Relativistic radiative decay of heavy-light mesons

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Main focus of this talk

- Transition amplitude of a bound state in the nonrelativistic limit

$$\neq$$

- A sum of nonrelativistic Transition amplitudes

  – True for some of M1,2 transitions

$$M_R = \langle f|V_R|i\rangle \xrightarrow{non-rel.} M_{NR} \neq \langle f|V_{NR}|i\rangle$$

Introduction 1

- Study on heavy-light mesons in a relativistic potential model
  - Mass spectra PRD 56, 5646 (1997); PTP 117, 1077 (2007); EPJ A31, 701 (2007)
  - (relativistic) wave functions
    - different from simple Harmonic oscillator w.f.
- Apply them to
  - Semi-leptonic decay form factor PTP 118, 1087 (2007)
  - π or K emitting hadronic decay width

PRD 85, 014036 (2012)

Introduction 2

- Former studies on radiative decay for heavy-light mesons
  - Bardeen, Eichten, Hill: PRD 68, 054024 (2003) … E1 and M1
  - Close and Swanson, PRD 72, 094004 (2005) … E1 and M1
  - Barnes, Godfrey, Swanson, PRD 72, 054029 (2005) … E1 only
  - All are nonrelativistic treatment

Former nonrelativistic method1

- Electric transition : interaction

$$V_E = \left\{ (-e_q)\frac{\vec{p}_{\bar{q}}}{m_q^*} + e_Q \frac{\vec{p}_Q}{m_Q^*} \right\} \cdot \vec{A} = -i\,[\,\vec{d}, H\,] \cdot \vec{A} \to -i\,\omega\,\vec{d} \cdot \vec{A}$$

- where ω is photon energy and classical electric current is

$$\vec{d} = (-e_q)\vec{x}_{\bar{q}} + e_Q \vec{x}_Q = \frac{(-e_q)m_Q^* - e_Q m_q^*}{m_Q^* + m_q^*}\vec{x}_{rel}$$

- Transition matrix with w.f. in the rest frame

$$T_{E1} \propto \left\langle f \middle| \vec{d} \middle| i \right\rangle \cdot \vec{\epsilon}$$

w.f. in the rest frame

Former nonrelativistic method2

- Magnetic transition : interaction

$$V_M = -\vec{\mu} \cdot \vec{H}$$

- magnetic moment

$$\vec{\mu} = \frac{2(-e_q)}{2m_q^*}\vec{s}_{\bar{q}} + \frac{2e_Q}{2m_Q^*}\vec{s}_Q$$

- Transition matrix with w.f. in the rest frame

$$T_{M1} \propto \langle f | \vec{\mu} | i \rangle \cdot (\vec{k} \times \vec{\epsilon})$$

w.f. in the rest frame

Puzzle

- ele.–mag. interaction : $V_R = -\sum_i \psi_i^\dagger \vec{\alpha} \cdot \vec{A} \psi_i$

$$\xrightarrow{\text{non-rel.}} (\text{E.T.}) - \boxed{\sum_i \frac{e_i}{2m_i} \psi_i^\dagger \vec{\sigma} \cdot \vec{\nabla} \times \vec{A} \psi_i}$$ magnetic int.

$$M_R = \langle P' | V_R | P \rangle_{\text{mag.}} \xrightarrow{\text{non-rel.}} \boxed{M_{NR} \neq \langle P' | V_{NR} | P \rangle_{\text{mag.}}}$$

$$\langle P' | V_{NR} | P \rangle_{\text{mag.}}$$

$$\propto \sum_{i=1,2} \frac{e_i}{2m_i} \int dx_{\text{rel}}\, e^{ik\cdot x_{\text{rel}}} \psi_i^\dagger (x_{\text{rel}}) \vec{\sigma}_i \cdot \left(\vec{k} \times \vec{\varepsilon} \right) \psi_i (x_{\text{rel}})$$

common or unique

i.e., we claim

- For magnetic transitions,

- Transition Amplitude of a bound state in the naïve nonrelativistic limit $(V \to 0)$

$$\neq$$

- A sum of nonrelativistic Transition Amplitudes

Comparison of relativistic and nonrelativistic formulations

- ${}^3P_2 \rightarrow {}^1S_0$ M2 transition
- Nonrelativistic calculation

$$\frac{1}{5}\sum_{m=-2}^{2}\sum_{p=1}^{2}|\langle f|\varepsilon_{1jl}k\varepsilon_j^{(p)*}V_l|i,m\rangle|^2=\frac{2k^2}{5}\sum_{m=-2}^{2}|\langle f|V_z|i,m\rangle|^2$$

$$\propto\left(\frac{k}{2M_i}\right)^2\left(\frac{e_q m_Q^*}{m_q^*}-\frac{e_Q m_q^*}{m_Q^*}\right)^2\langle r\rangle^2$$

$$\Gamma\propto\left(\frac{k}{M_i}\right)^2\left(\frac{e_q m_Q^*}{m_q^*}-\frac{e_Q m_q^*}{m_Q^*}\right)^2(k\langle r\rangle)^2$$

Comparison of relativistic and nonrelativistic formulations

- Relativistic calculation
  - Momentum conservation
    - assume $\boxed{k = 2m_q V, k = 2m_Q V}, k = 2\bar{M}V,$

$$\eta^{(7)} = \frac{V}{\sqrt{3}}\left(e_q\left(-2m_Q V\right) + e_Q\left(M_i + M_f - 2m_Q\right)V\right)$$ **Rel.**

$$= \frac{k^2}{\sqrt{3}}\left(-\frac{e_q}{2m_q}\frac{m_Q}{\bar{M}} + \frac{e_Q}{2m_Q}\frac{\bar{m}_q}{\bar{M}}\right)\langle r\rangle$$ **NonRel.**

$$\Gamma \propto \left(\eta^{(7)}\right)^2 \propto \left(-\frac{e_q}{2m_q}\frac{m_Q}{\bar{M}} + \frac{e_Q}{2m_Q}\frac{\bar{m}_q}{\bar{M}}\right)^2 \langle r\rangle^2$$

$$\bar{M} = \frac{M_i + M_f}{2}, \bar{m}_q = \frac{M_i + M_f - 2m_Q}{2}$$

Reltivistic -> Nonrelativistic

- Prescription how you get nonrelativistic expression from relativistic one :

Heavy Quark : $V \rightarrow k/(2m_q)$

Light Quark : $V \rightarrow k/(2m_Q)$

Relativistic expression for transition amplitudes

Wave function in the moving frame

$$\langle 0\,|\,q^c(\vec{x},t)\,Q(\vec{y},t)\,|\,P\rangle = \psi_P^{(\xi)}(\vec{x}-\vec{y})\,e^{-iP\cdot X_\xi}$$

(Internal coordinate)

$$X_\xi = \xi x + (1-\xi)y$$

(External coordinate)

$$\mathcal{M}_R = e_q \int d^4x \int d^3y \left\langle P' \left| Q^\dagger Q(y)\, q^{c\dagger} O^\mu q^c(x) \right| P \right\rangle \epsilon_\mu^*\, e^{ik\cdot x}$$
$$-e_Q \int d^4y \int d^3x\, {}_{\alpha\beta} \left\langle P' \left| Q^\dagger O^\mu Q(y)\, q^{c\dagger} q^c(x) \right| P \right\rangle \epsilon_\mu^*\, e^{ik\cdot y}$$

$$\approx e_q \int d^4x \int d^3z\, tr\left[\psi_f^{(\xi)\dagger}\, O^\mu\, \psi_i^{(\xi)}(\vec{z}) \right] \epsilon_\mu^*\, e^{ik\cdot x - i(P-P')\cdot X_\xi}$$
$$-e_Q \int d^4y \int d^3z\, tr\left[(O^\mu)^T \psi_f^{(\xi)\dagger} \psi_i^{(\xi)}(\vec{z}) \right] \epsilon_\mu^*\, e^{ik\cdot y - i(P-P')\cdot X_\xi}$$

Relativistic expression for transition amplitudes

$$= (2\pi)^4 \delta^4(P - P' - k) \int d^3z \, tr\Big[\psi_{P'}^{(\xi)\dagger}(\vec{z})$$

$$\Big\{ e_q e^{-i\vec{k}\cdot\vec{z}\,(1-\xi)}\, O^\mu \;-\; e_Q\, e^{i\vec{k}\cdot\vec{z}\,\xi}\, (O^\mu)\otimes \Big\} \psi_P^{(\xi)}(\vec{z}) \Big]\, \epsilon_\mu^*$$

Relation between w.f.'s in rest and moving frames

- In moving frame, two particles have the same time

$$
\begin{aligned}
& < 0 \,|\, q^c(\vec{x}, t)\, Q(\vec{y}, t) \,|\, P > \\
& = < 0 \,|\, q^c(\vec{x}, t)\, Q(\vec{y}, t) \left\{ \mathcal{G} \,|\, M > \right\} \\
& = < 0 \,|\, \left\{ G_q\, q^c(\vec{x}\,', t'_q) \right\} \left\{ G_Q\, Q(\vec{y}\,', t'_Q) \right\} |\, M >
\end{aligned}
$$

Boost operator

Boost matrix

- In rest frame, they have different times

$$\Delta t' \equiv t'_Q - t'_q = \gamma \vec{V} \cdot (\vec{x} - \vec{y}) \equiv \gamma \vec{V} \cdot \vec{z}$$

Velocity of the meson

Time translation of H.Q. $t'_Q \to t'_q$ Estimate it using $1/m_Q$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{V} \quad (\mathcal{H}_0 = \text{HQ mass term})$$

$$e^{i\mathcal{H}_0\Delta t}\, Q_a(x)\, e^{-i\mathcal{H}_0\Delta t} = \left(e^{-im_Q\Delta t}P_+ + e^{im_Q\Delta t}P_-\right)_{ab} Q_b$$

$$e^{i\mathcal{H}\Delta t}\, Q_a(x)\, e^{-i\mathcal{H}\Delta t} \approx T(\Delta t)_{ab}\, \{ Q_b + i\Delta t\, [\mathcal{V}, Q_b] \}$$

$$T(\Delta t)_{ab} \equiv \left(e^{-im_Q\Delta t}P_+ + e^{im_Q\Delta t}P_-\right)_{ab}$$

$$\psi_P^{(\xi)}(\vec{z})\, e^{-iP\cdot X_\xi} = G_q G_Q T(\Delta t')(1 - i\Delta t' H_Q)\, \psi_M(\vec{z}\,')\, e^{-iMt'_q}$$

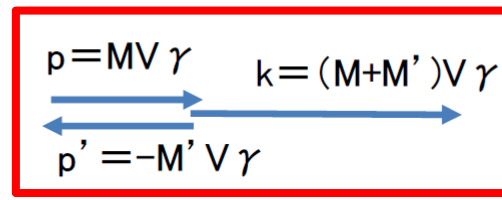
$$P \cdot X_\xi - Mt'_q = M(1-\xi)\Delta t'$$

$$\boxed{\psi_P^{(\xi)}(\vec{z}) = G_q G_Q T(\Delta t')(1 - i\Delta t' H_Q)\, \psi_M(\vec{z}\,')\, e^{iM(1-\xi)\Delta t'}}$$

tt. (time translation) Correction

Estimate Transition amplitude in Breit frame

$$
\begin{aligned}
\mathcal{M}_R \approx\ & (2\pi)^4 \delta^4(P - P' - k) \\
& \times \int d^3x\, tr\Big[\big\{ T(-\Delta t')(1 + i\Delta t' H_Q)\psi_{M'}(x_\perp, \gamma z) \big\}^\dagger \\
& \times \big\{ e_q e^{-ikz(1-\xi)} (G^{-1}\, O^\mu G) - e_Q\, e^{ikz\xi} (G^{-1}\, O^\mu G) \otimes \big\} \\
& \times T(\Delta t')(1 - i\Delta t' H_Q)\psi_M(x_\perp, \gamma z) \Big] e^{i(M+M')\Delta t'(1-\xi)} \epsilon^*_\mu \\
=\ & (2\pi)^4 \delta^4(P - P' - k)\gamma^{-1} \\
& \times \int d^3x\, tr\Big[\big\{ T(-\Delta t')(1 + i\Delta t' H_Q)\psi_{M'}(\vec{x}) \big\}^\dagger \\
& \times \big\{ e_q (G^{-1}\, O^\mu G) - e_Q\, e^{i\tilde{k}z} (G^{-1}\, O^\mu G) \otimes \big\} \\
& \times T(\Delta t')(1 - i\Delta t' H_Q)\psi_M(\vec{x}) \Big] \epsilon^*_\mu
\end{aligned}
$$



ξ independent

$$\tilde{k} = (M + M')V$$

Estimate (continued)

where

term due to boosting

$$G^{-1}O^1G = G^{-1}\rho_1\sigma_1 G = \gamma(\rho_1\sigma_1 - iV\sigma_2)$$

$$H_Q = \beta_q\beta_Q S(r) + V_c(r) + \rho_{Q1} W$$

$$W = -\vec{\sigma}_Q \cdot \vec{p} - \frac{1}{2}V_c(r)\left[\vec{\alpha}_q \cdot \vec{\sigma}_Q + (\vec{n}\cdot\vec{\alpha}_q)(\vec{n}\cdot\vec{\sigma}_Q)\right]$$

$$S(r) = \frac{r}{a^2}, \quad V_c(r) = -\frac{4\alpha_s}{3r}$$

w.f. correction due to $1/m_Q$

- wf. corr.

$$\psi_M(\vec{x}) = \left(1 + \frac{1}{2m_Q}\rho_{Q1} W \otimes\right) \frac{\sqrt{2M}}{\sqrt{4\pi} r} \begin{pmatrix} u(r) \\ -i(\vec{n}\cdot\vec{\sigma})v(r) \end{pmatrix} \frac{y}{\sqrt{2}}$$

- Dirac equation satisfied by $u(r), v(r)$

$$\begin{pmatrix} m_q + S + V_c & -\frac{d}{dr} + \frac{l}{r} \\ \frac{d}{dr} + \frac{l}{r} & -m_q - S + V_c \end{pmatrix} \begin{pmatrix} u(r) \\ v(r) \end{pmatrix} = E \begin{pmatrix} u(r) \\ v(r) \end{pmatrix}$$

Relativistic T.A. in Breit frame

$$\left\langle P' \left| j^{(em)1} \right| P \right\rangle$$

$$\approx \int d^3x \, \frac{1}{2} tr \Big[(-e_q) \psi^\dagger_{P'} \Big\{ (\rho_1 \sigma_1 - iV\sigma_2) - iVz\rho_1\sigma_1 (S\beta_q + V_c)$$

$$+ \cdots \Big\} \psi_P e^{iqz} - ie_Q V \psi^\dagger_{P'} \sigma_{Q2} \otimes \psi_P e^{ipz}$$

$$+ e_Q \left(-iVz + \frac{1}{2m_Q} \right) \Big\{ \psi^\dagger_{P'} (\sigma_{Q1} W \otimes \psi_P) + \cdots \Big\} e^{i\tilde{k}z} \Big]$$

$$q = -\frac{m_Q}{\overline{M}} \tilde{k} \quad p = \frac{\overline{m}_q}{\overline{M}} \tilde{k} \quad \tilde{k} = 2\overline{M}V$$

$$(\overline{M} = (M_1 + M_2)/2, \quad \overline{m}_q = \overline{M} - m_Q)$$

Nonrelativistic limit

$m_Q \gg m_q \gg \omega$ (photon energy)

- (ex1) M1 Transition $1^-({}^3S_1) \to 0^-({}^1S_0), \quad l_1 = l_2 = -1$

$$\mathcal{A}^1 \approx i\varepsilon_2 \int dr \Big[e_q \{ -f'(qr)(u_2 v_1 + v_2 u_1) + V f(qr) u_2 u_1 \}$$

$$+ e_Q V f(pr) u_2 u_1 \Big] + \text{(wf.corr+tt.corr)}$$

$$\boxed{\int dr\, r(u_2 v_1 + v_2 u_1) \approx \frac{l_1 + l_2 - 1}{2m_q} \int dr\, u_2 u_1 \quad S(r) = 0}$$

$$\approx i\varepsilon_2 V \left(e_q \frac{m_Q}{m_q} + e_q + e_Q \right) + \text{(wf.corr+tt.corr)}$$

$$\mathcal{A}^1 \equiv \frac{\langle P' | j^{(em)1} | P \rangle}{2\sqrt{MM'}}$$

(L.Q.) $\quad V \to k/(2m_q)$

(H.Q.) $\quad V \to k/(2m_Q)$

$$\mathcal{A}^1 \to i\varepsilon_2 k \left(\frac{e_q}{2m_q} + \frac{e_Q}{2m_Q} \right)$$

Neglecting constituents form a bound state -> coincide with nonrela. result

Other examples of M1,2 Transitions

- M1 transition

$$1^{+}(''^{3}P_{1}'') \to 0^{+}(^{3}P_{0}), \quad l_1 = l_2 = 1$$

$$\mathcal{A}^{1} \approx i\varepsilon_2 V\left(+ e_q \frac{m_Q}{3m_q} - \underline{\frac{1}{3}e_q + e_Q} \right) + (\text{wf.corr+tt.corr})$$

$$i\varepsilon_2 \left(- \frac{e_q}{6m_q} + \frac{e_Q}{2m_Q} \right) k$$

$$\boxed{\begin{array}{ll} \text{(L.Q.)} & V \to k/(2m_q) \\ \text{(H.Q.)} & V \to k/(2m_Q) \end{array}}$$

- M1 transition

$$1^{+}(''^{1}P_{1}'') \to 0^{+}(^{3}P_{0}), \quad \mathcal{A}^{1} \to i\varepsilon_2 \left(\frac{2\sqrt{2}}{3} \frac{e_q}{2m_q} k \times O.I. \right)$$

Other examples of M1,2 Transitions

- M2 transition

$$1^+(^3P_2) \to 0^-(^1S_0), \qquad \mathcal{A}^1 \to \varepsilon_{23}\frac{1}{\sqrt{3}}\left(-\frac{e_q}{2m_q}\frac{m_Q}{M}+\frac{e_Q}{2m_Q}\frac{m_q}{M}\right)k^2\langle r\rangle$$

$$1^+(^3D_1) \to 0^-(^1S_0), \qquad \mathcal{A}^1 \to i\varepsilon_2\frac{\sqrt{2}}{30}\left\{\frac{e_q}{2m_q}\left(\frac{m_Q}{M}\right)^2+\frac{e_Q}{2m_Q}\left(\frac{m_q}{M}\right)^2\right\}k^3\langle r^2\rangle$$

- Adding a term due to boosting and replacing the velocity as $\boxed{\begin{array}{ll}\text{(L.Q.)} & V \to k/(2m_q) \\ \text{(H.Q.)} & V \to k/(2m_Q)\end{array}}$,

$$\boxed{\text{relativistic exppression = nonrelativistic expression}}$$

Electric transition

- Ex. E1 transition $0^+(^3P_0) \to 1^-(^3S_1), \quad l_1 = 1,\ l_2 = -1$

$$\mathcal{A}^1 \approx i\varepsilon_1 \int dr \left\{ e_q(f u_2 v_1 - f'' v_2 u_1 + V f' u_2 u_1) - e_Q V f'(pr) u_2 u_1 \right\}$$
$$+ \frac{e_Q}{2m_Q} \int dr \left[(f + f'') u_2 u_1' - \frac{2f''}{r} u_2 u_1 \right.$$
$$\left. + \frac{V_c}{2} \{(3f + f'') u_2 v_1 - (f + 3f'') v_2 u_1\} \right] (\tilde{k}r) + \text{tt.corr}$$

- where we have used potential-independent identity

$$\int dr \, \{(l_1 - l_2 + 1) u_2 v_1 + (l_1 - l_2 -) v_2 u_1\} = k \langle r \rangle$$

Result of Electric transition

$$\mathcal{A}^1 \stackrel{S=0}{\rightarrow} i\varepsilon_1 \frac{1}{3}\left(e_q + e_Q \frac{m_q}{m_Q}\right) k \langle r \rangle$$

- Under the conditions
  - Up to O(k) for boost effects and time translation
  - First order correction to wave function
  - S(r)=0 (scalar confining potential)

Comparison with non-relativistic calculations

in units of keV

Process	Γ_{th}(this work)	Ref.1	Ref.2	Ref.3
$D^{*0} \to D^0 + \gamma$	11	33.5	32	-
$D^{*+} \to D^+ + \gamma$	0.56	1.63	1.8	-
$D_0^*(2400)^0 \to D^{*0} + \gamma$	282	-	304	274
$D_0^*(2400)^+ \to D^{*+} + \gamma$	51	-	17	28
$D_s^{*+} \to D_s^+ + \gamma$	0.72	0.43	0.2	-
$D_{s0}^*(2317)^+ \to D_s^{*+} + \gamma$	26	1.74	1.0	1.9
$D_{s1}(2460)^+ \to D_s^+ + \gamma$	11	5.08	-	15
$D_{s1}(2460)^+ \to D_s^{*+} + \gamma$	101	4.66	-	5.6

$$\mu \propto \frac{e_d m_c^* + e_c m_d^*}{m_c^* m_d^*} = \frac{e_d}{m_d^*}\left(1 - \frac{2m_d^*}{m_c^*}\right)$$

$$d \propto \frac{e_s m_c^* + e_c m_s^*}{m_c^* + m_s^*} = \frac{e_s m_c^*}{m_c^* + m_s^*}\left(1 - \frac{2m_s^*}{m_c^*}\right)$$

Ref.1…Bardeen et al., **PRD 68, 054024 (2003)**

Ref.2…Close and Swanson, **PRD 72, 094004 (2005)**

Ref.3…Close, Godfrey, Swanson, **PRD 72, 054029 (2005)**

D

TABLE I: Numerical evaluation of the electromagnetic decay widths of excited D mesons

Initial State($^{2S+1}L_J$)	Final State	k_R(MeV/c)	Γ_{th}(keV)	Γ_{exp}(keV)	$(\gamma/\pi^0)_{th}$	$(\gamma/\pi^0)_{exp}$
$D^{*0}(^3S_1)$	$D^0\gamma$	137.2	11		0.26	0.62
$D^{*+}(^3S_1)$	$D^+\gamma$	135.8	0.56	1.5	0.019	0.052
$D_0^*(2400)^0(^3P_0)$	$D^{*0}\gamma$	319.7	282		5.6×10^{-3}	
$D_0^*(2400)^+(^3P_0)$	$D^{*+}\gamma$	360.6	51		8.4×10^{-4}	
$D_1(2430)^0$	$D^0\gamma$	497.1	148			
("$^3P_1$")	$D^{*0}\gamma$	383.7	121		3.1×10^{-3}	
	$D_0^*(2400)^0\gamma$	73.8	0.19			
$D_1(2430)^+$	$D^+\gamma$	493.4	16			
("$^3P_1$")	$D^{*+}\gamma$	380.9	104			
	$D_0^*(2400)^+\gamma$	23.9	6.9×10^{-3}			
$D_1(2420)^0$	$D^0\gamma$	493.3	559			
("$^1P_1$")	$D^{*0}\gamma$	379.7	159		0.11	
	$D_0^*(2400)^0\gamma$	69.3	0.22			
$D_1(2420)^+$	$D^+\gamma$	490.5	20			
("$^1P_1$")	$D^{*+}\gamma$	377.9	41		2.9×10^{-2}	
	$D_0^*(2400)^+\gamma$	20.3	1.6×10^{-5}			
$D_2^*(2460)^0$	$D^0\gamma$	524.1	4.2		1.2×10^{-3}	
($^3P_2$)	$D^{*0}\gamma$	412.2	811		0.58	
$D_2^*(2460)^+$	$D^+\gamma$	519.6	0.82		2.4×10^{-4}	
($^3P_2$)	$D^{*+}\gamma$	408.7	26		0.020	
$D(2760)^0$	$D^0\gamma$	752.4	2.1		8.8×10^{-4}	
($^3D_1$)	$D^{*0}\gamma$	652.8	141		0.34	
$D(2760)^+$	$D^+\gamma$	749.2	22		9.2×10^{-3}	
($^3D_1$)	$D^{*+}\gamma$	650.4	41		0.11	

Ds

TABLE II: Numerical evaluation of the electromagnetic decay widths of excited D_s mesons

Initial State($^{2S+1}L_J$)	Final State	k_R(MeV/c)	Γ_{th}(keV)	$(\gamma/\pi^0)_{th}$	$(\gamma/\pi^0)_{exp}$
$D_s^{*+}(^3S_1)$	$D_s^+\gamma$	138.9	0.72	90	16
$D_{s0}^*(2317)^+(^3P_0)$	$D_s^{*+}\gamma$	196.4	26		
$D_{s1}(2460)^+$	$D_s^+\gamma$	442.1	11		
(”$^3P_1$”)	$D_s^{*+}\gamma$	322.8	101	25.9	
	$D_{s0}^*(2317)^+\gamma$	137.7	1.2		
$D_{s1}(2536)^+$	$D_s^+\gamma$	503.5	27		
(”$^1P_1$”)	$D_s^{*+}\gamma$	387.8	48		
	$D_{s0}^*(2317)^+\gamma$	208.3	4.2×10^{-3}		
$D_{s2}(2573)^+$	$D_s^+\gamma$	533.2	0.62		
(”$^3P_2$”)	$D_s^{*+}\gamma$	419.1	36		
$D_s(2818)^+$	$D_s^+\gamma$	721.5	16		
($^3D_1$)	$D_s^{*+}\gamma$	617.3	50		

B

TABLE III: Numerical evaluation of the electromagnetic decay widths of excited B mesons

Initial State($^{2S+1}L_J$)	Final State	k_R(MeV/c)	Γ_{th}(keV)	$(\gamma/\pi^0)_{th}$
$B^{*0}(^3S_1)$	$B^0\gamma$	45.4	4.6×10^{-2}	
$B^{*-}(^3S_1)$	$B^-\gamma$	45.7	0.11	
$B_0^*(5590)^0(^3P_0)$	$B^{*0}\gamma$	258.6	24	1.4×10^{-3}
$B_0^*(5590)^-(^3P_0)$	$B^{*-}\gamma$	258.6	47	
$B_1(5646)^0$	$B^0\gamma$	354.6	12	
("$^3P_1$")	$B^{*0}\gamma$	311.8	14	7.4×10^{-4}
	$B_0^*(5590)^0\gamma$	55.7	3.8×10^{-3}	
$B_1(5646)^-$	$B^-\gamma$	354.9	26	
("$^3P_1$")	$B^{*-}\gamma$	311.8	55	
	$B_0^*(5590)^-\gamma$	55.7	2.2×10^{-5}	
$B_1(5721)^0$	$B^0\gamma$	426.7	51	
("$^1P_1$")	$B^{*0}\gamma$	384.4	22	5.8×10^{-3}
	$B_0^*(5590)^0\gamma$	131.8	0.19	
$B_1(5721)^-$	$B^-\gamma$	427.0	108	
("$^1P_1$")	$B^{*-}\gamma$	384.4	70	
	$B_0^*(5590)^-\gamma$	131.8	0.70	
$B_2^*(5747)^0$	$B^0\gamma$	444.8	1.3	4.1×10^{-4}
($^3P_2$)	$B^{*0}\gamma$	402.7	77	2.7×10^{-2}
$B_2^*(5747)^-$	$B^-\gamma$	445.1	5.3	
($^3P_2$)	$B^{*-}\gamma$	402.7	169	
$B(5985)^0$	$B^0\gamma$	663.9	0.18	6.2×10^{-5}
($^3D_1$)	$B^{*0}\gamma$	623.5	21	1.9×10^{-2}
$B(5985)^-$	$B^-\gamma$	664.2	5.2	
($^3D_1$)	$B^{*-}\gamma$	623.5	27	

Bs

TABLE IV: Numerical evaluation of the electromagnetic decay widths of excited B_s mesons

Initial State($^{2S+1}L_J$)	Final State	k_R(MeV/c)	Γ_{th}(keV)	$(\gamma/\pi^0)_{th}$
$B_s^*(^3S_1)$	$B_s\gamma$	46.3	4.2×10^{-2}	
$B_{s0}^*(5615)^0(^3P_0)$	$B_s^*\gamma$	198.6	30	
$B_{s1}(5679)^0$	$B_s\gamma$	304.1	16	
("$^3P_1$")	$B_s^*\gamma$	260.0	20	11
	$B_{s0}^*(5615)^0\gamma$	63.6	1.1×10^{-2}	
$B_{s1}(5830)^0$	$B_s\gamma$	444.7	53	
("$^1P_1$")	$B_s^*\gamma$	401.7	23	
	$B_{s0}^*(5615)^0\gamma$	210.5	0.55	
$B_{s2}^*(5840)^0$	$B_s\gamma$	454.2	1.1	
("$^3P_2$")	$B_s^*\gamma$	411.3	80	
$B_s(6025)^0$	$B_s\gamma$	622.7	0.23	
($^3D_1$)	$B_s^*\gamma$	581.1	20	

Summary

- In general,
  - Magnetic transition Amplitude of a bound state in the naïve nonrelativistic limit $\neq$
  - A sum of nonrelativistic Transition Amplitudes
- Electric transition amplitude agree with each other when S(r)=0
- Is nonrelativistic calculation of magnetic transition amplitude like hydrogen atom, etc. correct?
- We have numerically given relativistic radiative decay widths of D, Ds, B, Bs

Hope

- We hope to do experiments which distinguish relativistic and nonrelativistic calculations.

Thanks for your attention

Other radiative decays

- Wave functions in the **moving frame** were related to those in the **rest frame**, and **formulas for the radiative decay widths** were expressed by the radial wave function in the rest frame.
- The **recoil effect** of the heavy-light mesons was taken into account.
- **1st order corrections** of wave functions in **$1/m_Q$ expansion** were also taken into account.
- For charged D*or D_s*, sizable decay widths were obtained by including the 1st order correction in $1/m_Q$ expansion.

$$\Gamma(D^{*0} \to D^0 + \gamma) = 11\, keV \text{ [6.4 wo corr.] } (\sim 30\, keV \text{ non-rel.})$$

$$\Gamma(D^{*+} \to D^+ + \gamma) = 0.56\, keV\ [4.9 \times 10^{-4}]\ (\sim 1.7\, keV)$$

$$\Gamma(D_s^{*+} \to D_s^+ + \gamma) = 0.72\, keV\ [2.8 \times 10^{-3}]\ (\sim 0.3\, keV)$$

- For D_{sJ}, large decay widths were obtained compared with non-relativistic works.

$$\Gamma(D_{s0}^*(2317)^+ \to D_s^{*+} + \gamma) = 26\, keV\ (\sim 1\, keV \text{ non-rel.})$$

$$\Gamma(D_{s1}(2460)^{*+} \to D_s^+ + \gamma) = 11\, keV\ (\sim 5\, keV)$$

$$\Gamma(D_{s1}(2460)^{*+} \to D_s^{*+} + \gamma) = 100\, keV\ (\sim 5\, keV)$$