Collider Physics

— From basic knowledge to new physics searches

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Basics of Collider physics

Lecture II:

Physics at an e^+e^- Collider

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Physics at Hadron Colliders

(and New Physics Searches)

LHC Run-II is in mission!

Running at $E_{cm} = 6.5 \oplus 6.5 = 13$ TeV.

New era in HEP and in science has begun!

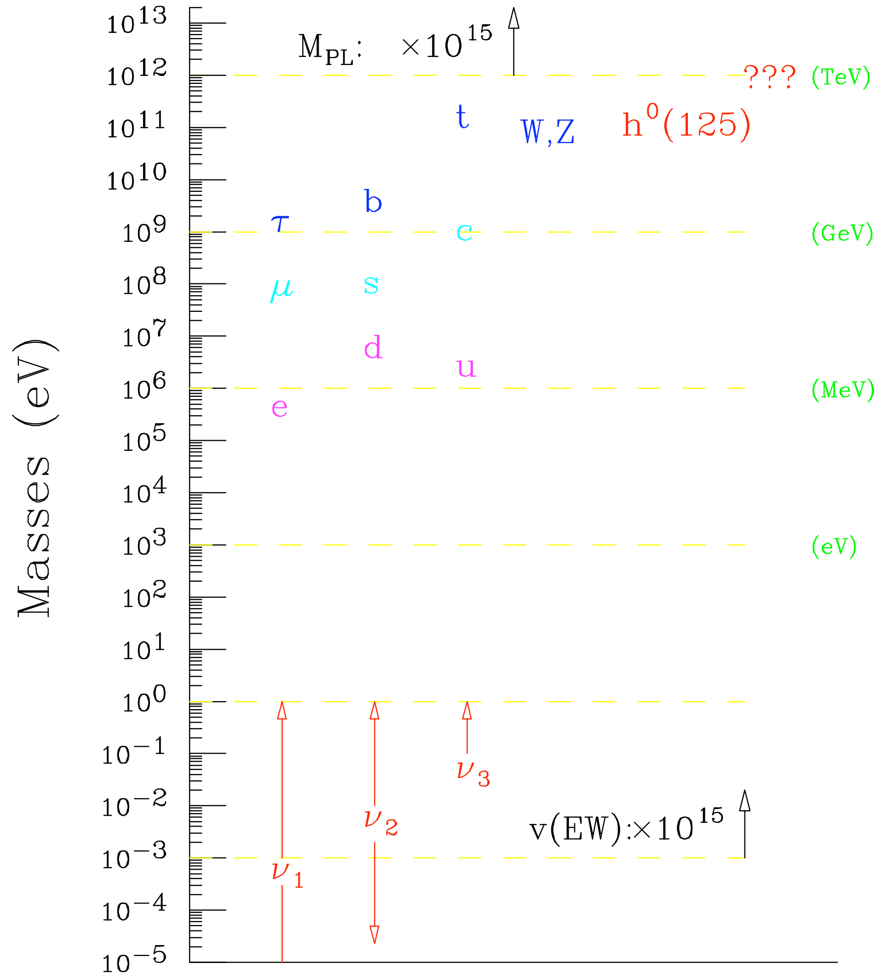
High Energy Physics IS at an extremely interesting time!

The completion of the Standard Model: With the discovery of the Higgs boson, for the first time ever, we have a consistent relativistic quantum-mechanical theory, weakly coupled, unitary, renormalizable, vacuum (quasi?) stable, valid up to an exponentially high scale!

Question: Where IS the next scale?

$\mathcal{O}(1$ TeV)? M_{GUT}? M_{Planck}?

Large spread of masses for elementary particles:



Large hierarchy: Electroweak scale ⇔ M_{Planck}? Conceptual.

Little hierarchy: Electroweak scale ⇔ Next scale at TeV? Observational.

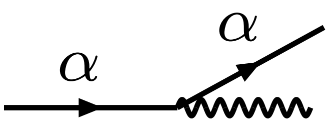
That motivates us to the new energy frontier!

I-A. Colliders and Detectors

(0). A Historical Count:

Rutherford's experiments were the first

to study matter structure:

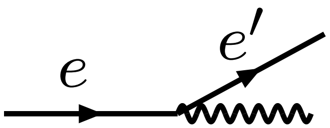


Gold foil target

discover the point-like nucleus:

$$\frac{d\sigma}{d\Omega} = \frac{(\alpha Z_1 Z_2)^2}{4E^2 \sin^4 \theta/2}$$

SLAC-MIT DIS experiments



Proton target

discover the point-like structure of the proton:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(\frac{F_1(x,Q^2)}{m_p} \sin^2 \frac{\theta}{2} + \frac{F_2(x,Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

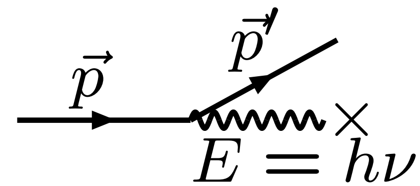
QCD parton model $\Rightarrow 2xF_1(x,Q^2) = F_2(x,Q^2) = \sum_i x f_i(x) e_i^2.$

Rutherford's legendary method continues to date!

(A). High-energy Colliders:

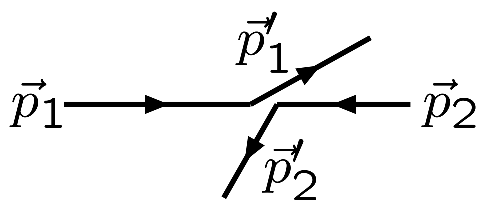
To study the deepest layers of matter,

we need the probes with highest energies.



Two parameters of importance:

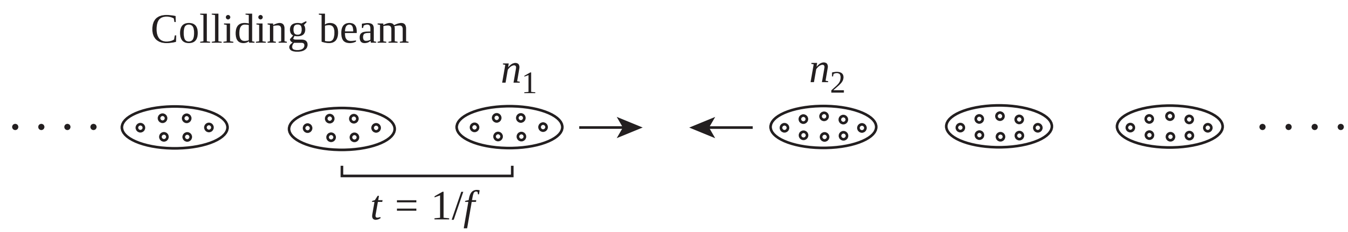
1. The energy:



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{p}_2 = 0. \end{cases}$$

2. The luminosity:



$$\mathcal{L} \propto f n_1 n_2 / a,$$

(a some beam transverse profile) in units of #particles/cm^2/s

$\Rightarrow 10^{33}$ cm^{-2}s^{-1} = 1 nb^{-1} s^{-1} $\approx$ 10 fb^{-1}/year.

Current and future high-energy colliders:

Hadron Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
LHC Run (I) II	(7,8) 13	(10^{32}) 10^{33}	0.01%	40	10.5	26.66
HL-LHC	14	7×10^{34}	0.013%	40	22	26.66
FCC$_{hh}$ (SppC)	100	1.2×10^{35}	0.01%	40	10	100

e^+e^- Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	polar.	L (km)
ILC	0.5–1	2.5×10^{34}	0.1%	3	80, 60%	14 – 33
CEPC	0.25–0.35	2×10^{34}	0.13%			50-100
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

(B). e^+e^- Colliders

The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,

$\Longrightarrow$ it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,

$\Longrightarrow$ the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties,

$\Longrightarrow$ the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol:

For $\sigma \approx 10$ pb $\Rightarrow 0.1$ Hz at 10^{34} cm^{-2}s^{-1}.

- Linear Collider: possible to achieve high degrees of beam polarizations,

$\Longrightarrow$ chiral couplings and other asymmetries can be effectively explored.

Disadvantages

- Large synchrotron radiation due to acceleration,

$$\Delta E \sim \frac{1}{R}\ \left(\frac{E}{m_e}\right)^4 .$$

  Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized;
  beamsstrahlung severe.

CEPC/FCC$_{ee}$ Higgs Factory

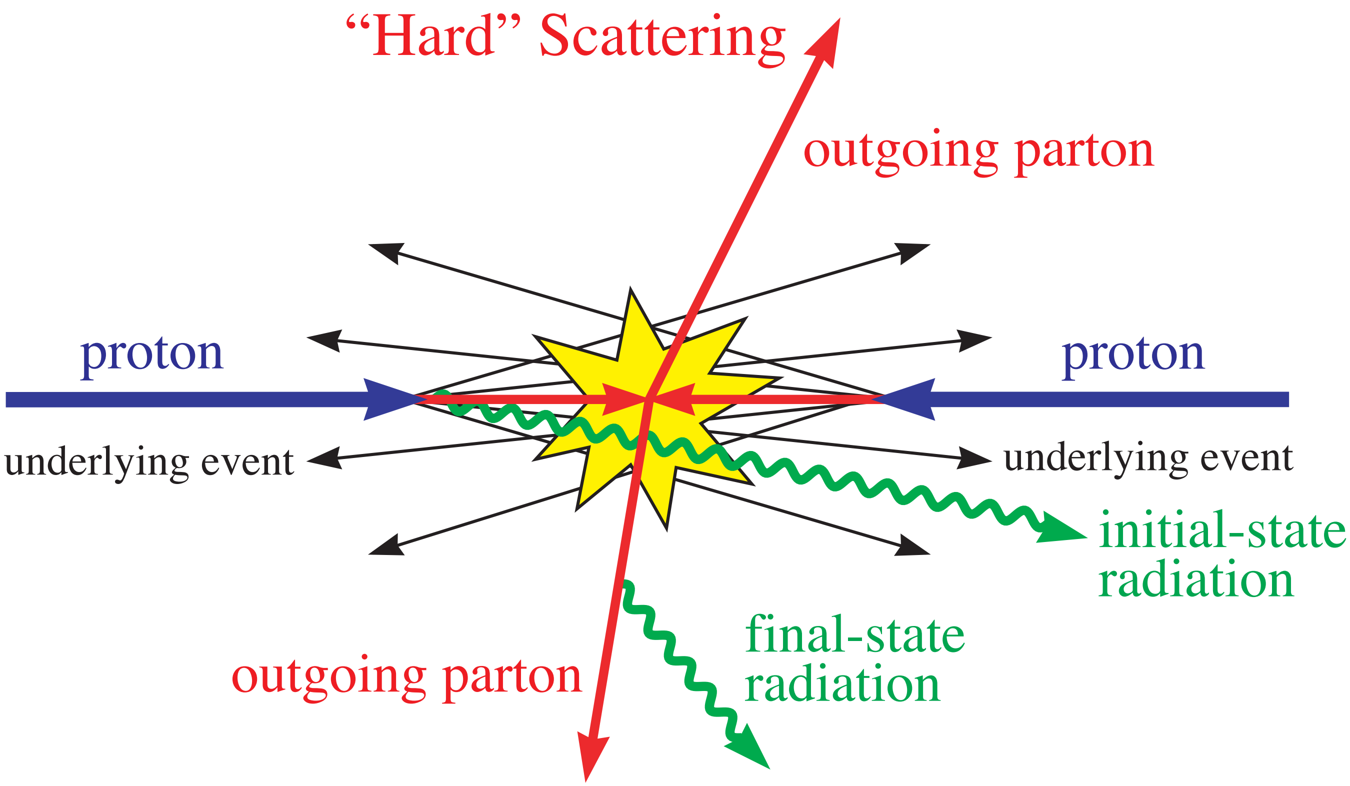
It has been discussed to build a circular e^+e^- collider

$$E_{cm} = 245 \text{ GeV}–350 \text{ GeV}$$

with multiple interaction points for very high luminosities.

(C). Hadron Colliders

LHC: the new high-energy frontier



Advantages

- Higher c.m. energy, thus higher energy threshold:

$\sqrt{S} = 14$ TeV: $\quad M^2_{new} \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.3\sqrt{S} \sim 4$ TeV.

- Higher luminosity: 10^{34}/cm^2/s $\Rightarrow$ 100 fb^{-1}/yr.
  Annual yield: 1B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...
- Multiple (strong, electroweak) channels:
  $q\bar{q}'$, gg, qg, $b\bar{b} \to$ colored; $Q = 0, \pm 1$; $J = 0, 1, 2$ states;
  WW, WZ, ZZ, $\gamma\gamma \to I_W = 0, 1, 2$; $Q = 0, \pm 1, \pm 2$; $J = 0, 1, 2$ states.

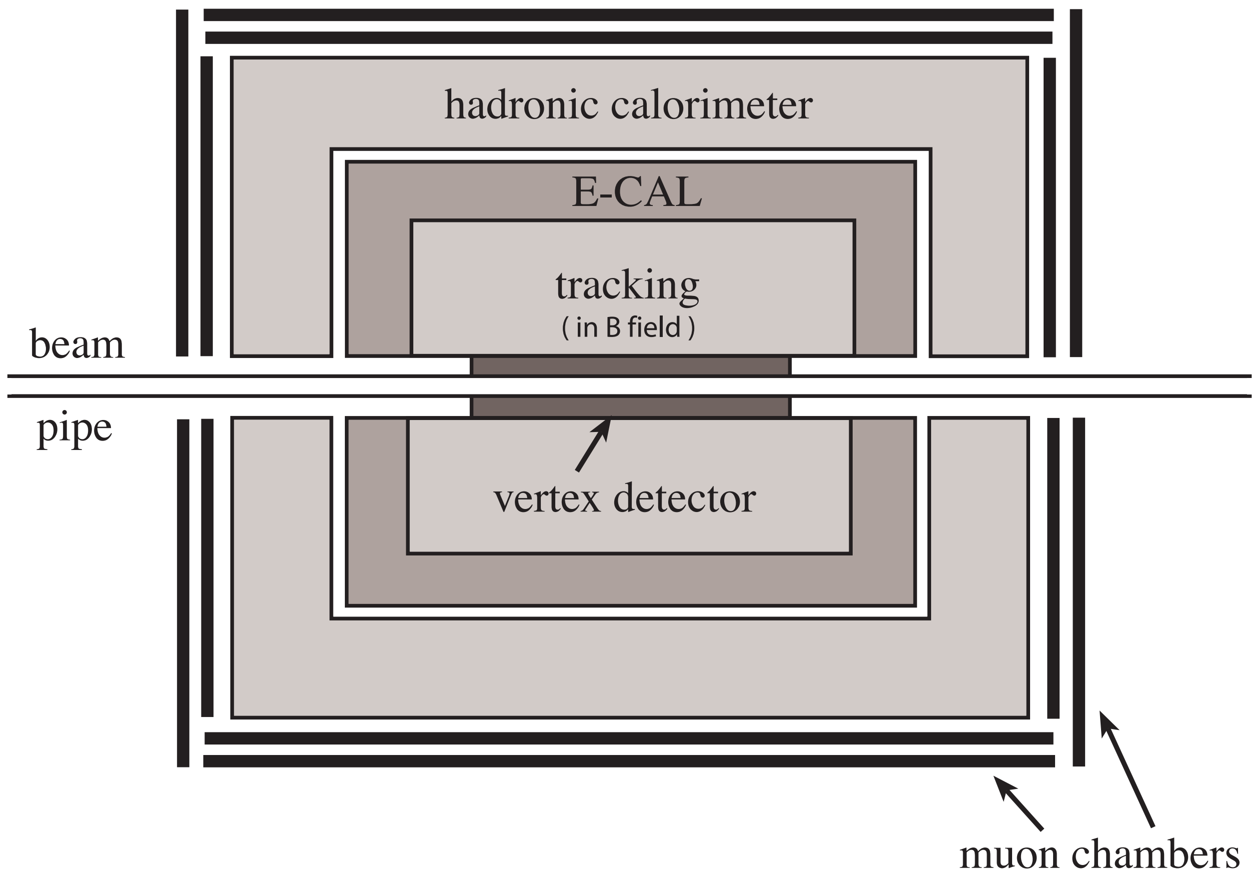
Disadvantages

- Initial state unknown:
  colliding partons unknown on event-by-event basis;
  parton c.m. energy unknown: $E^2_{cm} \equiv s = x_1 x_2 S$;
  parton c.m. frame unknown.
  $\Rightarrow$ largely rely on final state reconstruction.
- The large rate turns to a hostile environment:
  $\Rightarrow$ Severe backgrounds!

Our primary job !

(D). Particle Detection:

The detector complex:
Utilize the strong and electromagnetic interactions between detector materials and produced particles.



What we "see" as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c\ \tau)\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

- stable particles directly "seen":

$$p,\ \bar{p},\ e^{\pm},\ \gamma$$

- quasi-stable particles of a life-time $\tau \geq 10^{-10}$ s also directly "seen":

$$n, \Lambda, K_L^0, ...,\ \mu^{\pm},\ \pi^{\pm}, K^{\pm}...$$

- a life-time $\tau \sim 10^{-12}$ s may display a secondary decay vertex, "vertex-tagged particles":

$$B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}...$$

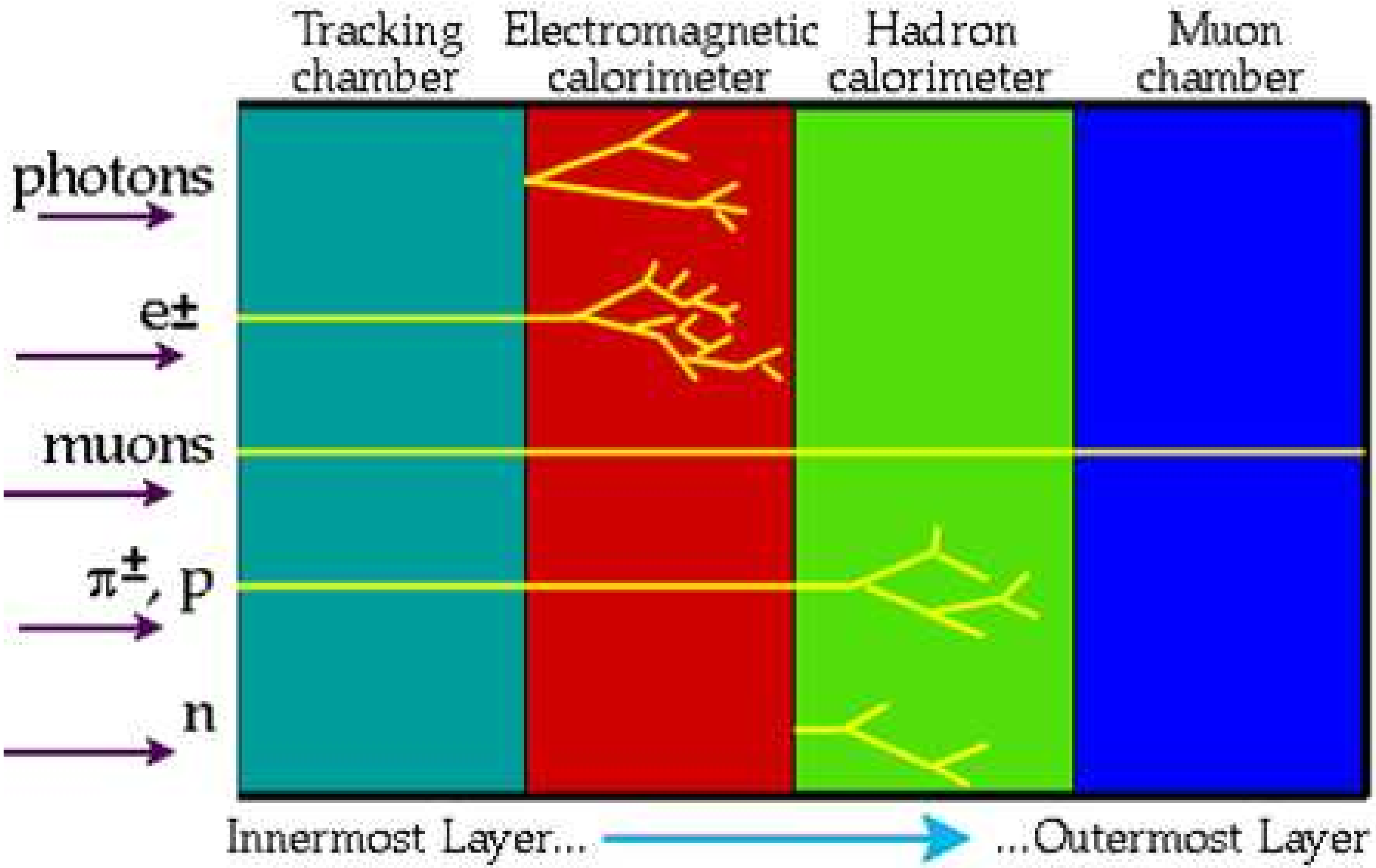
- short-lived not "directly seen", but "reconstructable":

$$\pi^0,\ \rho^{0,\pm}...\ ,\quad Z, W^{\pm}, t, H...$$

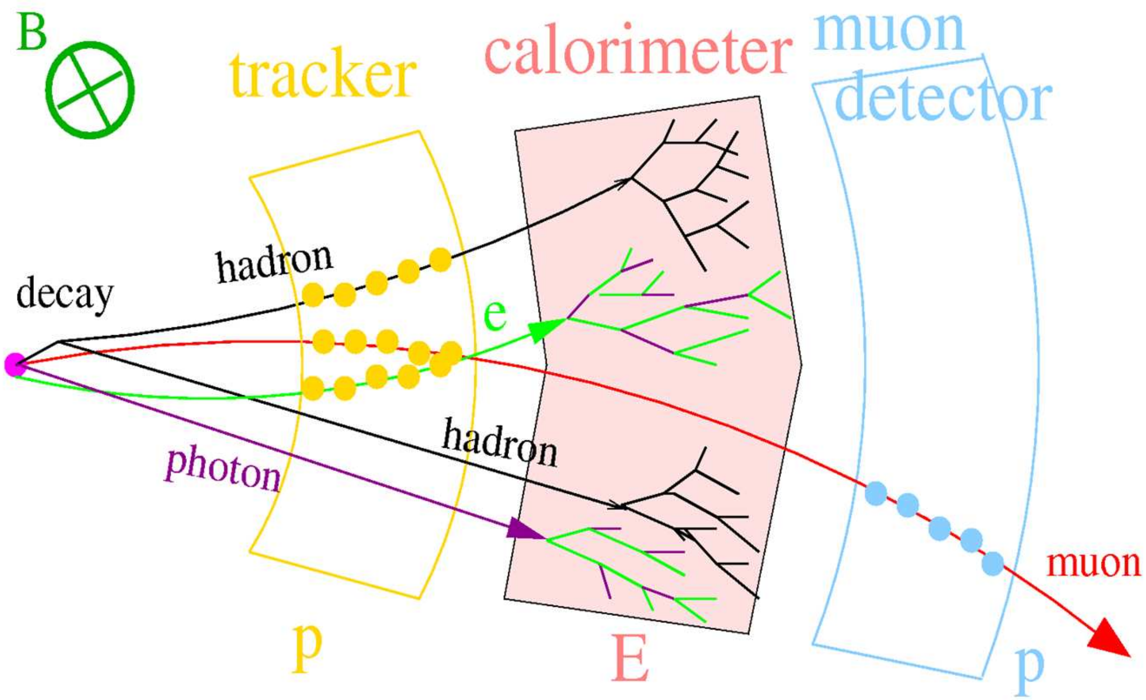
- missing particles are weakly-interacting and neutral:

$$\nu,\ \tilde{\chi}^0, G_{KK}...$$

† For stable and quasi-stable particles of a life-time $\tau \geq 10^{-10} - 10^{-12}$ s, they show up as



A closer look:

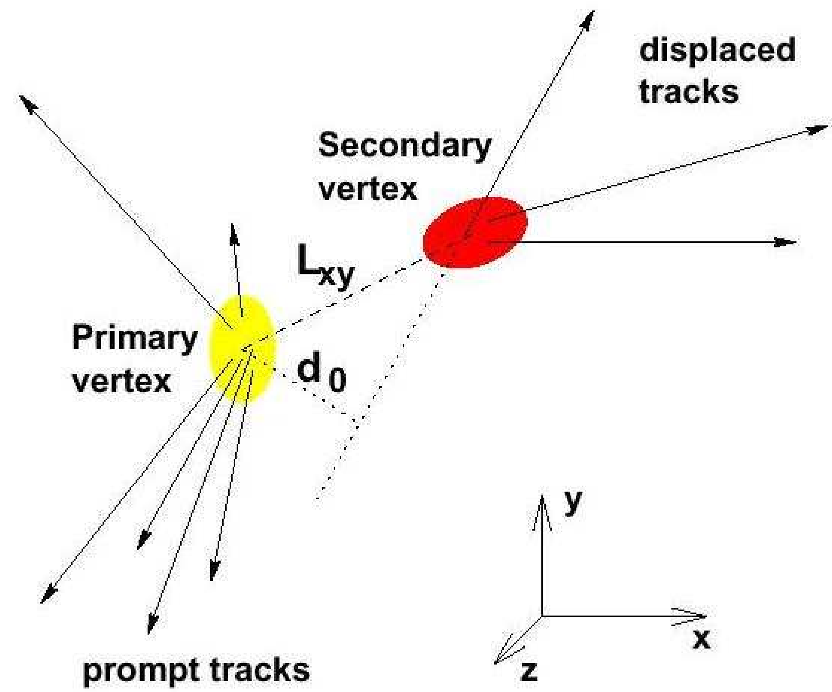


Theorists should know:

$$\text{For charged tracks}: \quad \Delta p/p \;\propto\; p,$$
$$\text{typical resolution}: \quad \sim\; p/(10^4\ \text{GeV}).$$
$$\text{For calorimetry}: \quad \Delta E/E \;\propto\; \frac{1}{\sqrt{E}},$$
$$\text{typical resolution}: \quad \sim\; (10\%_{ecal},\ 50\%_{hcal})/\sqrt{E/\text{GeV}}$$

† For vertex-tagged particles $\tau \approx 10^{-12}$ s,

heavy flavor tagging: the secondary vertex:



Typical resolution: $d_0 \sim 30 - 50\ \mu$m or so

$\Rightarrow$ Better have two (non-collinear) charged tracks for a secondary vertex;

Or use the "impact parameter" w.r.t. the primary vertex.

For theorists: just multiply a "tagging efficiency":

$$\epsilon_b \sim 70\%; \quad \epsilon_c \sim 40\%; \quad \epsilon_\tau \sim 40\%.$$

† For short-lived particles: $\tau < 10^{-12}$ s or so,

make use of final state kinematics to reconstruct the resonance.

† For missing particles:

make use of energy-momentum conservation to deduce their existence.

$$p_1^i + p_2^i = \sum_f^{obs.} p_f \ + \ p_{miss}.$$

But in hadron collisions, the longitudinal momenta unknown,
thus transverse direction only:

$$0 = \sum_f^{obs.} \vec{p}_{f\ T} \ + \ \vec{p}_{miss\ T}.$$

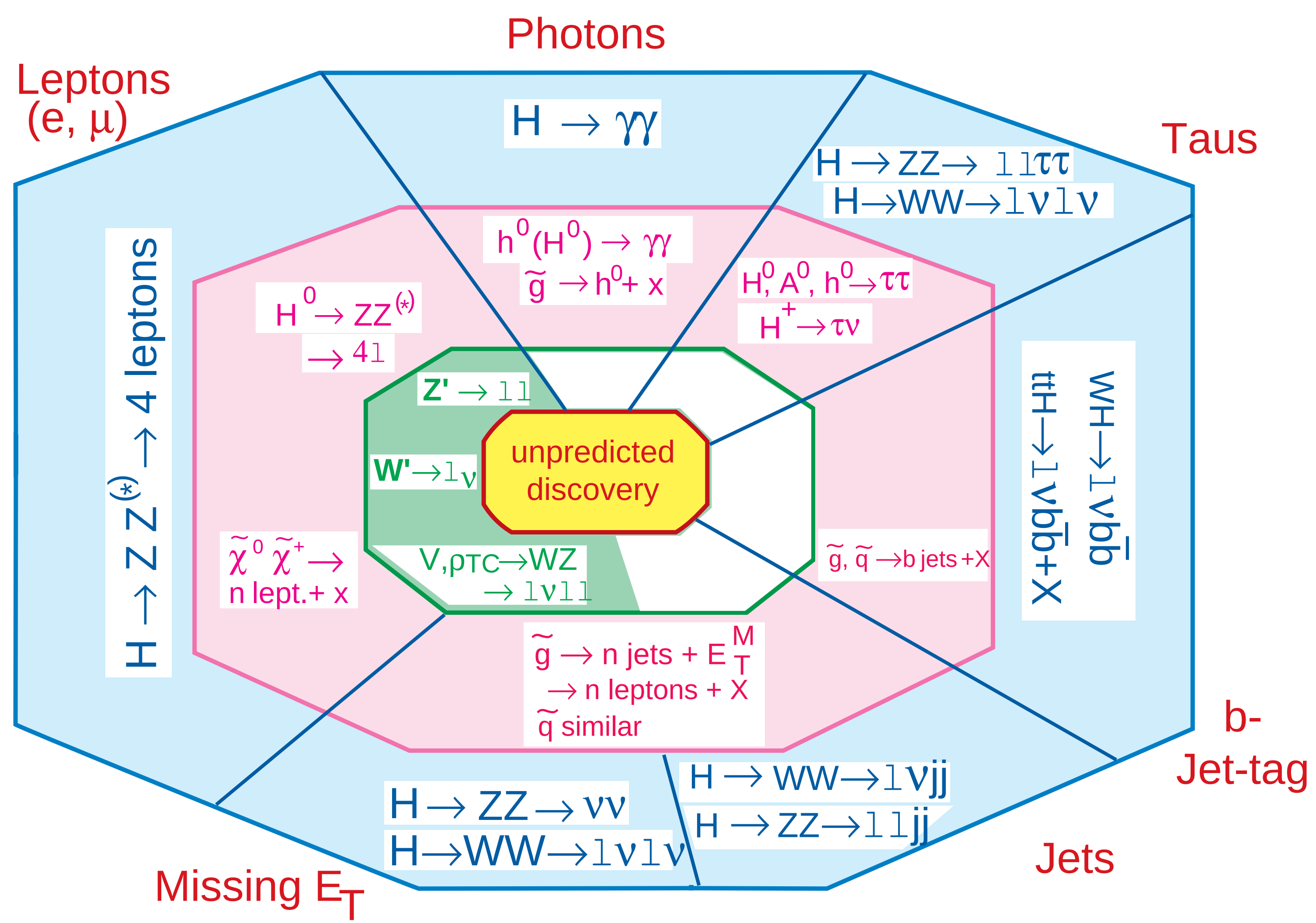
often called "missing p_T" ($\not{p}_T$) or (conventionally) "missing E_T" ($\not{E}_T$).

Note: "missing E_T" (MET) is *conceptually* ill-defined!
It is only sensible for massless particles: $\not{E}_T = \sqrt{\vec{p}^{\,2}_{miss\ T} + m^2}$.

What we "see" for the SM particles
(no universality!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b+$ 2 jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^\pm \to \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$
the Higgs boson					
$h^0 \to b\bar{b}$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$\to ZZ^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$
$\to WW^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$

How to search for new particles?



y98014_416dPauss rd

Homework:

Exercise 1.1: For a π^0, μ^-, or a τ^- respectively, calculate its decay length for $E = 10$ GeV.

Exercise 1.2: An event was identified to have a $\mu^+\mu^-$ pair, along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider both an e^+e^- and a hadron collider.

Exercise 1.3: Electron and muon measurements: Estimate the relative errors of energy-momentum measurements for an electron by an electromagnetic calorimetry ($\Delta E/E$) and for a muon by tracking ($\Delta p/p$) at energies of $E = 50$ GeV and 500 GeV, respectively.

Exercise 1.4: A 125 GeV Higgs boson will have a production cross section of 20 pb at the 14 TeV LHC. How many events per year do you expect to produce for the Higgs boson with an instantaneous luminosity $10^{33}/\text{cm}^2/\text{s}$? Do you expect it to be easy to observe and why?

I-B. Basic Techniques and Tools for Collider Physics

(A). Scattering cross section

For a $2 \to n$ scattering process:

$$\sigma(ab \to 1 + 2 + ...n) = \frac{1}{2s} \overline{\sum} |\mathcal{M}|^2 \, dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \, \delta^4 \left(P - \sum_{i=1}^{n} p_i \right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i \right)^2 ,$$

where $\overline{\sum}|\mathcal{M}|^2$: dynamics (dimension $4 - 2n$);
dPS_n: kinematics (Lorentz invariant, dimension $2n - 4$.)

For a $1 \to n$ decay process, the partial width in the rest frame:

$$\Gamma(a \to 1 + 2 + ...n) = \frac{1}{2M_a} \overline{\sum} |\mathcal{M}|^2 \, dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = (\sum_f \Gamma_f)^{-1}.$$

(B). Phase space and kinematics *

One-particle Final State $a + b \to 1$:

$$\begin{aligned} dPS_1 &\equiv (2\pi)\,\frac{d^3\vec{p}_1}{2E_1}\delta^4(P - p_1) \\ &\doteq \pi|\vec{p}_1|d\Omega_1\delta^3(\vec{P} - \vec{p}_1) \\ &\doteq 2\pi\ \delta(s - m_1^2). \end{aligned}$$

where the first and second equal signs made use of the identities:

$$|\vec{p}|d|\vec{p}| = EdE, \quad \frac{d^3\vec{p}}{2E} = \int d^4p\ \delta(p^2 - m^2).$$

Kinematical relations:

$$\begin{aligned} \vec{P} &\equiv \vec{p}_a + \vec{p}_b = \vec{p}_1, \quad E_1^{cm} = \sqrt{s} \text{ in the c.m. frame,} \\ s &= (p_a + p_b)^2 = m_1^2. \end{aligned}$$

The "dimensinless phase-space volume" is $s(dPS_1) = 2\pi$.

*E.Byckling, K. Kajantie: Particle Kinemaitcs (1973).

Two-particle Final State $a+b \to 1+2$:

$$
\begin{aligned}
dPS_2 &\equiv \frac{1}{(2\pi)^2}\, \delta^4 (P-p_1-p_2) \frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2} \\
&\doteq \frac{1}{(4\pi)^2}\, \frac{|\vec{p}_1^{cm}|}{\sqrt{s}}\, d\Omega_1 = \frac{1}{(4\pi)^2}\, \frac{|\vec{p}_1^{cm}|}{\sqrt{s}}\, d\cos\theta_1 d\phi_1 \\
&= \frac{1}{4\pi}\frac{1}{2}\, \lambda^{1/2}\left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2, \\
d\cos\theta_1 &= 2dx_1, \quad d\phi_1 = 2\pi dx_2, \quad 0 \le x_{1,2} \le 1,
\end{aligned}
$$

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

$$
|\vec{p}_1^{cm}| = |\vec{p}_2^{cm}| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}}, \;\; E_1^{cm} = \frac{s+m_1^2-m_2^2}{2\sqrt{s}}, \;\; E_2^{cm} = \frac{s+m_2^2-m_1^2}{2\sqrt{s}},
$$

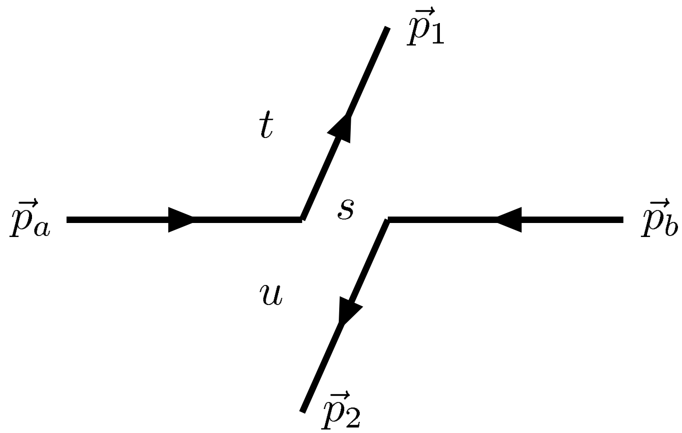
$$
\lambda(x,y,z) = (x-y-z)^2 - 4yz = x^2+y^2+z^2-2xy-2xz-2yz.
$$

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

$$
\frac{dPS_2}{s\ dPS_1} \approx \frac{1}{(4\pi)^2}.
$$

just like a “loop factor”.

Consider a $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$,



the (Lorentz invariant) Mandelstam variables are defined as

$$
\begin{aligned}
s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\
u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}), \\
&\quad s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2.
\end{aligned}
$$

The two-body phase space can be thus written as

$$dPS_2 = \frac{1}{(4\pi)^2} \frac{dt\ d\phi_1}{s\ \lambda^{1/2}\left(1, m_a^2/s, m_b^2/s\right)}.$$

Exercise 2.1: Assume that $m_a = m_1$ and $m_b = m_2$. Show that

$$t = -2p_{cm}^2(1 - \cos\theta_{a1}^*),$$
$$u = -2p_{cm}^2(1 + \cos\theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s},$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.
Note: t is negative-definite; $t \rightarrow 0$ in the collinear limit.

Exercise 2.2: A particle of mass M decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed β_z? Compare the result with your expectation for the shape change for a basket ball.

Three-particle Final State $a+b\to 1+2+3$:

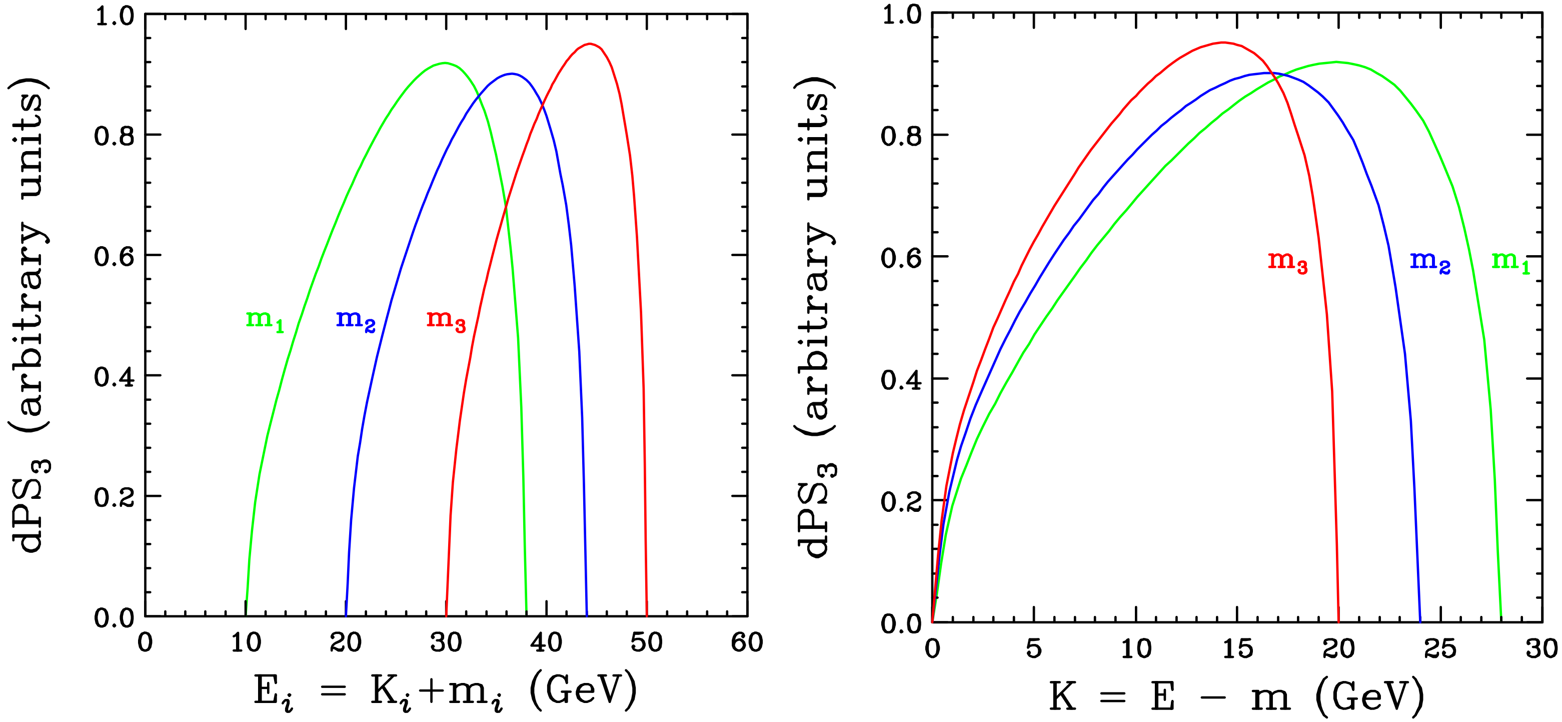
$$
\begin{aligned}
dPS_3 &\equiv \frac{1}{(2\pi)^5}\,\delta^4(P-p_1-p_2-p_3)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_3}{2E_3}\\
&\doteq \frac{|\vec{p}_1|^2\ d|\vec{p}_1|\ d\Omega_1}{(2\pi)^3\ 2E_1}\ \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_2^{\,(23)}|}{m_{23}}\ d\Omega_2\\
&= \frac{1}{(4\pi)^3}\ \lambda^{1/2}\left(1,\frac{m_2^2}{m_{23}^2},\frac{m_3^2}{m_{23}^2}\right)\ 2|\vec{p}_1|\ dE_1\ dx_2dx_3dx_4dx_5.
\end{aligned}
$$

$$
\begin{aligned}
d\cos\theta_{1,2} &= 2dx_{2,4},\quad d\phi_{1,2}=2\pi dx_{3,5},\quad 0\le x_{2,3,4,5}\le 1,\\
& |\vec{p}_1^{\,cm}|^2 = |\vec{p}_2^{\,cm}+\vec{p}_3^{\,cm}|^2=(E_1^{cm})^2-m_1^2,\\
& m_{23}^2 = s-2\sqrt{s}E_1^{cm}+m_1^2,\quad |\vec{p}_2^{\,23}|=|\vec{p}_3^{\,23}|=\frac{\lambda^{1/2}(m_{23}^2,m_2^2,m_3^2)}{2m_{23}},
\end{aligned}
$$

The particle energy spectrum is not monochromatic.
The maximum value (the end-point) for particle 1 in c.m. frame is

$$
\begin{aligned}
E_1^{max} &= \frac{s+m_1^2-(m_2+m_3)^2}{2\sqrt{s}},\quad m_1\le E_1\le E_1^{max},\\
|\vec{p}_1^{\,max}| &= \frac{\lambda^{1/2}(s,m_1^2,(m_2+m_3)^2)}{2\sqrt{s}},\quad 0\le p_1\le p_1^{max}.
\end{aligned}
$$

With $m_i = 10,\ 20,\ 30,\ \sqrt{s} = 100$ GeV.



More intuitive to work out the end-point for the kinetic energy,

– recall the direct neutrino mass bound in β-decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$

In general, the 3-body phase space boundaries are non-trivial.
That leads to the "Dalitz Plots".

One practically useful formula is:

Exercise 2.3: A particle of mass M decays to 3 particles $M \to abc$.
Show that the phase space element can be expressed as

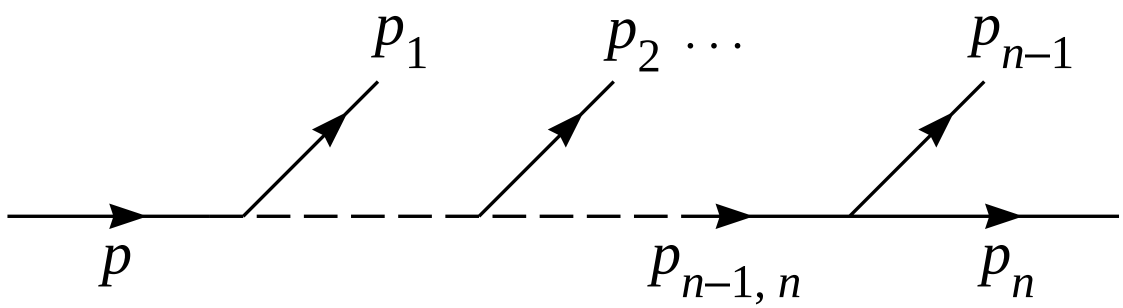
$$dPS_3 = \frac{1}{2^7\pi^3}\ M^2 dx_a dx_b.$$

$$x_i = \frac{2E_i}{M},\ \ (i = a, b, c,\ \sum_i x_i = 2).$$

where the integration limits for $m_a = m_b = m_c = 0$ are

$$0 \le x_a \le 1, \quad 1 - x_a \le x_b \le 1.$$

Recursion relation $P \to 1 + 2 + 3 ... + n$:



$$dPS_n(P; p_1, ..., p_n) = dPS_{n-1}(P; p_1, ..., p_{n-1,n}) \; dPS_2(p_{n-1,n}; p_{n-1}, p_n)\frac{dm^2_{n-1,n}}{2\pi}.$$

For instance,

$$dPS_3 = dPS_2(i)\ \frac{dm^2_{prop}}{2\pi}\ dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an s-channel particle propagation.

Breit-Wigner Resonance, the Narrow Width Approximation

An unstable particle of mass M and total width Γ_V, the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

Consider an intermediate state V^*

$$a \longrightarrow bV^* \longrightarrow b\ p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2=(m_1+m_2)^2}^{(m_*^{max})^2=(m_a-m_b)^2} dm_*^2.$$

Variable change

$$\tan\theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over θ

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

In the limit

$$(m_1+m_2)+\Gamma_V \ll M_V \ll m_a-\Gamma_V,$$
$$\theta^{min} = \tan^{-1}\frac{(m_1+m_2)^2-M_V^2}{\Gamma_V M_V} \to -\pi,$$
$$\theta^{max} = \tan^{-1}\frac{(m_a-m_b)^2-M_V^2}{\Gamma_V M_V} \to 0,$$

then the Narrow Width Approximation

$$\frac{1}{(m_*^2-M_V^2)^2+\Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\ \delta(m_*^2-M_V^2).$$

Exercise 2.4: Consider a three-body decay of a top quark, $t\to bW^*\to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

$$\Gamma(t\to bW^*\to b\ e\nu) \approx \Gamma(t\to bW)\cdot BR(W\to e\nu).$$

(C). Matrix element: The dynamics

Properties of scattering amplitudes $T(s,t,u)$

• Analyticity: A scattering amplitude is analytical except:
simple poles (corresponding to single particle states, bound states etc.);
branch cuts (corresponding to thresholds).

• Crossing symmetry: A scattering amplitude for a $2 \to 2$ process is symmetric among the s-, t-, u-channels.

• Unitarity:
S-matrix unitarity leads to :

$$-i(T - T^{\dagger}) = TT^{\dagger}$$

Partial wave expansion for $a + b \to 1 + 2$:

$$\mathcal{M}(s,t) = 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d^J_{\mu\mu'}(\cos\theta)$$
$$a_J(s) = \frac{1}{32\pi} \int_{-1}^{1} \mathcal{M}(s,t)\ d^J_{\mu\mu'}(\cos\theta) d\cos\theta.$$

where $\mu = s_a - s_b,\ \mu' = s_1 - s_2,\ J = \max(|\mu|, |\mu'|)$.

By Optical Theorem: $\sigma = \frac{1}{s}\mathrm{Im}\mathcal{M}(\theta = 0) = \frac{16\pi}{s} \sum_{J=M}^{\infty} (2J+1)|a_J(s)|^2$.

The partial wave amplitude have the properties:

(a). partial wave unitarity: $\mathrm{Im}(a_J) \geq |a_J|^2$, or $|\mathrm{Re}(a_J)| \leq 1/2$,

(b). kinematical thresholds: $a_J(s) \propto \beta_i^{l_i}\ \beta_f^{l_f}\quad (J = L + S)$.

$\Rightarrow$ well-known behavior: $\sigma \propto \beta_f^{2l_f+1}$.

`Exercise 2.5: Appreciate the properties (a) and (b) by explicitly calculating the helicity amplitudes for`

$$e^-_L e^+_R \to \gamma^* \to H^- H^+, \quad e^-_L e^+_{L,R} \to \gamma^* \to \mu^-_L \mu^+_R, \quad H^- H^+ \to G^* \to H^- H^+.$$

(D). Calculational Tools

Traditional "Trace" Techniques: (Good for simple processes)

- ∗ You should be good at this — QFT course!

With algebraic symbolic manipulations:

- ∗ REDUCE, FORM, MATHEMATICA, MAPLE ...

Helicity Techniques: (Necessary for multiple particles)

More suitable for direct numerical evaluations.

* Hagiwara-Zeppenfeld: best for massless particles... (NPB, 1986)
* CalCul Method (by T.T. Wu et al., Parke-Mangano: Phys. Report);
* New techniques in loop calculations (by Z.Bern, L.Dixon, W. Giele, N. Glover, K.Melnikov, F. Petriello ...)
* "Twisters" (string theory motivated organization) (by Britto, F.Chachazo, B.Feng, E.Witten ...)

`Exercise 2.6: Calculate the squared matrix element for` $\overline{\sum}|\mathcal{M}(f\bar{f}\to ZZ)|^2$, `in terms of` s, t, u, `in whatever technique you like.`

Calculational packages:

• Monte Carlo packages for phase space integration:

(1) VEGAS by P. LePage: adaptive important-sampling MC
http://en.wikipedia.org/wiki/Monte-Carlo_integration

(2) SAMPLE, RAINBOW, MISER ... (Rarely used.)

• Automated software for matrix elements:

(1) REDUCE — an interactive program designed for general algebraic computations, including to evaluate Dirac algebra, an old-time program,
http://www.uni-koeln.de/REDUCE;
http://reduce-algebra.com. (Rarely used.)

(2) FORM by Jos Vermaseren: A program for large scale symbolic manipulation, evaluate fermion traces automatically,
and perform loop calculations,s commercially available at
http://www.nikhef.nl/ form

(3) FeynCalc and FeynArts: Mathematica packages for algebraic calculations in elementary particle physics.
http://www.feyncalc.org;
http://www.feynarts.de

(4) MadGraph: Helicity amplitude method for tree-level matrix elements available upon request or
http://madgraph.hep.uiuc.edu

- Automated evaluation of cross sections:

(1) MadGraph/MadEvent and MadSUSY:
Generate Fortran codes on-line! http://madgraph.hep.uiuc.edu
(Now allows you to input new models.)

(2) CompHEP/CalHEP: computer program for calculation of elementary particle processes in Standard Model and beyond. CompHEP has a built-in numeric interpreter. So this version permits to make numeric calculation without additional Fortran/C compiler. It is convenient for more or less simple calculations.

— It allows your own construction of a Lagrangian model!

http://theory.npi.msu.su/k̃ryukov
(Now allows you to input new models.)

(3) GRACE and GRACE SUSY: squared matrix elements (Japan)
http://minami-home.kek.jp

(4) AlpGen: higher-order tree-level SM matrix elements (M. Mangano ...):
http://mlm.home.cern.ch/mlm/alpgen/

(5) SHERPA (F. Krauss et al.): (Gaining popularity)
Generate Fortran codes on-line! Merging with MC generators (see next).
http://www.sherpa-mc.de/

(6) Pandora by M. Peskin:
C++ based package for e^+e^-, including beam effects.
http://www-sldnt.slac.stanford.edu/nld/new/Docs/Generators/PANDORA.htm
The program pandora is a general-purpose parton-level event generator which includes beamstrahlung, initial state radiation, and full treatment of polarization effects. (An interface to PYTHIA that produces fully hadronized events is possible.)

- Cross sections at NLO packages: (Gaining popularity)

(1) MC(at)NLO (B. Webber et al.):
http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
Combining a MC event generator with NLO calculations for QCD processes.

(2) MCFM (K. Ellis et al.):
http://mcfm.fnal.gov/
Parton-level, NLO processes for hadronic collisions.

(3) BlackHat (Z.Bern, L.Dixon, D.Kosover et al.):
http://blackhat.hepforge.org/
Parton-level, NLO processes to combine with Sherpa

- Numerical simulation packages: Monte Carlo Event Generators

  Reading: http://www.sherpa-mc.de/

(1) PYTHIA:

PYTHIA is a Monte Carlo program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between e^+, e^-, p and $\bar{p}$ in various combinations.

They contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.

— It can be combined with MadGraph and detector simulations.

http://www.thep.lu.se/ torbjorn/Pythia.html

Already made crucial contributions to Tevatron/LHC.

(2) HERWIG

HERWIG is a Monte Carlo program which simulates $pp, p\bar{p}$ interactions at high energies. It has the most sophisticated perturbative treatments, and possible NLO QCD matrix elements in parton showing.

http://hepwww.rl.ac.uk/theory/seymour/herwig/

(3) ISAJET

ISAJET is a Monte Carlo program which simulates $pp, \bar{p}p$, and ee interactions at high energies. It is largely obsolete.

ISASUSY option is still useful.

http://www.phy.bnl.gov/ isajet (Rarely used these days.)

- "Pretty Good Simulation" (PGS):

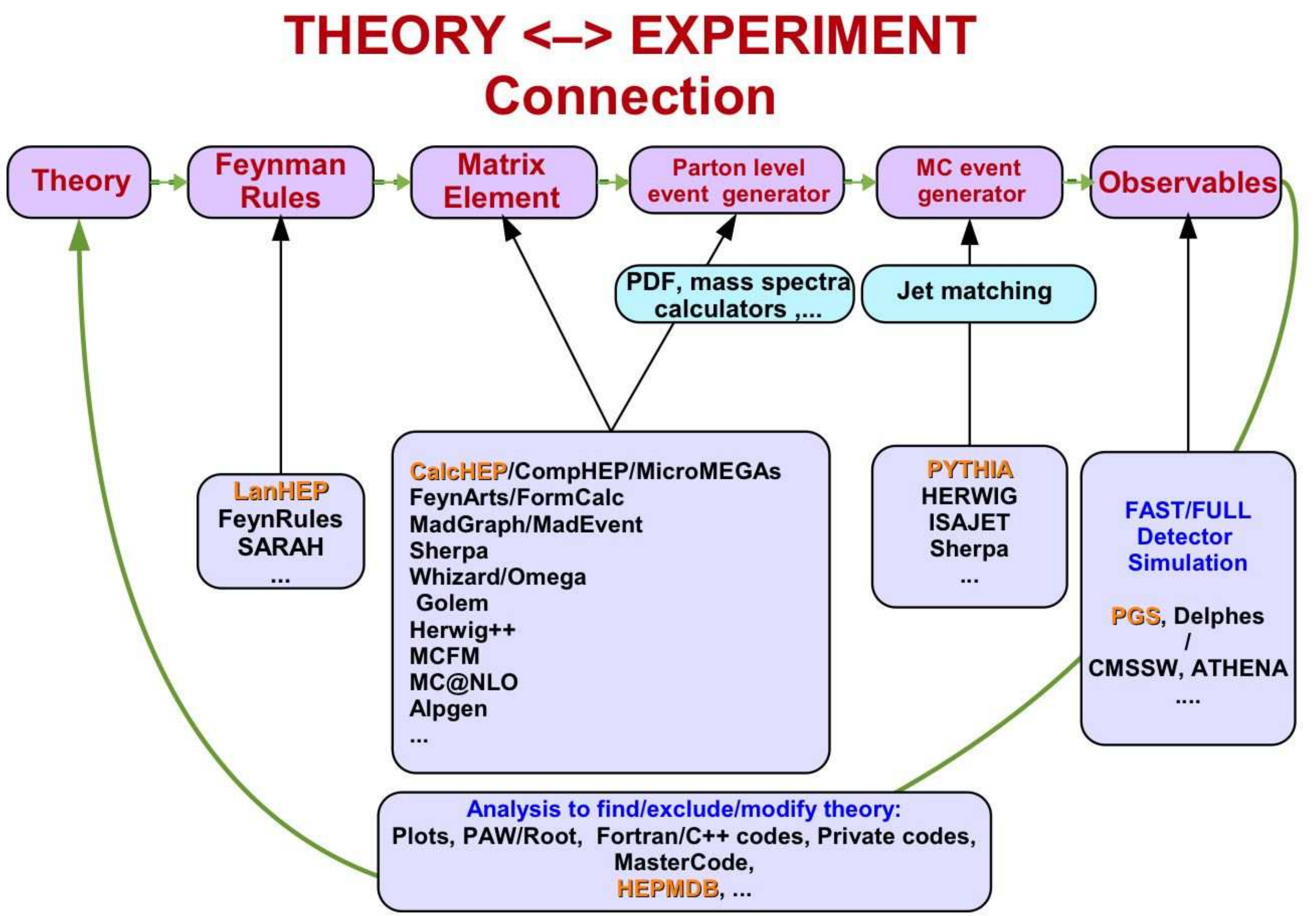
By John Conway: A simplified detector simulation, mainly for theorists to estimate the detector effects.

http://www.physics.ucdavis.edu/ conway/research/software/pgs/pgs.html

PGS has been adopted for running with PYTHIA and MadGraph.

(but just a "toy".)

Over all:


THEORY <–> EXPERIMENT
Connection
Theory
Feynman Rules
Matrix Element
Parton level event generator
MC event generator
Observables
PDF, mass spectra calculators ,...
Jet matching
LanHEP
FeynRules
SARAH
...
CalcHEP/CompHEP/MicroMEGAs
FeynArts/FormCalc
MadGraph/MadEvent
Sherpa
Whizard/Omega
Golem
Herwig++
MCFM
MC@NLO
Alpgen
...
PYTHIA
HERWIG
ISAJET
Sherpa
...
FAST/FULL Detector Simulation
PGS, Delphes
/
CMSSW, ATHENA
....
Analysis to find/exclude/modify theory:
Plots, PAW/Root, Fortran/C++ codes, Private codes,
MasterCode,
HEPMDB, ...

II. Physics at an e^+e^- Collider

(A.) Simple Formalism

Event rate of a reaction:

$$R(s) = \sigma(s)\mathcal{L}, \quad \text{for constant } \mathcal{L}$$
$$= \mathcal{L}\int d\tau \frac{dL(s,\tau)}{d\tau}\,\sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}.$$

As for the differential production cross section of two-particle a, b,

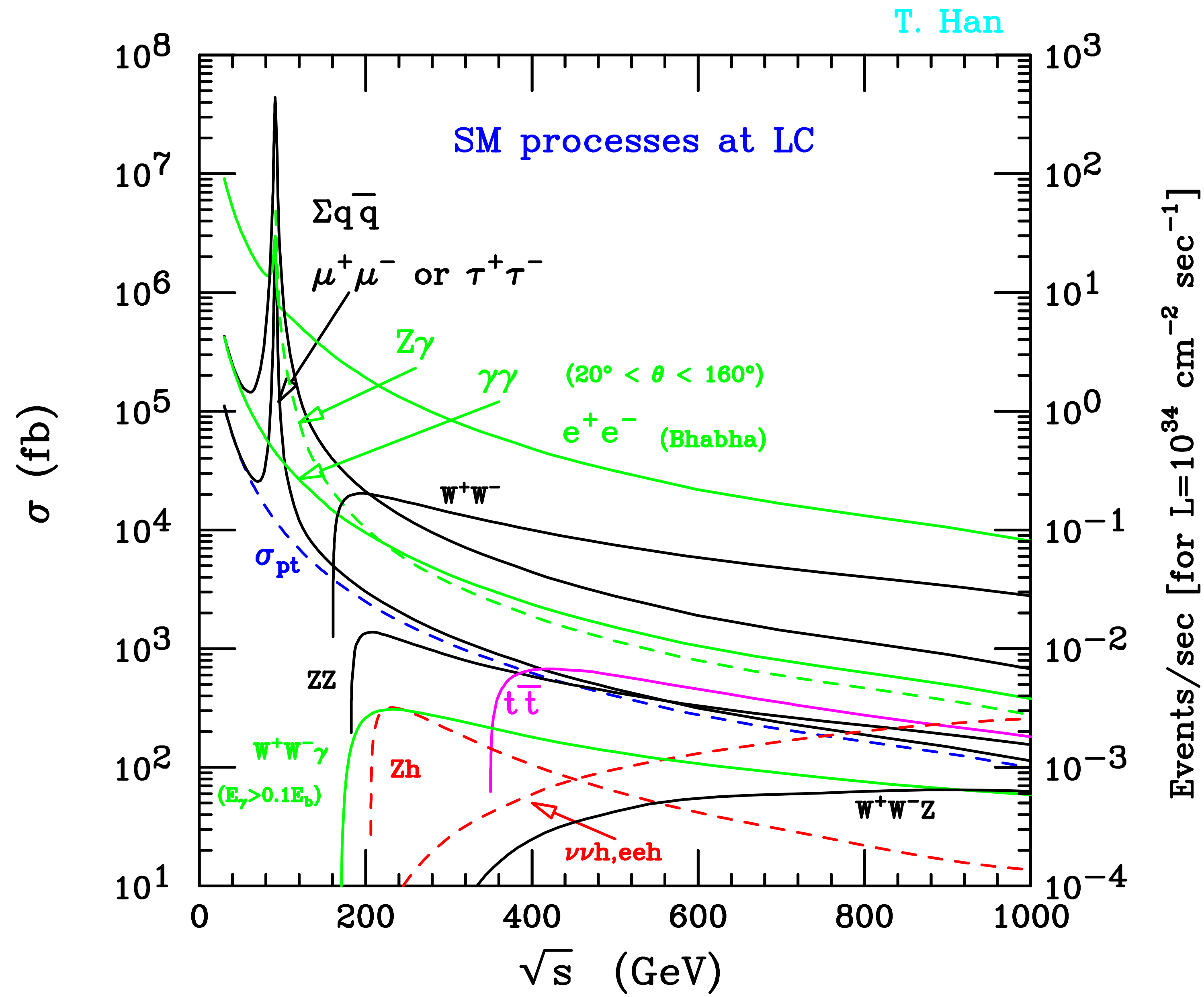
$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum}|\mathcal{M}|^2$$

where

• $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,

• $\overline{\sum|\mathcal{M}|^2}$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)

• unpolarized beams so that the azimuthal angle trivially integrated out, Total cross sections and event rates for SM processes:



(B). Resonant production: Breit-Wigner formula

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}$$

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \to V^* \to X) = \frac{4\pi(2j+1)\Gamma(V\to e^+e^-)\Gamma(V\to X)}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}\ \frac{s}{M_V^2},$$

If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2} \quad\to\quad \frac{\pi}{M_V\Gamma_V}\ \delta(s-M_V^2),$$

$$\sigma(e^+e^- \to V^* \to X) \quad=\quad \frac{2\pi^2(2j+1)\Gamma(V\to e^+e^-)BF(V\to X)}{M_V^2}\frac{dL(\hat{s}=M_V^2)}{d\sqrt{\hat{s}}}$$

Exercise 3.1: sketch the derivation of these two formulas, assuming a Gaussian distribution for

$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\ \Delta}\exp[\frac{-(\sqrt{\hat{s}}-\sqrt{s})^2}{2\Delta^2}].$$

Note: Away from resonance

For an s-channel or a finite-angle scattering:

$$\sigma \sim \frac{1}{s}.$$

For forward (co-linear) scattering:

$$\sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

(C). Fermion production:

Common processes: $e^- e^+ \to f\bar{f}$.
For most of the situations, the scattering matrix element can be casted into a $V \pm A$ chiral structure of the form (sometimes with the help of Fierz transformations)

$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta}\ [\bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1)]\ [\bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where $P_\mp = (1 \mp \gamma_5)/2$ are the L, R chirality projection operators, and $Q_{\alpha\beta}$ are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.
With this structure, the scattering matrix element squared:

$$\begin{aligned} \overline{\sum}|\mathcal{M}|^2 &= \frac{e^4}{s^2}\left[(|Q_{LL}|^2 + |Q_{RR}|^2)\ u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2)\ t_i t_j \right. \\ &+ \left. 2Re(Q^*_{LL} Q_{LR} + Q^*_{RR} Q_{RL}) m_f m_{\bar{f}} s\right], \end{aligned}$$

where $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$ and $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$.

`Exercise 3.2: Verify this formula.`

(D). Typical size of the cross sections:

- The simplest reaction

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact, $\sigma_{pt} \approx 100 \text{ fb}/(\sqrt{s}/\text{TeV})^2$ has become standard units to measure the size of cross sections.

- The Z resonance prominent (or other M_V),
- At the ILC $\sqrt{s} = 500$ GeV,

$$\sigma(e^+e^- \to e^+e^-) \sim 100\sigma_{pt} \sim 40 \text{ pb}.$$

(anglular cut dependent.)

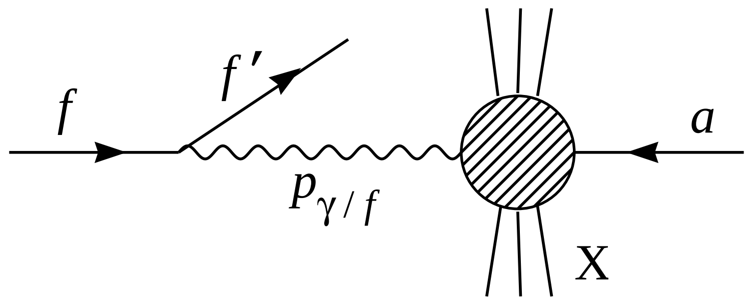
$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400 \text{ fb};$$
$$\sigma(u, d, s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb};$$
$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb}.$$

and

$$\sigma(ZH) \sim \sigma(WW \to H) \sim \sigma_{pt}/4 \sim 100 \text{ fb};$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40 \text{ fb}.$$

(E). Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+,\ \gamma^* e^- \to e^+e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

`Exercise 3.3: Try to derive this splitting function.`

We see that:

- m_e enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^{\pm}, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1 - x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

(F). Recoil mass technique:

One of the most important techniques, that distinguishes an e^+e^- collisions from hadronic collisions.

Consider a process:

$$e^+ + e^- \to V + X,$$

where V: a (bunch of) visible particle(s); X: unspecified.

Then:

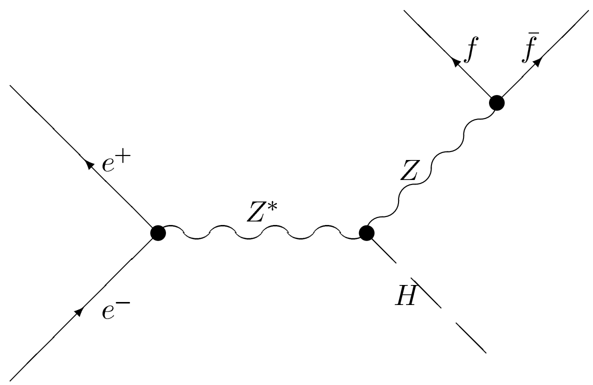
$$p_{e^+} + p_{e^-} = p_V + p_X, \ (p_{e^+} + p_{e^-} - p_V)^2 = p_X^2,$$
$$M_X^2 = (p_{e^+} + p_{e^-} - p_V)^2 = s + M_V^2 - 2\sqrt{s}E_V.$$

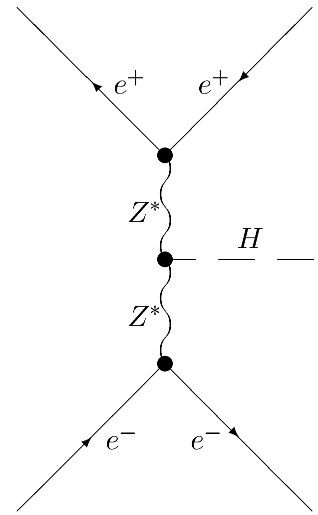
One thus obtain the "model-independent" inclusive measurements

a. mass of X by the recoil mass peak

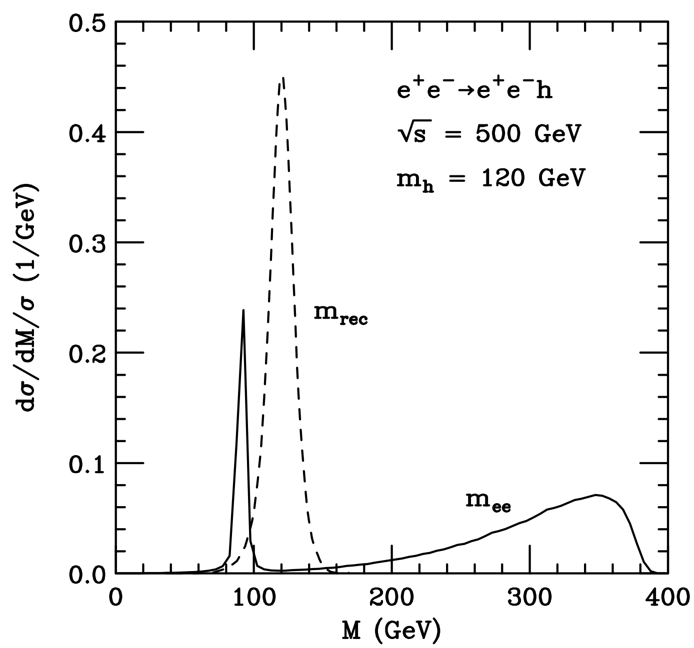
b. coupling of X by simple event-count at the peak

The key point for a Higgs factory: $e^+ + e^- \to f\bar{f} + h.$





Then: $$M_h^2 = (p_{e^+} + p_{e^-} - p_f - p_{\bar{f}})^2 = s + M_V^2 - 2\sqrt{s}E_{f\bar{f}}.$$



Model-independent, kinematical selection of signal events!

(G). Beam polarization:

One of the merits for an e^+e^- linear collider is the possible high polarization for both beams.
Consider first the longitudinal polarization along the beam line direction. Denote the average $e^\pm$ beam polarization by $P^L_\pm$, with $P^L_\pm = -1$ purely left-handed and $+1$ purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $\mathcal{M}_{\sigma_{e-}\sigma_{e+}}$:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{4}[(1-P^L_-)(1-P^L_+)|\mathcal{M}_{--}|^2 + (1-P^L_-)(1+P^L_+)|\mathcal{M}_{-+}|^2 \\ +(1+P^L_-(1-P^L_+)|\mathcal{M}_{+-}|^2 + (1+P^L_-)(1+P^L_+)|\mathcal{M}_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral:
Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: *e.g.*, $W^\pm$ exchange ...

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.

If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P^T_\pm < 1$, then there will be one additional term in the limit $m_e \to 0$,

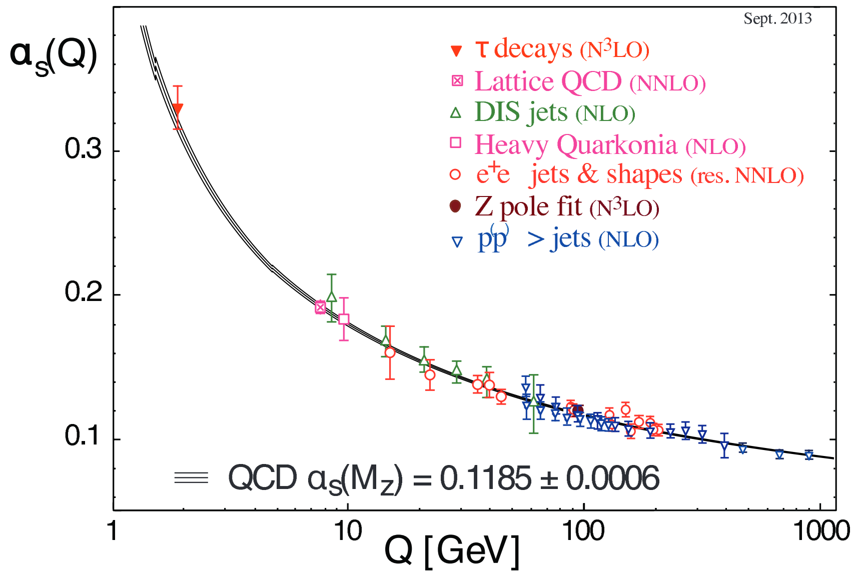
$$\frac{1}{4}\, 2\; P^T_- P^T_+ \; \mathrm{Re}(\mathcal{M}_{-+}\mathcal{M}^*_{+-}).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.

III-A. Perturbative QCD at a Glance

(A). Running of the strong coupling:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi} \ln \frac{Q^2}{Q_0^2}} \quad (33 - 2n_f > 0), \quad \{\alpha_{em}(Q^2) = \frac{\alpha_{em}(Q_0^2)}{1 - \frac{\alpha_{em}(Q_0^2)}{3\pi} \ln \frac{Q^2}{Q_0^2}}\}$$



Significant implications (D. Gross, D. Politzer, F. Wilczek, Nobel Prize 2004):

† Confinement at low energies (hadrons: the observable world);

† Asymptotic freedom at high energies (quarks, gluons and perturbation techniques);

† Possibility of Grand Unification; Description of the early universe.

(B). Parton Distribution Functions (PDF)

• Factorization theorem:

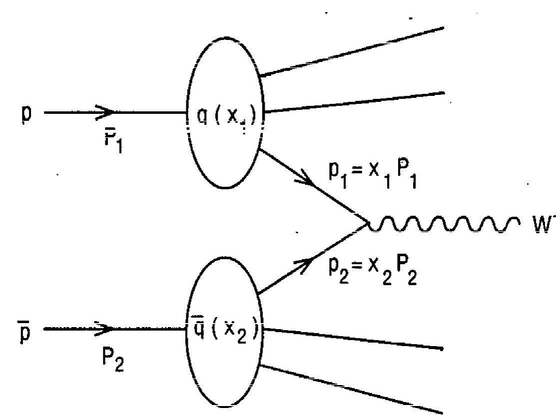
In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

(1). hard subprocess of parton scattering with a large scale $\mu^2 \gg \Lambda^2_{QCD}$;

(2). "parton distribution functions" (hadronic structure with $Q^2 < \mu^2$.)

Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$



† $\hat{\sigma}_{parton}(s)$ is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;
Infra-red (IR) divergence is cancelled by soft gluon emissions;
Co-linear divergence (massless) is factorized into PDF
– The essence of "factorization theorem".

† $P(x, Q^2)$ is the "Parton Distribution Functions" (PDF): The probability of finding a parton P with a momentum fraction x inside a proton.

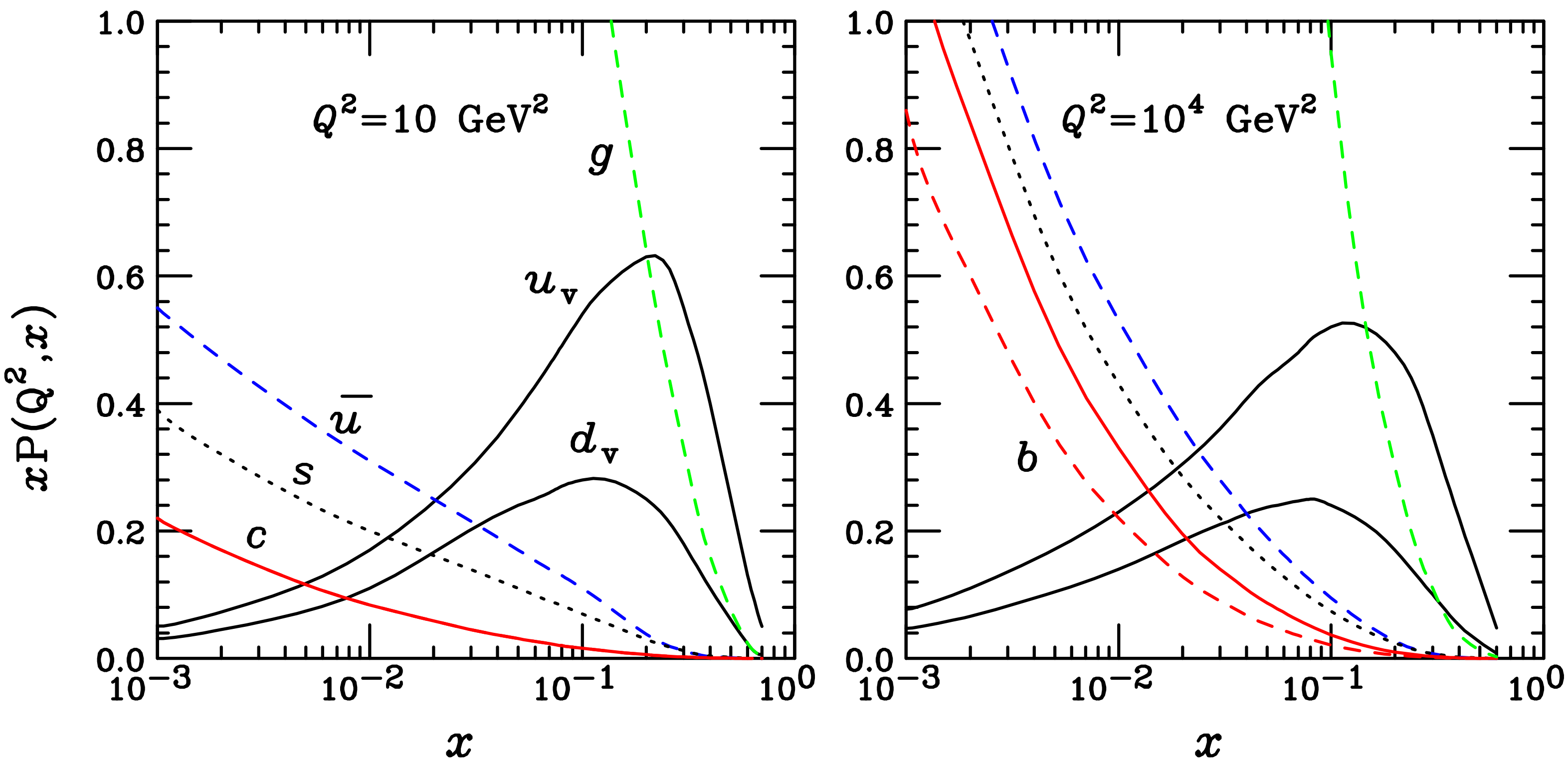
$P(x, Q^2)$ cannot be calculated from first principles, only extracted by fitting data, assuming a boundary condition at $Q_0^2 \sim$(2 GeV)^2.

The PDF's should match the parton-level cross section $\hat{\sigma}_{parton}(s)$ at a given order in α_s.

† Q^2 is the "factorization scale", below which it is collinear physics. It is NOT uniquely determined, leading to intrinsic uncertainty in QCD perturbation predictions. But its uncertainty is reduced with higher order calculations.

Several dedicated groups are developing PDF's:
CTEQ (Michigan State U.); MRSxxx (Durham U.) ... ...

Typical quark/gluon parton distribution functions:



(CTEQ-5)

Better understanding of the SM cross section, in particular in QCD are crucial for observing new physics as deviations from the SM.

(C). Jets and fragmentation functions

Upon production of a colored parton (quark/gluon):

† At the scale $\Lambda_{QCD} \sim 10^{-24}$s or 1 fm, the parton "hadronizes (fragments)" into massive, color-neutral, hadrons $\pi,\ n,\ p,\ K\ \ldots$

The "fragmentation function" is like the reverse of the PDF:

$$\frac{d\sigma(pp \to hX)}{dE_h} = \sum_q \int \frac{d\sigma(pp \to qX)}{dE_q} \frac{dE_q}{E_q} f_q^h(z, Q^2)$$

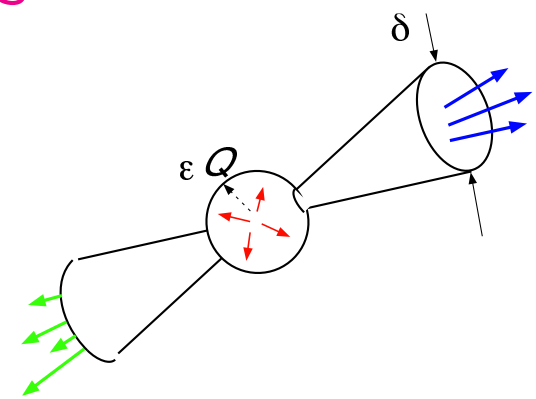
where $z = E_h/E_q$.

Non-perturbative and cann't be calculated from first principles.

† For most of the purposes in high energy collisions, we do not need to keep track of the individual hadrons, and thus the "inclusive processes".

Jets

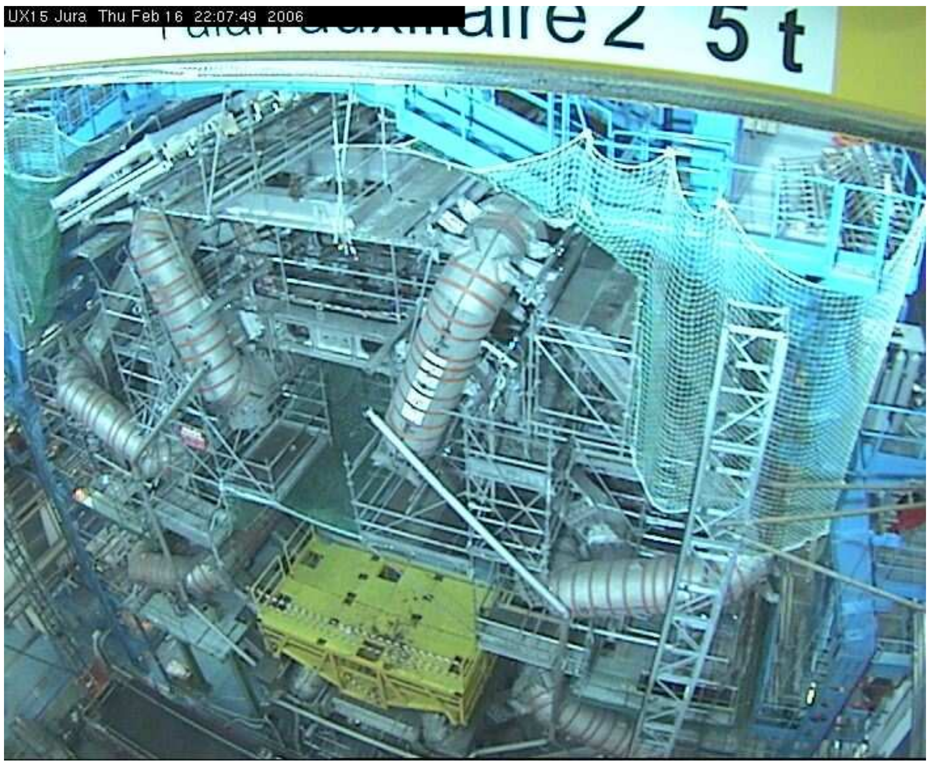
When $E_q \gg m_q$, then $\delta \approx \frac{2}{\gamma} = \frac{2m_q}{E_q}$.
It becomes a "jet", kinematically:



Need realistic algorithms to define the observable jets.

III-B. Hadron Collider Physics

(A). New HEP frontier: the LHC
The Higgs discovery and more excitements ahead ...

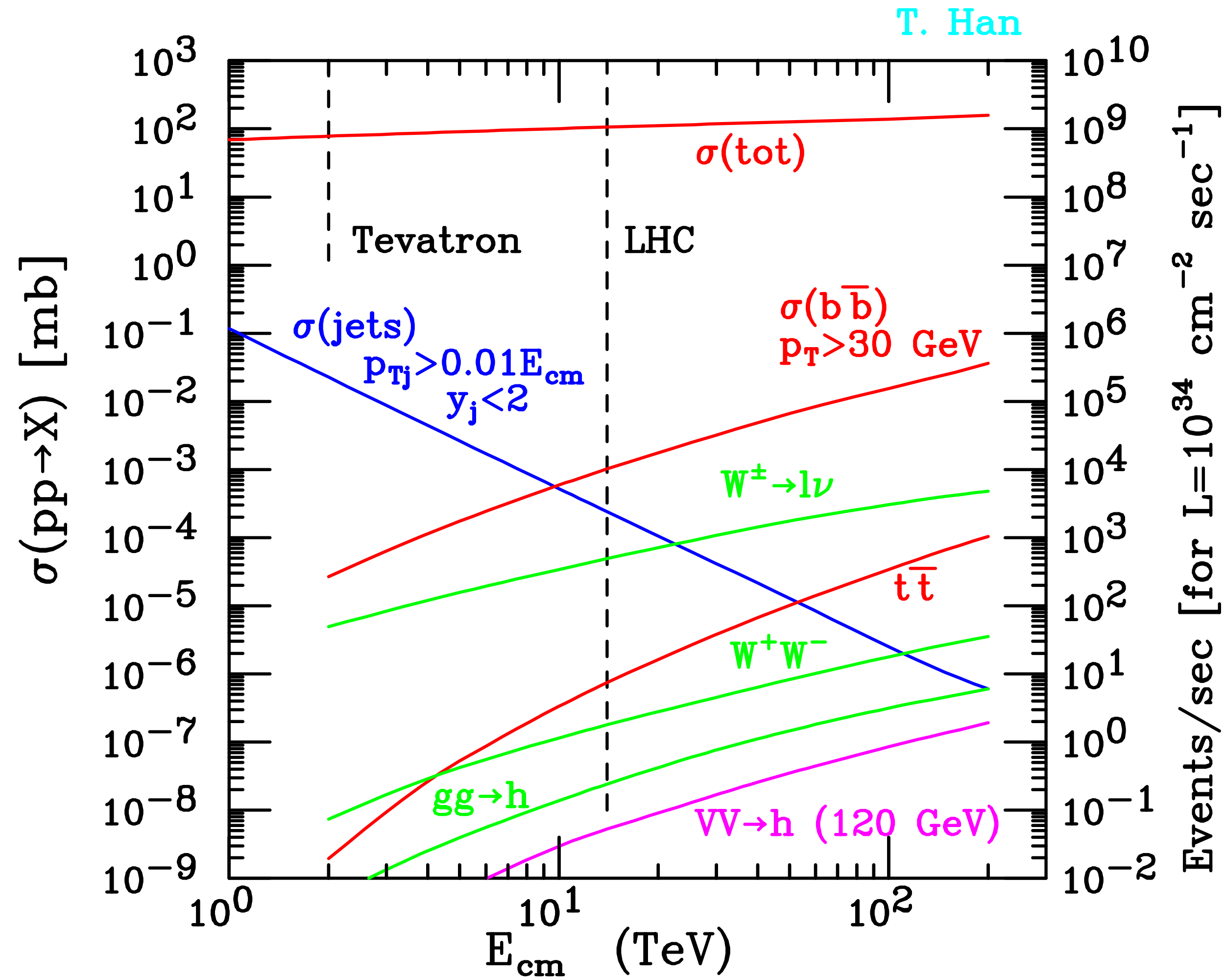


ATLAS (90m underground)

CMS

(New mission started in June 2015.)

LHC Event rates for various SM processes:



$10^{34}/\text{cm}^2/\text{s} \Rightarrow 100\ \text{fb}^{-1}/\text{yr}.$

Annual yield # of events $= \sigma \times L_{int}$:

10B $W^{\pm}$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...

Discovery of the Higgs boson opened a new chapter of HEP!

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.

The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi / m_\pi^2 \sim 120 \text{ mb}.$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7 \ (\frac{s}{\text{GeV}^2})^{0.0808} \ \text{ mb}, & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \ \ln^2 \frac{s}{s_0}, & \text{Froissart bound.} \end{cases}$$

(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2) \ \hat{\sigma}_{parton}(s).$$

- Accurate (higher orders) partonic cross sections $\hat{\sigma}_{parton}(s)$.
- Parton distribution functions to the extreme (density):

$$Q^2 \sim (a \ few \ TeV)^2, \quad x \sim 10^{-3} - 10^{-6}.$$

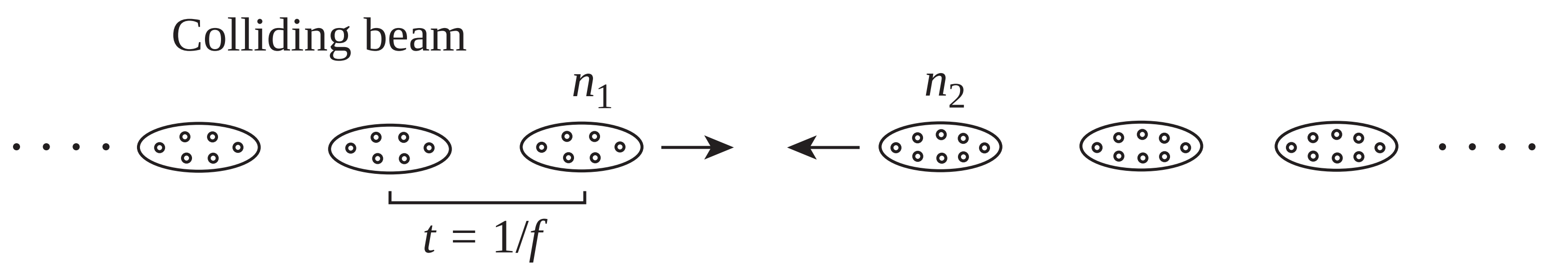
Experimental challenges:

- The large rate turns to a hostile environment:

  ≈ 1 billion event/sec: impossible read-off !

  ≈ 1 interesting event per $1,000,000$: selection (triggering).

  ≈ 25 overlapping events/bunch crossing:



$\Rightarrow$ Severe backgrounds!

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

$(\eta = 2.5 \Rightarrow 10^\circ; \qquad \eta = 5 \Rightarrow 0.8^\circ.)$

With optimal triggering and kinematical selections:

$p_T \geq 30 - 100$ GeV, $\quad |\eta| \leq 3 - 5; \quad \not{E}_T \geq 100$ GeV.

(B). Special kinematics for hadron colliders

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$
$$y_{cm} = \frac{1}{2}\ln\frac{1+\beta_{cm}}{1-\beta_{cm}} = \frac{1}{2}\ln\frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

The four-momentum vector transforms as

$$\begin{pmatrix} E' \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\ \beta_{cm} \\ -\gamma\ \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} = \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}.$$

This is often called the "boost".

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \\ p^\mu &= (E_T \cosh y,\ p_T \sin\phi,\ p_T \cos\phi,\ E_T \sinh y), \\ \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy. \end{aligned}$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

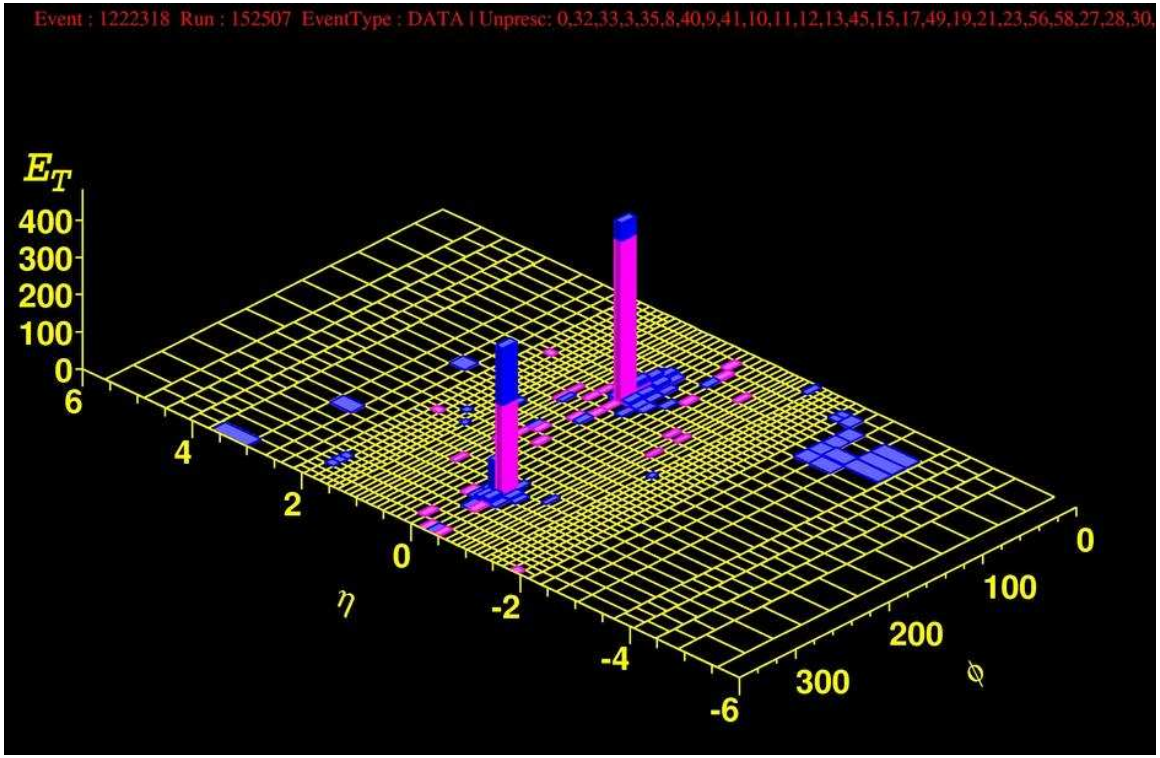
$$y' = \frac{1}{2}\ln\frac{E'+p'_z}{E'-p'_z} = \frac{1}{2}\ln\frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm}.$$

In the massless limit, rapidity $\to$ pseudo-rapidity:

$$y \to \eta = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \ln\cot\frac{\theta}{2}.$$

`Exercise 4.1:  Verify all the above equations.`

The "Lego" plot:



A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

$\phi, \Delta y = y_2 - y_1$ is boost-invariant.

Thus the "separation" between two particles in an event

$$\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$$ is boost-invariant,

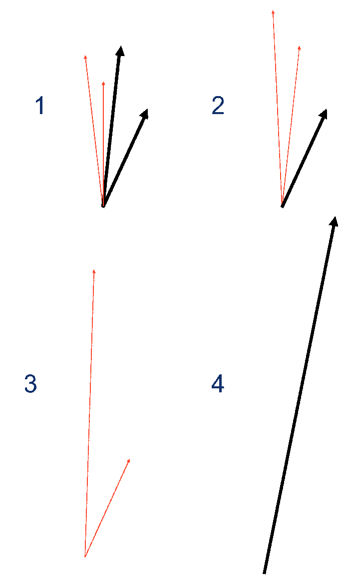
and lead to the "cone definition" of a jet.

The Jets! Alternative algorithms: Successive combination

- Given a cluster of proto-jets, $i = 1, 2, ..., n$, pick an initial pair i, j.
- Calculate their "beam distance" $d_i =$
and angular separation $\Delta R^2 = \Delta\phi^2 +$
- With respect to an angular resolutio
define a "pair distance"

$$d_{ij} = min(d_i, d$$

- If $d_{ij} < d_i, d_j$, then combine $p_i + p_j$ in
- If d_i $(d_j) < d_{ij}$, then leave the proto-jet i (j) alone as a "finished jet".



Repeat this procedure until every proto-jet becomes a finished jet.

† Cambridge-Aachen algorithm: $d_i = 1$. (the cone algorithm)

† k_T-algorithm: $d_i = p_{Ti}^2$. (d_{ij} is the relative p_T^2 between i and j)

† Anti-k_T-algorithm: $d_i = p_{Ti}^{-2}$. (higher pT proto-jet serves as the seed)

(C). Characteristic observables:
Crucial for uncovering new dynamics.

Selective experimental events

$\Longrightarrow$ Characteristic kinematical observables

(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

Energy momentum observables $\Longrightarrow$ mass parameters

Angular observables $\Longrightarrow$ nature of couplings;

Production rates, decay branchings/lifetimes $\Longrightarrow$ interaction strengths.

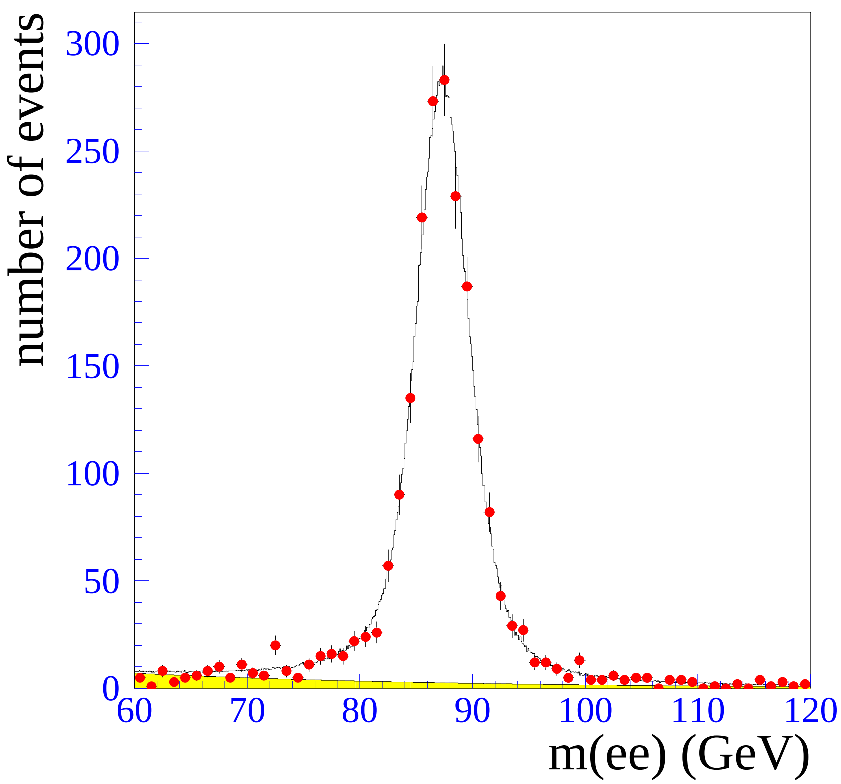
(D). Kinematical features:

(a). s-channel singularity: bump search we do best.

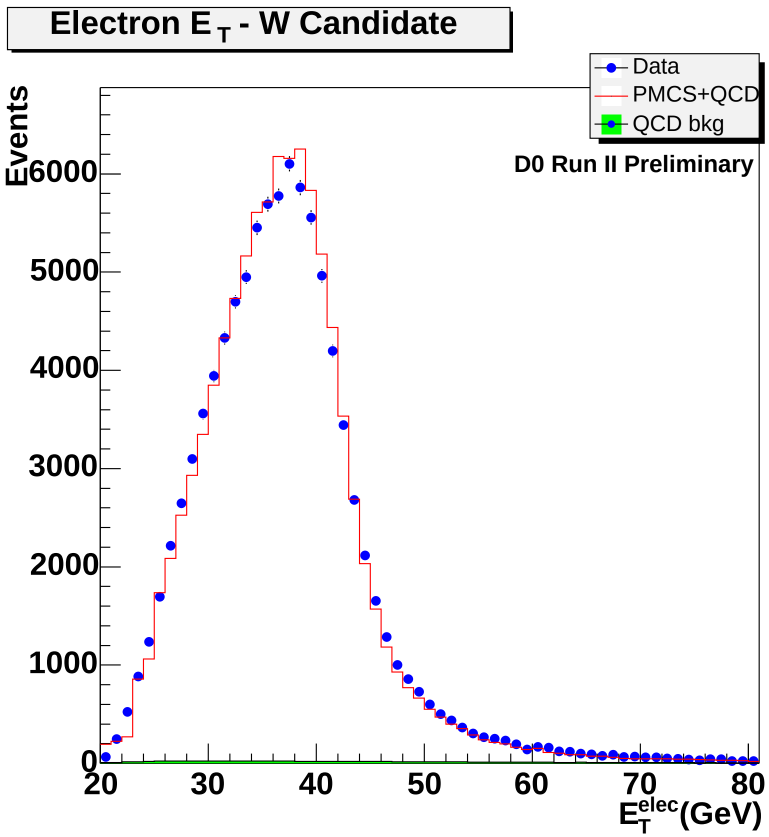
- invariant mass of two-body $R \to ab: \; m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2 \; dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \, \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$



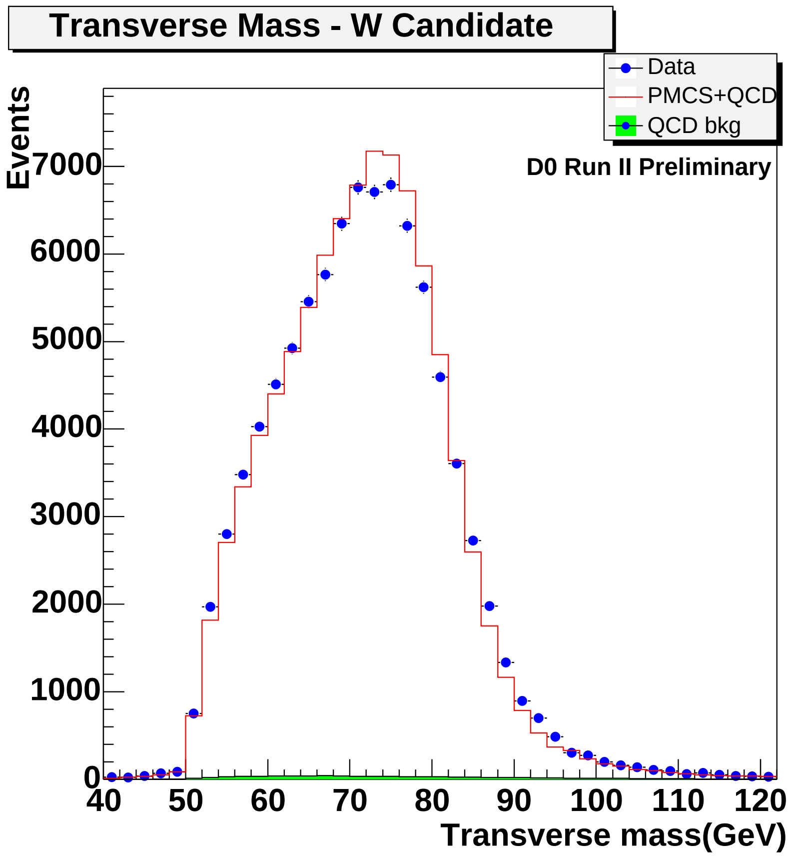
$Z \to e^+e^-$

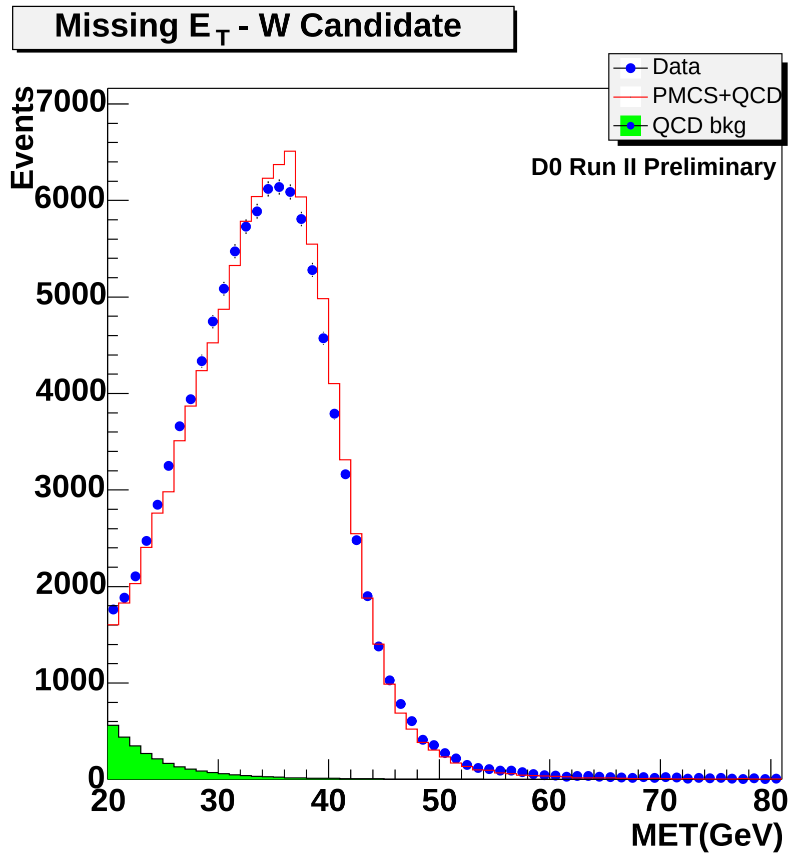


$W \to e\nu$

- "transverse" mass of two-body $W^- \to e^- \bar{\nu}_e$:

$$m_{e\nu\ T}^2 = (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2$$
$$= 2 E_{eT} E_T^{miss} (1 - \cos\phi) \le m_{e\nu}^2.$$





If $p_T(W) = 0$, then $m_{e\nu\ T} = 2E_{eT} = 2E_T^{miss}$.

Exercise 5.1: For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}}\ \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

where $p_{eT} = p_e \sin\theta^*$ is the transverse momentum and θ^* is the polar angle in the c.m. frame. Comment on the apparent singularity at $p_{eT}^2 = s/4$.

Exercise 5.2: Show that for an on-shell decay $W^- \to e^-\bar{\nu}_e$:

$$m_{e\nu\ T}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \leq m_{e\nu}^2.$$

Exercise 5.3: Show that if W/Z has some transverse motion, δP_V, then:

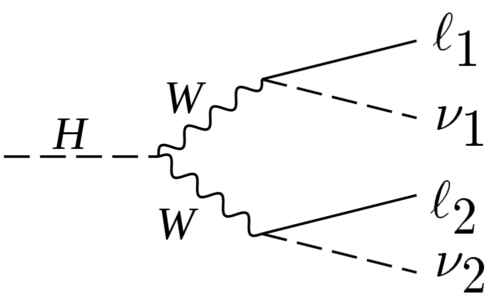
$$p'_{eT} \sim p_{eT}\ [1 + \delta P_V/M_V],$$
$$m'^2_{e\nu\ T} \sim m^2_{e\nu\ T}\ [1 - (\delta P_V/M_V)^2],$$
$$m'^2_{ee} = m^2_{ee}.$$

- $H^0 \to W^+W^- \to j_1 j_2\ e^-\bar{\nu}_e$:

cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2$$
$$= \left(\sqrt{p^2_{jjT} + M^2_W} + \sqrt{p^2_{e\nu T} + M^2_W}\right)^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \le M^2_H.$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs}\ \vec{p}_T^{\,obs}$.



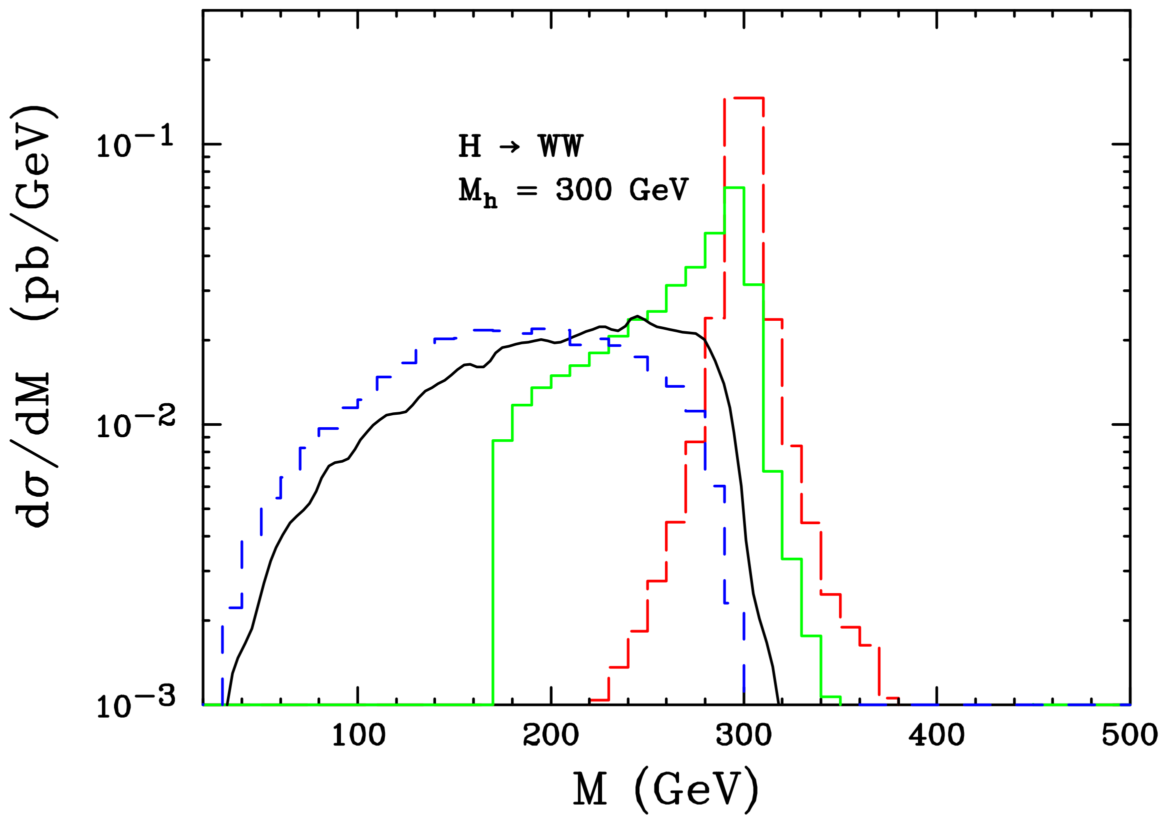
- $H^0 \to W^+W^- \to e^+\nu_e\ e^-\bar{\nu}_e$:

"effecive" transverse mass:

$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2$$
$$m_{eff\ T} \approx E_{e1T} + E_{e2T} + E_T^{miss}$$

cluster transverse mass (II):

$$m^2_{WW\ C} = \left(\sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T\right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not{p}}_T)^2$$
$$m_{WW\ C} \approx \sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T$$



M_{WW} invariant mass (WW fully reconstructable): - - - - - - - - -

$M_{WW,\,T}$ transverse mass (one missing particle ν): ————————

$M_{eff,\,T}$ effetive trans. mass (two missing particles): - - - - - - - -

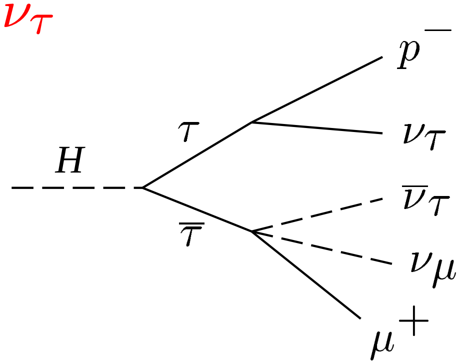
$M_{WW,\,C}$ cluster trans. mass (two missing particles): ————————

YOU design an optimal variable/observable for the search.

- cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+\ \bar\nu_\tau\ \nu_\mu, \quad \rho^-\ \nu_\tau$$

A lot more complicated with (many) more $\nu's$?



Not really!

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau/E_\tau = 2m_\tau/m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

$$\vec{p}_{\tau^+} = \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu's}, \quad \vec{p}_+^{\,\nu's} \approx c_+ \vec{p}_{\mu^+}.$$
$$\vec{p}_{\tau^-} = \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu's}, \quad \vec{p}_-^{\,\nu's} \approx c_- \vec{p}_{\rho^-}.$$

where $c_\pm$ are proportionality constants, to be determined.

This is applicable to any decays of fast-moving particles, like

$$T \to Wb \to \ell\nu,\ b.$$

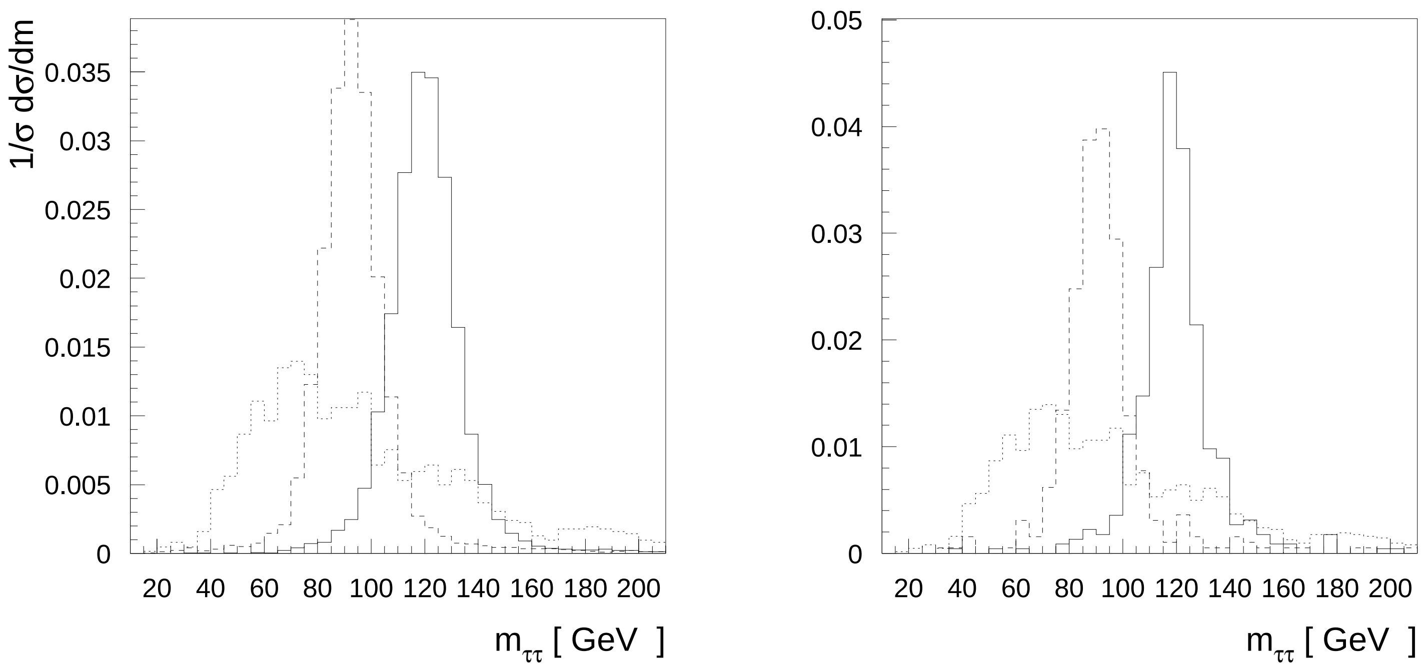
Experimental measurements: p_{ρ^-}, p_{μ^+}, $\not{p}_T$:

$$c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x = (\not{p}_T)_x,$$
$$c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y = (\not{p}_T)_y.$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle, or the Higgs should have a non-zero transverse momentum.



(b). Two-body versus three-body kinematics

- Energy end-point and mass edges:

utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \ \tilde{\mu}_R^-$$
$$\text{with two} - \text{body decays}: \ \ \tilde{\mu}_R^+ \to \mu^+ \tilde{\chi}_0, \ \ \ \tilde{\mu}_R^- \to \mu^- \tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

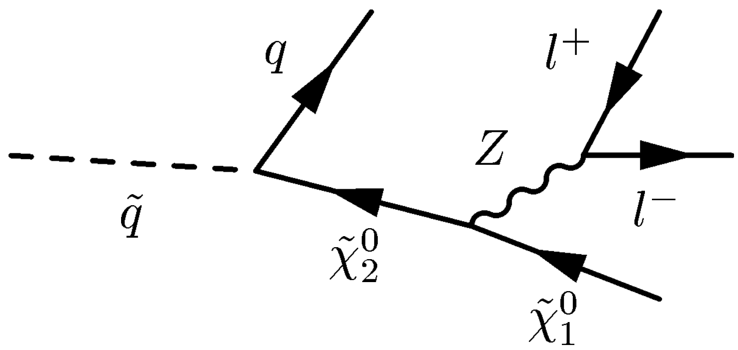
$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$
$$\text{with} \ \ \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \ \ \ \gamma = (1-\beta)^{-1/2}.$$

Energy end-point: $E_\mu^{lab} \Rightarrow M_{\tilde{\mu}_R}^2 - m_\chi^2$.

Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

Same idea can be applied to hadron colliders ...

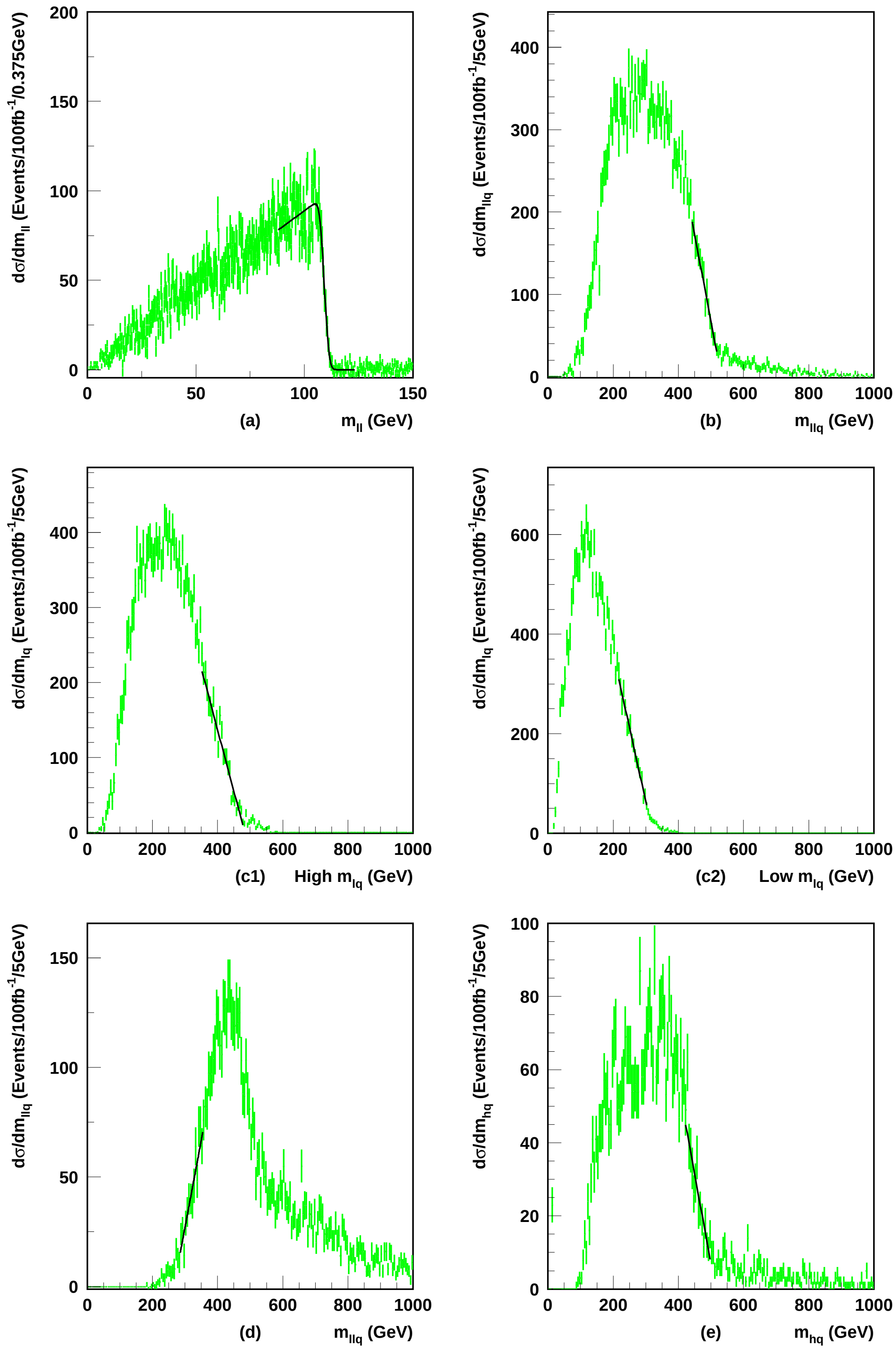
Consider a squark cascade decay:



$$1^{\rm st}\ \text{edge}: \quad M^{max}(\ell\ell) = M_{\chi_2^0} - M_{\chi_1^0};$$

$$2^{\rm nd}\ \text{edge}: \quad M^{max}(\ell\ell j) = M_{\tilde{q}} - M_{\chi_1^0}.$$

`Exercise 5.4:  Verify these relations.`

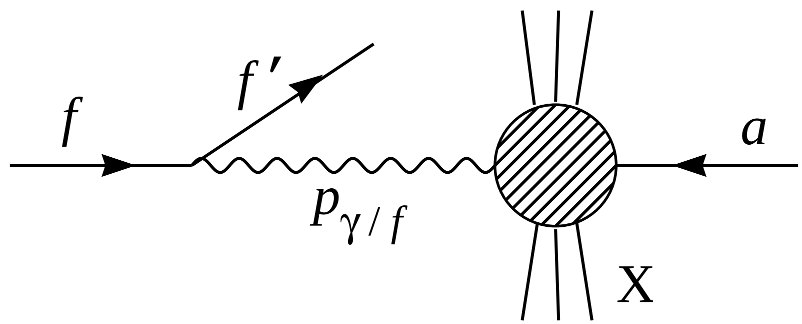


dσ/dm_ll (Events/100fb^-1/0.375GeV)
m_ll (GeV)
(a)
dσ/dm_llq (Events/100fb^-1/5GeV)
m_llq (GeV)
(b)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c1)
High m_lq (GeV)
(c2)
Low m_lq (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(d)
m_llq (GeV)
dσ/dm_hq (Events/100fb^-1/5GeV)
(e)
m_hq (GeV)

(c). t-channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \to f'X) \approx \int dx\, dp_T^2\, P_{\gamma/f}(x, p_T^2)\, \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi} \frac{1+(1-x)^2}{x} \left(\frac{1}{p_T^2}\right) |_{m_e}^{E}.$$

† The kernel is the same as $q \to qg^*$ $\Rightarrow$ generic for parton splitting;

† The form $dp_T^2/p_T^2 \to \ln(E^2/m_e^2)$ reflects the collinear behavior.

- Generalize to massive gauge bosons:

$$P^T_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{8\pi^2}\,\frac{1+(1-x)^2}{x}\,\frac{p_T^2}{(p_T^2+(1-x)M_V^2)^2},$$

$$P^L_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{4\pi^2}\,\frac{1-x}{x}\,\frac{(1-x)M_V^2}{(p_T^2+(1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low-p_{jT},

$$\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}$$

At high-p_{jT},

$$\left.\begin{array}{l} \frac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

(E). Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

$$i\sum_{\tau}^{L,R} g_\tau^f\ \gamma^\mu\ P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i\mathcal{A}_f,$$

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1) P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1) P_q(x_2) \Big) \, \mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1) P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1) P_q(x_2) \Big)}.$$

Perfectly fine for Z/Z'-type:

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

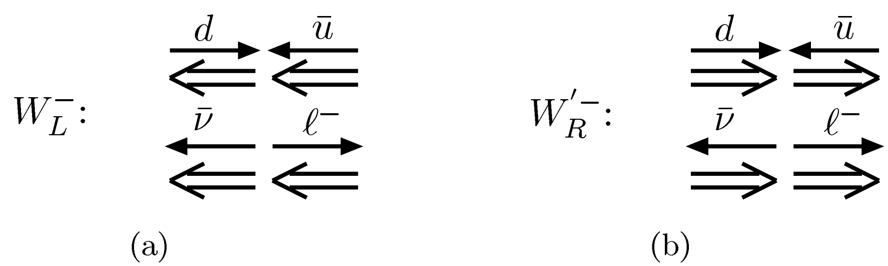
In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

It is the boost-direction of $\ell^+\ell^-$.

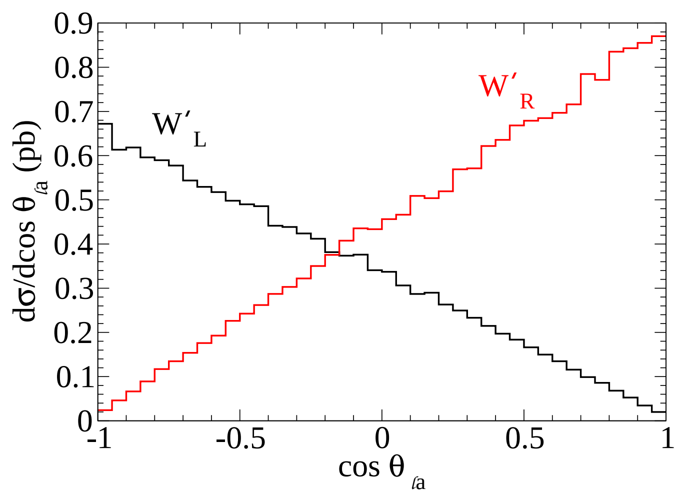
How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
AND ℓ^+ (ℓ^-) along the direction with $\bar{q}$ (q) $\Rightarrow$ OK at the Tevatron,

But: (1). cann't get the boost-direction of $\ell^{\pm}\nu$ system;
(2). Looking at $\ell^{\pm}$ alone, no insight for W_L or W_R!



In $p\bar{p}$ collisions: (1). a reconstructable system
(2). with spin correlation $\rightarrow$ only tops $W' \rightarrow t\bar{b} \rightarrow \ell^{\pm}\nu\ \bar{b}$:



(F). CP asymmetries A_{CP}:

To non-ambiguously identify CP-violation effects,
one must rely on CP-odd variables.

Definition: A_{CP} vanishes if CP-violation interactions do not exist (for the relevant particles involved).

This is meant to be in contrast to an observable:
that'd be *modified* by the presence of CP-violation,
but is *not zero* when CP-violation is absent.

$$\text{e.g. } M_{(\chi^\pm\ \chi^0)}, \quad \sigma(H^0,\ A^0), \ldots$$

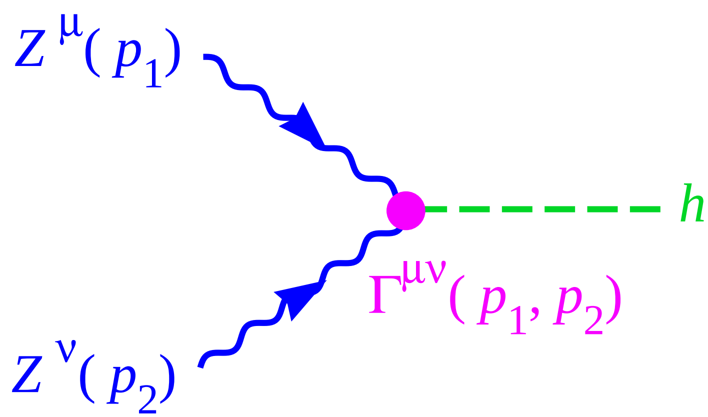
Two ways:
a). Compare the rates between a process and its CP-conjugate process:

$$\frac{R(i \to f) - R(\bar{i} \to \bar{f})}{R(i \to f) + R(\bar{i} \to \bar{f})}, \quad \text{e.g.} \quad \frac{\Gamma(t \to W^+ q) - \Gamma(\bar{t} \to W^- \bar{q})}{\Gamma(t \to W^+ q) + \Gamma(\bar{t} \to W^- \bar{q})}.$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin\theta,$$
$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

E.g. 1: $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$



$$\Gamma^{\mu\nu}(p_1,p_2) = i\frac{2}{v}\ h[a\ M_Z^2 g^{\mu\nu} + b\ (p_1^\mu p_2^\nu - p_1 \cdot p_2 g^{\mu\nu}) + \tilde{b}\ \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}]$$

$a = 1,\ b = \tilde{b} = 0$ for SM.

In general, $a,\ b,\ \tilde{b}$ complex form factors,

describing new physics at a higher scale.

For $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$, define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$
$$\text{or}\ \ \cos\theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2||\vec{q}_1 \times \vec{q}_2)|}.$$

E.g. 2: $H \to t(p_t)\bar{t}(p_{\bar{t}}) \to e^+(q_1)\nu_1 b_1,\ e^-(q_2)\nu_2 b_2.$

$$-\frac{m_t}{v}\bar{t}(a + b\gamma^5)t\ H$$
$$O_{CP} \sim (\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{p}_{e^+} \times \vec{p}_{e^-}).$$

thus define an asymmetry angle.