



# How to distinguish an elementary particle from a molecular state

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- ▶ Definition of a “molecule” and an “elementary” particle?
  - ▶ Only unambiguous in Quantum mechanics (or quantum mechanical scattering theory)!
1. Morgan's pole counting rule ( D. Morgan, Nucl. Phys. A543, 632(1992).)  
Many analyses on XYZ states
  2. Some remarks on the properties of  $f_0(600)$  and  $f_0(980)$ .

It has been a longstanding problem for particle physicists to judge whether a particle is elementary or composite, from experimentally known cross-section or phase shift data.

In some special cases, it is however possible to solve the problem model independently, by counting the number of poles near a threshold, in an  $s$ -wave amplitude.

Any scattering amplitude can always be written as,

$$T = \frac{1}{M - ik} \quad (1)$$

and (resonance) poles correspond to the zeros of  $M - ik$ , and  $M$  can be expanded in powers of  $k^2$ ,

$$M(k^2) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}}^2 k^2 + o(k^4) \quad (2)$$

where  $a$  the scattering length,  $r_{\text{eff}}$  the effective range parameter. When  $r_{\text{eff}} \simeq R$ ,  $M - ik$  only contains one zero near threshold.

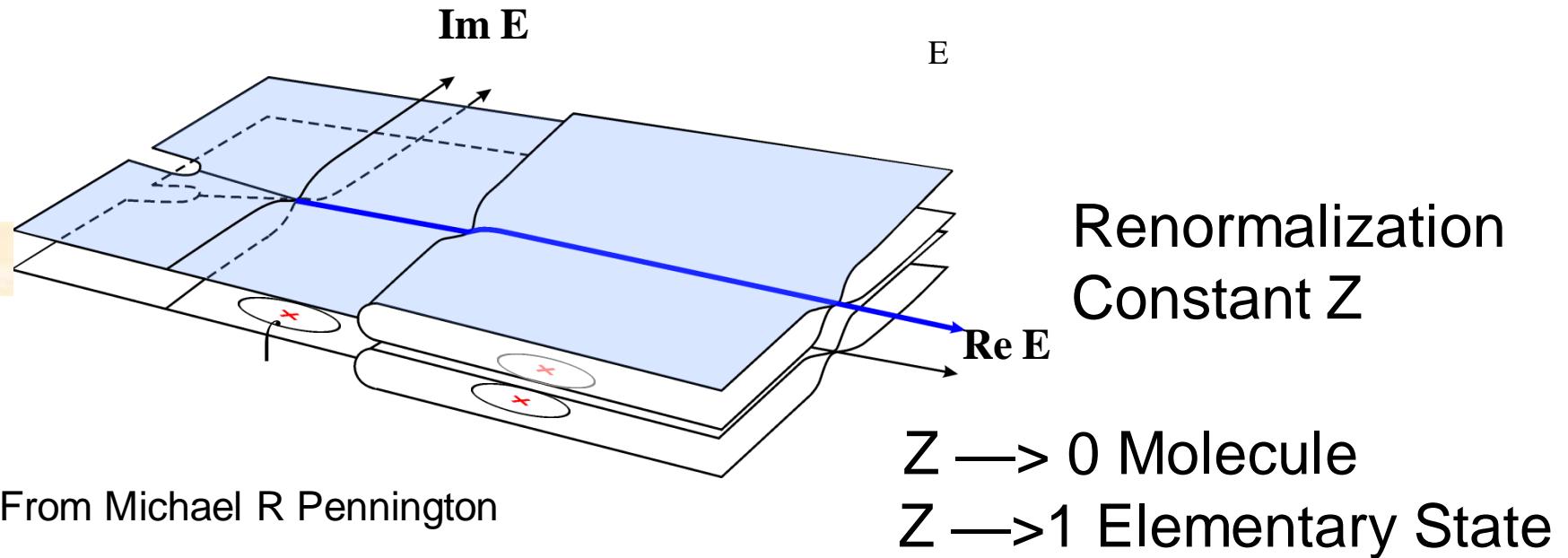
On the other hand when there is a CDD pole weakly couples to this channel, there will be two poles near the threshold,  $M(k^2)$  can be expressed as,

$$M(k^2) = \frac{k^2 - k_0^2}{g^2} + \text{rescattering corrections} \quad (3)$$

where  $g$  is small. In this situation, there appears a pair of poles on the  $k$  plane, which implies the occurrence of an elementary particle.

Using the pole counting method, it is found that  $X(3872)$  contains two poles near threshold. For this reason it is argued that  $X(3872)$  contains large  $c\bar{c}$  component (O. Zhang, C. Meng, H. Q. Zheng, PLB 680,(2009),453).

# Elementary state vs Molecule



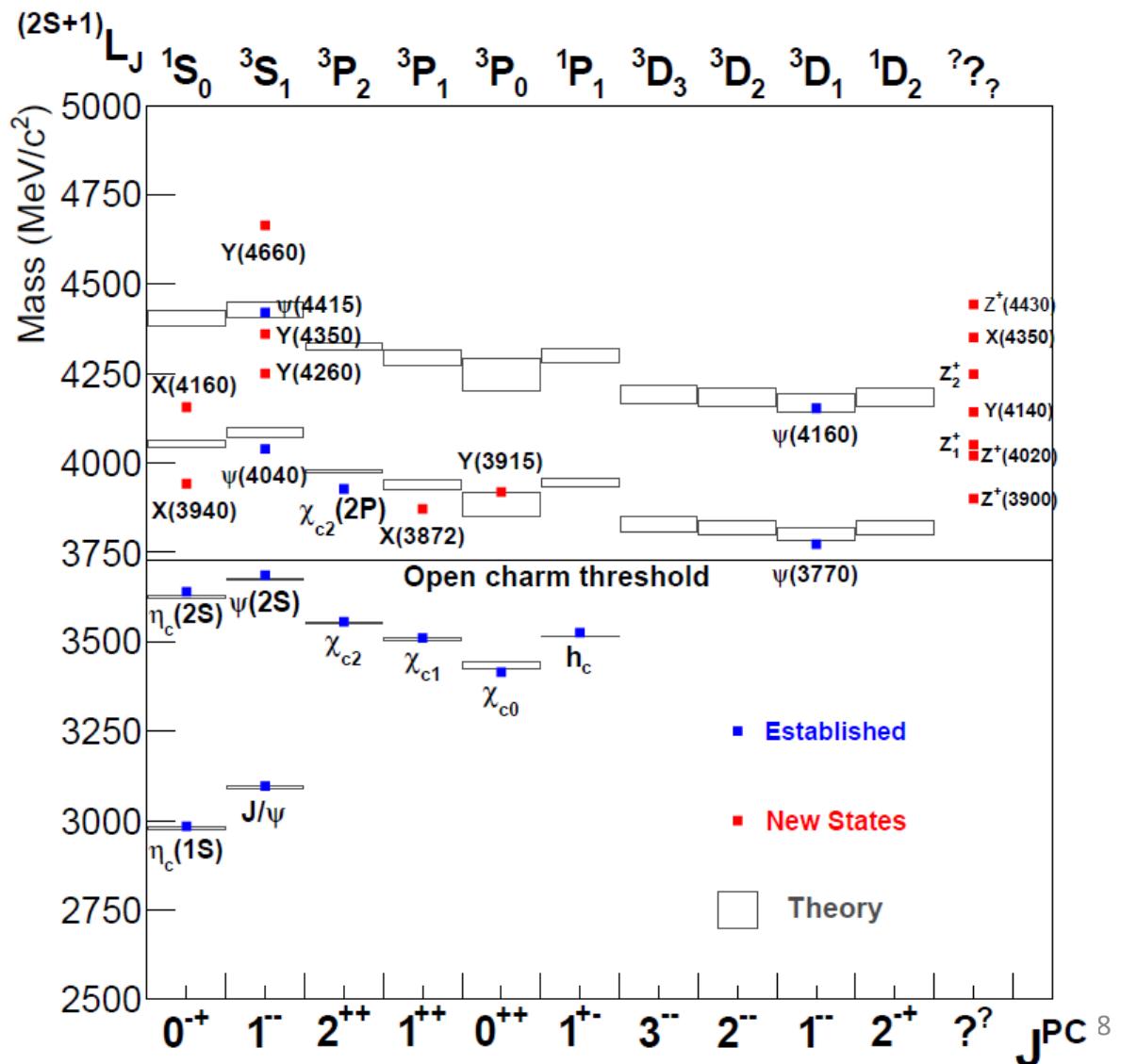
Weinberg 1963,1964  
Hanhart, Kalashnikova et al 2004,2007

# Outline

- X(4260)
- X(3872)
- Zc(3900)
- Zb(10610) and Zb(10650)
- $f_0(600)$  and  $f_0(980)$

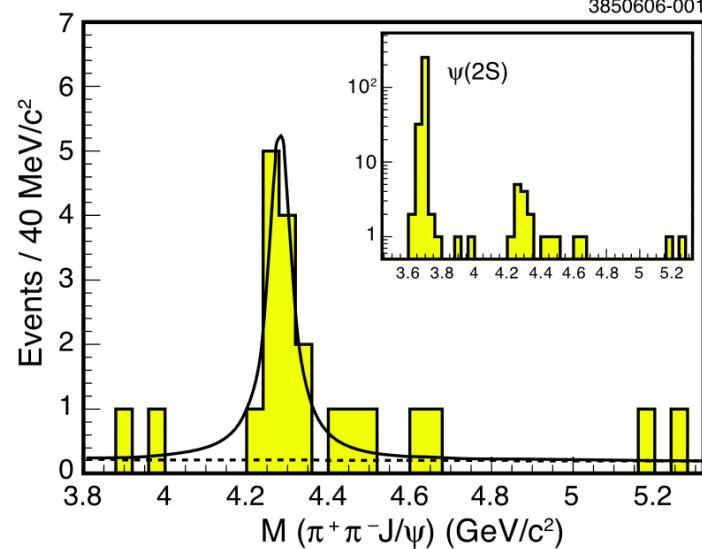
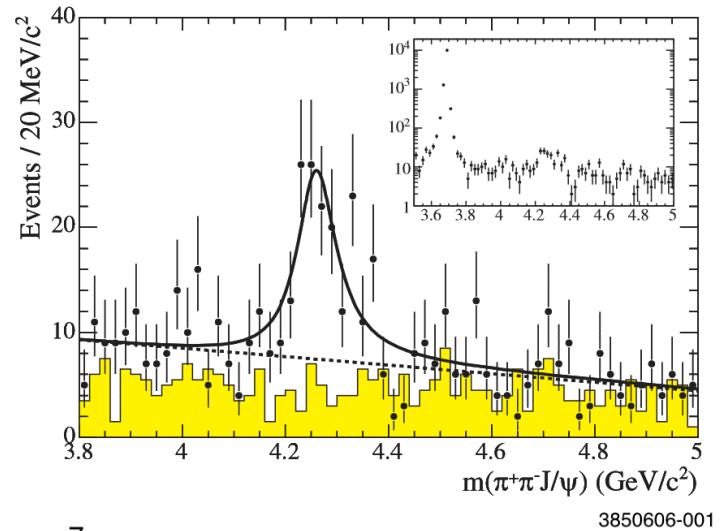
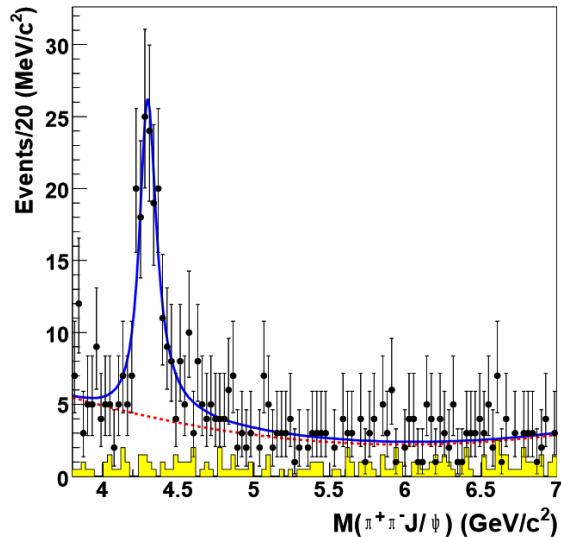
# X(4260)

- Charmonium spectrum.
  - [Eur. Phys. J. C74 \(2014\) 3026](#)



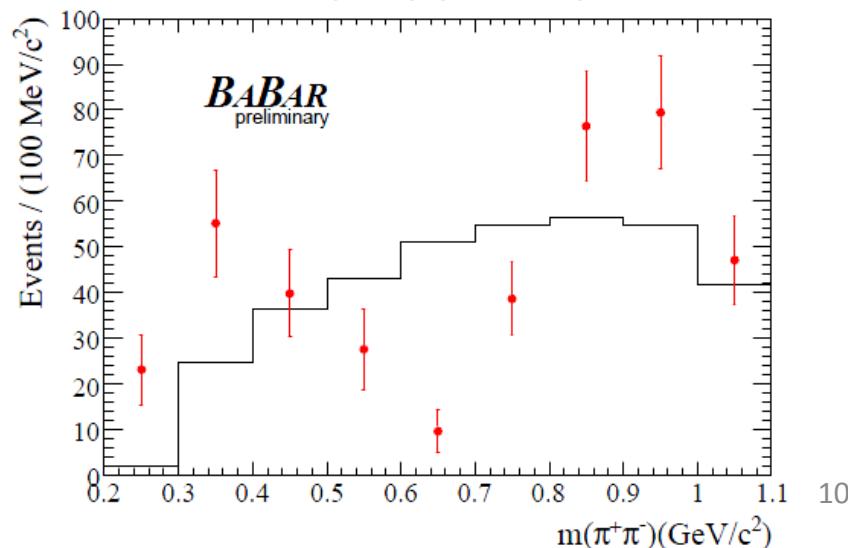
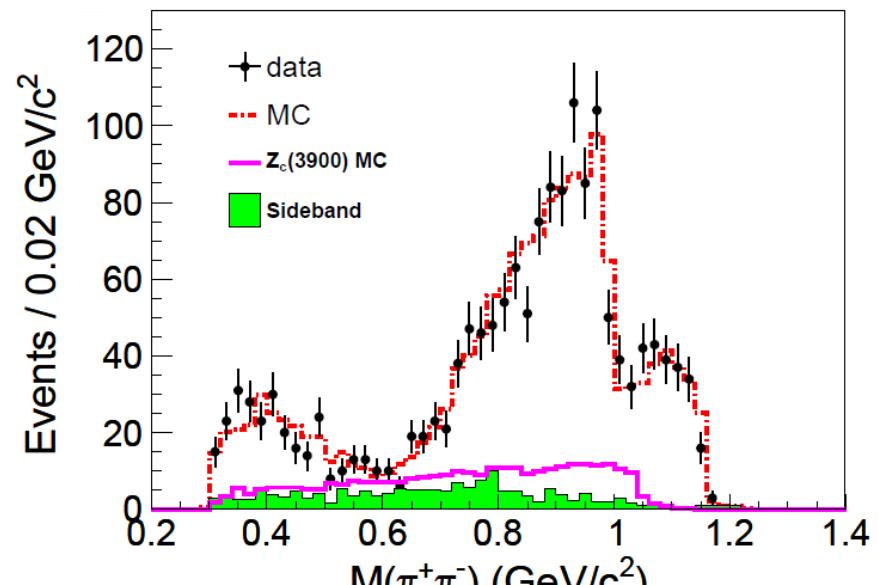
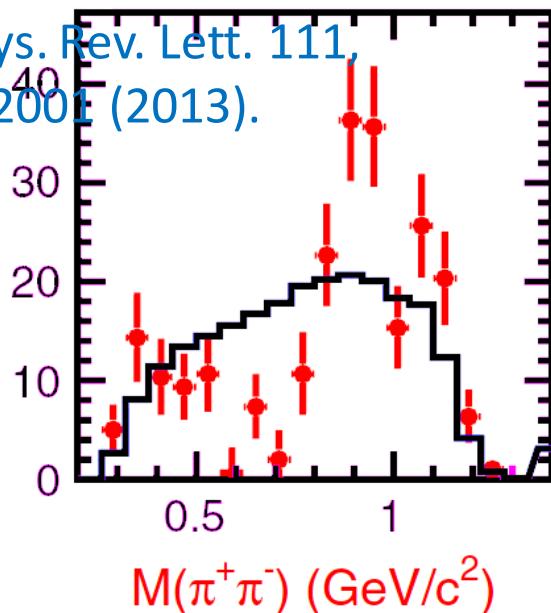
# $X(4260)$ in $e^+e^- \rightarrow \gamma_{ISR}\pi\pi J/\psi$

- BaBar first observed in initial state radiation process.
  - Phys. Rev. Lett. 95, 142001 (2005)
- Confirmed by Belle and CLEO.

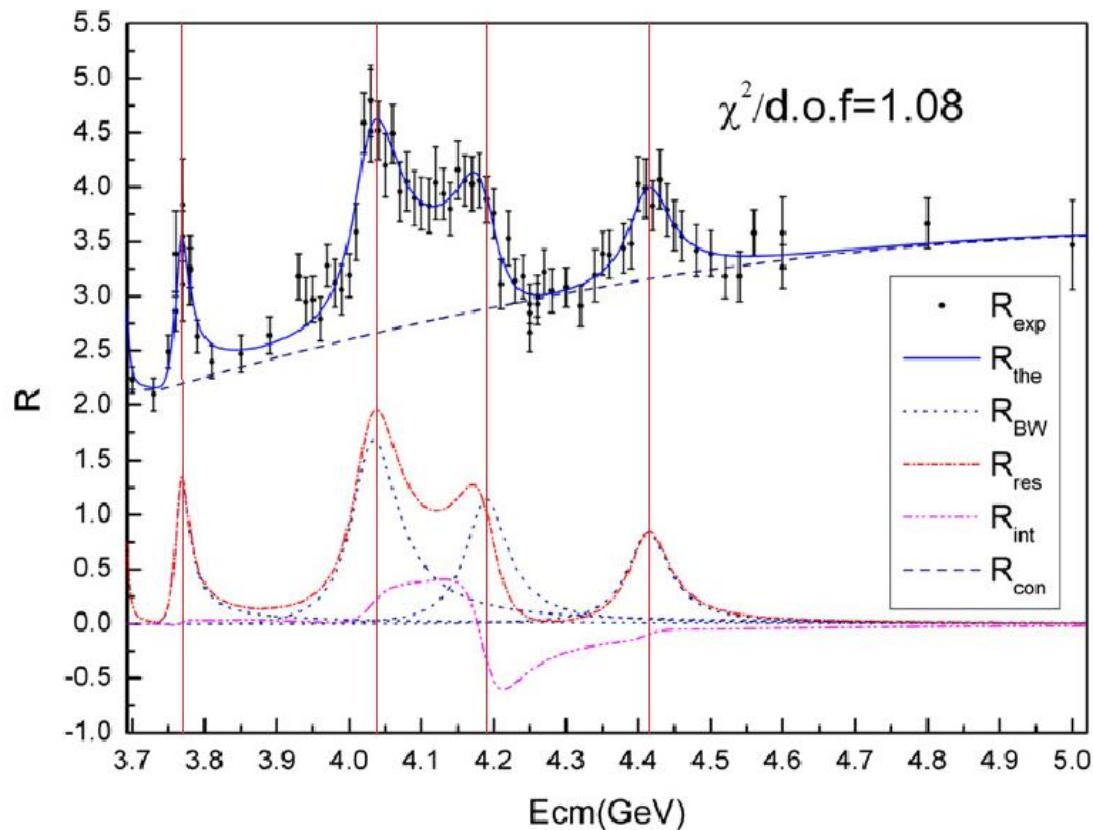


# $\pi\pi$ system in $e^+e^- \rightarrow (\gamma_{isr})\pi\pi J/\psi$

- In signal region.
  - Phys. Rev. Lett. 110, 252001 (2013).
  - PRL 99, 182004 (2007).
  - Phys. Rev. Lett. 111, 242001 (2013).



# $\chi(4260)$ in hadronic decay

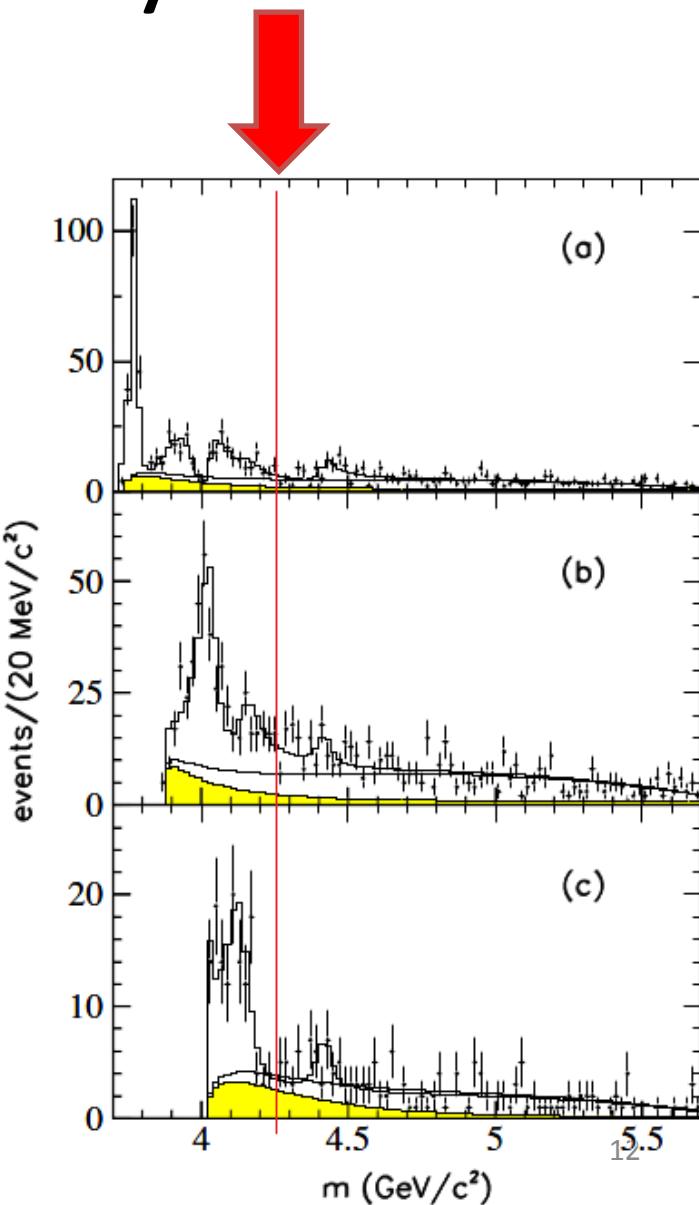
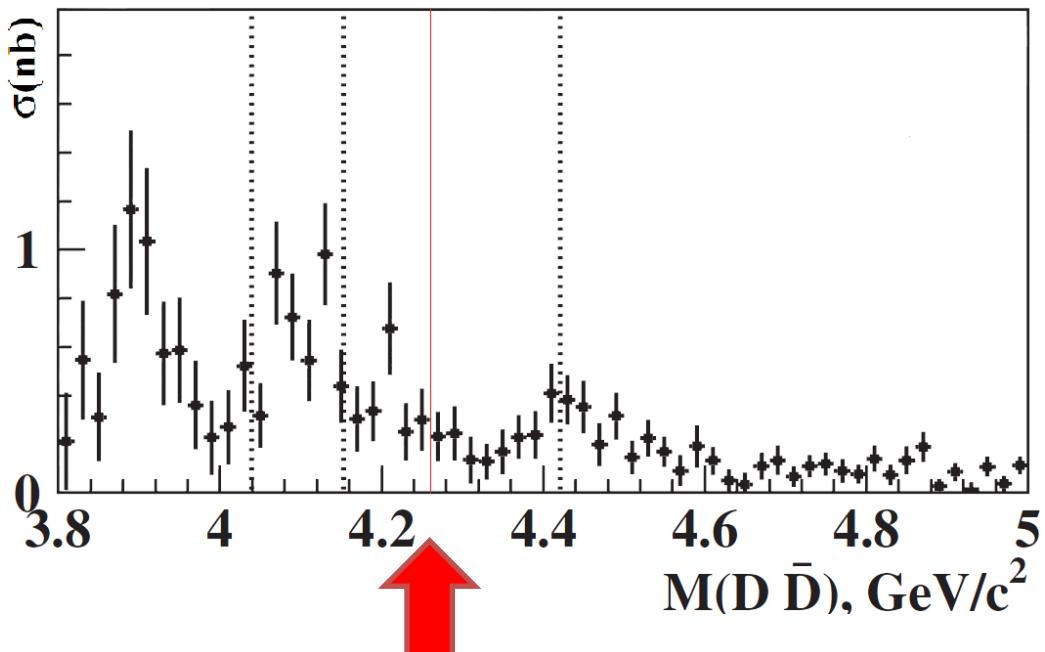


4 resonance in this Fig. :  
 $\psi(3770)$ ,  
 $\psi(4040)$ ,  
 $\psi(4160)$ ,  
 $\psi(4415)$ .

Physics Letters B 660  
(2008) 315–319

# $X(4260)$ in hadronic decay

- $D\bar{D}$ ,  $D\bar{D}^*$ ,  $D^*\bar{D}^*$  channels.
  - Phys. Rev. D79, 092001 (2009).
  - PRL 98, 092001 (2007).



(a)

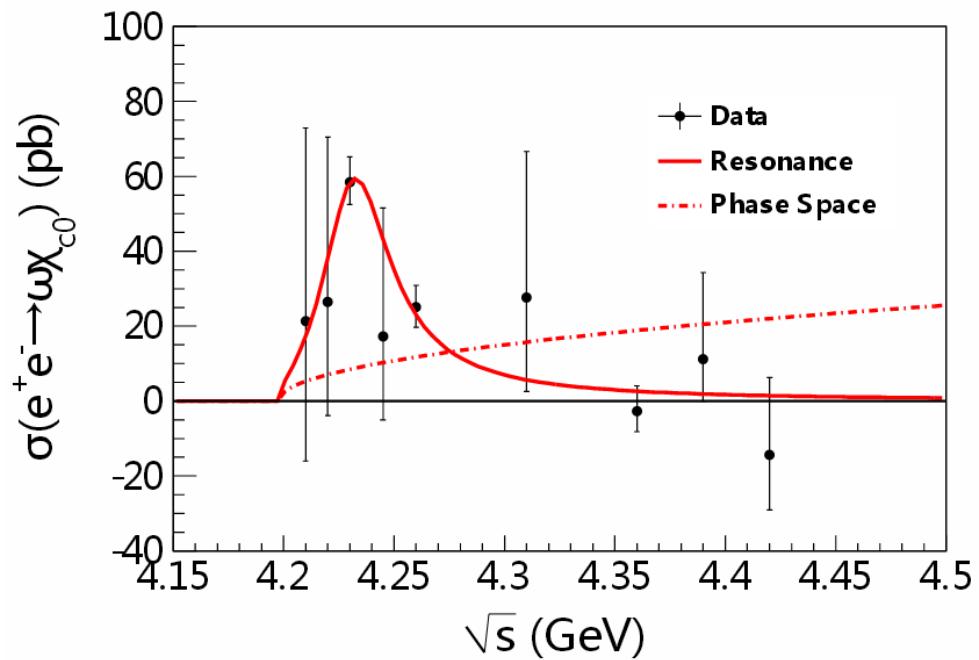
(b)

(c)

# $\omega\chi_{c0}$ channel

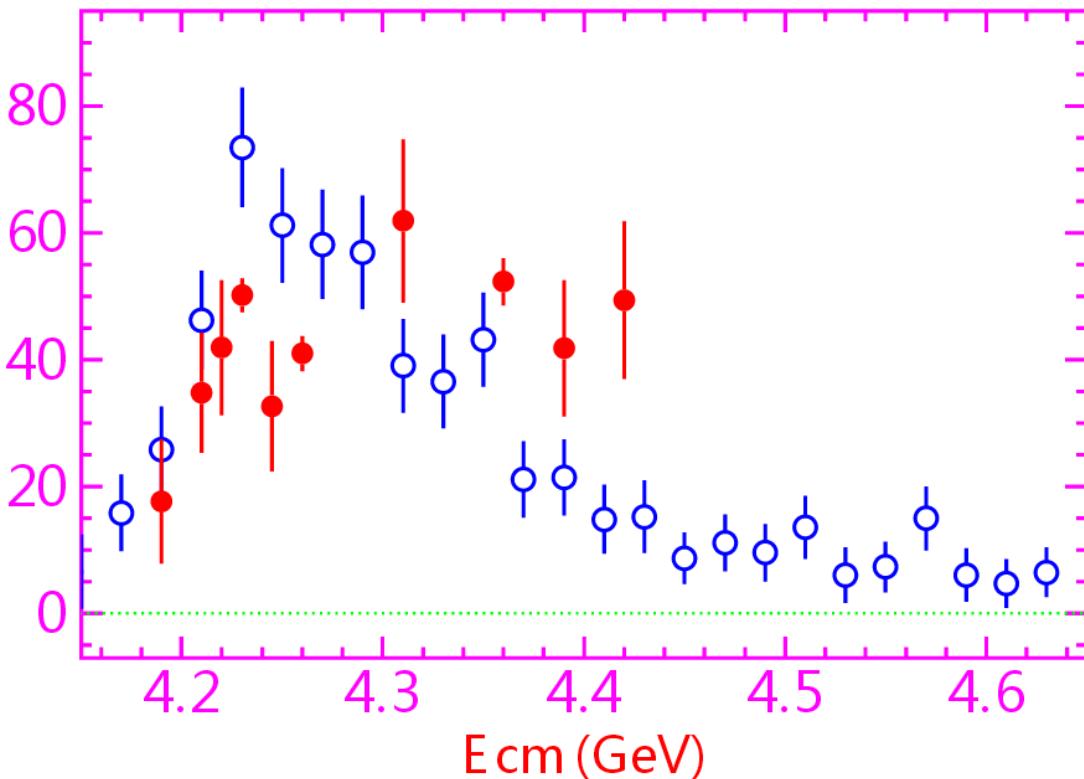
L. Y. Dai, Archive 1206.6911

- BES III measurement:  
[PRL 114 092003](#)



# $h_c\pi\pi$ channel

- BESIII measurement
  - Phys. Rev. Lett. 111 242001.
- Maybe resonant structures
  - C. Z. Yuan, Chin. Phys. C38 043001.



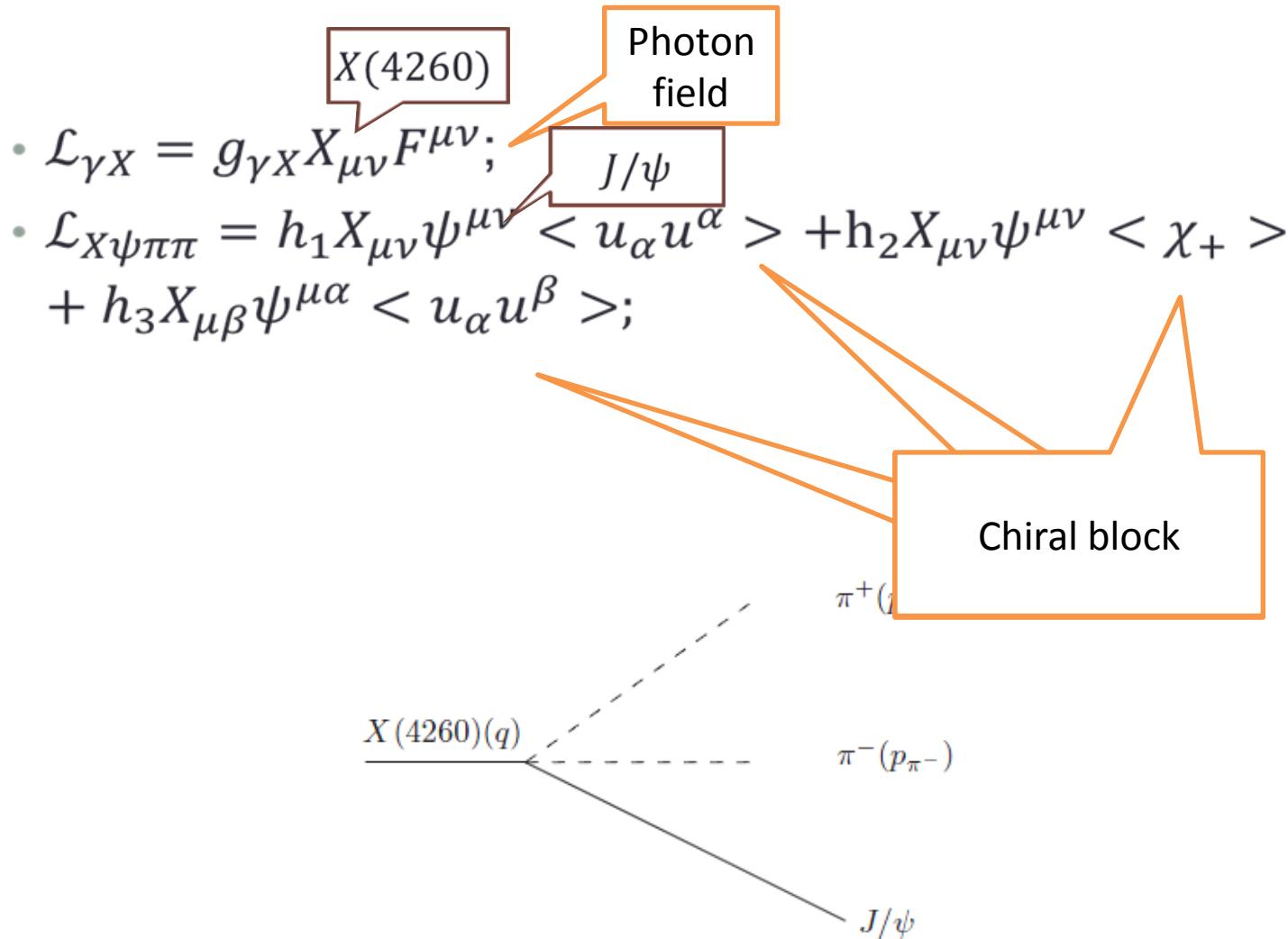
Red:  $h_c\pi\pi$   
Blue:  $J/\psi\pi\pi$

# Theoretical explanation on X(4260)

- Charmonium:
  - B.-Q. Li and K.-T. Chao, Phys. Rev. D79 094004;
  - F. J. Llanes-Estrada, Phys. Rev. D72 031503.
- $DD_1$  molecule:
  - G.-J. Ding, Phys. Rev. D79 (2009) 014001;
  - Q. Wang, C. Hanhart, and Q. Zhao, Phys. Rev. Lett. 111 132003.
- $\chi_{c0}\rho^0$  molecule
- $\omega\chi_{c1}$  molecule:
  - C. Z. Yuan, P. Wang, and X. H. Mo, Phys. Lett. B634 399

- $c\bar{c}g$  hybrid:
  - S.-L. Zhu, (2005), Phys. Lett. B625 212;
  - Y. Chen, PoS LATTICE2013, 215.
- $\Lambda_c \bar{\Lambda}_c$  bayronium
- Non-resonance
  - E. van Beveren, G. Rupp, Phys. Rev. D79 111501.
  - D. Y. Chen, J. He, X. Liu, Phys. Rev. D83 054021.
- Tetraquark
- .....

# Our consideration

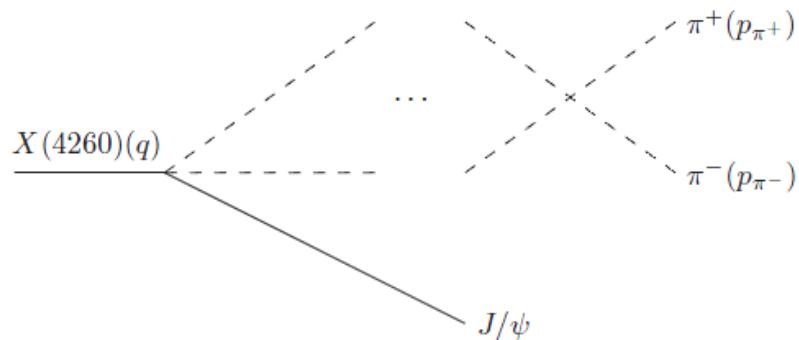


# Final state interaction between pions

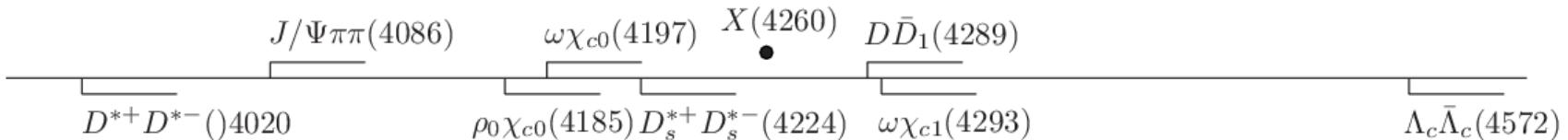
- Use Padé amplitude to describe  $\pi\pi$  inv. mass spectrum.
- Connect Padé amplitude to  $J/\psi\pi\pi$  amplitude as follow:

$$\mathcal{A}_{direct} = \mathcal{A}_1^{tree} \alpha_1(s) T_{11}(s) + \mathcal{A}_2^{tree} \alpha_2(s) T_{21}(s)$$

- $\alpha_{11}(s), \alpha_{21}(s)$  are polynomials. Subscript 1(2) denotes  $\pi\pi(KK)$  channel.
- K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D35 1633 (1987); D. Morgan and M. R. Pennington, Phys. Rev. D48 1185 (1993).



# Propagator

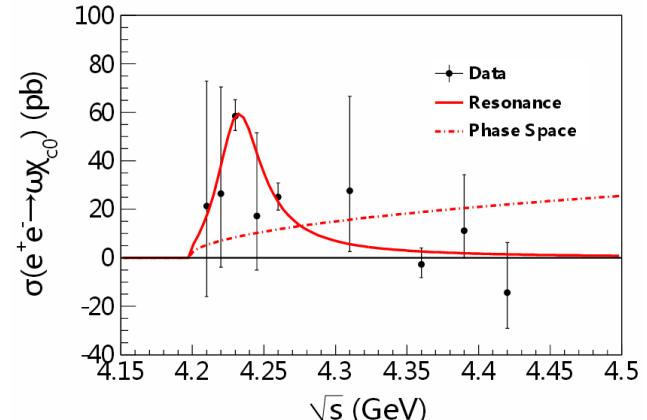


- $X(4260)$  total width:

$$\Gamma_X = \Gamma_{J/\psi\pi\pi} + g_{\omega\chi_{c0}} k_{\omega\chi_{c0}} + g_{D\bar{D}_1} k_{D\bar{D}_1} + g_{D_s^* D_s^*} k_{D_s^* D_s^*}^3 + \Gamma_{h_c\pi\pi} + \Gamma_0.$$

- $\omega\chi_{c0}$  channel cross section:

$$\sigma_{e^+e^- \rightarrow X(4260) \rightarrow \omega\chi_{c0}}(q^2) = \frac{3\pi}{4q^2} \frac{\Gamma_{ee}\Gamma_{\omega\chi_{c0}}}{|D_X(q^2)|^2}$$



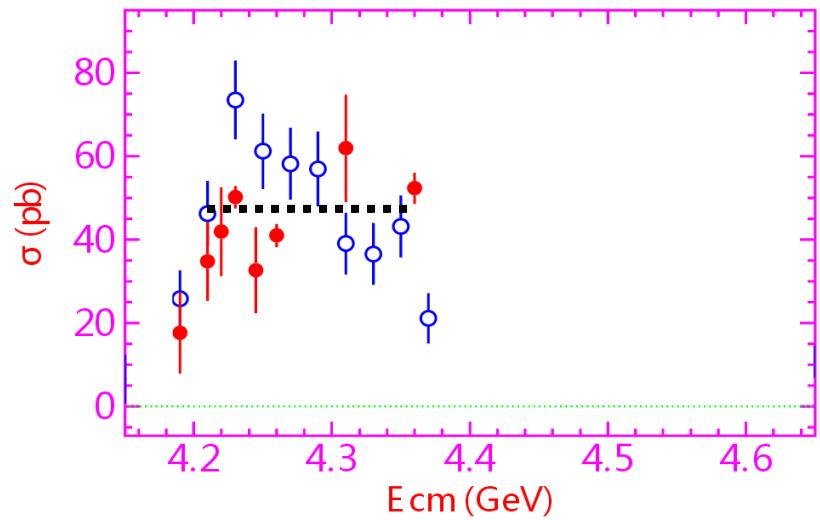
- $\Gamma_{h_c\pi\pi}$ : a constant, but the value is determined as follow,

$$R \times \Gamma_{J/\psi\pi\pi} |_{q=4.26\text{GeV}}.$$

- Upper bound

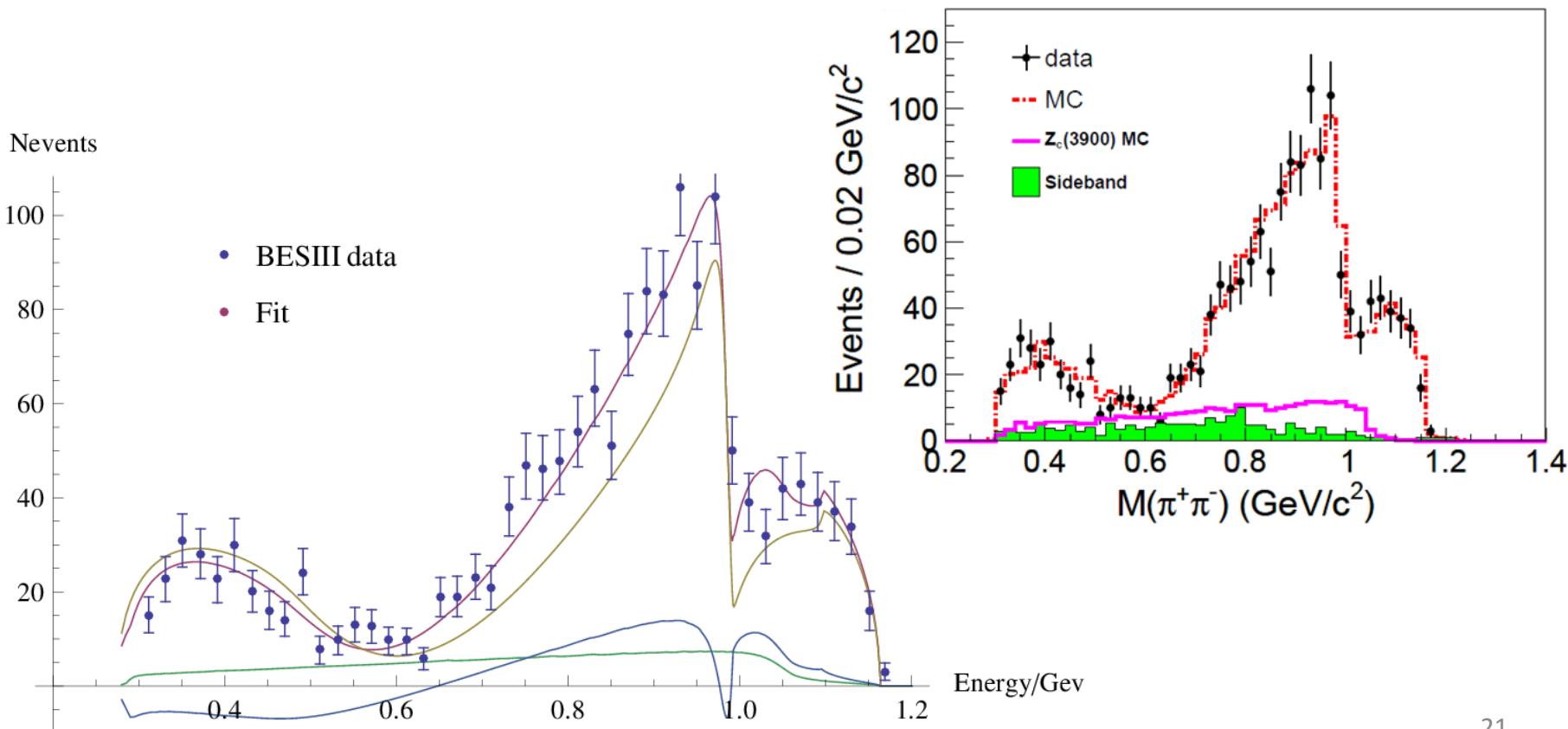
$$R_{upper} = \frac{\Gamma_{h_c\pi\pi}}{\Gamma_{J/\psi\pi\pi}} \sim \frac{\sigma_{h_c\pi\pi}}{\sigma_{J/\psi\pi\pi}} |_{q=4.26\text{GeV}} \approx 0.66.$$

- $R = 0.66, 0.56, \dots, 0$ .



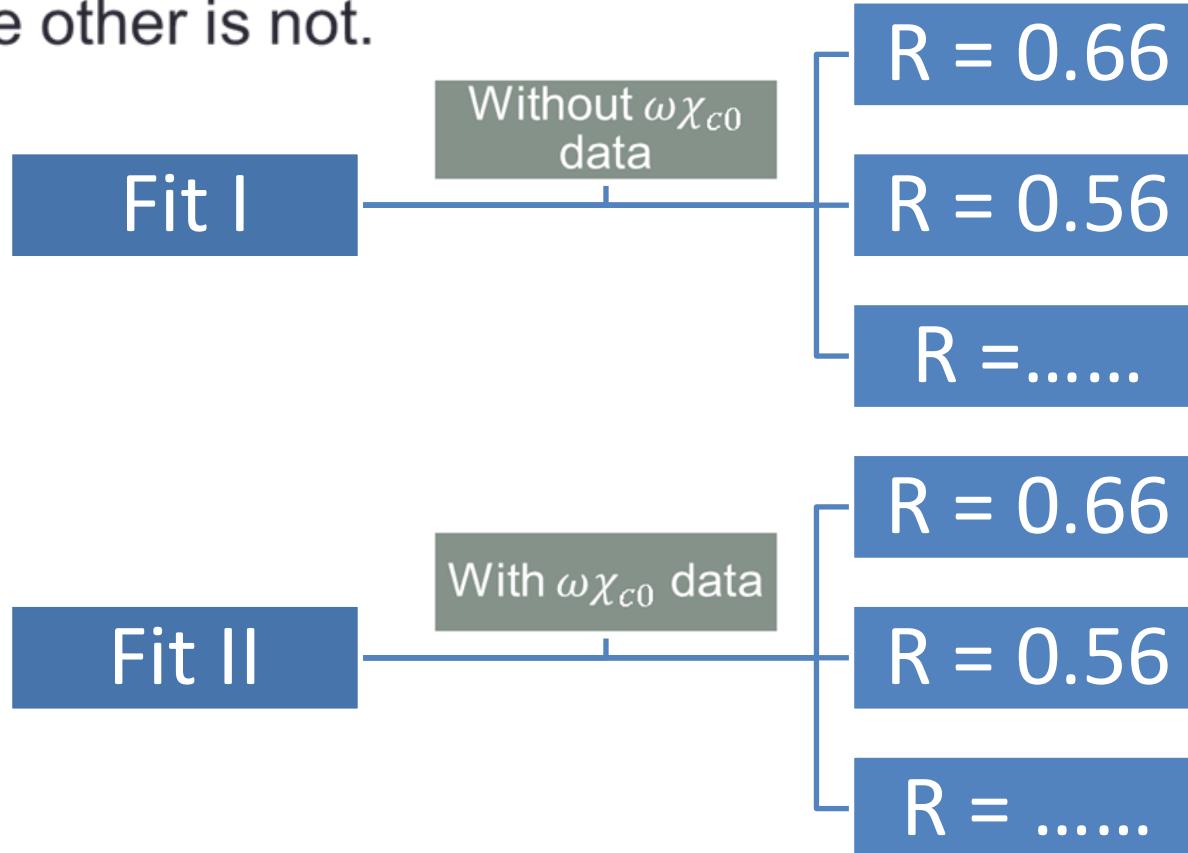
# What about $Z_c$ in $X(4260) \rightarrow J/\psi\pi\pi$

- The contribution of  $X \rightarrow Z_c\pi \rightarrow J/\psi\pi\pi$  can be absorbed into  $X \rightarrow J/\psi\pi\pi$  contact interaction.



# Numerical Fit

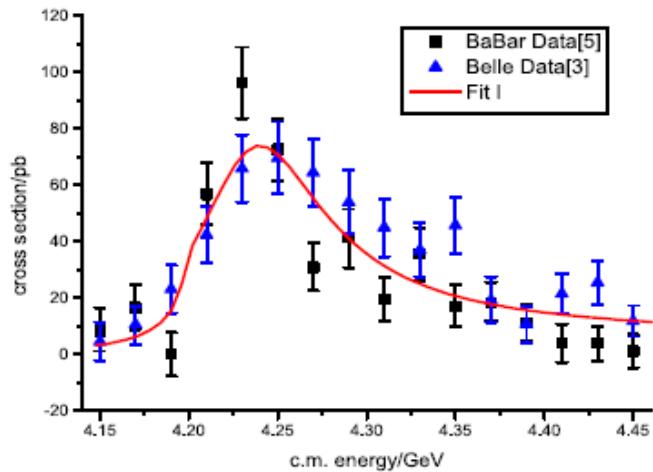
- Two fits:
  - One is using the data of cross section of  $\omega\chi_{c0}$  channel.  
The other is not.



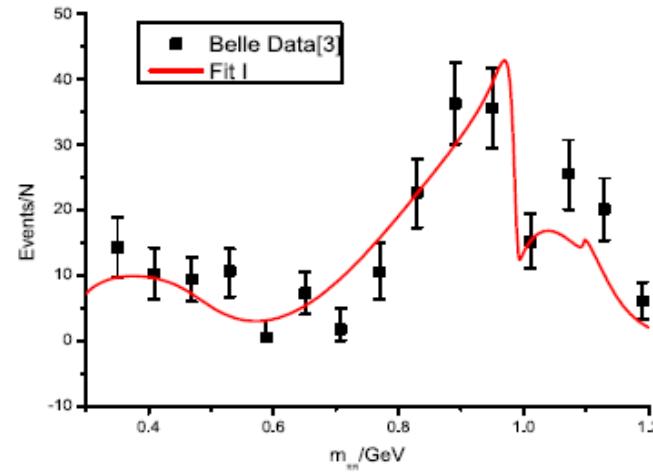
- All fits are not sensitive to  $\Gamma_{h_c\pi\pi}$ .

	Fit I					
$\chi^2/d.o.f.$	185.58/(145-14)	185.58/(145-14)	185.57/(145-14)	185.57/(145-14)	185.57/(145-14)	185.56/(145-14)
$g_0(MeV)$	$23.7197 \pm 1.6426$	$23.7129 \pm 1.6367$	$23.7194 \pm 1.6132$	$23.7293 \pm 1.6368$	$23.7229 \pm 1.6175$	$23.7382 \pm 1.6312$
$g_{X\omega\chi_{c0}}$	$0.3637 \pm 0.0271$	$0.3639 \pm 0.0270$	$0.3644 \pm 0.0270$	$0.3650 \pm 0.0271$	$0.3652 \pm 0.0269$	$0.3661 \pm 0.0270$
$h_1$	$0.0411 \pm 0.0045$	$0.0412 \pm 0.0045$	$0.0412 \pm 0.0044$	$0.0412 \pm 0.0045$	$0.0412 \pm 0.0044$	$0.0412 \pm 0.0044$
$h_2$	$-0.1361 \pm 0.0067$	$-0.1361 \pm 0.0067$	$-0.1361 \pm 0.0065$	$-0.1361 \pm 0.0066$	$-0.1361 \pm 0.0065$	$-0.1361 \pm 0.0066$
$h_3$	$0.15297 \pm 0.0080$	$0.1530 \pm 0.0080$	$0.1530 \pm 0.0078$	$0.15298 \pm 0.0080$	$0.1530 \pm 0.0078$	$0.1530 \pm 0.0079$
$M_X(GeV)$	$4.2541 \pm 0.0032$	$4.2542 \pm 0.0032$				
$\Gamma_0(GeV)$	$0.0000 \pm 0.0005$	$0.0000 \pm 0.0005$	$0.0000 \pm 0.0010$	$0.0000 \pm 0.0006$	$0.0000 \pm 0.0004$	$0.0000 \pm 0.0005$
ratio	0.56	0.46	0.36	0.26	0.16	0.00
	Fit II					
$\chi^2/d.o.f.$	193.07/(154-14)	192.74/(154-14)	192.50/(154-14)	193.07/(154-14)	192.35/(154-14)	192.26/(154-14)
$g_0(MeV)$	$3.2640 \pm 0.1099$	$3.2093 \pm 0.1136$	$3.1594 \pm 0.1188$	$3.1155 \pm 0.1254$	$3.1041 \pm 0.1116$	$3.1039 \pm 0.1602$
$g_{X\omega\chi_{c0}}$	$0.0650 \pm 0.0128$	$0.0713 \pm 0.0139$	$0.0789 \pm 0.0152$	$0.0882 \pm 0.0167$	$0.0913 \pm 0.0209$	$0.0914 \pm 0.0227$
$h_1$	$0.0272 \pm 0.0011$	$0.0275 \pm 0.0011$	$0.0279 \pm 0.0011$	$0.0313 \pm 0.0012$	$0.0315 \pm 0.0013$	$0.0316 \pm 0.0015$
$h_2$	$0.0330 \pm 0.0014$	$0.0335 \pm 0.0014$	$0.0340 \pm 0.0014$	$0.0381 \pm 0.0016$	$0.0383 \pm 0.0017$	$0.0385 \pm 0.0019$
$h_3$	$-0.0903 \pm 0.0034$	$-0.915 \pm 0.0035$	$0.0928 \pm 0.0035$	$-0.1041 \pm 0.0039$	$-0.1047 \pm 0.0042$	$-0.1052 \pm 0.0049$
$M_X(GeV)$	$4.2444 \pm 0.0033$	$4.2458 \pm 0.0034$	$4.2475 \pm 0.0037$	$4.2496 \pm 0.0040$	$4.2504 \pm 0.0048$	$4.2504 \pm 0.0053$
$\Gamma_0(GeV)$	$0.0000 \pm 0.0001$	$0.0000 \pm 0.0002$	$0.0000 \pm 0.0002$	$0.0000 \pm 0.0007$	$0.0049 \pm 0.0094$	$0.0162 \pm 0.0093$
ratio	0.56	0.46	0.36	0.26	0.16	0.00

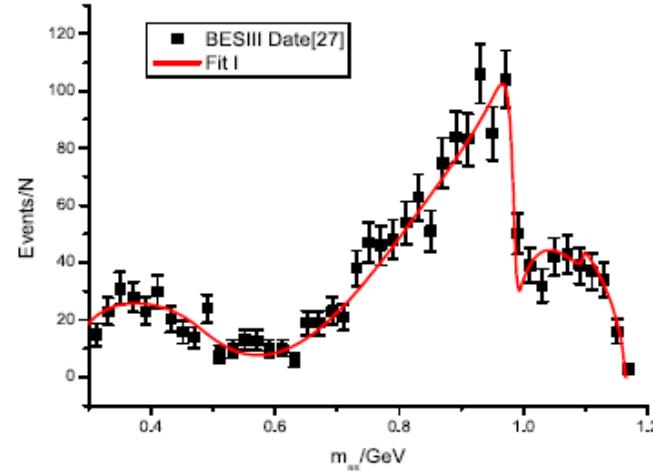
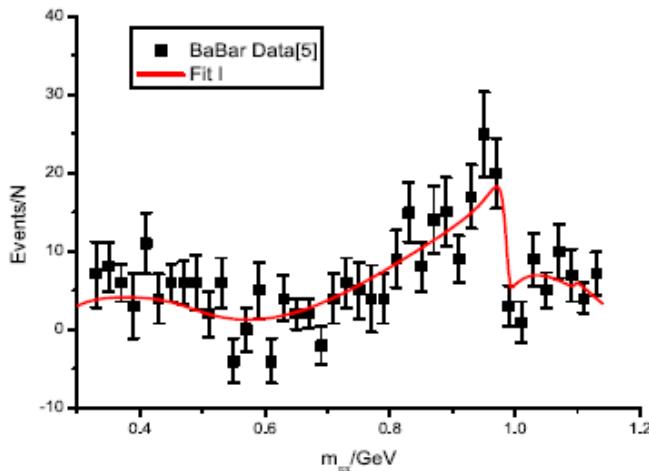
# Numerical: Fit I



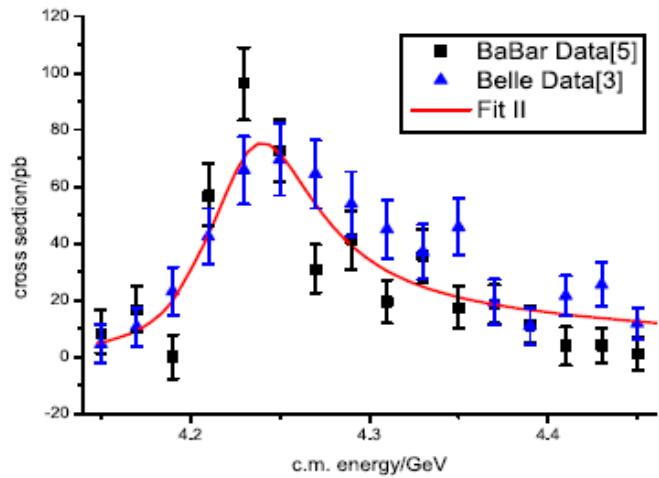
(a)



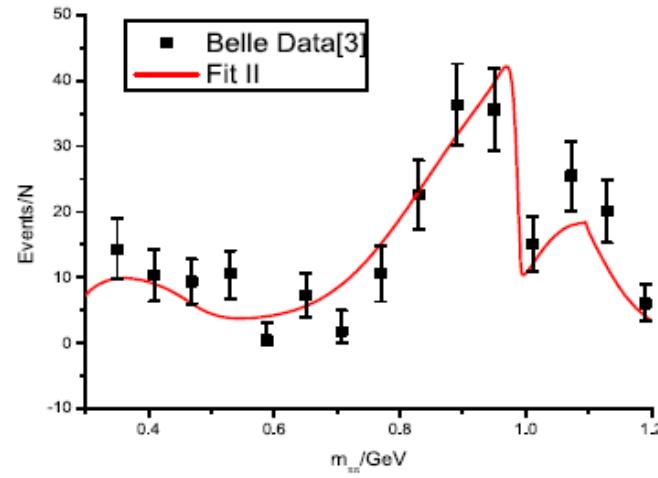
(b)



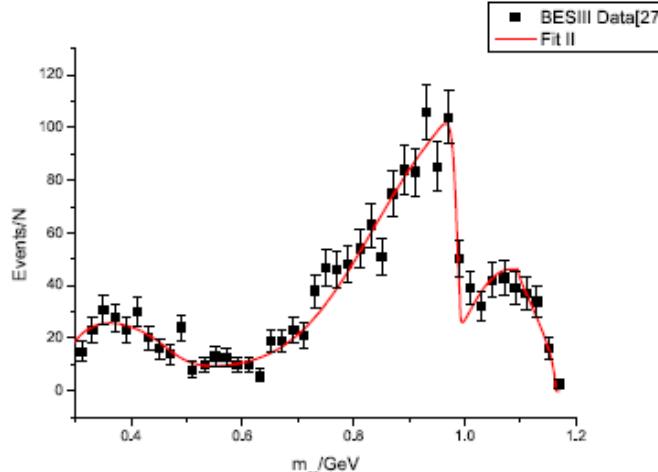
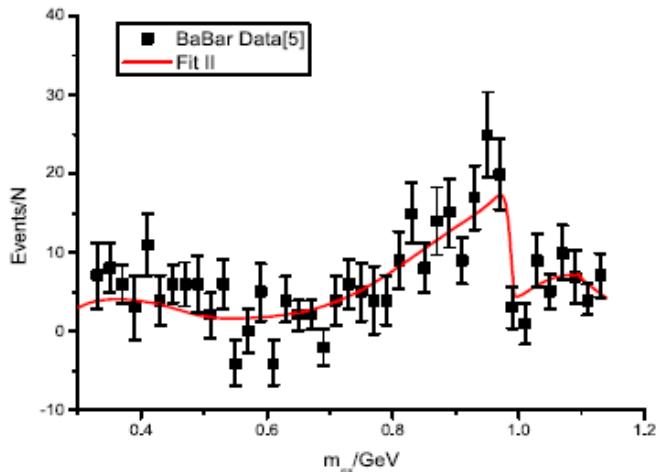
# Numerical: Fit II



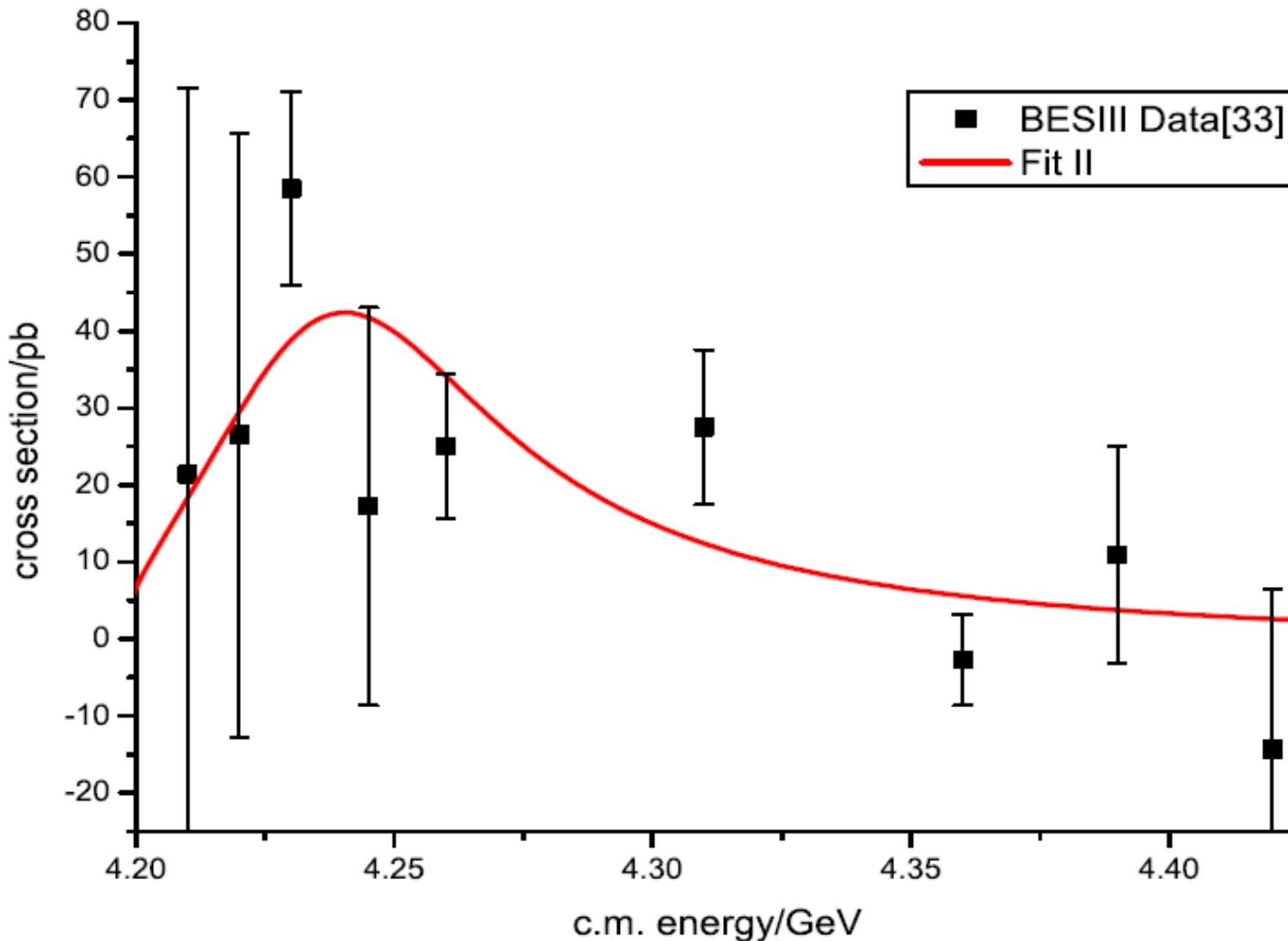
(a)



(b)

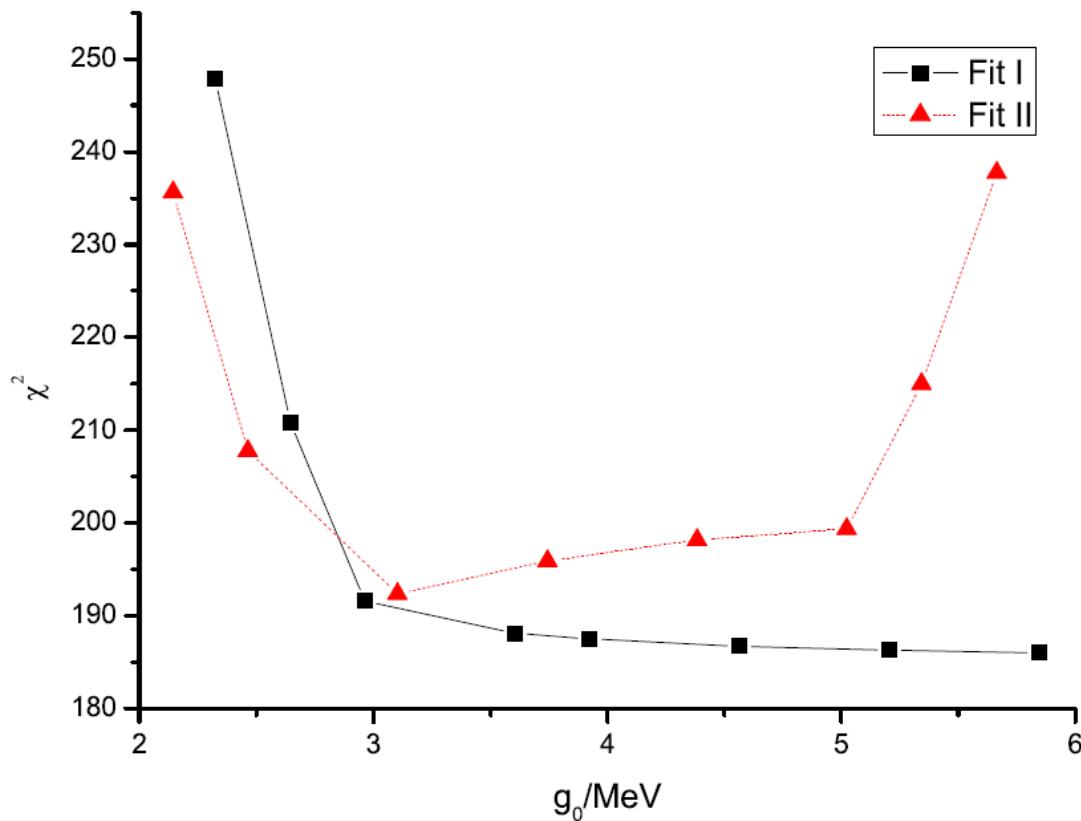


# Numerical: Fit II



# Numerical

- Scaling law exists in Fit I.



# Numerical

- Riemann Sheets and pole position:

	sheet I	sheet II	sheet III	sheet IV
$\Gamma_{J/\psi\pi\pi} + \Gamma_{h_c\pi\pi}$	+	-	-	+
$\Gamma_{\omega\chi_{c0}}$	+	+	-	-

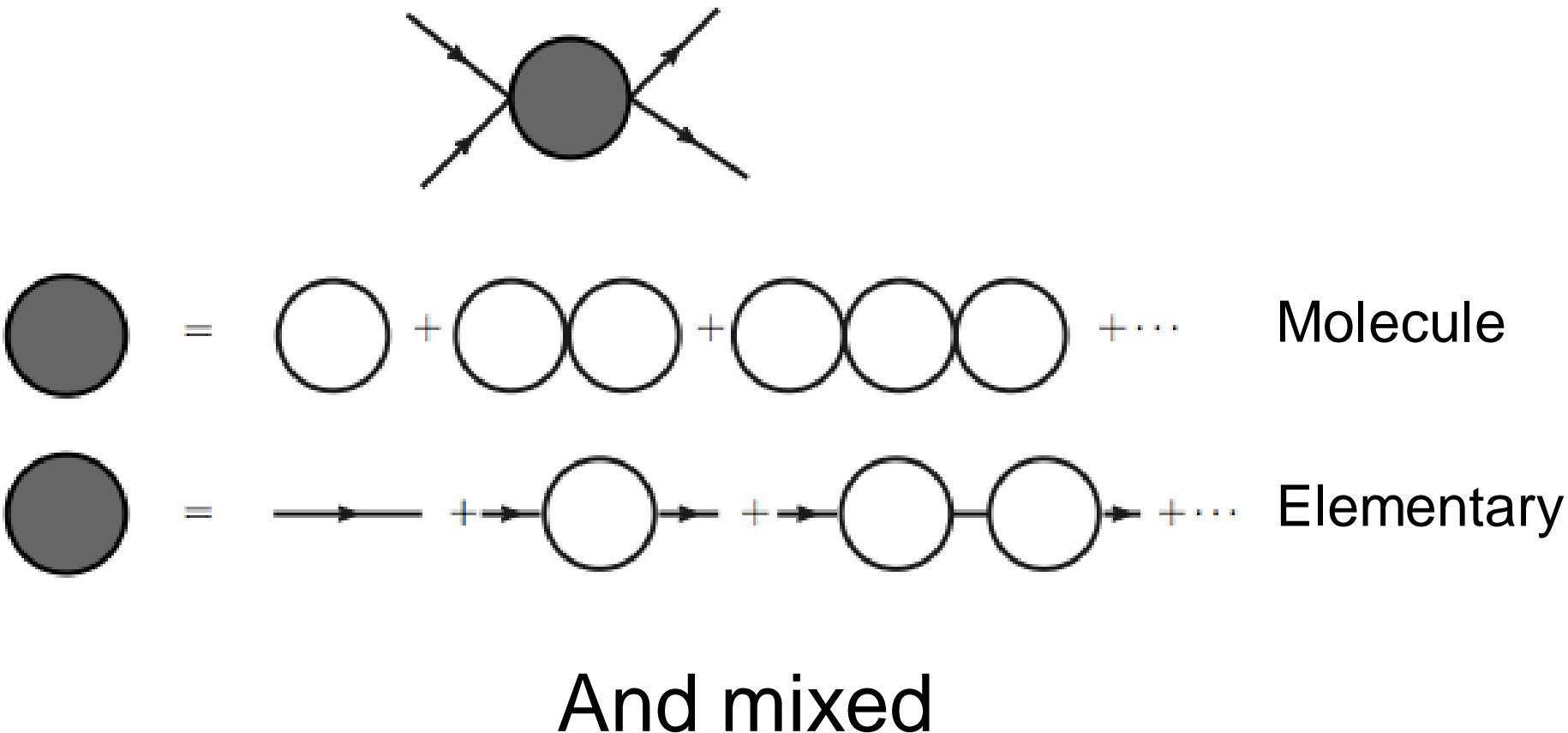
	sheet I	sheet II	sheet III	sheet IV
Fit I	—	—	4231.9-44.2i	4233.2-42.5i
Fit II	—	4241.5-24.4i	4232.8-36.3i	—

# X(4260): conclusion

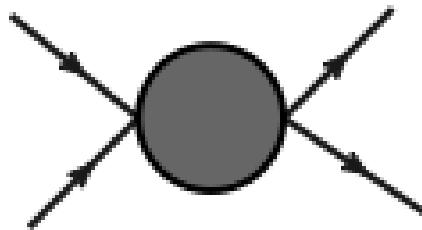
- All fits are not sensitive to the  $\Gamma_{h_c\pi\pi}$ .
- $\frac{\chi^2}{dof} \sim 1.4$
- $\Gamma_0$ ,  $g_{DD_1}$  and  $g_{D_S^* D_S^*}$  tend to vanish.
- Sizeable  $\omega\chi_{c0}$  coupling;
  - In Fit I,  $\Gamma_{\omega\chi_{c0}} \geq 49.8\text{MeV}$ . If there is no other decay channel.
  - In Fit II,  $\Gamma_{\omega\chi_{c0}} = 16.9\text{MeV}$ .
- In Fit II,  $\Gamma_{ee} = 23.3 \pm 3.5\text{eV}$ .

Two nearby poles indicate an “elementary” particle

# X(3872): Including bubble chains



# Elementary state & Molecule

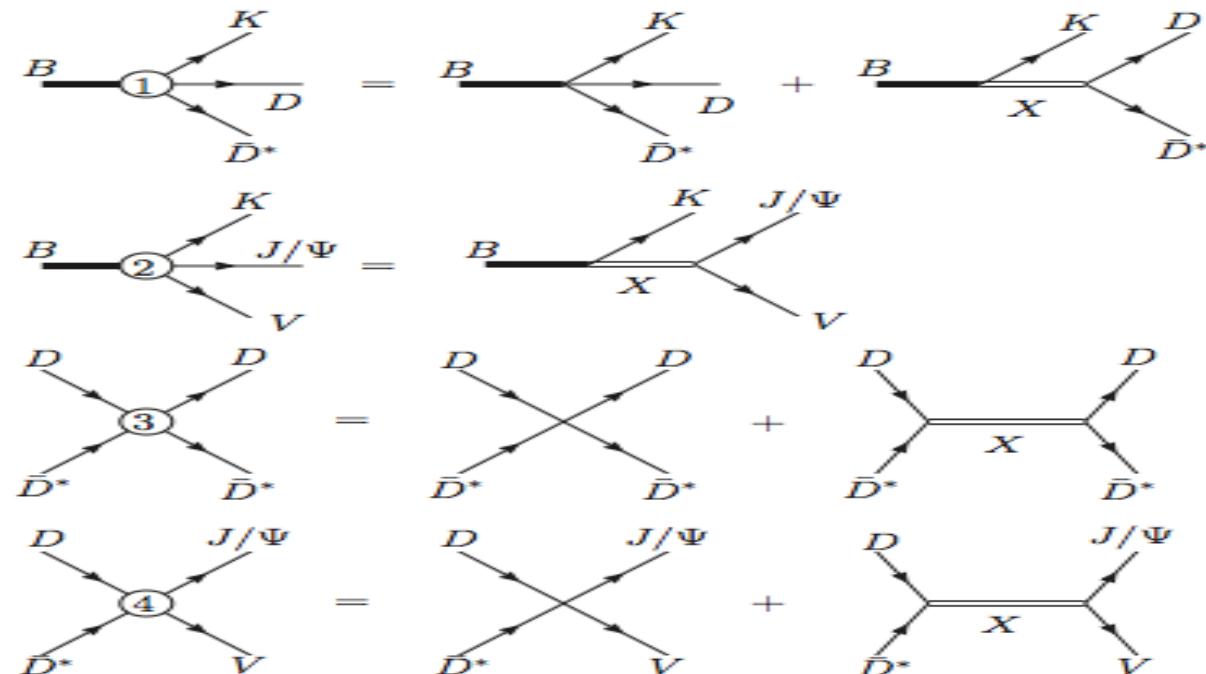


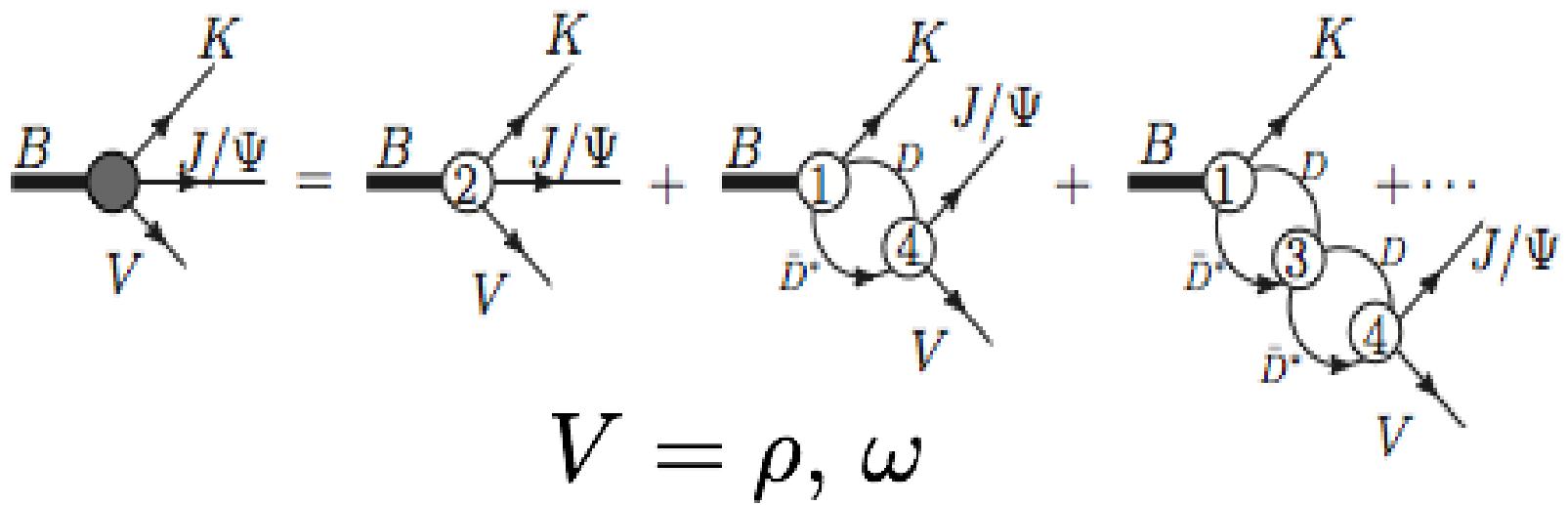
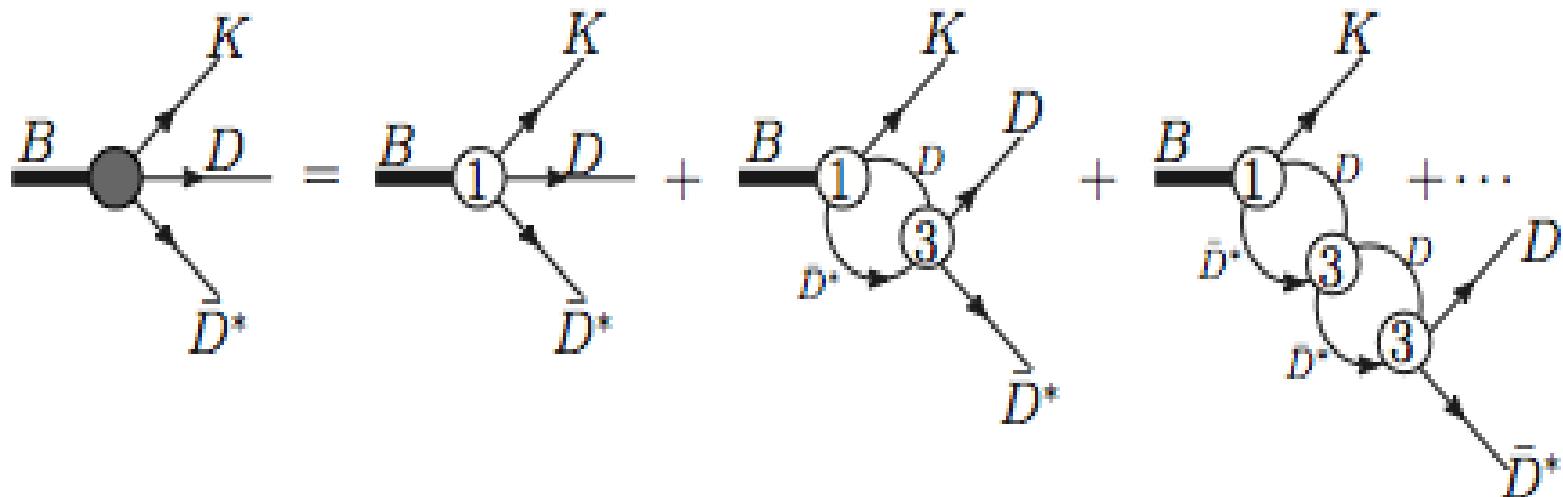
$$\text{Large Gray Circle} = \text{Small Circle } M2 + \text{Small Circle } M2 \text{ connected to a chain of circles } M2 + \text{Small Circle } M2 \text{ connected to a chain of circles } M2 + \dots$$

$$\text{Small Circle } M2 = \text{Crossed Lines} + \text{Y-shaped Line}$$

$$\begin{aligned}
\mathcal{L}_{DD^*} &= \lambda_1 (\bar{D}^{*\mu} D \bar{D}^{*\mu} D + \bar{D} D^{*\mu} \bar{D} D^{*\mu}) + \lambda_2 (\bar{D}^{*\mu} D \bar{D} D^{*\mu}), \\
\mathcal{L}_{XDD^*} &= g_1 X^\mu (\bar{D} D_\mu^* - \bar{D}_\mu^* D), \\
\mathcal{L}_{BXK} &= ig_2 X^\mu (\bar{B} \partial_\mu K + \text{h.c.}), \\
\mathcal{L}_{BKDD^*} &= ig_3 (\bar{D} D_\mu^* - \bar{D}_\mu^* D) (\bar{B} \partial^\mu K + \text{h.c.}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{X\Psi V} &= ig_4 X^\mu \Psi^\nu \partial^\alpha V^\beta \epsilon_{\mu\nu\alpha\beta}, \\
\mathcal{L}_{\Psi V D \bar{D}^*} &= ig_5 (\bar{D} D^{*\mu} - \bar{D}^{*\mu} D) \Psi^\nu \partial^\alpha V^\beta \epsilon_{\mu\nu\alpha\beta},
\end{aligned}$$





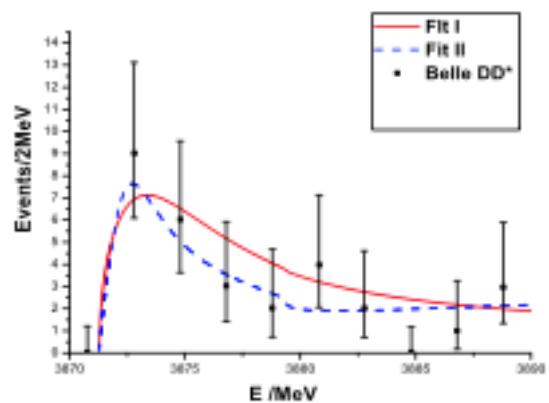
$$V = \rho, \omega$$

$$\mathcal{M}_{D^0\bar{D}^{*0}} = - \frac{(g_3 + \frac{g_1 g_2}{s - M_X^2}) p_K^\mu \epsilon_{D^*}^\nu}{1 - (i\lambda_2 + i\frac{g_1^2}{s - M_X^2}) \hat{\Pi}_T(s)} P_{T\mu\nu}(p) + \frac{(g_3 + \frac{g_1 g_2}{M_X^2}) p_K^\mu \epsilon_{D^*}^\nu}{1 - (i\lambda_2 + i\frac{g_1^2}{M_X^2}) \hat{\Pi}_L(s)} P_{L\mu\nu}(p)$$

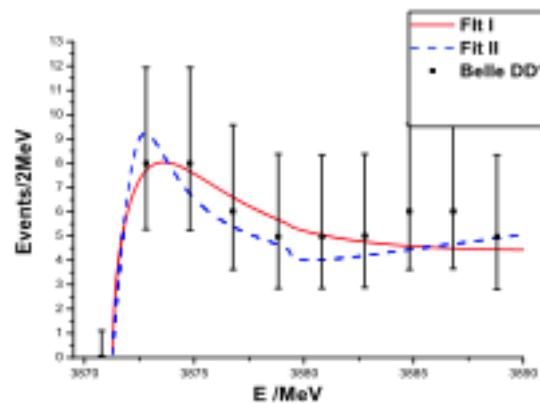
$$\int \frac{d^D k}{(2\pi)^D} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_{D^{*0}}^2}}{(k^2 - m_{D^{*0}}^2)((p-k)^2 - m_{D^0}^2)} = P_{T\mu\nu}(p) \Pi_{T_{D^0\bar{D}^{*0}}}(s) + P_{L\mu\nu}(p) \Pi_{L_{D^0\bar{D}^{*0}}}(s)$$

-

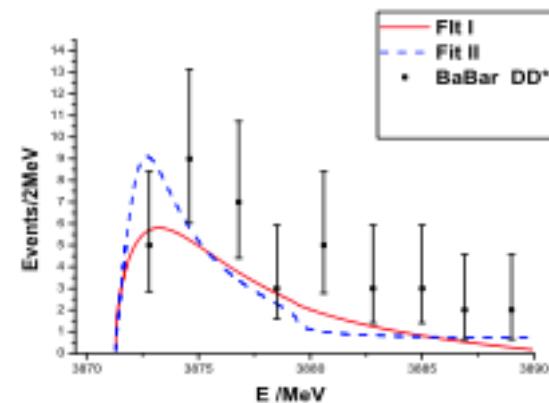
$$s - M_X^2 \Rightarrow s - M_X^2 + iM_X(\Gamma_{J/\Psi\pi\pi}(s) + \Gamma_{J/\Psi\pi\pi\pi}(s) + \Gamma_0),$$



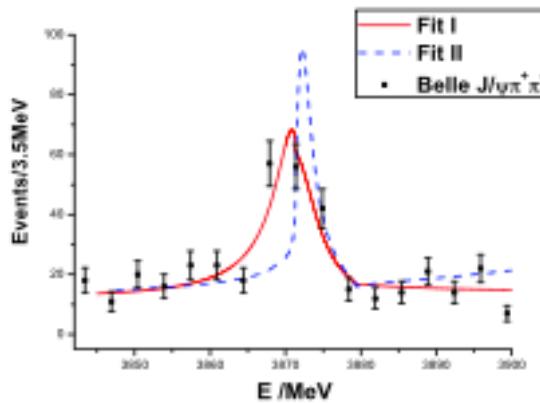
(a)



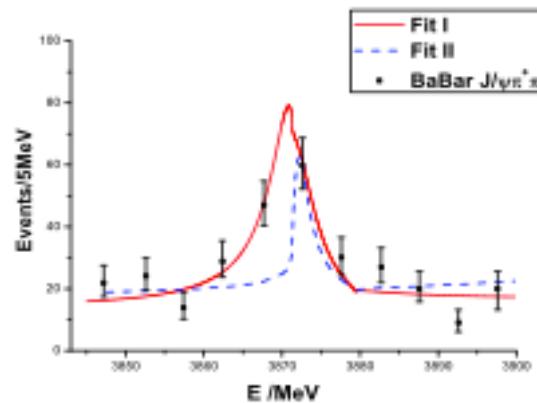
(b)



(c)



(d)



(e)

**Fit I Elementary State**  
**Fit II Molecule**

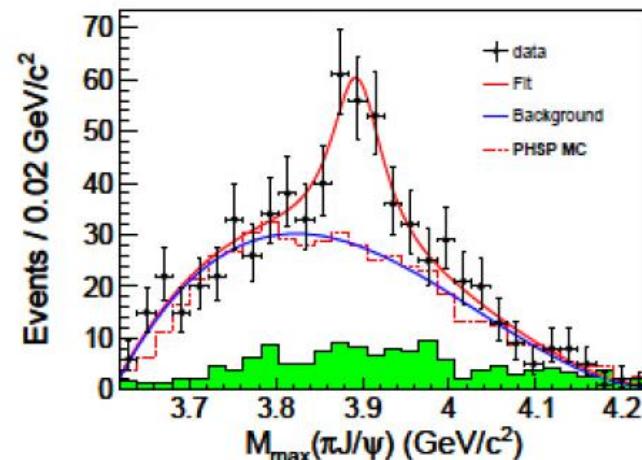
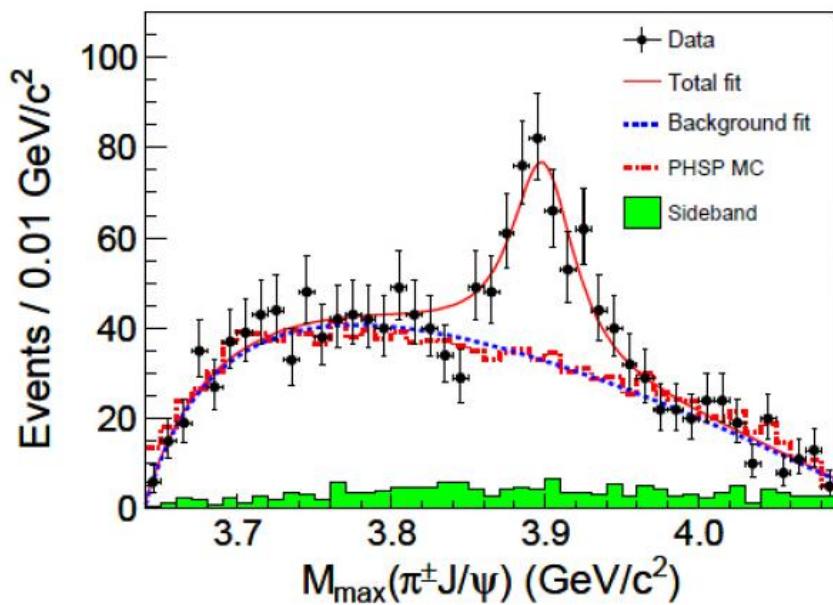
**Two poles**  
**One pole**

arxiv:1411.3106

	Fit I	Fit II
	$\chi^2/dof = 47.1/(60 - 17)$	$\chi^2/dof = 83.3/(60 - 12)$
$\lambda_2$	—	$552.7 \pm 1.1$
$c_0$	—	$(1.70 \pm 0.01) \times 10^{-4}$
$g_1$ (MeV)	$1977 \pm 908$	—
$g_2/g_3$ (MeV)	$196 \pm 52$	—
$g_4$	$0.27 \pm 0.08$	—
$g'_4$	$0.44 \pm 0.11$	—
$g_5$ (MeV $^{-1}$ )	$0.016 \pm 0.014$	1.0 (fixed)
$M_X$ (MeV)	$3870.3 \pm 0.5$	—
$\Gamma_0$ (MeV)	$4.3 \pm 1.5$	—
$N_{11} \cdot g_3^2$ (10 $^{-3}$ MeV $^{-3}$ )	$9.2 \pm 5.0$	$159 \pm 55$
$N_{12} \cdot g_3^2$ (10 $^{-3}$ MeV $^{-3}$ )	$8.1 \pm 4.0$	$181 \pm 53$
$N_{13} \cdot g_3^2$ (10 $^{-3}$ MeV $^{-3}$ )	$9.1 \pm 4.7$	$143 \pm 48$
$N_{21} \cdot g_3^2$ (10 $^{-5}$ MeV $^{-4}$ )	$4.7 \pm 1.3$	$63 \pm 35$
$N_{22} \cdot g_3^2$ (10 $^{-5}$ MeV $^{-4}$ )	$3.9 \pm 1.1$	$116 \pm 33$
$c_{11} \times 10^5$	$3.4 \pm 1.7$	$3.6 \pm 1.4$
$c_{12} \times 10^5$	$1.9 \pm 1.0$	$0.4 \pm 0.2$
$c_{13} \times 10^5$	$1.6 \pm 1.2$	$1.1 \pm 1.0$
$c_{21}$	$15.5 \pm 2.1$	$15.1 \pm 2.0$
$c_{22}$	$13.1 \pm 3.5$	$12.6 \pm 1.4$

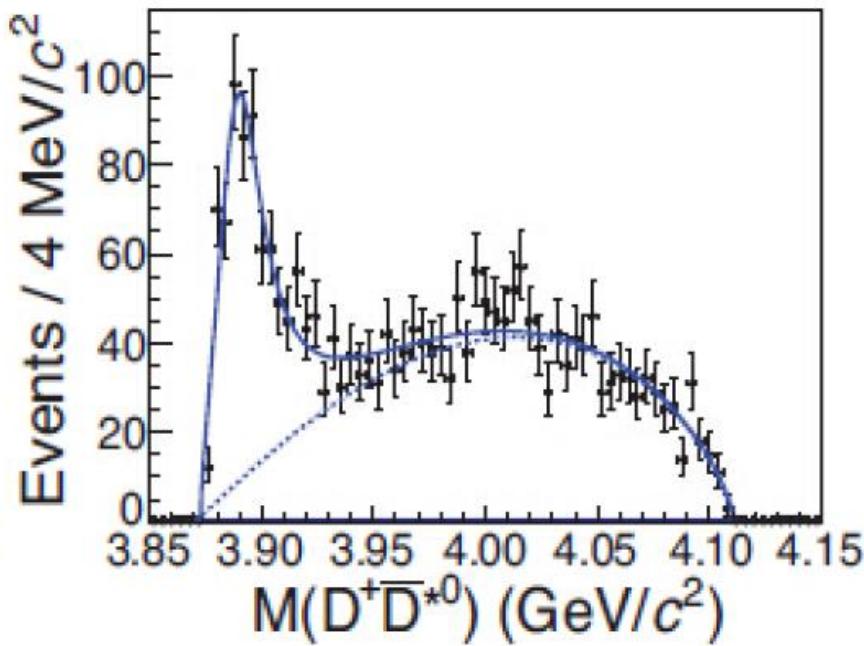
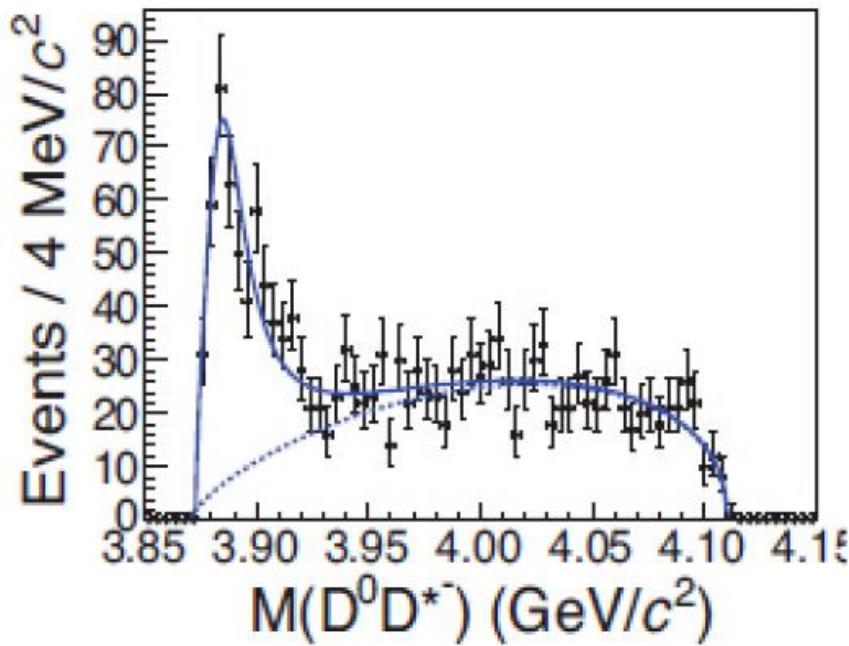
# $Z_c(3900)^\pm$

- BESIII  $e^+e^- \rightarrow \pi\pi J/\psi$  Phys. Rev. Lett. 110, 252001 (2013).
  - $M_z = 3899.0 \pm 3.6 \pm 4.9 \text{ MeV}$ ,  $\Gamma_z = 46 \pm 10 \pm 20 \text{ MeV}$
- Belle  $e^+e^- \rightarrow \pi\pi J/\psi$  Phys. Rev. Lett. 110, 252001 (2013).
  - $M_z = 3894.5 \pm 6.6 \pm 4.5 \text{ MeV}$ ,  $\Gamma_z = 37 \pm 4 \pm 8 \text{ MeV}$



# $Z_c(3900)^\pm$

- BESIII  $e^+e^- \rightarrow \pi D\bar{D}^*$  Phys. Rev. Lett. 112, 022001 (2014).
  - $M_z = 3883.9 \pm 1.5 \pm 4.2 \text{ MeV}$ ,  $\Gamma_z = 24.8 \pm 3.3 \pm 11.0 \text{ MeV}$



# Picture about $Z_c(3900)$

- Hadronic molecule:
  - E. Wilbring, et al., Phys. Lett. B726 (2013) 326,
  - J. R. Zhang, Phys. Rev. D87 116004(2013).
- Tetraquark:
  - J. M. Dias, Phys. Rev. D88 016004(2013),
  - Y. Chen, Phys. Rev. D89 094506(2014).
- Cusp effects:
  - Adam P. Szczepaniak, arXiv:1501.01691.
- .....
- Molecule? Confining state(tetraquark,...)?

# Theoretical model

- $X \rightarrow \pi\pi J/\psi, \pi\pi h_c$

$$\mathcal{L}_{XJ/\psi\pi\pi} = g_1 X_\mu \psi_\nu < u^\mu u^\nu > + g_2 X_\mu \psi^\mu < u^\nu u_\nu > + g_3 X_\mu \psi^\mu < \chi_+ > + \dots \quad (2)$$

$$\mathcal{L}_{XZ_c\pi} = g_4 \nabla_\nu X_\mu < Z_c^\mu u^\nu > + \dots \quad (3)$$

$$\mathcal{L}_{ZcJ/\Psi\pi} = g_7 \nabla_\nu \psi_\mu < Z_c^\mu u^\nu > + \dots \quad (4)$$

$$\mathcal{L}_{Xh_c\pi\pi} = f_8 \nabla^\lambda \nabla_\rho X_\mu H_\nu < u_\lambda u_\sigma > \epsilon^{\mu\nu\rho\sigma} + \dots \quad (5)$$

$$\mathcal{L}_{Z_ch_c\pi} = f_9 \nabla_\alpha H_\nu < Z_\mu u_\beta > + \dots \quad (6)$$

# Theoretical model

- $X \rightarrow \pi D \bar{D}^*$

$$\begin{aligned}\mathcal{L}_{XDD^*\pi} = & f_1 \nabla^\nu X^\mu \langle \bar{D}_\mu^* Du_\nu \rangle + f_2 X^\mu \langle \nabla^\nu \bar{D}_\mu^* Du_\nu \rangle \\ & + f_3 \nabla^\nu X^\mu \langle \bar{D}_\nu^* Du_\mu \rangle + f_4 X^\mu \langle \nabla_\mu \bar{D}^{*\nu} Du_\nu \rangle + \dots\end{aligned}\quad (7)$$

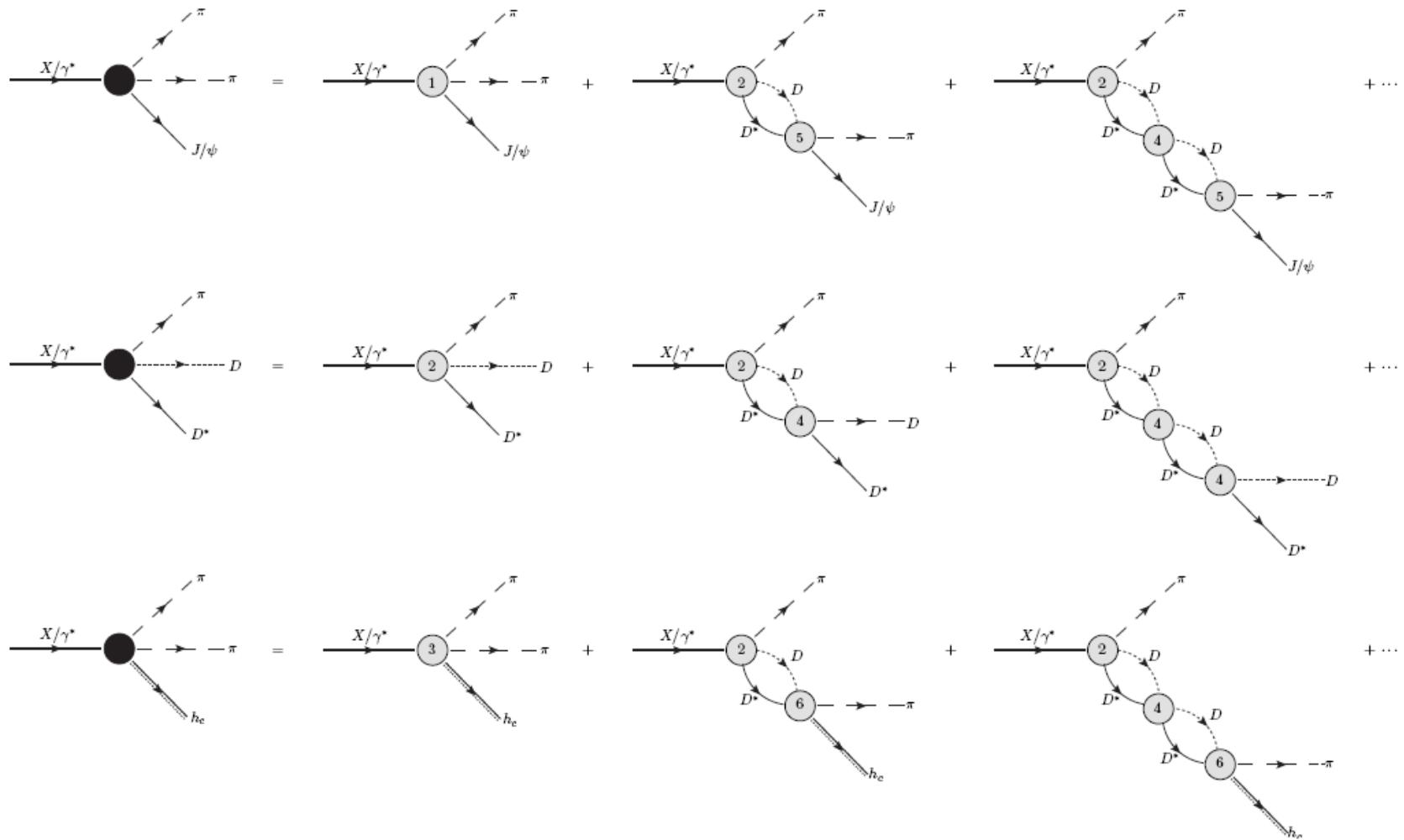
$$\mathcal{L}_{ZcDD^*} = f_7 [(\bar{D}_\mu^{*0} D^+ + D_\mu^{*+} \bar{D}^0) Z_c^{-\mu} + (D_\mu^{*-} D^0 + D_\mu^{*0} D^-) Z_c^{+\mu}] \quad (8)$$

$$\begin{aligned}\mathcal{L}_{D\bar{D}^*D\bar{D}^*} = & \lambda_1 (D^{*+\mu} \bar{D}^0 D_\mu^{*-} D^0 + D^+ \bar{D}^{*0\mu} D^- D_\mu^{*0} + D^{*-\mu} D^0 D_\mu^{*+} \bar{D}^0 + D^- D^{*0\mu} D^+ \bar{D}_\mu^{*0}) \\ & + \lambda_1 (D^{*+\mu} \bar{D}^0 D^- D_\mu^{*0} + D^+ \bar{D}^{*0\mu} D_\mu^{*-} D^0 + D^{*-\mu} D^0 D^+ \bar{D}_\mu^{*0} + D^- D^{*0\mu} D_\mu^{*+} \bar{D}^0)\end{aligned}\quad (9)$$

$$\begin{aligned}\mathcal{L}_{DD^*J/\psi\pi} = & \lambda_3 \nabla^\nu \psi^\mu \langle \bar{D}^{*\mu} Du_\nu \rangle + \lambda_4 \psi^\mu \langle \nabla^\nu \bar{D}^{*\mu} Du_\nu \rangle \\ & + \lambda_5 \nabla^\nu \psi^\mu \langle \bar{D}^{*\nu} Du_\mu \rangle + \lambda_6 \psi^\mu \langle \nabla^\mu \bar{D}^{*\nu} Du_\nu \rangle + \dots\end{aligned}\quad (10)$$

$$\mathcal{L}_{D\bar{D}^*h_c\pi} = \lambda_9 \nabla^\alpha H^\nu \langle \bar{D}^{*\mu} Du^\beta \rangle + \lambda_{10} H^\nu \langle \nabla^\alpha \bar{D}^{*\mu} Du^\beta \rangle + \dots \quad (11)$$

# Theoretical model

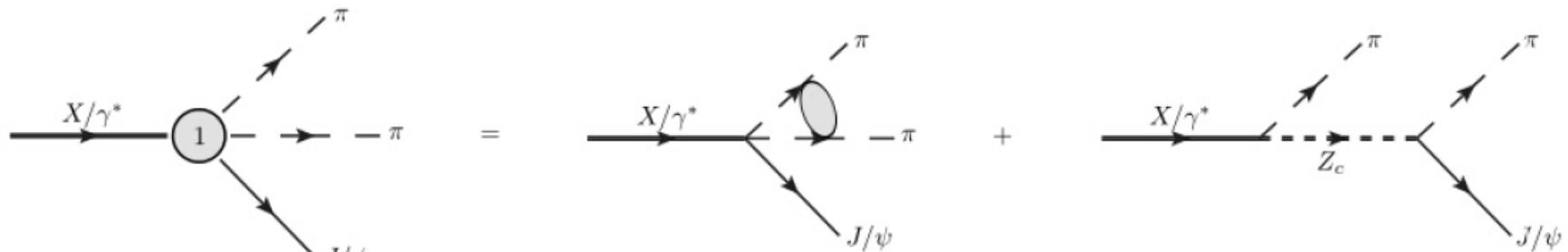


# Amplitude

- Transverse and longitudinal components respectively
  - $1 - i(\lambda_1 + \frac{f_7^2}{p^2 - M_Z^2})\Pi_T, 1 - i\left(\lambda_1 - \frac{f_7^2}{M_Z^2}\right)\Pi_L;$
  - Only focus on the pole in the transverse part, because the pole in the longitudinal component is far from  $3.9\text{GeV}(Z_c)$ .
- One-loop integration of  $D$  and  $D^*$  propagators can be written as:
$$\Pi_{\mu\nu} = \int \frac{d^D k}{(2\pi)^D} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_D^2}}{(k^2 - m_{D^*}^2)[(p-k)^2 - m_D^2]} = P_{T\mu\nu}\Pi_T + P_{L\mu\nu}\Pi_L$$
$$P_{T\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, P_{L\mu\nu} = \frac{p_\mu p_\nu}{p^2}$$
  - Both  $\Pi_T$  and  $\Pi_L$  are divergent and  $\mu$  dependent.
- Pure bubble chain:  $f_7 = 0$ ; Pure Breit-Wigner:  $\lambda_1 = 0$ ; Mixing:  $f_7 \neq 0, \lambda_1 \neq 0$ .

# Partial wave analysis

- Extract S-wave form contact tree vertices



- $\Gamma_{Z_c} \approx 40 \text{ MeV}$ , the final state interaction of two pions in the last diagram is very weak.

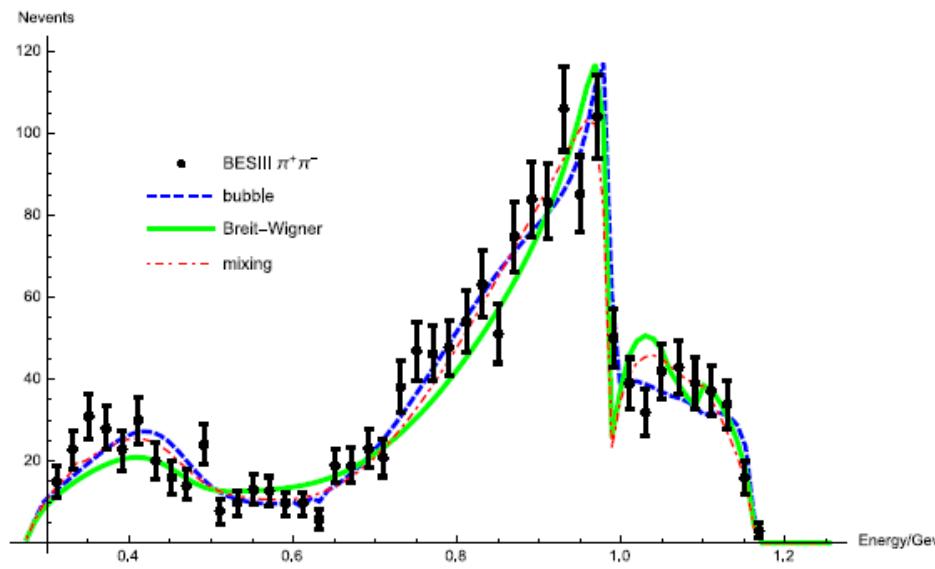
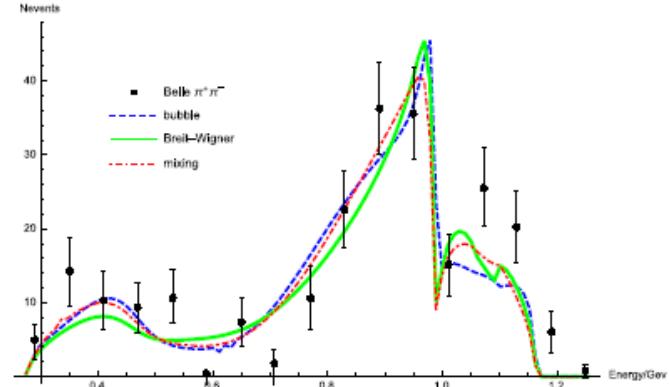
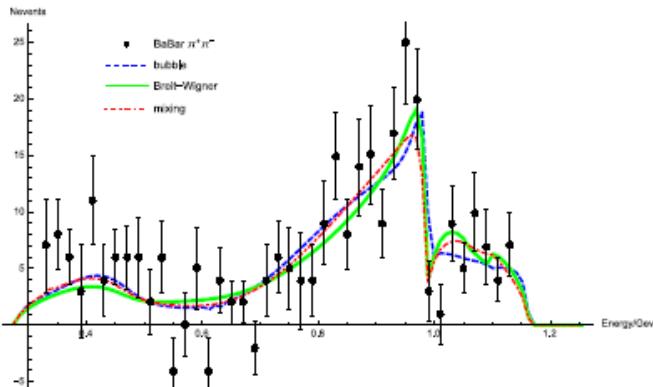
- $X \rightarrow \pi\pi J/\psi, \pi\pi h_c$ :

- $M = M_S^{\pi\pi-tree} \alpha_1(s) T_{\pi\pi \rightarrow \pi\pi} + M_S^{KK-tree} \alpha_1(s) T_{KK \rightarrow \pi\pi} + M'$

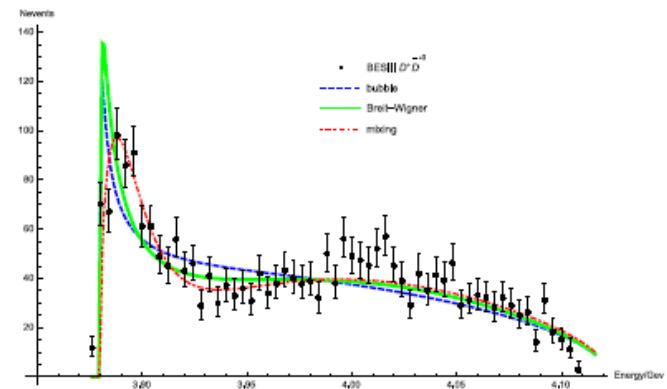
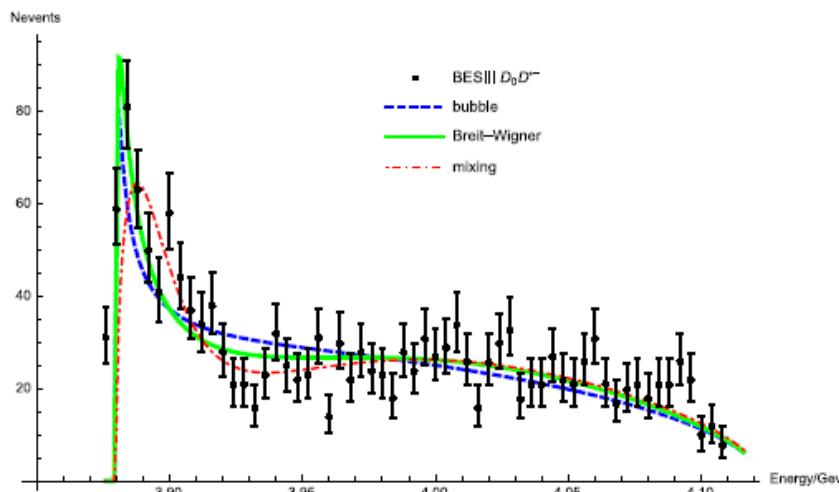
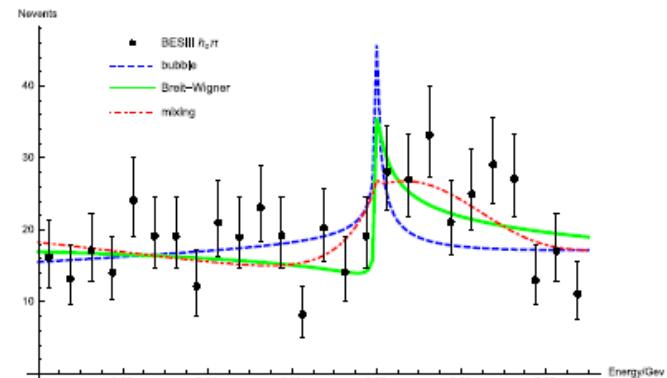
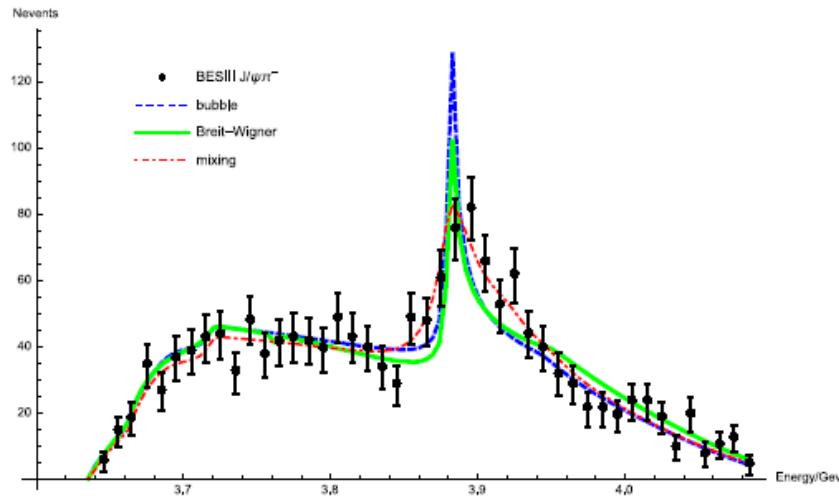
- $\alpha_i(s) = \frac{c_0^i}{s-s_A} + c_1^i + c_2^i s + \dots$  (s is the energy square of  $\pi\pi$  system in  $\pi\pi$  rest frame.  $s_A$  is Adler zero.)

- $M'$ : amplitude correspond to the last diagram.

# $\pi\pi$ inv. mass spectrum



# $\pi J/\psi$ , $\pi h_c$ and $DD^*$ inv. mass spectrum



# Preliminary numerical results

Riemann Sheets	Sheet I	Sheet II	Sheet III	Sheet IV
$\text{Im}\Gamma_{h_c\pi+J/\psi\pi}$	+	-	-	+
$\text{Im}\rho_{DD^*}$	+	+	-	-

$\chi^2 / d.o.f$

Fit I,  $456.5/(332-28)=1.5$

Fit II,  $491/(332-25)=1.874$

	sheet I	sheet II	sheet III	sheet IV
Fit I	-	$3.8869 \pm 0.0035i$	-	-
Fit II	-	-	-	$3.8791 \pm 0.0014i$

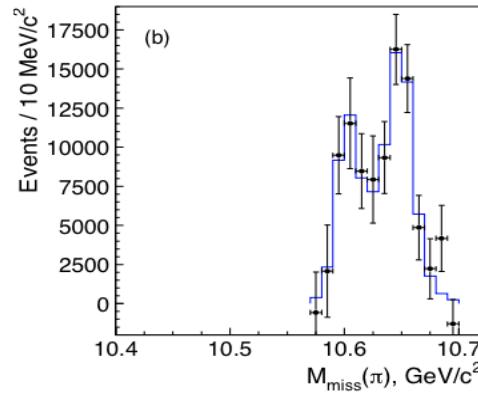
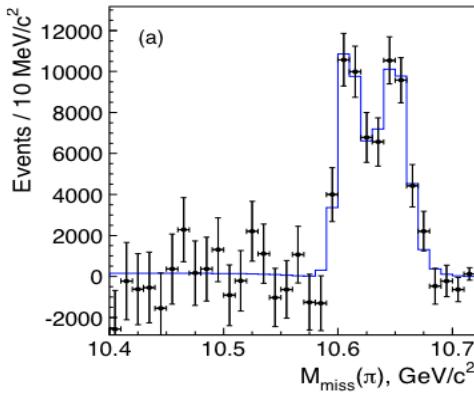
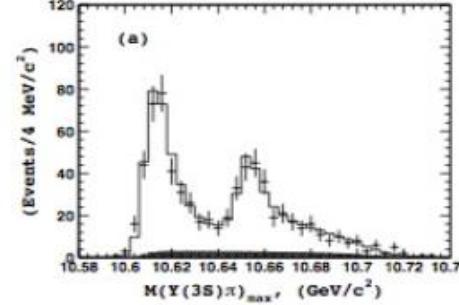
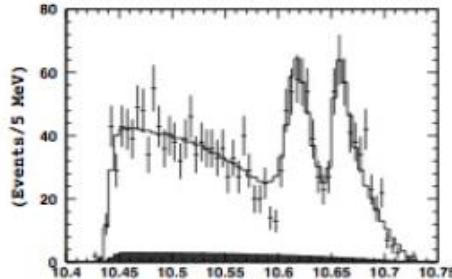
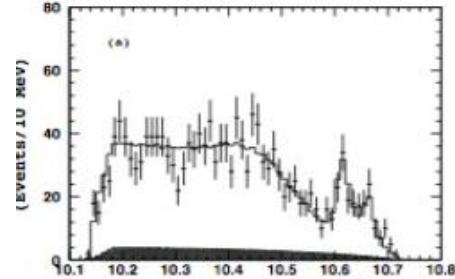
- Compared with X(3872) (in part II)

	mixing	bubble
$\chi^2/dof$	$47.1/(60 - 17)$	$83.3/(60 - 12)$

Sheet	Fit I	Fit II
I	$3871.1-3.3i$	—
II	$3870.5-3.7i$	$3871.7-0.9i$
III	$3869.0-4.0i$	—
IV	$3869.8-3.5i$	—

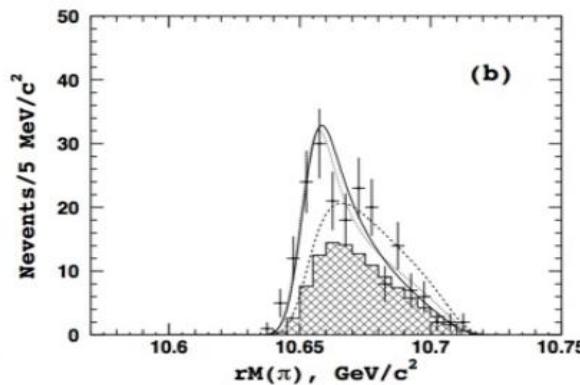
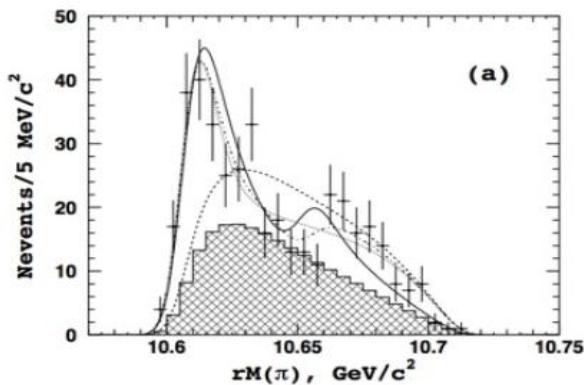
- $Z_c(3900)$  may be very different with X(3872).

# Zb(10610) and Zb(10650)



$$\frac{\Gamma[\Upsilon(5S) \rightarrow h_b(nP) \pi^+ \pi^-]}{\Gamma[\Upsilon(5S) \rightarrow \Upsilon(2S) \pi^+ \pi^-]} = \begin{cases} 0.45 \pm 0.08^{+0.07}_{-0.12} & \text{for } h_b(1P) \\ 0.77 \pm 0.08^{+0.22}_{-0.17} & \text{for } h_b(2P) \end{cases}$$

no flip! spin-flip!



$\Upsilon(nS)\pi\pi$   
n=1,2,3

$BB^*\pi$   
and  
 $B^*B^*\pi$

- Many Evidences point to a molecule Zb state

M. B. Voloshin

$$Z_b(10610) \sim (B^* \bar{B} - \bar{B}^* B) \sim \frac{1}{\sqrt{2}} \left( 0_H^- \otimes 1_{SLB}^- + 1_H^- \otimes 0_{SLB}^- \right) ,$$

$$Z_b(10650) \sim B^* \bar{B}^* \sim \frac{1}{\sqrt{2}} \left( 0_H^- \otimes 1_{SLB}^- - 1_H^- \otimes 0_{SLB}^- \right) ,$$

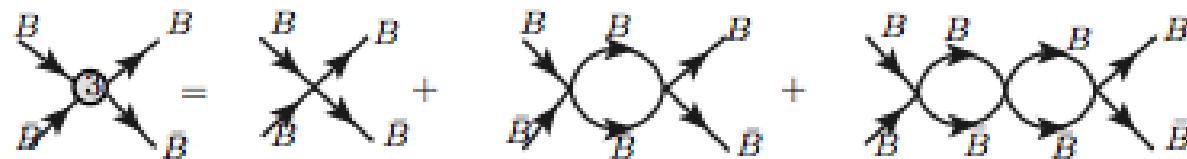
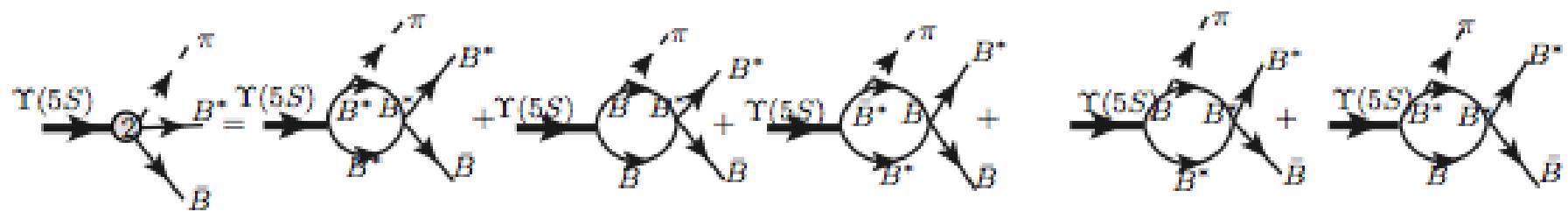
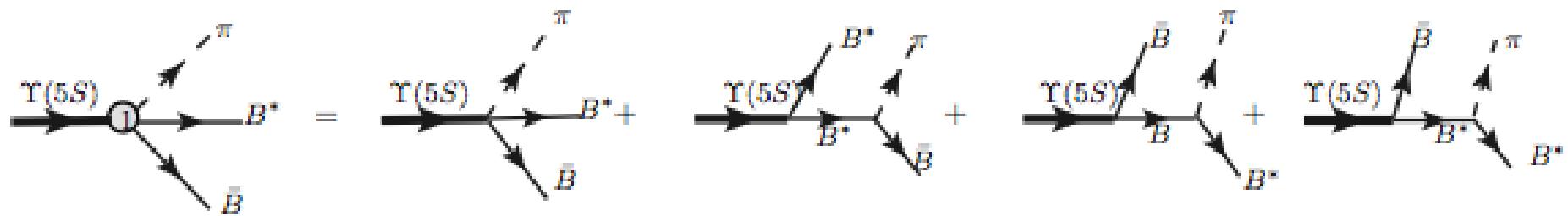
T. Mehen

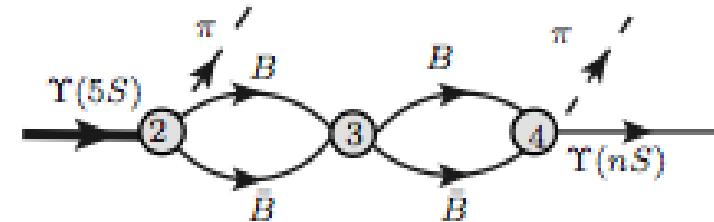
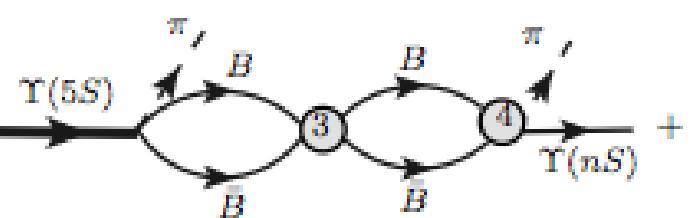
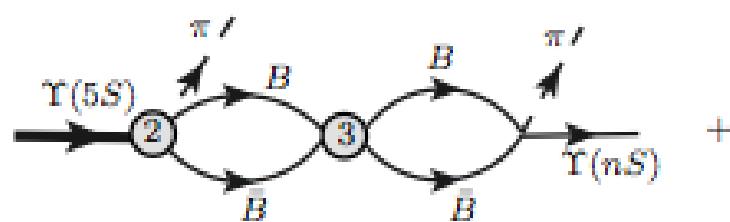
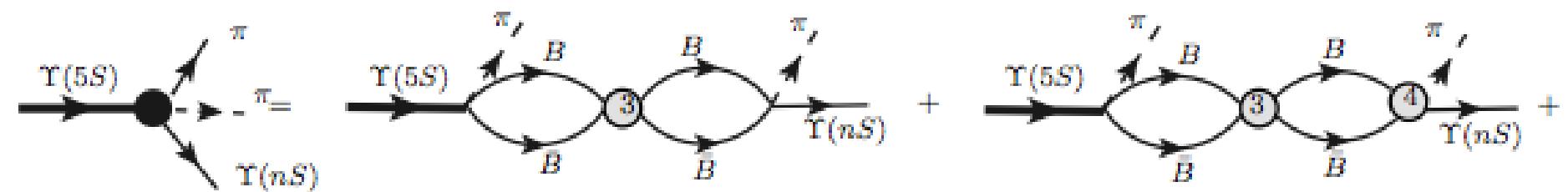
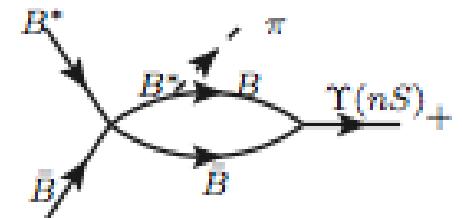
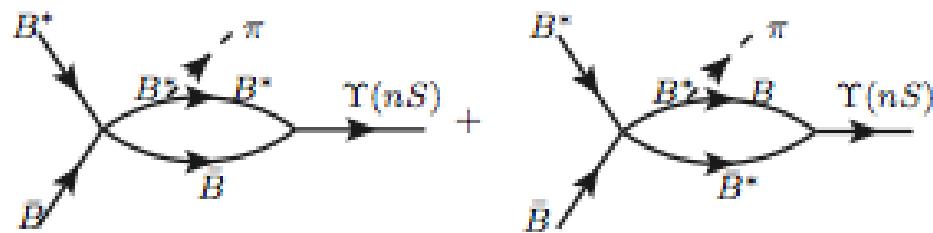
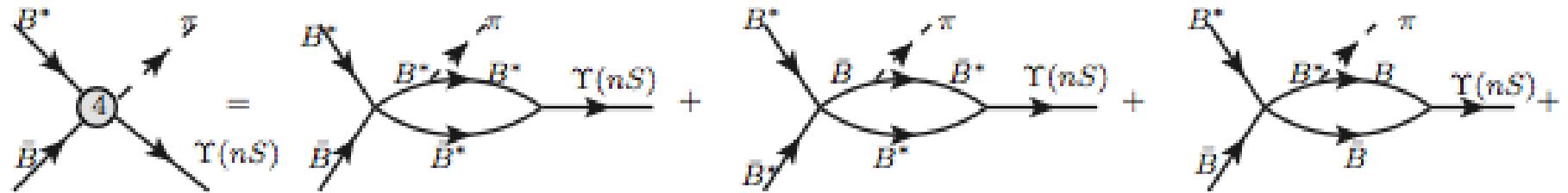
$$Z^{Ai} = \frac{1}{\sqrt{2}} (V_a^i \tau_{ab}^A \bar{P}_b - P_a \tau_{ab}^A \bar{V}_b^i), \quad Z'^{Ai} = \frac{i}{\sqrt{2}} \epsilon^{ijk} V_a^j \tau_{ab}^A \bar{V}_b^k,$$

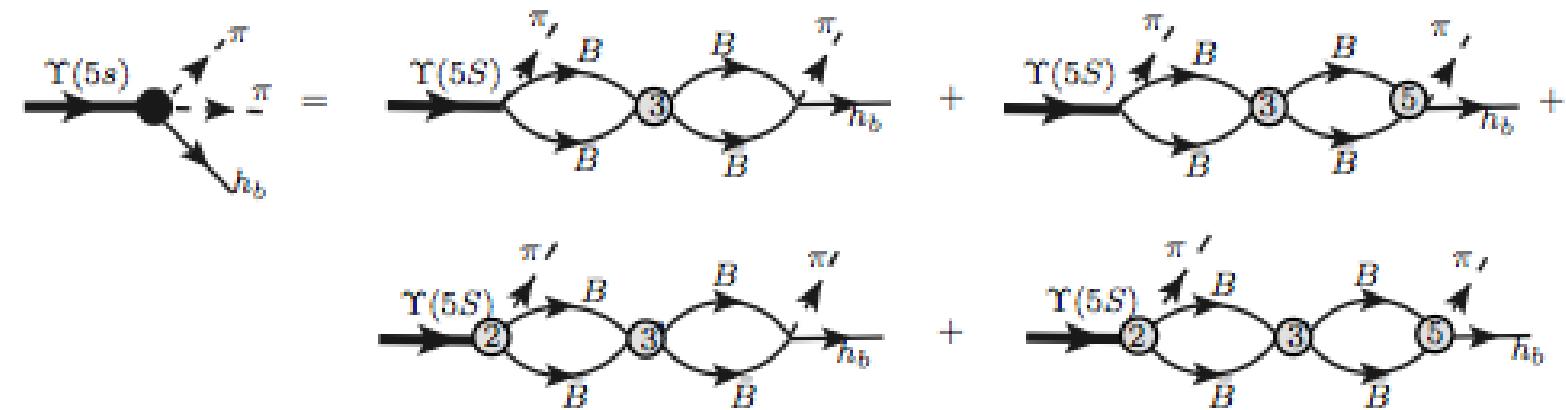
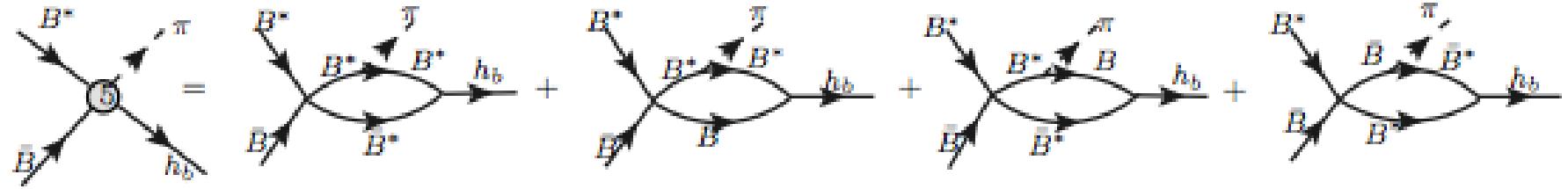
$$\begin{aligned}
\mathcal{L} = & g_\pi Tr[\bar{H}_a^\dagger \sigma^i \bar{H}_b] A_{ab}^i - g_\pi Tr[\bar{H}_a^\dagger \sigma^i \bar{H}_b] A_{ab}^i \\
& + \frac{1}{2} g_{\pi\Upsilon,n} Tr[\Upsilon_n^\dagger H_a \bar{H}_b] A_{ab}^0 + \frac{1}{2} g_{\Upsilon,n} Tr[\Upsilon_n^\dagger H_a \sigma^j \overset{\leftrightarrow}{\partial}_j i \bar{H}_a] + H.c. \\
& + \frac{1}{2} g_{\pi\chi,n} Tr[\chi_{n,i}^\dagger H_a \sigma^j \bar{H}_b] \epsilon_{ijk} A_{ab}^k + \frac{i}{2} g_{\chi,n} Tr[\chi_{n,i}^\dagger H_a \sigma^i \bar{H}_a] + H.c. \\
& + \frac{1}{4} g'_{\pi\Upsilon,n} Tr[(\Upsilon \sigma^i + \sigma^i \Upsilon) \bar{H}_a^\dagger \sigma^i H_a^\dagger] A^0 + H.c..
\end{aligned}$$

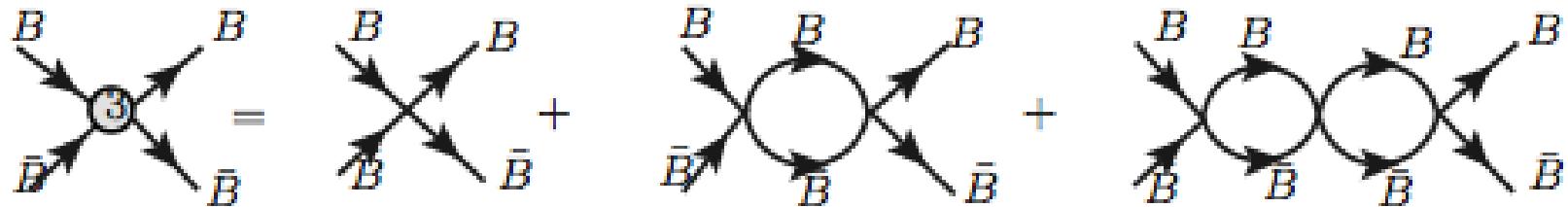
$$\begin{aligned}
H_a = & P_a + V_a^i \sigma^i, \quad \bar{H}_a = \bar{P}_a - \bar{V}_a^i \sigma^i \\
\Upsilon_n = & \sigma^i \Upsilon^i + \eta_b, \quad \chi_n^i = \sigma_l (\chi_{b2}^{il} + \frac{1}{\sqrt{2}} \epsilon^{ilm} \chi_{b1}^m + \frac{1}{\sqrt{3}} \delta^{il} \chi_{b0}) + h_b^i.
\end{aligned}$$

$$Z^{Ai} = \frac{1}{\sqrt{2}}(V_a^i \tau_{ab}^A \bar{P}_b - P_a \tau_{ab}^A \bar{V}_b^i), \quad Z'^{Ai} = \frac{i}{\sqrt{2}} \epsilon^{ijk} V_a^j \tau_{ab}^A \bar{V}_b^k.$$









$$\begin{aligned}
 T_Z &= -C_Z + C_Z \Sigma_Z C_Z - C_Z \Sigma_Z C_Z \Sigma_Z C_Z + \dots \\
 &= -(1 + T_Z \Sigma_Z) C_Z.
 \end{aligned}$$

$$T_Z^{-1} = -C_Z^{-1} - \Sigma_Z(E),$$

$$C_Z = \begin{pmatrix} C_+ & C_- \\ C_- & C_+ \end{pmatrix} = \begin{pmatrix} C_{11} + C_{10} & C_{11} - C_{10} \\ C_{11} - C_{10} & C_{11} + C_{10} \end{pmatrix}$$

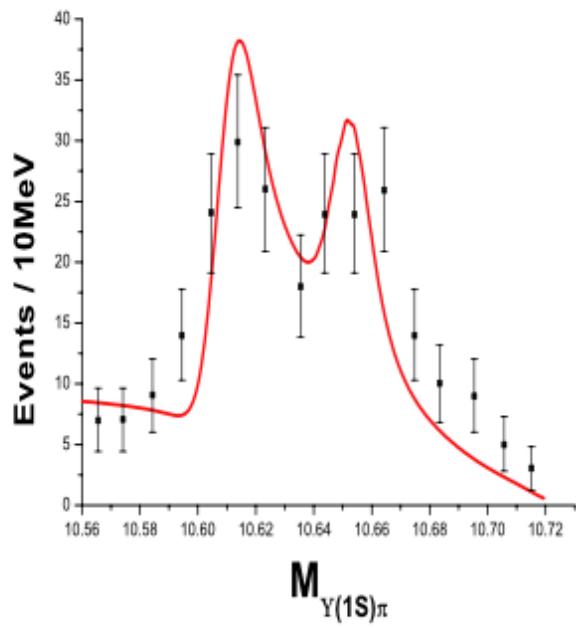
$$\begin{aligned}
 C_{10} &= \frac{2\pi}{M} \frac{1}{-\Lambda + \gamma_{10}}, & \Sigma_Z(E) &= \begin{pmatrix} \Sigma_{B^* B^*}(E) & 0 \\ 0 & \Sigma_{B^* B}(E) \end{pmatrix} \\
 C_{11} &= \frac{2\pi}{M} \frac{1}{-\Lambda + \gamma_{11}}. & &= \frac{M}{4\pi} \begin{pmatrix} \Lambda - \sqrt{M(2\Delta - E) - i\epsilon} & 0 \\ 0 & \Lambda - \sqrt{M(\Delta - E) - i\epsilon} \end{pmatrix}
 \end{aligned}$$

$$T_Z=\frac{4\pi}{M}\left(\begin{array}{cc} \frac{\Delta_1-\gamma_+}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}&\frac{\gamma_-}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}\\ \frac{\gamma_-}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}&\frac{\Delta_2-\gamma_+}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}\end{array}\right)$$

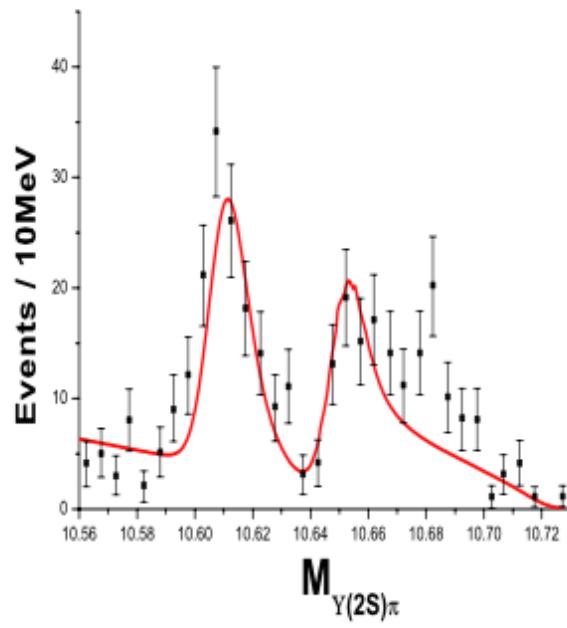
$$\Delta_1 = \sqrt{M(\Delta-E) - i\epsilon} \quad \Delta_2 = \sqrt{M(2\Delta-E) - i\epsilon}$$

$$\gamma_{\pm}=(\gamma_{11}\pm\gamma_{10})/2$$

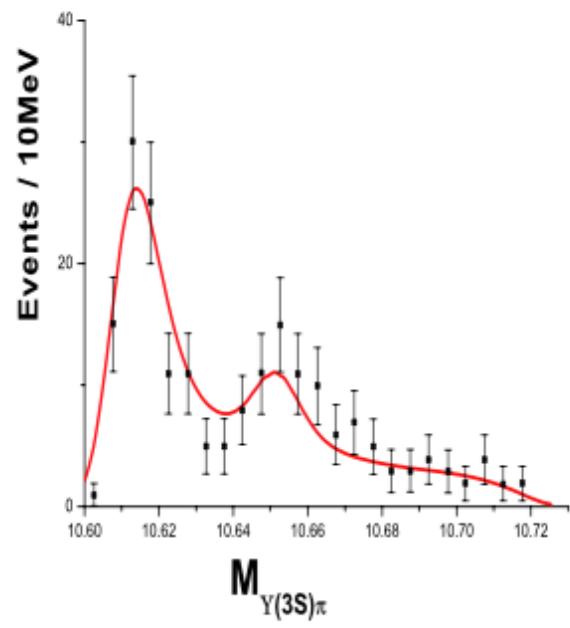
$$\Delta=m_{B^\ast}-m_B$$



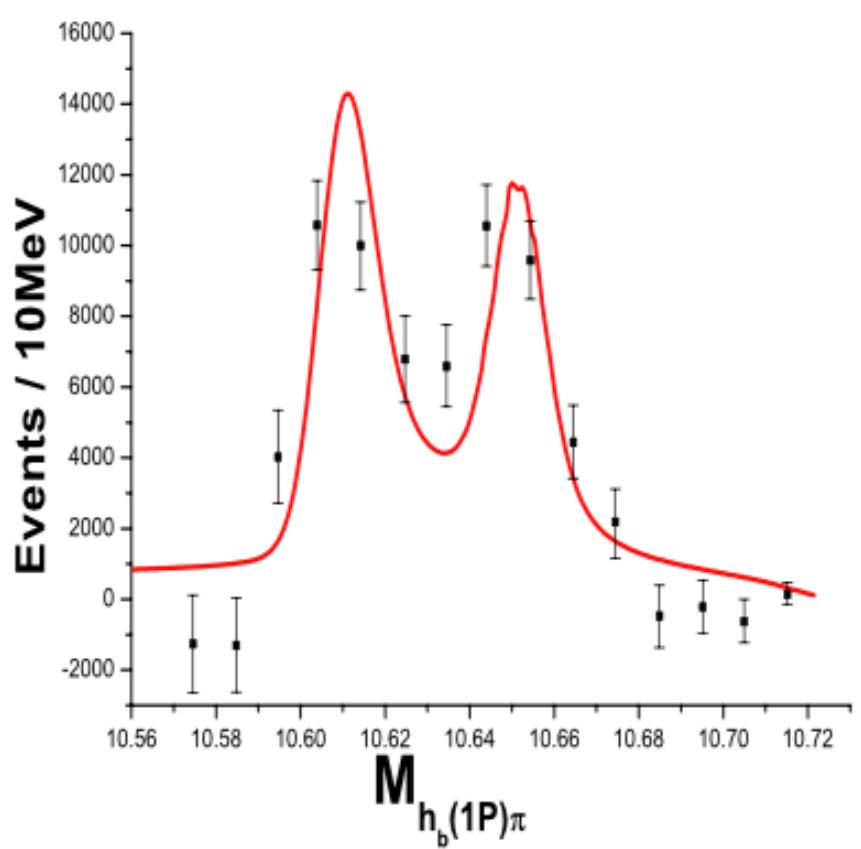
(a)



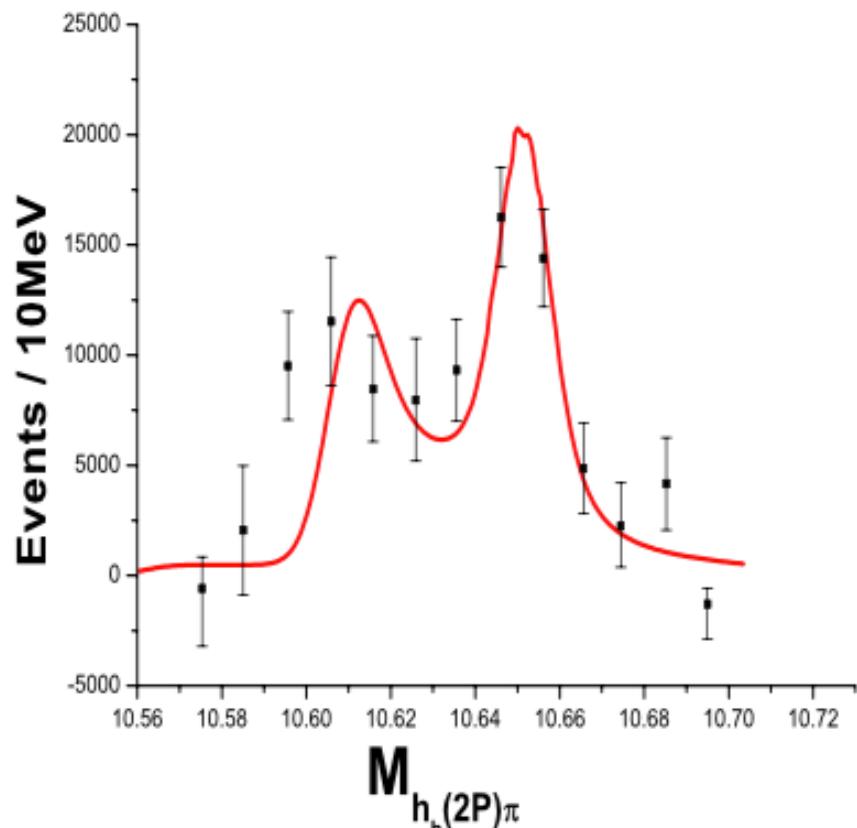
(b)



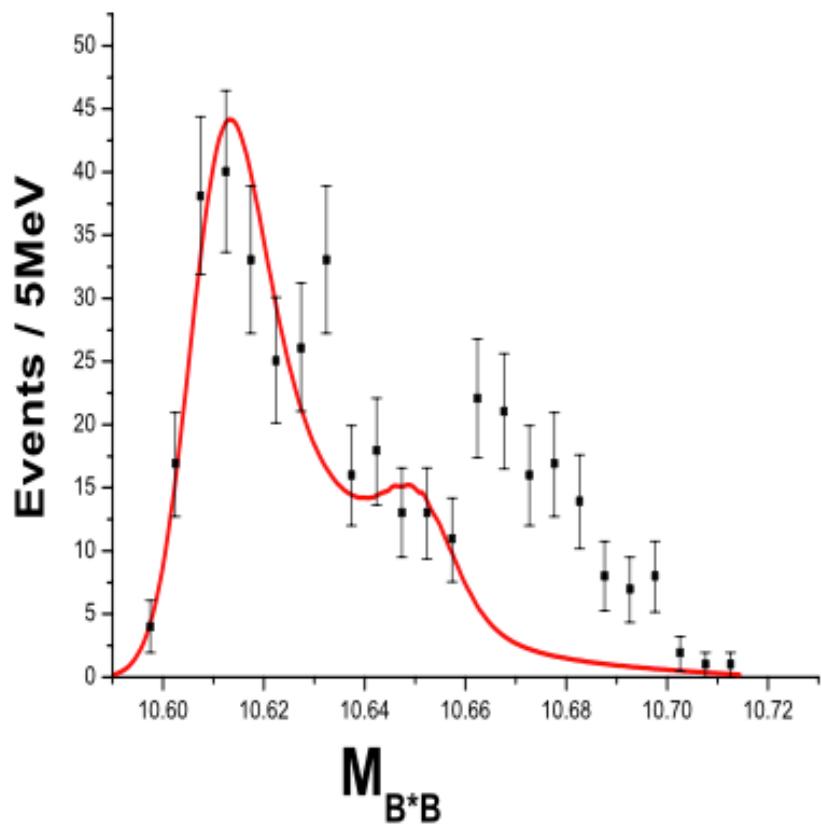
(c)



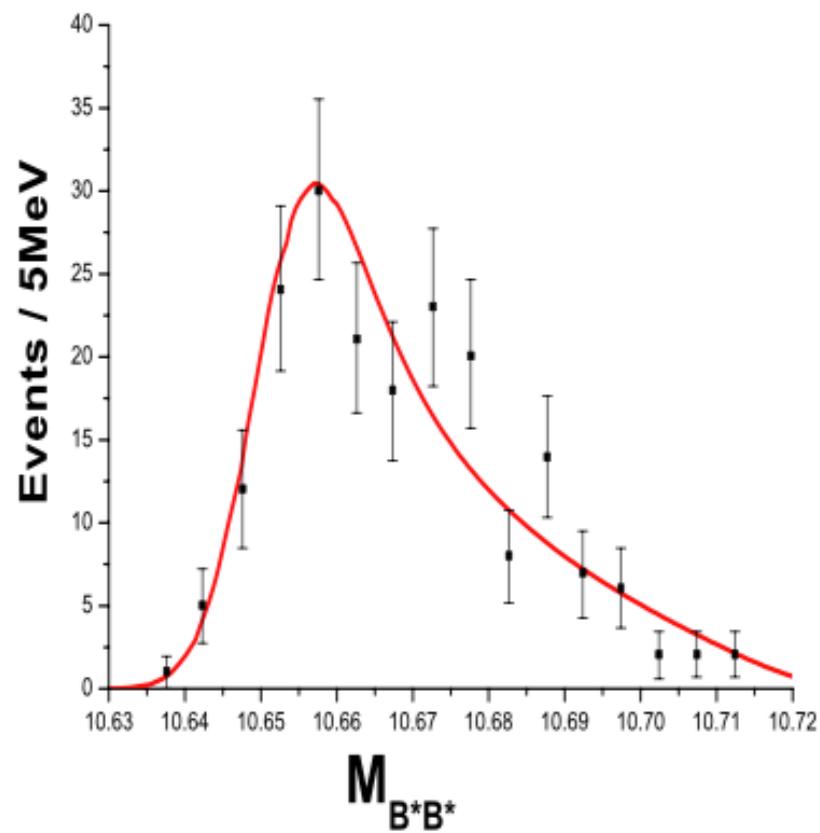
(d)



(e)



(f)



(g)

$\text{Re}[\gamma_+] = -143 \pm 3 \text{ MeV}$	$\text{Im}[\gamma_+] = -138 \pm 2 \text{ MeV}$
$\text{Re}[\gamma_-] = -222 \pm 3 \text{ MeV}$	$\text{Im}[\gamma_-] = 10.2 \pm 1.8 \text{ MeV}$
$g_{\pi\Upsilon,1} = 0.017 \pm 0.005 \text{ GeV}^{-\frac{3}{2}}$	$g_{\pi\Upsilon,2} = 0.235 \pm 0.023 \text{ GeV}^{-\frac{3}{2}}$
$g_{\pi\Upsilon,3} = 0.510 \pm 0.063 \text{ GeV}^{-\frac{3}{2}}$	$g_{\pi\Upsilon,5} = 3.408 \pm 0.216 \text{ GeV}^{-\frac{3}{2}}$
$g_{\Upsilon,1} = 0.022 \pm 0.004 \text{ GeV}^{-\frac{3}{2}}$	$g_{\Upsilon,2} = -0.150 \pm 0.071 \text{ GeV}^{-\frac{3}{2}}$
$g_{\Upsilon,3} = 0.654 \pm 0.321 \text{ GeV}^{-\frac{3}{2}}$	$g_{\Upsilon,5} = -0.112 \text{ GeV}^{-\frac{3}{2}} (\text{fixed})$
$g_{\sigma,1} = 0.367 \pm 0.028$	$g_{f_0,1} = 1.526 \pm 0.610$
$g_{\sigma,2} = 0.957 \pm 0.141$	$g_{\sigma,3} = 0.514 \pm 0.021$
$g_{\pi\chi,1} = -0.784 \pm 0.039 \text{ GeV}^{-\frac{3}{2}}$	$g_{\chi,1} = -0.258 \pm 0.031 \text{ GeV}^{-\frac{1}{2}}$
$g_{\pi\chi,2} = -1.172 \pm 0.280 \text{ GeV}^{-\frac{3}{2}}$	$g_{\chi,2} = -0.445 \pm 0.133 \text{ GeV}^{-\frac{1}{2}}$

Channel	our model	PDG [24]
$\text{BR}(\Upsilon(1S)\pi\pi)$	$(3.4 \pm 2.1) \times 10^{-3}$	$(5.3 \pm 0.6) \times 10^{-3}$
$\text{BR}(\Upsilon(2S)\pi\pi)$	$(1.3 \pm 0.5) \times 10^{-2}$	$(7.8 \pm 1.3) \times 10^{-3}$
$\text{BR}(\Upsilon(3S)\pi\pi)$	$(2.5 \pm 2.2) \times 10^{-3}$	$(4.8_{-1.7}^{+1.9}) \times 10^{-3}$
$\text{BR}(h_b(1P)\pi\pi)$	$(6.5 \pm 2.4) \times 10^{-3}$	$(3.5_{-1.3}^{+1.0}) \times 10^{-3}$
$\text{BR}(h_b(2P)\pi\pi)$	$(7.6 \pm 6.6) \times 10^{-3}$	$(6.0_{-1.8}^{+2.1}) \times 10^{-3}$
$\text{BR}(B\bar{B}^*\pi + \bar{B}B^*\pi)$	$(11.6 \pm 2.1)\%$	$(7.3 \pm 2.3)\%$
$\text{BR}(B^*\bar{B}^*\pi)$	$(2.4 \pm 0.4)\%$	$(1.0 \pm 1.4)\%$

$$\Gamma_{Z_b \rightarrow \Upsilon(1S)\pi} : \Gamma_{Z_b \rightarrow \Upsilon(2S)\pi} : \Gamma_{Z_b \rightarrow \Upsilon(3S)\pi} = 0.10 : 2.76 : 1 ,$$

$$\Gamma_{Z'_b \rightarrow \Upsilon(1S)\pi} : \Gamma_{Z'_b \rightarrow \Upsilon(2S)\pi} : \Gamma_{Z'_b \rightarrow \Upsilon(3S)\pi} = 0.07 : 1.95 : 1 ,$$

$$\Gamma_{Z_b \rightarrow h_b(1P)\pi} : \Gamma_{Z_b \rightarrow h_b(2P)\pi} = 4.2 ,$$

$$\Gamma_{Z'_b \rightarrow h_b(1P)\pi} : \Gamma_{Z'_b \rightarrow h_b(2P)\pi} = 3.4 .$$

$$T_Z=\frac{4\pi}{M}\left(\begin{array}{cc} \frac{\Delta_1-\gamma_+}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}&\frac{\gamma_-}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}\\ \frac{\gamma_-}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}&\frac{\Delta_2-\gamma_+}{(\gamma_+-\Delta_1)(\gamma_+-\Delta_2)-\gamma_-^2}\end{array}\right)$$

$$\Delta_1 = \sqrt{M(\Delta-E) - i\epsilon} \quad \Delta_2 = \sqrt{M(2\Delta-E) - i\epsilon}$$

$$\gamma_{\pm}=(\gamma_{11}\pm\gamma_{10})/2$$

	sheet I	sheet II	sheet III	sheet IV
$\Delta_1$	+	-	-	+
$\Delta_2$	+	+	-	-

	$Z_b(B\bar{B}^*)$	$Z'_b(B^*\bar{B}^*)$
Sheet I	10595.3-14.7i	
Sheet II	—	10655.5-10.2i
Sheet III	—	—
Sheet IV		—

# Final Conclusions

Careful studies using pole counting rule:

- 1)  $X(3872)$  is mainly a cc bar state
- 2)  $X(4260)$  is an elementary particle
- 3)  $Zc(3900)$  (and  $Zb$ ) is a molecular state

## A few comments on $f_0(980)$

Renewed interests...

Confusions in the literature...

“A Dispersive Analysis on the  $f_0(600)$  and  $f_0(980)$  Resonances in  $\gamma\gamma \rightarrow \pi^+\pi^-$ ,  $\pi^0\pi^0$  Processes” Y. Mao et al., PRD79(2009)116008 A K-matrix fit gives:

pole	sheet-II	sheet-III
$\sigma$	$0.549 - 0.230i$	$0.705 - 0.327i$
$f_0(980)$	$0.999 - 0.021i$	$0.977 - 0.060i$

Table: The poles's location on the  $\sqrt{s}$ -plane, in units of GeV.

Not quite stable

It has a tiny di-photon width (not a  $\bar{q}q$  state):

	Pole-positions(GeV)	$\Gamma(f_J \rightarrow \gamma\gamma)$ (keV)
$f_0^{II}(980)$	$0.999 - 0.021i$	0.12
$f_0^{III}(980)$	$0.977 - 0.060i$	0.35
$f_0(600)$	$0.549 - 0.230i$	0.76
$f_2(1270)(\lambda = 0)$	$1.272 - 0.087i$	0.66
$f_2(1270)(\lambda = 2)$		3.70

It also maintains some odd properties which is hard to understand:

pole position	$g_{\pi\pi}^2$	$g_{KK}^2$
$\sqrt{s_{II}} = 0.999 - 0.021i$	$-0.07 - 0.01i$	$-0.10 + 0.09i$
$\sqrt{s_{III}} = 0.977 - 0.060i$	$-0.10 + 0.02i$	$-0.02 - 0.09i$

$f_0(600)$  also has a negative residue, but that seems to be well understood (see my talk at Chiral dynamics 09, Bern). (It's broad!) K– Matrix analysis for  $f_0(600)$  can be refined using the production representation (PKU)

## Further evidence in support of an odd $f_0(980)$

"On the scalar nonet in the extended Nambu Jona-Lasinio model"  
M. X. Su et al., Nucl.Phys. A792 (2007) 288-305

ENJL model + Heat Kernel expansion  $\Rightarrow$  Effective chiral lagrangian in a linear realization.

With an unnaturally small  $\Lambda$ , and a bare  $M_\sigma = 1 \text{ GeV}$ . Using a K-matrix unitarization amplitude, one predicts the mass and width of  $\sigma$  ( $f_0(600)$ ),  $\kappa$  ( $K^*(700)$ ) and  $a_0(980)$  simultaneously. No way to put the  $f_0(980)$  in the game.

Here scalars are chiral partners of Nambu-Goldstone bosons, suggesting  $f_0(980)$  may be different.

"Pole analysis on unitarized  $SU(3) \times SU(3)$  one loop  $\chi$ PT amplitudes" Dai L.Y. et al., Commun. Theor. Phys. 58 (2012) 410-414

We analyze  $\pi\pi - K\bar{K}$  and  $\pi\eta - K\bar{K}$  couple channel [1,1] matrix Padé amplitudes of  $SU(3) \times SU(3)$  chiral perturbation theory. By fitting phase shift and inelasticity data, we determine pole positions in different channels ( $f_0(980)$ ,  $a_0(980)$ ,  $f_0(600)$ ,  $K_0^*(800)$ ,  $K^*(892)$ ,  $\rho(770)$ ) and trace their  $N_c$  trajectories. We stress that a couple channel Breit–Wigner resonance should exhibit two poles on different Riemann sheets that reach the same position on the real axis when  $N_c = \infty$ . (Extended pole counting rule) Poles are hence classified using this criteria and we conclude that  $K^*(892)$  and  $\rho(770)$  are unambiguous Breit–Wigner resonances. For scalars the situation is much less clear. We find that  $f_0(980)$  is a molecular state rather than a Breit–Wigner resonance, while  $a_0(980)$ , though behaves oddly when varying  $N_c$ , does maintain a twin pole

Conclusions on  $f_0(980)$ :

- ▶ No sound conclusion
- ▶ Not like a  $q\bar{q}$ , neither a tetra quark
- ▶ Does not like to be in a chiral multiplet
- ▶ Prefer a molecule interpretation.

Thank you!