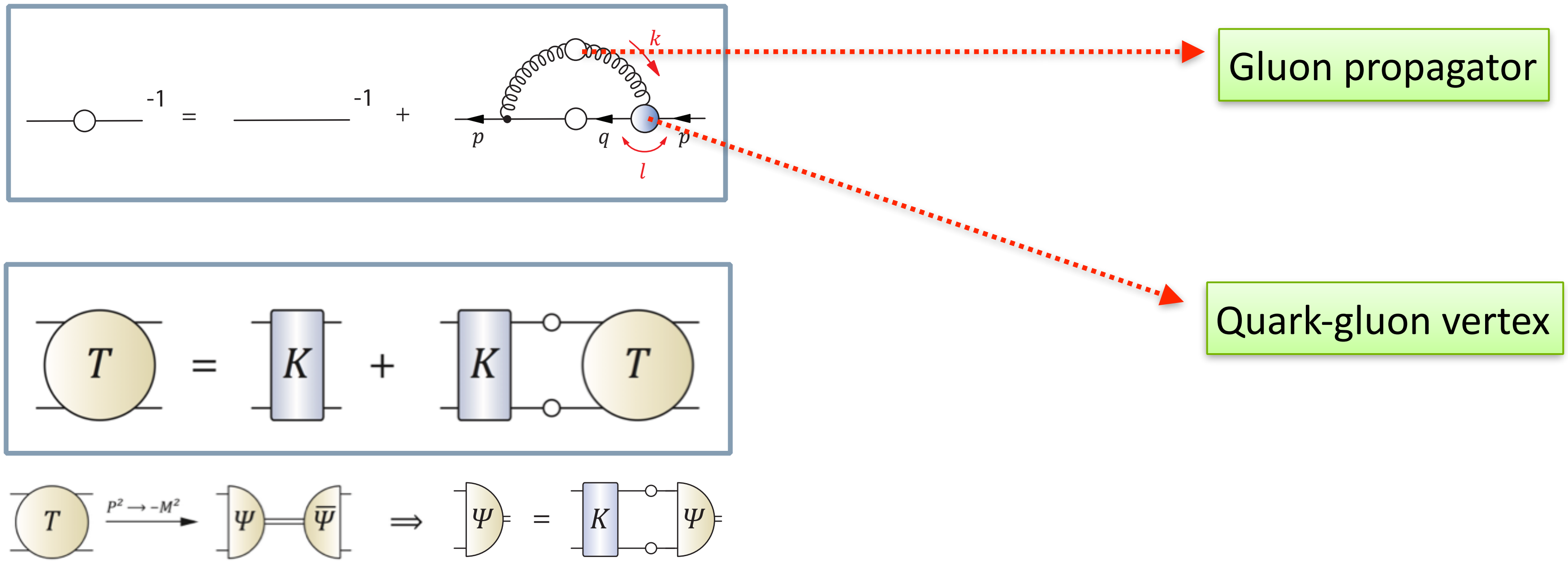

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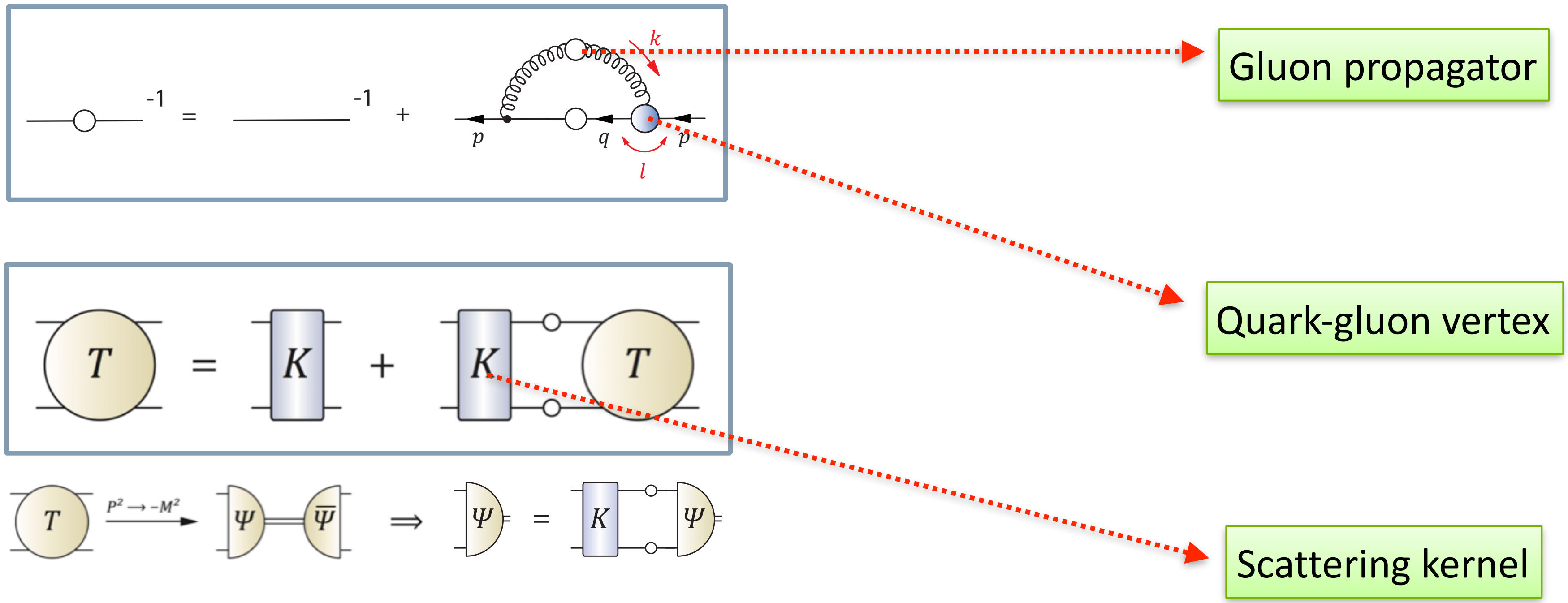

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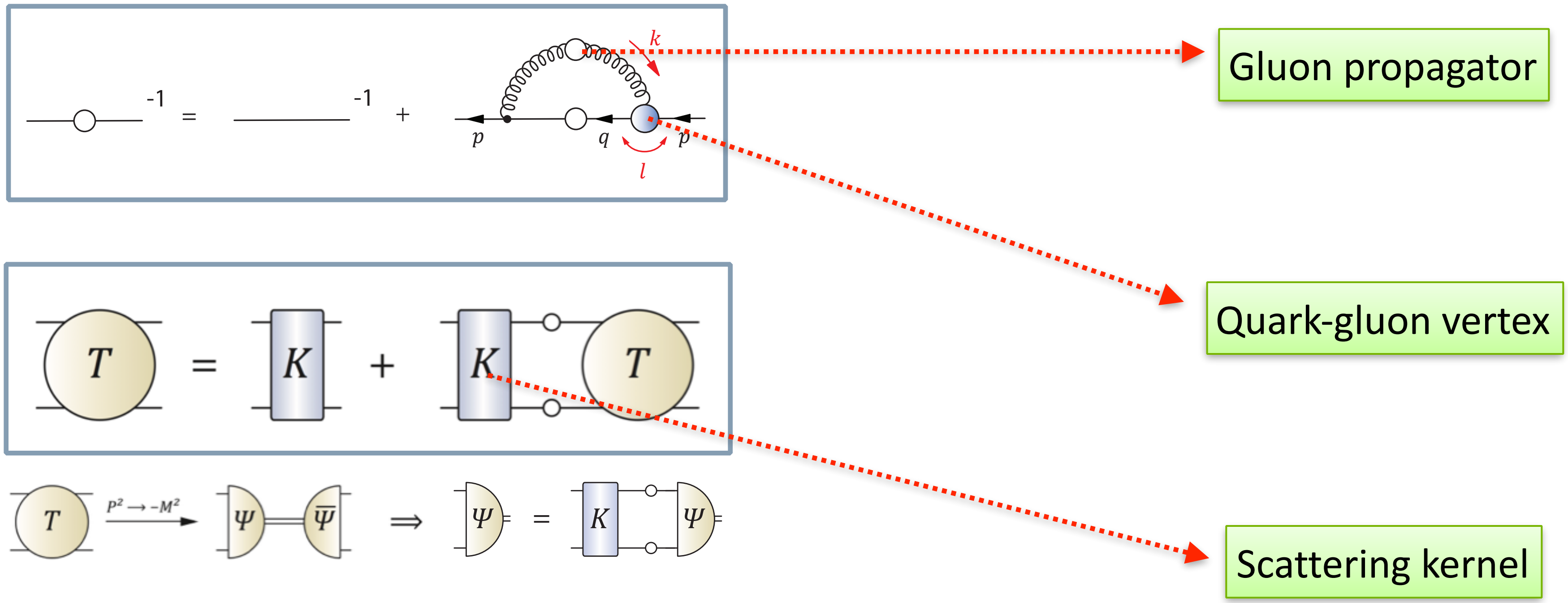
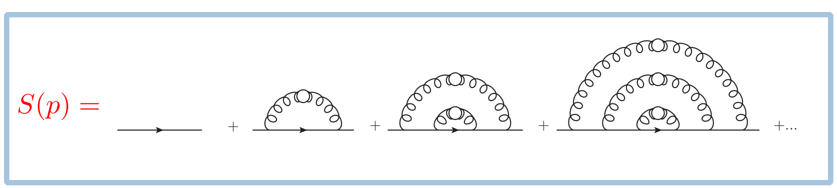
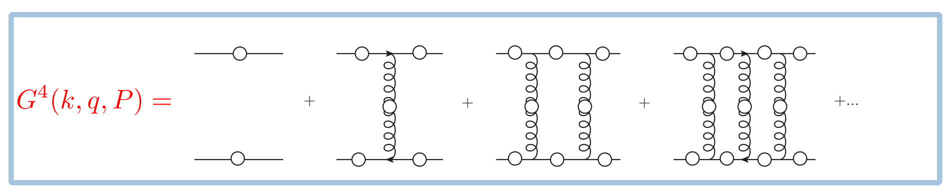


Rainbow diagrams of quark propagator:



Ladder diagrams of 4-point Green function:



Dyson-Schwinger Equations: A systematic truncation

- **Gluon propagator**: Solve the DSE of gluon or Extract information from lattice QCD. The dressing function of gluon has a mass scale as that of quark.

- **Quark-gluon vertex**: Solve the WGTIs which come from the Lagrangian symmetries (gauge, chiral, and Lorentz symmetries). The dressed vertex is significantly modified by DCSB.

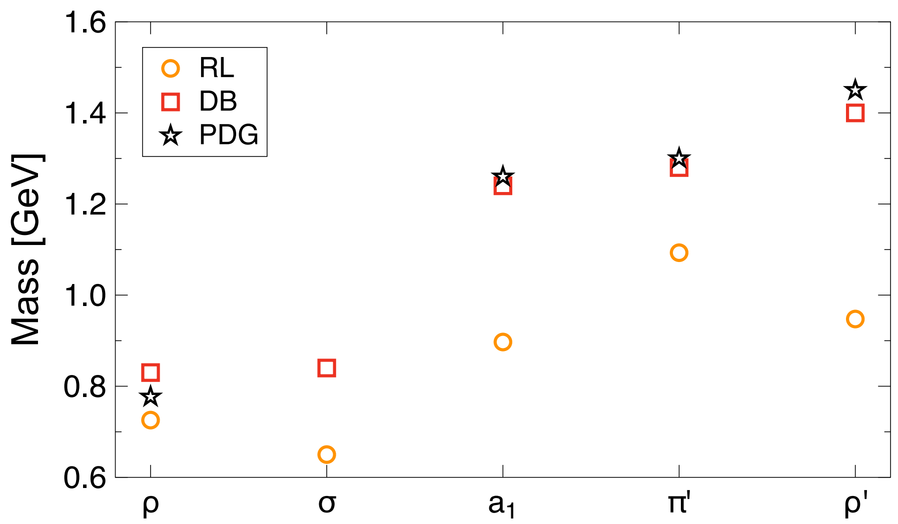
- **Scattering kernel**: Solve the color-singlet vector and axial-vector WGTIs. The kernel preserves the chiral symmetry which makes pion to play a twofold role: Bound-state and Goldstone boson.

Meson spectroscopy: From ground to radial excitation states

Let the quark-gluon vertex include both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma^{\rm L}_\mu(p,q) + \Gamma^{\rm T}_\mu(p,q)$$

then the spectrum from ground to radial excitation states can be well produced:



The mass ordering reads:

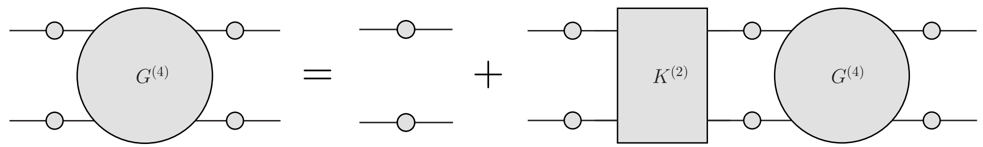
$$m_{\rho'} > m_{\pi'} > m_{a_1} > m_\sigma > m_\rho > m_\pi$$

	$-\langle\bar{q}q\rangle_0^{1/3}$	f_π	m_σ	m_ρ	m_{a_1}	$m_{\pi'}$	$m_{\rho'}$
this work	0.220	0.092	0.84	0.83	1.24	1.28	1.40
PDG	-	0.093	0.50	0.78	1.26	1.30	1.45

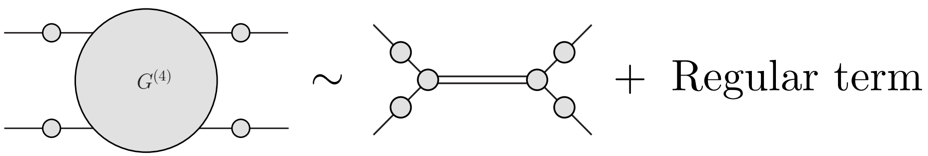
TABLE I: The fitted spectrum and its comparison with PDG data (Full vertex, $(D\omega)^{1/3} = 0.484$ GeV, $\omega = 0.55$ GeV, $\eta = 0.5$ and $\xi = 1.15$, in the chiral limit where pion is always massless).

Off-shell pion: Bound-state as a pole of Green function

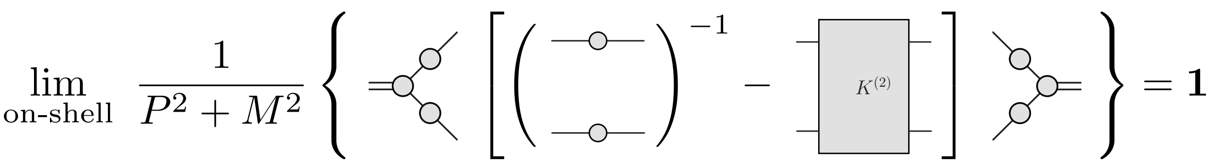
The Dyson-Schwinger equation of the **four-point Green function** is written as



Assuming that there is a **bound state**



the **wave function** of the bound state has to satisfy the following condition



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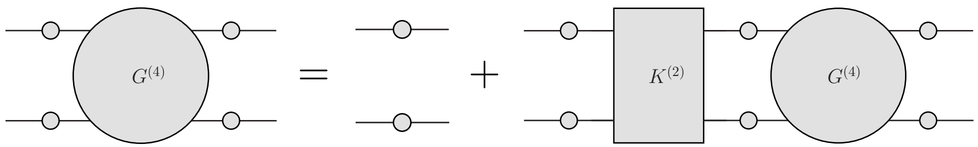

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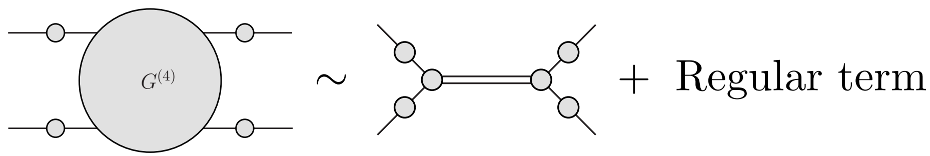

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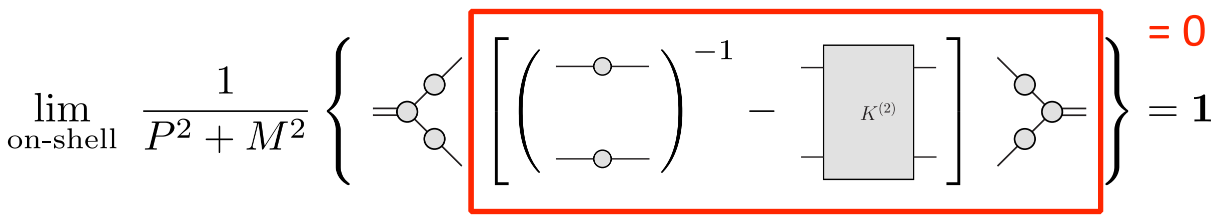


- ✦ The wave function of the on-shell bound state satisfies the Bethe-Salpeter equation.
- ✦ The physical wave function must be normalized (elementary particle vs. bound state).

Off-shell pion: Off-shell state decomposition of Green function

For any total momentum *P*, the BSE can be generalized as ($\lambda_i = 1$ on-shell state)

$$\lambda_i |v_i\rangle = K^{(2)} |v_i\rangle$$

The kernel can be decomposed by the orthonormal eigenbasis:

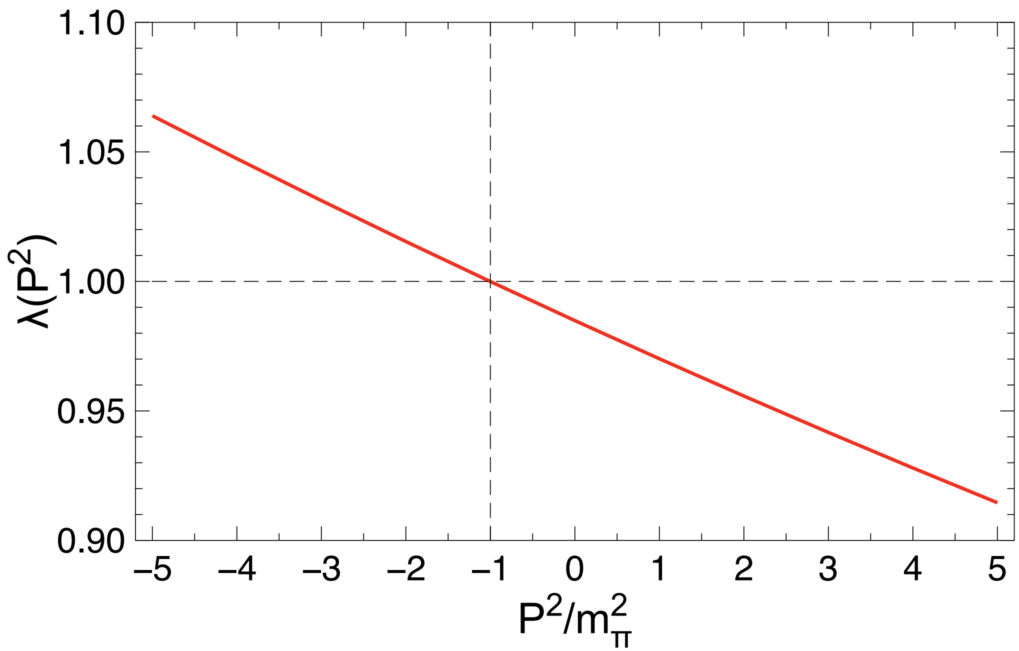
$$K^{(2)} = \sum_i \lambda_i |v_i\rangle\langle v_i|$$

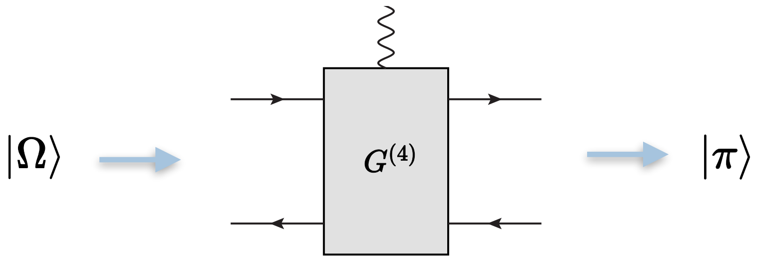
with

$$\langle v_i | v_j \rangle = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Accordingly, the four-point Green function can be decomposed as

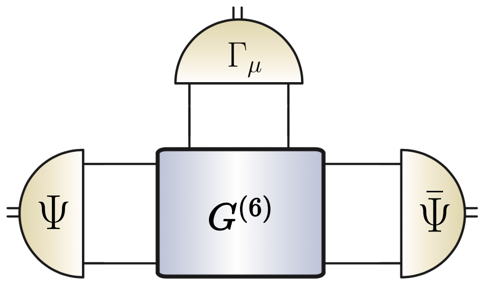
$$G^{(4)} = G_0^{(4)} + K^{(2)} \cdot G^{(4)} = \sum_i \frac{|v_i\rangle\langle v_i| \cdot G_0^{(4)}}{1 - \lambda_i}$$

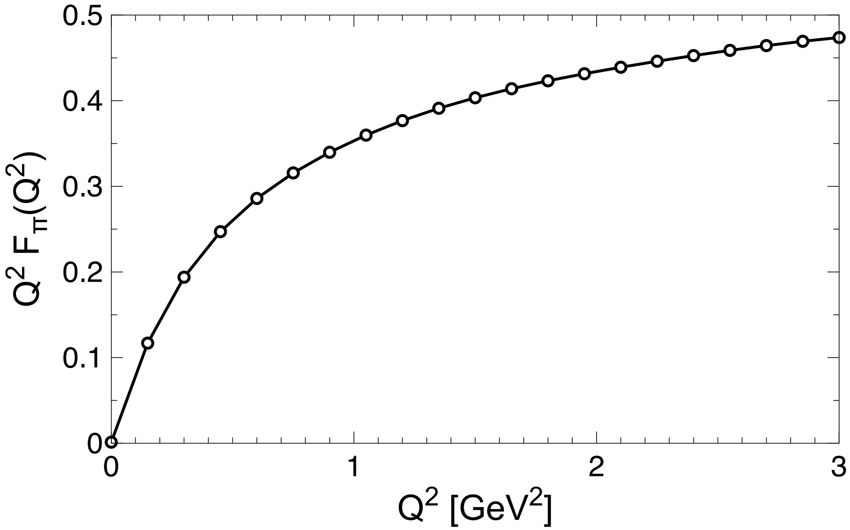




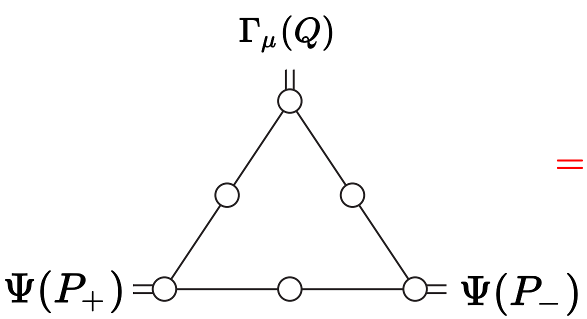
Form factor: On-shell pion in the simplest approximation

The coupling between photon and meson is described by the diagram:





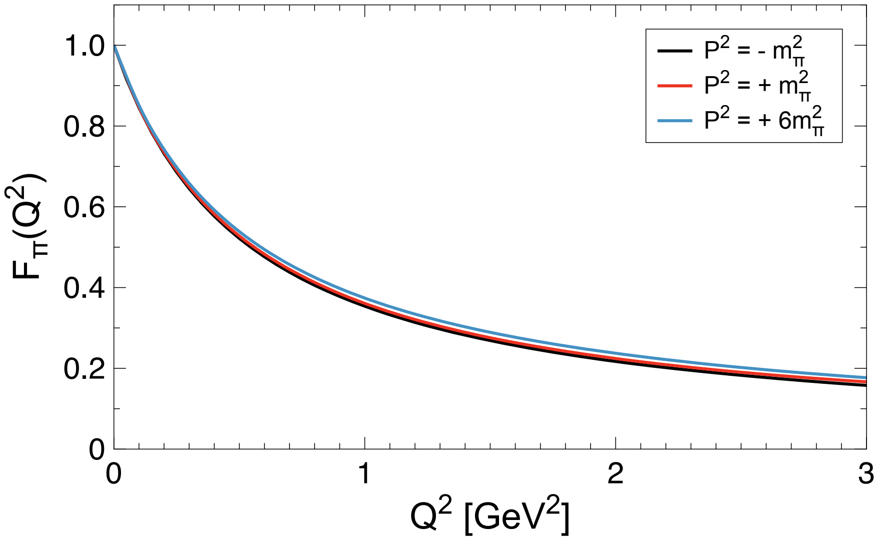
The triangle diagram of the form factor ($P_\pm = P \pm \frac{Q}{2}$):

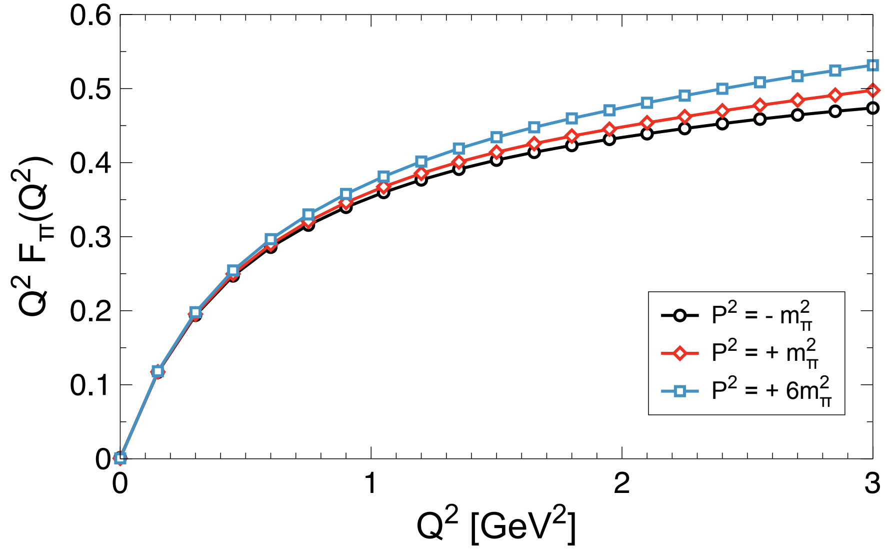


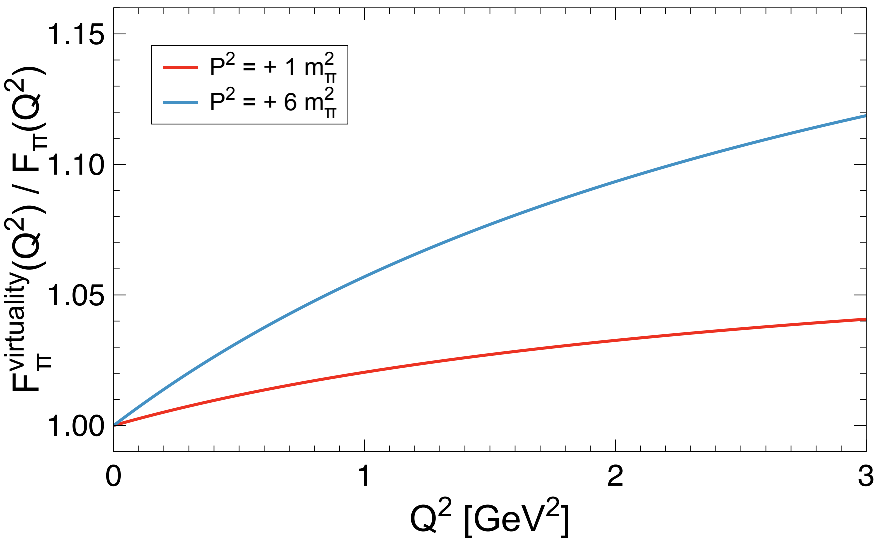
$$= \Lambda_\mu(P,Q) = 2P_\mu F(Q^2)$$

- ✦ $Q^2 F_\pi(Q^2)$ keeps increasing with Q^2 increasing. The monotonic behavior is inconsistent with the sum rules of the form factor.
- ✦ The obtained form factor is almost identical to the monopole behavior of vector meson.

Form factor: Off-shell pion in the simplest approximation







- ✦ With the virtuality increasing, the pion has a smaller radius and becomes more point-like.
- ✦ With the momentum increasing, the difference of the form factor increases (~ 10% for 6 virtuality in the medium momentum region).

How to formulate the **form factor** of bound state in a more sophisticated form?

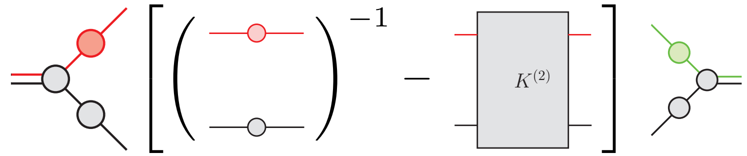
How to formulate the **form factor** of bound state in a more sophisticated form?

I. Wavefunction of bound state

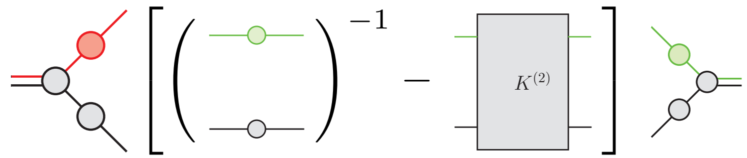
II. Quark-photon vertex

Form factor: Wavefunction of bound state — normalization condition

Introduce two functions depending on ***(P, Q)*** as

$\mathcal{G}_+(P,Q) =$ 

$q_+ + \frac{Q}{2}$

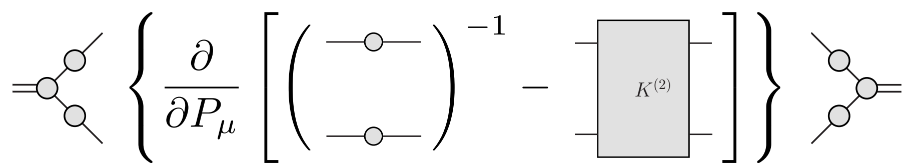
$\mathcal{G}_-(P,Q) =$ 

$q_+ - \frac{Q}{2}$

Then the difference between the two functions

$$\mathcal{G}(P,Q) \equiv \mathcal{G}_+(P,Q) - \mathcal{G}_-(P,Q)$$

satisfies the following condition

$\lim_{Q \to 0} \frac{\mathcal{G}(P,Q)}{Q_\mu} =$ 

$= 2P_\mu$

Form factor: Wavefunction of bound state — normalization condition

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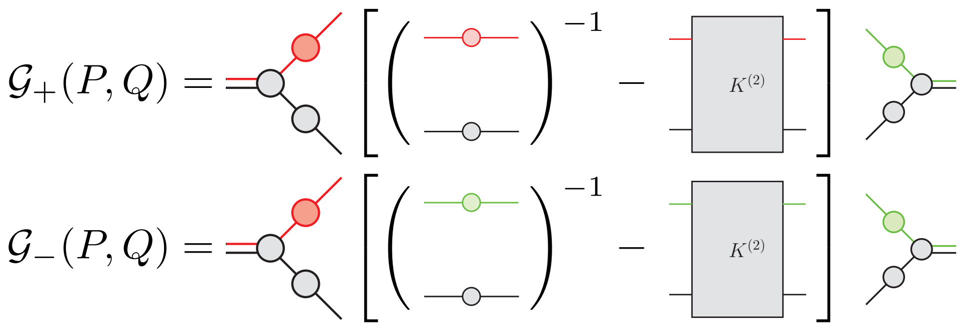

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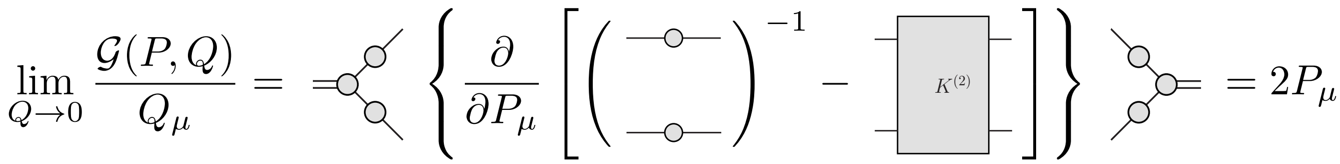

How to take advantage of the normalization condition?

Form factor: Quark-photon vertex — current conservation

Inserting the **color-singlet** **vector Ward identity**

$$Q_\mu \Gamma_\mu \left(q_+ + \frac{Q}{2}, q_+ - \frac{Q}{2}\right) = S^{-1}\left(q_+ + \frac{Q}{2}\right) - S^{-1}\left(q_+ - \frac{Q}{2}\right)$$

into the **normalization condition**, we can have

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Eventually, the **form factor** can be defined as $\Lambda_\mu(P,Q) = 2P_\mu F(Q^2)$ with $F(Q^2=0) = 1$

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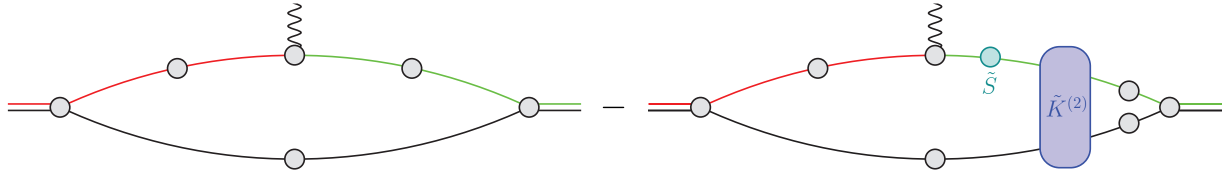

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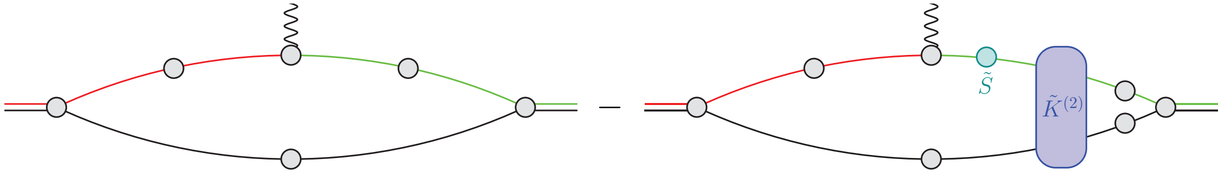


Numerical results are in progress…

Summary

- ✦ A **systematic** and **self-consistent** method to construct **the gluon propagator**, **the quark-gluon vertex**, and **the scattering kernel** is summarized.

- ✦ A **model-independent** scheme to define the **off-shell** bound state is proposed. A demonstration in the simplest approximation is presented.

- ✦ A **general** scheme to compute the **form factor** of bound state is proposed.

Outlook

- ◆ With the **most sophisticated** truncation scheme to solve the DSEs, we can compute the form factor of **on-shell** and **off-shell** pion and work with new data in JLab12GeV.

- ◆ Using the **diquark** picture, **proton** can be reduced as a two-body problem. Then the scheme can be adopted to study proton from factor.

Backups

Gluon propagator:

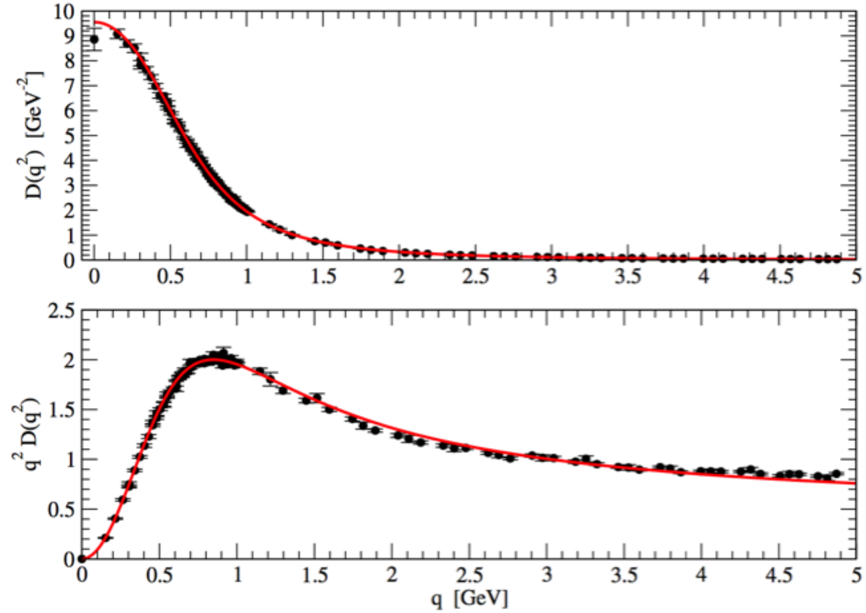
➢ In Landau gauge (a fixed point of the renormalization group):

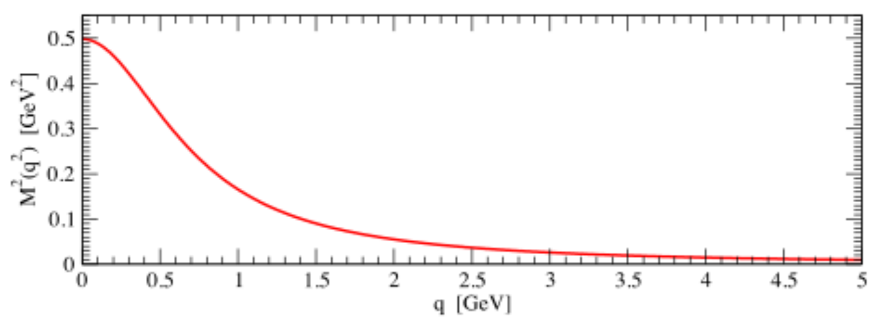
$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

➢ Modeling the dress function: ***gluon mass scale*** + ***effective running coupling constant***

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)},$$

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2},$$





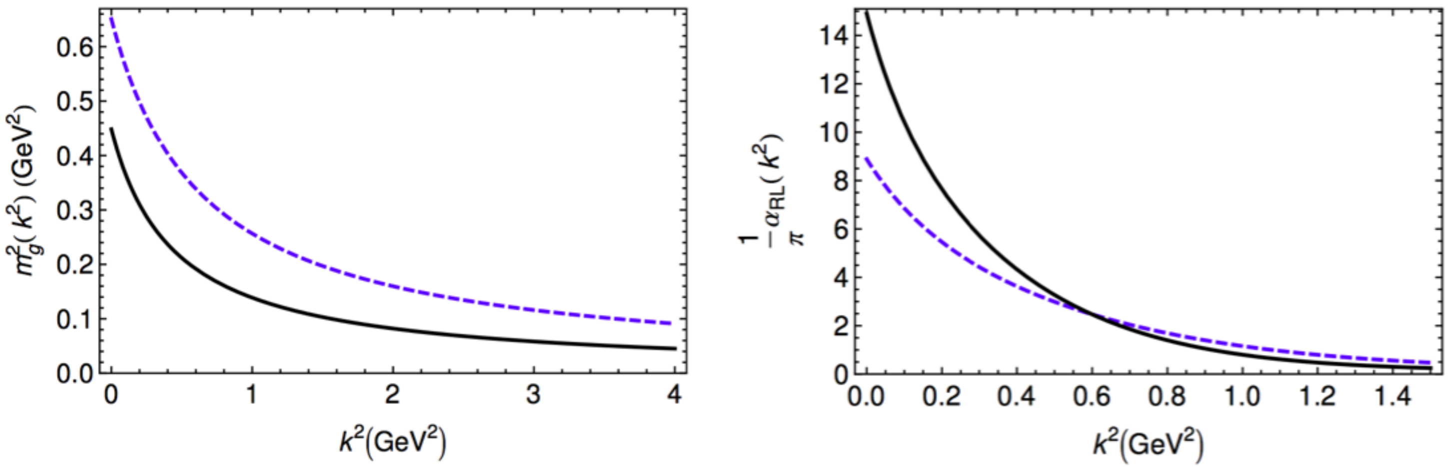
O. Oliveira et. al., arXiv:1002.4151

Gluon propagator:

Model the gluon propagator as two parts: Infrared + Ultraviolet. The former is an expansion of delta function; The latter is a form of one-loop perturbative calculation.

$$\delta^4(k) \overset{\omega\sim 0}{\approx} \frac{1}{\pi^2}\frac{1}{\omega^4}e^{-k^2/\omega^2} \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln\left[\tau + \left(1 + s/\Lambda_{QCD}^2\right)^2\right]}$$

- ❑ The gluon mass scale is *typical values of lattice QCD* in our parameter range: *Mg* in [0.6, 0.8] GeV.
- ❑ The gluon mass scale is inversely proportional to the confinement length.



Quark-Gluon Vertex: (Abelian) Ward-Green-Takahashi Identities

- **Gauge symmetry (vector current conservation):** vector WGTI

$$\psi(x) \rightarrow \psi(x) + ig\alpha(x)\psi(x),$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x)$$

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry (axial-vector current conservation):** axial-vector WGTI

$$\psi(x) \rightarrow \psi(x) + ig\alpha(x)\gamma^5\psi(x),$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}(x) + ig\alpha(x)\bar{\psi}(x)\gamma^5,$$

$$q_\mu \Gamma^A_\mu(k,p) = S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

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$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry (axial-vector current conservation):** axial-vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\gamma^5\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) + ig\alpha(x)\bar{\psi}(x)\gamma^5,$$

$$q_\mu \Gamma^A_\mu(k,p) = S^{-1}(k)i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

- **Lorentz symmetry + (axial-)vector current conservation:** transverse WGTIs

$$\delta_T \phi^a(x) = \delta_{\text{Lorentz}}(\delta\phi^a(x)) = -\frac{i}{2}\epsilon^{\mu\nu}S^{(\delta\phi^a)}_{\mu\nu}(\delta\phi^a(x)).$$
$$S^{(\text{spinor})}_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu}, \qquad (S^{(\text{vector})}_{\mu\nu})^\alpha_\beta = i(\delta^\alpha_\mu g_{\nu\beta} - \delta^\alpha_\nu g_{\mu\beta});$$

He, PRD, 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p), \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k) \\ & + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p), \qquad \sigma^5_{\mu\nu} = \gamma_5\sigma_{\mu\nu} \end{aligned}$$

Quark-Gluon Vertex: (Abelian) Ward-Green-Takahashi Identities

- **Gauge symmetry (vector current conservation):** vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x)$$

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry (axial-vector current conservation):** axial-vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\gamma^5\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) + ig\alpha(x)\bar{\psi}(x)\gamma^5,$$

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He, PRD, 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p), \end{aligned}$$

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The longitudinal and transverse WGTIs express the vertex divergences and curls, respectively.

$$\nabla \cdot \Phi \qquad \nabla \times \Phi$$

Quark-Gluon Vertex: Solution of WGTIs

Define two projection tensors and contract them with the transverse WGTIs,

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_\mathrm{D}, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta.$$

one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

$$\begin{aligned} q\cdot tt\cdot\Gamma(k,p) &= T^1_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + t^2 q\cdot\Gamma(k,p) + T^1_{\mu\nu}V^A_{\mu\nu}(k,p), \\ q\cdot t\gamma\cdot\Gamma(k,p) &= T^2_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p). \end{aligned}$$

Qin et. al., PLB 722, 384 (2013)

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$$q\cdot t\gamma\cdot\Gamma(k,p) = T^2_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)] + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p).$$

They are a group of full-determinant linear equations. Thus, a unique solution for the vector vertex is exposed:

$$\Gamma^{\mathrm{Full}}_\mu(k,p) = \Gamma^{\mathrm{BC}}_\mu(k,p) + \Gamma^{\mathrm{T}}_\mu(k,p) + \Gamma^{\mathrm{FP}}_\mu(k,p).$$

Qin et. al., PLB 722, 384 (2013)

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$$\Gamma^{\rm Full}_\mu(k,p) = \Gamma^{\rm BC}_\mu(k,p) + \Gamma^{\rm T}_\mu(k,p) + \Gamma^{\rm FP}_\mu(k,p).$$

❖ The quark propagator contributes to the longitudinal and transverse parts. The DCSB-related terms are highlighted.

$$\Gamma^{\rm BC}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B,$$

$$\Gamma^{\rm T}_\mu(k,p) = -\sigma_{\mu\nu}q_\nu\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x)+\phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x)-\phi(y)}{x-y}.$$

$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

❖ The unknown high-order terms only contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

Qin et. al., PLB 722, 384 (2013)

Scattering kernel: Color-singlet vector and axial-vector WGTIs

✦ The Bethe-Salpeter equation and the quark gap equation are written as

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

$$S^{-1}(k) = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

✦ The color-singlet axial-vector and vector WGTIs are written as

$$P_\mu\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),$$

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$$iP_\mu\Gamma_\mu(k,P) = S^{-1}(k_+) - S^{-1}(k_-).$$

✦ The kernel satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

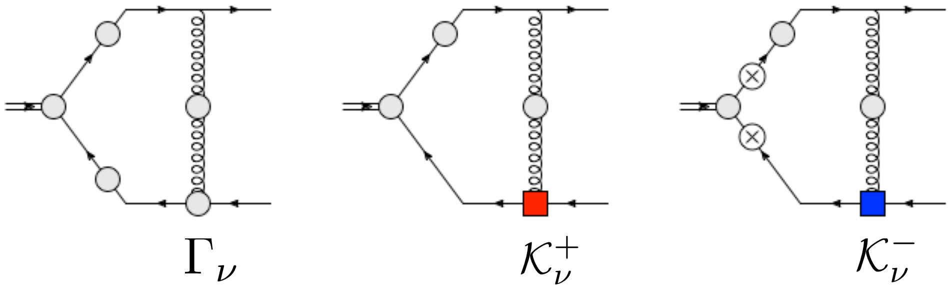
$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

Scattering kernel: An ansatz for the kernel

Assuming the scattering kernel has the following structure:

$$\begin{aligned}\mathcal{K}_{\alpha\alpha',\beta'\beta}(q_\pm,k_\pm)[S(q_+)\ \bigcirc\ S(q_-)]_{\alpha'\beta'} = &-D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \bigcirc\ S(q_-)\Gamma_\nu(q_-,k_-)\\ &+D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \bigcirc\ \mathcal{K}^+_\nu(q_\pm,k_\pm)\\ &+D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \gamma_5 \bigcirc \gamma_5\ \mathcal{K}^-_\nu(q_\pm,k_\pm),\end{aligned}$$

which has three terms including two unknown objects.



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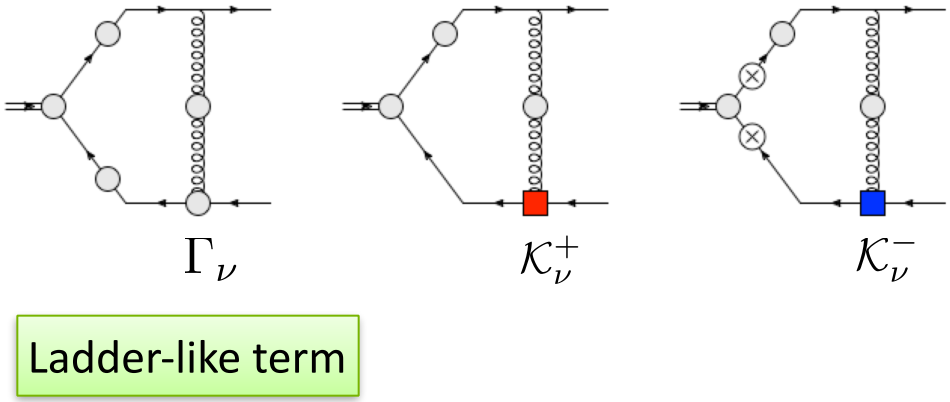

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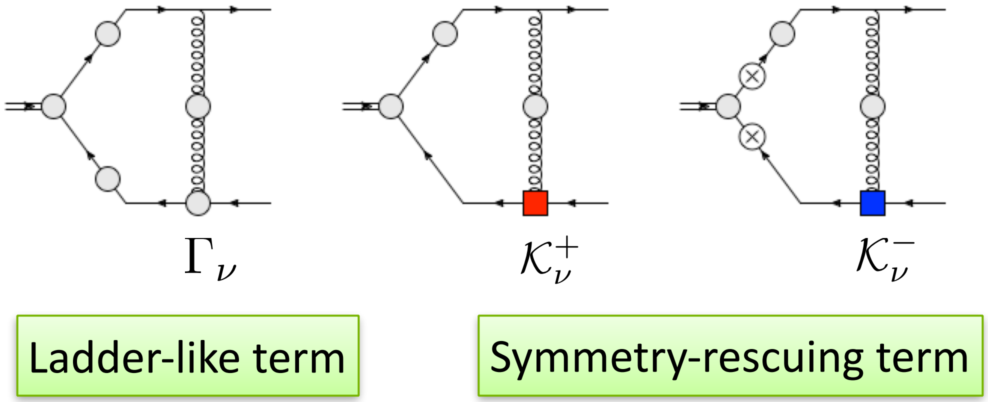

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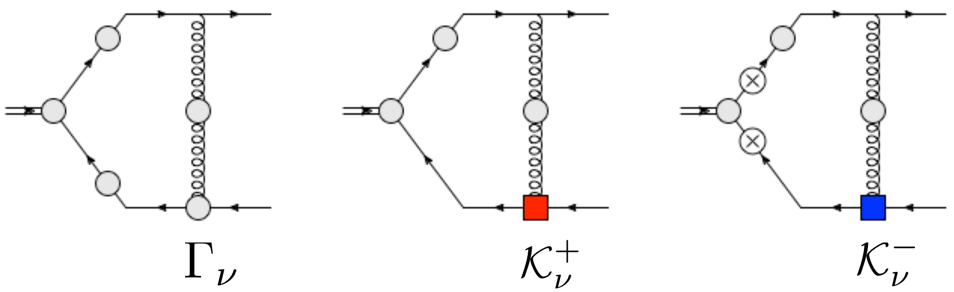

Ladder-like term

Symmetry-rescuing term

Inserting the assumed form of the kernel into its WGTIs, we have

$$\int_q D_{\mu\nu}\gamma_\mu S_+(\Gamma^+_\nu - \Gamma^-_\nu) = \int_q D_{\mu\nu}\gamma_\mu S_+(S_+^{-1} - S_-^{-1})\mathcal{K}^+_\nu + \int_q D_{\mu\nu}\gamma_\mu S_+\gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}^-_\nu$$

$$\int_q D_{\mu\nu}\gamma_\mu S_+(\Gamma^+_\nu\gamma_5 + \gamma_5\Gamma^-_\nu) = \int_q D_{\mu\nu}\gamma_\mu S_+(S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}^+_\nu + \int_q D_{\mu\nu}\gamma_\mu S_+(\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}^-_\nu$$

Scattering kernel: A solution with propagators and vertices

Since the integral WGTIs are satisfied for any model of the gluon propagator, the integral kernels must be identical, e.g.,

$$\int_x f(x)g(x) = \int_x f(x)g'(x)$$

Algebraic version of the WGTIs, which the kernel satisfies, are written as

$$\begin{aligned}
\Gamma_\nu^+ - \Gamma_\nu^- &= (S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}_\nu^-\,, \\
\Gamma_\nu^+\gamma_5 + \gamma_5\Gamma_\nu^- &= (S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}_\nu^+ + (\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}_\nu^-\,.
\end{aligned}$$

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$$\Gamma_\nu^+ - \Gamma_\nu^- = (S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}_\nu^-,$$
$$\Gamma_\nu^+\gamma_5 + \gamma_5\Gamma_\nu^- = (S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}_\nu^+ + (\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}_\nu^-.$$

The solution is following:

$$\mathcal{K}_\nu^\pm = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma_\nu^\Sigma \pm B_\Sigma \Gamma_\nu^\Delta].$$

$$\Gamma_\nu^\Sigma = \Gamma_\nu^+ + \gamma_5\Gamma_\nu^+\gamma_5 \qquad \Gamma_\nu^\Delta = \Gamma_\nu^+ - \Gamma_\nu^-$$
$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$
$$A_\Delta = i(\gamma\cdot q_+)A_+ - i(\gamma\cdot q_-)A_-$$

Scattering kernel: with elements of quark gap equation

Rearranging the scattering kernel as the left- and right-hand forms

$$\mathcal{K}_{\alpha\alpha',\beta'\beta}(q_\pm,k_\pm)[S(q_+)\bigcirc S(q_-)]_{\alpha'\beta'} = -D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\bigcirc S(q_-)\Gamma_\nu(q_-,k_-) + D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\boxed{\tfrac{1}{2}(\bigcirc+\gamma_5\bigcirc\gamma_5)}\mathcal{K}^L_\nu(q_\pm,k_\pm) + D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\boxed{\tfrac{1}{2}(\bigcirc-\gamma_5\bigcirc\gamma_5)}\mathcal{K}^R_\nu(q_\pm,k_\pm),$$

we have the solution as

$$\boxed{\begin{aligned}\mathcal{K}^L_\nu &= B_\Sigma^{-1}\Gamma^\Sigma_\nu,\\ \mathcal{K}^R_\nu &= (B_\Sigma A_\Delta)^{-1}(B_\Sigma\Gamma^\Delta_\nu - B_\Delta\Gamma^\Sigma_\nu).\end{aligned}}$$

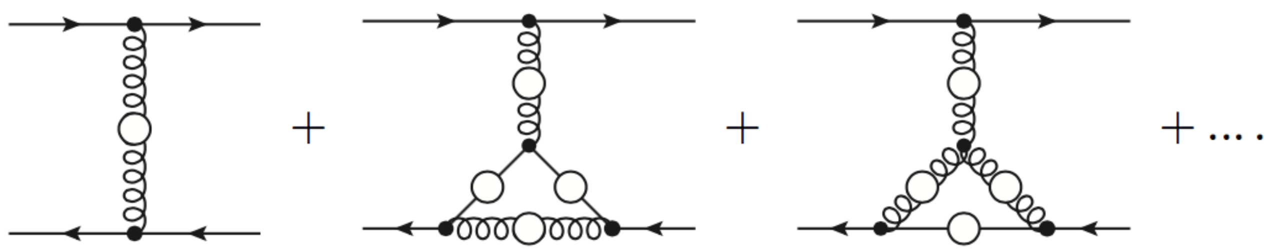
For a given Dirac structure, only one of K^L and K^R can survive, e.g.,

$\bigcirc = \gamma_\mu$	$\gamma_5\bigcirc\gamma_5 = -\bigcirc$	K^R
$\bigcirc = \gamma_5$	$\gamma_5\bigcirc\gamma_5 = \bigcirc$	K^L

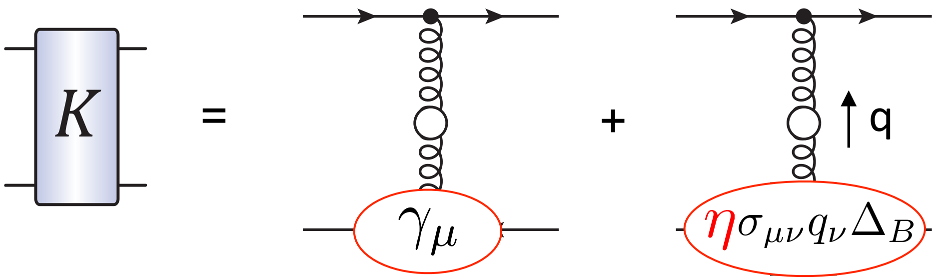
✦ The form of scattering kernel is simple.
✦ The kernel has no kinetic singularities.
✦ All channels share the same kernel.

Scattering kernel: A demonstration

In Feynman diagrams, the scattering kernel can have many pieces:



As an example, the kernel is written as two parts (bare + ACM), phenomenologically:



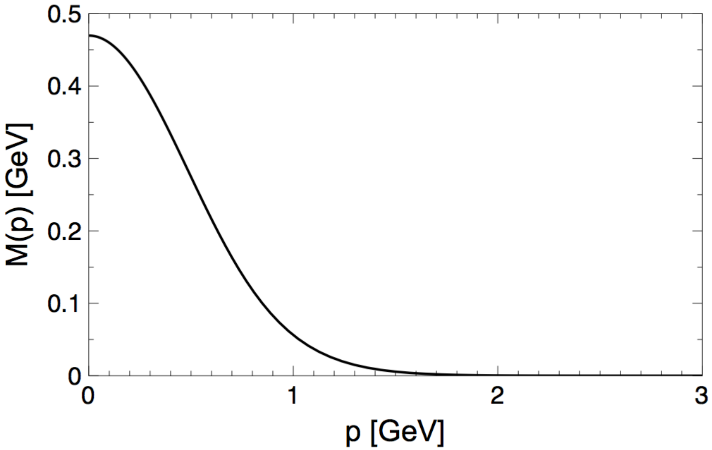
Meson spectroscopy: From ground to radial excitation states

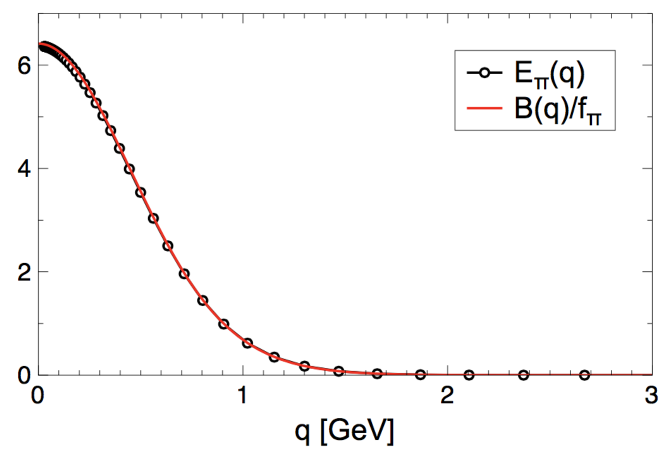
Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\rm BC}(p,q) + \Gamma_\mu^{\rm T}(p,q) \quad \Gamma_\mu^{\rm T}(p,q) = \eta\Delta_B\tau_\mu^5 + \xi\Delta_B\tau_\mu^8 + 4(\eta+\xi)\Delta_A\tau_\mu^4$$

$$\begin{aligned}\tau_\mu^4 &= l_\mu^{\rm T}\gamma\cdot k + i\gamma_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho,\\ \tau_\mu^5 &= \sigma_{\mu\nu}k_\nu,\\ \tau_\mu^8 &= 3\, l_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho/(l^{\rm T}\cdot l^{\rm T}).\end{aligned}$$

- ✦ The longitudinal part is the Ball-Chiu vertex—an exact piece from symmetries.
- ✦ The transverse part is the Anomalous Chromomagnetic Moment (ACM) vertex.





To generate the quark mass scale which is comparable to that of LQCD, the coupling strength can be so small that the Rainbow-ladder approximation has NO DCSB at all.