Remarks on resonances

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What is a resonance ?

The S–matrix is characterized by its analytic structure (in s).

$\rightarrow$ branch points (and the corresponding cuts)

$\longrightarrow$ at each channel opening for $s > s^{\text{thres}}$: right hand cut

$\longrightarrow$ in the crossed channels for $s < s^{\text{thres}}$: left hand cut

$\longrightarrow$ inside the un-physical sheet (see below)

$\rightarrow$ poles on the physical sheet: bound states

$\longrightarrow$ only for real $s < s_{\text{min}}^{\text{thres}}$ (no other singul. allowed here)

$\rightarrow$ poles on the un-physical sheet (closest to the physical one)

$\longrightarrow$ for real $s < s_{\text{min}}^{\text{thres}}$: virtual state

$\longrightarrow$ for complex s: resonance

For bound states

Weinberg PR 130 (1963) 776

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix} ,$$

here $|\psi_0\rangle$ = elementary state and $|h_1 h_2\rangle$ = two–hadron cont., then

λ^2 equals probability to find the bare state in the physical state

$\rightarrow$ λ^2 is the quantity of interest!

After some algebra we get for the residue at the pole

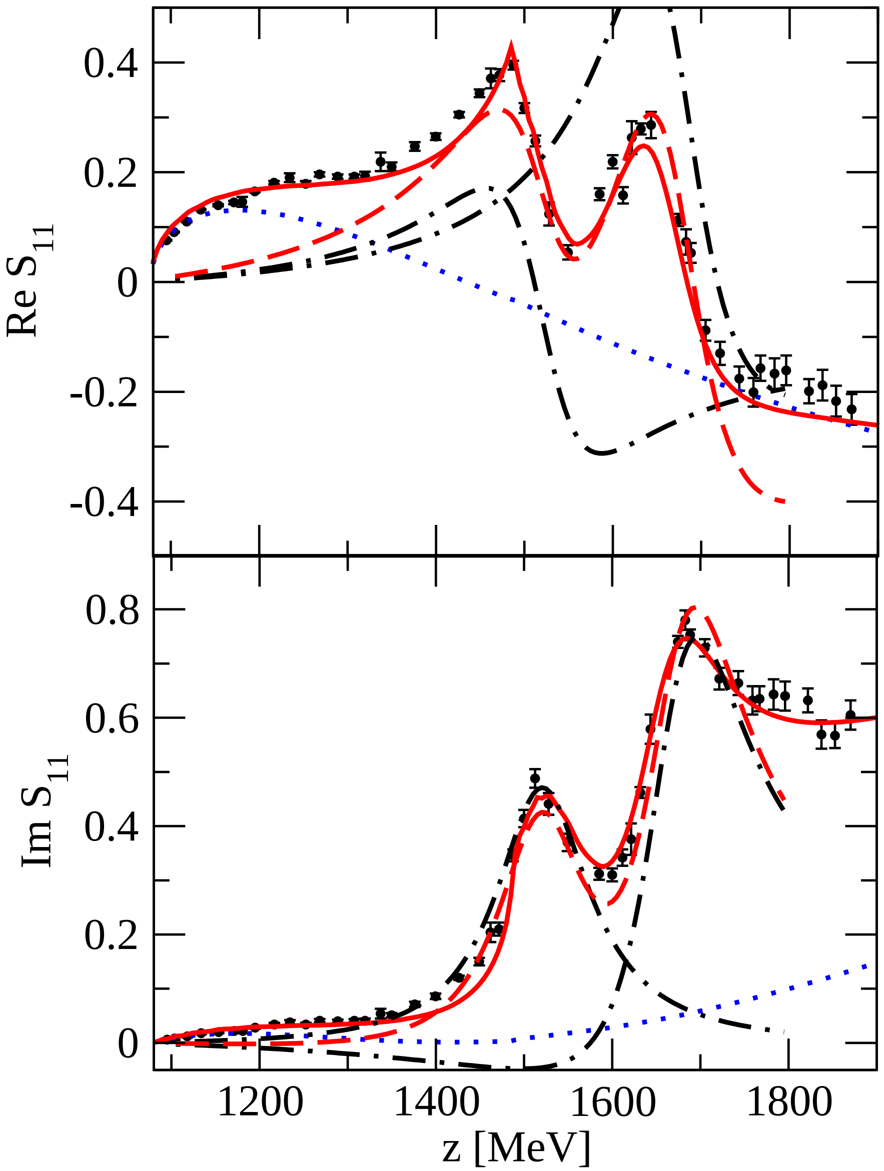
$$\boxed{g_{\rm eff}^2 = 2(1-\lambda^2)\sqrt{\epsilon/m} \le 2\sqrt{\epsilon/m}}$$

For bound state low E amplitude fixed in hh channel!

Picture not changed by far away threshold Baru et al. PLB586 (2004) 53

Equivalent to, e.g., Morgan NPA543 (1992) 63; Törnqvist PRD51 (1995) 5312

More on Resonances



A resonance is uniquely and unambiguously characterized by its pole position and residues

$$\boxed{BR_{\rm pole}(i) = \frac{|{\rm res}_i|^2 \sigma_i}{2\,|m_{\rm pole}|\,(\Gamma_{\rm pole}/2)}}$$

with σ_i =phase space channel i

used e.g. for $f_0(500) \to \gamma\gamma$

Thus, naively one may write

$$T_{ij} = -\sum_r \frac{{\rm res}_{\rm i}^{\rm r}\,{\rm res}_{\rm j}^{\rm r}}{s - s_r}$$

which is a sum of Breit-Wigners

But, in general this is wrong!

Reason I: Unitarity

For one channel only one has: $\text{Im}(T) = \sigma |T|^2$

where $\sigma = \sqrt{1 - 4m^2/s}$ is the phase space. Then for

$$T = -\frac{\text{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1} - \frac{\text{res}_{(2)}{}^2}{s - M_2^2 + iM_1\Gamma_2}$$

we get using $\sigma\,\text{res}_{(\text{i})}{}^2 = M_i\Gamma_i$ (implies $\text{res}_{(i)}$ real)

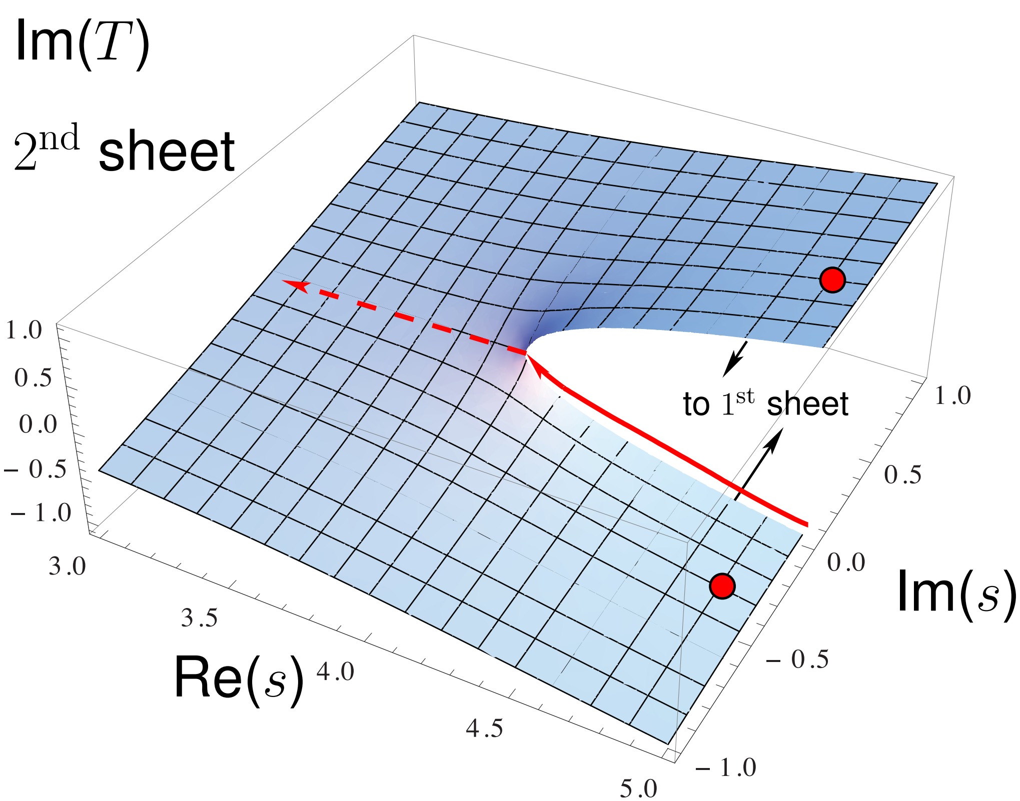
$$\begin{aligned}
\text{Im}(T) &= \frac{\text{res}_{(1)}{}^2\Gamma_1\text{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}{}^2\Gamma_2\text{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2} \\
\sigma|T|^2 &= \frac{\text{res}_{(1)}{}^2\Gamma_1\text{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}{}^2\Gamma_2\text{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2} \\
&\quad + 2\sigma\text{Re}\left(\frac{\text{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1}\frac{\text{res}_{(2)}{}^2}{s - M_2^2 - iM_1\Gamma_2}\right)
\end{aligned}$$

Interference term violates unitarity!

Reason II: Analyticity

$\rightarrow$ For real $s < s_{\rm min}^{\rm thres}$, S is real $\rightarrow$ Branchpoint at $s = s^{\rm thres}$

$\rightarrow$ $S(s^*) = S^*(s) \longrightarrow$ pole at s implies pole at s^*



For narrow resonances:

In resonance region:
only lower pole matters

At threshold:
both poles important!

For broad resonances:

always both important

Keep track of the cuts!

Structure of S–matrix

C.H., Pelaez, Rios, PLB739(2014) 375

lets assume there were a pole at $k = k_0 - i\gamma$, then

$\rightarrow$ there is also a pole at $k = -k_0 - i\gamma$ (analyticity)

$\rightarrow$ there are zeros at $k = \pm k_0 + i\gamma$ (unitarity)

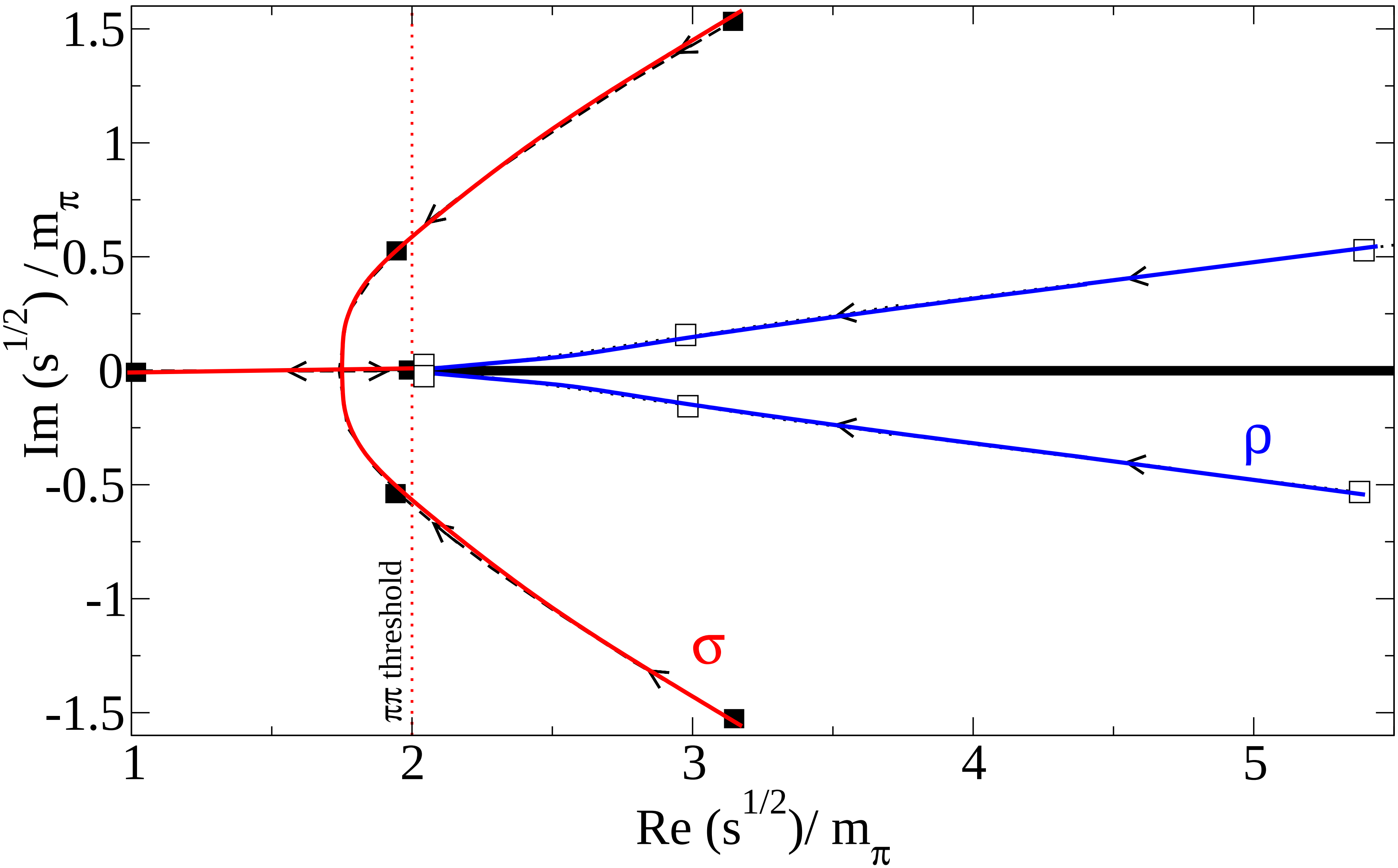
and we get for the simplest S–matrix with one pole

$$\boxed{\begin{aligned} S(k) &= \frac{(k-k_0-i\gamma)(k+k_0-i\gamma)}{(k-k_0+i\gamma)(k+k_0+i\gamma)} \\ &= \frac{s-s_0-i\gamma\sqrt{s-4m^2}}{s-s_0+i\gamma\sqrt{s-4m^2}} \end{aligned}}$$

with $s = 4(m^2 + k^2)$ and $s_0 = 4(k_0^2 + \gamma^2 + m^2)$ and $\gamma = \gamma_0 k^{2L}$.

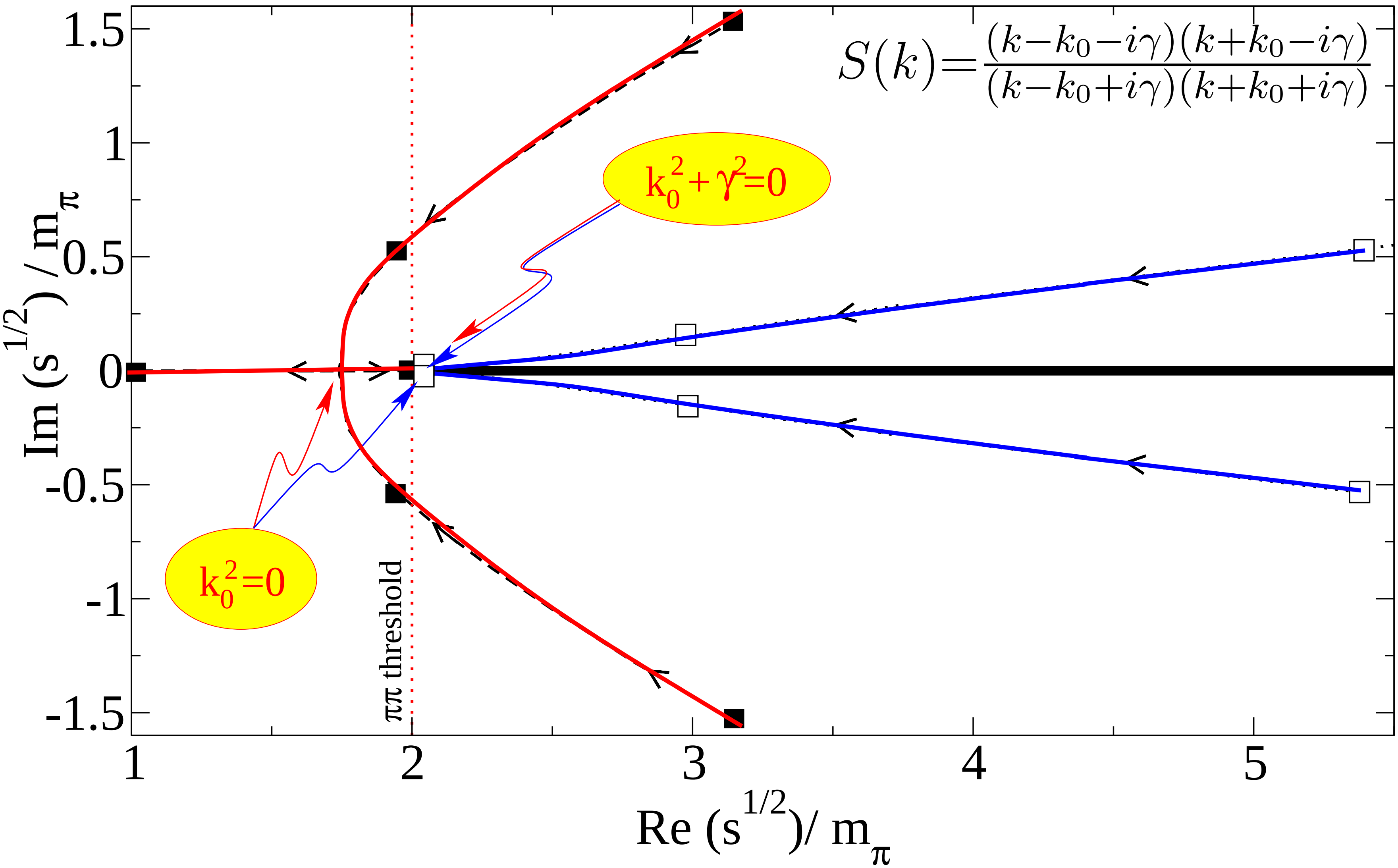
Pole Movement

On the second sheet (as k_0^2 is reduced):



Pole Movement

On the second sheet (as k_0^2 is reduced):



Thus ...

For a meson–meson molecule: necessary condition that follows from Weinberg criterion:

$$\gamma|_{k_0^2=0} \geqslant \Lambda_{QCD}$$

Then poles located very assymmetric.

To have a meson–meson molecule pole above threshold:

k^2/s–dependent interaction needed

$\rightarrow$ natural for $\pi\pi$–interaction in scalar–isoscalar channel:

Adler zero at $s = m_\pi^2/2$ S. L. Adler, Phys. Rev. 137, B1022 (1965).

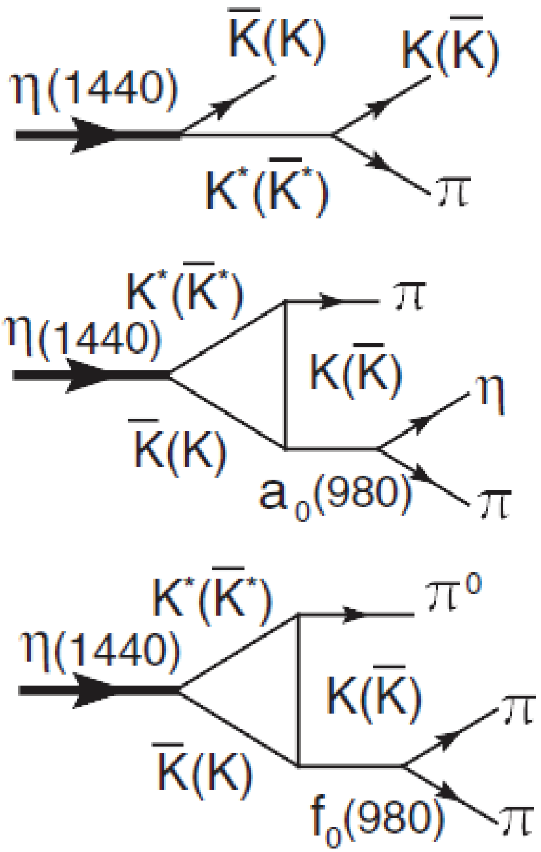
$\rightarrow$ follows naturally from coupled channel dynamics

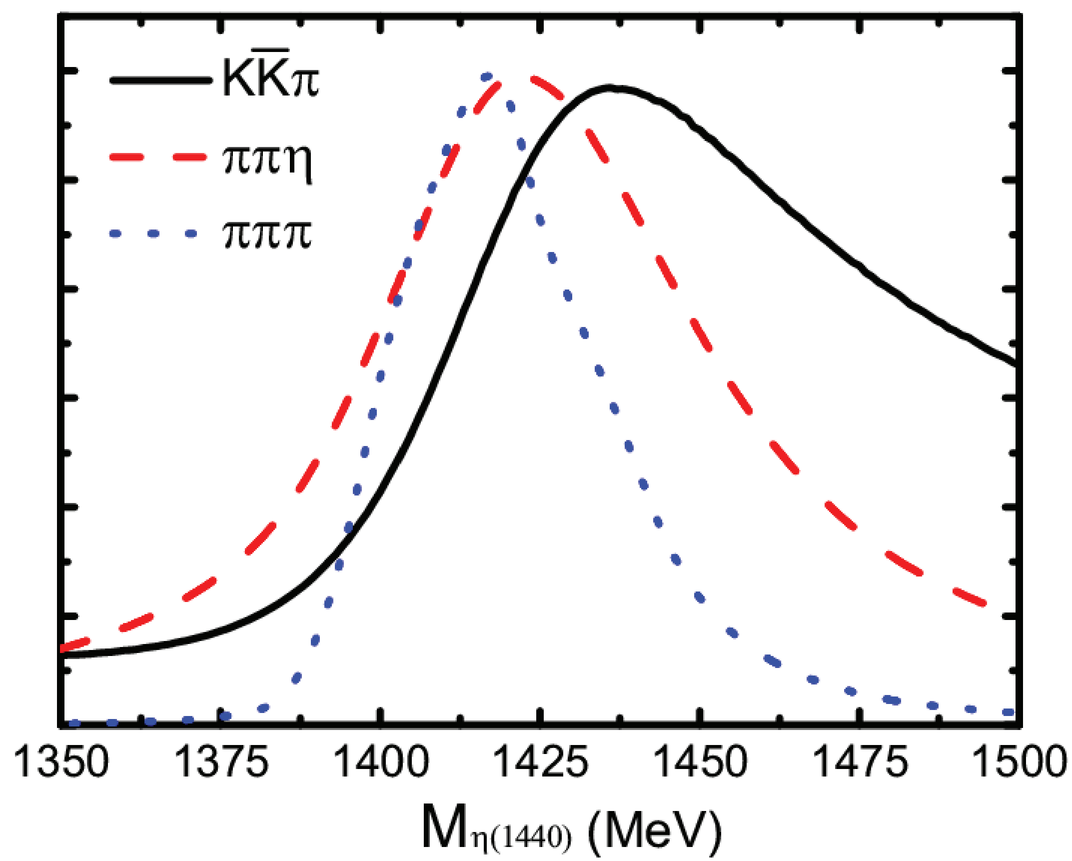
T. Hyodo, Int.J.Mod.Phys. A28 (2013) 1330045

Only poles are physical

Line shapes and peak positions are channel dependent

Only pole–locations are physical!

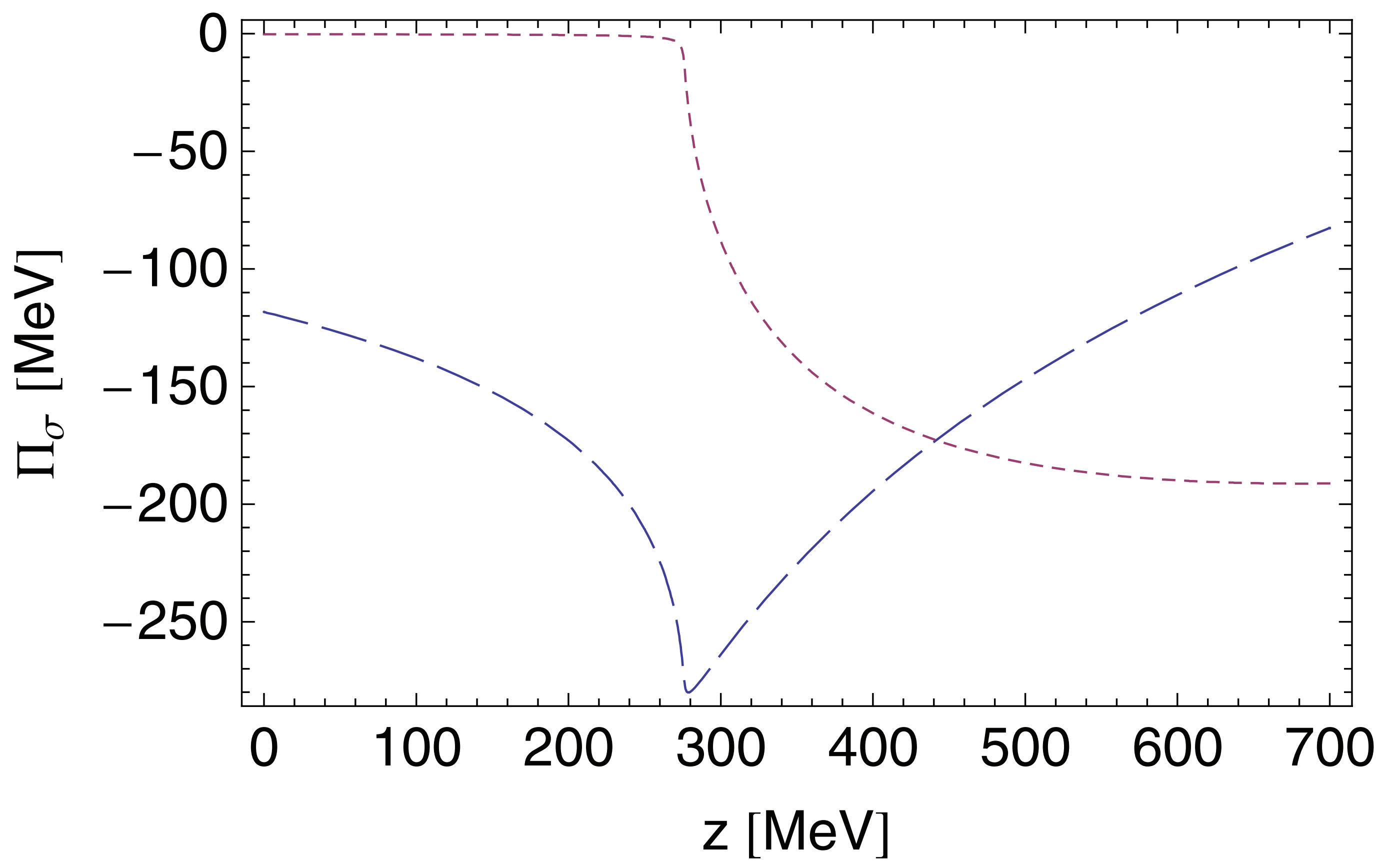




J. J. Wu et al., PRL**108**, 081803 (2012)

Crossing a threshold

M. Döring et al. NPA 829(2009)170

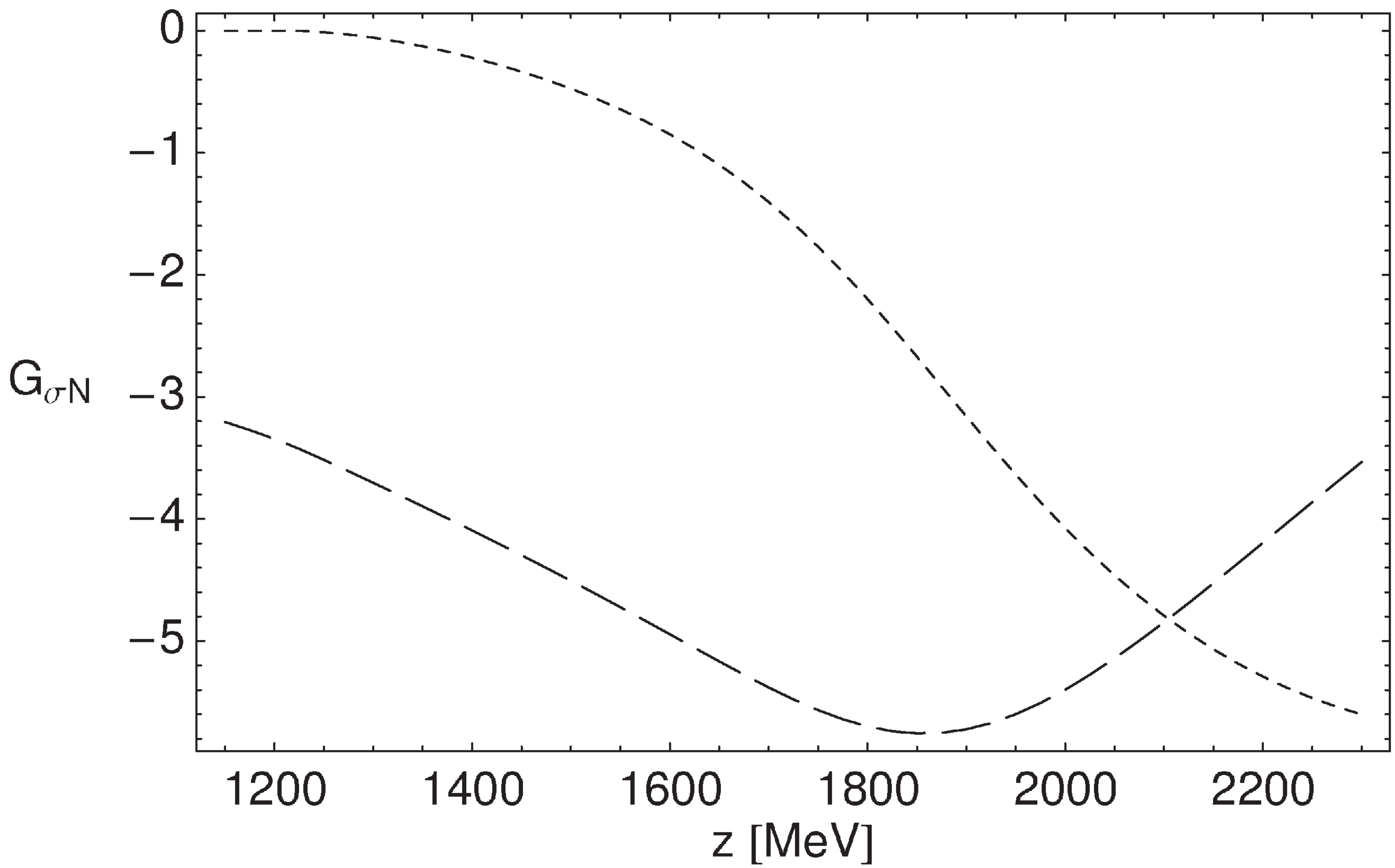


→ The amplitude is non–analytic in s at the threshold

→ The imaginary part rises as $\sqrt{1 - s^{\text{thres}}/s}^{(2L+1)}$
very steep for s–waves; no cusp for $L > 0$

... with unstable constituents

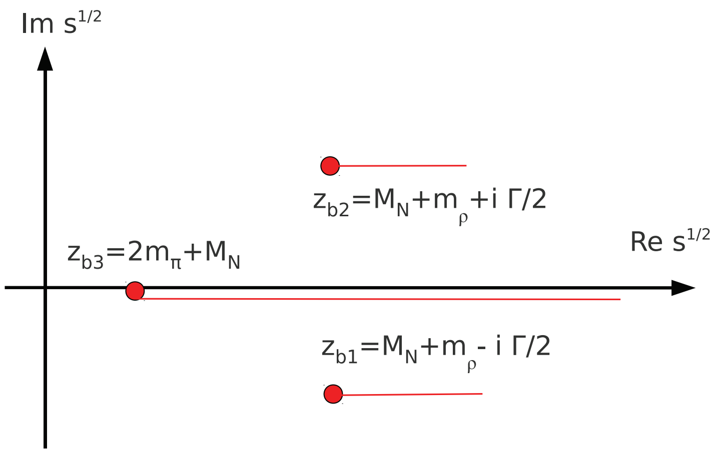
M. Döring et al. NPA 829(2009)170

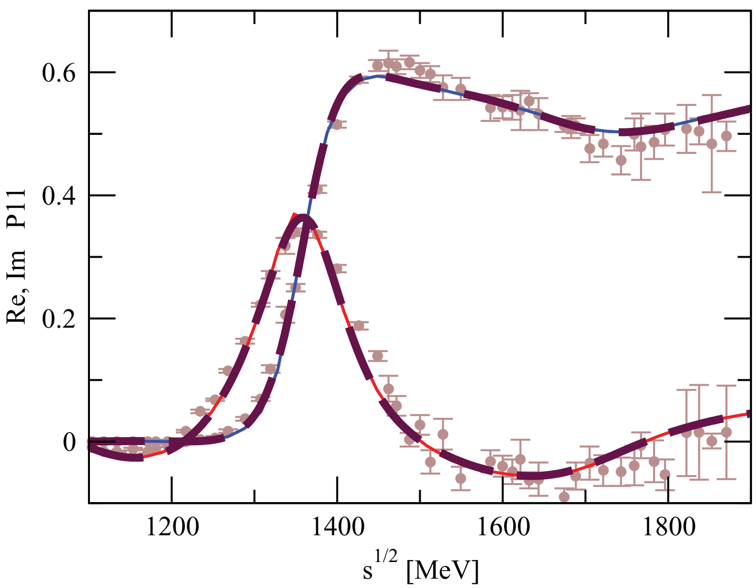


→ Non–analyticity in s is moved into complex plane

→ The shorter the life time the weaker the structure

Not every bump is a resonance ...





In the complex plane there are

- $\rightarrow$ poles = states
- $\rightarrow$ branch points
  - $\triangleright$ on the real axis = channel with stable particles
  - $\triangleright$ in the complex plane = channel with un-stable particles
- $\rightarrow$ triangle-singularities

See talk by Xiao-Hai Liu

Analysis without ρN–channel gives pole at 1698-130 i MeV

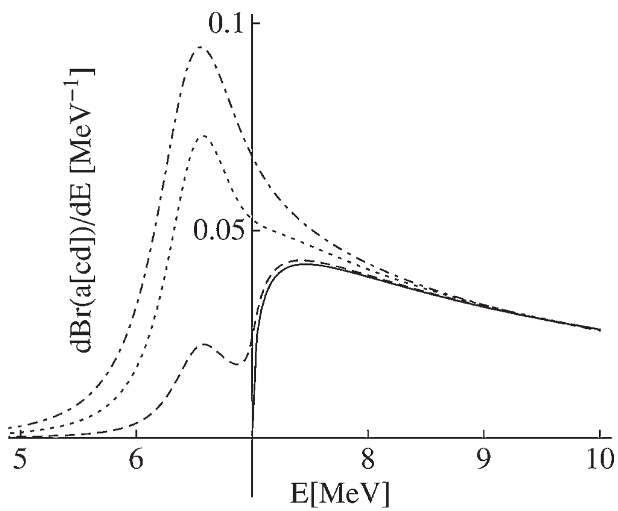
S. Ceci et al. PRC84 (2011) 015205

Lineshapes of near threshold states

Braaten & Lu PRD76 (2007) 094028; C.H. et al. PRD81(2010)094028

Direct channel | Inelastic channel

bound state:

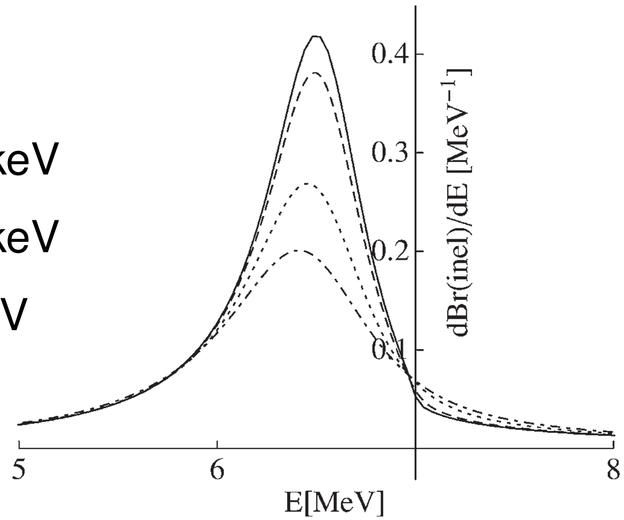


$\Gamma = 0$

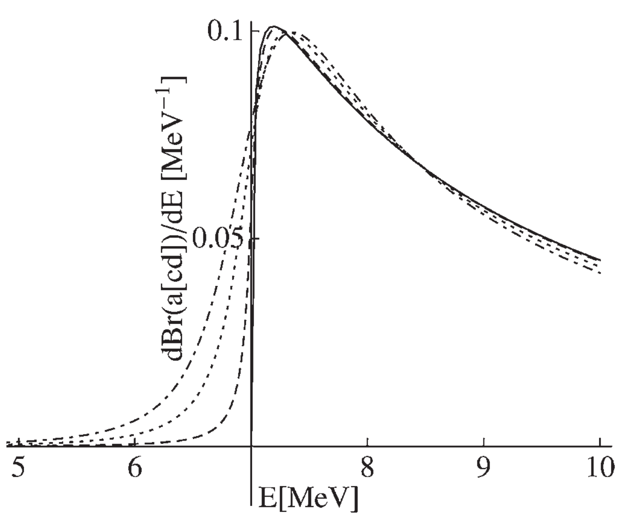
$\Gamma = 100$ keV

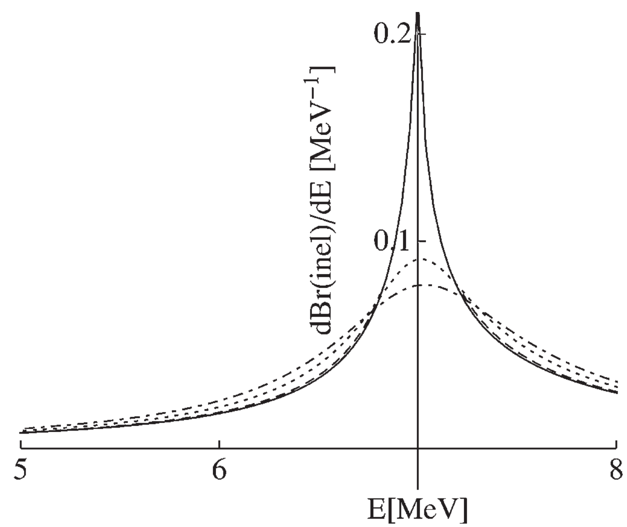
$\Gamma = 500$ keV

$\Gamma = 1$ MeV



virtual state:





Only molecules can appear as virtual states!

Summary

Physical states show up as poles in the S–matrix

Poles appear as

$\rightarrow$ on the physical sheet (bound states)

$\rightarrow$ or on the unphysical sheets (virtual states/resonances)

In addition there are

$\rightarrow$ branch points (on the real axis or in the unphysical sheets)

$\rightarrow$ triangle singularities ...

To understand QCD in the non–perturbative regime

its singularity structure needs to be mapped out and understood