Quantum Chromodynamics (QCD)

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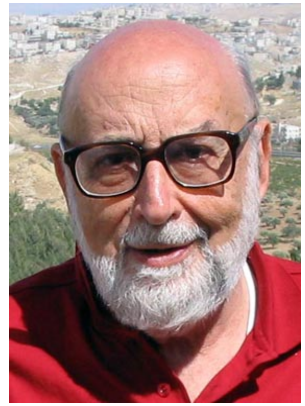
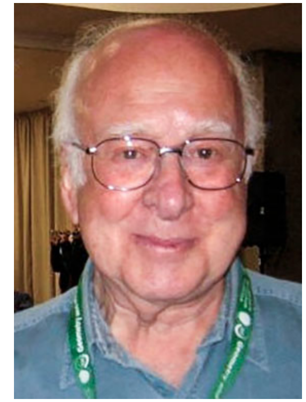
Today's big story - discovery of Higgs boson

Nobel Prize, 2013



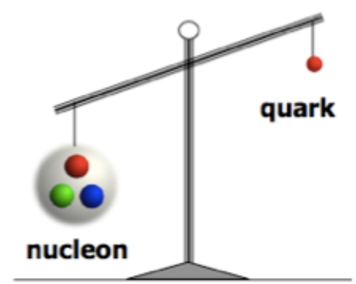
François Engler



Peter W. Higgs

"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

But, the Higgs mechanism generates
Too little to be relevant for
the mass of our world of visible matter!



$m_q \sim 10$ MeV

$m_N \sim 1000$ MeV

It is QCD that is responsible for the mass of the visible world!

The plan for my four lectures

- The Goal:

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- The Plan (approximately):

  Fundamentals of QCD, factorization, evolution,

  and elementary hard processes

  Two lectures

  Role of QCD in high energy collider phenomenology

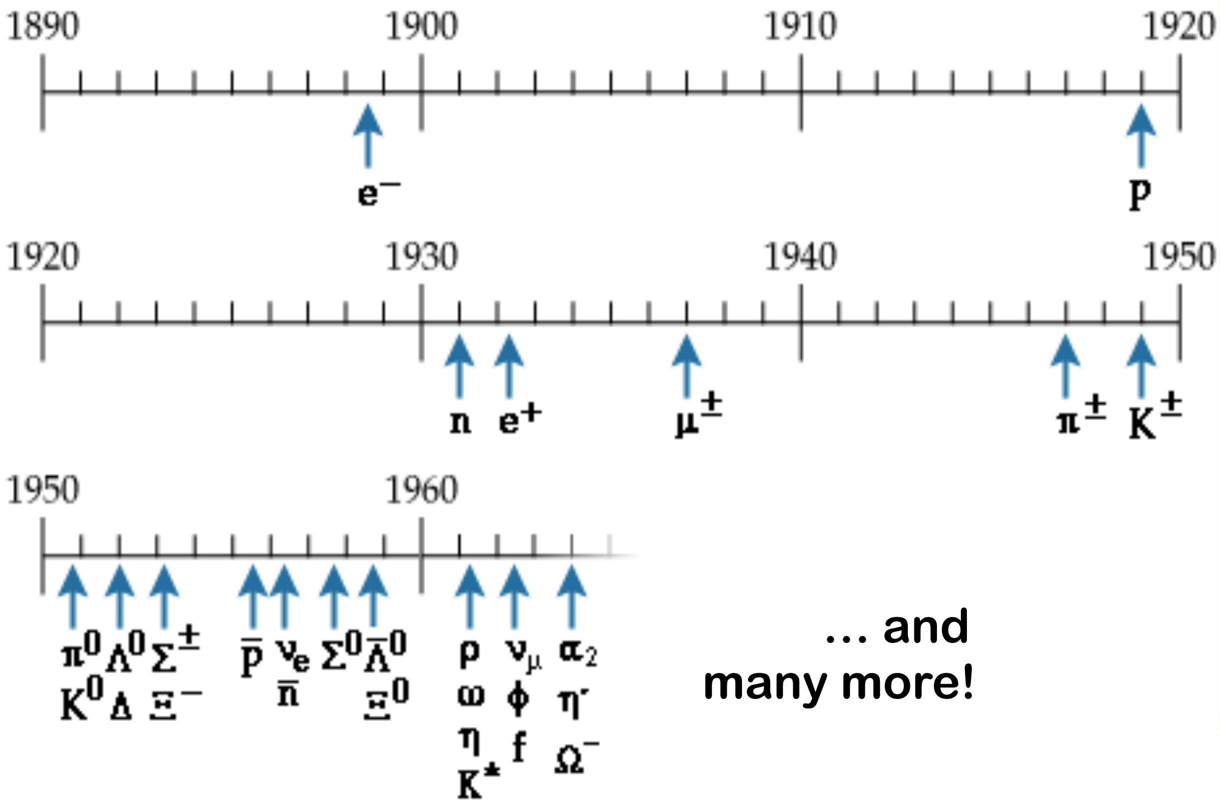
  One lecture

  QCD and hadron structure and properties

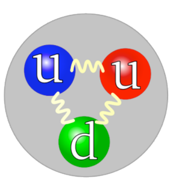
  One lecture

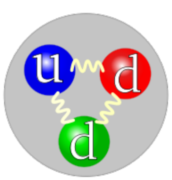
New particles, new ideas, and new theories

- Early proliferation of new particles – "particle explosion":



→ Quark Model

Proton 

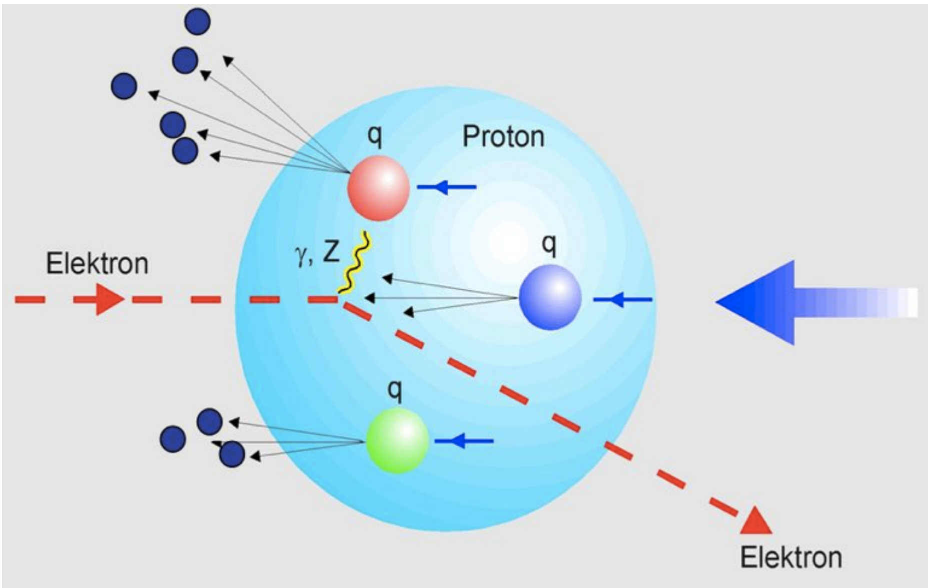
Neutron 

Nobel Prize, 1969

Deep inelastic scattering (DIS)

❑ Modern Rutherford experiment – DIS (SLAC 1968)

$$e(p) + h(P) \to e'(p') + X$$



✧ **Localized probe:**

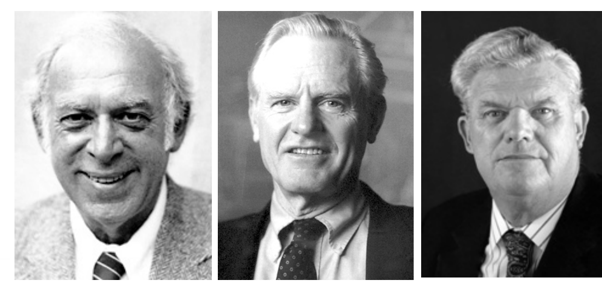
$$Q^2 = -(p-p')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \text{fm}$$

✧ **Two variables:**

$$Q^2 = 4EE'\sin^2(\theta/2)$$
$$x_B = \frac{Q^2}{2m_N\nu}$$
$$\nu = E - E'$$

→ **Discovery of spin ½ quarks, and partonic structure!**



Nobel Prize, 1990

⇒ **The birth of QCD (1973)**

– Quark Model + Yang-Mill gauge theory

Quantum Chromo-dynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:**

$\psi_i^f(x)$ — **Quark fields: spin-½ Dirac fermion (like electron)**

Color triplet: $i = 1, 2, 3 = N_c$

Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$ — **Gluon fields: spin-1 vector field (like photon)**

Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f$$
$$- \frac{1}{4} \left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc} A_{\mu,b} A_{\nu,c} \right]^2$$
$$+ \text{ gauge fixing} + \text{ghost terms}$$

- **QED – force to hold atoms together:**

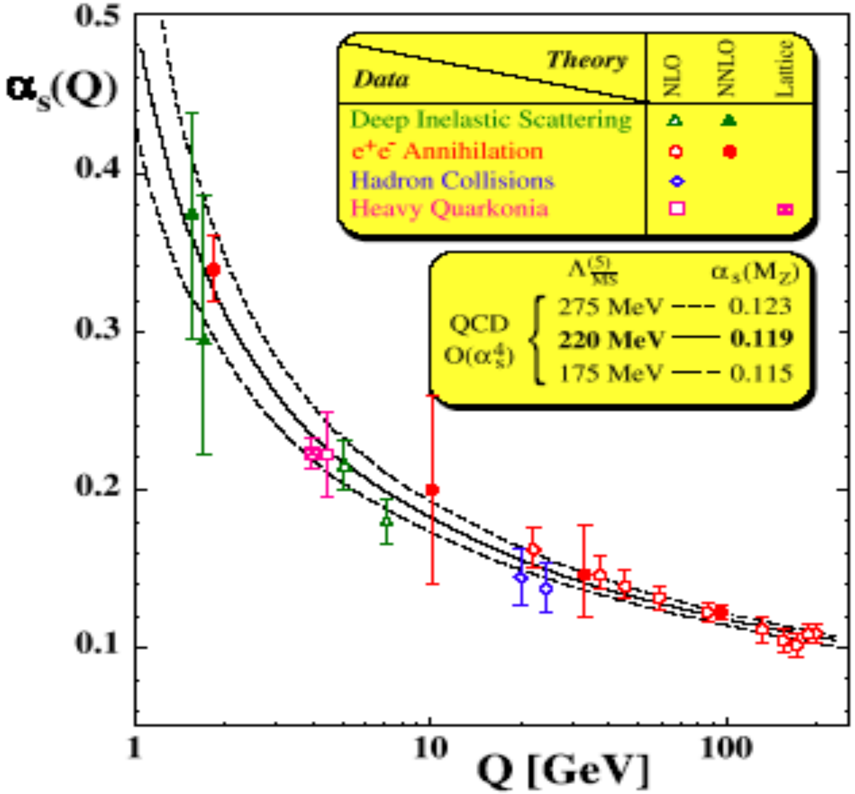
$$\mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}^f \left[(i\partial_\mu - eA_\mu)\gamma^\mu - m_f \right] \psi^f - \frac{1}{4} \left[\partial_\mu A_\nu - \partial_\nu A_\mu \right]^2$$

- **QCD Color confinement:**

Gluons are dark, No free quarks or gluons ever been detected!

QCD Asymptotic Freedom

- **Interaction strength:** $\alpha_s(\mu_2) = \dfrac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \dfrac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{\rm QCD}^2}\right)}$



μ_2 and μ_1 not independent

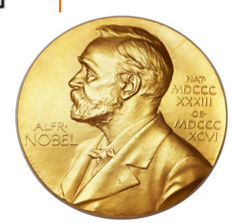
Asymptotic Freedom ⇔ antiscreening

$$\text{QCD:}\ \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED:}\ \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)



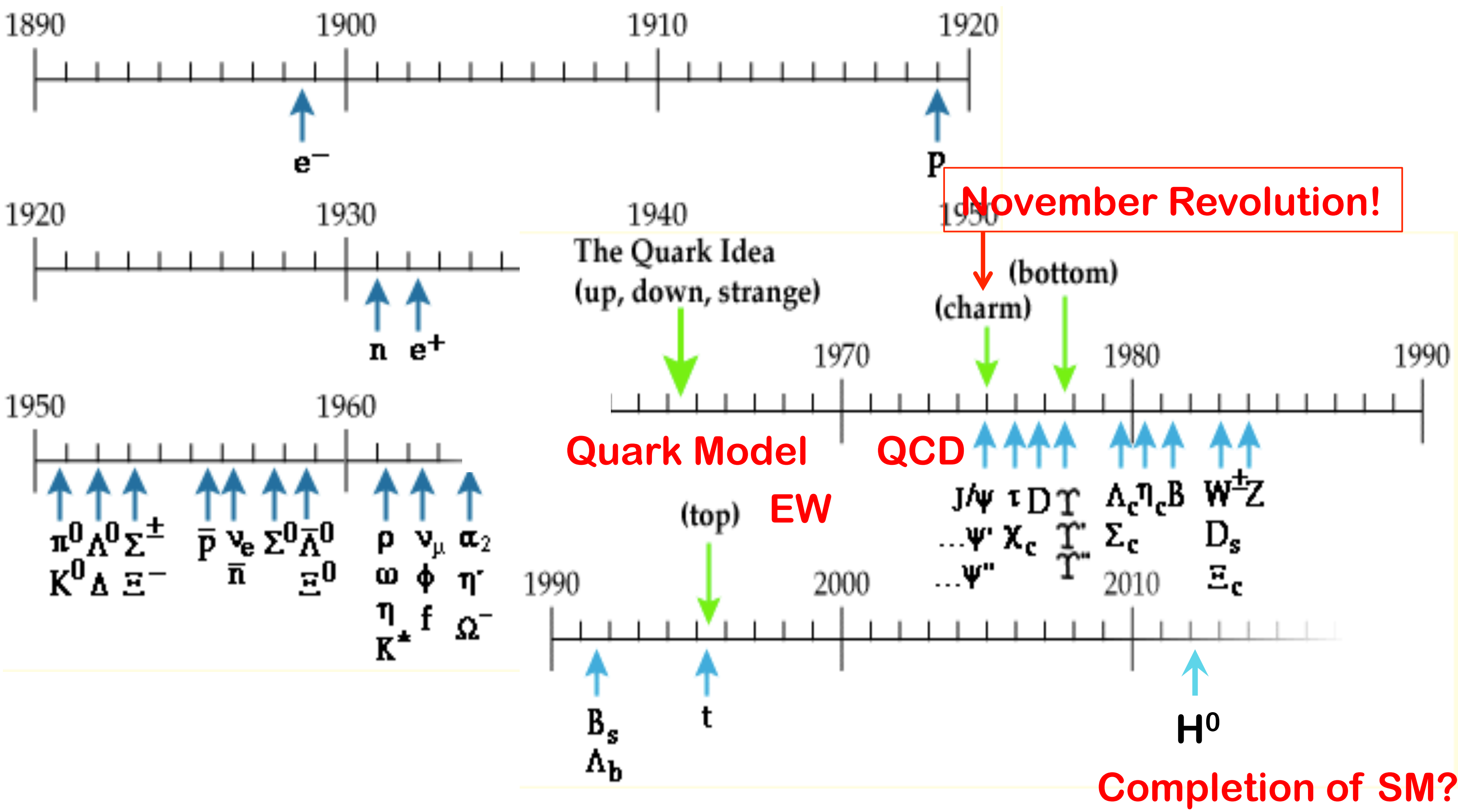
→ **Discovery of QCD Asymptotic Freedom**

→ **Collider phenomenology**
– *Controllable perturbative QCD calculations*

Nobel Prize, 2004

New particles, new ideas, and new theories

- **Proliferation of new particles – "November Revolution":**



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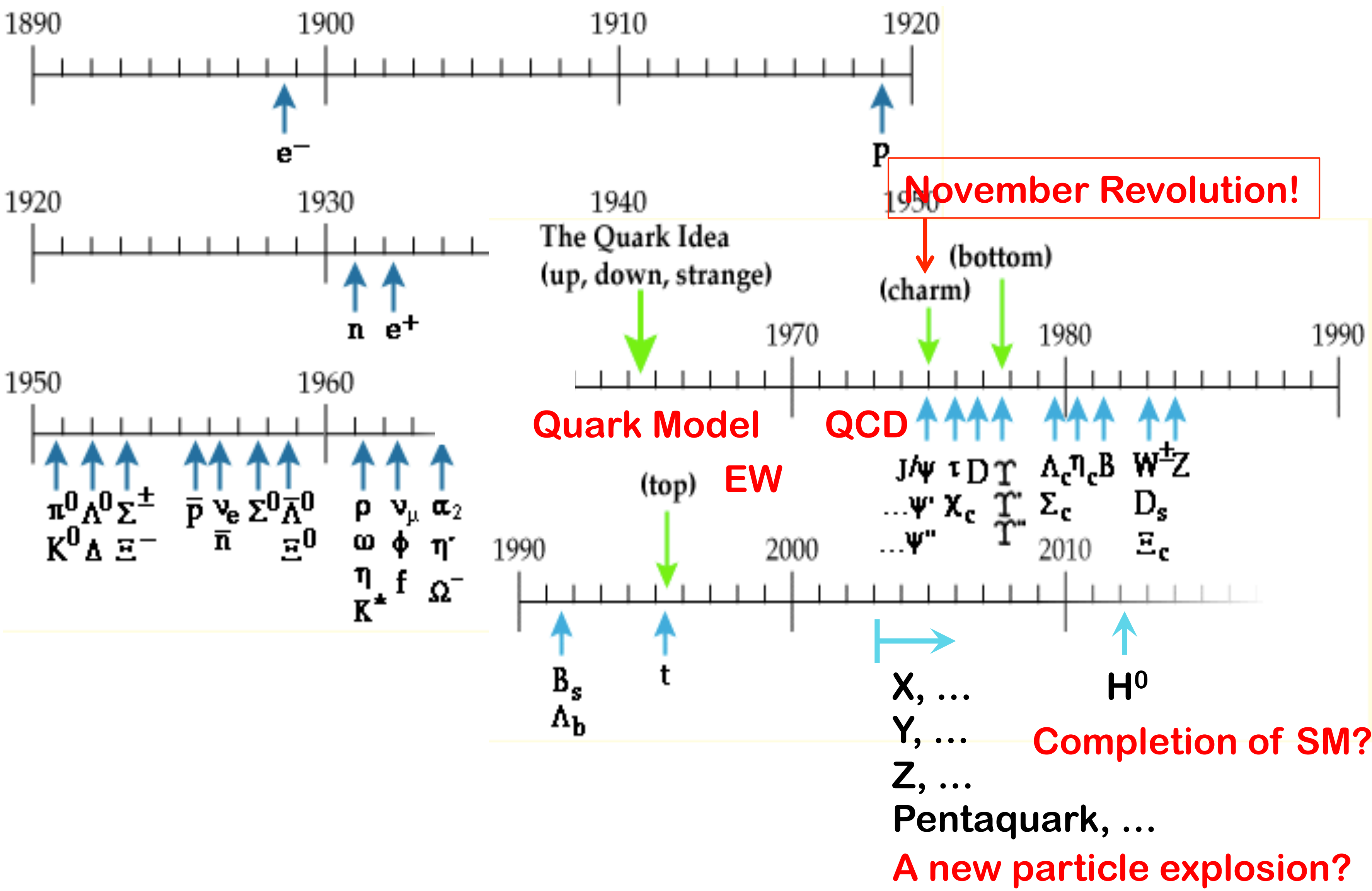
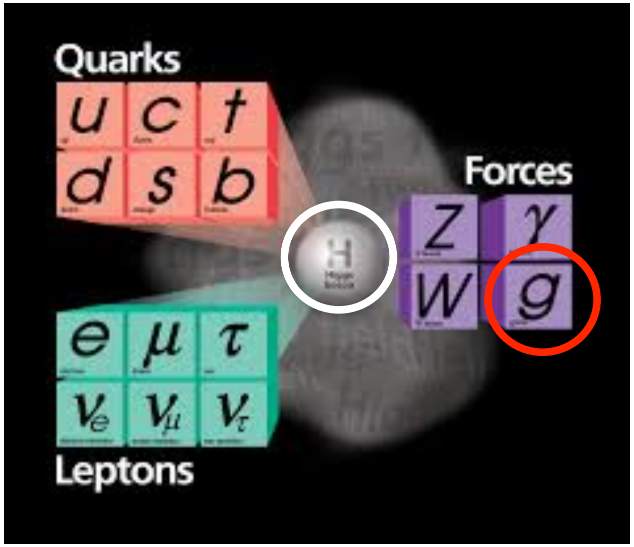
❑ Proliferation of new particles – "November Revolution":



The Standard Model

The "most" successful story to tell

Found every elementary particles in the table but, not even one extra!



All of them behave in the way that they are supposed to behave

We found the "lonely" Higgs too!

What is *beyond* the Higgs and the SM?

The true nature of the effective theory?

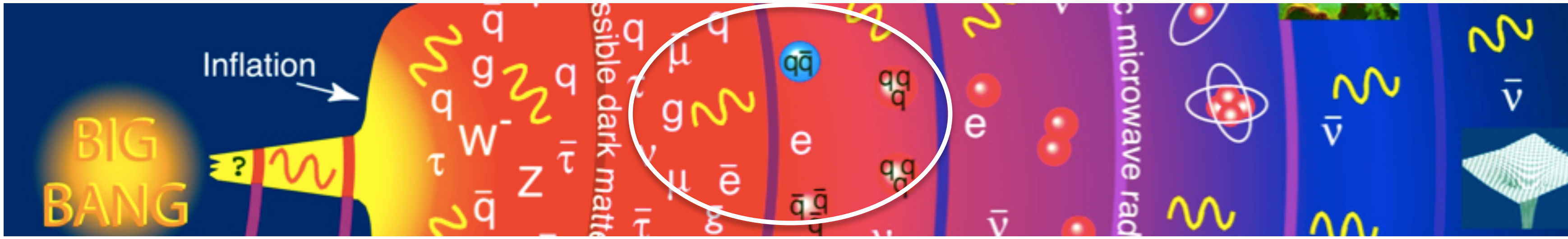
What we have not been able to address *within* the SM?

The magic of glue that binds us all

Without gluons, no hadrons, no atomic nuclei, no human, …, no visible world!

Overarching questions for QCD

- ❑ What is the role of QCD in the evolution of the universe?



- ❑ How hadrons are emerged from quarks and gluons?
- ❑ How does QCD make up the properties of hadrons?

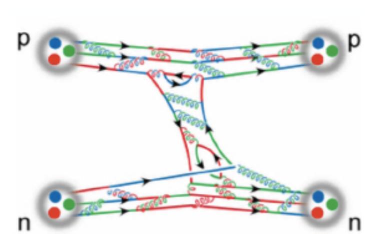
  Their mass, spin, magnetic moment, …

- ❑ What is the QCD landscape of nucleon and nuclei?

Color Confinement — Asymptotic freedom

Q (GeV)

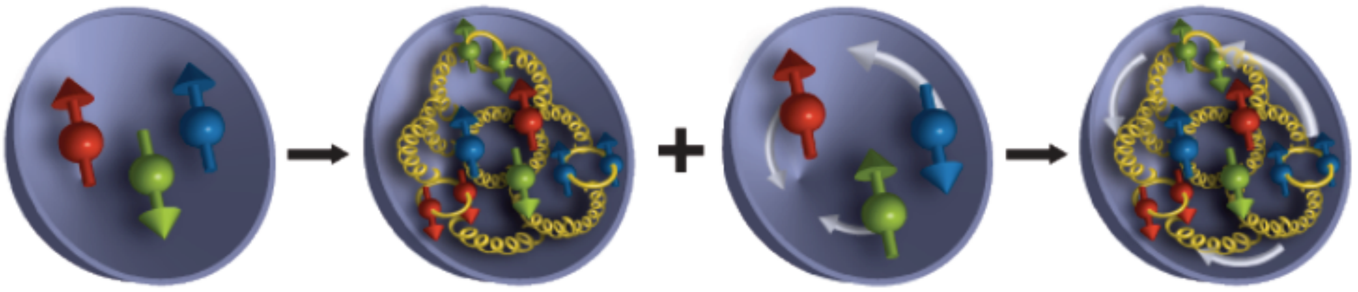
200 MeV (1 fm) — 2 GeV (1/10 fm) — Probing momentum



- ❑ How do the nuclear force arise from QCD?
- ❑ …

Example: The proton

- Our understanding of the proton evolves



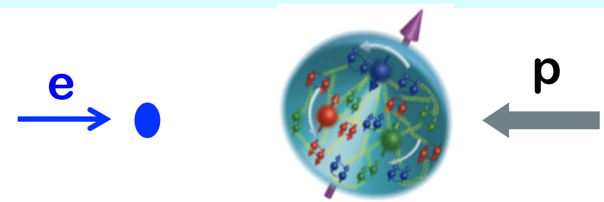
  - Proton is a strongly interacting, relativistic bound state of quarks and gluons
  - Neither quarks nor gluons appear in isolation!
  - Understanding such systems completely is still beyond the capability of the best minds in the world

- The great intellectual challenges:

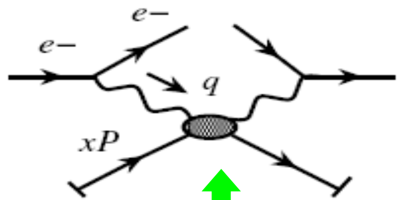
  - *Probe QCD dynamics without "seeing" quarks and gluons?*
  - Cross section involving an identified proton (or a hadron) is NOT IR safe and is NOT perturbatively calculable!

Theoretical approaches – approximations

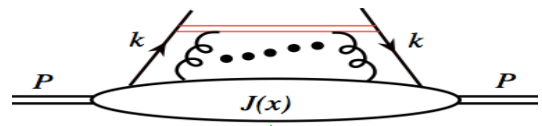
- **Perturbative QCD Factorization:**
  - *– Approximation at Feynman diagram level*



$\sigma_{\text{tot}}^{\text{DIS}}$:



$\otimes$



$+ \, O\left(\frac{1}{QR}\right)$

Probe Hard-part	Structure Parton-distribution	Approximation Power corrections

- **Effective field theory (EFT):**
  - *– Approximation at the Lagrangian level*

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Other approximation or model approaches:**

  Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

- **Lattice QCD:**
  - *– Approximation mainly due to computer power*

  Hadron structure, hadron spectroscopy, nuclear structure, …

Foundation of perturbative QCD

- ❑ **Renormalization**
  - – QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- ❑ **Asymptotic freedom**
  - – weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Welczek

- ❑ **Infrared safety and factorization**
  - – calculable short distance dynamics
  - – pQCD factorization – connect the partons to physical cross sections

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe and factorizable observables!

Effective quark mass

- **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since} \quad \gamma_m(g(\lambda)) > 0$$

- **Choice of renormalization scale:**

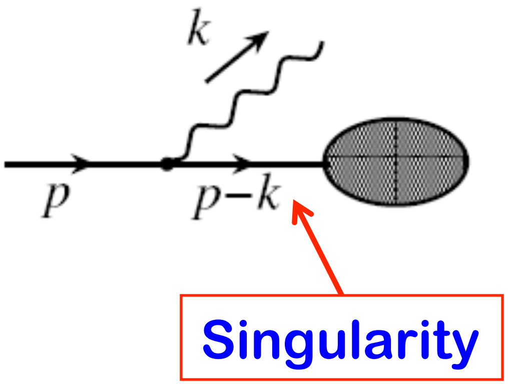
$\mu \sim Q$ **for small logarithms in the perturbative coefficients**

- **Light quark mass:** $m_f(\mu) \ll \Lambda_{\rm QCD} \quad \text{for } f = u, d, \text{ even } s$

QCD perturbation theory ($Q \gg \Lambda_{QCD}$)
is effectively a massless theory

Infrared and collinear divergences

❑ **Consider a general diagram:**



$p^2 = 0, \quad k^2 = 0$ **for a massless theory**

✧ $k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$

$\Longrightarrow$ **Infrared (IR) divergence**

✧ $k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1$

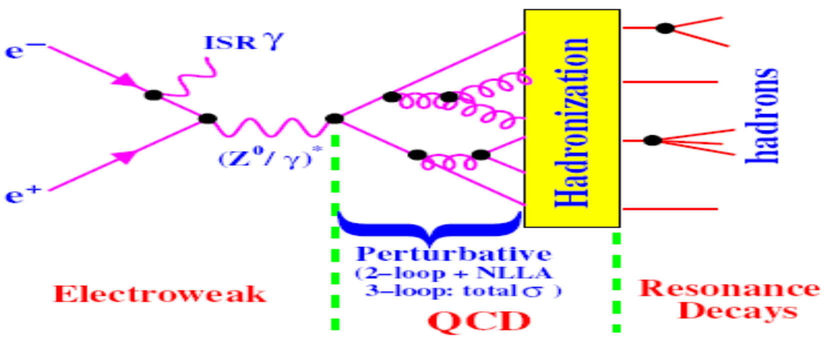
$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2 p^2 = 0$

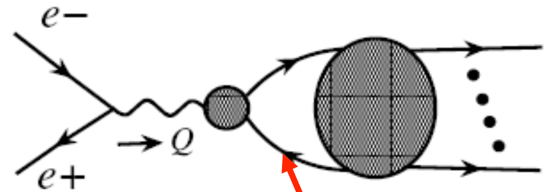
$\Longrightarrow$ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

Observables without identified hadron(s)

- $e^+e^- \rightarrow$ hadron **total** cross section – not a specific hadron!





Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{tot}_{e^+e^-\rightarrow \text{hadrons}} \propto \sum_n P_{e^+e^-\rightarrow n} = \sum_n \sum_m P_{e^+e^-\rightarrow m} P_{m\rightarrow n} = \sum_m P_{e^+e^-\rightarrow m} \sum_n P_{m\rightarrow n} = 1$$

Unitarity

$$\sigma^{tot}_{e^+e^-\rightarrow \text{partons}} \propto \sum_m P_{e^+e^-\rightarrow m}$$

$$\Rightarrow \quad \sigma^{tot}_{e^+e^-\rightarrow \text{hadrons}} = \sigma^{tot}_{e^+e^-\rightarrow \text{partons}}$$

Finite in perturbation theory – KLN theorem

- $e^+e^- \rightarrow$ parton total cross section:

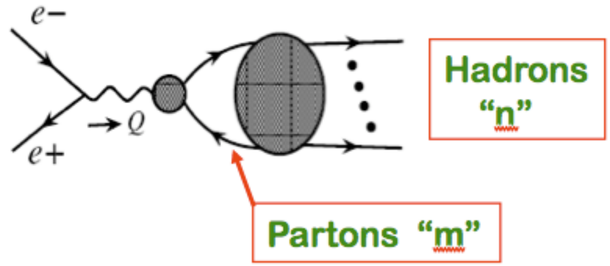
$$\sigma^{tot}_{e^+e^-\rightarrow \text{partons}}(s = Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

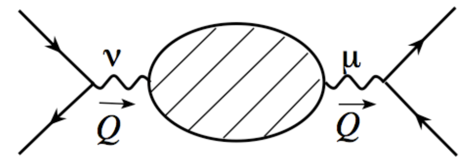
Calculable in pQCD

Infrared Safety of e⁺e⁻ Total Cross Sections

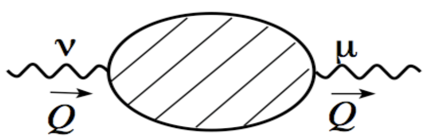
❑ Optical theorem:

$$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\left| \cdots \right|^2 \propto \text{Im} \cdots$$





❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

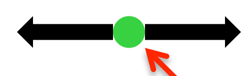
IR safety of $\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$ = IR safety of $\Pi(Q^2)$ with $Q^2 > 0$

❑ IR safety of $\Pi(Q^2)$

If there were pinched poles in $\Pi(Q^2)$,

✧ real partons moving away from each other

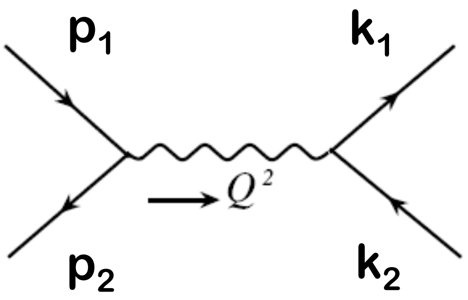
✧ cannot be back to form the virtual photon again!



Rest frame of the virtual photon

Lowest order (LO) perturbative calculation

- Lowest order Feynman diagram:



- Invariant amplitude square:

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]$$

$$s=(p_1+p_2)^2$$
$$t=(p_1-k_1)^2$$
$$u=(p_2-k_1)^2$$

- Lowest order cross section:

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt}=\frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \quad \text{where } s=Q^2$$

Threshold constraint

$$\sigma_2^{(0)}=\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}=\sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

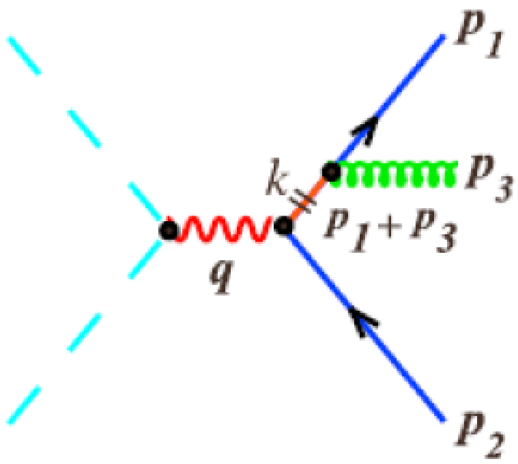
Next-to-leading order (NLO) contribution

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

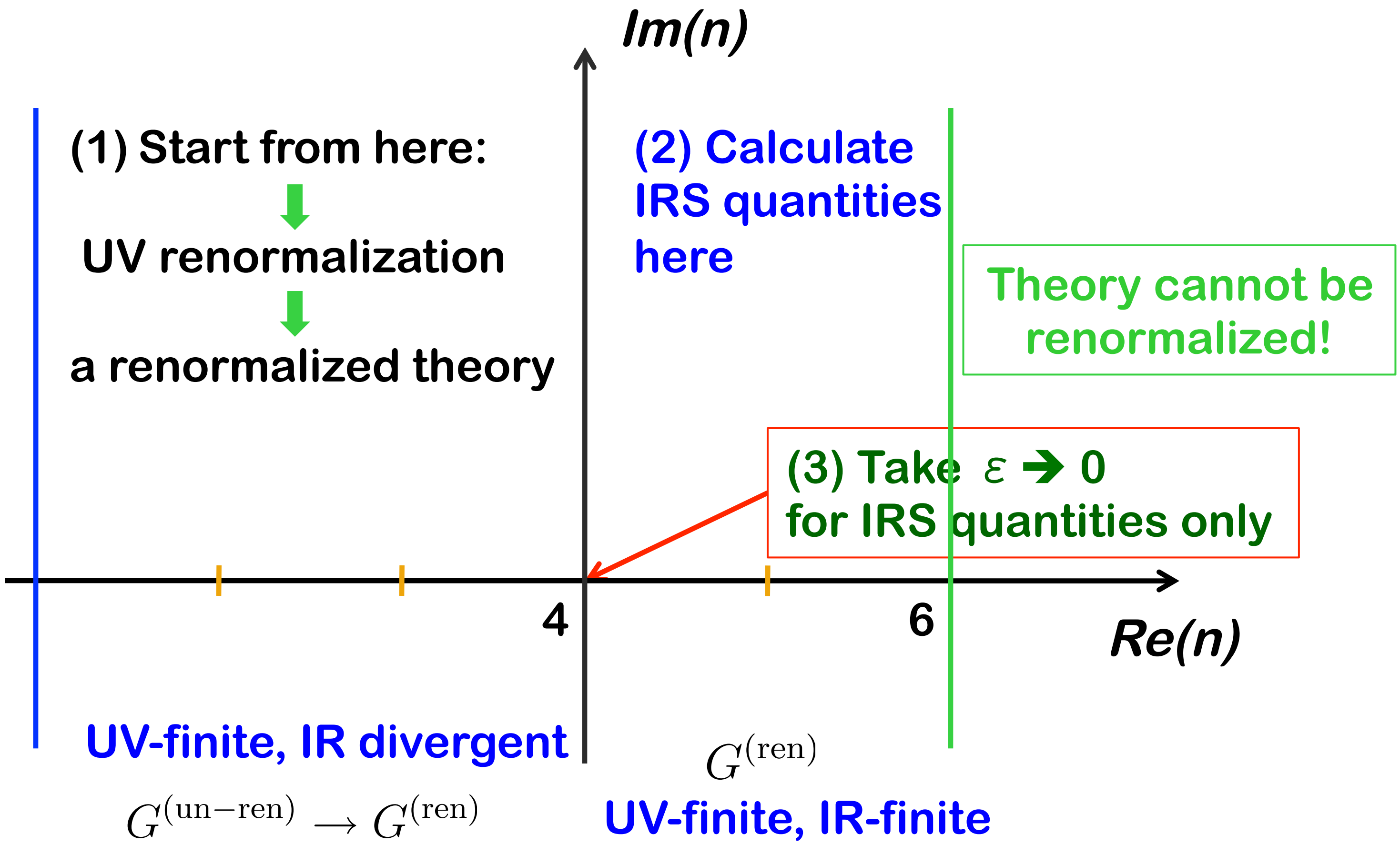
IR as $x3 \to 0$
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

- **Complex n-dimensional space:** $\int d^n k \, F(k, Q)$



Dimensional regularization for both IR and CO

- NLO with a dimensional regulator:

  - **Real:** $$\sigma^{(1)}_{3,\varepsilon} = \sigma^{(0)}_{2,\varepsilon}\,\frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

  - **Virtual:** $$\sigma^{(1)}_{2,\varepsilon} = \sigma^{(0)}_{2,\varepsilon}\,\frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

  - **NLO:** $$\sigma^{(1)}_{3,\varepsilon}+\sigma^{(1)}_{2,\varepsilon} = \sigma^{(0)}_{2}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]$$

    No ε dependence!

  - **Total:** $$\sigma^{\text{tot}} = \sigma^{(0)}_{2}+\sigma^{(1)}_{3,\varepsilon}+\sigma^{(1)}_{2,\varepsilon}+O\left(\alpha_s^2\right) = \sigma^{(0)}_{2}\left[1+\frac{\alpha_s}{\pi}\right]+O\left(\alpha_s^2\right)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Go beyond the inclusive total cross section?

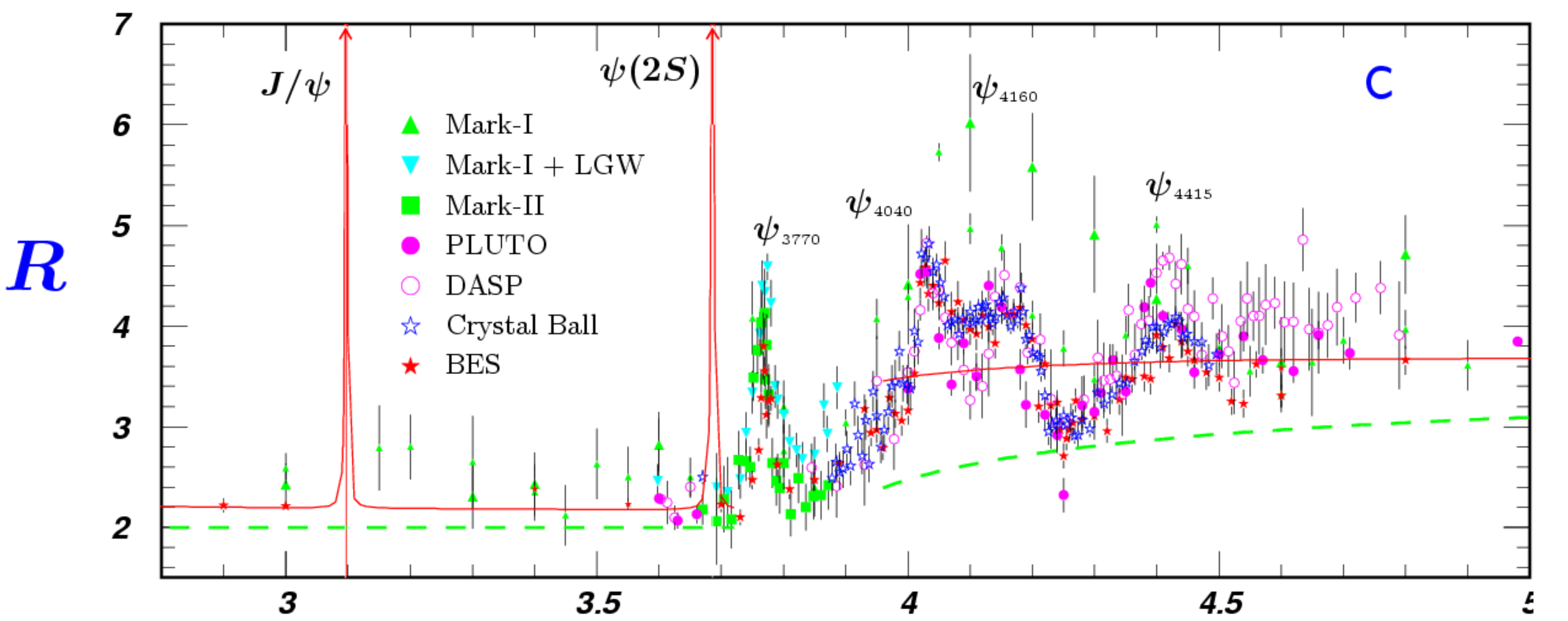
Hadronic cross section in e+e- collision

- Normalized hadronic cross section:

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Jets – trace of partons

- Jets – "total" cross-section with a limited phase-space

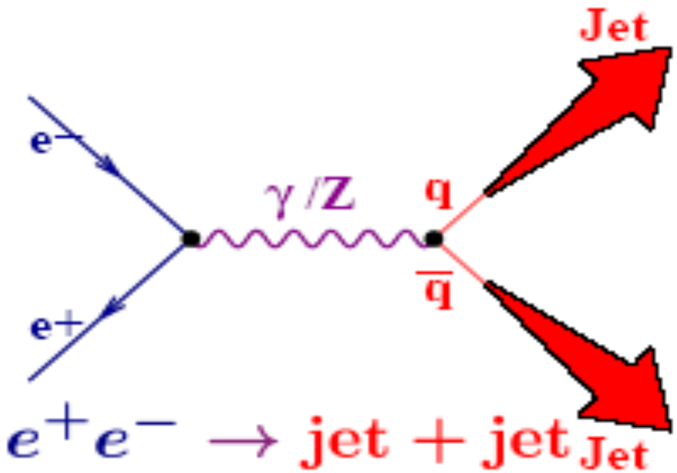
  Not any specific hadron!

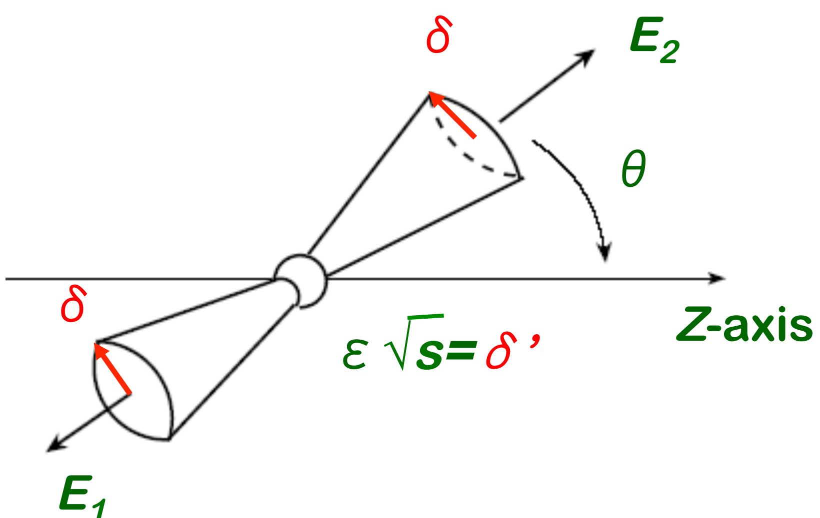
- Q: will IR cancellation be completed?

  - Leading partons are moving away from each other

  - Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton

- Many Jet algorithms

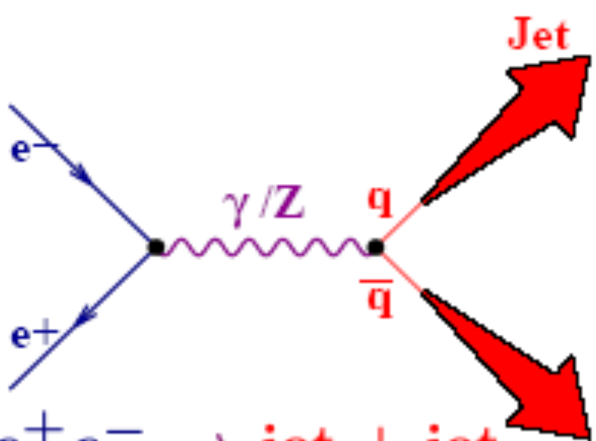




Sterman-Weinberg Jet

An early clean two-jet event

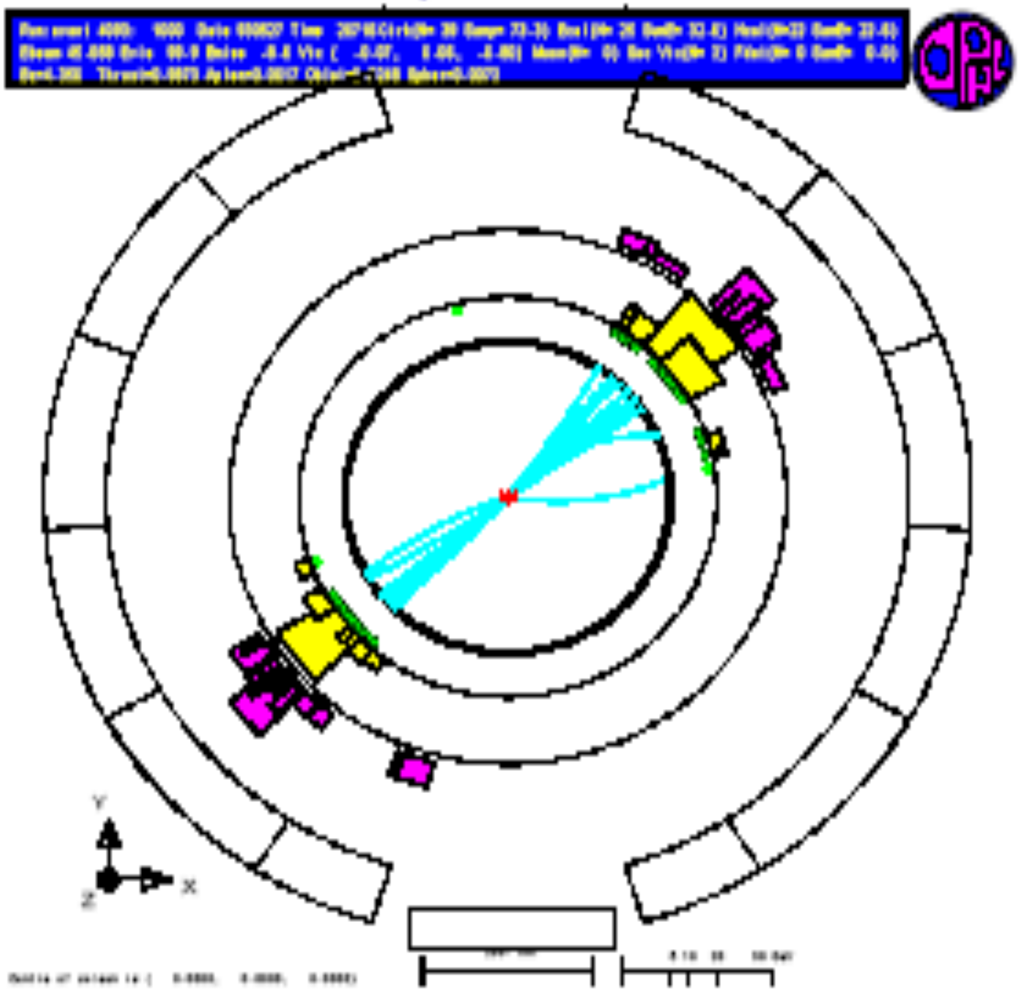
Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

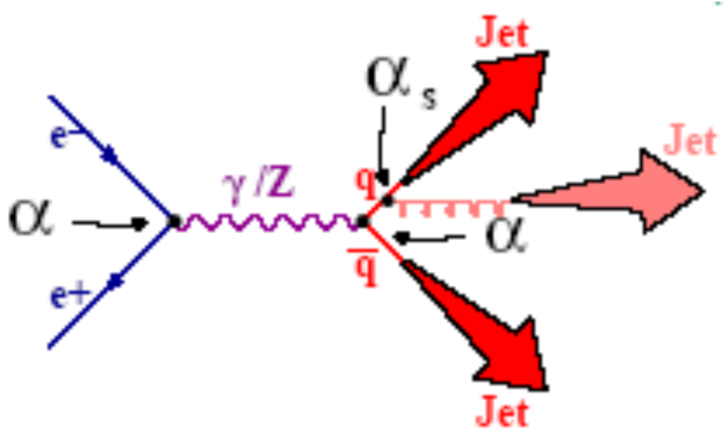
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



Discovery of a gluon jet

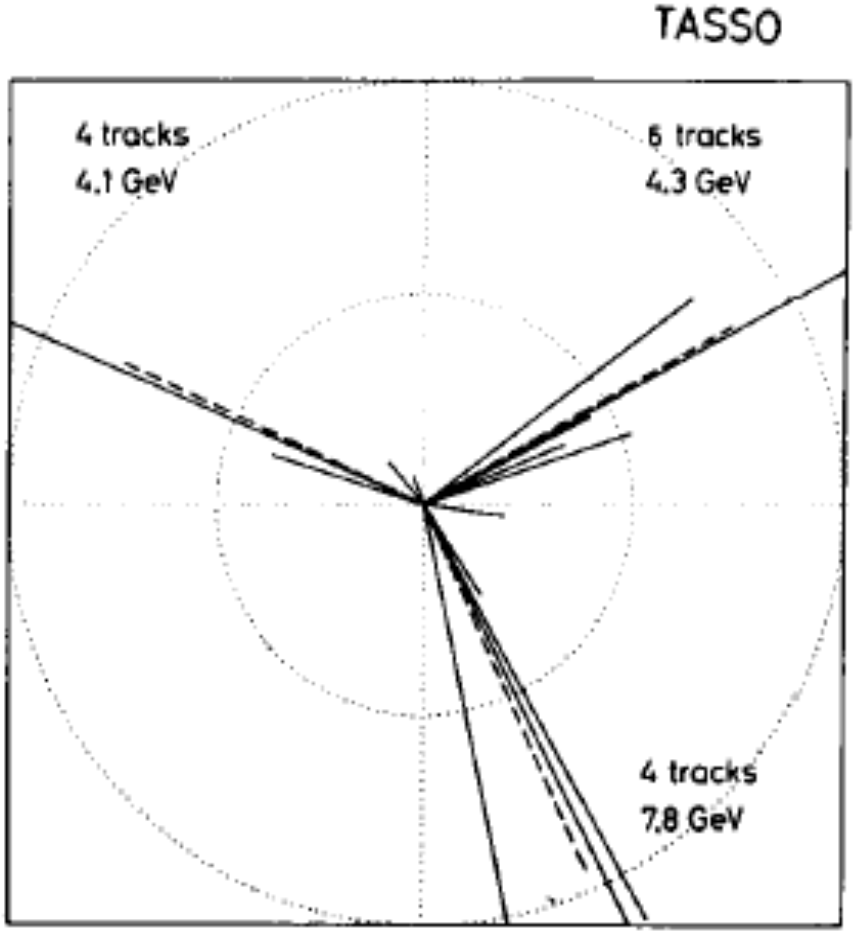
First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



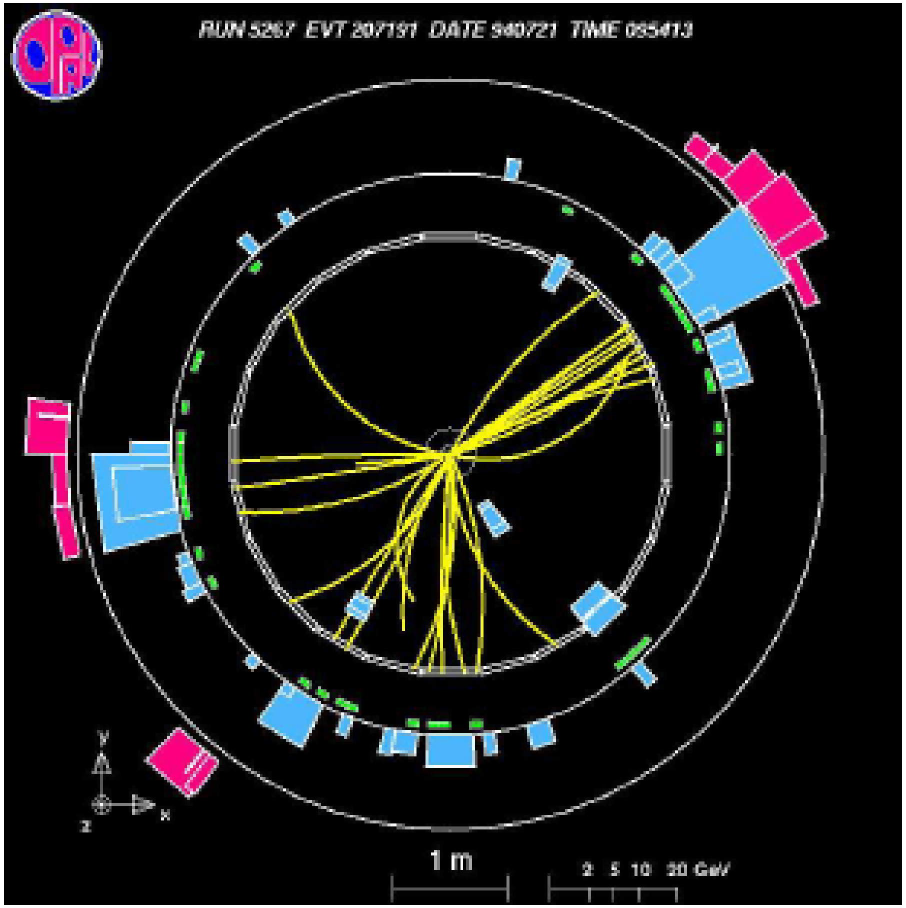
TASSO Collab., Phys. Lett. B86 (1979) 243

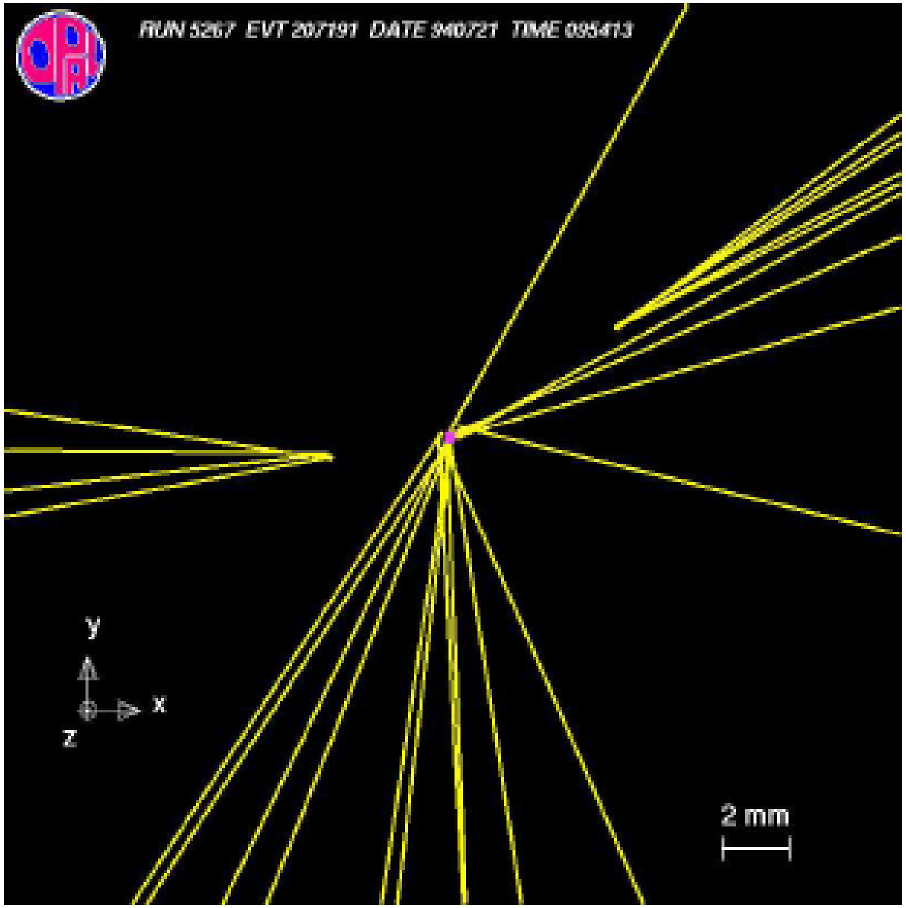
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged three-jet event from LEP





Gluon Jet

Basics of jet finding algorithms

- Recombination jet algorithms (almost all e+e- colliders):

Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$ $\quad M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

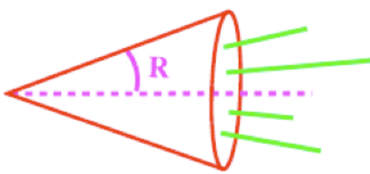
for Durham k_T

- ✧ different algorithm = different choice of M_{ij}^2 :
- ✧ Combine the particle pair (i, j) with the smallest y_{ij} : $(i, j) \to k$

$$e.g.\ \mathrm{E\ scheme}: \quad p_k = p_i + p_j$$

- ✧ iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$

- Cone jet algorithms (CDF, …, colliders):



- ✧ Cluster all particles into a cone of half angle R to form a jet:
- ✧ Require a minimum visible jet energy: $E_{jet} > \epsilon$

Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$

with $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

- ✧ Classical choices: ***p=1*** – "k_T algorithm", ***p= -1*** – "anti-k_T", …

Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \quad 0 \le \lambda \le 1$$

Physical meaning:

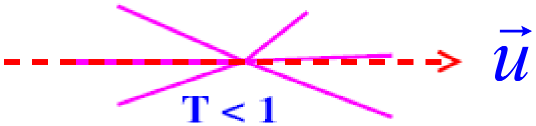
Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without the parton

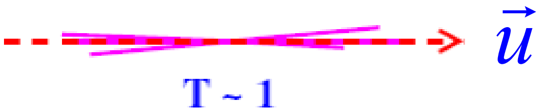
Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1$ for all $n \quad \Rightarrow \quad \sigma^{(\mathrm{tot})}$

Another example: Thrust distribution

- ❑ **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}}\left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|}\right)$$





- ❑ **Phase space constraint:**

$$\Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

✧ Contribution from p=0 particles drops out the sum

✧ Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

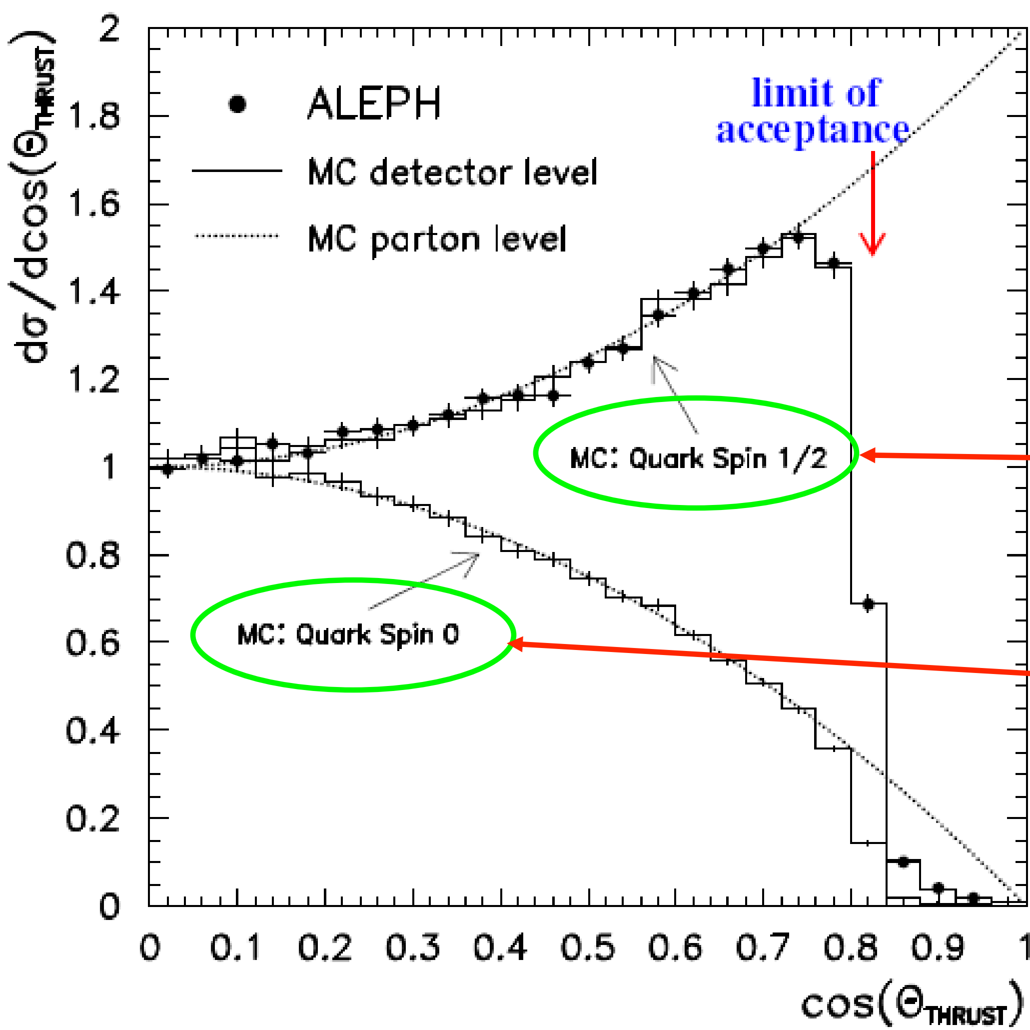
and

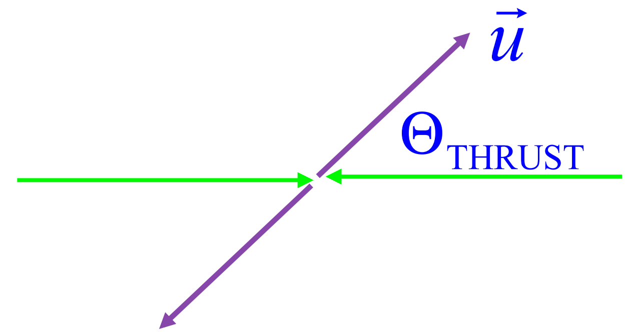
$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

Another test of quark spin

- Angle between the thrust axis and the beam axis:

[ALEPH Collab., Phys. Rep. 294 (1998) 1]





At LO:

$1 + \cos^2 \Theta_{THRUST}$

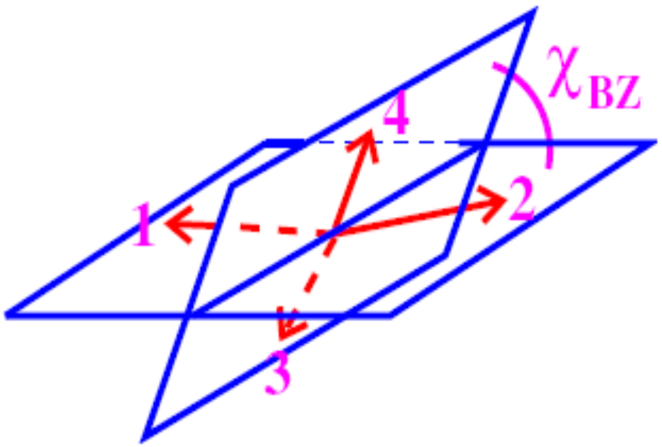
$1 - \cos^2 \Theta_{THRUST}$

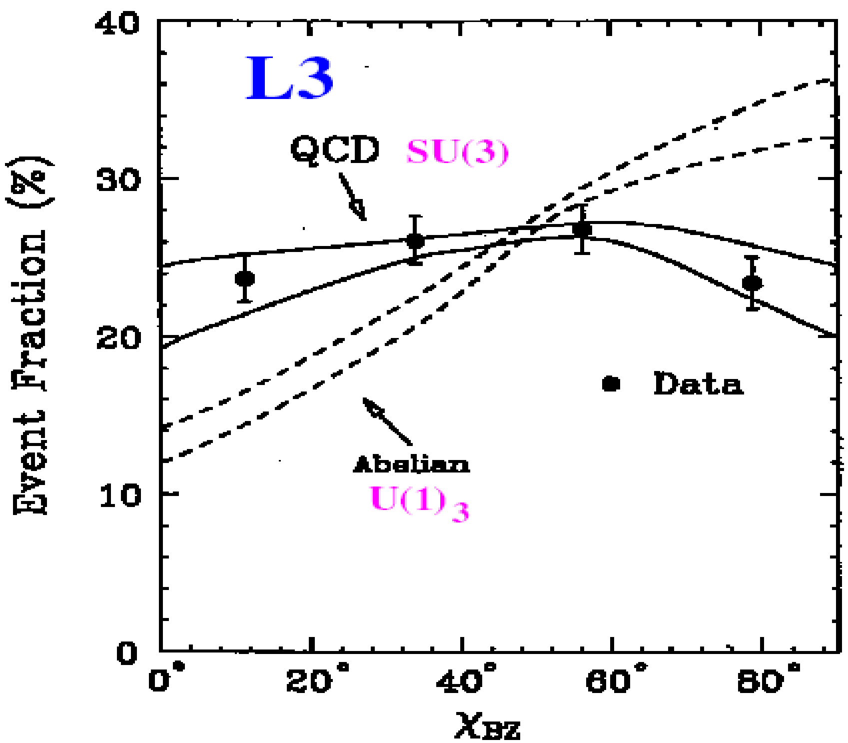
Another test of SU(3) color

- Select 4-jet events: $E_1 > E_2 > E_3 > E_4$

jet-3 and jet-4 are more likely from radiation

- Bengtsson-Zerwas angle:





The harder question

- **Question:**

  How to test QCD in a reaction with identified hadron(s)?
  – to probe the quark-gluon structure of the hadron

- **Facts:**

  Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

  Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

- **Solution – Factorization:**

  ✧ Isolate the calculable dynamics of quarks and gluons

  ✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
  – provide information on the partonic structure of the hadron

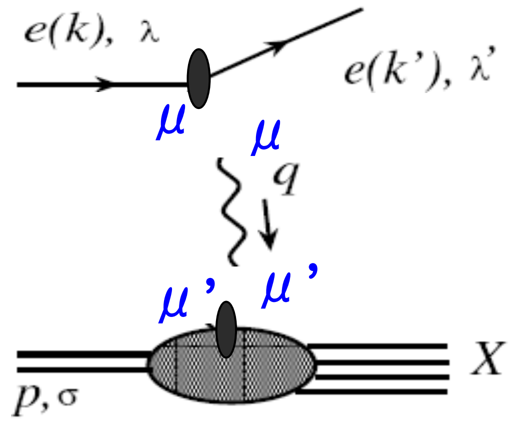
Inclusive lepton-hadron DIS – one hadron

- **Scattering amplitude:**

$$M(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_\mu\right]u_\lambda(k)$$

$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$

$$* \langle X|eJ^{em}_{\mu'}(0)|p,\sigma\rangle$$



- **Cross section:**

$$d\sigma^{\text{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|M(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X} \frac{d^3 l_i}{(2\pi)^3 2E_i}\right] \frac{d^3k'}{(2\pi)^3 2E'} (2\pi)^4 \delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\Rightarrow \quad E'\frac{d\sigma^{\text{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

- **Leptonic tensor:**

– known from QED $\quad L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}\right)$

DIS structure functions

- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\ \mathrm{e}^{iq\cdot z}\ \left\langle p,S\left|J_\mu^\dagger(z)J_\nu(0)\right|p,S\right\rangle$$

- **Symmetries:**

  - ✧ Parity invariance (EM current) → $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
  - ✧ Time-reversal invariance → $W_{\mu\nu} = W_{\mu\nu}^*$ real
  - ✧ Current conservation → $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2 = -q^2 \qquad x_B = \frac{Q^2}{2p\cdot q}$$

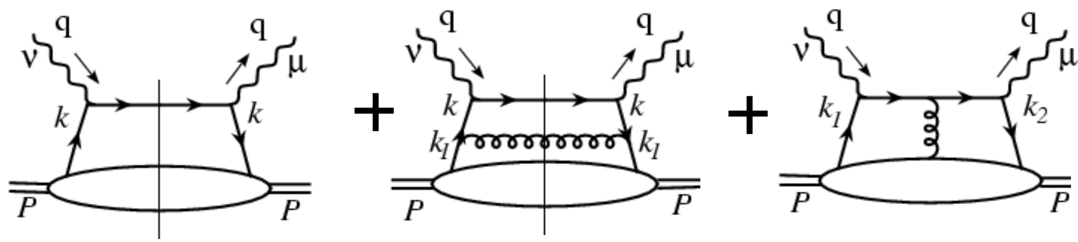
- **Structure functions – infrared sensitive:**

$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

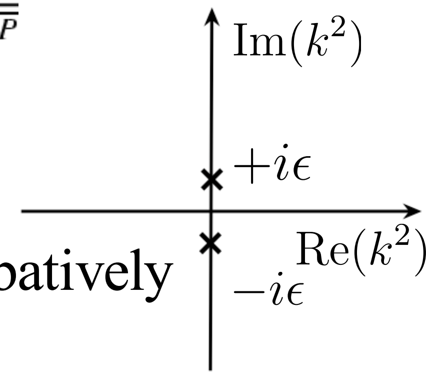
Long-lived parton states

❑ Feynman diagram representation:

$$W^{\mu\nu} \propto$$  $$+ \ldots$$

❑ Perturbative pinched poles:

$$\int d^4k \; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right) \Rightarrow \infty \text{ perturbatively}$$



❑ Perturbative factorization:

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

Nonperturbative matrix element

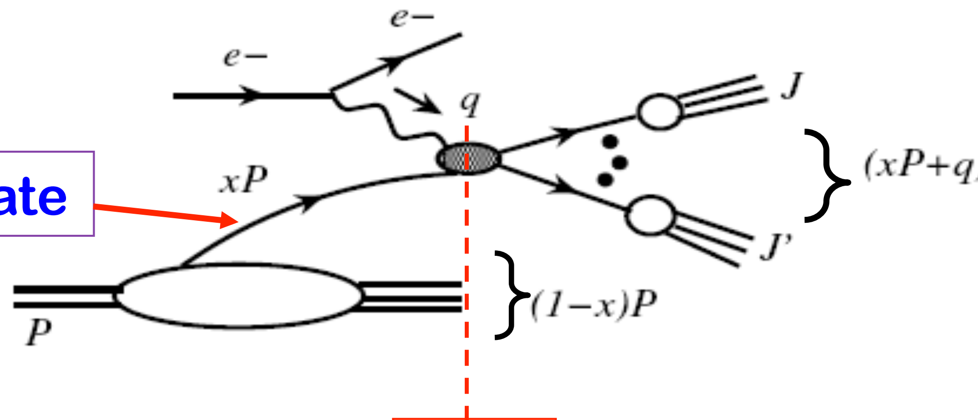
$$\int\frac{dx}{x}d^2k_T\;\mathrm{H}(Q,k^2=0)\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right)$$

Short-distance

Picture of factorization for DIS

- **Time evolution:**

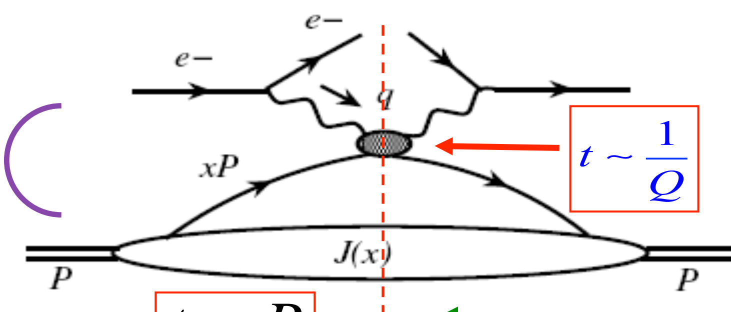
Long-lived parton state



Time: "Past" **Now** "Future"

- **Unitarity – summing over all hard jets:**

$$\sigma_{\rm tot}^{\rm DIS} \propto \mathrm{Im}\Big(\quad \Big)$$



$t \sim \frac{1}{Q}$

$t \sim R$

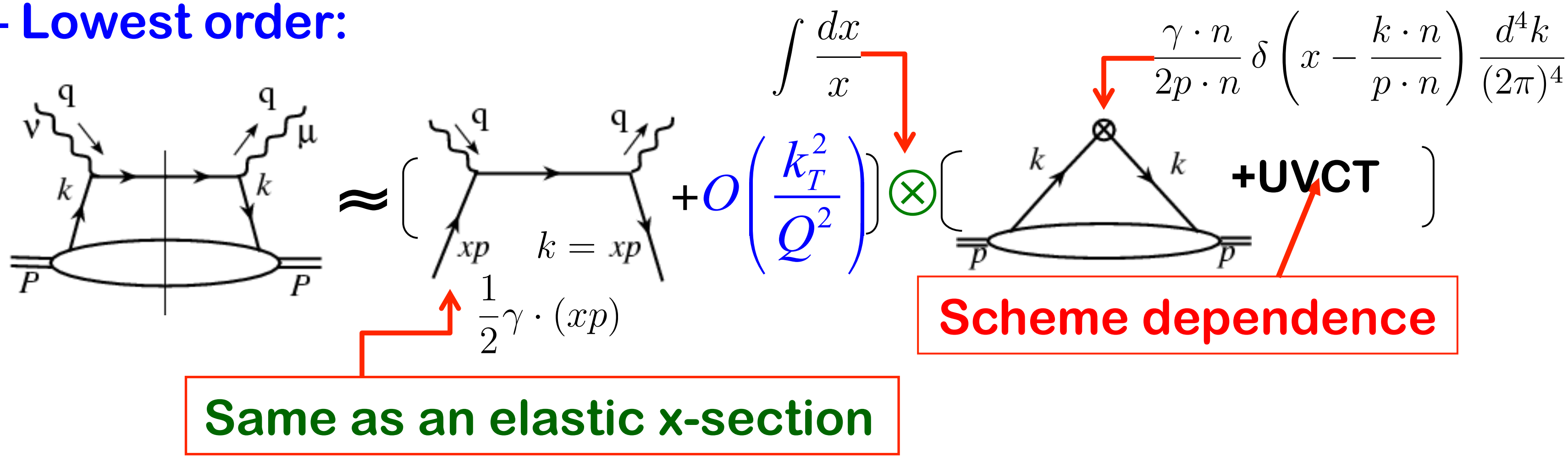
Not IR safe

Interaction between the "past" and "now" are suppressed!

Collinear factorization - approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T, \sqrt{k^2}$

  – **Lowest order:**



Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \to \infty$, while x_B fixed

Feynman's parton model and Bjorken scaling

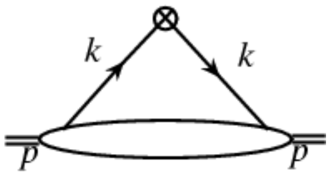
$$F_2(x_B,Q^2) = x_B \sum_f e_f^2\, \phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

Spin-½ parton!

- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle/Q^2\right)$

Parton distribution functions (PDFs)

❑ PDFs as matrix elements of two parton fields:



– **combine the amplitude & its complex-conjugate**

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle\, \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

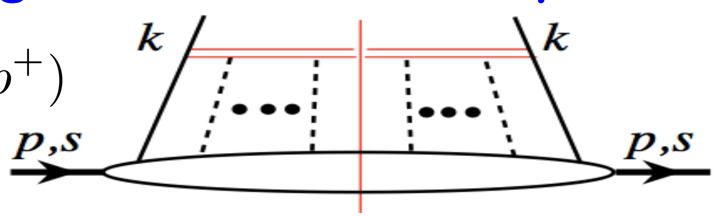
But, it is not gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– **need a gauge link:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– **corresponding diagram in momentum space:**

$$\int \frac{d^4k}{(2\pi)^4}\, \delta(x - k^+/p^+)$$



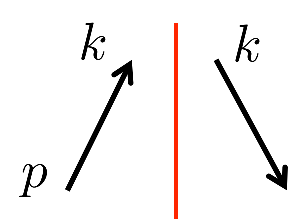
$$+\,\mathrm{UVCT}(\mu^2)$$

μ **-dependence**

Universality – process independence – predictive power

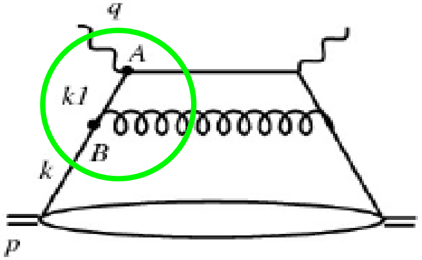
❑ Quark distribution of a quark – LO:

$$\int \frac{d^4k}{(2\pi)^4} \delta(x - k^+/p^+)(2\pi)^4\delta^4(k-p) = \delta(x-1)$$



QCD high order corrections

- **NLO partonic diagram to structure functions:**

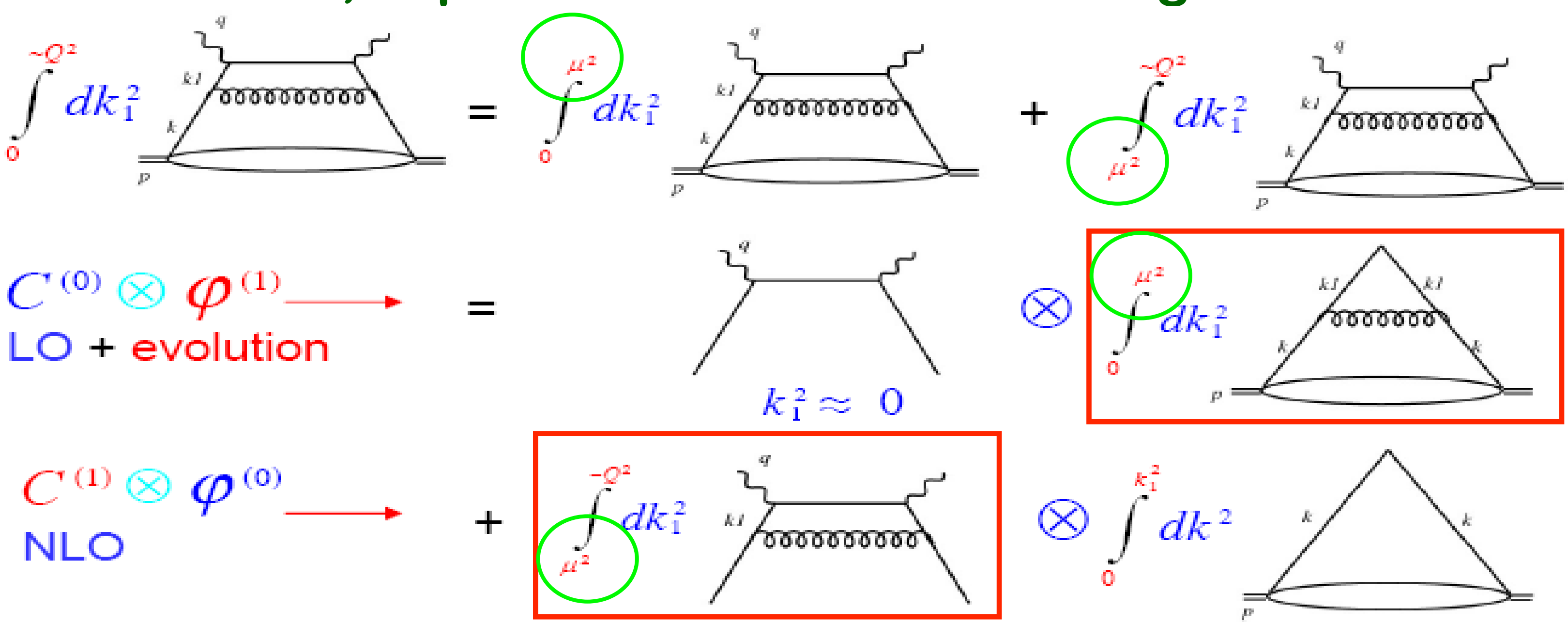


$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by $\begin{cases} k_1^2 \sim 0 \\ t_{AB} \rightarrow \infty \end{cases}$

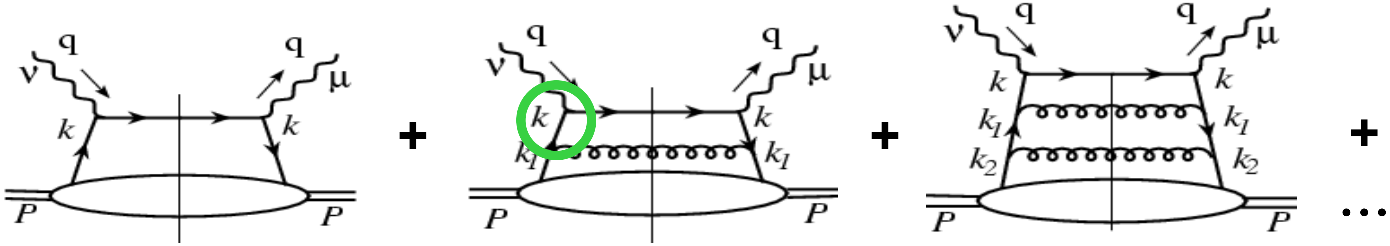
Diagram has both long- and short-distance physics

- **Factorization, separation of short- from long-distance:**

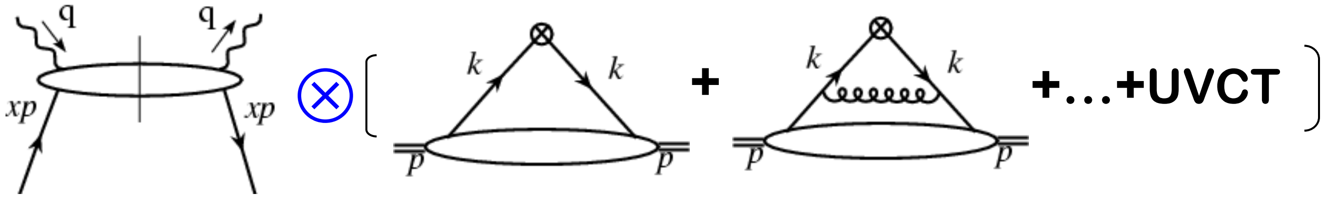


QCD high order corrections

- QCD corrections: pinch singularities in $\int d^4 k_i$



- Logarithmic contributions into parton distributions:



$$\Rightarrow \quad F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- Factorization scale: μ_F^2

→ To separate the collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution

How to calculate the perturbative parts?

❑ **Use DIS structure function F_2 as an example:**

$$F_{2h}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

✧ **Apply the factorized formula to parton states:** $h\to q$

Feynman diagrams → $$F_{2q}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ ← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of** α_s**:**

0th order: $$F_{2q}^{(0)}(x_B,Q^2)=C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

⇒ $$C_q^{(0)}(x)=F_{2q}^{(0)}(x) \qquad \varphi_{q/q}^{(0)}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order: $$F_{2q}^{(1)}(x_B,Q^2)=C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$
$$+C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

⇒ $$C_q^{(1)}(x,Q^2/\mu^2)=F_{2q}^{(1)}(x,Q^2)-F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

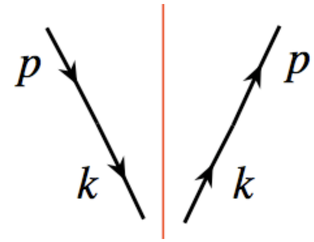
PDFs of a parton

- **Change the state without changing the operator:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\,\psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\mathrm{parton}(p)\rangle$ ⇒ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

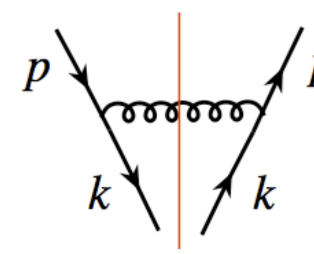
- **Lowest order quark distribution:**



  - ✧ **From the operator definition:**

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

- **Leading order in α_s quark distribution:**



$+\ldots$

  - ✧ **Expand to $(g_s)^2$ – logarithmic divergent:**

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

Partonic cross sections

❑ Projection operators for SFs:

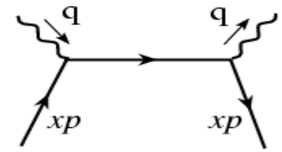
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x, Q^2\right)$$

$$F_1(x, Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

$$F_2(x, Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left[\frac{1}{4\pi} \right.$$



$$\left.\right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2} \gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x \delta(1-x)$$

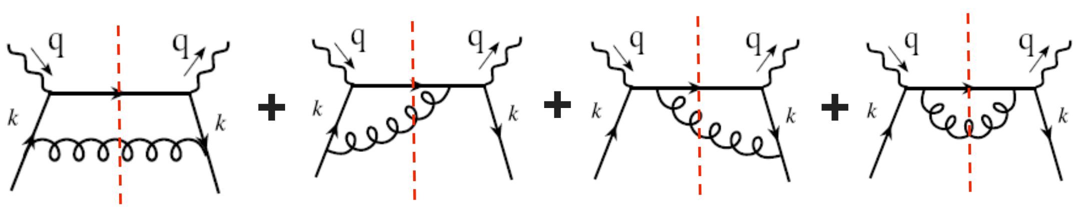
NLO coefficient function – complete example

$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

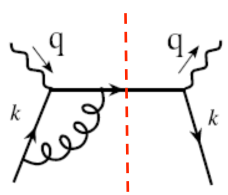
- **Projection operators in n-dimension:** $g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

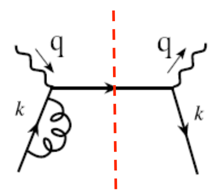
$$\left(1-\varepsilon\right) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}$$

- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$ 

} **Real**

Virtual { + 

+ c.c. (+ 

+ c.c. + UV CT)

- **Calculation:** $-g^{\mu\nu} W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

- **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ln(1-z) f(1)$$

❑ One loop contribution to the trace of W$_{\mu\,\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+C_F\left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$
$$\left.\left. +3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

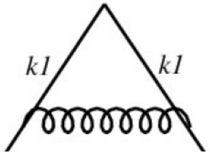
❑ One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ **MS scheme:** $\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

✧ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$

✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu^2)\big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x) \ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ **Evolution (differential-integral) equation for PDFs**

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\rm QCD}^2\right)$

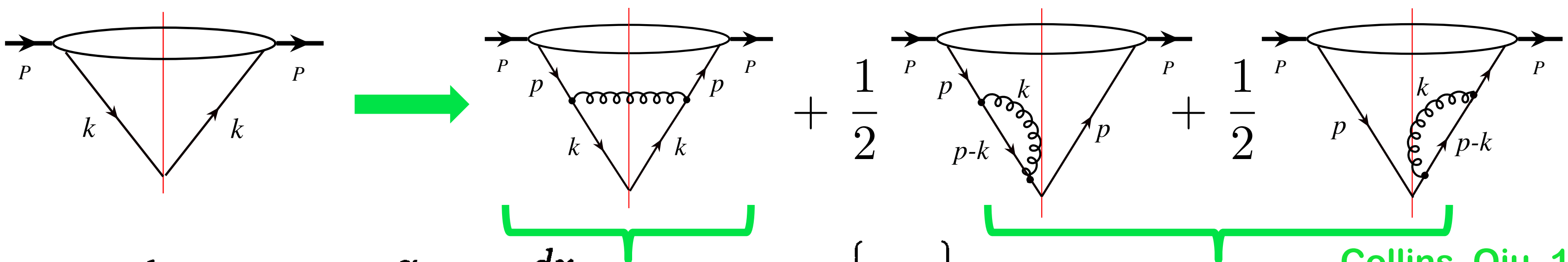
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ **DGLAP evolution equation:**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

Calculation of evolution kernels

- ❑ Evolution kernels are process independent
  - ✧ Parton distribution functions are universal
  - ✧ Could be derived in many different ways
- ❑ Extract from calculating parton PDFs’ scale dependence



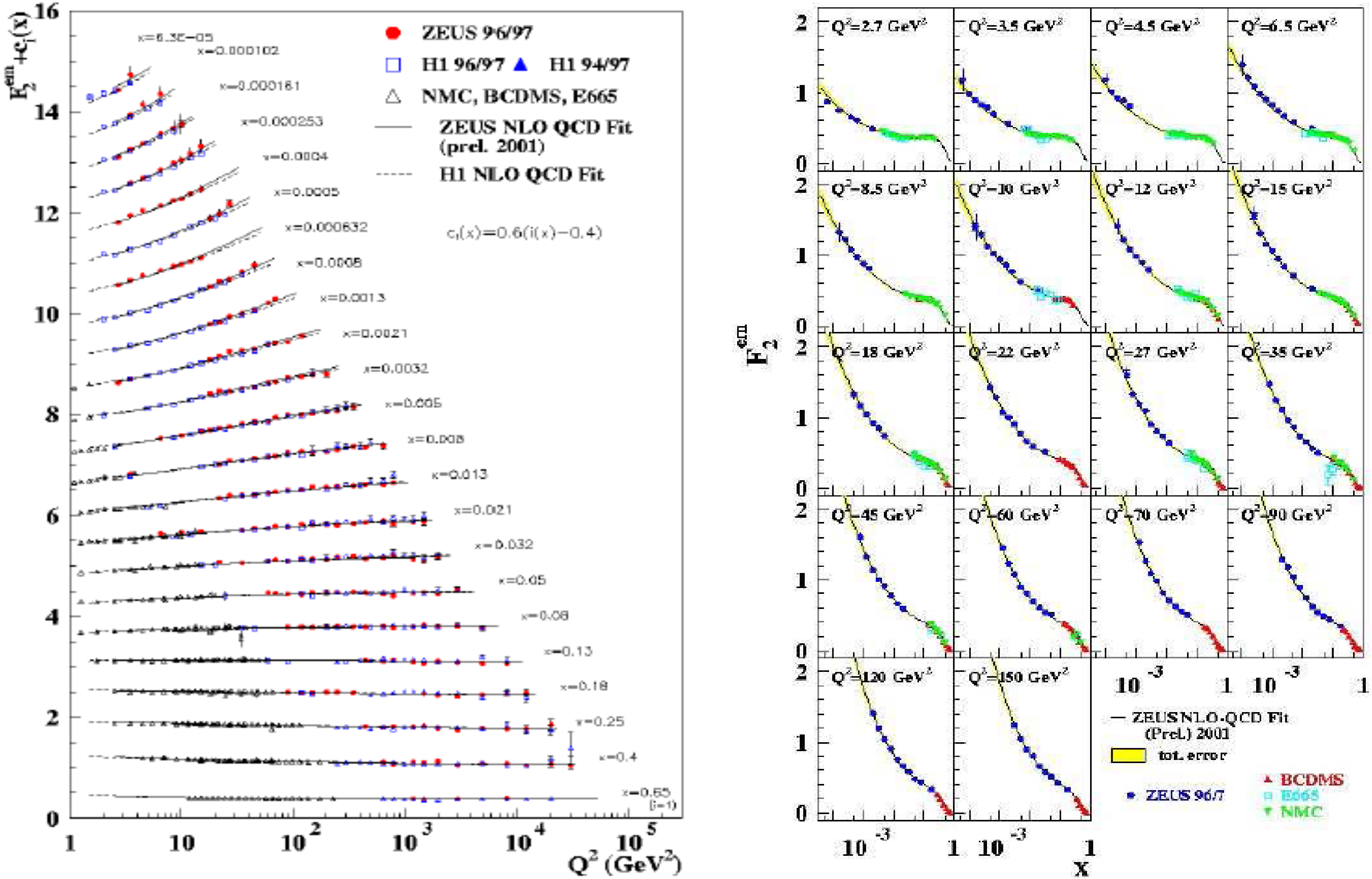
Collins, Qiu, 1989

$$Q^2 \frac{d}{dQ^2} q_i(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_1}{x_1} q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right) - \frac{\alpha_s}{2\pi} q_i(x,Q^2) \int_0^1 dz\, \gamma_{qq}(z)$$

Change “Gain” “Loss”

  - ✧ Same is true for gluon evolution, and mixing flavor terms
- ❑ One can also extract the kernels from the CO divergence of partonic cross sections

Scaling and scaling violation



Q^2-dependence is a prediction of pQCD calculation

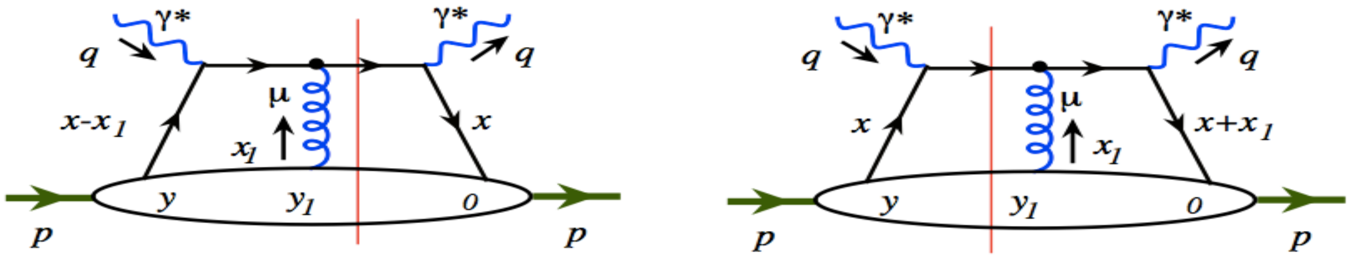
Summary of lecture one

- QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields
- QCD perturbation theory works at high energy because of the Asymptotic Freedom
- Perturbative QCD calculations make sense only for infrared safe (IRS) quantities – e^+e^- total cross section
- Jets in high energy collisions provide us the "trace" of energetic quarks and gluons
- Factorization is necessary for pQCD to treat observables (cross sections) with "identified hadrons"
- Predictive power of QCD factorization relies on the universality of PDFs (or TMDs, GPDs, …), the calculations of perturbative coefficient functions – hard parts

Backup slides

Gauge link – 1st order in coupling "g"

❑ Longitudinal gluon:



❑ Left diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a) \frac{\gamma\cdot p}{p^+} \frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

❑ Right diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+y_1^-} n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon} (+igt^a) \frac{\gamma\cdot p}{p^+} \mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1p^+y_1^-}\right] \mathcal{M} \;= ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

❑ Total contribution:

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$