Quantum Chromodynamics (QCD)

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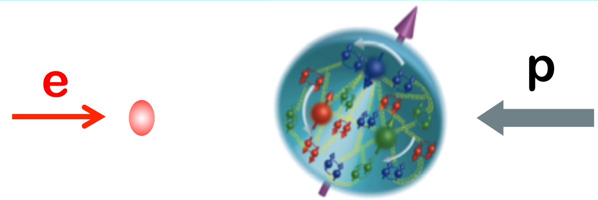
Summary of lecture one

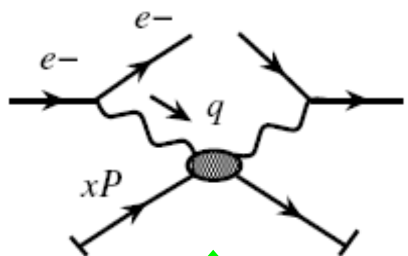
- QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields
- QCD perturbation theory works at high energy because of the Asymptotic Freedom
- Perturbative QCD calculations make sense only for infrared safe (IRS) quantities – e^+e^- total cross section
- Jets in high energy collisions provide us the "trace" of energetic quarks and gluons
- Factorization is necessary for pQCD to treat observables (cross sections) with "identified hadrons"
- Predictive power of QCD factorization relies on the universality of PDFs (or TMDs, GPDs, …), the calculations of perturbative coefficient functions – hard parts

From one hadron to two hadrons

- **One hadron:**



$$\sigma_{\text{tot}}^{\text{DIS}} \sim$$



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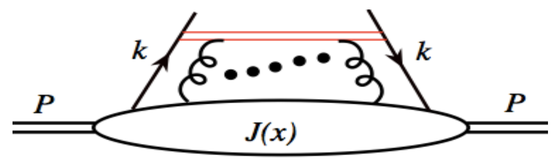
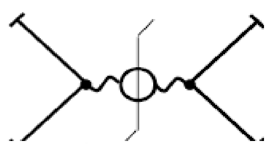


$$+ \, O\left(\frac{1}{QR}\right)$$

Hard-part	Parton-distribution	Power corrections
Probe	Structure	Approximation

- **Two hadrons:**

$$\sigma_{\text{tot}}^{\text{DY}} \sim$$



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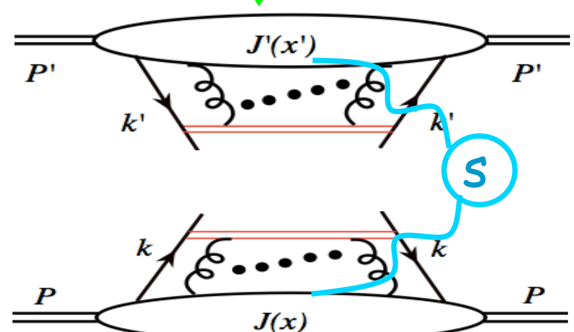
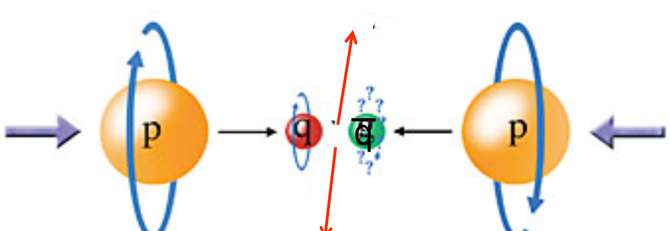


$$+ \, O\left(\frac{1}{QR}\right)$$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

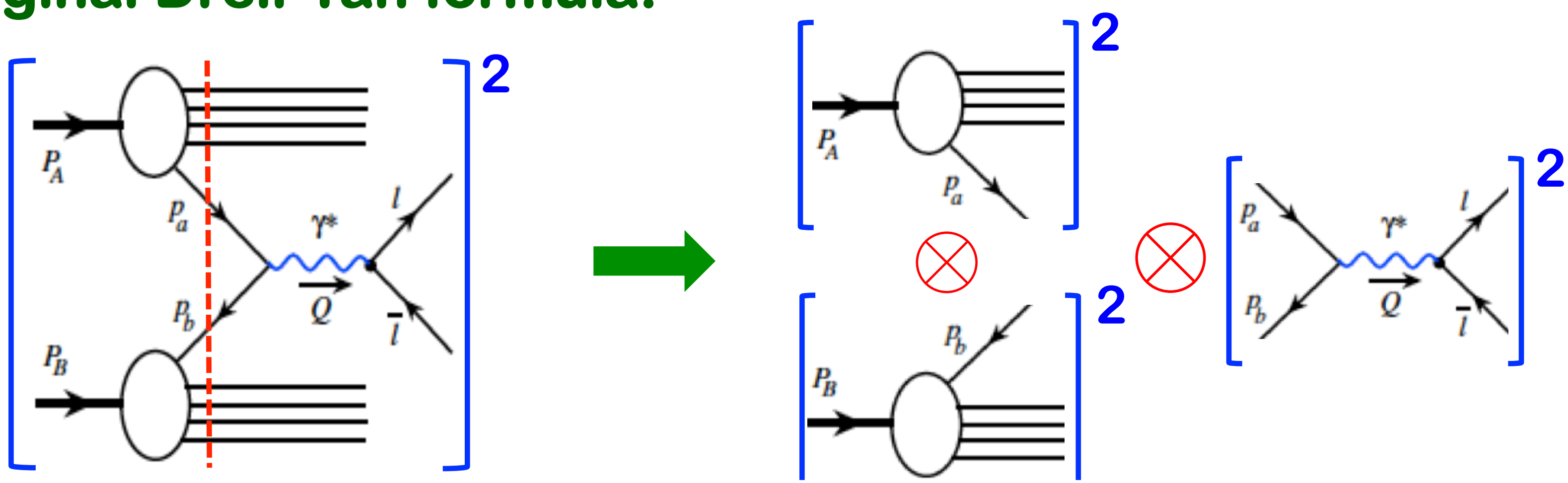
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

❑ Drell-Yan mechanism:

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \qquad \text{with} \quad q^2 \equiv Q^2 \gg \Lambda_{\text{QCD}}^2 \sim 1/\text{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:



$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2dy} = \frac{4\pi\alpha_{em}^2}{3Q^4}\sum_{p,\bar{p}} x_A\phi_{p/A}(x_A)\, x_B\phi_{\bar{p}/B}(x_B)$$

No color yet!

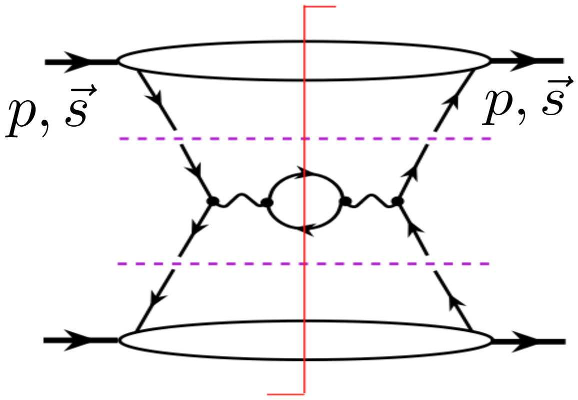
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}}e^{y} \qquad x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD

- **Spin decomposition – cut diagram notation:**



$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

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- **Factorized cross section:**

$$\sigma(Q,\vec{s}) \pm \sigma(Q,-\vec{s}) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle \pm \langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

- **Parity-Time reversal invariance:**

$$\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle = \langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle$$

- **Good operators:**

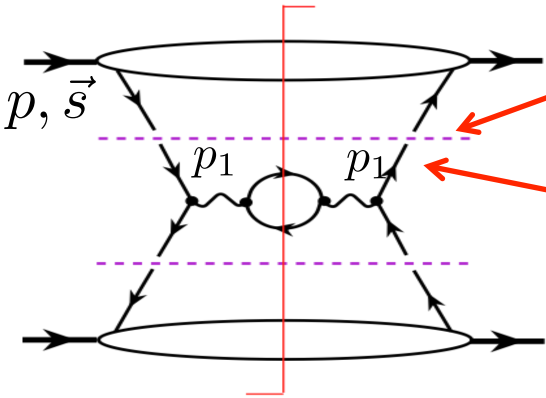
$$\langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle = \pm\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

"+" for spin-averaged cross section $\rightarrow$ PDFs:

$$\langle p,\vec{s}|\overline{\psi}(0)\gamma^+\psi(y^-)|p,\vec{s}\rangle, \qquad \langle p,\vec{s}|F^{+i}(0)F^{+j}|p,\vec{s}\rangle(-g_{ij})$$

Drell-Yan process in QCD – LO

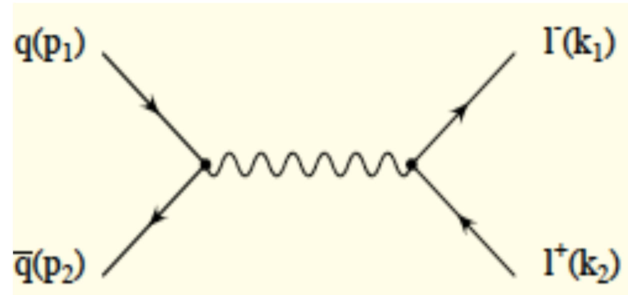
❑ Spin-averaged cross section – Lowest order:



$$\frac{1}{2p^+}\gamma^+\,\delta(x - p_1^+/p^+)dx$$

$$\frac{1}{2}\gamma\cdot p = \frac{1}{2}\sum_s u_s(p)\overline{u}_s(p)$$

$$\hat{s} = (p_1 + p_2)^2 = Q^2$$



❑ Lowest order partonic cross section:

$$\overline{\Sigma}\,|M|^2 = \frac{e_q^2 e^4}{\hat{s}^2}\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]3\left\{\frac{1}{3}\right\}\left\{\frac{1}{3}\right\}\mathrm{Tr}[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu]\mathrm{Tr}[\not{k}_1\gamma_\nu\not{k}_2\gamma_\mu] = \left\{\frac{1}{3}\right\}e_q^2 e^4(1+\cos^2\theta)$$

$$PS^{(2)} = \frac{d^2k_1}{(2\pi)^3 2E_1}\frac{d^2k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2) = \frac{1}{16\pi}d\cos(\theta)$$

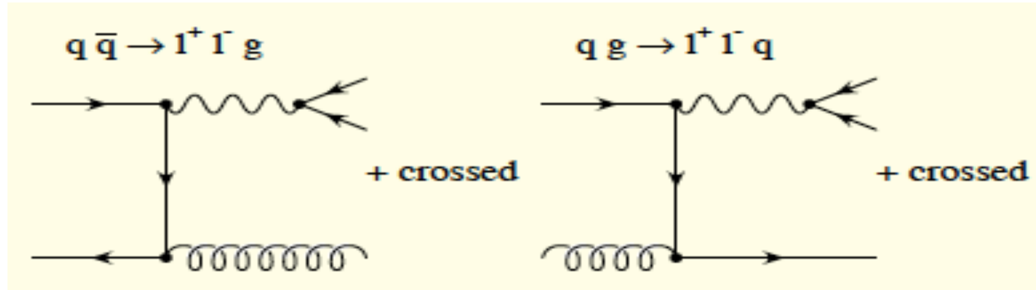
$$\sigma(q\bar{q}\to l^+l^-) = \left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2 \equiv \sigma_0$$

❑ Drell-Yan cross section:

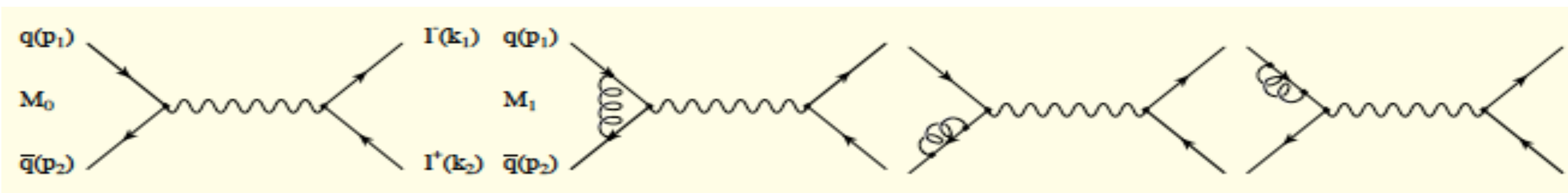
$$\frac{d\sigma}{dQ^2dy} = \Sigma_q\int dx_A\,dx_B\,\phi_{q/A}(x_A)\phi_{\bar{q}/B}(x_B)\left[\left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2\right]\delta(Q^2-\hat{s})\,\delta(y-\frac{1}{2}\ln(\frac{x_A}{x_B}))$$

Drell-Yan process in QCD – NLO

- **Real contribution:**



- **Virtual contribution:**



- **NLO contribution:**

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \qquad q\bar{q} \to l^+l^-$$

$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(-\frac{A}{\epsilon^2} + \frac{B'}{\epsilon} + C'\right) \qquad q\bar{q} \to l^+l^-g$$

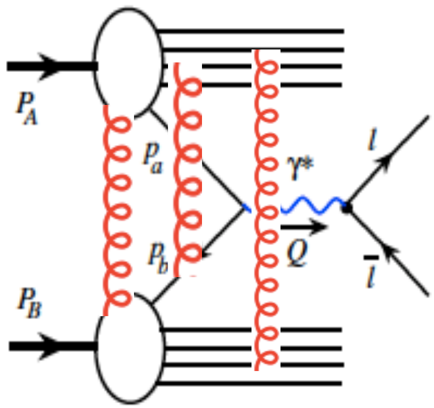
$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(\frac{B''}{\epsilon} + C''\right) \qquad qg \to l^+l^-q$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{B + B' + B''}{\epsilon} + C + C' + C''\right)$$

Absorbed into PDFs – scheme dependence

Drell-Yan process in QCD – factorization

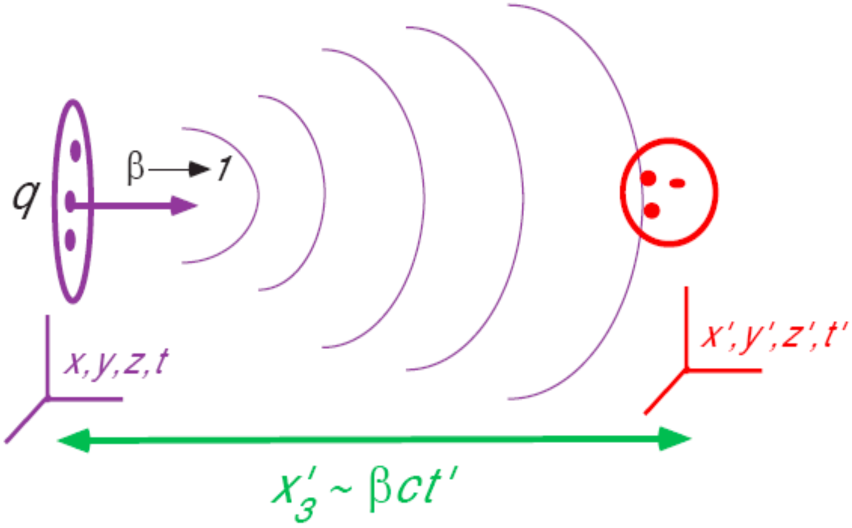
❑ **Beyond the lowest order:**



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ **Factorization – power suppression of soft gluon interaction:**



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$ $\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2 + \gamma^2\Delta^2)^{3/2}}$ $\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

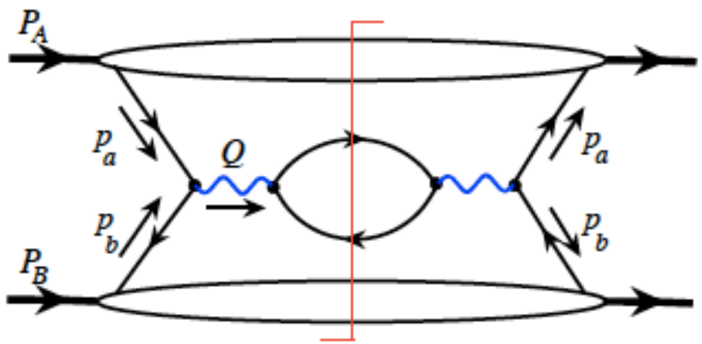
Drell-Yan process in QCD – factorization

Collins, Soper, Sterman, 1988

❑ Factorization – approximation:

- ✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

- ✧ Maintain the universality of PDFs:

  Long-range soft gluon interaction has to be power suppressed

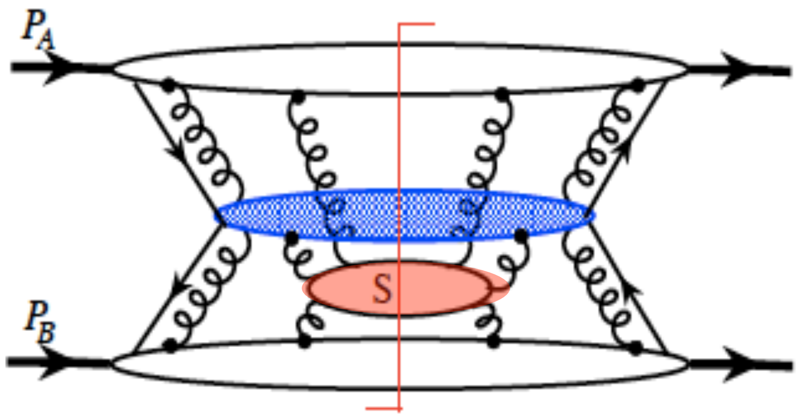
- ✧ Infrared safe of partonic parts:

  Cancelation of IR behavior
  Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} &\quad p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} &\quad p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} &\quad p_a^- \ll q^-; \quad \text{and} \\ &\quad p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD – factorization

❑ Leading singular integration regions (pinch surface):



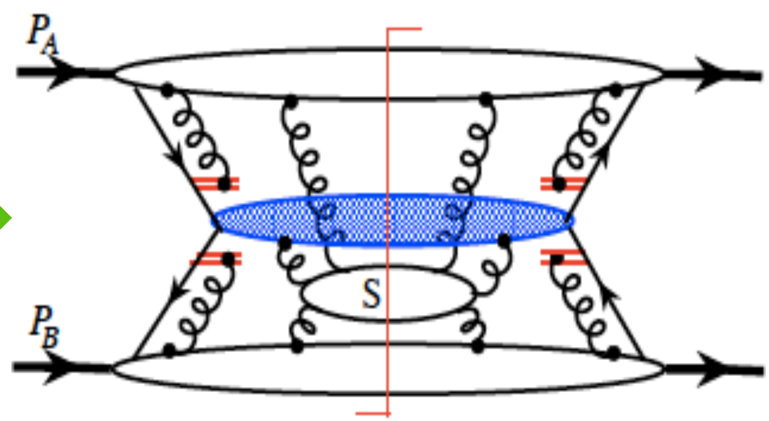
Hard: all lines off-shell by Q

Collinear:
- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

❑ Collinear gluons:

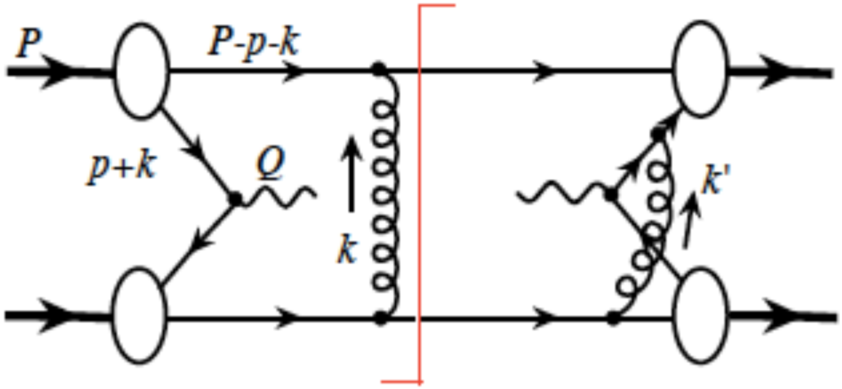
- ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
- ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

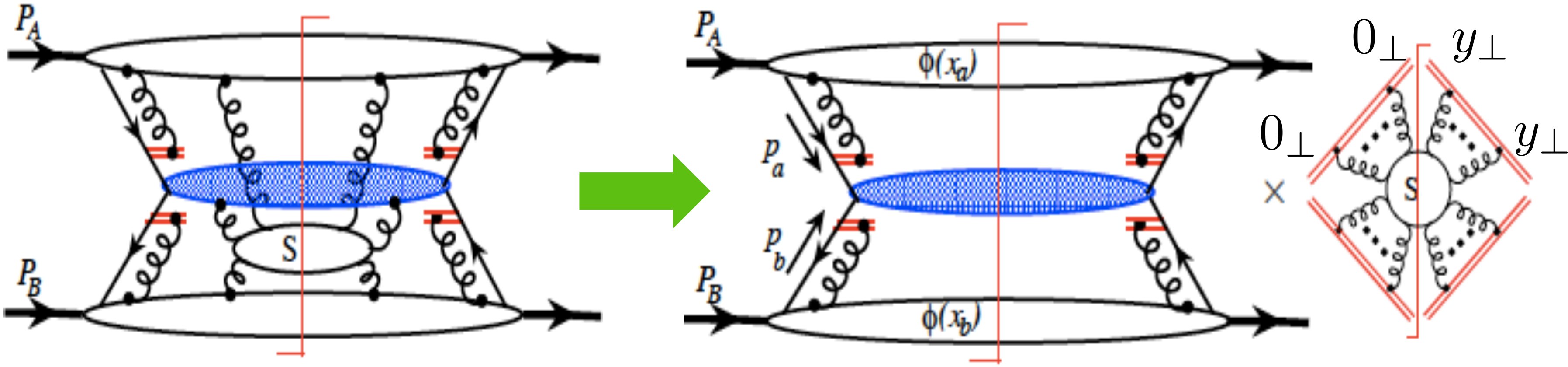
- ✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

- ✧ **The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

❑ Most difficult part of factorization:



- ✧ Sum over all final states to remove all poles in one-half plane
  – no more pinch poles
- ✧ Deform the $k^\pm$ integration out of the trapped soft region
- ✧ Eikonal approximation ⟶ soft gluons to eikonal lines
  – gauge links
- ✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^y \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

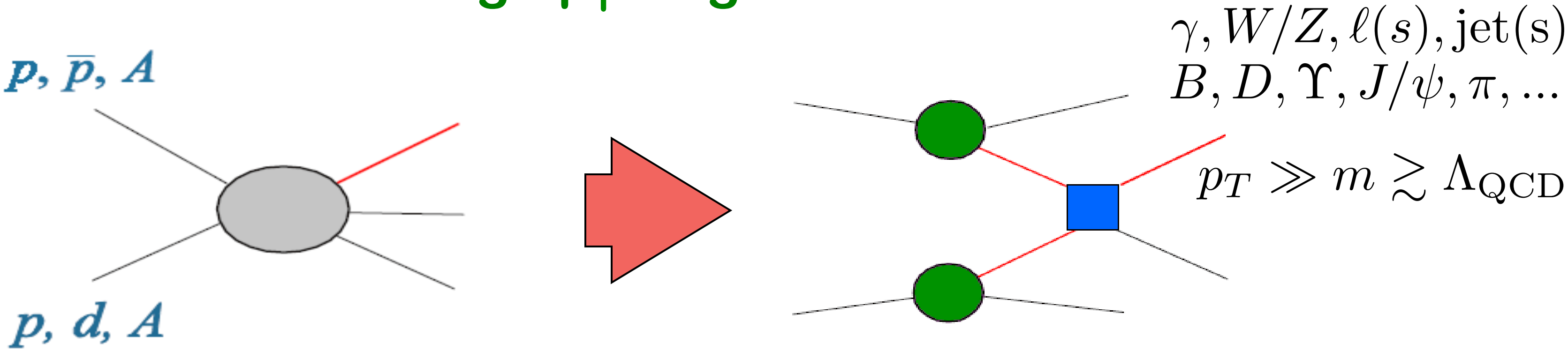
The factorization arguments are independent of the spin states of the colliding hadrons

→ **same formula with polarized PDFs for γ^*, W/Z, H^0...**

Factorization for more than two hadrons

Nayak, Qiu, Sterman, 2006

❑ Factorization for high p_T single hadron:



$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

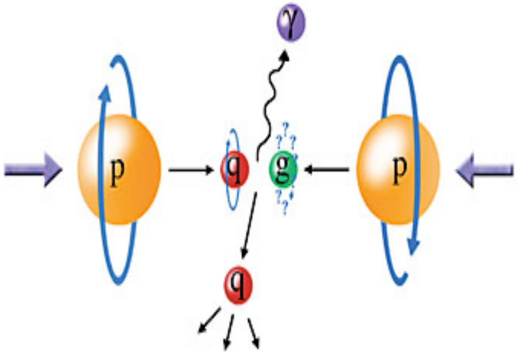
✧ **Fragmentation function:** $D_{c\to C}(z, \mu_F^2)$

✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Example: direct photon production

- Production mechanism – leading power factorization:



$$\frac{d\sigma_{AB}}{dy dp_T^2} = \int dx f_{a/A}(x,\mu) \int dx' f_{b/B}(x',\mu) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu))}{dy dp_T^2} + \text{frag contribution} + \mathcal{O}\left(\frac{1}{p_T^n}\right)$$

Hard part: $\hat{\sigma}_{ab}(\alpha_s(\mu)) = \hat{\sigma}^0_{ab}\alpha_s^m(\mu) + \hat{\sigma}^1_{ab}(\log(\mu))\alpha_s^{m+1}(\mu) + \ldots$

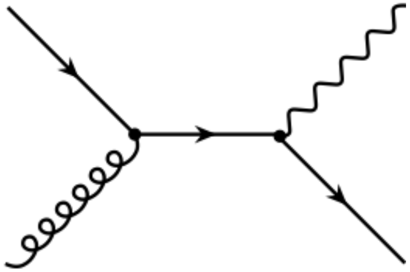
- Predictive power:
  - ✧ Short-distance part is infrared-Safe, and calculable
  - ✧ Long-distance part at the leading power is Universal – PDFs, FFs
- Factorization and renormalization scale dependence:
  - ✧ NLO is necessary
- Power correction could be important at low p_T

Direct photon is sensitive to gluon

❑ Sensitive to gluon at the leading order – hadronic collision:

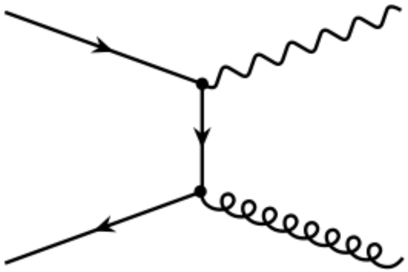
✧ Lowest order direct $\mathcal{O}(\alpha_{em}\alpha_s)$:

Compton: $q(\bar{q}) + g \to \gamma + q(\bar{q})$



+ …

Annihilation: $q + \bar{q} \to \gamma + g$



+ …

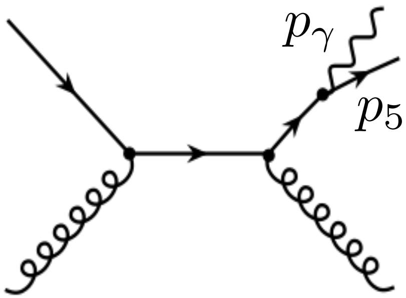
✧ Compton dominates in pp collision:

$f_{g/p}(x,\mu^2) \gg f_{\bar{q}/p}(x,\mu^2)$ **for all x**

Direct photon production could be a good probe of gluon distribution

Complication from high orders

❑ Final-state collinear singularity:



$$\overline{\sum}|M(qg\to\gamma qg)|^2 \approx \frac{\alpha_{em}}{2\pi}\mathcal{P}^{(0)}_{q\to\gamma}(z)\,\frac{1}{s_{\gamma q}}\,\overline{\sum}|M(qg\to qg)|^2$$

$$\mathcal{P}^{(0)}_{q\to\gamma}(z) = \frac{1+(1-z)^2}{z}$$

$$s_{\gamma q} = (p_\gamma + p_5)^2 \to 0 \qquad \textbf{when} \quad p_\gamma \parallel p_5$$

An internal quark line goes on-shell signaling long-distance physics

❑ Fragmentation contribution:

$$\frac{d\sigma^{\rm Frag}_{AB\to\gamma}}{dy\,dp_T^2} = \sum_{abc}\int\frac{dz}{z^2}D_{c\to\gamma}(z,\mu)\int dx\,f_{a/A}(x,\mu)\int dx'\,f_{b/B}(x',\mu)\,\frac{d\hat{\sigma}^{\rm Frag}_{ab\to c}}{dy\,dp_T^2}$$

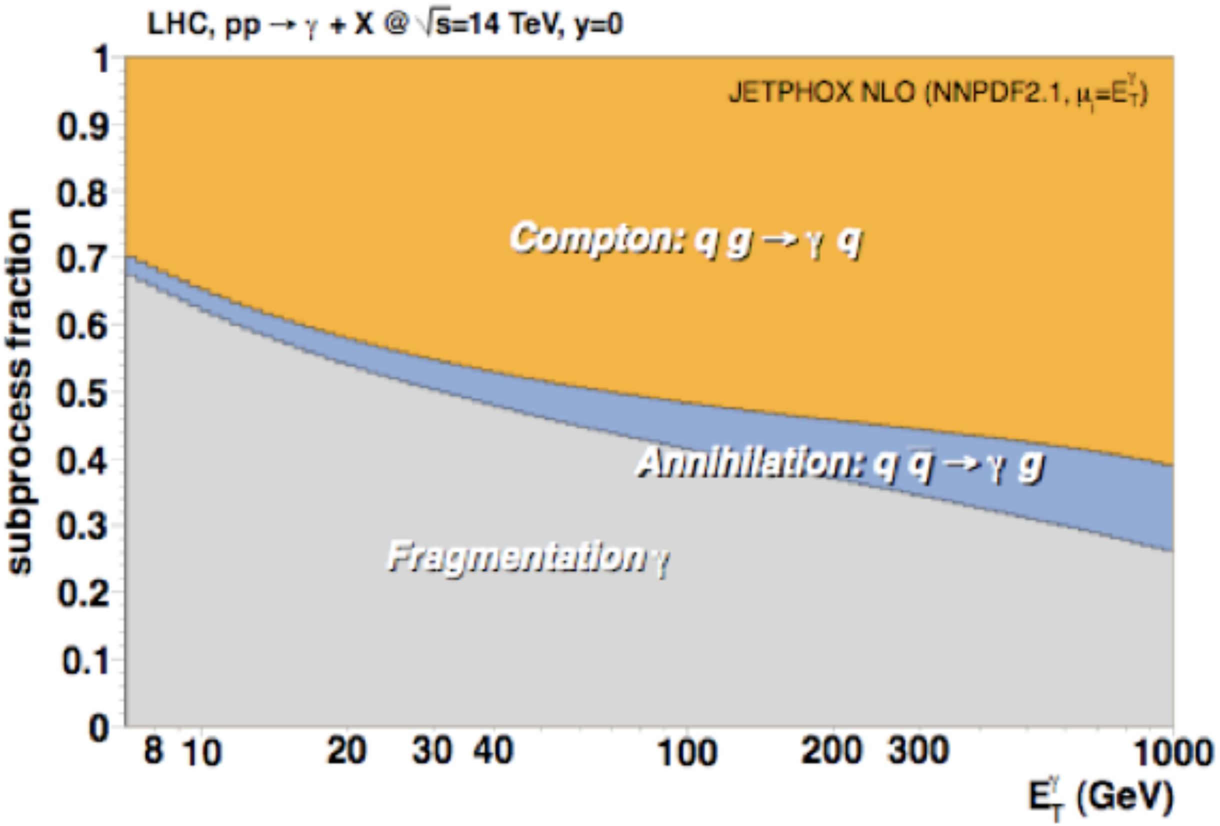
❑ Photon fragmentation functions – inhomogeneous evolution:

$$\frac{\partial D_{c\to\gamma}(z,\mu)}{\partial\log(\mu)} = \frac{\alpha_{em}}{2\pi}\mathcal{P}_{c\to\gamma}(z) + \sum_{a=q\bar{q}g}\frac{\alpha_s}{2\pi}P_{ac}(z)\otimes D_{a\to\gamma}(z,\mu)$$

Size of fragmentation

Campbell, CTEQ SS2013

❑ Inclusive direct photon:



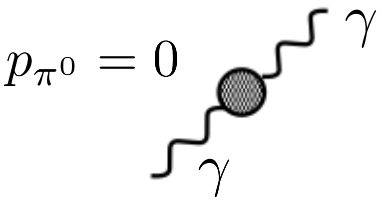
✧ Production at NLO – available, e.g., in MCFM and JETPHOX (shown here)

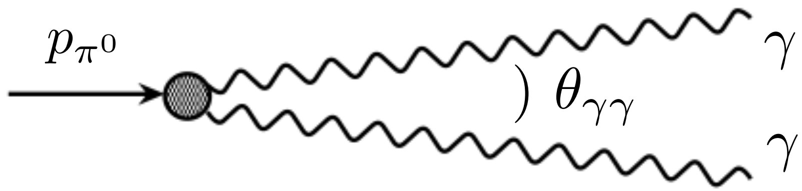
✧ Fragmentation contribution is huge for inclusive production:

$\sigma^{Frag} / \sigma^{Total}$ > 50% at pT=20 GeV @ LHC (role of FF!)

Complication from the measurement

- Separation the signal photon from $\pi^0 \to \gamma\gamma$:





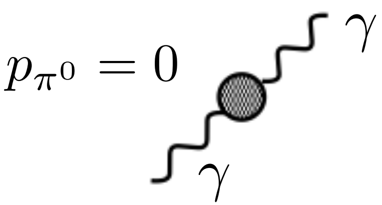
  - When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases
  - Two photons could be misidentified as one photon at high p_T

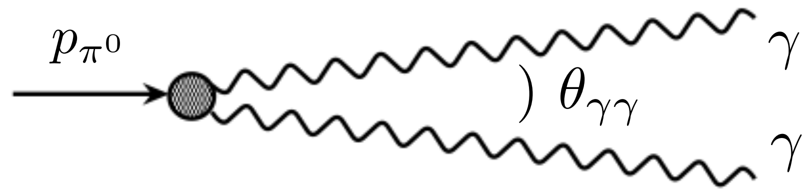
- Isolation cut – algorithms (like jet):
  - Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Complication from the measurement

❑ **Separation the signal photon from $\pi^0 \to \gamma\gamma$:**





✧ **When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases**

✧ **Two photons could be misidentified as one photon at high p_T**

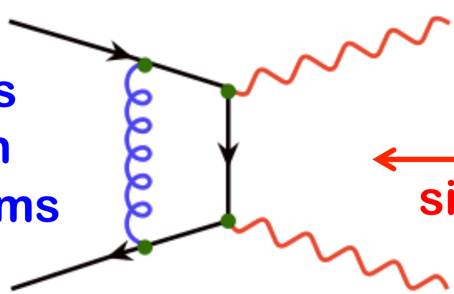
❑ **Isolation cut – algorithms:**

Needed for IR safety

✧ **Cone algorithm – reduction of fragmentation contribution**

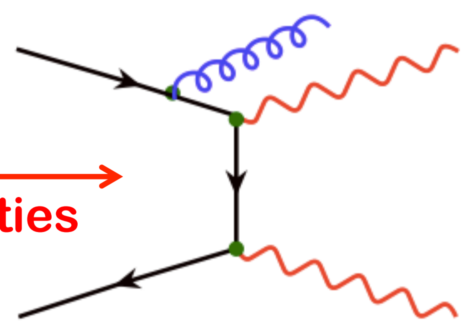
Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Isolation has No effect on virtual diagrams



Soft

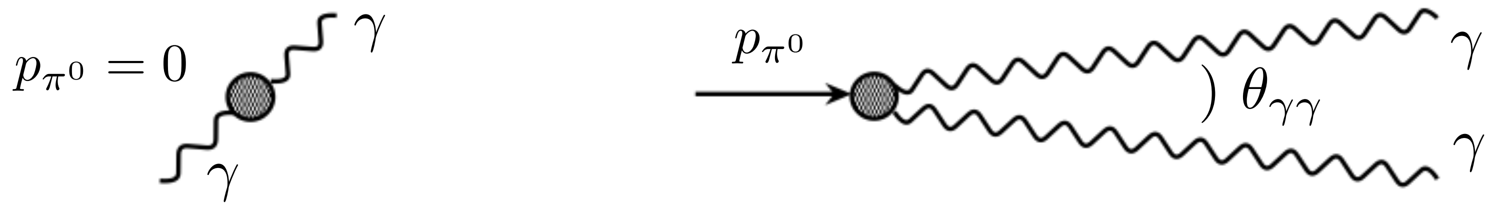
singularities



Isolation limits phase space of real diagrams

Complication from the measurement

❑ Separation the signal photon from $\pi^0 \to \gamma\gamma$:



✧ When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases

✧ Two photons could be misidentified as one photon at high p_T

❑ Isolation cut – algorithms:

✧ Cone algorithm – reduction of fragmentation contribution

Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

✧ Modified cone algorithm – NO fragmentation contribution

$$\sum_{R_{j\gamma} \in R_0} E_T(\text{had}) < \epsilon_h p_T^\gamma \left(\frac{1 - \cos R_{j\gamma}}{1 - \cos R_0} \right)$$

✧ Parton is softer as it closer to photon

✧ No contribution at CO singularity

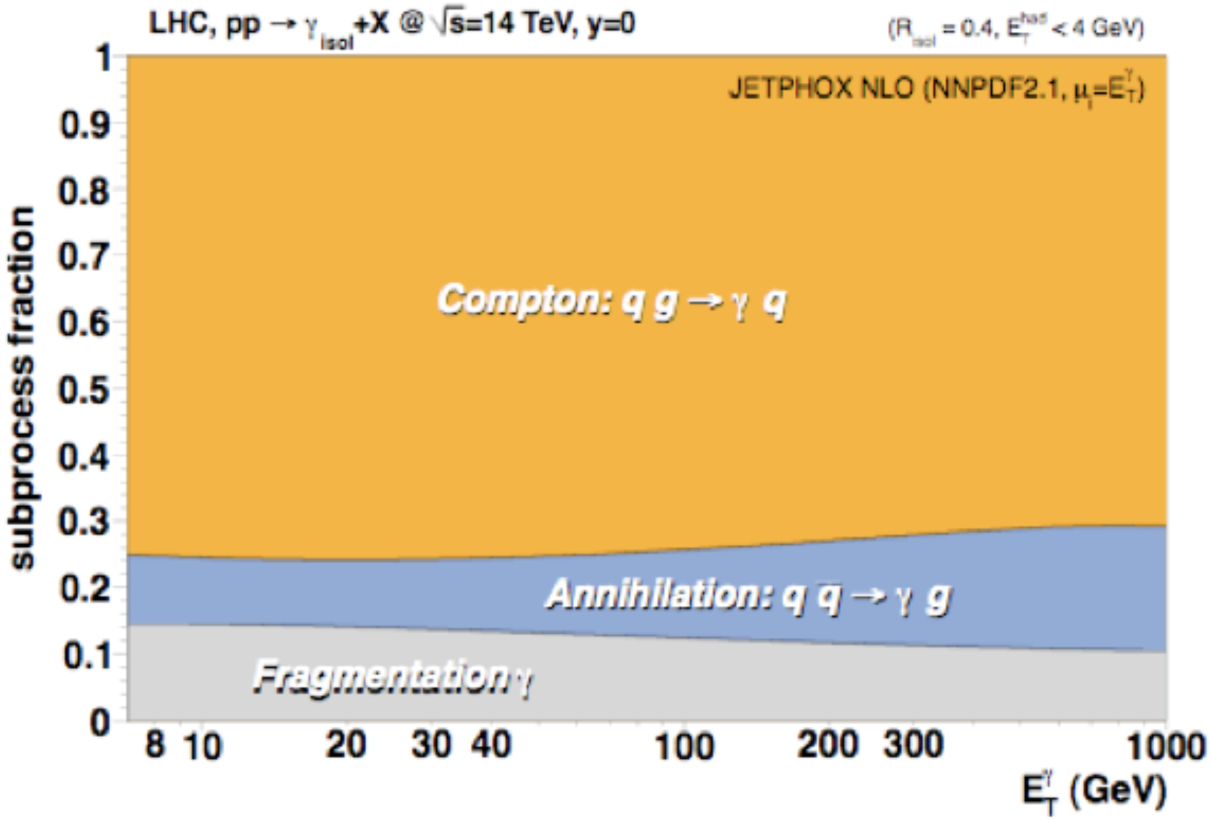
Hard to implement experimentally (detector resolution)

S. Frixione, 1998

Size of fragmentation

❑ Isolated direct photon:

Campbell, CTEQ SS2013



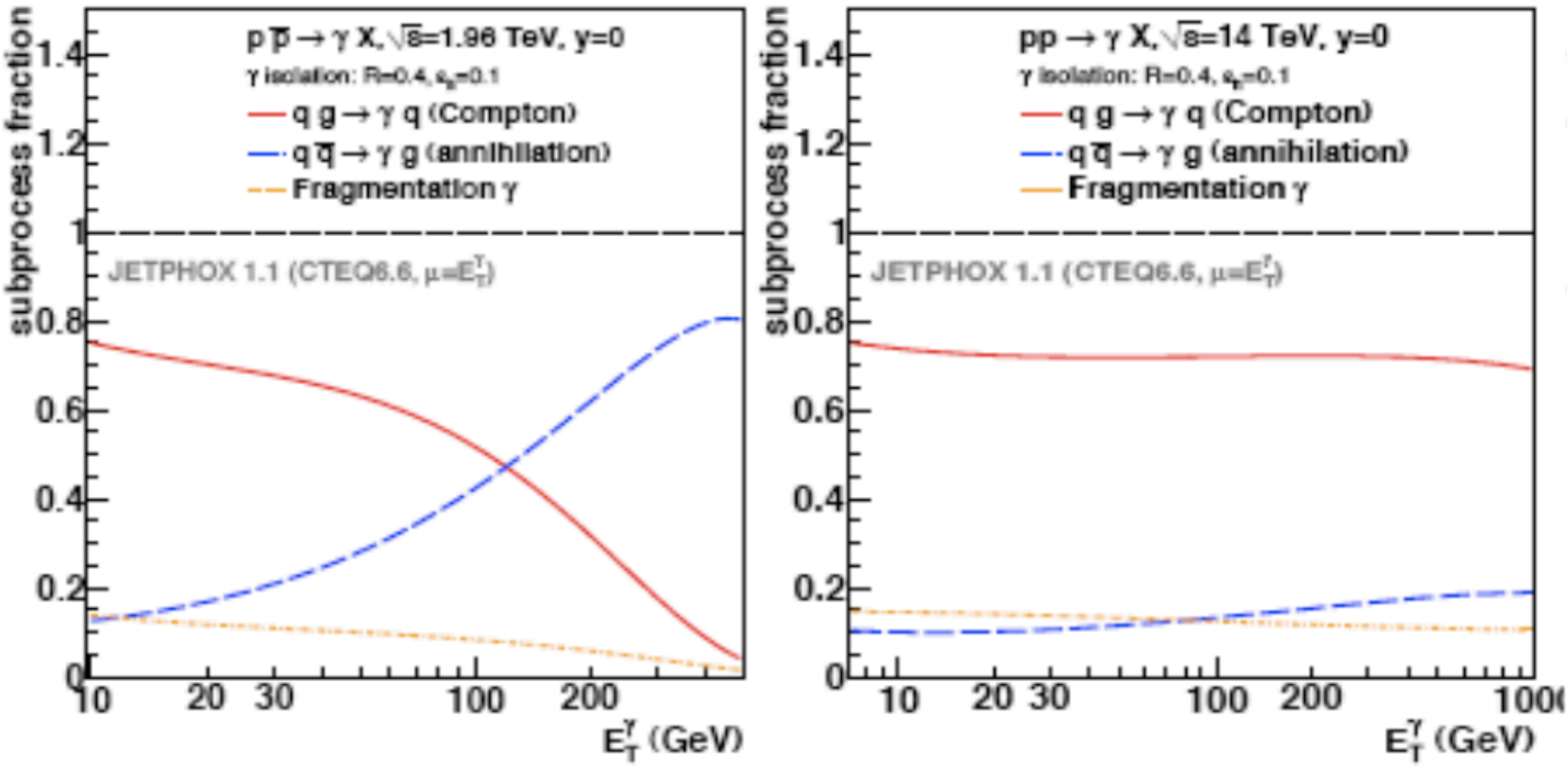
✧ Isolation removes the most of fragmentation contribution! (down to 10%)

✧ About 75% of production rate is from gluon initiated subprocesses

Potentially, a useful probe of gluon PDF

Role of gluon in pp collision

❑ pp vs p$\bar{p}$:



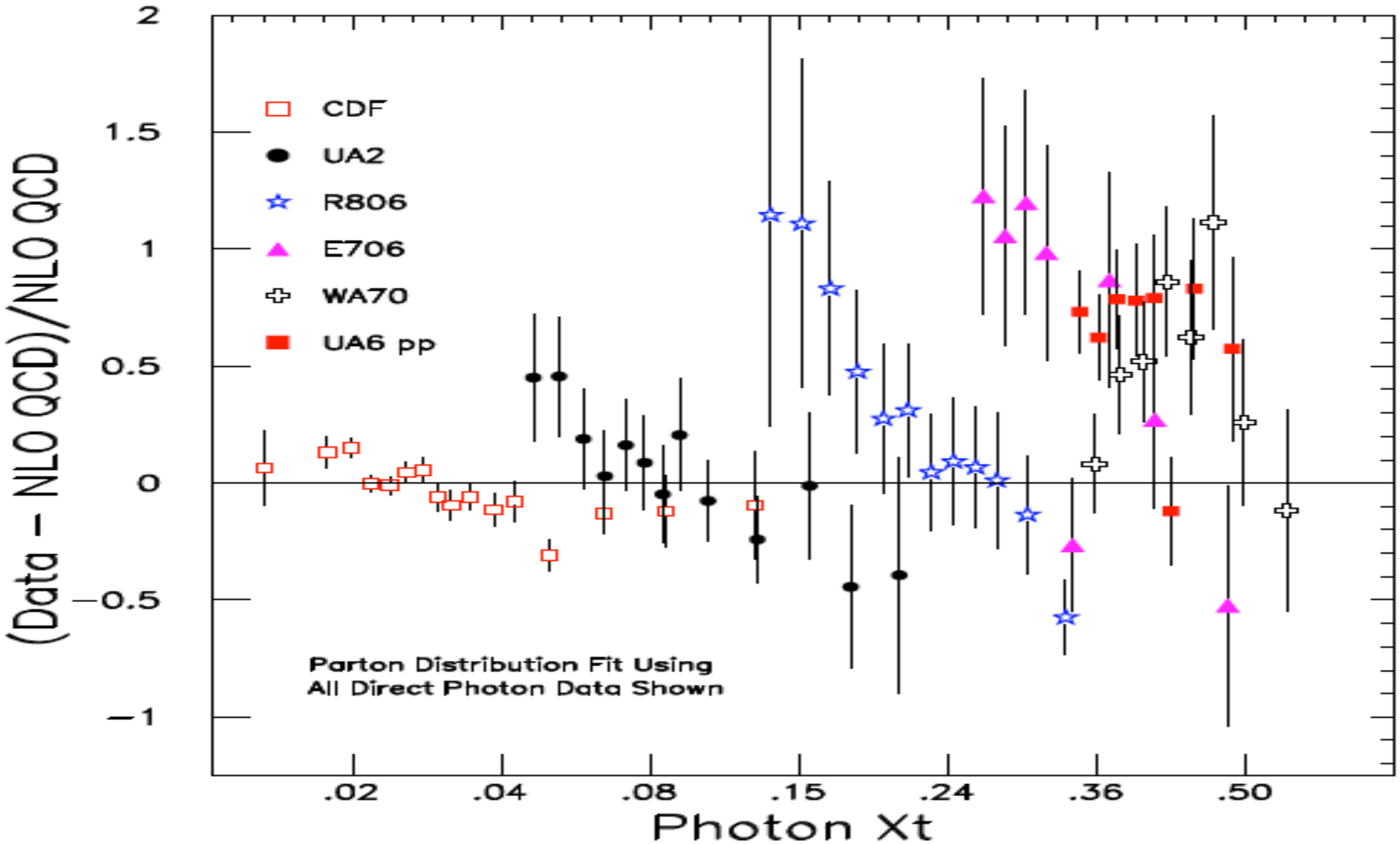
✧ Dominant role of the gluon in pp collision!

✧ Even more dominance in the forward region!

Compare with data from different expt's

❑ **CTEQ global analysis:**

CTEQ Huston et al.



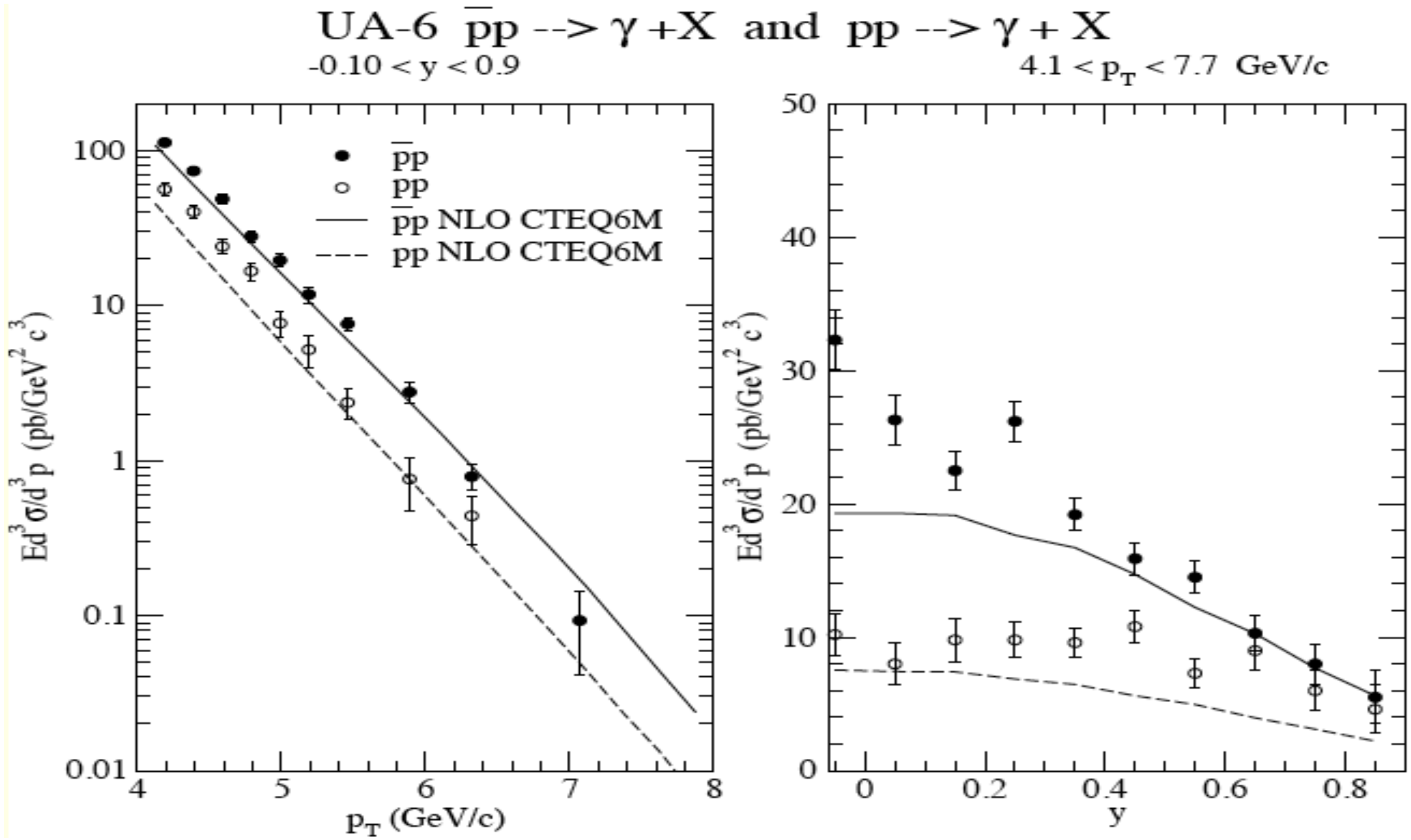
$$x_T = \frac{2p_T}{\sqrt{s}}$$

✧ **Neither PDFs nor photon FFs can significantly improve the shape**

✧ **Direct photon data were excluded from most global fits**

Experiments with both pp and $p\bar{p}$

❑ **UA6:** both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV

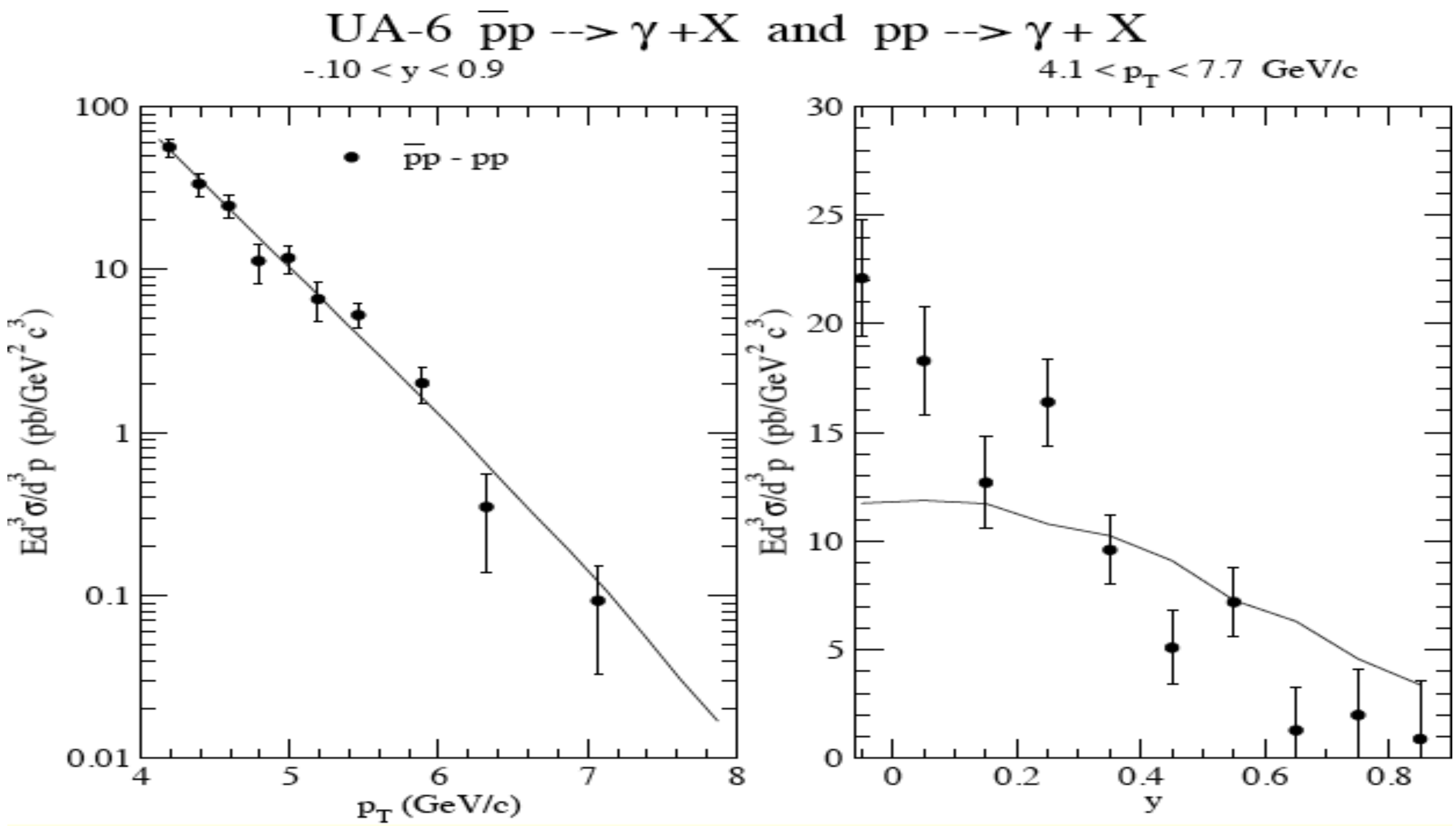


✧ **Theory curves are below the data**

✧ **Rapidity curves are flatter**

Role of gluon distribution?

❑ **UA6: $\bar{p}p$ - pp**   both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV

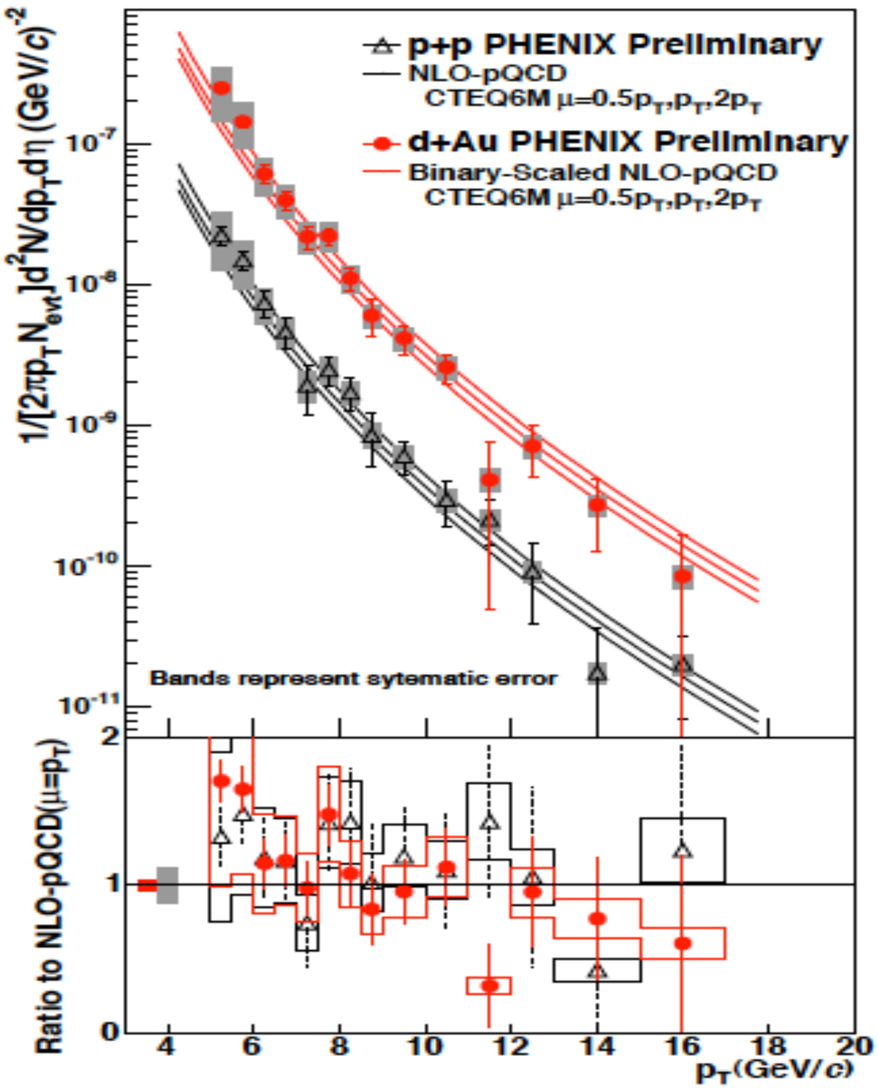


✧ **NO gluon contribution to the difference!**

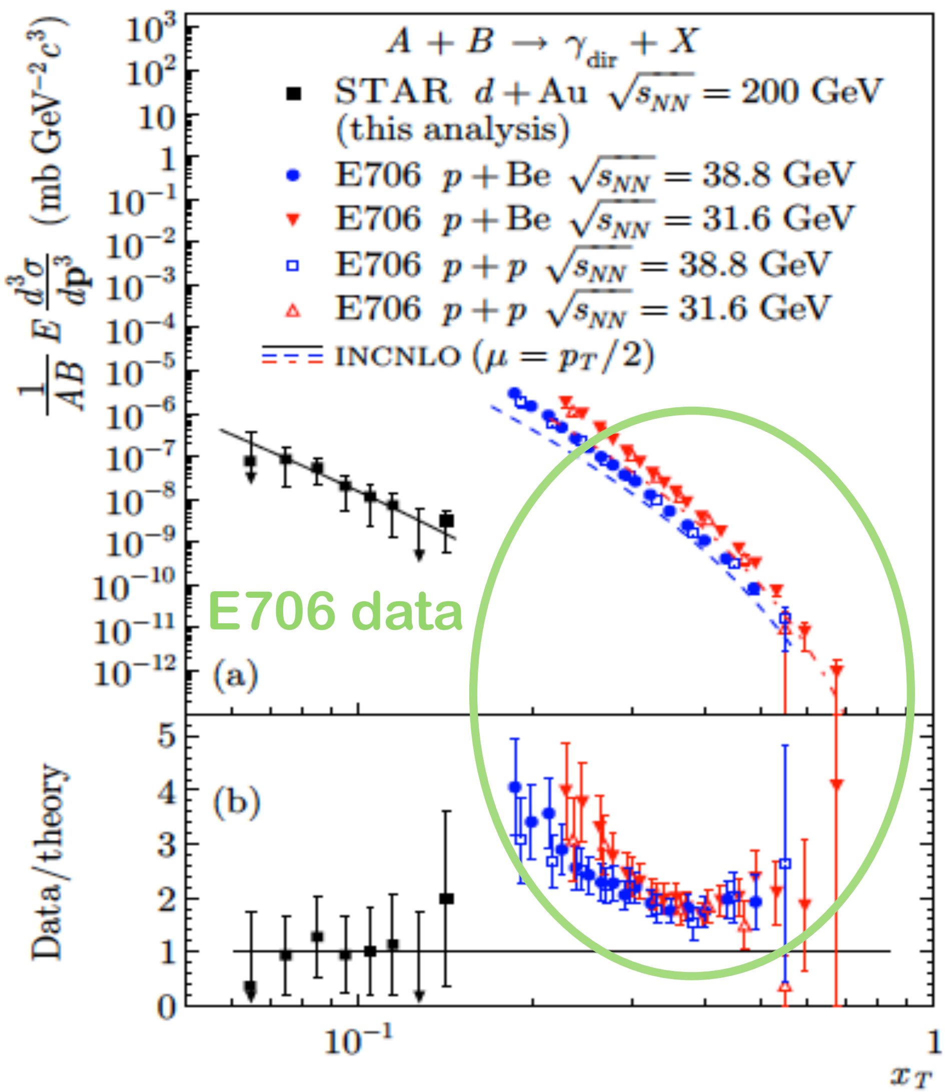
✧ **Theory matches the data better – role of gluon?**

Theory works well at RHIC energy

PHENIX



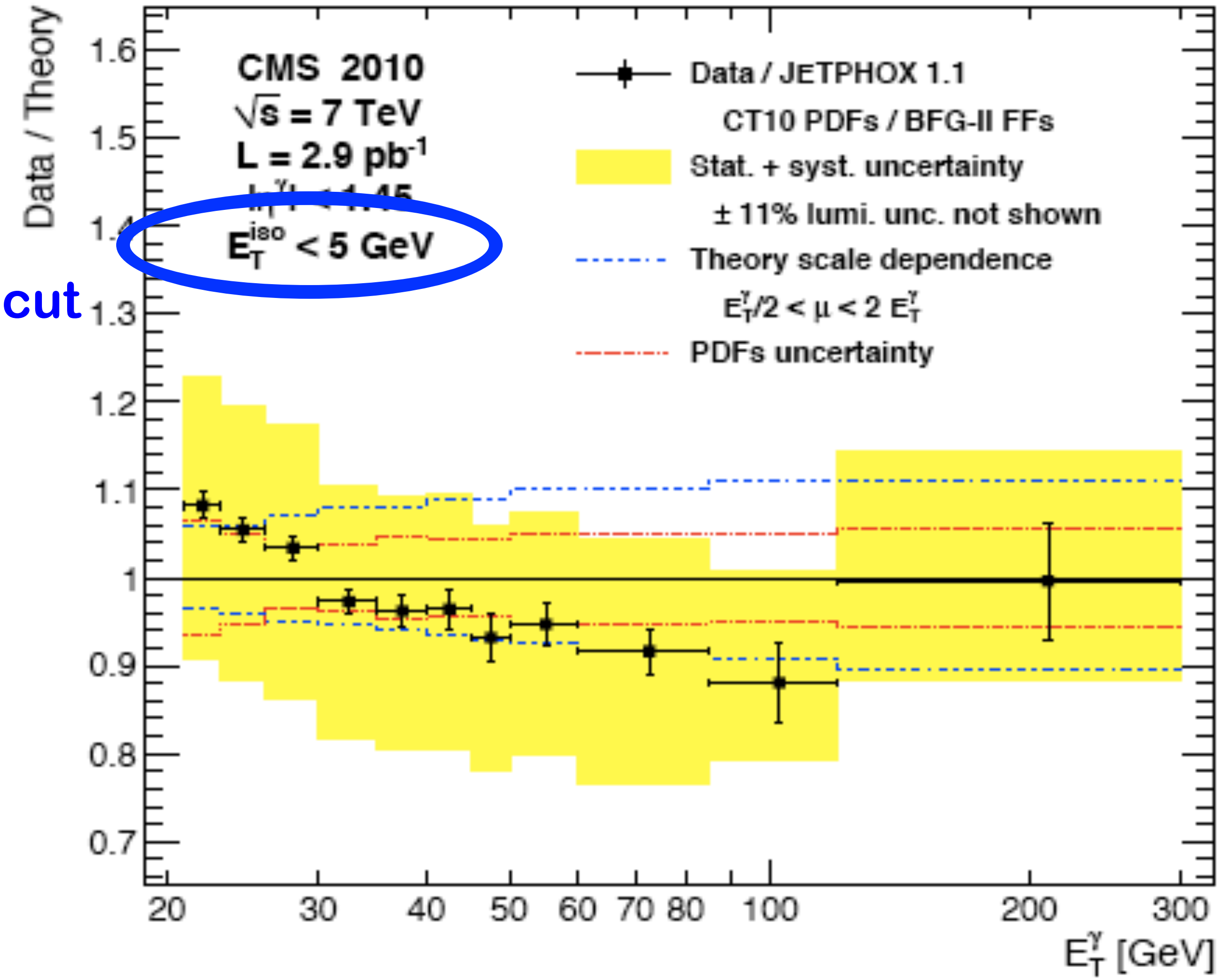
STAR



How about at the LHC?

- CMS:

Isolation cut

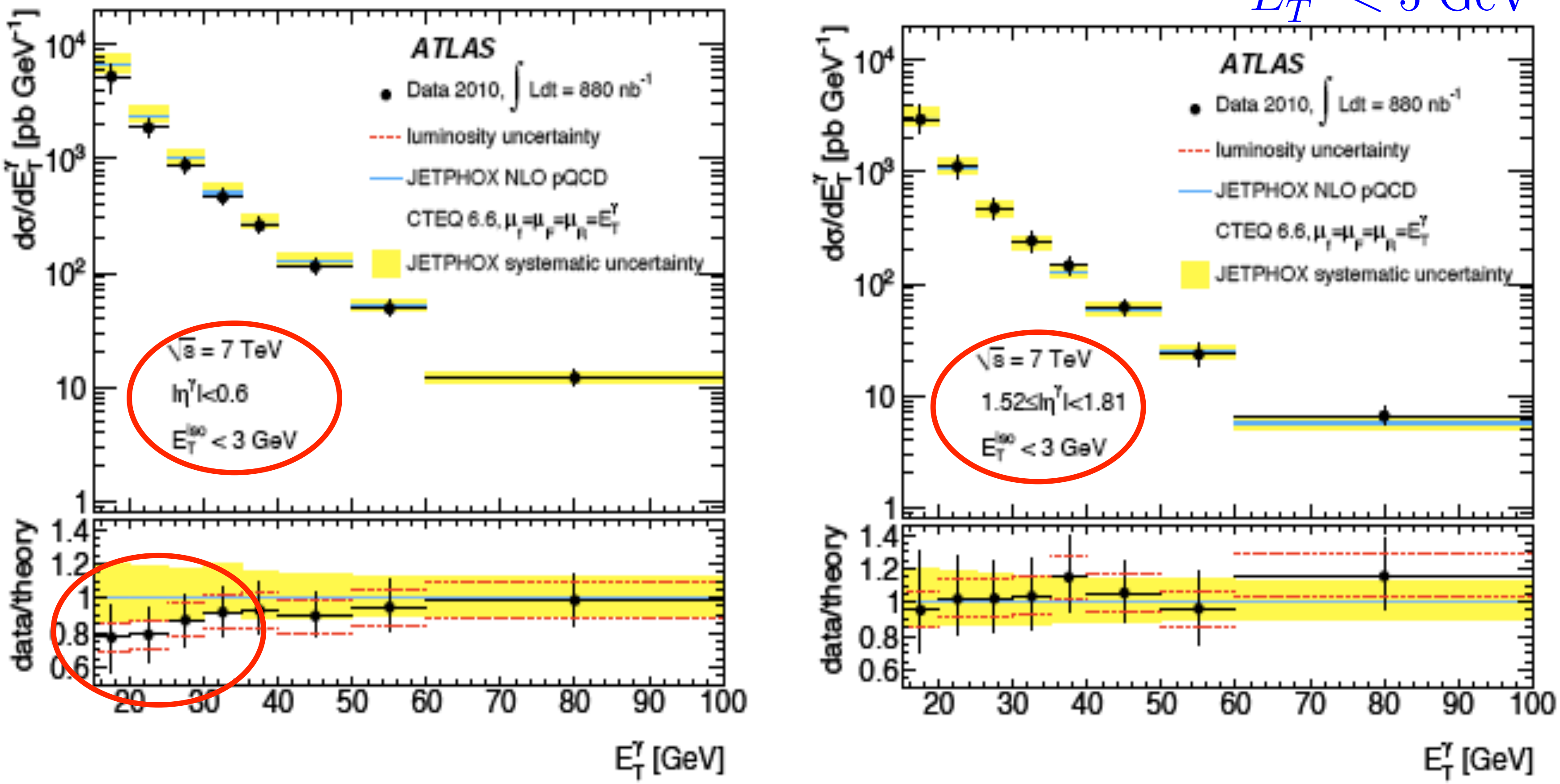


✧ Shape in x_T – within the PDF uncertainty?

Rapidity dependence at the LHC

- ATLAS:

Note: CMS has $E_T^{\rm iso} < 5$ GeV



✧ **Data seems to be lower than theory at central** η^γ **and small** E_T^γ

Overall consistency is better at collider energies!

Where do we stand?

- Agreement between theory and data improves with increasing energy and is excellent at $\sqrt{s}$ = 200 GeV

- Situation with fixed target direct photon data is confusing:
  - ✧ Disagreement between experiments
  - ✧ A reassessment of systematic errors on the existing fixed target photon experiments might help resolve the discrepancies

- We need an improved method of calculating single particle inclusive cross sections in the fixed target energy
  – Threshold resummation helps

- All experiments see an excess of data over theory at fixed target energies, but, less than theory at low pT at the LHC

More data from the LHC should help (the gluon dominance)!

Global QCD analyses – test of pQCD

- **Factorization for observables with identified hadrons:**
  - **One-hadron (DIS):**

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - **Two-hadrons (DY, Jets, W/Z, …) :**

$$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

  - **DGLAP Evolution:**

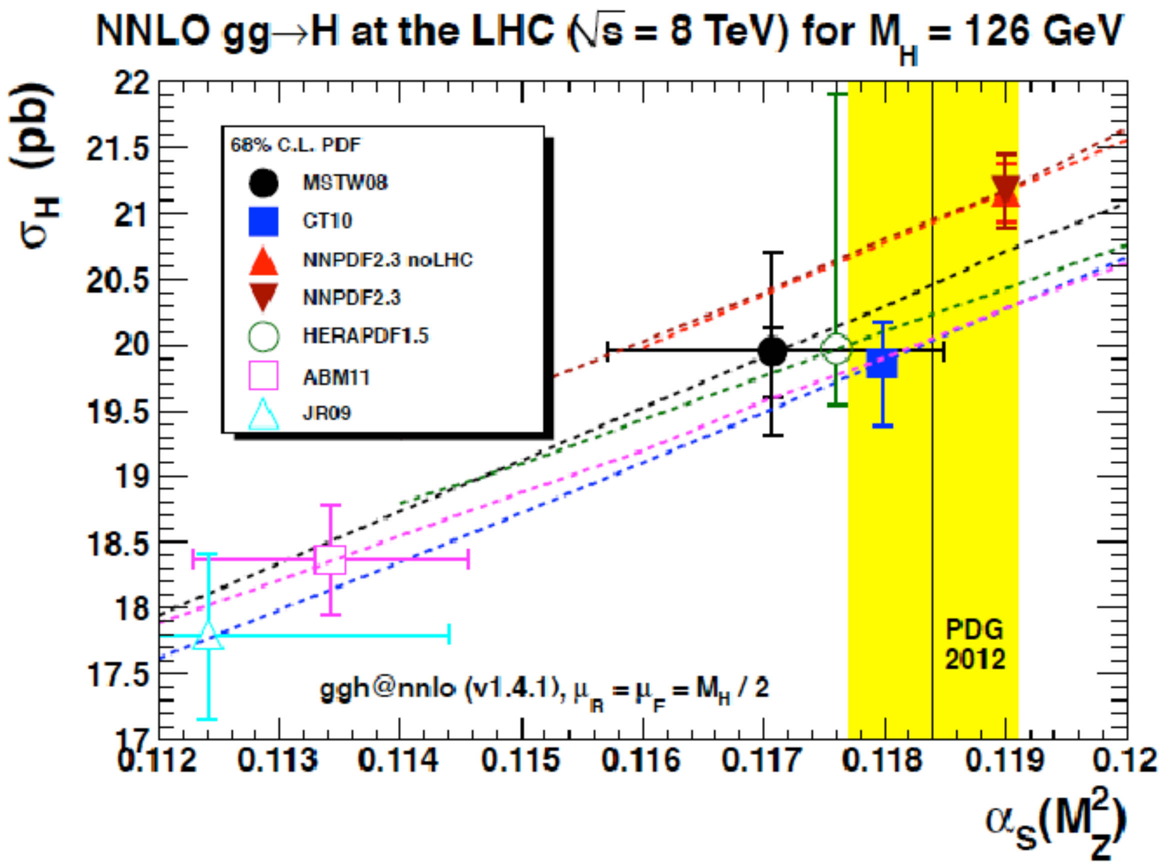
$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

→ **Solve for PDFs**

- **The key to test pQCD:**
  - **To show the existence of a universal set of PDFs, from which we can interpret or predict all data from high energy hadronic scattering, with pQCD calculated partonic hard parts**

Global QCD analysis – PDFs

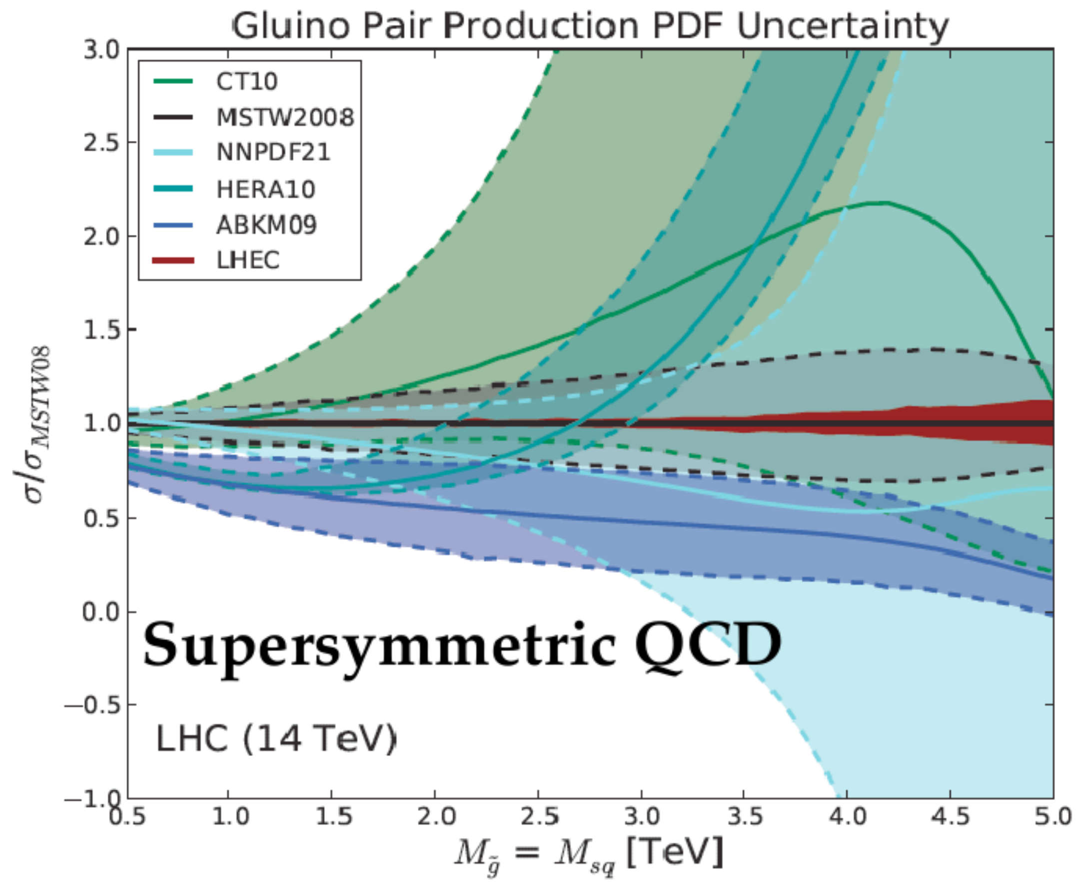
- Critical importance to have the precise PDFs
  - ✧ Without PDFs, we cannot calculate or interpret hadronic cross sections
  - ✧ PDF uncertainties limit the predictions of Higgs production, as well as the predictive power for signals of new physics



Strong impact on the measurement of Higgs coupling?

Global QCD analysis – PDFs

- **Critical importance to have the precise PDFs**
  - **Without PDFs, we cannot calculate or interpret hadronic cross sections**
  - **PDF uncertainties limit the predictions of Higgs production, as well as the predictive power for signals of new physics**

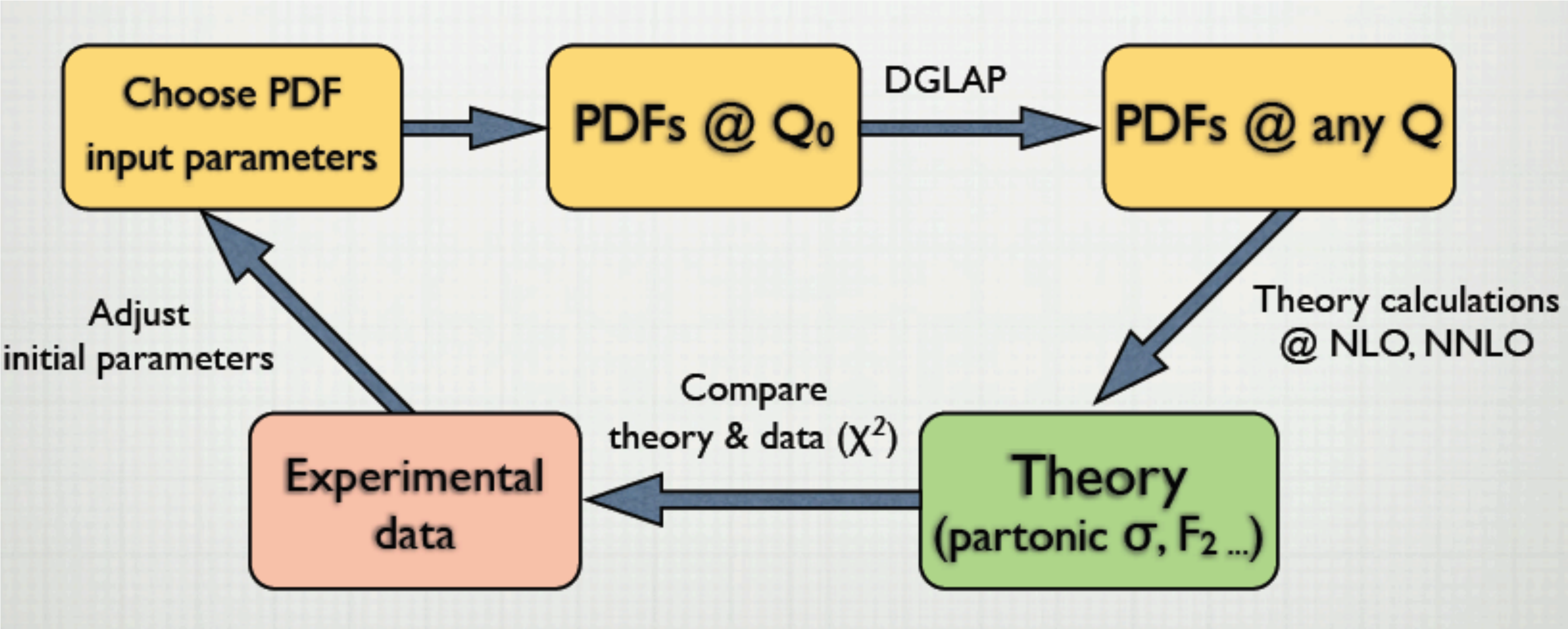


Precision of partonic flux for producing new particle with a large invariant mass

Global QCD analysis – PDFs

KAROL KOVAŘÍK @ CTEQ SS2015

❑ Routine to determine the PDFs:



✧ Choose input PDFs: functional forms and initial parameters $\{a_j\}$

✧ Select the fitting set of world data with "Q" > 2 GeV

✧ pQCD calculated partonic hard parts, at least NLO, better, NNLO

PDFs at an input scale

- Generic parameterization for input PDFs:

$$x f_k(x, Q_0) = x^{c_1}(1-x)^{c_2} P_k(x)$$

Different for various global fittings

- Popular PDF sets:

CTEQ6

hep-ph/0201195

$$x\, f_k(x, Q_0) = c_0\, x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0\, x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

$$s = \bar{s} = 0.2\,(\bar{u}+\bar{d}) \qquad k = u_v, d_v, g, \bar{u}+\bar{d}$$

CT10

arXiv:1007.2241

$$x\, f_k(x, Q_0) = c_0\, x^{c_1}(1-x)^{c_2} e^{-c_3(1-x)^2 + c_4 x^2}$$

$$k = u_v, d_v$$

$$x\, g(x, Q_0) = c_0\, x^{c_1}(1-x)^{c_2} e^{c_3 x + c_4 x^2} e^{-c_6 x^{-c_7}}$$

CT14

arXiv:1506.07443

$$k = u_v, d_v, g$$

$$x\, f_k(x, Q_0) = x^{c_1}(1-x)^{c_2}\left(c_3 + c_4\sqrt{x} + c_5\, x + c_6\, x^{3/2} + c_7\, x^2\right)$$

PDFs at an input scale

- Additional popular PDF sets:

CTEQ-JLAB '12

arXiv:1212.1702

$$k = u_v, d_v, g, \bar{u} + \bar{d}, \bar{d} - \bar{u}$$

$$x\, f_k(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} \left(1 + c_3 \sqrt{x} + c_4\, x\right)$$

$$d_v \to c_0^{d_v} \left(\frac{d_v}{c_0^{d_v}} + b\, x^c u_v \right)$$

MSTW

arXiv:0901.0002

$$k = u_v, d_v, 2(\bar{u} + \bar{d}) + s + \bar{s}, s + \bar{s}$$

$$x\, f_k(x, Q_0) = A_k\, x^{\eta_1} (1-x)^{\eta_2} (1 + \epsilon_k \sqrt{x} + \gamma_k x)$$

$$x\, g(x, Q_0) = A_g\, x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'}\, x^{\delta_{g'}} (1-x)^{\eta_{g'}}$$

- Input scale:

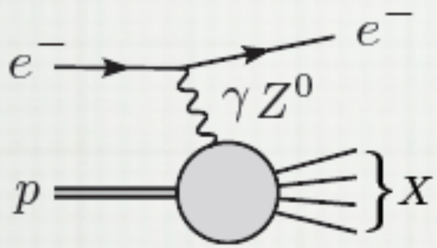
$Q_0 = 1.3$ GeV	$Q_0 = 1.0$ GeV	$Q_0 = \sqrt{2}$ GeV
CTEQ	**MSTW**	**NNPDF**

Processes, data, and theory

❑ Selection of data sets:

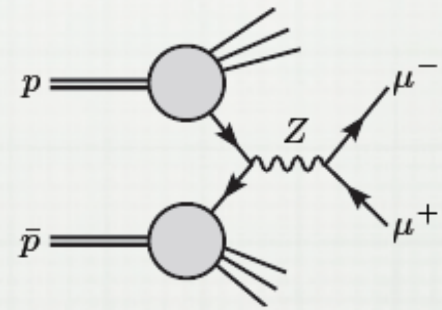
☑ *Neutral current DIS*

(HERA, SLAC, NMC, BCDMS)



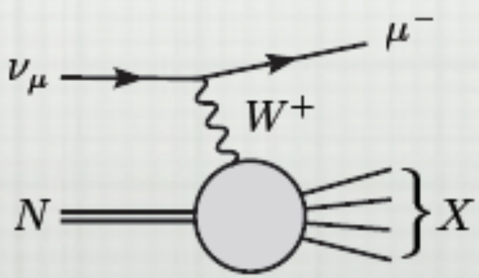
☑ *DY data (W,Z production) from Tevatron & LHC*

(E602, E866, D0, CDF, ATLAS, CMS)



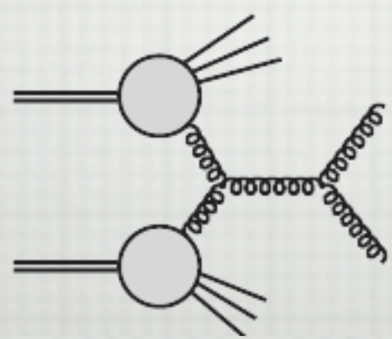
☑ *Neutrino DIS & di-muon*

(CDHSW, CHORUS,NuTeV,



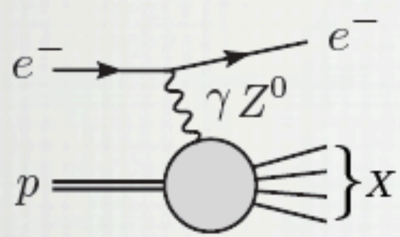
☑ *Jet data from Tevatron & LHC*

(D0,CDF,ATLAS, CMS)



Physics behind the data choice

☑ *DIS data from HERA & fixed target experiments (SLAC,NMC,BCDMS)*



- DIS @ low Q - dominated by photon exchange
- DIS @ high Q - dominated by photon-Z interference

$$F_{1,2} = F_{1,2}^{\gamma\gamma} + \frac{2g_V}{4s_W^2 c_W^2}\frac{Q^2}{Q^2+M_Z^2}F_{1,2}^{\gamma Z} + \frac{g_V^2+a_V^2}{16 s_W^4 c_W^4}\frac{Q^4}{(Q^2+M_Z^2)^2}F_{1,2}^{ZZ}$$

$$F_3 = \frac{2a_V}{4s_W^2 c_W^2}\frac{Q^2}{Q^2+M_Z^2}F_3^{\gamma Z} + \frac{2g_V a_V}{16 s_W^4 c_W^4}\frac{Q^4}{(Q^2+M_Z^2)^2}F_3^{ZZ},$$

sensitive to quark & anti-quark PDF @ LO

- quarks & anti-quarks enter together with different weights depending on the exchange vector boson

NC DIS

$$F_2^{\gamma\gamma}(x,Q^2) = x\sum_q e_q^2\left[q(x,Q^2)+\bar{q}(x,Q^2)\right]$$ **Photon**

$$F_2^{\gamma Z}(x,Q^2) = x\sum_i B_i\left[q_i(x,Q^2)+\bar{q}_i(x,Q^2)\right]$$ **Photon-Z interference**

$$xF_3^{\gamma Z}(x,Q^2) = x\sum_i D_i\left[q_i(x,Q^2)-\bar{q}_i(x,Q^2)\right]$$ **Photon-Z interference**

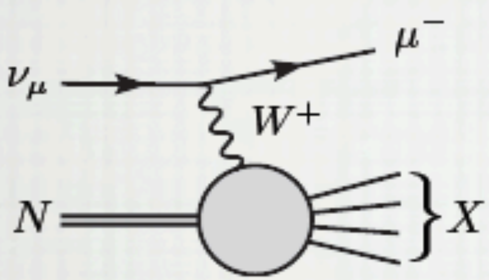
Physics behind the data choice

☑ *Charge current DIS on proton (HERA)*

- neutrino DIS can be replaced by CC DIS on protons (still experimentally challenging)

$$F_2^{W^\pm}(x,Q^2) = x(\bar{u} \pm d \pm s + \bar{c})$$

☑ *Neutrino DIS & di-muon (CDHSW, CHORUS,NuTeV)*



- neutrino DIS contributes to $F_2(x,Q^2)$ and $F_3(x,Q^2)$
- different PDF combinations contribute to flavor separation

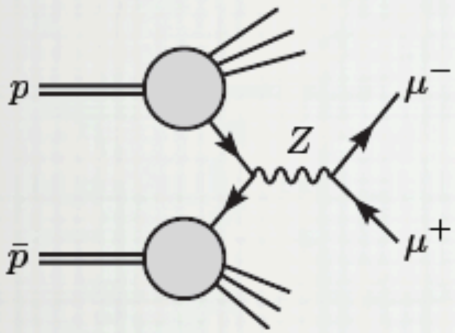
$$F_2(x,Q^2) = x\sum_q \left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

$$xF_3(x,Q^2) = x\sum_q \left[q(x,Q^2) - \bar{q}(x,Q^2)\right]$$

- neutrino DIS data on protons are scarce and hard to come by (WA21/22)
- neutrino DIS typically taken on nuclei - need for nuclear corrections

Physics behind the data choice

Drell-Yan lepton pair production (W,Z production)



- DY dominated by photon exchange away from W & Z resonances

$$\frac{d\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{9Q^2 s}\sum_i e_i^2 \left[q_i(x_a, Q^2)\bar{q}_i(x_b, Q^2) + a \leftrightarrow b\right]$$

$$Q^2 = (p_l + p_{\bar{l}})^2 \qquad x_{a,b} = \frac{Q}{\sqrt{s}}\exp(\pm y)$$

- DY at the W & Z resonances - different PDF combinations

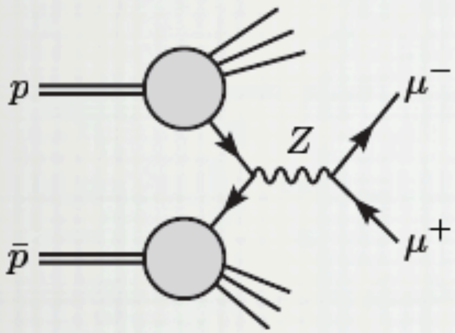
$$\frac{d\sigma^W}{dy} = \frac{\sqrt{2}\pi G_F m_W^2}{3s}\sum_{i,j}|V_{ij}^{\mathrm{CKM}}|\left[q_i(x_a, Q^2)\bar{q}_j(x_b, Q^2) + a \leftrightarrow b\right]$$

$$\frac{d\sigma^Z}{dy} = \frac{\sqrt{2}\pi G_F m_Z^2}{3s}\sum_i (V_i^2 + A_i^2)\left[q_i(x_a, Q^2)\bar{q}_i(x_b, Q^2) + a \leftrightarrow b\right]$$

- DY helps better determine (anti-)quark PDFs

Physics behind the data choice

Drell-Yan lepton pair production (W,Z production)

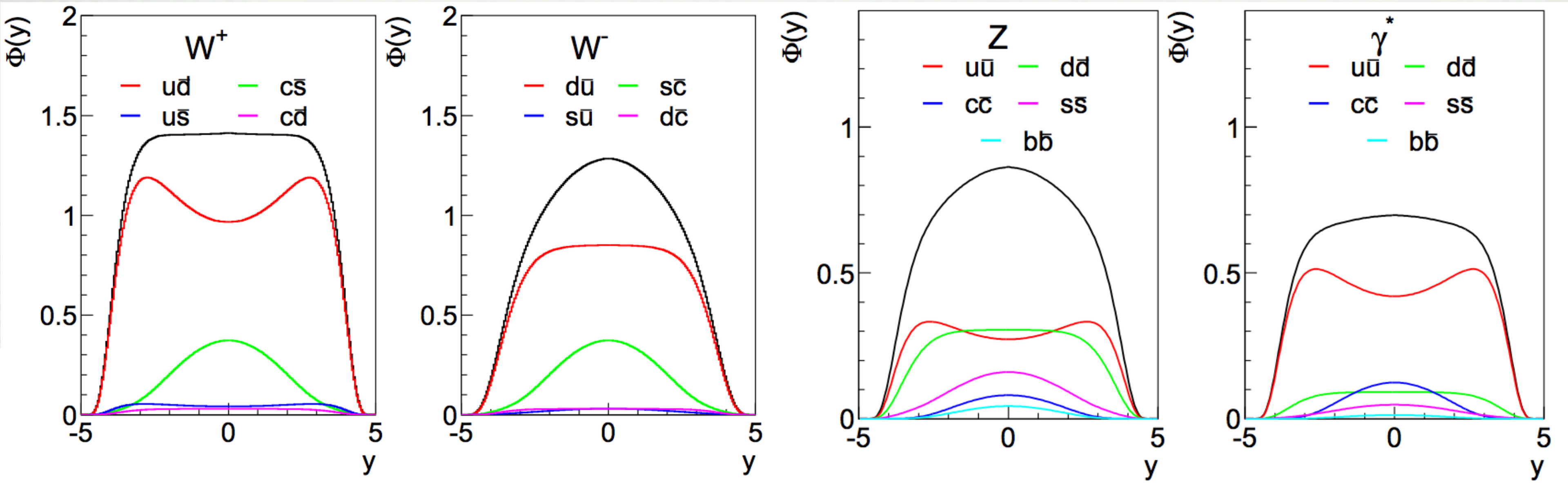


- DY dominated by photon exchange away from W & Z resonances

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y} = \frac{4\pi\alpha^2}{9Q^2 s}\sum_i e_i^2\left[q_i(x_a,Q^2)\bar{q}_i(x_b,Q^2) + a \leftrightarrow b\right]$$

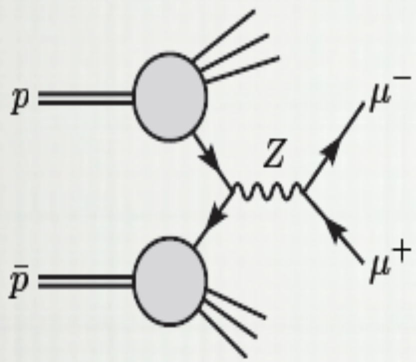
$$Q^2 = (p_l + p_{\bar{l}})^2 \qquad x_{a,b} = \frac{Q}{\sqrt{s}}\exp(\pm y)$$

- DY at the W & Z resonances - different PDF combinations



Physics behind the data choice

Drell-Yan lepton pair production (W,Z production)



- different asymmetries & ratios sensitive to different combinations of PDFs

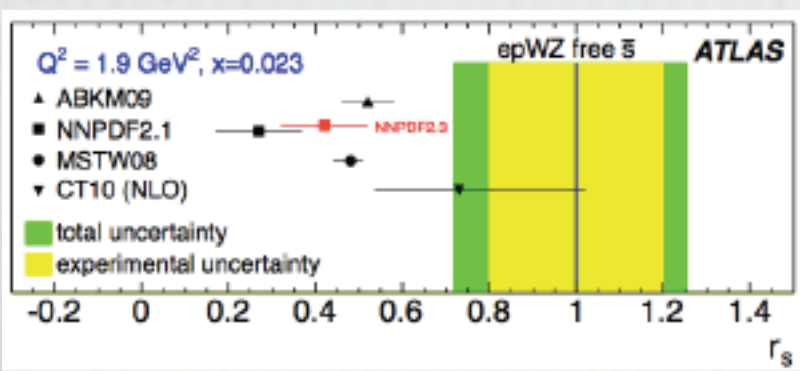
$$\frac{\sigma_{W^+} - \sigma_{W^-}}{\sigma_{W^+} + \sigma_{W^-}} \sim \frac{u_v(x_1) - d_v(x_1)}{u(x_1) + d(x_1)} \qquad \frac{\sigma_{W^+}}{\sigma_{W^-}} \sim \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \sim \frac{u(x_1)}{d(x_1)}$$

$$\frac{\sigma_{W^+} + \sigma_{W^-}}{\sigma_Z} \sim \frac{u(x_1) + d(x_1)}{0.29u(x_1) + 0.37d(x_1)}$$

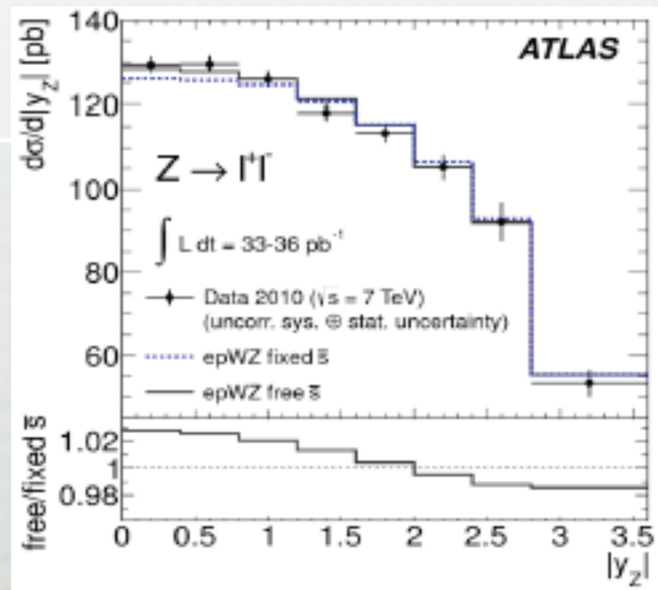
- Z cross-section + rapidity distribution used to extract strange quark PDF

typical assumptions related to strange PDF (motivated by di-muon DIS data)

$$s(x,Q) = \bar{s}(x,Q)$$
$$s(x,Q) = r_s d(x,Q) \qquad r_s \sim 0.75$$

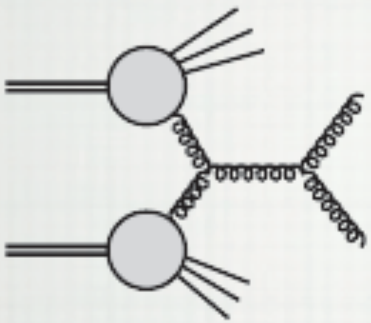


arXiv:1203.4051 [hep-ex]



Physics behind the data choice

☑ *Jet data from Tevatron & LHC (D0, CDF, ATLAS, CMS)*



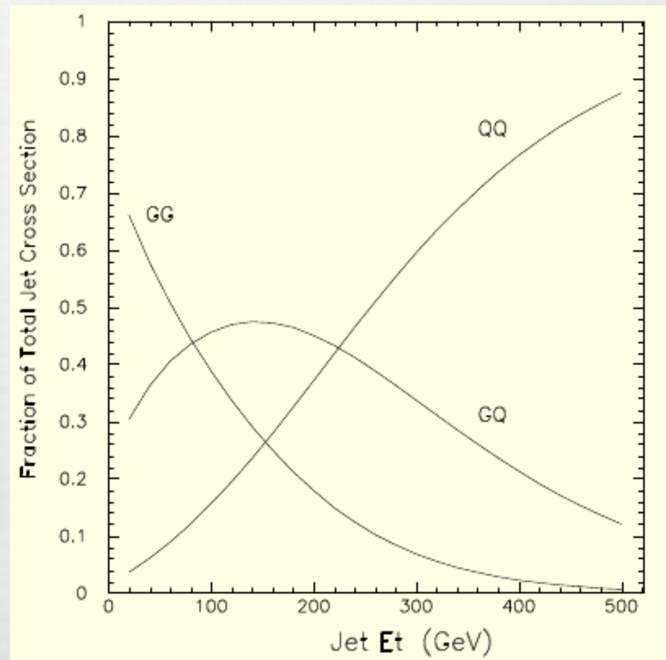
- hadronic jet production at leading order proceeds through

$$qq \to qq \qquad qg \to qg \qquad gg \to gg$$

- qq subprocess dominates high-Et jets but gluon important enough to allow jet data to put constraints on large-x gluon PDF

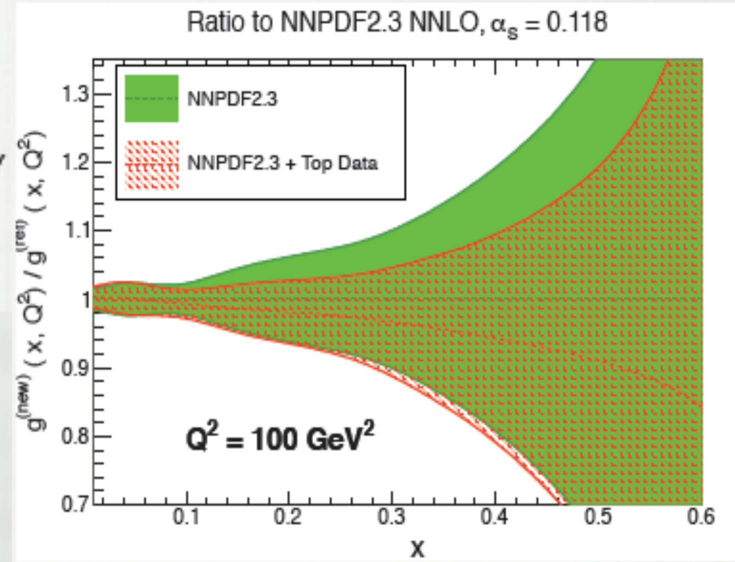
- combined with low-x constraints on gluon PDF from DIS and with sum rules one has strong constraints on the gluon PDF

- additional direct probes of gluon PDF needed to constrain the gluon PDF at mid-x and large-x for future searches e.g. SUSY @ LHC



Physics behind the data choice

arXiv:1301.7215



Top pair production

sensitive to gluon PDF at high-x

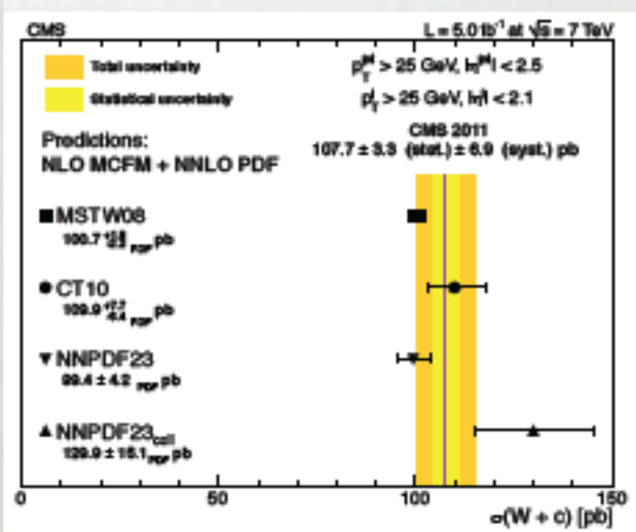
- very precise top pair production expected from LHC top-factory
- ratios of top/anti-top cross-sections sensitive also to u/d

"Yesterday's signal is today's background"

W+charm production

sensitivity to strange quark PDF (difficult to extract elsewhere)

CMS-SMP-12-002



Direct photons

additional, complementary probe of gluon PDF

(same x as gg Higgs production)

Chi^2 – fits and errors

- Most PDF fitters use χ^2- function to measure the goodness of the fit

standard definition

$$\chi^2 = \sum_{i=1}^{N_{\mathrm{dat}}} \left(\frac{D_i - T_i}{\sigma_i} \right)^2$$

definition with correlated errors

$$\chi^2 = \sum_{i=1}^{N_{\mathrm{dat}}} \sum_{j=1}^{N_{\mathrm{dat}}} (D_i - T_i)(V^{-1})_{ij}(D_j - T_j)$$

$$V_{ij} = \delta_{ij}(\sigma_i^{\mathrm{uncorr}})^2 + \sum_{k=1}^{N_{\mathrm{corr}}} \sigma_{k,i}^{\mathrm{corr}} \sigma_{k,j}^{\mathrm{corr}}$$

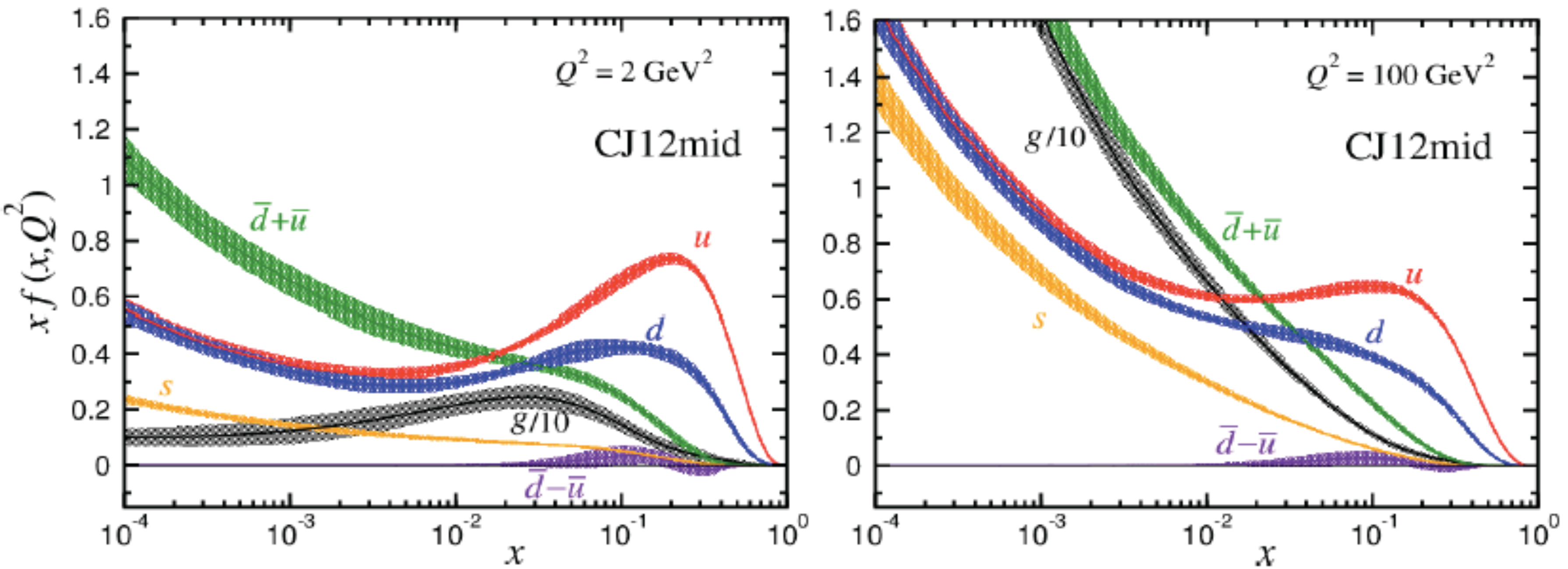
covariance matrix

- Try to use all possible experimental information available
  - statistical errors
  - systematic errors - (un)correlated
  - normalisation uncertainty (might be multiplicative)

Features of PDFs

- **☑ *u&d quarks - valence & sea***
  - valence part causes u&d dominate all other PDF at large-x where u>d
  - symmetric sea-quark: q & anti-q comparable at low-x
  - at high Q - contribution of the sea component increases through gluon radiation (DGLAP)
- **☑ *strange quarks***
  - strange quark PDF suppressed at initial scale but enhanced at high-Q

arXiv:1212.1702

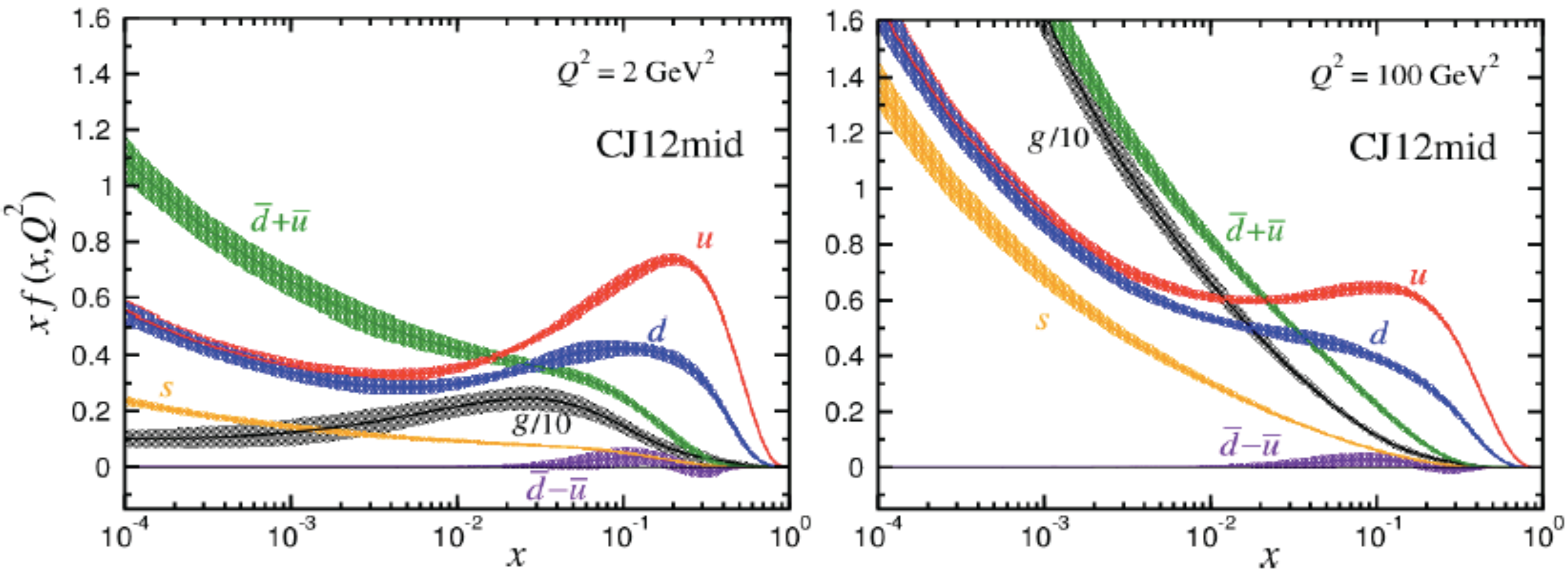


Consistently fit almost all data with Q > 2GeV

Features of PDFs

- *gluon*
  - dominate at small-x but fall off steeply as x increases
  - going to high-Q - gluon radiation reduces momenta of partons - everything shifts to smaller x
  - gluon radiates q-qbar pairs or additional gluons - at small-x gluon PDF and sea quark PDF get steeper
  - gluon can radiate even heavy quarks at high-Q so charm and bottom PDF are non-zero

arXiv:1212.1702



Consistently fit almost all data with Q > 2GeV

PDF uncertainties

❑ Sources of uncertainties:

☑ *error PDF*

- uncertainty of experimental data can be interpreted as uncertainty of the underlying PDF parameters
- different approaches how to translate experimental uncertainties to PDFs

☑ *other uncertainties (not in error PDFs)*

- choice of data sets or observables (include neutrino DIS or not, LHC or not ...)
- choice of kinematic cuts (looser cuts might constrain PDF better but ...)
- parameterisation bias
- pQCD choices (NLO vs NNLO, strong coupling)
- heavy-quark schemes (FFS, ZM-VFNS, VF-VFNS)
- higher-twist terms, nuclear corrections etc...

PDF uncertainties

- error PDFs are experimental errors translated to errors of free PDF parameters
- all approaches to determine error PDFs give approx. the same results in regions with data

☑ *Hessian method*

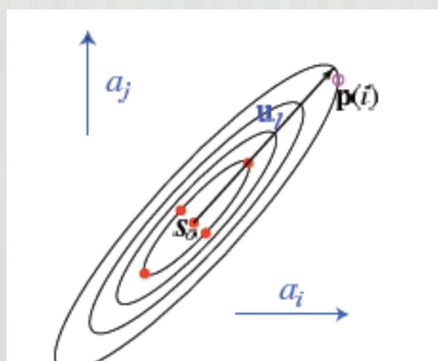
the most widely used technique to determine error PDFs

Fitting parameters

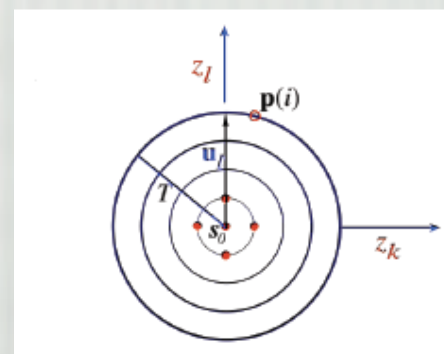
Expansion of χ^2:

Hessian — diagonal Hessian

$$\chi^2(a) = \chi_0^2 + \frac{1}{2}\frac{\partial \chi^2}{\partial a_i \partial a_j}(a-a_0)_i(a-a_0)_j + \ldots \rightarrow \chi_0^2 + \sum_i z_i^2$$



(a)
Original parameter basis



(b)
Orthonormal eigenvector basis

hep-ph/0201195

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Choice of $\Delta\chi^2 = \chi^2 - \chi_0^2$:

ideal choice $\Delta\chi^2 = 1$ for one sigma (68% CL)

pragmatic choice $\Delta\chi^2 \gg 1$ $\Delta\chi^2 \sim 50 - 100$

Construct error PDFs for each parameter in 2 directions:

error PDFs

$$z_i = \pm\sqrt{\Delta\chi^2} \qquad X_i^{\pm}(z) = X_i^{\pm}(0,0,\ldots,\pm\sqrt{\Delta\chi^2},\ldots,0,0)$$

Calculate PDF uncertainty of cross-section

$$(\Delta\sigma)^2 \approx \frac{1}{4}\sum_i^{N_p}\left(\sigma(X_i^+) - \sigma(X_i^-)\right)^2$$

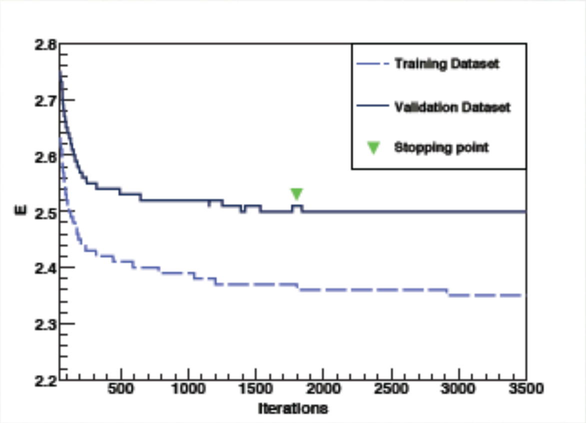
PDF uncertainties

- error PDFs are experimental errors translated to errors of free PDF parameters
- all approaches to determine error PDFs give approx. the same results in regions with data

☑ *Monte Carlo method (as used by NNPDF)*

new technique which allows for more flexible PDF x-shapes

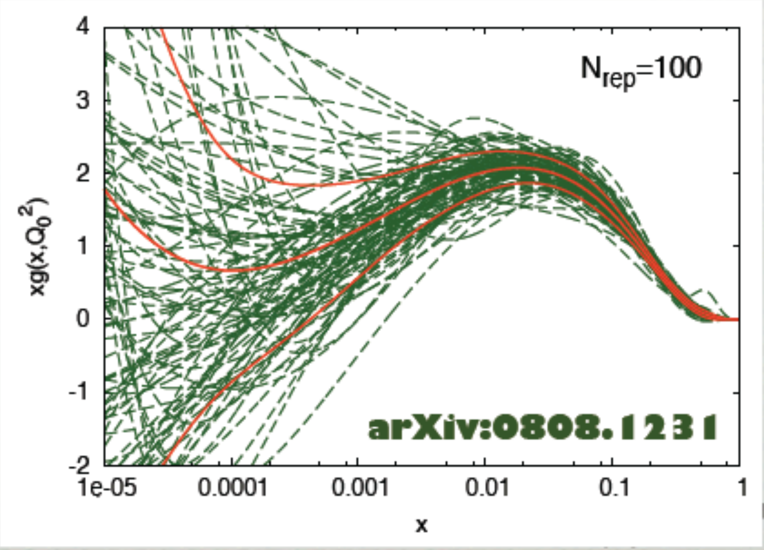
- neural network is used to (over-)parametrize PDFs @ Q_0
- N_{set} artificial replicas of data points generated assuming multi-Gaussian probability distribution
- random separation into training & validation data subsets
- minimize error function (not χ^2) for training set
- stop before overlearning



$$\langle\sigma(X)\rangle = \frac{1}{N_{\text{set}}}\sum_{i=1}^{N_{\text{set}}}\sigma(X^i)$$

error/replica PDFs

$$\Delta\sigma(X) = \left(\sum_{i=1}^{N_{\text{set}}}\left[\sigma(X^i) - \langle\sigma(X)\rangle\right]^2\right)^{1/2}$$



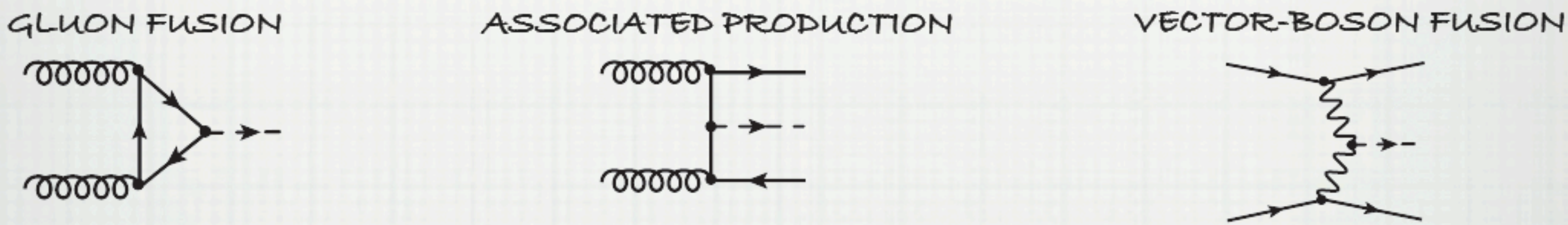
Current PDF sets

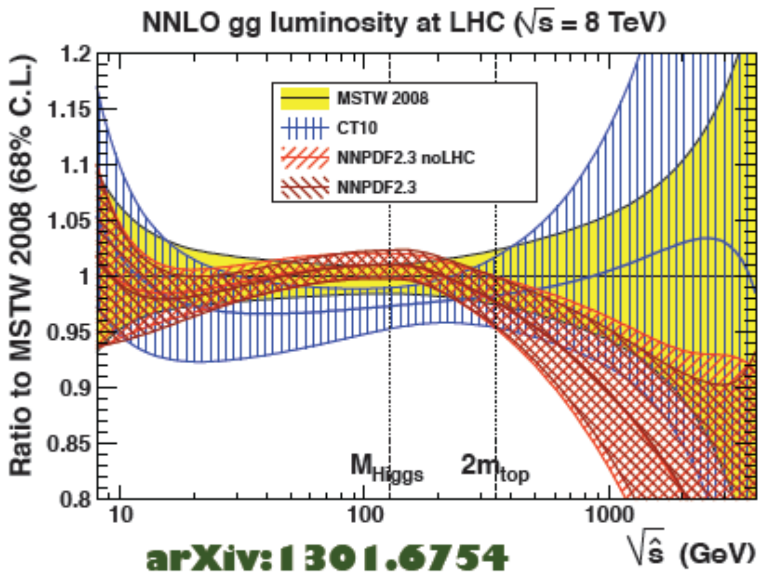
	DATA	Order	HQ	α_s	Params.	Uncert.
CT14	global	LO,NLO, NNLO	GM-VFNS (s-ACOT)	external	6 indep. PDFs (26 params)	Hessian ($\Delta\chi^2 \sim 100$)
MSTW08	global	LO,NLO, NNLO	GM-VFNS (TR)	fit	7 indep. PDFs (20 params)	Hessian ($\Delta\chi^2 \sim 25$)
NNPDF	global	LO,NLO, NNLO	GM-VFNS (FONLL)	external	7 indep. PDFs (259 params)	Monte Carlo
CJ12	global	LO,NLO	ZM-VFNS	external	5 indep. PDFs (22 params)	Hessian ($\Delta\chi^2=100$)
HERApdf	DIS (HERA)	NLO NNLO	GM-VFNS (TR)	external	5 indep. PDFs (14 params)	Hessian ($\Delta\chi^2=1$)
ABM11	DIS+DY	NLO NNLO	FFN	fit	6 indep. PDFs (25 params)	Hessian ($\Delta\chi^2=1$)

PDF impact

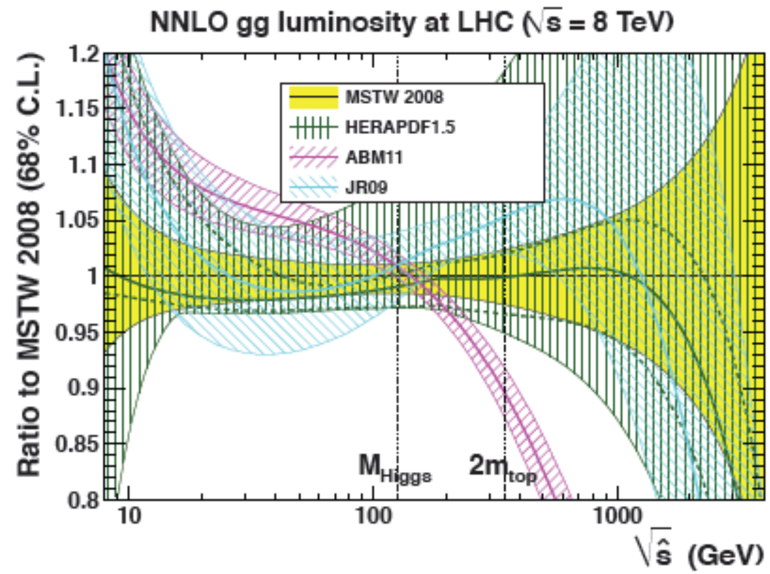
- *Parton Distribution Functions in Higgs production*
  - Higgs is pre-dominantly produced through gluon fusion -

    gluon PDFs at $x = M_H/\sqrt{s} \sim 0.02$ are crucial
  - sub-leading Higgs production via VBF is sensitive to quark & anti-quark PDFs





arXiv:1301.6754



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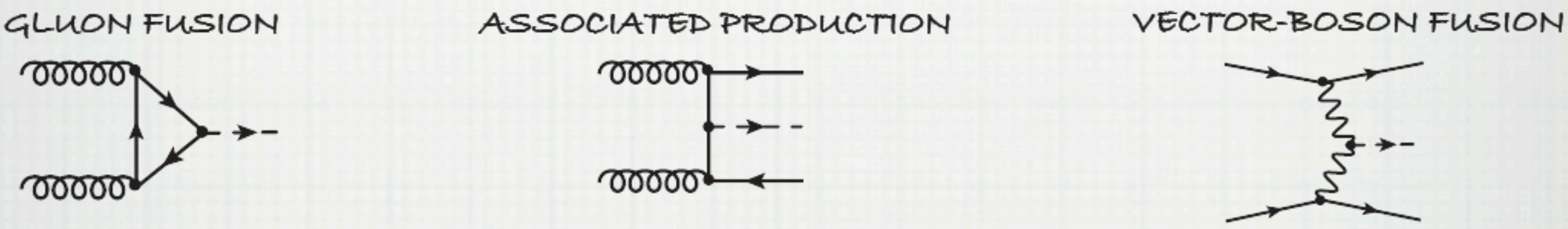
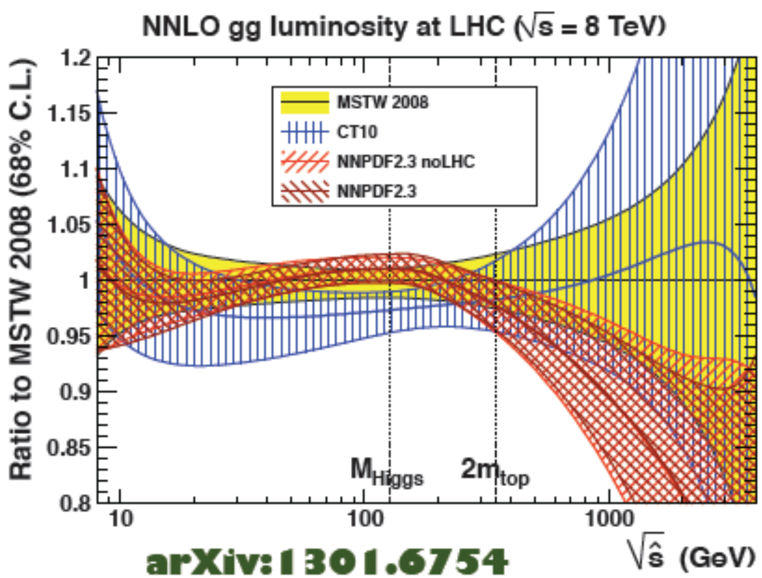
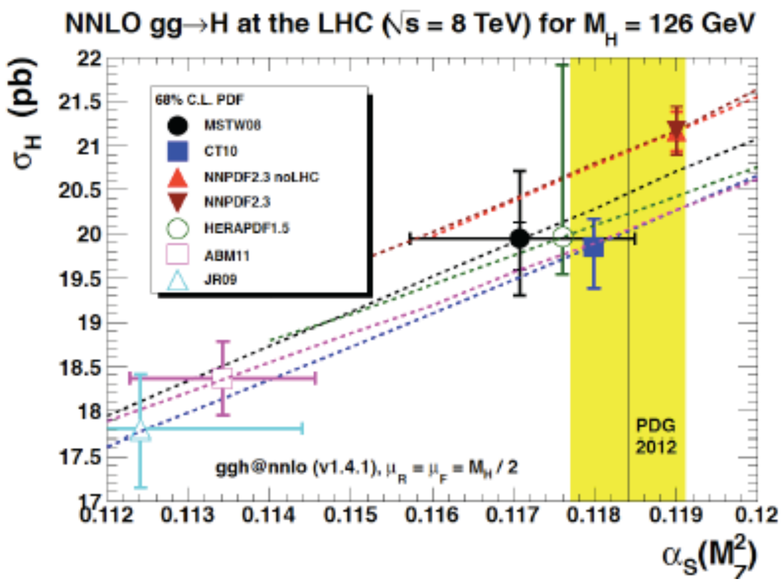


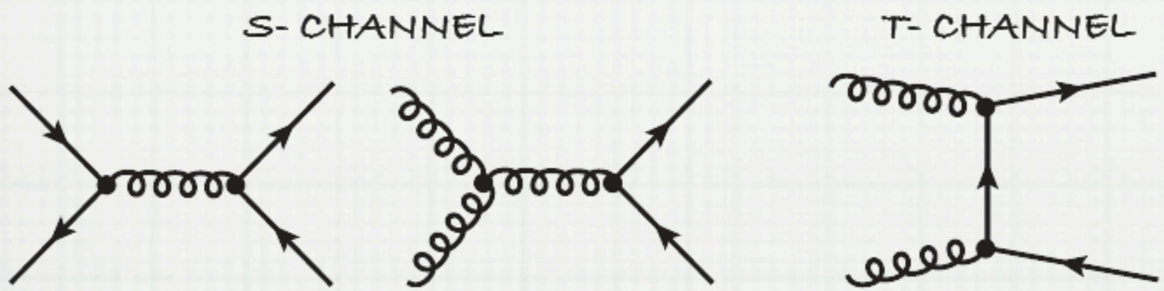


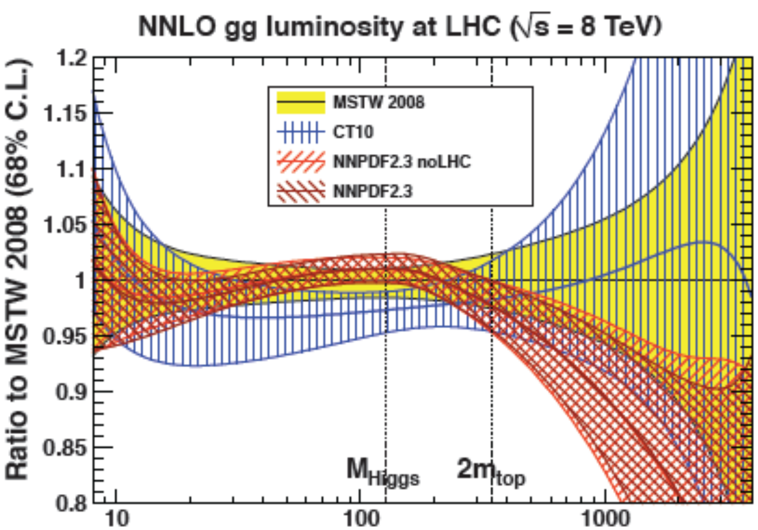


arXiv:1301.6754

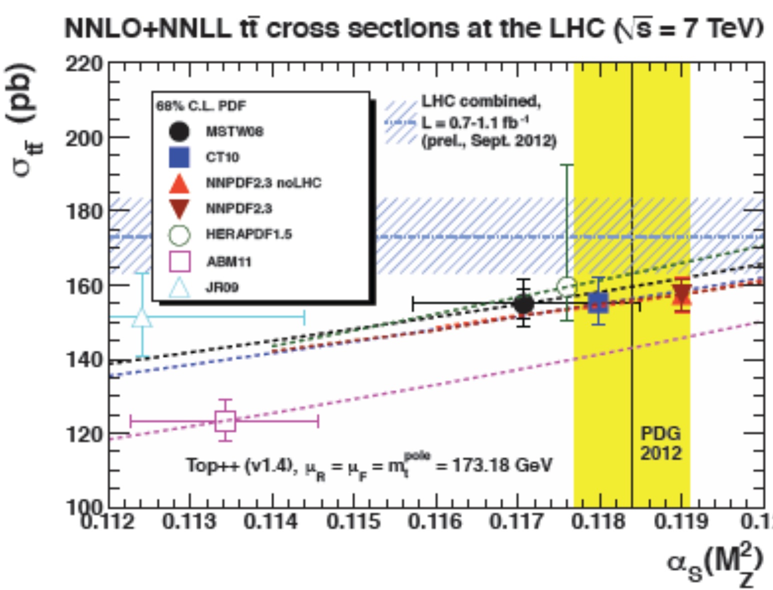
PDF impact

- *Parton Distribution Functions in top quark pair production*
  - Top quark pair production is dominated by s-channel diagrams where valence quarks & gluons are important at **$x=2m_t/\sqrt{s}$ ~ 0.05**





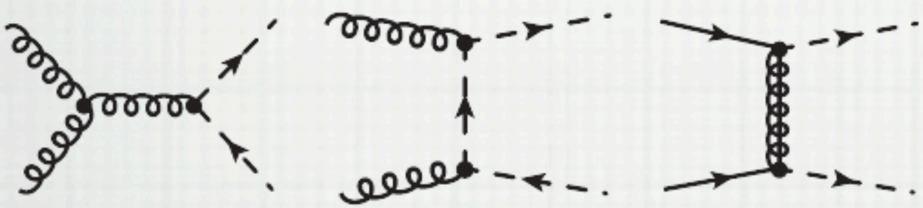
arXiv:1301.6754



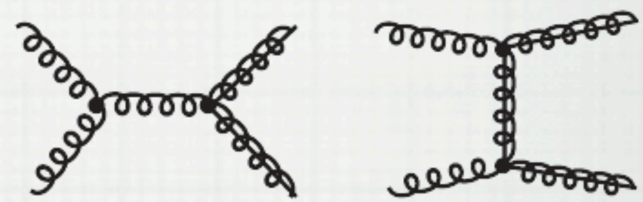
PDF impact

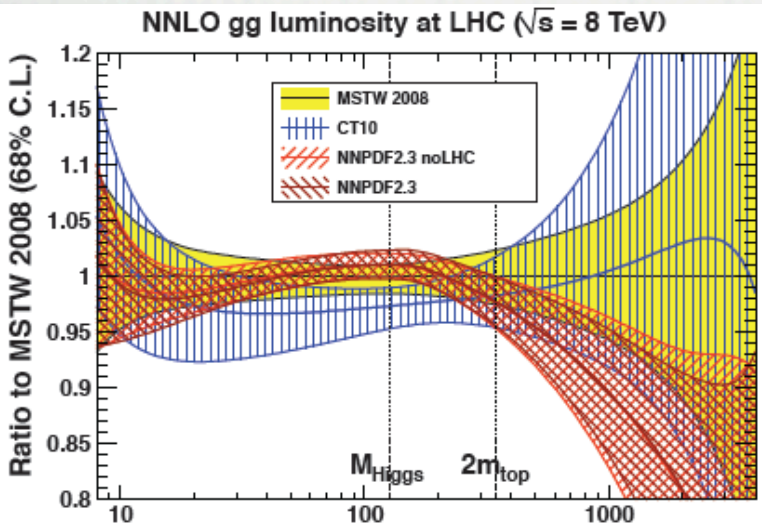
- *Parton Distribution Functions in SUSY production*
  - production of SUSY coloured particles (squarks & gluinos) very sensitive to gluon PDF at very high **$x = 2m_X / \sqrt{s} \sim 0.2\text{-}0.7$** ← very problematic

SQUARK PRODUCTION

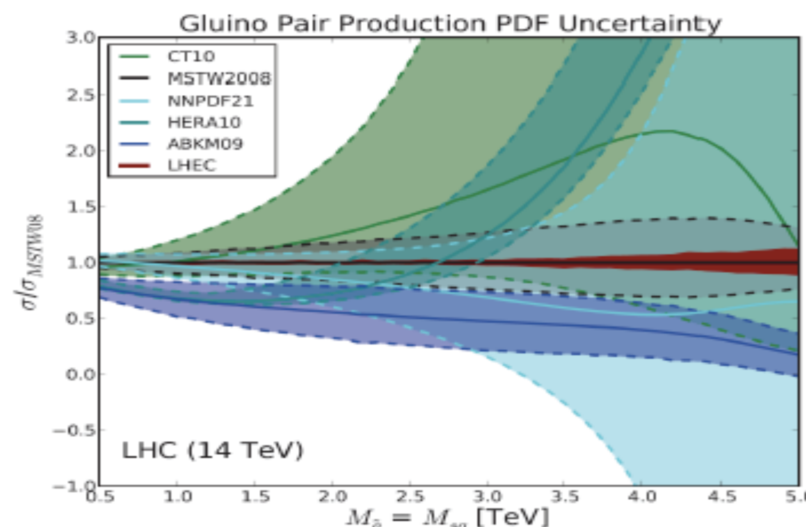


GLUINO PRODUCTION





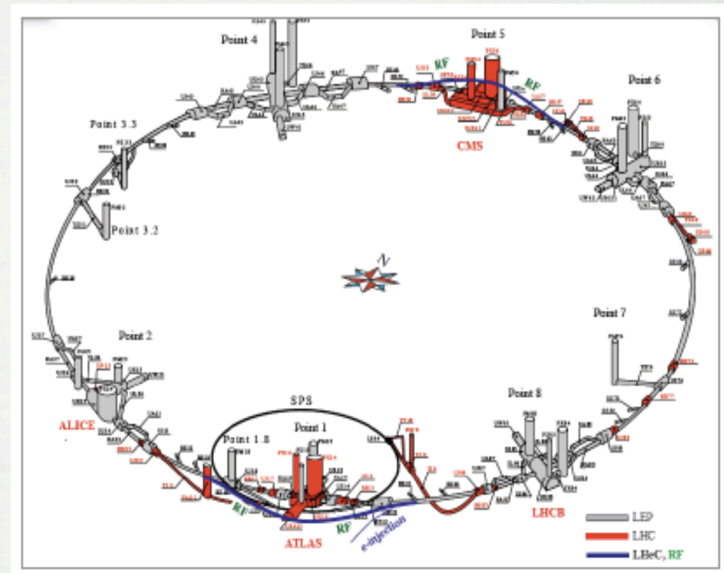
arXiv:1301.6754



PDFs after the LHC

- *Parton Distribution Functions - new dedicated data*
  - new projects with large possible impact on PDFs

arXiv:1206.2913



LHeC

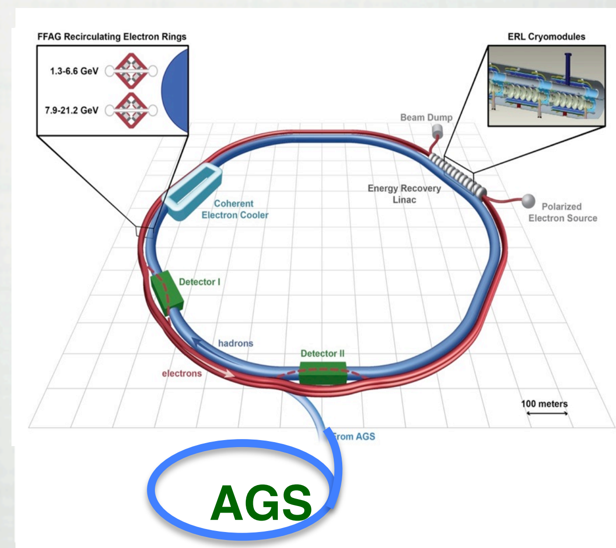
colliding electrons / positrons with LHC protons / nuclei

- unprecedented coverage in x-Q^2 plane
- precise determination of the gluon PDF
- interesting also for Higgs & BSM physics programs
- breakthrough machine for nuclear PDFs

EIC

electron ion collider

- high-intensity precision machine with polarized beams
- good coverage in x-Q^2 plane (down to x~10^{-4})
- precise determination of the gluon PDF
- breakthrough machine for nuclear PDFs, saturation, polarized PDFs...

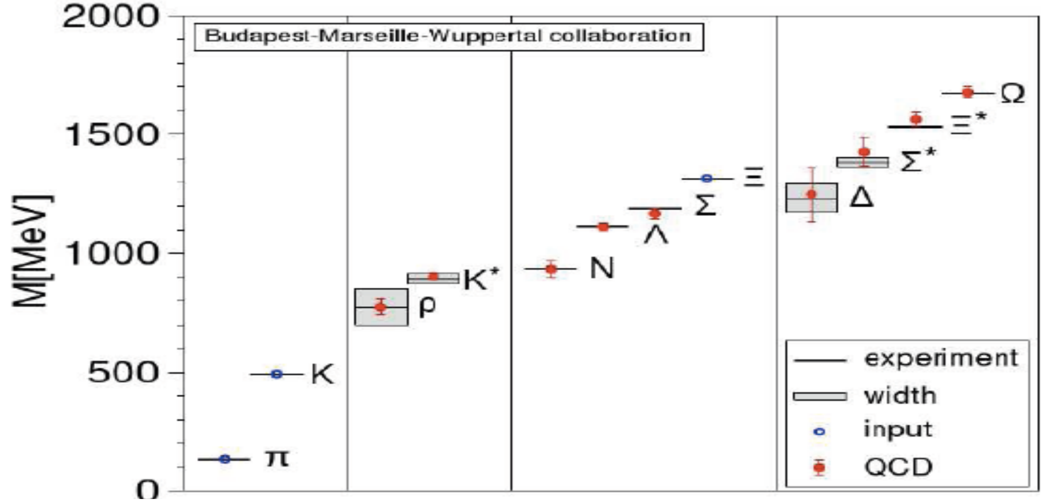


arXiv:1212.1701

We believe QCD because ...

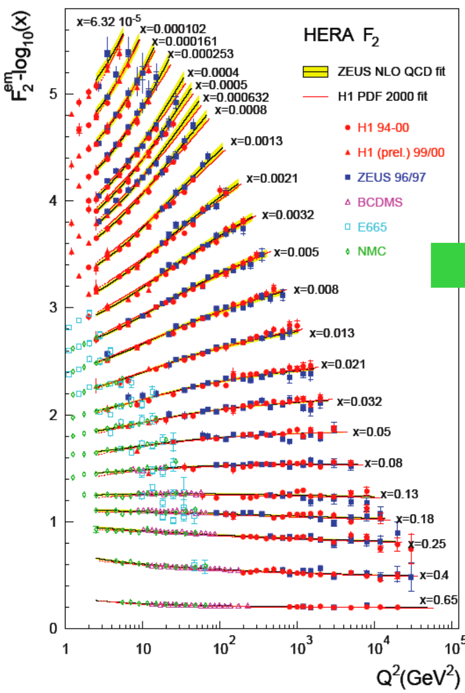
- **@low energy:**

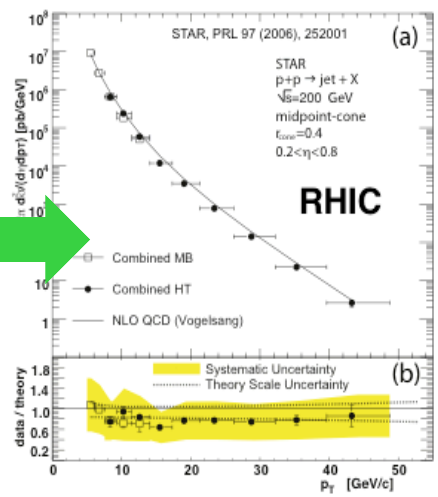
  Hadron mass spectrum from lattice QCD

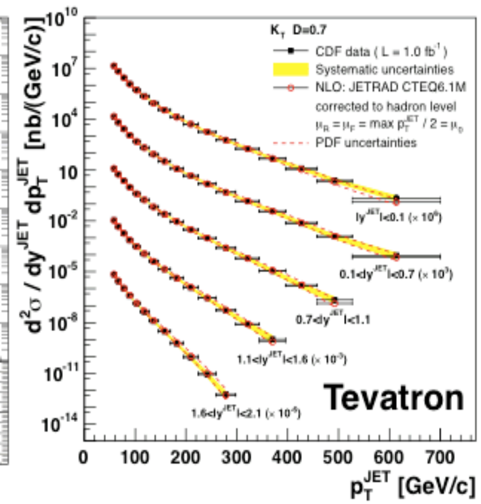


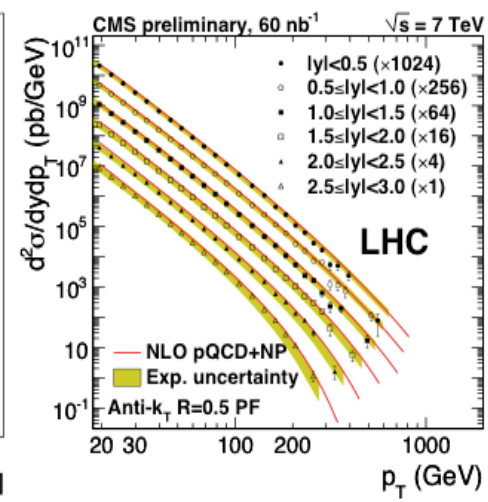
- **@high energy:**

  Asymptotic freedom + perturbative QCD









Measure e-p at 0.3 TeV (HERA)
Predict p-p and p-p̄ at 0.2, 1.96, and 7 TeV

Summary of lecture two

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- NLO calculations are available for most observables, NNLO are becoming available for the search of new physics

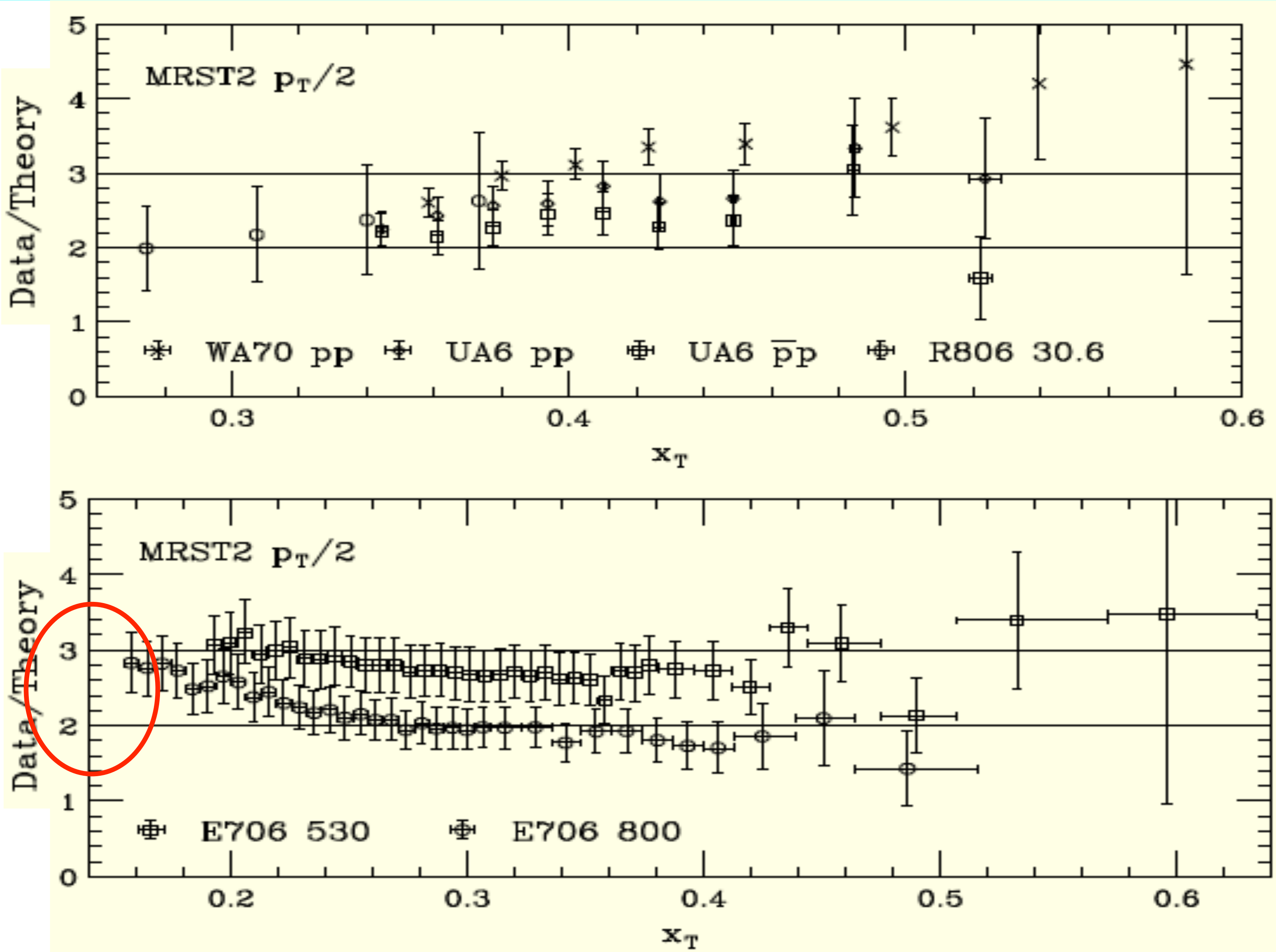
- Direct photon data are still puzzling and challenging, has a good potential for extracting the gluon distribution

- NLO PDFs are very stable now, and NNLO PDFs are becoming available

- Multi-scale observables could be valuable for new physics search – new factorization formalism, resummation, …

Backup slides

Same excess seen in π^0 production



But, works at RHIC energy