Precision calculations of semileptonic B decays

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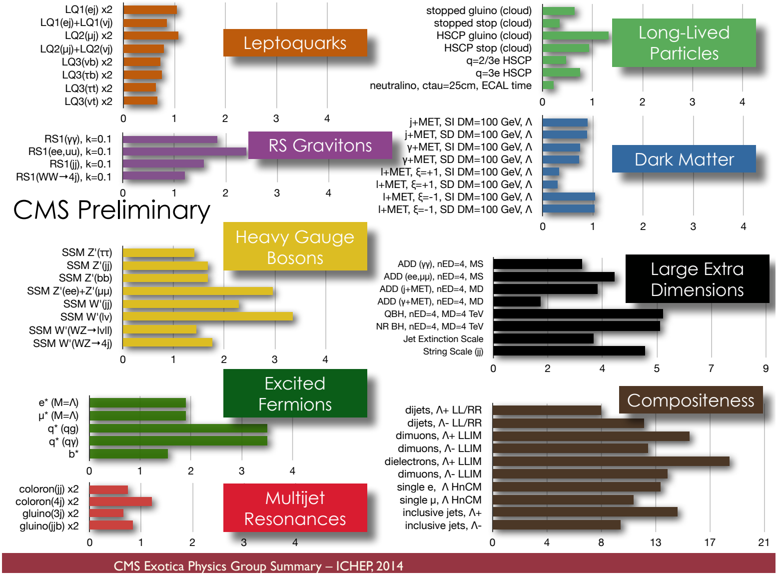
Where do we stand now?

- Big success of the SM and no evidences of NP $\Longrightarrow$ Precision physics (Higgs, flavor ...).



Triumphs of flavor physics

- 1963: concept of flavor mixing [Cabibbo].
- 1973: quark-flavor mixing with 3 generations incudes CP violation [KM mechanism].
- 1974: prediction of the charm mass from $K_0 - \overline{K}_0$ mixing. [Gaillard and Lee].
- 1987: prediction of the top mass from $B_0 - \overline{B}_0$ mixing observed by AUGUS (DESY) and UA1 (CERN).
- 2001: large CP violation in B meson decays [BaBar and Belle].
- 2004: direct CP violation in B meson decays [BaBar and Belle].

Why flavor matters in the LHC era?

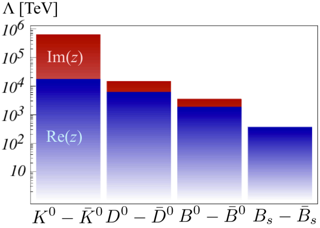
- Indirect probe of BSM physics beyond direct reach.
  - EFT parametrization of BSM physics:

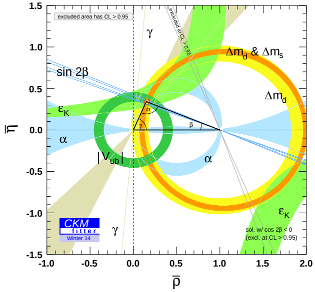
$$\mathscr{L} = \mathscr{L}^{SM}_{dim\,4} + \sum_{n>4} \sum_{i} z^{n}_{(i)} \frac{1}{\Lambda^{n-4}} Q^{(n)}_{i} \,.$$

  - Dimension-n operator $Q^{(n)}_i$ is $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant.
  - Higher dimension operator $Q^{(n)}_i$ suppressed by the large scale.
  - Two examples: (a) leading NP operators of $D = 6$ for $\Delta F = 2$ processes

$$Q^{(6)}_{AB,ij} = \left[\bar{q}_i \Gamma^A q_j\right] \otimes \left[\bar{q}_i \Gamma^B q_j\right] \,,$$

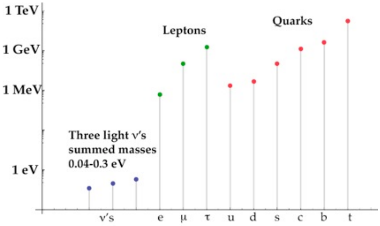
    (b) unitarity of the CKM triangles.



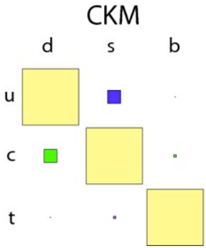


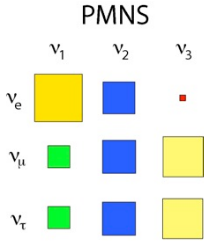
Why flavor matters in the LHC era?

- Find the underlying principle for the flavor structure.
  Suggestive pattern of masses and mixings.



S. Stone, 1212.6374

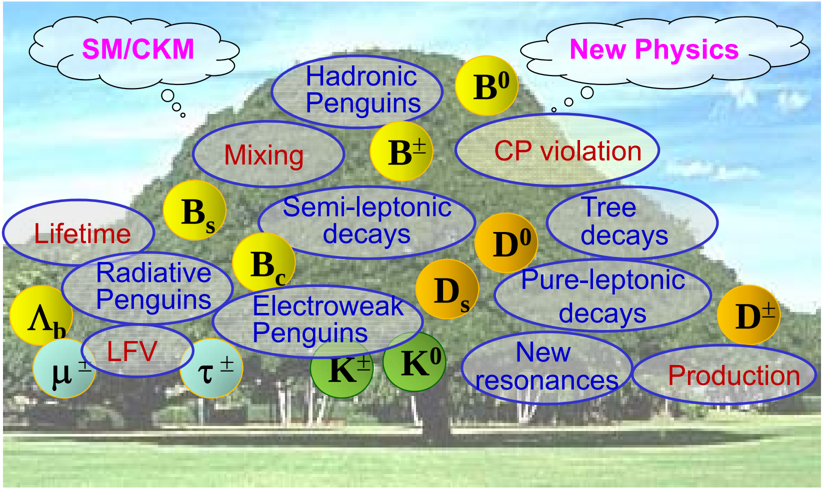




  - Why are the quark masses (except the top) so small compared with the vev?
  - Why is the CKM matrix hierarchical?
  - Why is CKM so different from the PMNS?
  - Why do we have three families?
  - Sources of flavor symmetry and violation?

Why flavor matters in the LHC era?

- Excellent opportunities to explore the strong interaction dynamics:



QCD factorization theorems, effective field theories, resummation techniques, non-perturbative QCD dynamics, QCD sum rules.

Theory tools for precision flavor physics

New Physics: $\mathscr{L}_{NP}$

$\downarrow$

EW scale (m_W): $\mathscr{L}_{SM} + \mathscr{L}_{D>4}$

$\downarrow$

Heavy-quark scale (m_b): $\mathscr{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_i C_i Q_i + \mathscr{L}_{eff,D>6}$

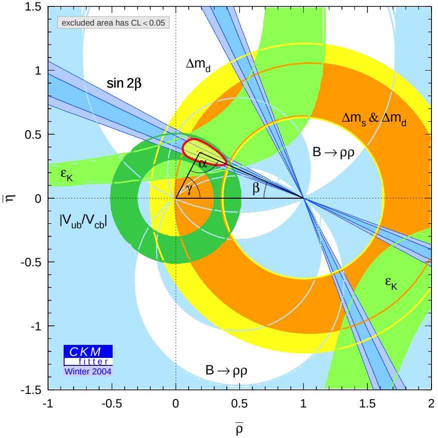
$\downarrow$

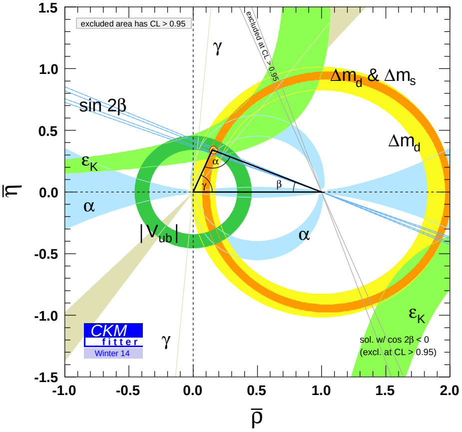
QCD scale ($\Lambda_{\rm QCD}$)

- Aim: $\langle f | Q_i | \bar{B} \rangle = ?$
- QCD factorization [BBNS approach].
- TMD factorization.
- SCET factorization.
- (Light-cone) QCD sum rules.
- Lattice QCD.

Ten years of flavour factories

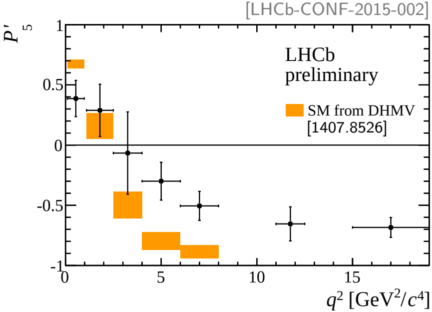
2004 $\Longrightarrow$ 2014

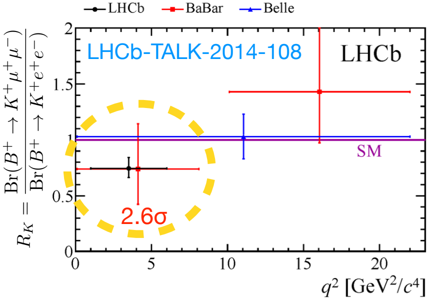




Anomalies in FCNC processes

A few "anomalies" exist in $B \to K^{(*)} \ell^+ \ell^-$.

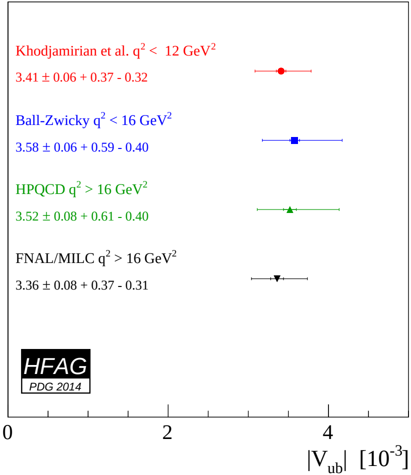


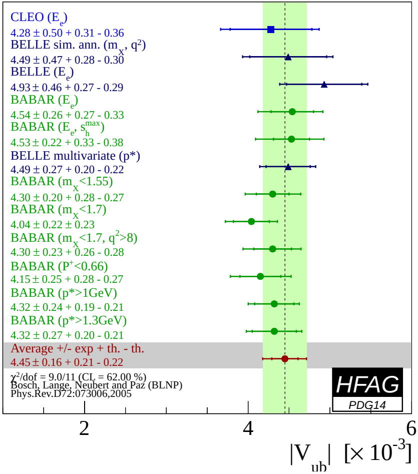


- Indication of BSM physics or ignorance of QCD dynamics?
- P'_5 anomaly below 6 GeV^2 more serious [power corrections].
- Violation of lepton flavor universality [QED corrections].
- Need more data and theoretical efforts.

$|V_{ub}|$ puzzle

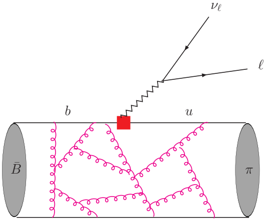
$3\,\sigma$ tension between exclusive and inclusive $|V_{ub}|$.





BSM physics or ignorance of the strong interaction dynamics?

Semileptonic $B \to \pi \ell \nu$ decays



Hadronic matrix element:

$$\langle \pi(p) | \bar{u} \gamma_\mu b | \bar{B}(p+q) \rangle = f_{B\pi}^{+}(q^2) \left[p_B + p - \frac{m_B^2 - m_\pi^2}{q^2} q \right]_\mu + f_{B\pi}^{0}(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \, .$$

- Lepton spectrum:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24 \pi^3 q^4 m_B^2} (q^2 - m_l^2)^2 \, |\vec{p}_\pi| \times \left[\left(1 + \frac{m_l^2}{q^2} \right) m_B^2 \, |\vec{p}_\pi|^2 \, |f_{B\pi}^{+}(q^2)|^2 + \frac{3 \, m_l^2}{8 q^2} \, (m_B^2 - m_\pi^2)^2 \, |f_{B\pi}^{0}(q^2)|^2 \right] .$$

- Still the best way to determine $|V_{ub}|$ exclusively in the continuum approach!

- $\Lambda_b \to p \ell \nu$ decays also become important now [LHCb, arXiv:1504.01568].

$$|V_{ub}| = \left(3.27 \pm 0.23 \right) \times 10^{-3} \, .$$

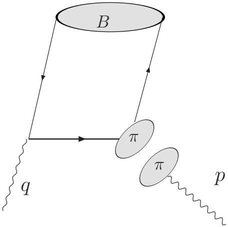
$B \to \pi$ form factors from the B-meson LCSR

- Starting point: vacuum-to-B-meson correlation function

$$\begin{aligned}\Pi_\mu(p,q) &= \int d^4x\, e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\not{n}\gamma_5 u(x), \bar{u}(0)\gamma_\mu b(0)\right\}|\bar{B}(p+q)\rangle \\ &= \Pi(n\cdot p, \bar{n}\cdot p)n_\mu + \widetilde{\Pi}(n\cdot p, \bar{n}\cdot p)\bar{n}_\mu\,, \\ n\cdot p &= \frac{m_B^2+m_\pi^2-q^2}{m_B}, \qquad \bar{n}\cdot p \sim O(\Lambda_{\rm QCD})\,, \qquad p+q \equiv m_B v = \frac{m_B}{2}(n+\bar{n})\,.\end{aligned}$$

- Inserting complete set of pion states $\to$ hadronic sum:

$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) =$



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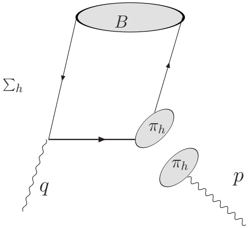
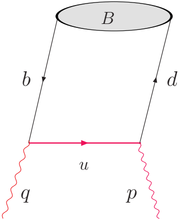


$$\frac{f_\pi(n\cdot p)m_B}{2(m_\pi^2-p^2)}\underbrace{\left[\frac{n\cdot p}{m_B}f_{B\pi}^+(n\cdot p)+f_{B\pi}^0(n\cdot p)\right]}_{\text{relative sign changes for }\Pi(n\cdot p,\bar{n}\cdot p)} \qquad \int_{\omega_s}^{+\infty} d\omega'\,\frac{\widetilde{\rho}^h(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p}$$

OPE calculation of the correlation function



Factorization at tree level:

$$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) = \tilde{f}_B\, m_B \int_0^{+\infty} d\omega' \, \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p} + O(\alpha_s)\,,$$

$$\Pi(n\cdot p, \bar{n}\cdot p) = O(\alpha_s)\,,$$

$$\Rightarrow f_{B\pi}^0(n\cdot p) = \frac{n\cdot p}{m_B} f_{B\pi}^+(n\cdot p) + O(\alpha_s)\,.$$

- Light-cone DAs of B-meson:

$$m_B \tilde{f}_B\, \phi_B^+(\omega) = \int \frac{d\tau}{2\pi}\, e^{i\omega\tau} \langle 0|\bar{q}(\tau n)\,[\tau n, 0]\, \not{n}\, \gamma_5\, h_v(0)|\bar{B}(m_B v)\rangle\,.$$

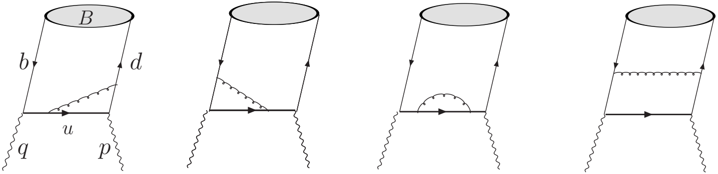
  [$\phi_B^-(\omega)$ defined in a similar way.]

- QCD correction involving $\phi_B^+(\omega')$ at NLO must be IR finite.

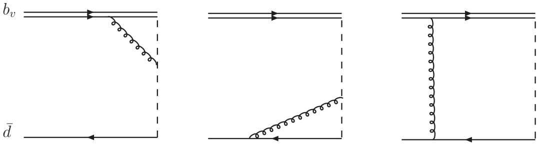
- Symmetry breaking of the form-factor relation at NLO must be IR finite.

Factorization of the correlation function

- Light-cone OPE: $0 > \bar{n} \cdot p \sim O(\Lambda_{\rm QCD})$.



- Cancellation of the soft divergences.



- Diagrammatic factorization:

$$\begin{aligned} \Pi_\mu &= \Pi_\mu^{(0)} + \Pi_\mu^{(1)} + \ldots = \Phi_B \otimes T \\ &= \Phi_B^{(0)} \otimes T^{(0)} + \left[\Phi_B^{(0)} \otimes T^{(1)} + \Phi_B^{(1)} \otimes T^{(0)} \right] + \ldots . \end{aligned}$$

$$\Downarrow$$

$$\Phi_B^{(0)} \otimes T^{(1)} = \Pi_\mu^{(1)} - \Phi_B^{(1)} \otimes T^{(0)} .$$

Sample calculation: the weak vertex diagram

- Strategy:
  (i) Identify the leading regions of the scalar integrals.
  (ii) Evaluate the contribution of each region using the method of regions.
  (iii) Performe the soft subtraction [the same as the QCD amplitude in the soft region].

- QCD amplitude:

$$\Pi^{(1)}_{\mu,weak} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[(p-k+l)^2+i0][(m_b v+l)^2-m_b^2+i0][l^2+i0]} \bar{d}(k)\not{n}\,\gamma_5\,\not{\bar{n}}\,\underbrace{\gamma_\rho\,(\not{p}-\not{k}+\not{l})\,\gamma_\mu\,(m_b\not{v}+\not{l}+m_b)\,\gamma^\rho}_{\text{soft}\,\Downarrow\,\text{region}}\,b(p_b)\,,$$

$$2\,n\cdot p\,m_b\,\gamma_\mu$$

- Soft subtraction [Wilson-line Feynman rules]:

$$\Phi^{(1)}_{B,weak}\otimes T^{(0)} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[\bar{n}\cdot(p-k+l)+i0][v\cdot l+i0][l^2+i0]} \bar{d}(k)\not{n}\,\gamma_5\,\not{\bar{n}}\,\gamma_\mu\,b(p_b)\,.$$

- Compute the hard and hard-collinear contributions with the light-cone projector.

$$\mathscr{M}^B_{\beta\,\alpha} = -\frac{i\tilde{f}_B\,m_B}{4}\left\{\frac{1+\not{v}}{2}\left[\phi_B^+(\omega)\not{n}+\phi_B^-(\omega)\not{\bar{n}}-\frac{2}{D-2}\,\omega\,\phi_B^-(\omega)\,\gamma_\perp^\mu\,\frac{\partial}{\partial k_\perp^\mu}\right]\right\}_{\alpha\beta}\,.$$

Factorization of the correlation function

- Aim: Factorization of the correlation function.

$$\widetilde{\Pi}(n\cdot p,\bar{n}\cdot p) = \tilde{f}_B\, m_B \sum_{k=\pm} \int \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{T}^{(k)}\,(n\cdot p,\bar{n}\cdot p,\omega,\mu)\, \phi_B^{(k)}(\omega,\mu)$$

$$= \tilde{f}_B\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p,\mu) \int \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)\,.$$

  Similar factorization formula for $\Pi(n\cdot p,\bar{n}\cdot p)$.

- Hard functions [Y.M.W and Y.L. Shen, 2015]:

$$C^{(+)}(n\cdot p,\mu) = \tilde{C}^{(+)}(n\cdot p,\mu) = 1, \qquad C^{(-)}(n\cdot p,\mu) = \frac{\alpha_s\, C_F}{4\pi}\, \frac{1}{\bar{r}} \left[\frac{r}{\bar{r}}\ln r + 1\right], \qquad r = \frac{n\cdot p}{m_b}\,,$$

$$\tilde{C}^{(-)}(n\cdot p,\mu) = 1 - \frac{\alpha_s\, C_F}{4\pi}\left[2\ln^2\frac{\mu}{n\cdot p} + 5\,\ln\frac{\mu}{n\cdot p} - \ln^2 r - 2\mathrm{Li}_2\left(\frac{r-1}{r}\right) + \frac{2-r}{r-1}\ln r + \frac{\pi^2}{12} + 5\right].$$

- Hard matching coefficient of the QCD weak current [Bauer et al, 2001; Beneke et al, 2004]:

$$\bar{q}\,\gamma_\mu\, b \to \left[C_4\,\bar{n}_\mu + C_5\, v_\mu\right]\, \bar{\xi}_{\bar{n}}\, b_v + \ldots\,.$$

  Perturbative matching coefficients independent of the external states ⇒

$$C^{(-)} = \frac{1}{2}\, C_5, \qquad \tilde{C}^{(-)} = C_4 + \frac{1}{2}\, C_5\,.$$

Factorization of the correlation function

- Jet functions [Y.M.W and Y.L. Shen, 2015]:

$$J^{(+)}(\bar{n}\cdot p,\omega,\mu) = \frac{1}{r}\tilde{J}^{(+)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left(1-\frac{\bar{n}\cdot p}{\omega}\right)\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right),$$

$$J^{(-)}(\bar{n}\cdot p,\omega,\mu) = 1,$$

$$\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) = 1+\frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)} - 2\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}\ln\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)} - \ln^2\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p} - \left(1+\frac{2\bar{n}\cdot p}{\omega}\right)\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p} - \frac{\pi^2}{6} - 1\right].$$

In agreement with the jet functions computed in SCET [De Fazio, Feldmann and Hurth, 2008].

- Cancellation of the factorization-scale dependence:

$$\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{n\cdot p}+5\right]\tilde{C}^{(-)}(n\cdot p,\mu),$$

$$\frac{d}{d\ln\mu}\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu^2}{n\cdot p\,\omega}\right]\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) + \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\,\tilde{J}^{(-)}(\bar{n}\cdot p,\omega',\mu),$$

$$\frac{d}{d\ln\mu}\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{\omega}-5\right]\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] - \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\left[\tilde{f}_B\,\phi_B^-(\omega',\mu)\right],$$

$B \to \pi$ form factors from the B-meson LCSR

- B-meson LCSR @ NLO:

$$f_\pi \; e^{-m_\pi^2/(n\cdot p\,\omega_M)} \left\{ \frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p)\,,\; f_{B\pi}^{0}(n\cdot p) \right\}$$

$$= \tilde{f}_B(\mu) \int_0^{\omega_s} d\omega' \; e^{-\omega'/\omega_M} \Big[r\,\widetilde{C}^{(+)}(n\cdot p,\mu)\,\phi_{B,\rm eff}^{+}(\omega',\mu) + \widetilde{C}^{(-)}(n\cdot p,\mu)\,\phi_{B,\rm eff}^{-}(\omega',\mu)$$

$$\pm \frac{n\cdot p - m_B}{m_B} \left(C^{(+)}(n\cdot p,\mu)\,\phi_{B,\rm eff}^{+}(\omega',\mu) + C^{(-)}(n\cdot p,\mu)\,\phi_B^{-}(\omega',\mu) \right) \Big] .$$

- Effective DAs:

$$\phi_{B,\rm eff}^{+}(\omega',\mu) = 0 + \frac{\alpha_s\,C_F}{4\pi} \int_{\omega'}^{\infty} \frac{d\omega}{\omega}\; \phi_B^{+}(\omega,\mu)\;,$$

$$\phi_{B,\rm eff}^{-}(\omega',\mu) = \phi_B^{-}(\omega',\mu) + \frac{\alpha_s\,C_F}{4\pi} \left\{ \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} - 4\ln\frac{\omega'-\omega}{\omega'} \right) \right]_{+} \phi_B^{-}(\omega,\mu) \right.$$

$$\left. - \int_{\omega'}^{\infty} d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega} - \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} + 3 \right) \ln\frac{\omega-\omega'}{\omega'} + 2\ln\frac{\omega}{\omega'} + \frac{\pi^2}{6} - 1 \right] \frac{d\phi_B^{-}(\omega,\mu)}{d\omega} \right\} .$$

- Power counting:

$$\omega_s \sim \omega_M \sim O(\Lambda_{\rm QCD}^2/m_b)\,.$$

The *B*-meson LCDAs

- Light-cone distribution amplitudes of the *B* meson:

$$\phi^+_{B,\mathrm{I}}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,, \qquad \text{[Grozin and Neubert, 1997]}$$

$$\phi^+_{B,\mathrm{II}}(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\,\ln k\right],\quad k=\frac{\omega}{1\text{ GeV}}\,, \qquad \text{[Braun et al, 2004]}$$

$$\phi^+_{B,\mathrm{III}}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\,\omega_1^2}\, e^{-(\omega/\omega_1)^2}\,,\quad \omega_1=\frac{2\omega_0}{2\sqrt{\pi}}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

$$\phi^+_{B,\mathrm{IV}}(\omega,\mu_0) = \frac{\omega}{\omega_0\,\omega_2}\,\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}}\,,\quad \omega_2=\frac{4\omega_0}{4-\pi}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

- The shape of $f^+_{B\pi}(q^2)$ less model dependent.

blue curve from pion LCSR, solid, dotted, dashed and dot-dashed curves from Model-I, II, III and IV.

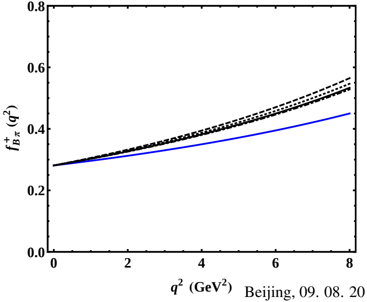
fitting $f^+_{B\pi}(q^2=0)=0.28\pm0.03$ from pion LCSR $\Rightarrow$

Model-I: $\omega_0 = 360^{+40}_{-30}\,\text{MeV}$,

Model-II: $\omega_0 = 375^{+40}_{-35}\,\text{MeV}$,

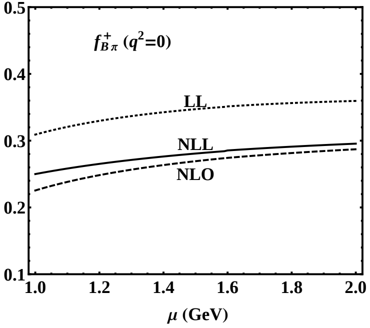
Model-III: $\omega_0 = 395^{+35}_{-30}\,\text{MeV}$,

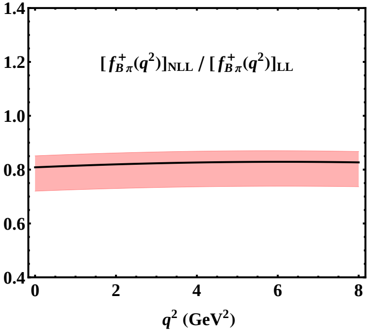
Model-IV: $\omega_0 = 310^{+40}_{-30}\,\text{MeV}$.



$B \to \pi$ form factors from the B-meson LCSR

- Factorization scale dependence and radiative correction:





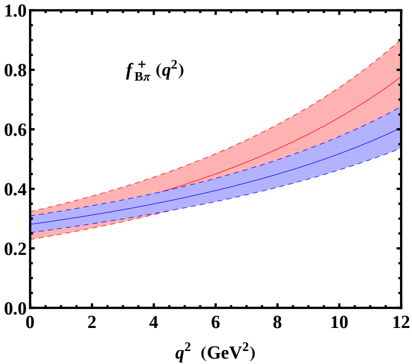
  - Dominant radiative effect from the NLO QCD correction instead of the QCD resummation.
  - Resummation improvement does stabilize the factorization scale dependence.
  - Radiative effect can induce 20 % reduction of the form factor.

$B \to \pi$ form factors from the B-meson LCSR

- The predicted form factor $f^+_{B\pi}(q^2)$:

Pink band: B-meson LCSR @ NLO,
Blue band: pion LCSR @ NLO.

Rapidly increasing $f^+_{B\pi}(q^2)$ from B-meson LCSR:
(i) Different pattern of the subleading contribution?
(ii) Different quark-hadron quality ansatz?



- Exclusive $|V_{ub}|$ from B-meson LCSR @ NLO [Y.M.W and Y.L. Shen, 2015]:

$$|V_{ub}| = \left(3.05^{+0.54}_{-0.38}\Big|_{\rm th.} \pm 0.09\Big|_{\rm exp.}\right) \times 10^{-3}.$$

- Exclusive $|V_{ub}|$ from $B \to \tau\nu$ [Belle, combined two tagging methods, arXiv: 1503.05613]:

$$|V_{ub}| = \left(3.28^{+0.37}_{-0.42}\right) \times 10^{-3}.$$

Concluding Remarks

- Irrespective of LHC13 discoveries, flavour physics is and will remain a powerful probe of BSM physics.
- *B* decays provide excellent platforms to understand the strong interaction dynamics.
- Still a lot to learn from the golden channel to extract the exclusive $|V_{ub}|$.
- Conceptual breakthrough for computing power corrections in demand.
- Exciting times are just ahead of us!