Quantum Chromodynamics (QCD)

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The plan for my four lectures

- The Goal:

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- The Plan (approximately):

  Fundamentals of QCD, factorization, evolution,

  and elementary hard processes

  Two lectures

  Role of QCD in high energy collider phenomenology

  One lecture

  QCD and hadron structure and properties

  One lecture

Summary of lecture two

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- NLO calculations are available for most observables, NNLO are becoming available for the search of new physics

- Direct photon data are still puzzling and challenging, has a good potential for extracting the gluon distribution

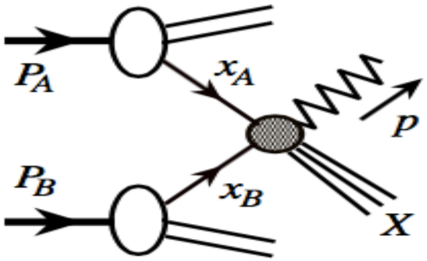
- NLO PDFs are very stable now, and NNLO PDFs are becoming available

- Multi-scale observables could be valuable for new physics search – new factorization formalism, resummation, …

A complete example – "Drell-Yan"

❑ Heavy boson production in hadronic collisions:

$$A(P_A) + B(P_B) \to V[\gamma^*, W/Z, H^0, ...](p) + X$$



✧ **Cross section with single hard scale:** $p_T \sim M_V$

$$\frac{d\sigma_{AB\to V}}{dy dp_T^2}(p_T \sim M_V), \qquad \frac{d\sigma_{AB\to V}}{dy}(M_V), \qquad \sigma_{AB\to V}(M_V)$$

$$\sigma_{AB\to V}(M_V) = \sum_{ff'} \int dx_A\, f(x_A,\mu^2) \int dx_B\, f(x_B,\mu^2)\, \hat{\sigma}_{ff'\to V}(x_A, x_B, \alpha_s(\mu); M_V)$$

– Fixed order pQCD calculation

✧ **Cross section with two different hard scales:**

$$\frac{d\sigma_{AB\to V}}{dy dp_T^2}(p_T \gg M_V)$$

– Resummation of single logarithms:

$$\alpha_s^n \ln^n(p_T^2/M_V^2)$$

$$\frac{d\sigma_{AB\to V}}{dy dp_T^2}(p_T \ll M_V)$$

– Resummation of double logarithms:

$$\alpha_s^n \ln^{2n}(M_V^2/p_T^2)$$

Same discussions apply to production of Higgs, and other heavy particles

Total cross section – single hard scale

- **Partonic hard parts:**

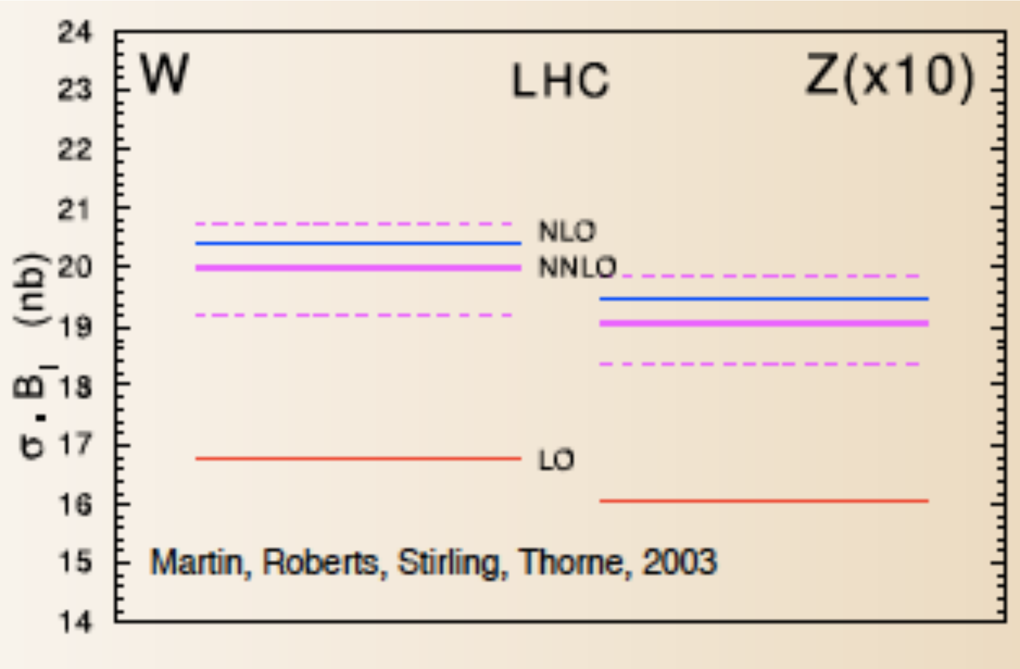
$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

LO NLO NNLO

- **NNLO total x-section** $\sigma(AB \to W, Z)$:

(Hamberg, van Neerven, Matsuura; Harlander, Kilgore 1991)

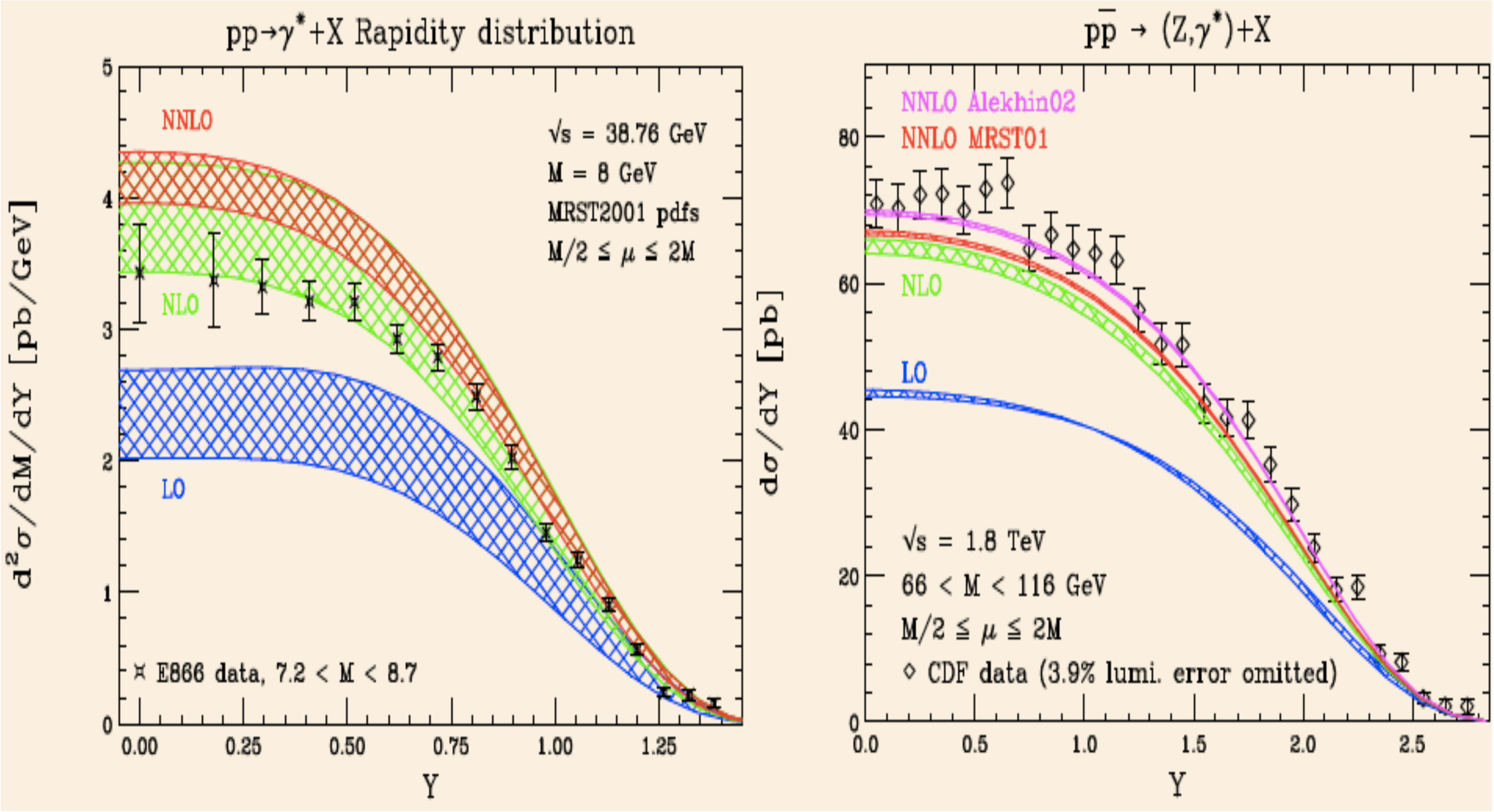
- ✧ **Scale dependence:** a few percent
- ✧ **NNLO K-factor is about 0.98 for LHC data, 1.04 for Tevatron data**



Rapidity distribution – single hard scale

- NNLO differential cross-section:

Anastasiou, Dixon, Melnikov, Petriello, 2003-05



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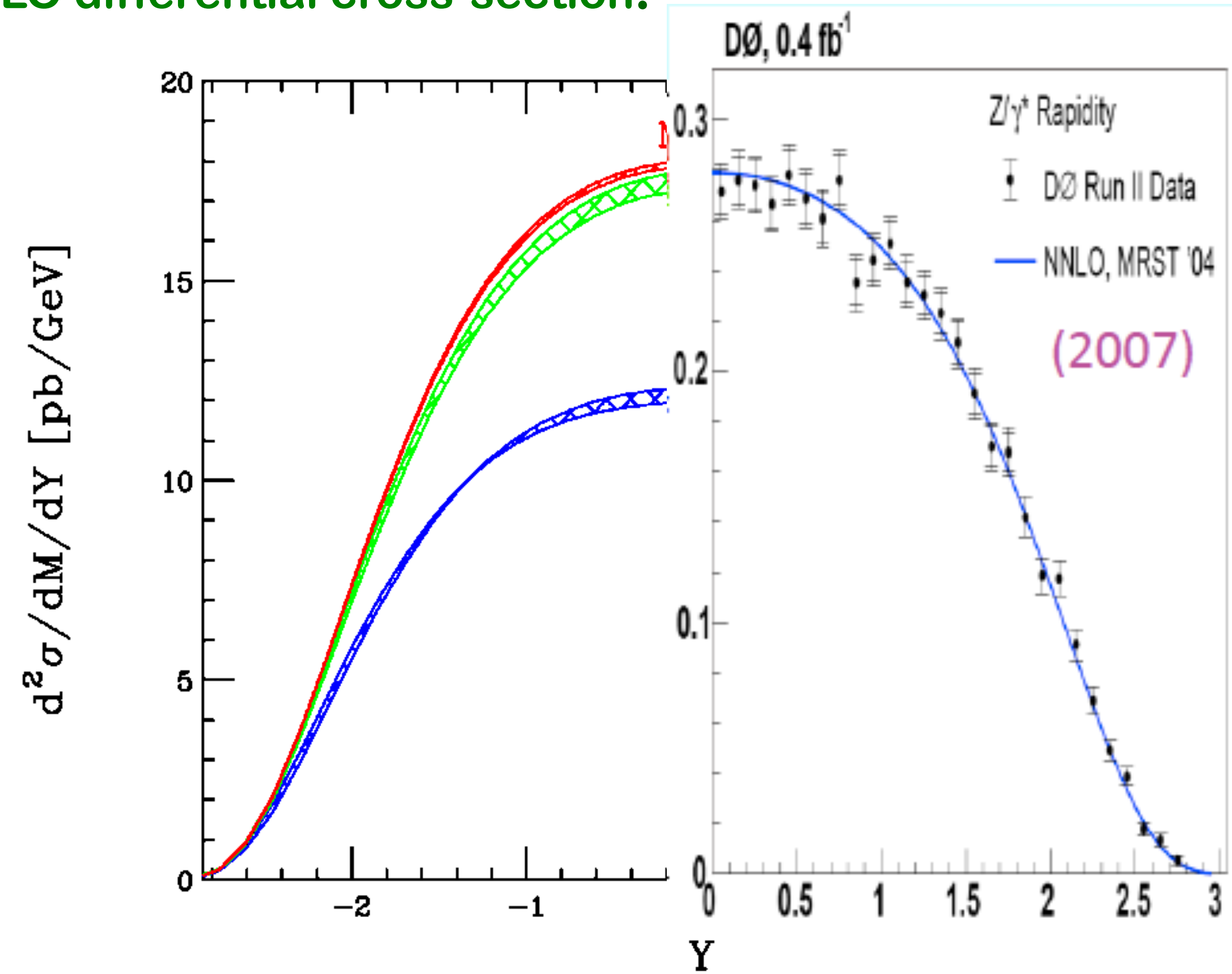
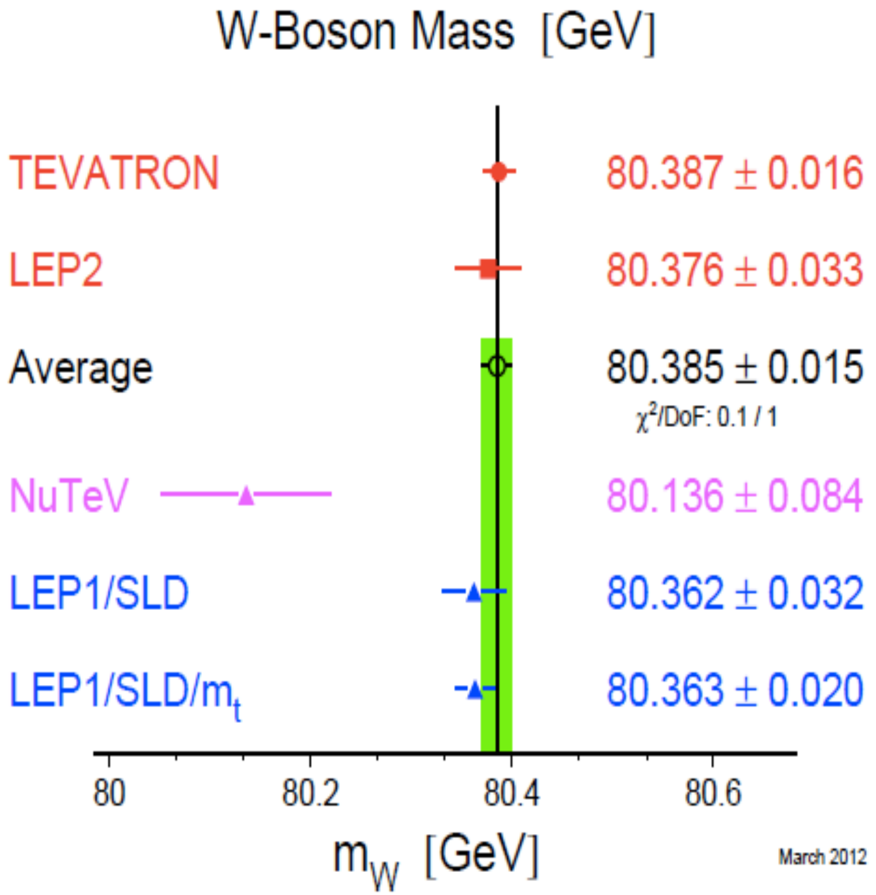
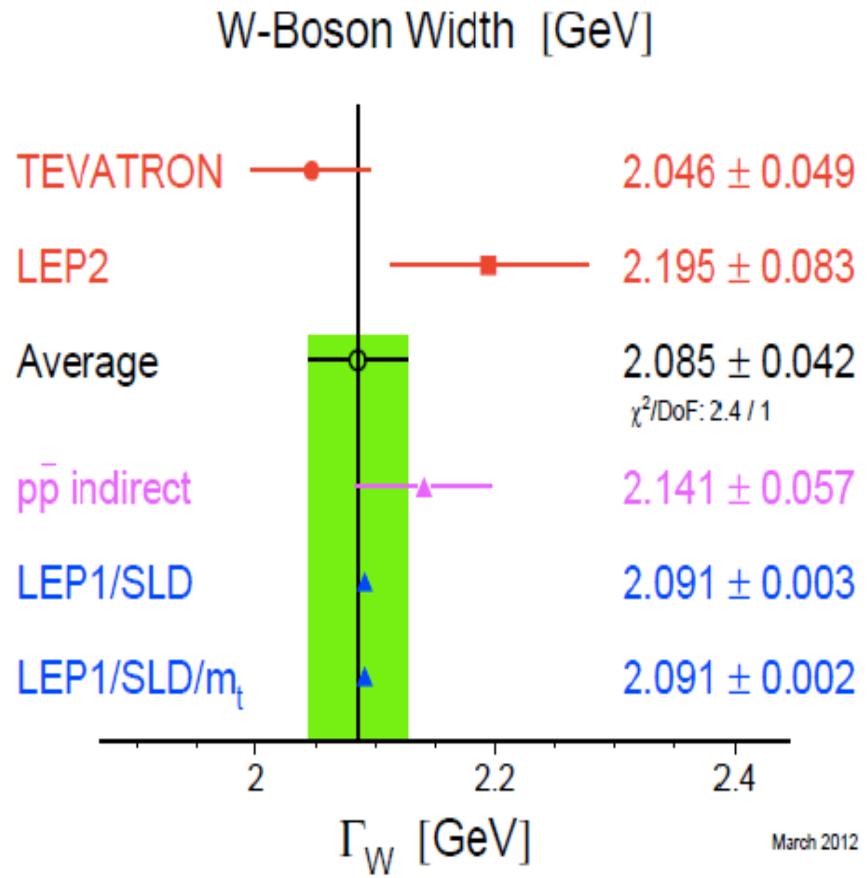


Determination of mass and width

Fernando Febres Cordero, CTEQ SS2012

- **W mass & width:**

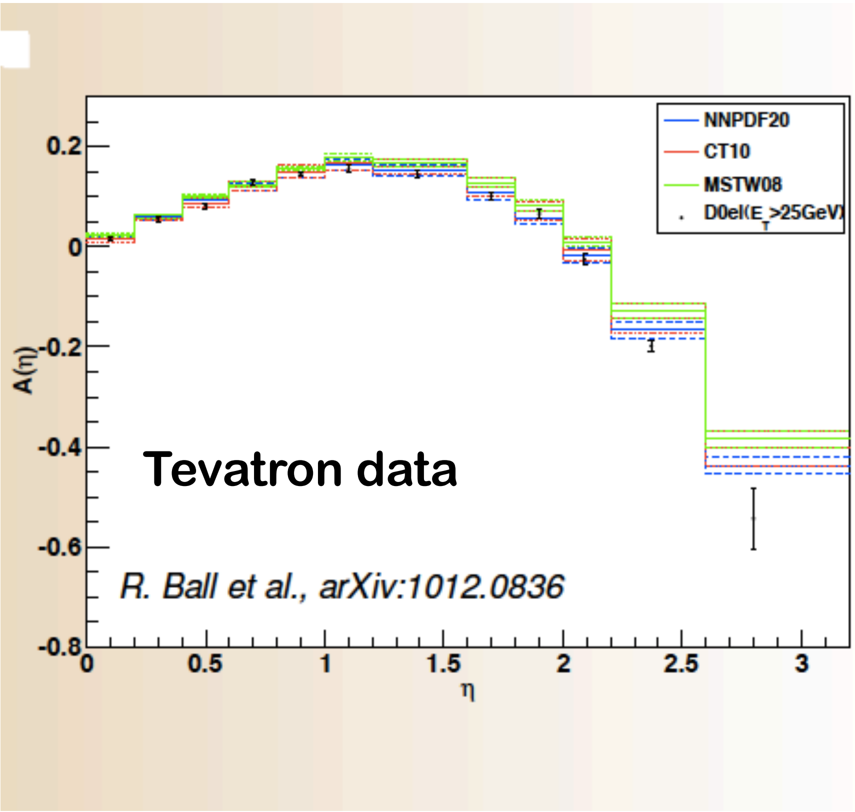


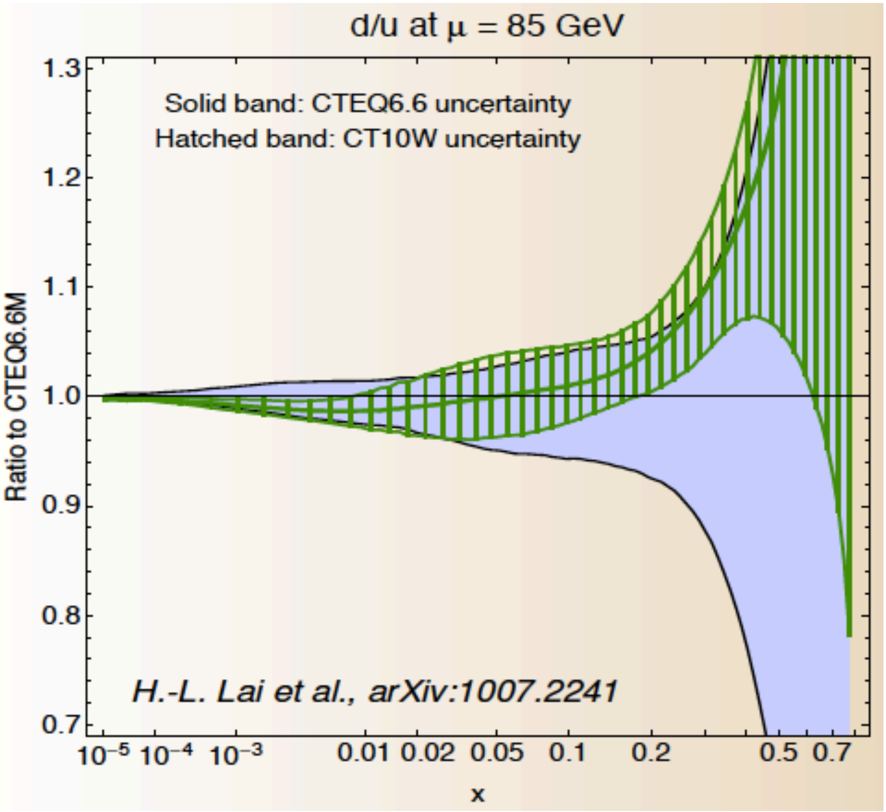


Charge asymmetry – single hard scale

- **Charged lepton asymmetry:** $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B,M_W)/u(x_B,M_W) - d(x_A,M_W)/u(x_A,M_W)}{d(x_B,M_W)/u(x_B,M_W) + d(x_A,M_W)/u(x_A,M_W)}$$





The A_{ch} data distinguish between the PDF models, reduce the PDF uncertainty

D0 – W charge asymmetry

Charge asymmetry – single hard scale

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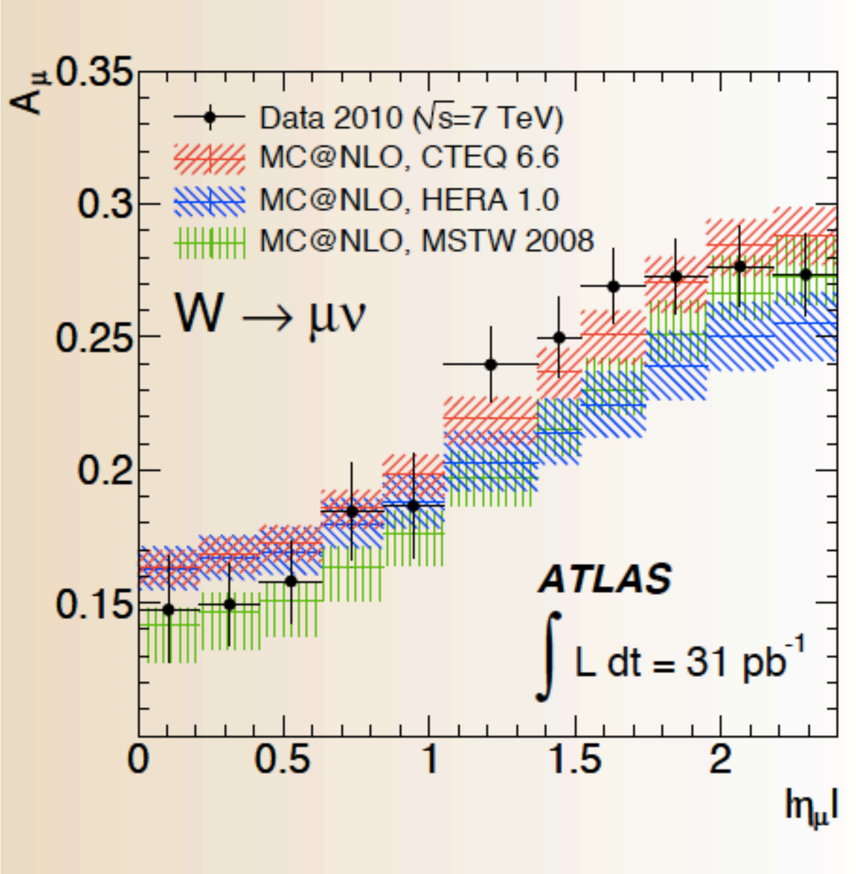
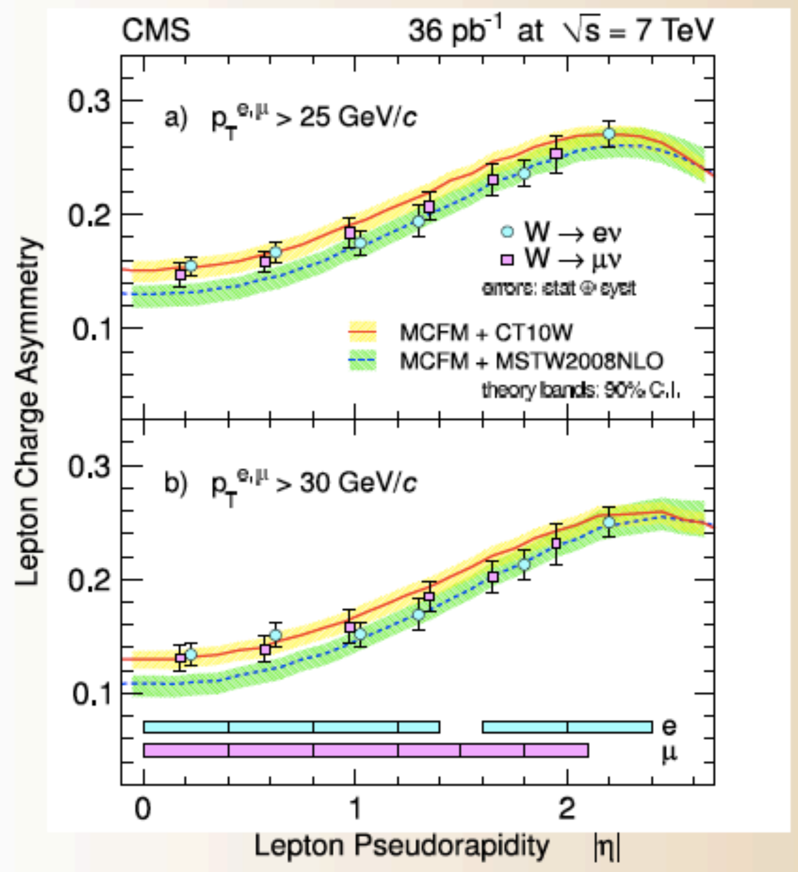


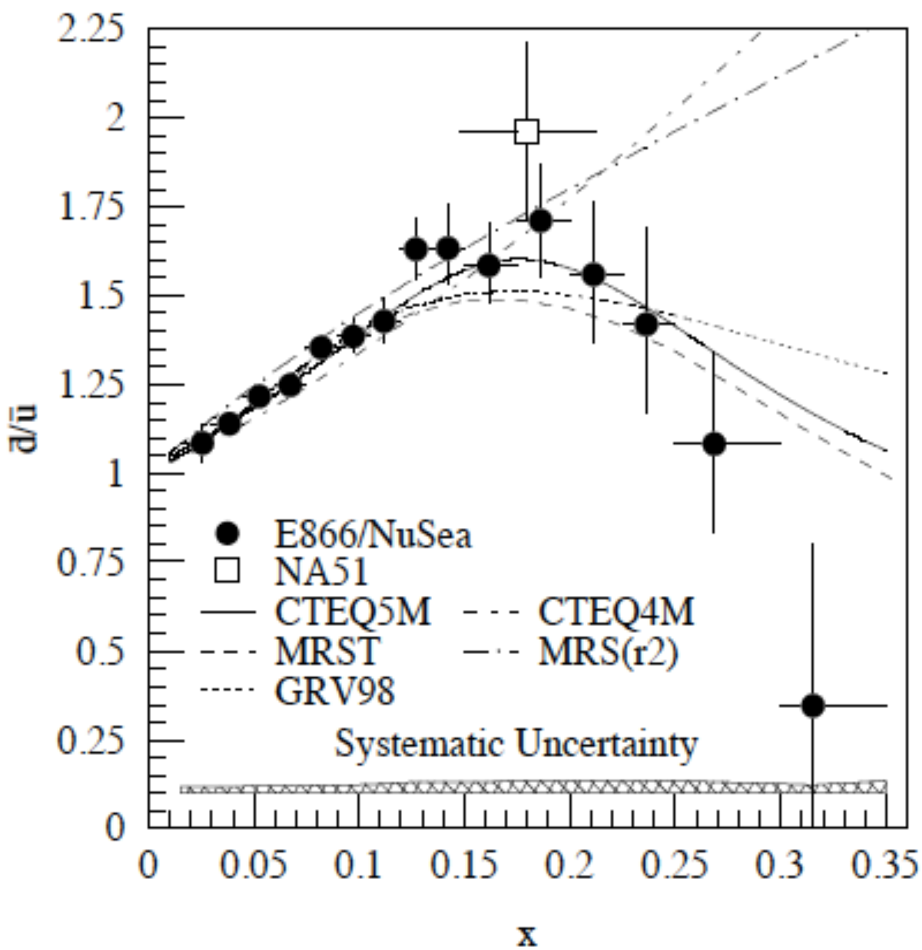


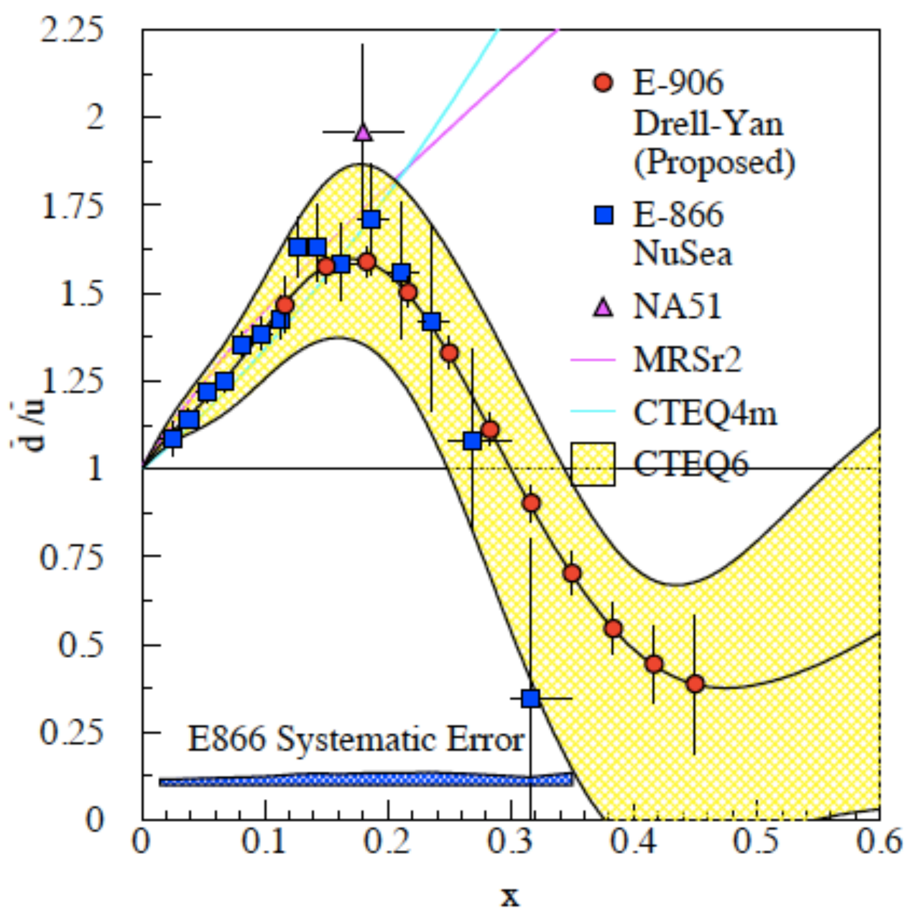
Sensitive both to d/u at x > 0.1 and u/d at x ~ 0.01

Flavor asymmetry – single hard scale

❑ **Flavor asymmetry of the sea:**

$$\sigma_{DY}(p+d)/2\sigma_{DY}(p+p) \simeq \left[1+\bar{d}(x)/\bar{u}(x)\right]/2$$



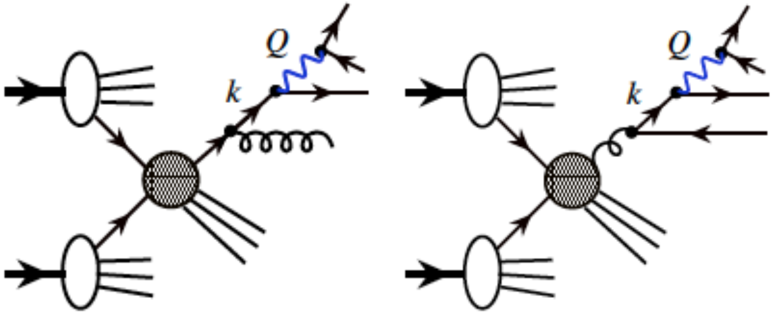


Could QCD allow ubar(x) > dbar(x)?

P_T–distribution (P_T >> M) – two hard scales

- **P_T-distribution – factorizable if M >> Λ_{QCD}:**

$$\frac{d\sigma_{AB}}{dy\,dp_T^2\,dQ^2} = \sum_{a,b}\int dx_a\, f_{a/A}(x_a)\int dx_b\, f_{b/B}(x_b)\,\frac{d\hat{\sigma}_{ab}}{dy\,dp_T^2\,dQ^2}(x_a, x_b, \alpha_s)$$



$$\hat{\sigma}^{\mathrm{F}}/\hat{\sigma}_f^{\mathrm{LO}} \sim \frac{\alpha_s(\mu)}{2\pi}P_{f\to q}(z)\int_{k^2_{\min}}^{k^2_{\max}}\frac{dk^2}{k^2}$$
$$\sim \frac{\alpha_s(\mu)}{2\pi}\ln\left(\frac{p_T^2}{Q^2}\right)$$

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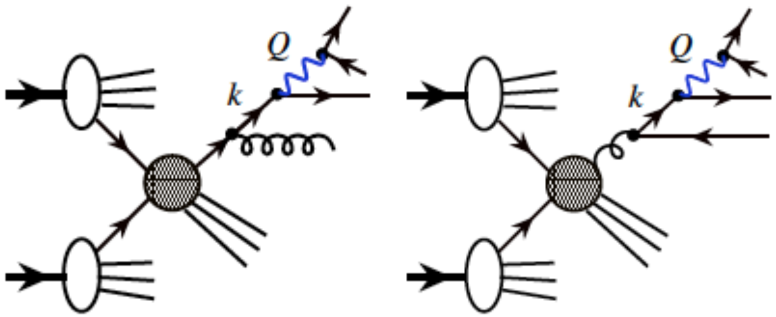

$$\hat{\sigma}^{F}/\hat{\sigma}_f^{LO} \sim \frac{\alpha_s(\mu)}{2\pi} P_{f\to q}(z) \int_{k_{min}^2}^{k_{max}^2} \frac{dk^2}{k^2}$$

$$\sim \frac{\alpha_s(\mu)}{2\pi} \ln\left(\frac{p_T^2}{Q^2}\right)$$

$$\lesssim \frac{\alpha_s(Q)}{2\pi} \ln\left(\frac{p_T^2}{Q^2}\right) \lesssim 10\%$$

How big is the logarithmic contribution?

P_T–distribution (P_T >> M) – two hard scales

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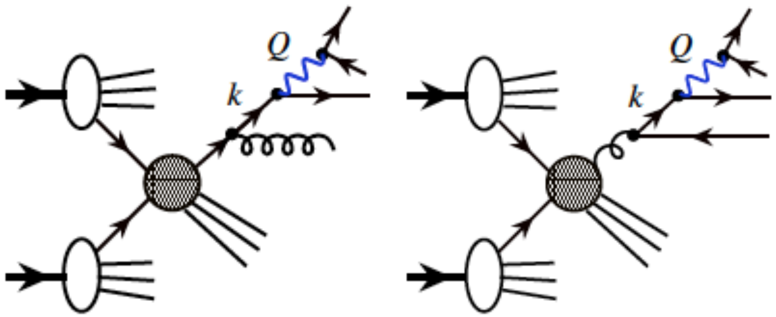

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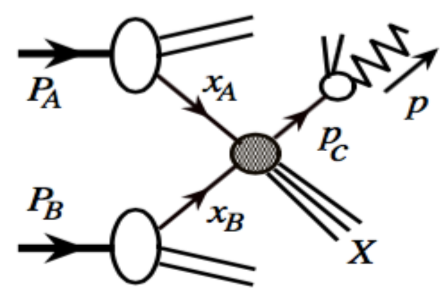
How big is the logarithmic contribution?

- **Improved factorization:**

$$\frac{d\sigma_{AB\to V(Q)X}}{dp_T^2\,dy} \equiv \frac{d\sigma^{\rm Dir}_{AB\to V(Q)X}}{dp_T^2\,dy} + \frac{d\sigma^{\rm Frag}_{AB\to V(Q)X}}{dp_T^2\,dy}$$

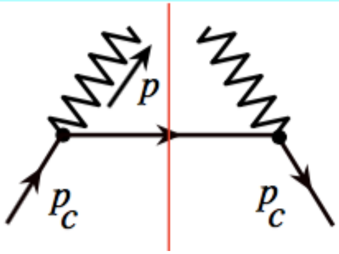
$$\frac{d\sigma^{\rm Frag}_{AB\to V(Q)X}}{dp_T^2\,dy} = \sum_{a,b,c}\int dx_1 f_a^A(x_1,\mu)\int dx_2 f_b^B(x_2,\mu)$$

$$\times\int\frac{dz}{z^2}\left[\frac{d\hat{\sigma}^{\rm Frag}_{ab\to cX}}{dp_{c_T}^2\,dy}(x_1,x_2,p_c;\mu_D)\right] D_{c\to V}(z,\mu_D^2;Q^2)$$



P_T–distribution (P_T >> M) – two hard scales

- **Fragmentation functions of elementary particles:**



$$D^{(0)}_{g\to V}(z,\mu_D^2;Q^2) = 0$$

$$D^{(0)}_{q\to V}(z,\mu_D^2;Q^2) = \frac{(|g_L^{Vq}|^2+|g_R^{Vq}|^2)}{2}\left(\frac{\alpha_{em}}{2\pi}\right)\left[\frac{1+(1-z)^2}{z}\ln\left(\frac{z\mu_D^2}{Q^2}\right)-z\left(1-\frac{Q^2}{z\mu_D^2}\right)\right]$$

- **Evolution equations:**

$$\mu_D^2\frac{d}{d\mu_D^2}D_{c\to V}(z,\mu_D^2;Q^2) = \left(\frac{\alpha_{\rm em}}{2\pi}\right)\gamma_{c\to V}(z,\mu_D^2,\alpha_s;Q^2)+\left(\frac{\alpha_s}{2\pi}\right)\sum_d\int_z^1\frac{dz'}{z'}P_{c\to d}\left(\frac{z}{z'},\alpha_s\right)D_{d\to V}(z',\mu_D^2;Q^2)$$

$$D_{c\to V}(z,\mu_D^2\le Q^2/z;Q^2) = 0$$

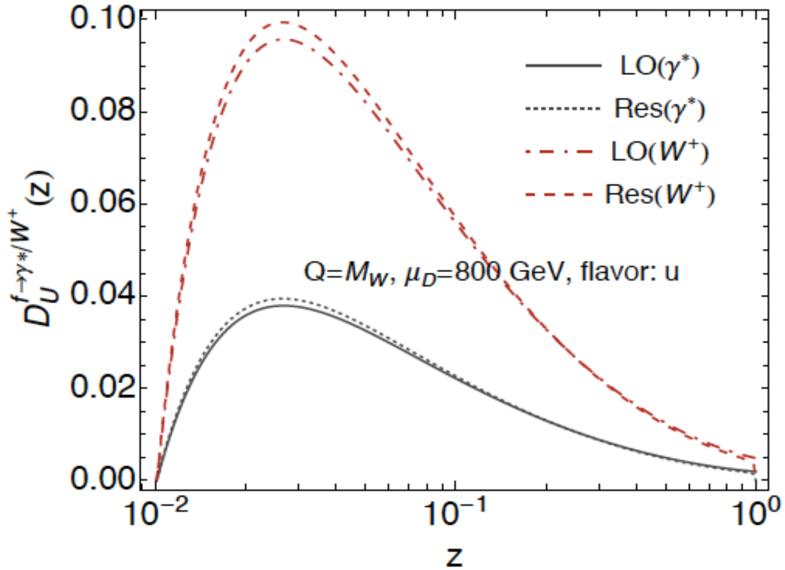
- **Evolution kernels:**

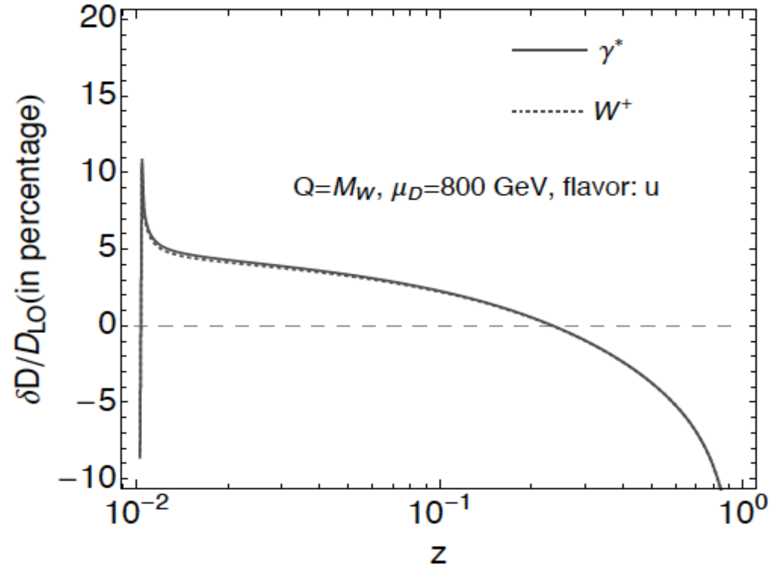
$$\gamma^{(0)}_{q\to V}(z,k^2;Q^2) = \frac{(|g_L^{Vq}|^2+|g_R^{Vq}|^2)}{2}\left[\frac{1+(1-z)^2}{z}-z\left(\frac{Q^2}{zk^2}\right)\right]\theta\left(k^2-\frac{Q^2}{z}\right)$$

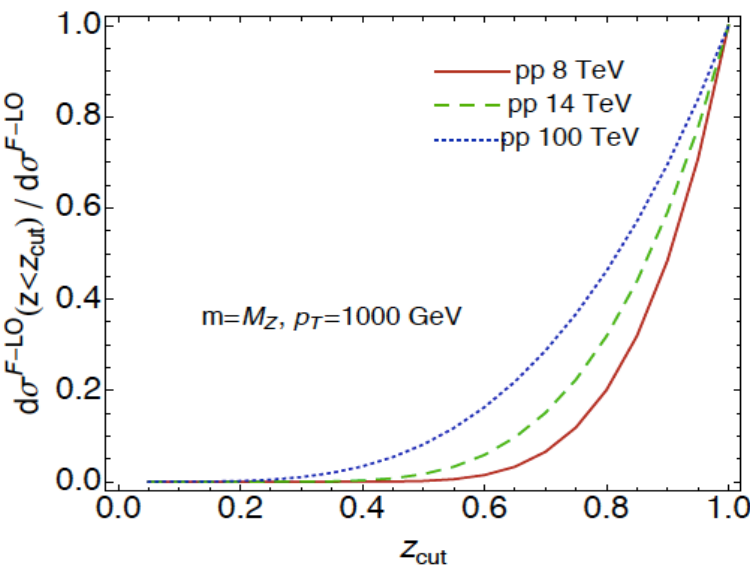
$$\gamma^{(0)}_{g\to V}(z,k^2;Q^2) = 0$$

If $Q \gg \Lambda_{\rm QCD}$, reorganization of perturbative expansion to remove all logarithms of hard parts

P_T–distribution (P_T >> M) – two hard scales



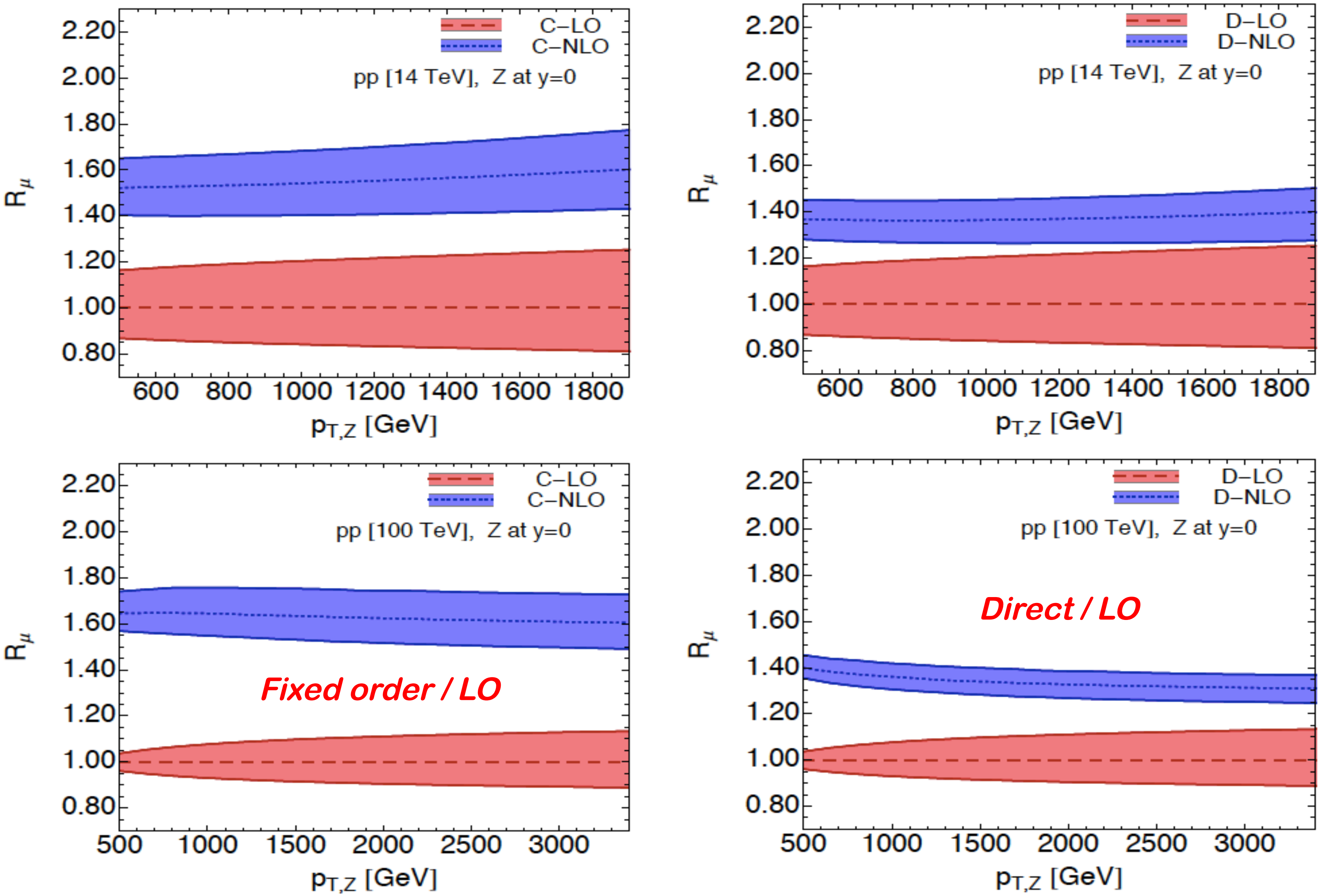




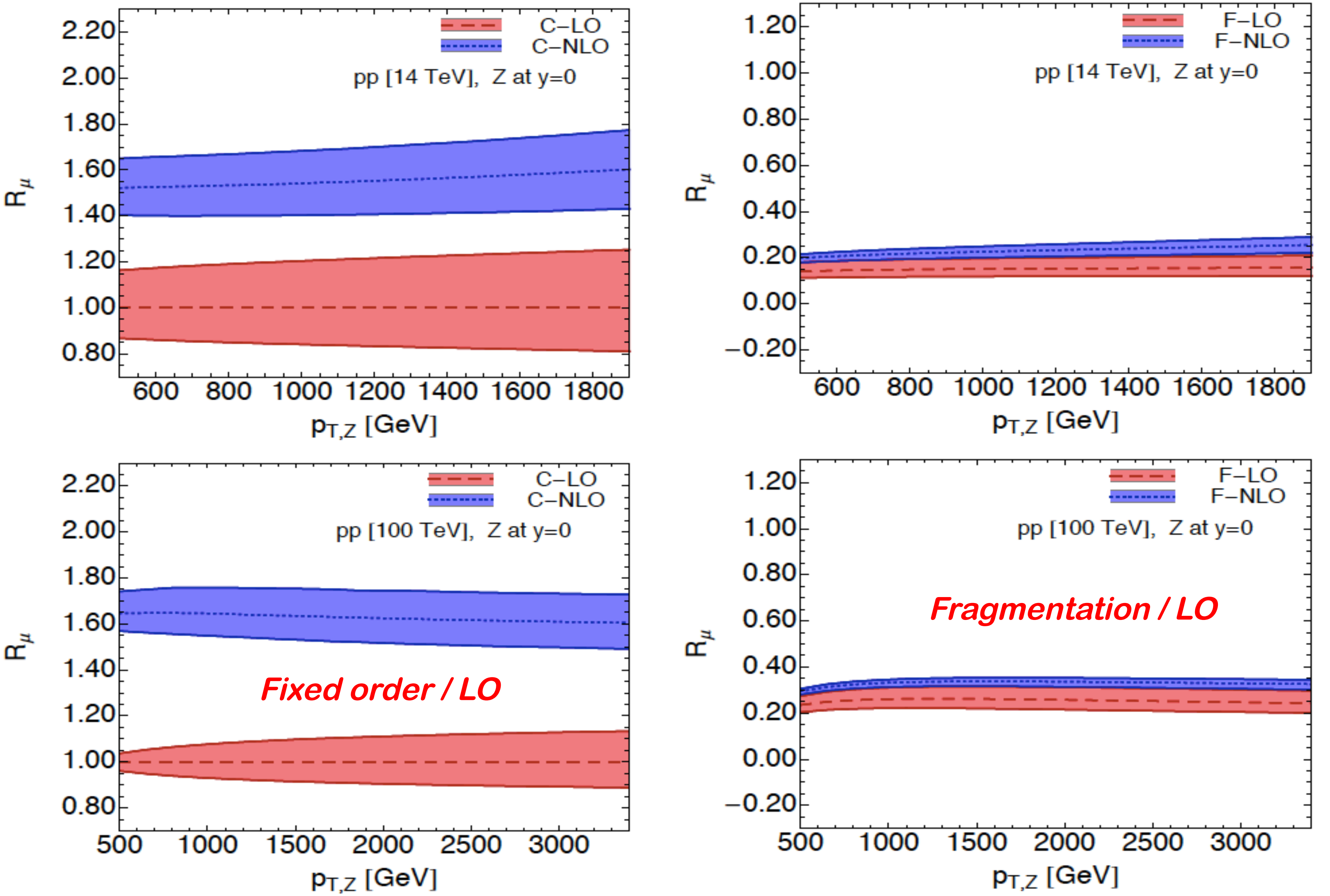
$\sigma_{p_T>800\,\mathrm{GeV}}$	VB	C-LO	C-NLO	NLO[modified]
13 TeV [fb]	Z	74.1	$117.4^{+12.0}_{-11.5}$	$120.5^{+9.2}_{-10.4}$
	W^+	126.2	$199.4^{+20.1}_{-19.3}$	$204.4^{+15.4}_{-17.3}$
	W^-	55.8	$90.2^{+9.6}_{-9.2}$	$92.7^{+7.4}_{-8.2}$
100 TeV [pb]	Z	11.48	$19.68^{+1.53}_{-1.30}$	$20.16^{+1.00}_{-0.98}$
	W^+	15.08	$26.23^{+2.14}_{-1.79}$	$26.86^{+1.41}_{-1.35}$
	W^-	10.50	$18.18^{+1.47}_{-1.23}$	$18.61^{+0.96}_{-0.92}$

Fragmentation logs are under control!

P_T–distribution (P_T >> M) – two hard scales

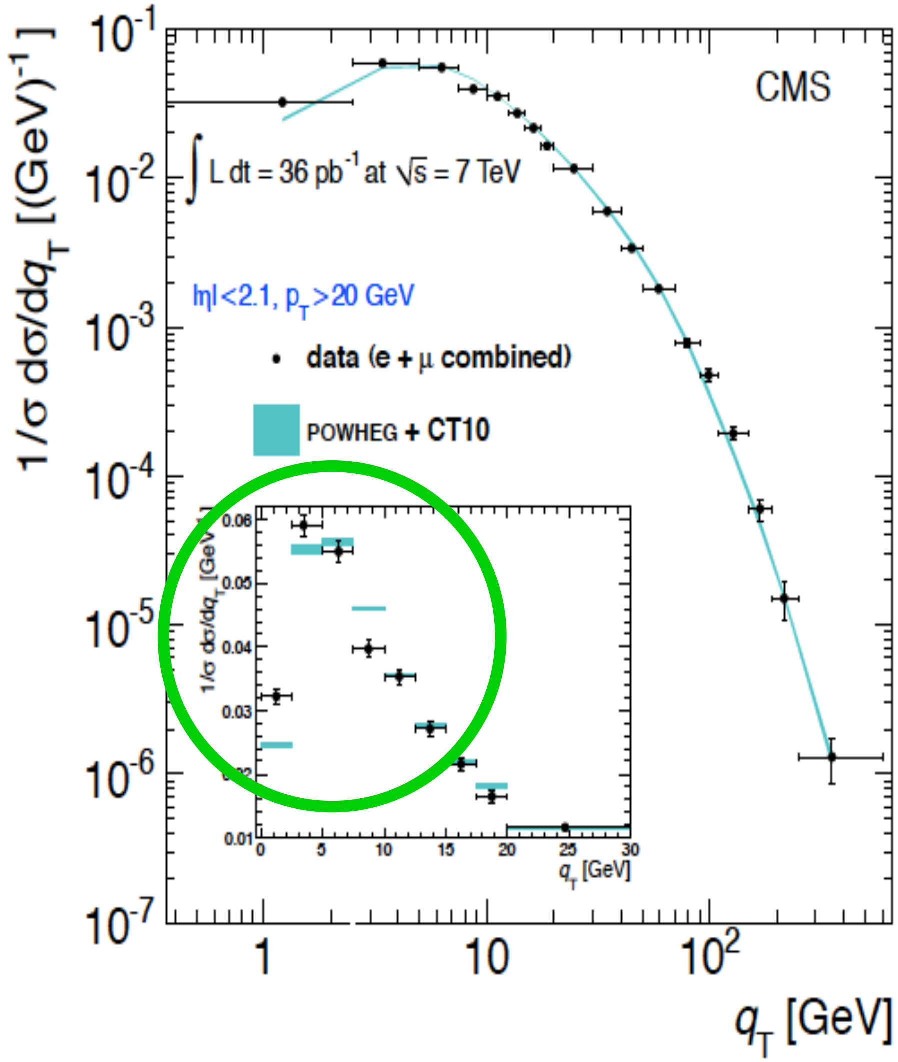


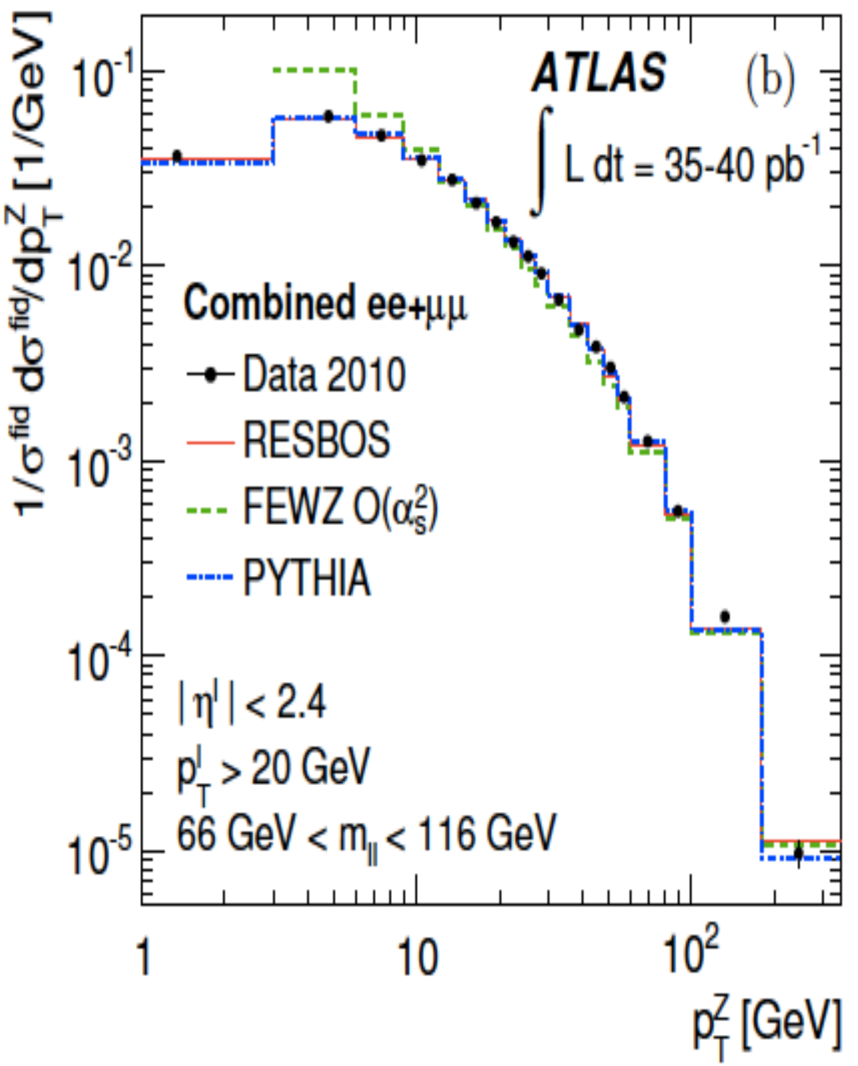
P_T–distribution (P_T >> M) – two hard scales



P_T–distribution (P_T << M) – two scales

- **Z^0-PT distribution in pp collisions:**





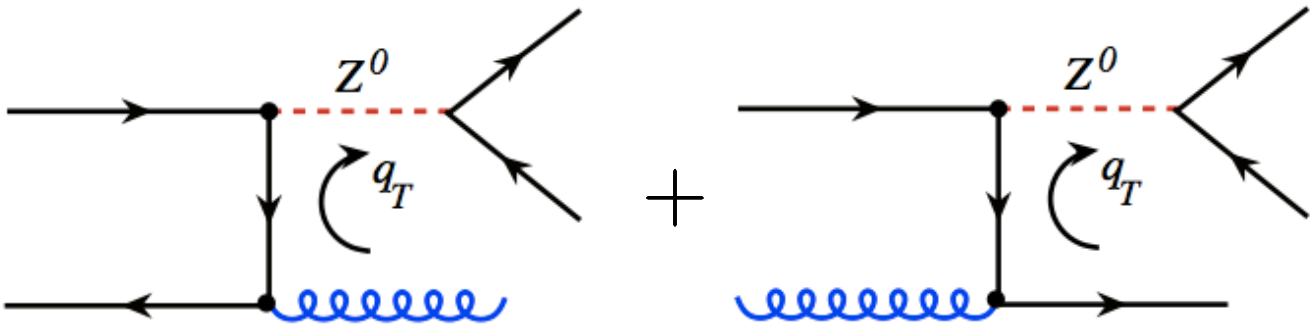
P_T as low as [0,2.5] GeV bin (or about 1.25 GeV)

P_T–distribution (P_T << M) – two scales

- **Interesting region – where the most data are:**

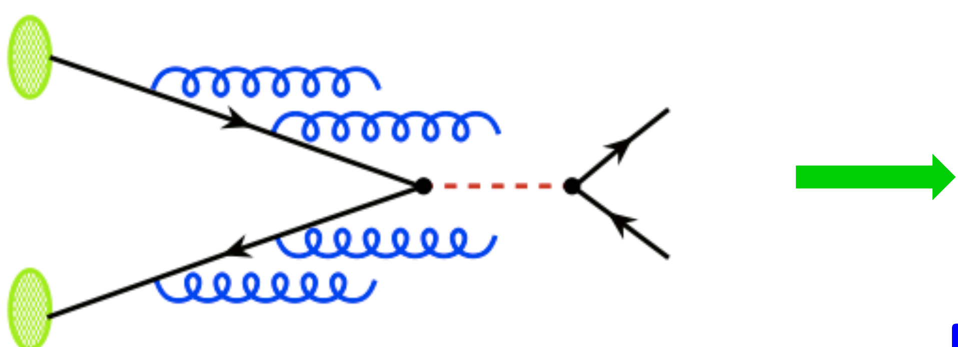
  $P_T \ll M_Z \sim 91$ GeV Two observed, but, very different scales

- **Fixed order pQCD calculation is not stable!**



$$\propto \frac{1}{q_T^2} \to \infty$$

- **Large logarithmic contribution from gluon shower:**



$$\left[\alpha_s \ln^2\left(\frac{M_Z^2}{q_T^2}\right)\right]^n$$

Resummation is necessary!

Cross section with two scales – resummation

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\rm QCD}^2, \quad Q_1^2 \gg Q_2^2 \gtrsim \Lambda_{\rm QCD}^2$$

- **Large perturbative logarithms:**

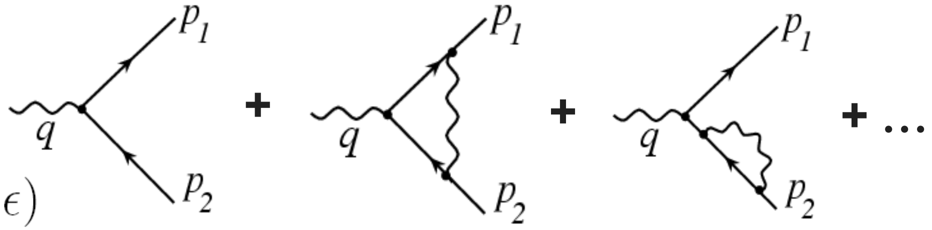
  $\alpha_s(\mu^2 = Q_1^2)$ is small, But, $\alpha_s(Q_1^2)\ln(Q_1^2/Q_2^2)$ is not necessary small!

- **Massless theory:**

  Two powers of large logs for each order in perturbation theory

  $\alpha_s(Q_1^2)\ln^2(Q_1^2/Q_2^2)$ due to overlap of IR and CO regions

- **Example – EM form factor:**



$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\, \bar{u}\,(p_1)\gamma_\mu v(p_2)\; \rho(q^2,\epsilon)$$

$$\rho(q^2,\epsilon) = -\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{1}{(-\epsilon)^2}-\frac{3}{2(-\epsilon)}+4\right\}$$

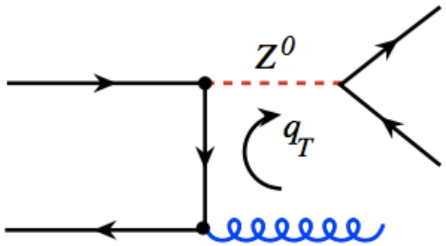
$$= 1 - \frac{\alpha_s}{4\pi}C_F\,\ln^2(q^2/\mu^2) + \ldots$$ Sudakov double logarithms

Common to all massless theories

"Drell-Yan" - leading double log contribution

❑ LO Differential Q_T-distribution as $Q_T \to 0$:

$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{LO} \approx \left(\frac{d\sigma}{dy}\right)_{Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$



$$\Longrightarrow \int_0^{Q^2} \left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{real+virutal} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{Born} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_Z^2$$

❑ Integrated Q_T-distribution:

$$\int_0^{Q_T^2} \left.\frac{d\sigma}{dy\,dp_T^2}\right|_{real+virutal} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right] \left.\frac{d\sigma}{dy\,dp_T^2}\right|_{real+virutal} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{Born} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F \frac{\alpha_s}{\pi} \frac{\ln\left(Q^2/p_T^2\right)}{p_T^2} dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{Born} \times \left[1 - C_F \frac{\alpha_s}{\pi} \ln^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{Born} \times \exp\left[-C_F \frac{\alpha_s}{\pi} \ln^2\left(Q^2/Q_T^2\right)\right]$$

Effect of gluon emission

Assume this exponentiates

Resumed Q_T distribution

- **Differentiate the integrated Q_T-distribution:**

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \times \exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

Resummed Q_T distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2}\times\exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- Compare to the explicit LO calculation:

$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

Resummed Q_T distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dydQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2}\times\exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- Compare to the explicit LO calculation:

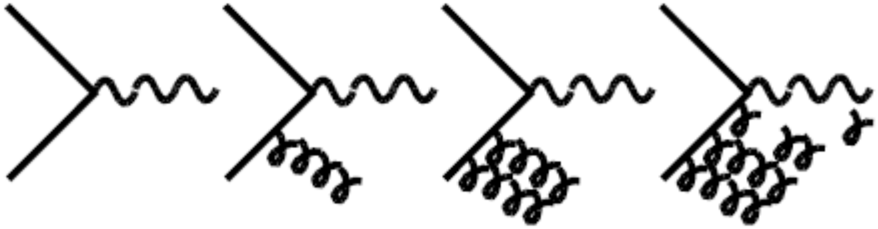
$$\frac{d\sigma}{dydQ_T^2}_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ \ldots$$

$$L \propto \ln\left(Q^2/Q_T^2\right)$$



Soft gluon emission treated as uncorrelated

Still a wrong Q_T-distribution

- **Experimental fact:**

$$\frac{d\sigma}{dy\,dQ_T^2} \Rightarrow \text{finite [neither } \infty \text{ nor } 0!] \text{ as } Q_T \to 0$$

- **Double Leading Logarithmic Approximation (DLLA):**

  - Radiated gluons are both soft and collinear with strong ordering in their transverse momenta
  - Ignores the overall vector momentum conservation
  - Double logs ~ random work ~ zero probability to be $Q_T = 0$

DLLA over suppress small Q_T region

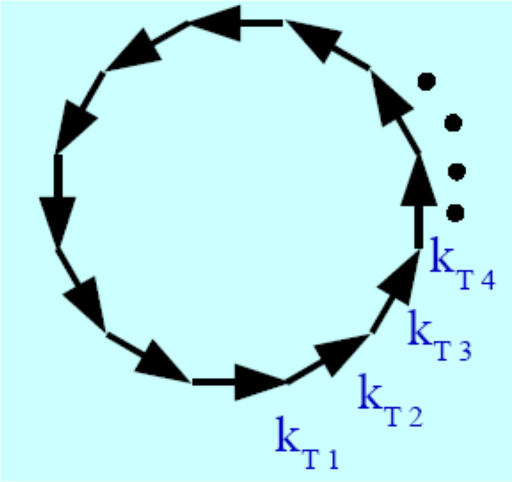
Resummation of uncorrelated soft gluon emission leads to a too strong suppression at $Q_T = 0$!

Still a wrong Q_T-distribution

- Why?

  Particle can receive many finite k_T kicks via soft gluon radiation yet still have $Q_T = 0$

  – Need a vector sum!



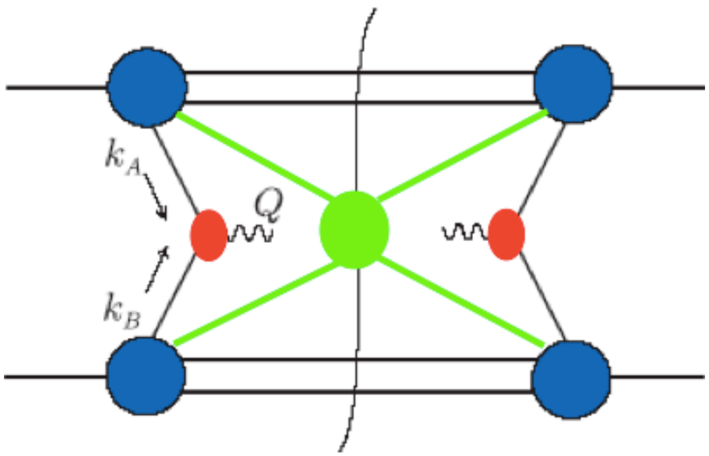
- Subleading logarithms are equally important at $Q_T = 0$
- Solution:

  To impose the 4-momentum conservation at each step of soft gluon resummation → TMD factorization

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

- **TMD-factorized cross section:**



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T} d^2k_{B_T} d^2k_{s,T}}{(2\pi)^6}$$

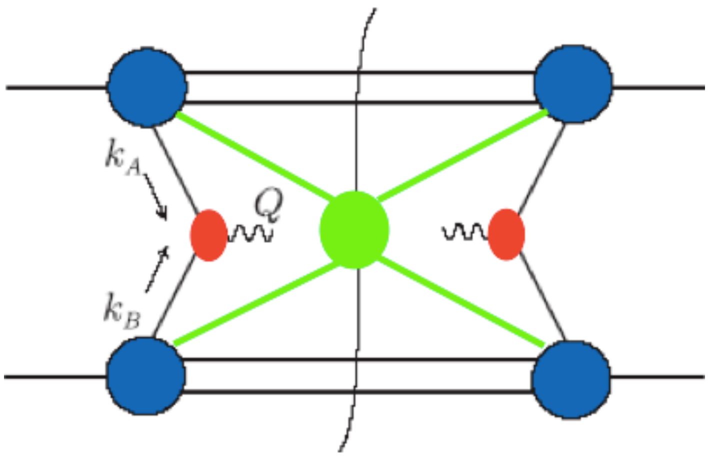
$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{f\bar{f}}(Q^2) S(k_{s,T})$$

$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

❑ TMD-factorized cross section:



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$$\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2} \int d^2b \, \mathrm{e}^{i\vec{b}\cdot\vec{Q}_T} \prod_i \mathrm{e}^{-i\vec{b}\cdot\vec{k}_{i,T}}$$

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

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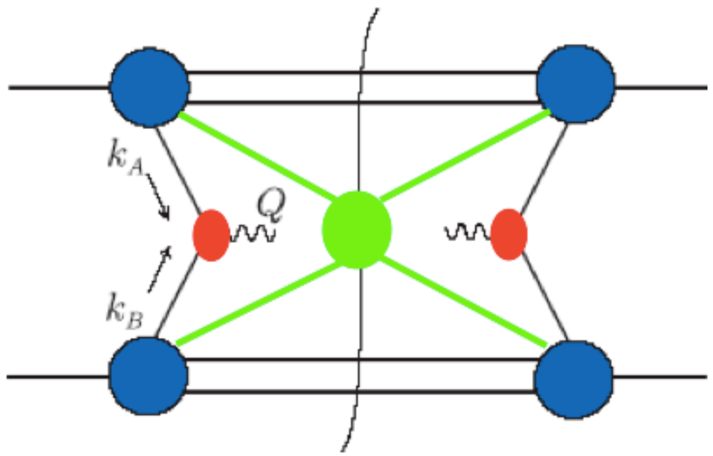

$$\frac{d\sigma_{AB}}{dQ^2\,dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T}\,d^2k_{B_T}\,d^2k_{s,T}}{(2\pi)^6}$$

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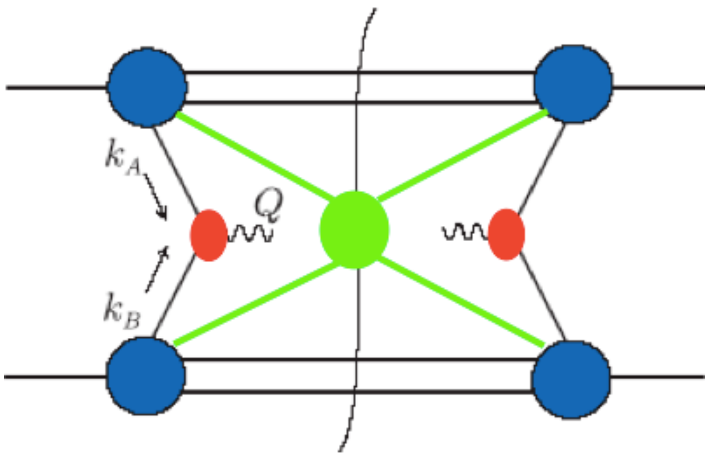
- Factorized cross section in "impact parameter b-space":

$$\frac{d\sigma_{AB}(Q,b)}{dQ^2} = \sum_f \int d\xi_a d\xi_b \overline{P}_{f/A}(\xi_a, b, n) \overline{P}_{\bar{f}/B}(\xi_b, b, n) H_{f\bar{f}}(Q^2) U(b,n)$$

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

- **TMD-factorized cross section:**



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- **Resummation:** Two equations, resummation of two log's

$$\mu_{\rm ren} \frac{d\sigma}{d\mu_{\rm ren}} = 0 \qquad\qquad n^{\nu} \frac{d\sigma}{dn^{\nu}} = 0$$

CSS b-space resummation formalism

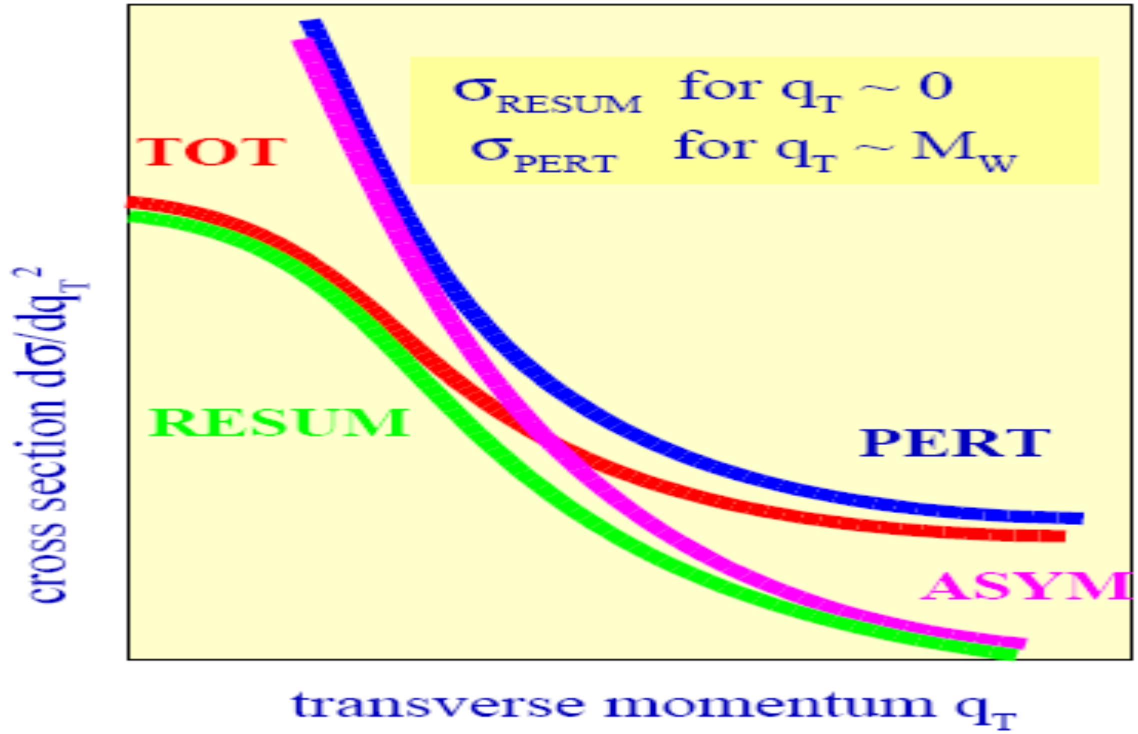
- Solve those two equations and transform back to Q_T:

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2} \int d^2b \; e^{i\vec{b}\cdot\vec{Q}_T} \tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2, Q^2)$$

resummed

No large log's

$$= \frac{1}{(2\pi)} \int_0^\infty db \; J_0(bQ_T) \; b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\mathrm{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\mathrm{Asym})}}{dQ^2 dQ_T^2} \right]$$



CSS b-space resummation formalism

- b-space distribution:

$$W_{AB}(b,Q) = \sum_{ij} W_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$$

- Collins-Soper equation:

$$\frac{\partial}{\partial \ln Q^2}\tilde{W}_{ij}(b,Q) = [K(b\mu,\alpha_s) + G(Q/\mu,\alpha_s)]\,\tilde{W}_{ij}(b,Q) \quad (1)$$

- Evolution kernels satisfy RG equation:

$$\frac{\partial}{\partial \ln \mu^2}K(b\mu,\alpha_s) = -\frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (2)$$

$$\frac{\partial}{\partial \ln \mu^2}G(Q/\mu,\alpha_s) = \frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (3)$$

- Solution - resummation:

$$W_{ij}(b,Q) = W_{ij}(b,1/b)\,\mathrm{e}^{-S_{ij}(b,Q)}$$

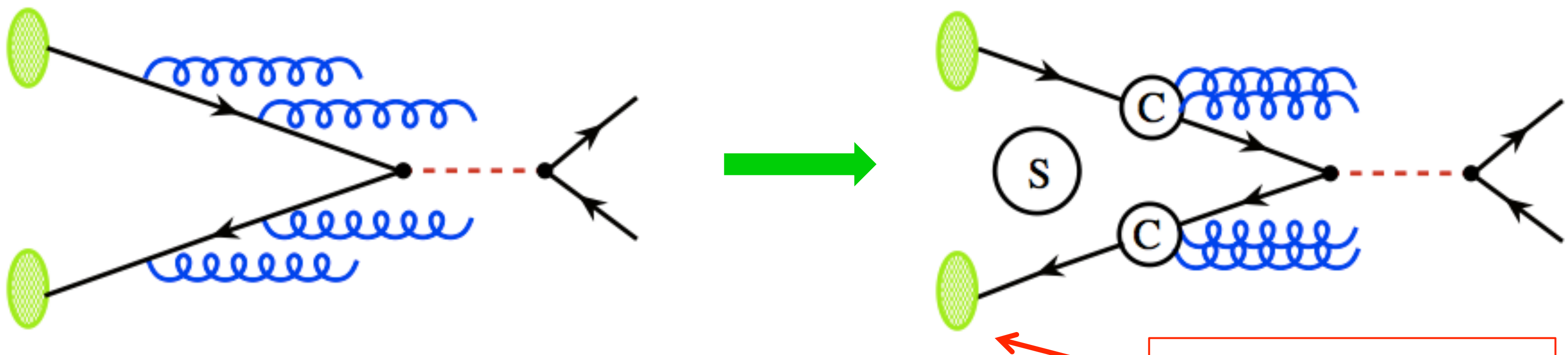
Sudakov form factor
All large logs

Boundary condition – perturbative if b is small!

CSS b-space resummation formalism

- Boundary condition – collinear factorization:

$$W_{ij}(b,Q) = \sum_{a,b} \sigma_{ij\to Z} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right]$$



- Perturbative solution:

$$W_{AB}^{\rm pert}(b,Q) = \sum_{a,b,i,j} \sigma_{ij\to Z} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right] \times e^{-S_{ij}(b,Q)}$$

Only valid when b << $1/\Lambda_{\rm QCD}$

- Extrapolation to large-b?
  - ✧ Non-perturbative
  - ✧ Predictive power?

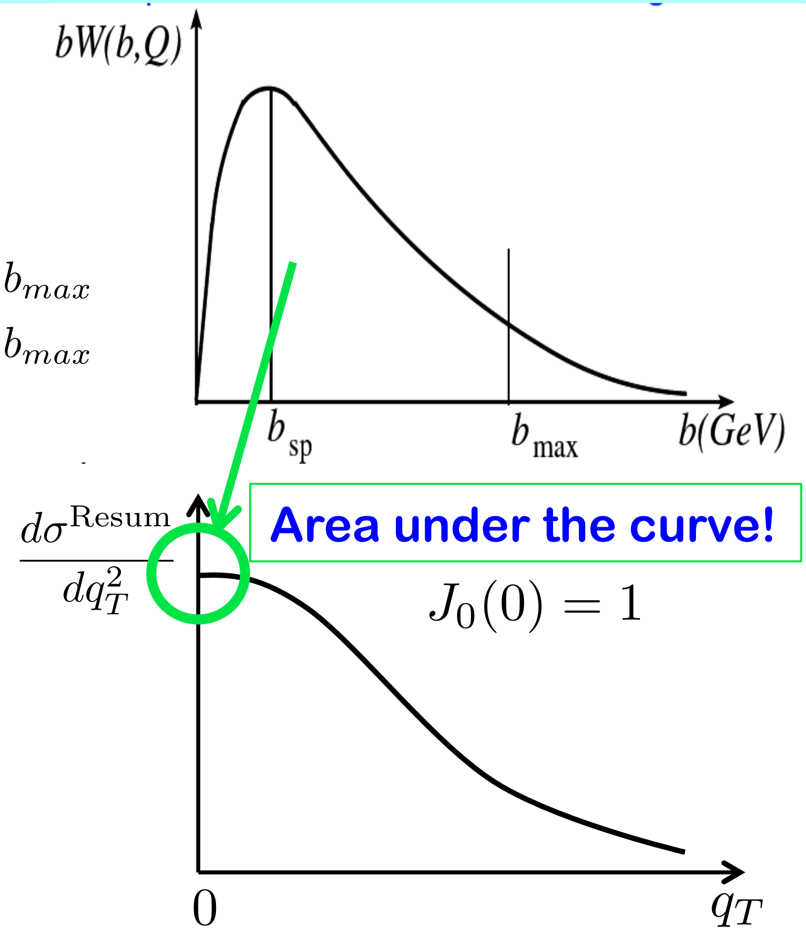
$$\sigma^{\rm Resum} \propto \int_0^\infty db\, J_0(q_T\, b) b\, W(b,Q)$$

Phenomenology – predictive power

❑ Resummed cross section:

$$\frac{d\sigma^{\text{resum}}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

$$W(b,Q) = \begin{cases} W^{\text{pert}}(b,Q) & b \le b_{max} \\ \color{red}{?} & b > b_{max} \end{cases}$$



Phenomenology – predictive power

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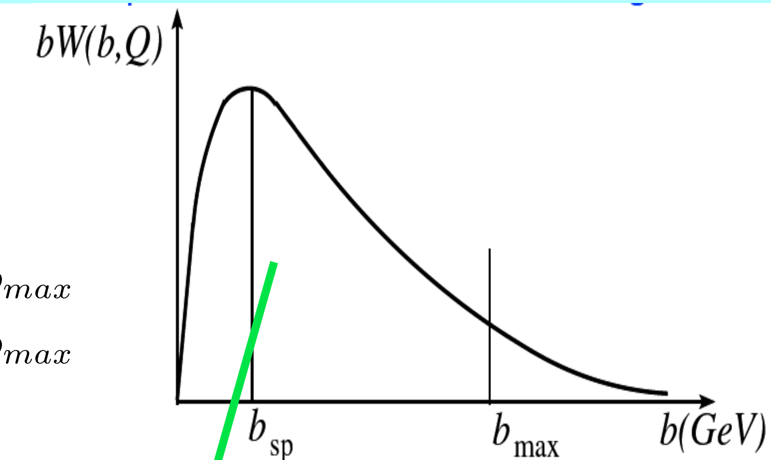
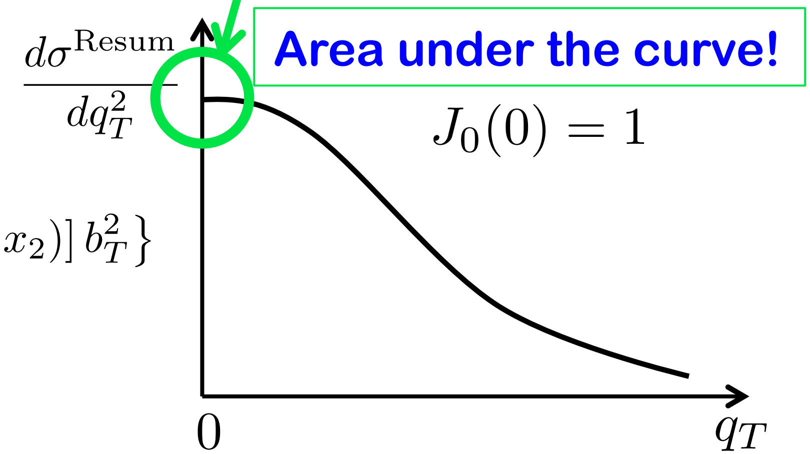


❑ CSS b*-prescription:

$$W(b,Q) \equiv W^{\rm pert}(b^*,Q)\, F^{\rm NP}(b,Q)$$

$$b^* \equiv \frac{b}{\sqrt{1+(b/b_{max})^2}} \quad \begin{array}{ll} \to b & \text{when } b\to 0 \\ \to b_{max} & \text{when } b \to \infty \end{array}$$

$$F^{\rm NP} \equiv \exp\left\{-\left[g_1 + g_2 \ln(Q/2Q_0) + g_1 g_3 \ln(100 x_1 x_2)\right] b_T^2\right\}$$



Phenomenology – predictive power

❑ Resummed cross section:

$$\frac{d\sigma^{\rm resum}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

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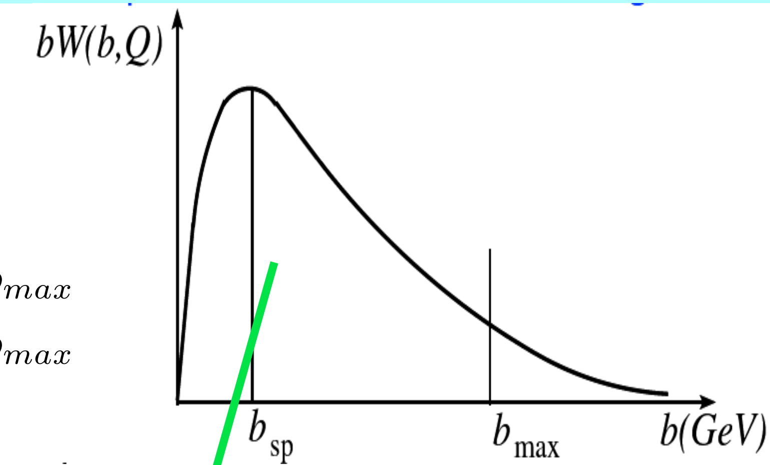

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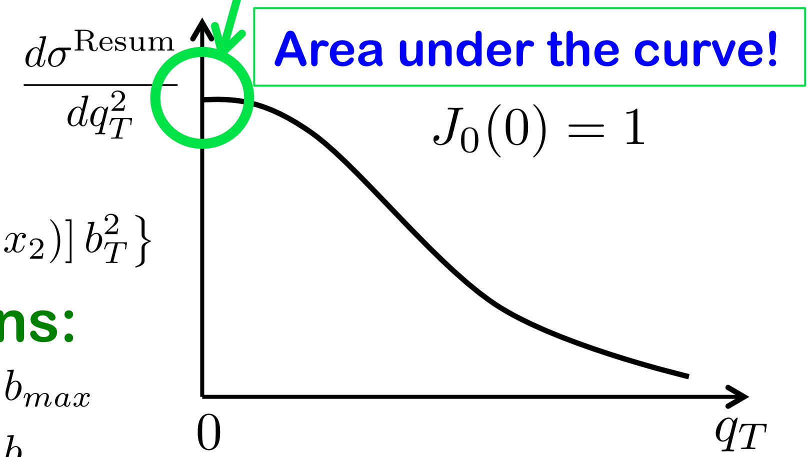


❑ Extrapolation with power corrections:

$$W(b,Q) = \begin{cases} W^{\rm pert}(b,Q) & b \le b_{max} \\ W^{\rm pert}(b_{max},Q) F^{NP}_{QZ}(b,Q,b_{max}) & b > b_{max} \end{cases}$$

$$F^{NP}_{QZ}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b^2_{max}}{c^2}\right) \left[g_1 \left((b^2)^\alpha - (b^2_{max})^\alpha \right) + g_2 \left(b^2 - b^2_{max} \right) \right] - \bar{g}_2 \left(b^2 - b^2_{max} \right) \right\}$$

Resummed leading power

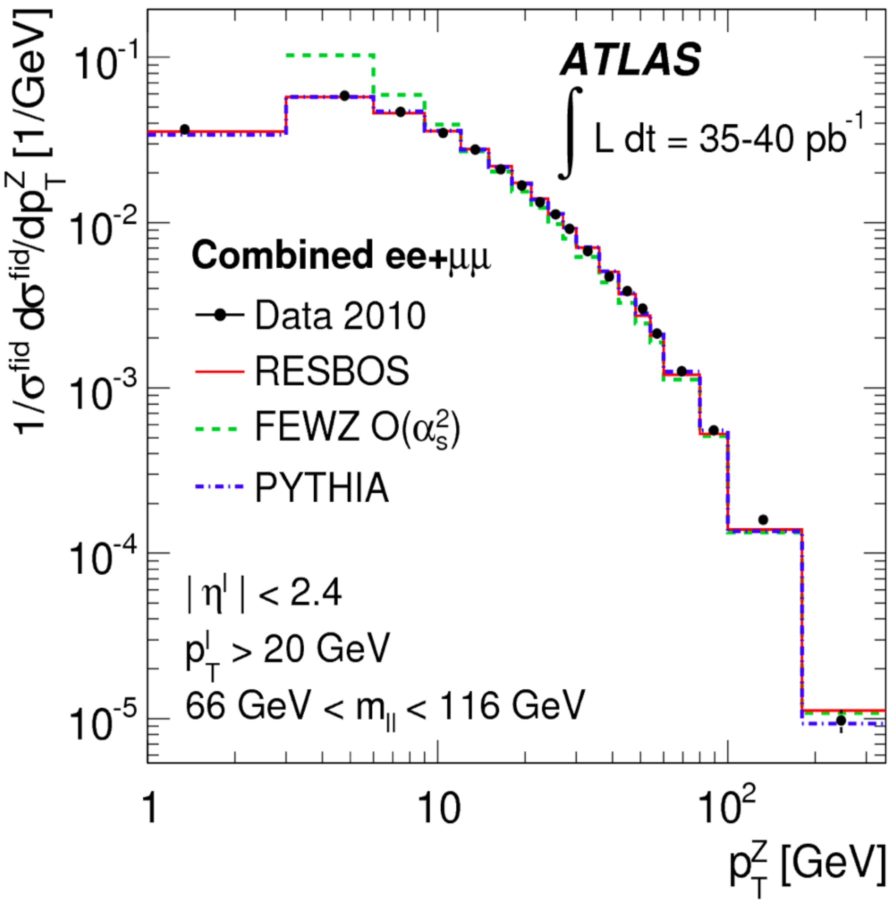
Intrinsic power corrections

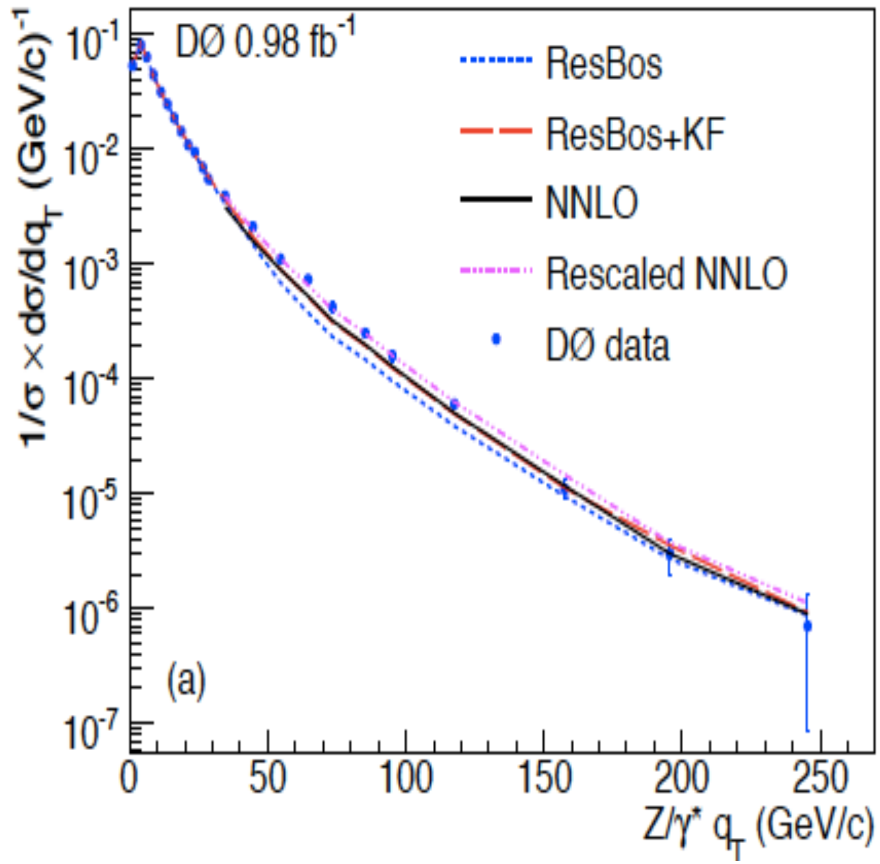
Dynamical power corrections

Qiu, Zhang, 2001

Phenomenology

- Compare with the LHC data:

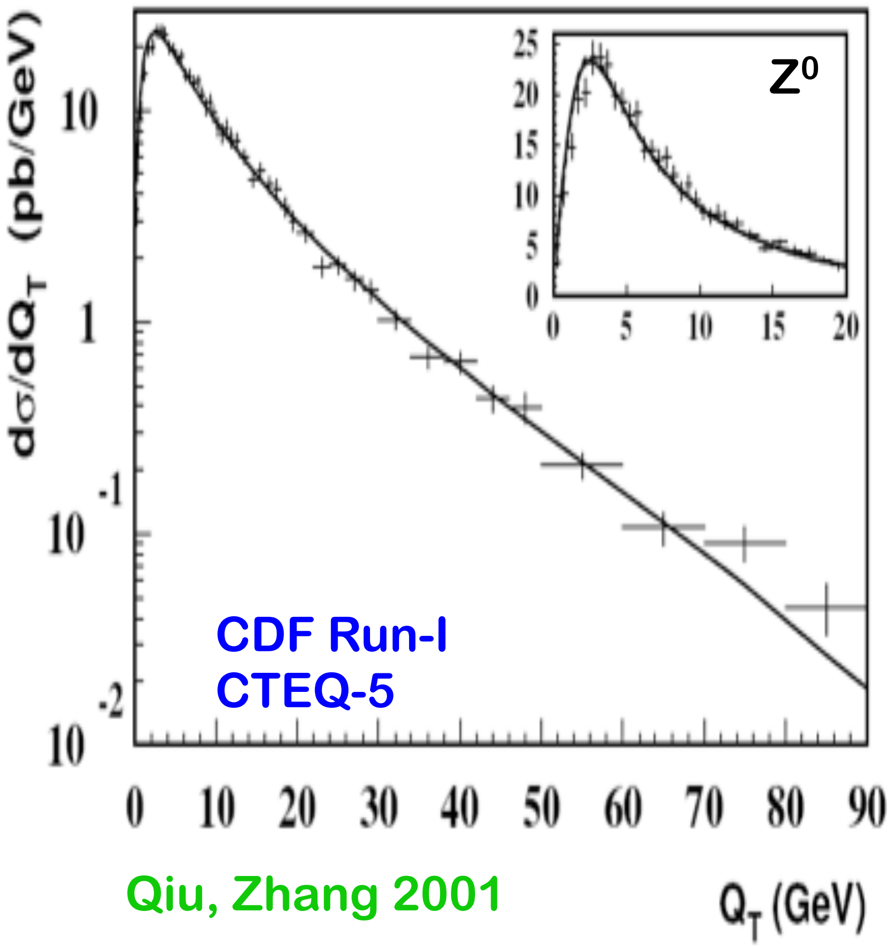




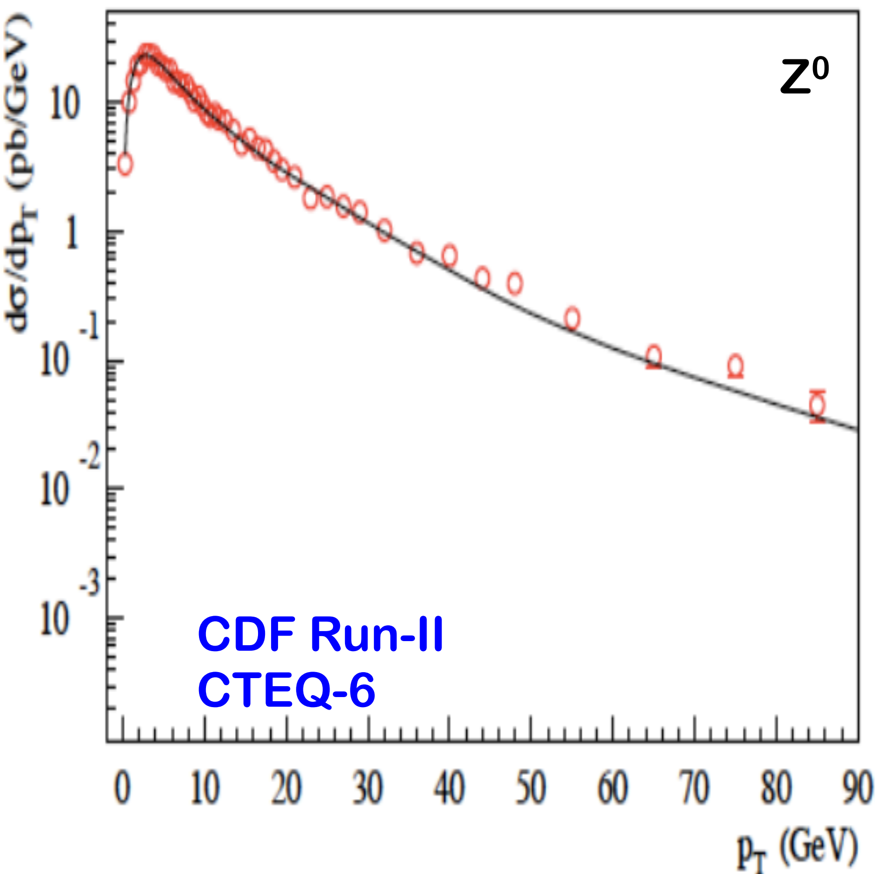
ResBos: CSS b*-prescription – fitting g_1, g_2, g_3,Q_0

Phenomenology

- Compare with the Tevatron data:



Qiu, Zhang 2001



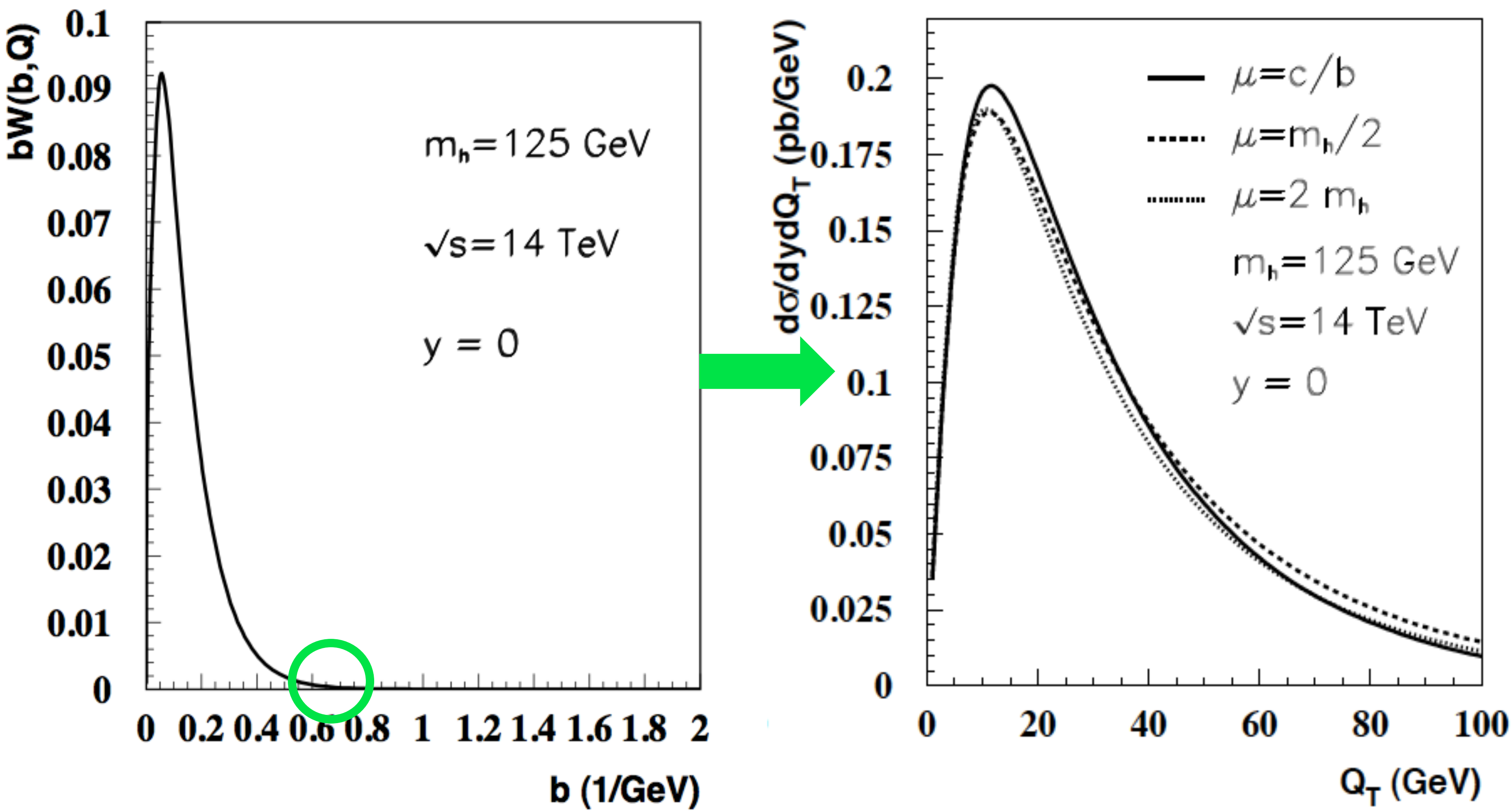
Kang, Qiu 2012

No free fitting parameter!

Phenomenology - Higgs

Berger, Qiu, 2003

- Prediction for Higgs spectrum:

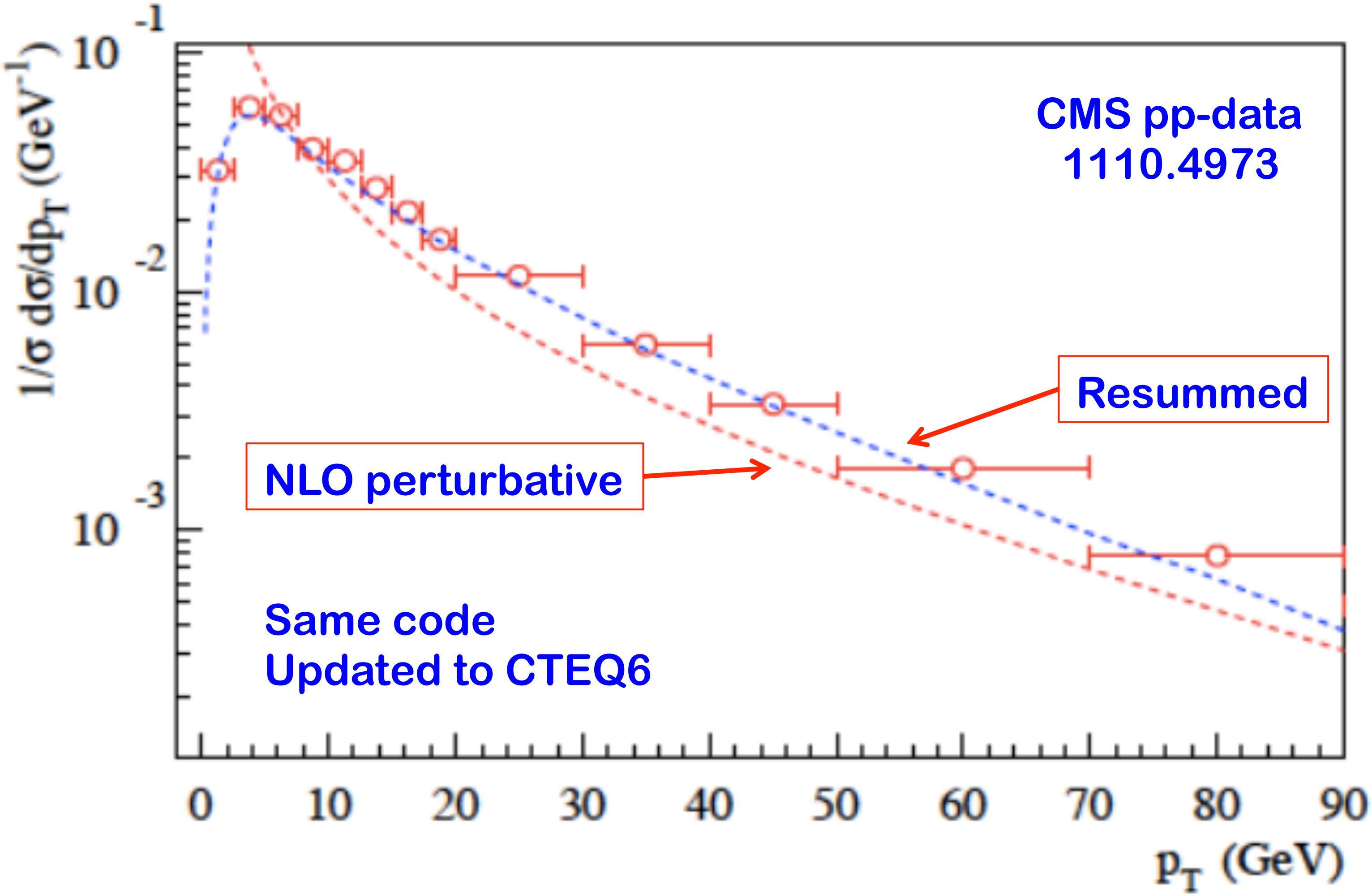


Effectively NO non-perturbative uncertainty – Shower dominates!

Phenomenology

Kang, Qiu, 2012

- ❑ **Prediction for Z^0@LHC:**

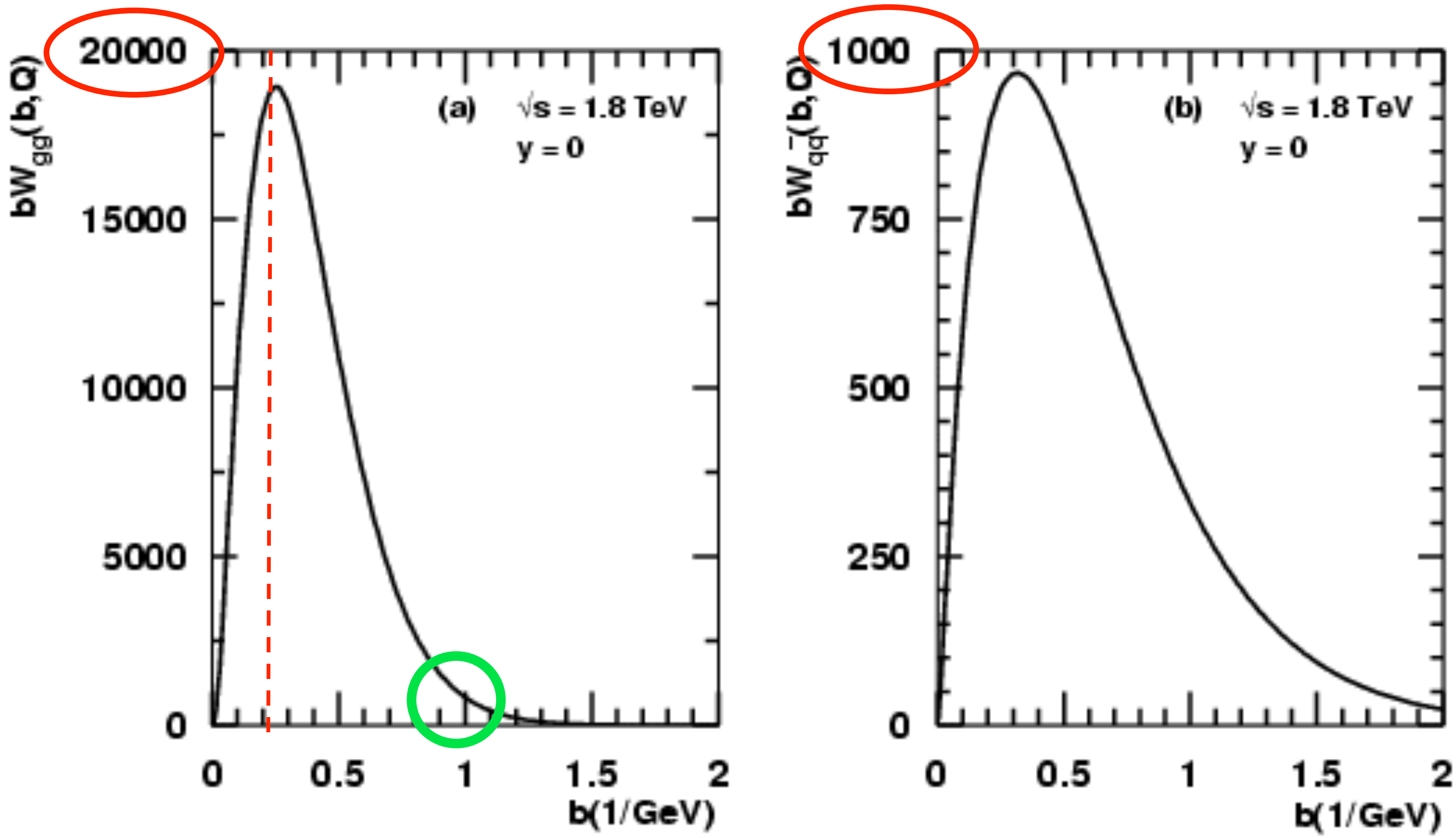


Effectively no non-perturbative uncertainty!

Phenomenology

Berger, Qiu, Wang, 2005

❑ **Upsilon production (low Q, large phase space):**



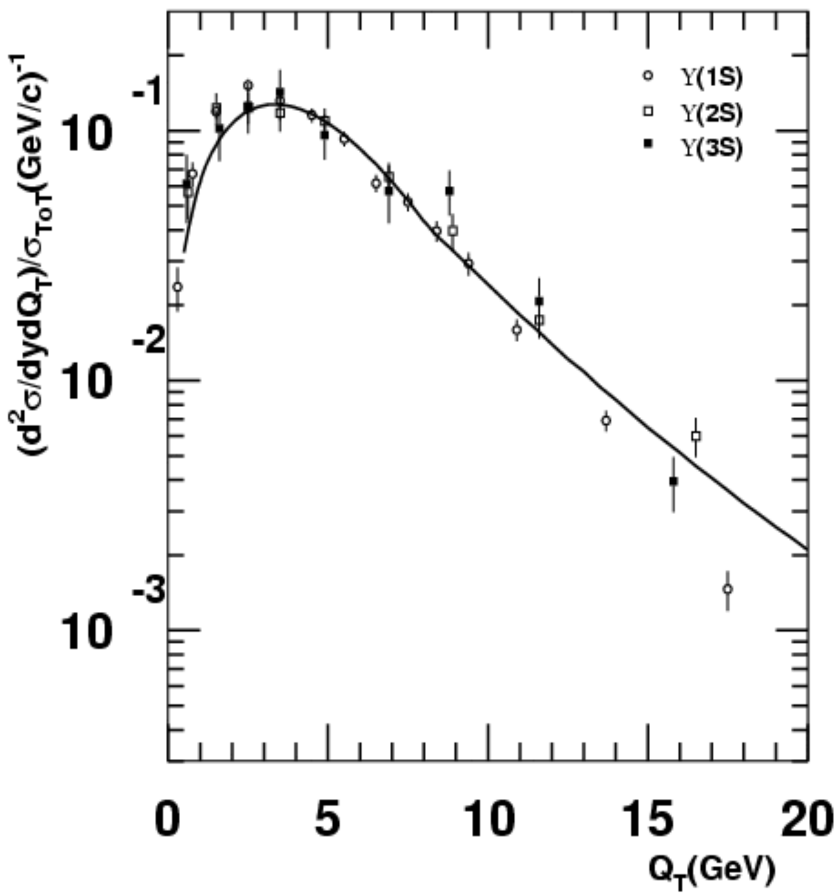
Gluon-gluon dominate the production

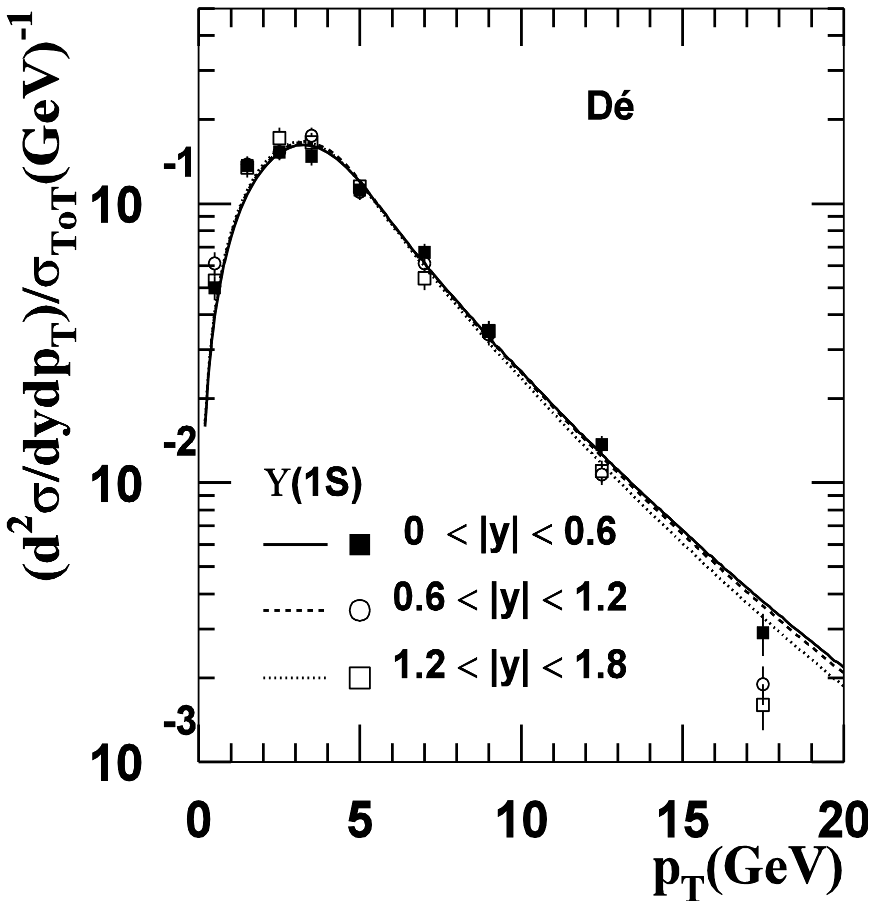
Dominated by perturbative contribution even M_Υ~10 GeV

Phenomenology

Berger, Qiu, Wang, 2005

- Prediction vs Tevatron data:



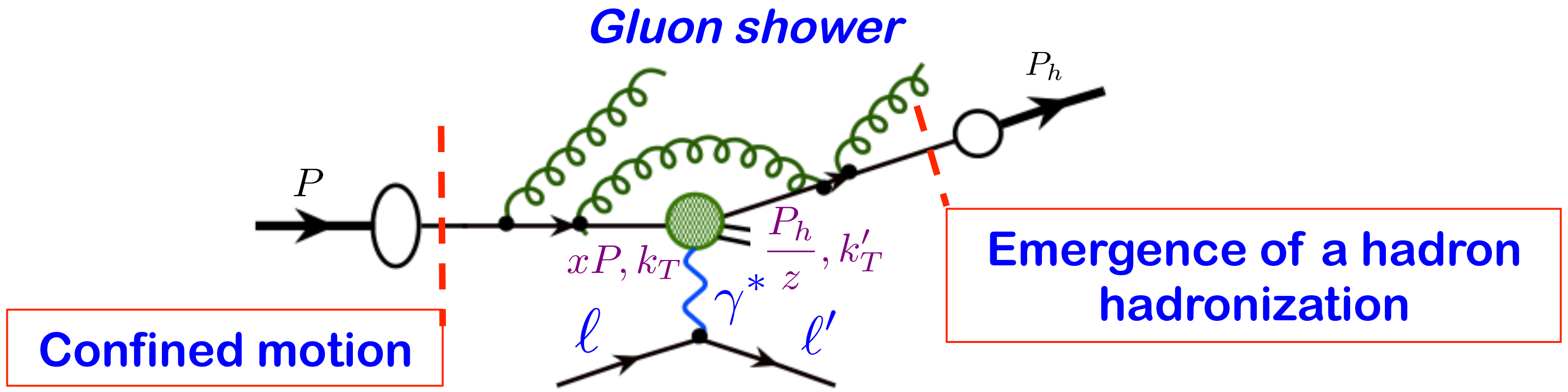


CDF Run-I data

DO Run-II data

Parton k_T at the hard collision

- Sources of parton k_T at the hard collision:



- Large k_T generated by the shower (caused by the collision):
  - ✧ Q^2-dependence – linear evolution equation of TMDs in b-space
  - ✧ The evolution kernels are perturbative at small b, but, not large b
  - ➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

- Challenge: to extract the "true" parton's confined motion:
  - ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs!

Di-photon production

- **Principle background to Higgs production channel $H^0 \to \gamma\gamma$:**

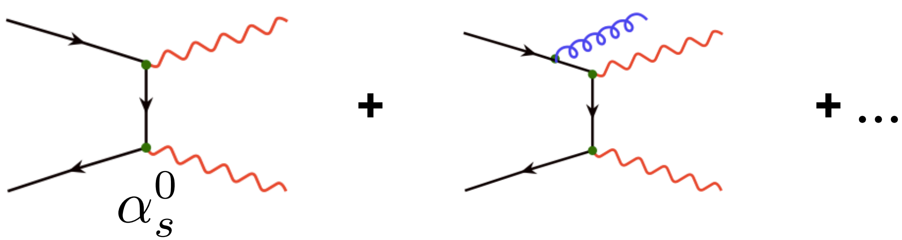
  Although the background is subtracted with a fitting procedure, it is also important to have some control of this process ab initio

- **Experimentally,**

  Significant contamination from the production of jets, or photon +jet, where jets are mis-identified as photons

  Jet production rate is so much higher photon, care is needed even with mis-identification rate as small as 10^{-4}!

- **Theoretically,**



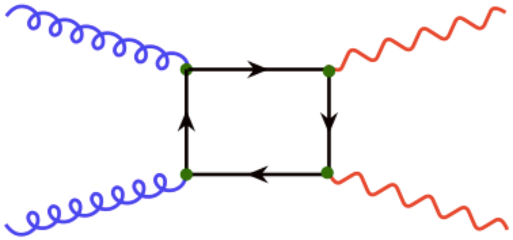
  Implementation of isolation cut with two photons

  Back-to-back kinematics – angular distribution – TMD factorization?

Di-photon production

❑ High order corrections:

- ✧ NLO corrections included in DIPHOX and MCFM
- ✧ A particular class of NNLO contributions is separately gauge-invariant, and, numerically important at the LHC – more gluons



Contribute at $\mathcal{O}(\alpha_s^2)$ to the x-section

NO tree-level $gg \to \gamma\gamma$

N^3LO correction with NLO technology

- ✧ Contributes approximately 15-25% of the NLO total, depending on exact choice of photon cuts, scale choice, etc.
- ✧ TMD factorization vs collinear factorization?

Qiu et al. PRL 2011

$$\frac{d\sigma}{d^4q_{\gamma\gamma}d\Omega_{\gamma\gamma}}$$

When $q_{T\gamma\gamma} \ll \sqrt{q^2_{\gamma\gamma}}$, or imposing photon pT cut

Linear polarized gluon impacts $\Omega_{\gamma\gamma}$ distribution

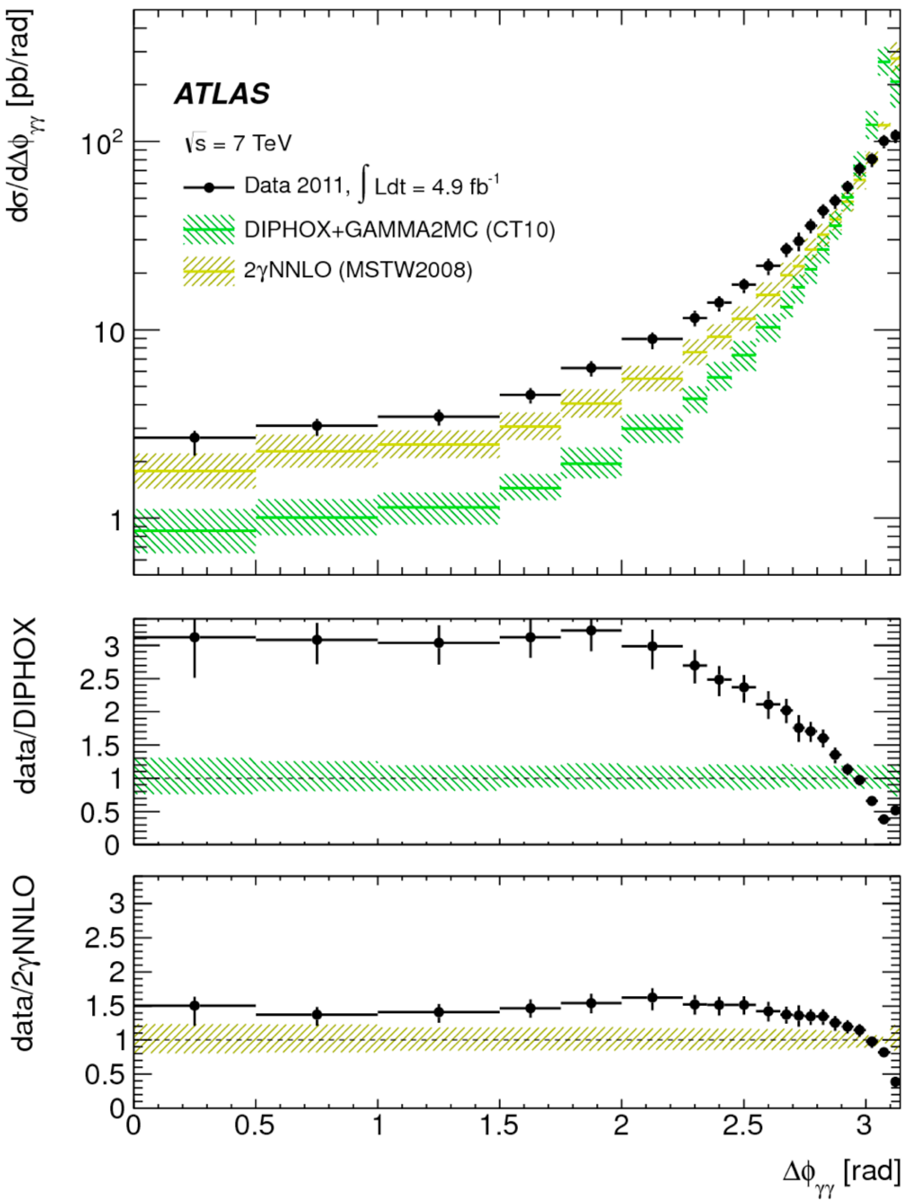
NNLO results

- Full NNLO calculation performed in the “Frixione” scheme, i.e. no need for fragmentation contributions

  Catani et al (2012)

- Better description of kinematic regions that are poorly described or inaccessible at NLO, e.g., azimuthal angle between photons

- Even better description would require either higher orders or inclusion in parton shower
  → not yet feasible.



Photon + jet angular distribution

- **QCD Compton and annihilation subprocess:**

$$\frac{d\sigma}{d\hat{t}} \sim (1 - \cos(\theta^*))^{-1} \quad \text{as} \quad \cos(\theta^*) \to 1$$

- **Other QCD subprocess,** $qq \to qq, qg \to qg, gg \to gg, \ \text{etc.}$ **more relevant to jet+jet angular distribution:**

$$\frac{d\sigma}{d\hat{t}} \sim (1 - \cos(\theta^*))^{-2}$$

$$\text{as} \ \ \cos(\theta^*) \to 1$$

- **Prediction:**

  Photon-jet angular distribution should be flatter than that observed in jet-jet final states

$$\cos(\theta^*) = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$$

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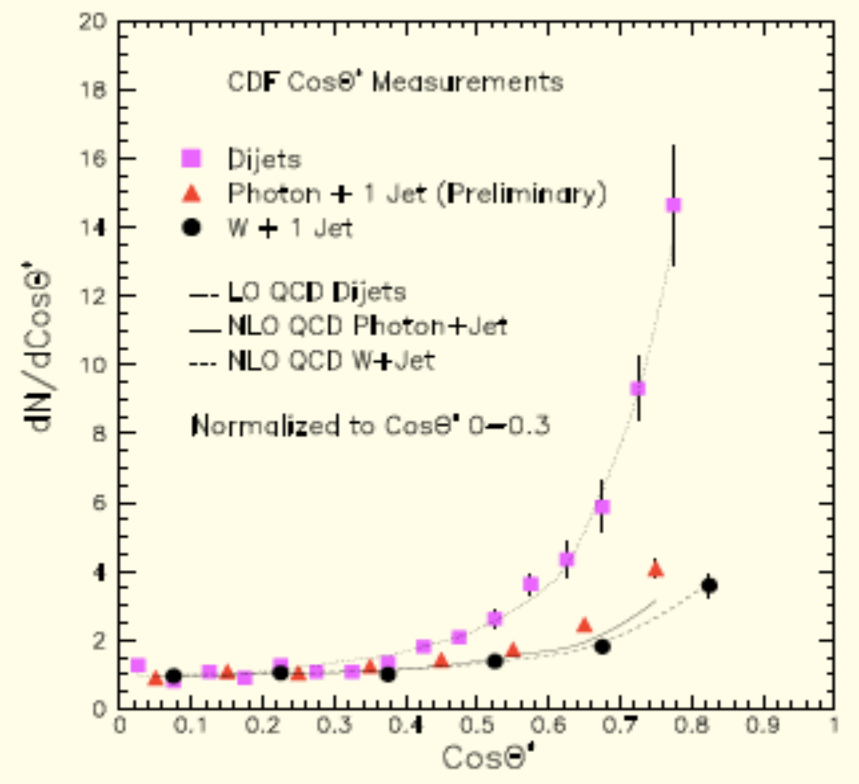
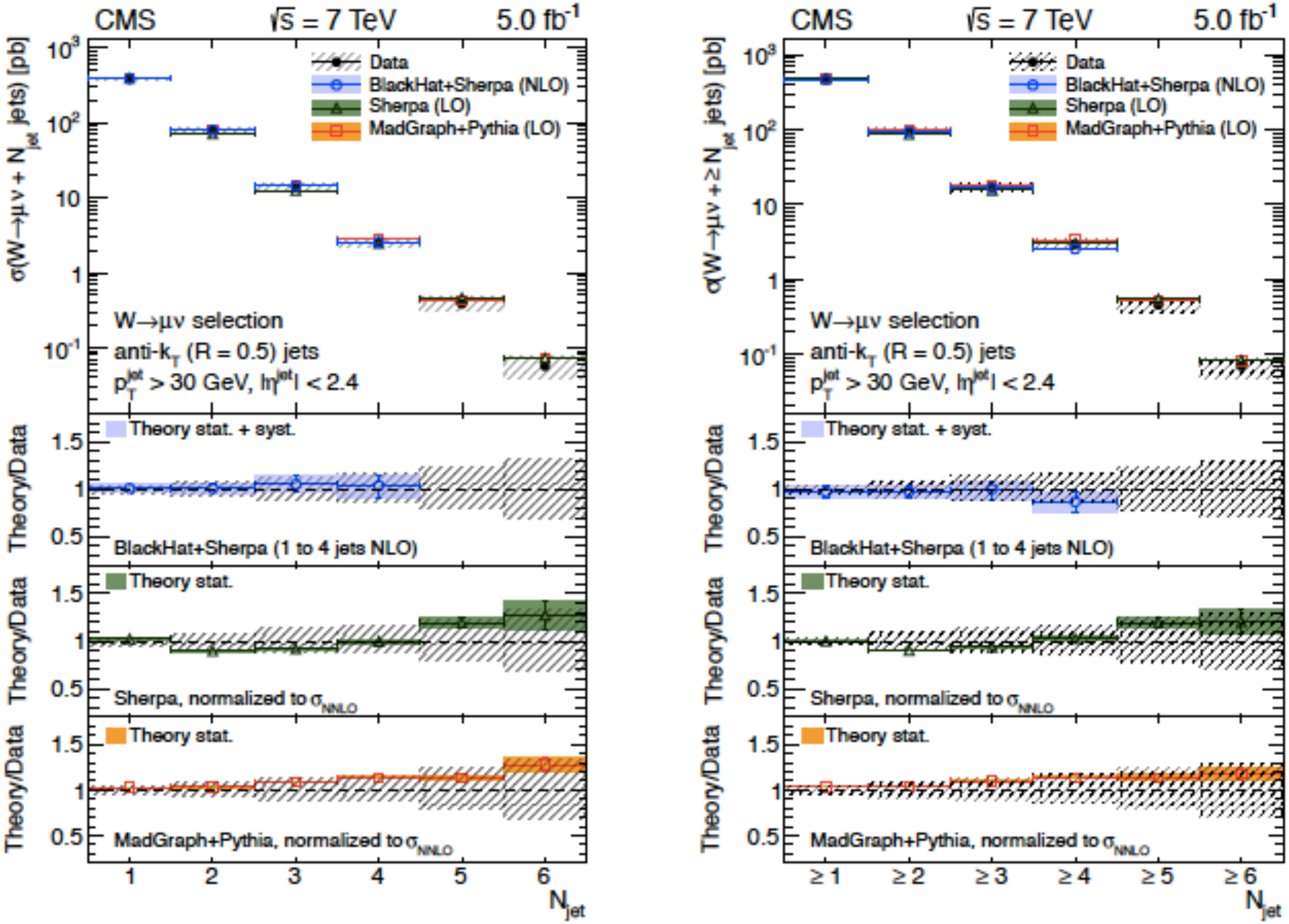


W-boson + jets

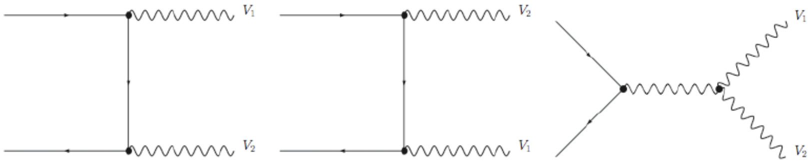
CMS – 1406.7533

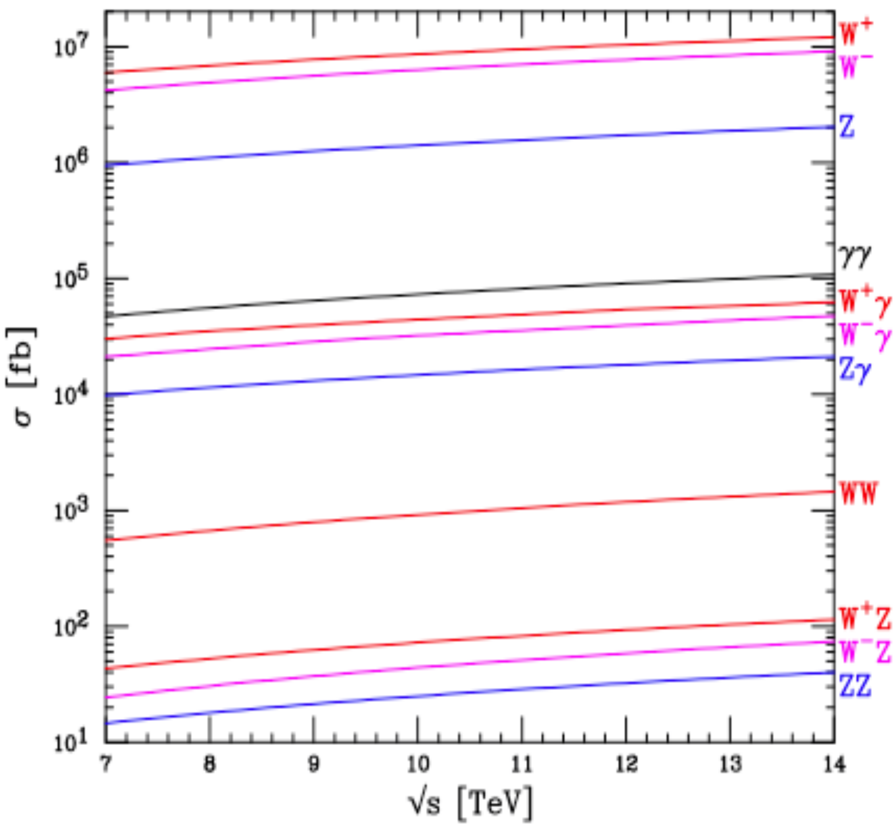


Di-boson hadronic production

Campbell, CTEQ SS2013

- Triple gauge boson interaction:





✧ Triple gauge coupling present for all processes except $Z\gamma$

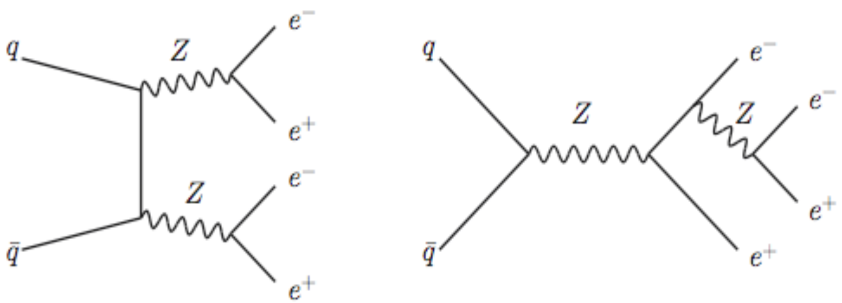
✧ Processes involving photons dependent on photon pT (and rapidity) cut, strongly

✧ NLO corrections known analytically, included in MCFM, VBFNLO (also POWHEG NLO MC)

Two bosons with single-resonant

Campbell, CTEQ SS2013

- **Two Z's:** $q\bar{q} \to ZZ \to e^+e^-e^+e^-$



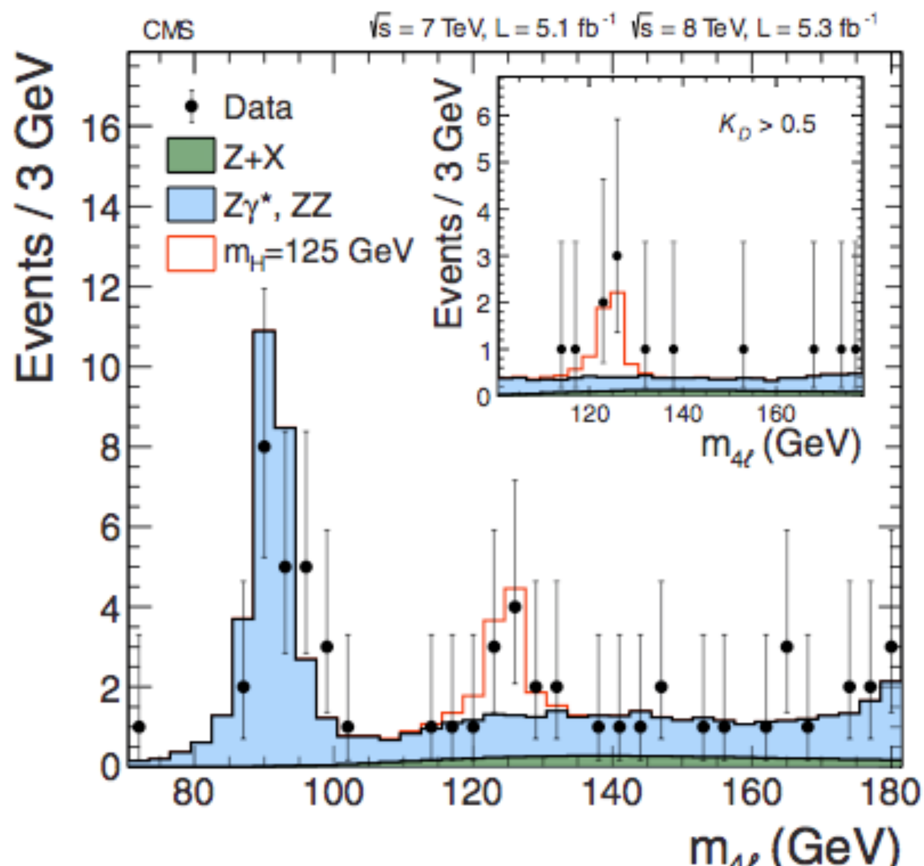
"double"-resonant **"single"-resonant**

Plus diagrams with Z replaced by photon

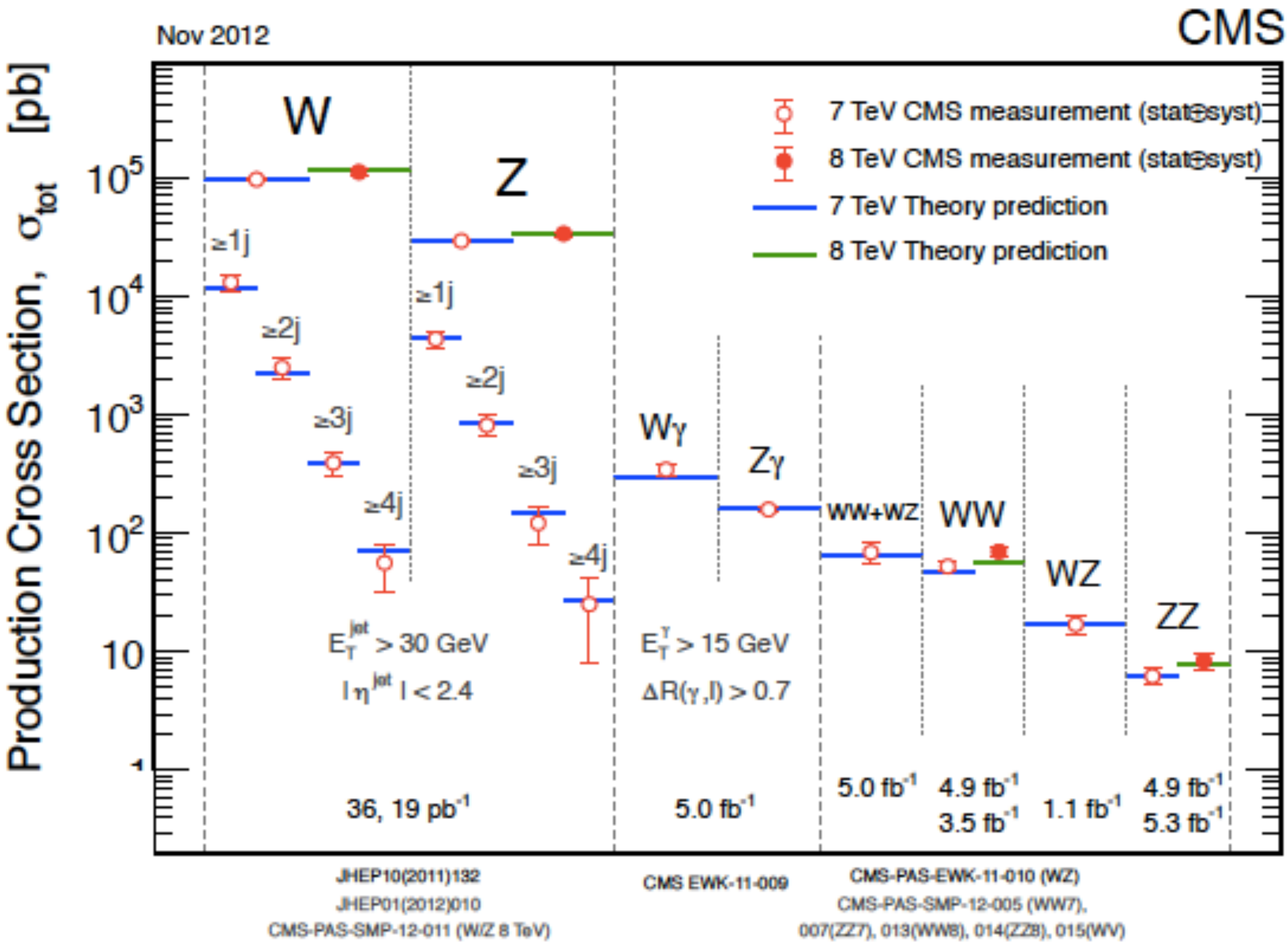
- **Inclusive cross section is dominated by the double-resonant contribution**

- **Notably: invariant mass of 4 leptons**

- **One of the cross-checks in Higgs search**

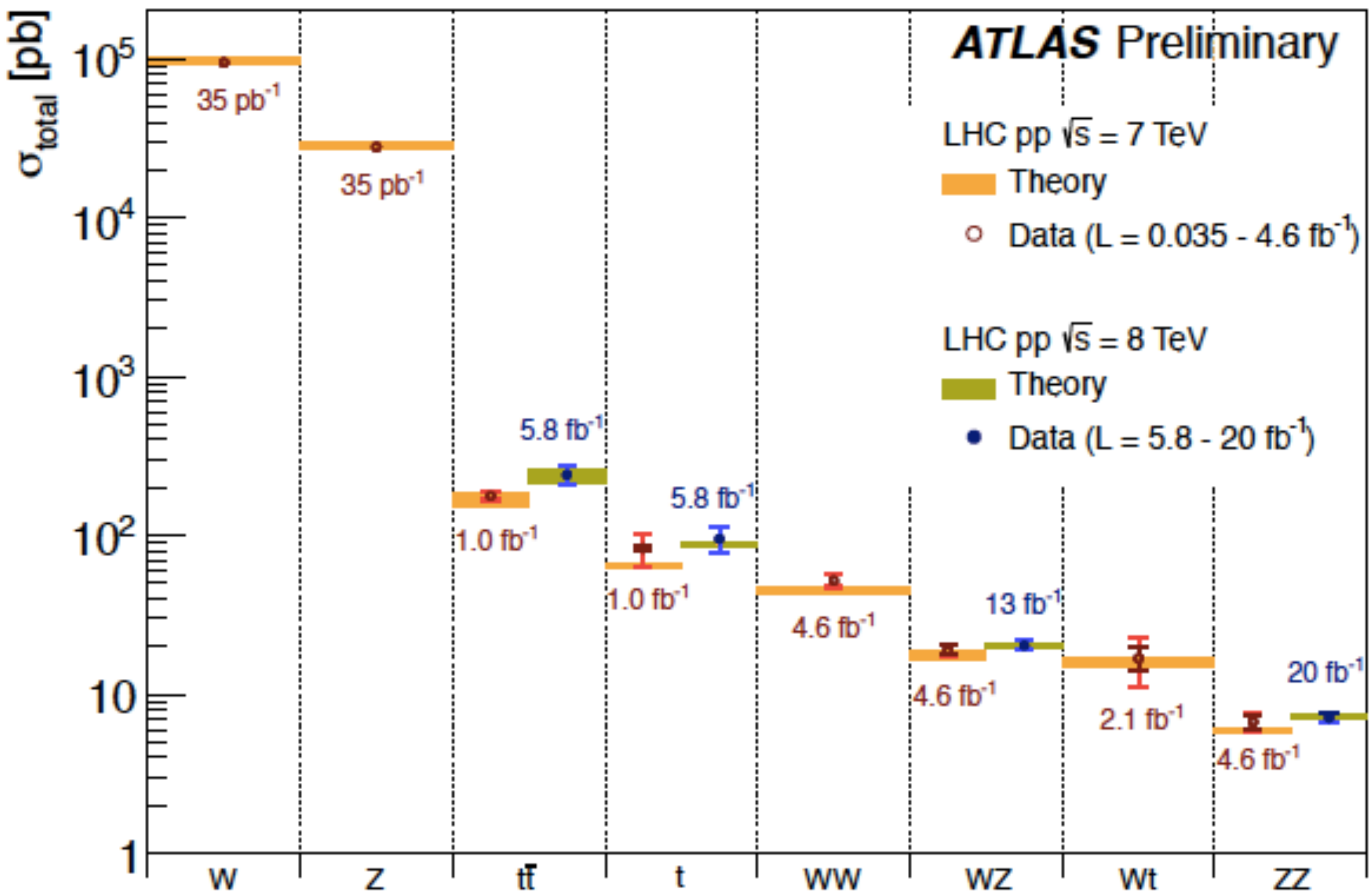


Vector bosons: experimental summary



Good consistency with theory expectations of NNLO (W/Z), and NLO (di-bosons) for all processes in both experiments

Vector bosons: experimental summary



Good consistency with theory expectations of NNLO (W/Z), and NLO (di-bosons) for all processes in both experiments

Improvement from resummation

- Beyond the Born term (lowest order), partonic hard-parts are NOT unique, due to the PDFs' scheme dependence

- Same parton-level PDFs should be used for calculations of partonic parts of all observables

- All partonic hard parts have: $P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)$

  Suggests to choose the scale: $\mu_F^2 \sim Q^2$

- Hard parts have potentially large logarithms:

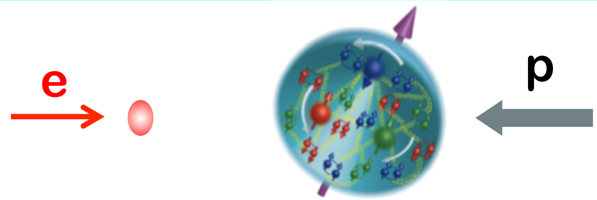
$$\ell n(x), \quad \frac{1}{(1-x)_+}, \quad \left(\frac{\ell n(1-x)}{1-x}\right)_+$$

⟹ Resummation of the large logarithms

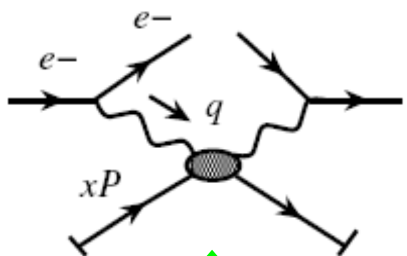
Lot of progresses in recent years

QCD power corrections

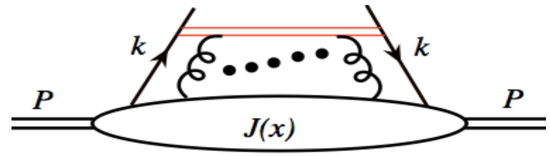
❑ QCD factorization:



$\sigma_{\text{tot}}^{\text{DIS}} \sim$



$\otimes$



$+ \, O\left(\frac{1}{QR}\right)$

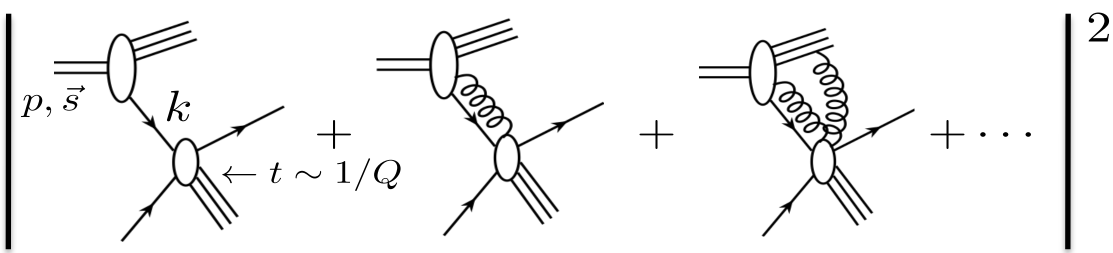
Hard-part
Probe

Parton-distribution
Structure

Power corrections
Approximation

❑ QCD power corrections:

$\sigma(Q, \vec{s}) \propto$



$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

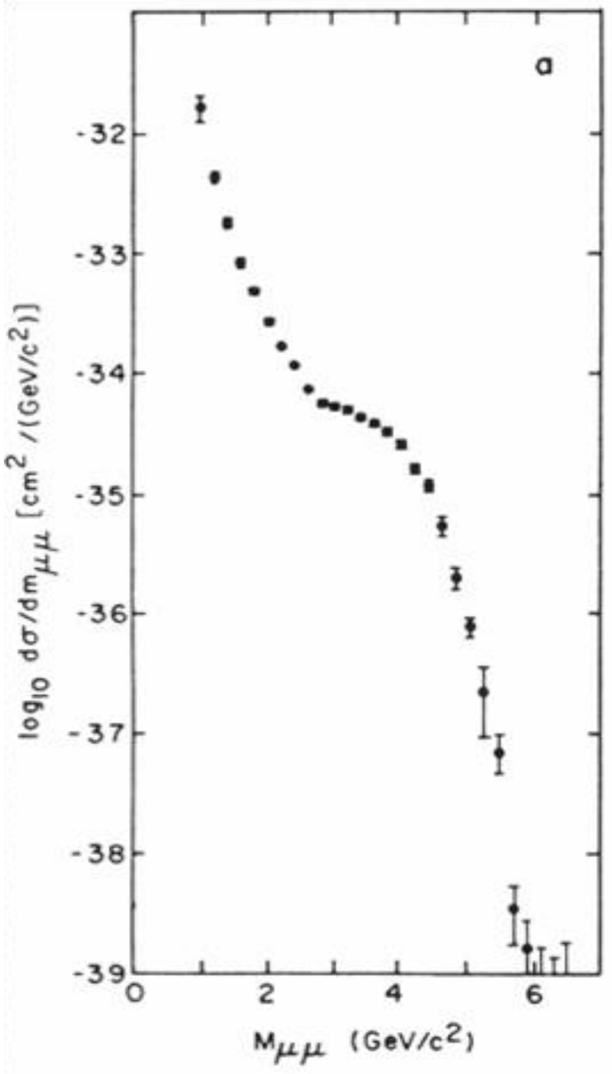
Three-parton correlation

Multi-parton correlation functions

No probability interpretation!

Heavy quarkonium production

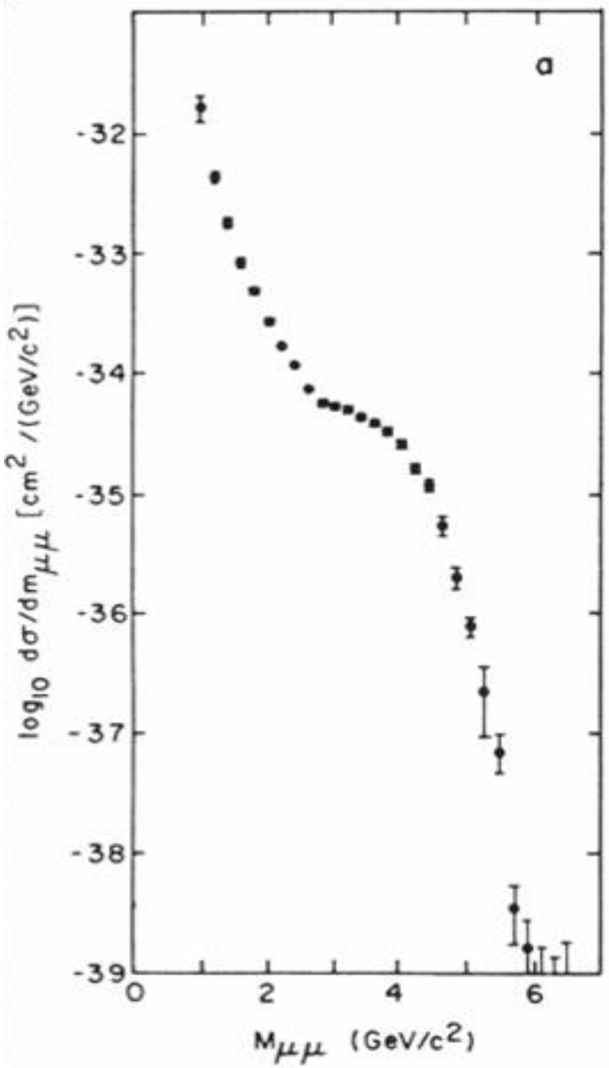
Lederman's Shoulder



Phys. Rev. Lett. 25, 1523 (1970)

Heavy quarkonium production

Lederman's Shoulder

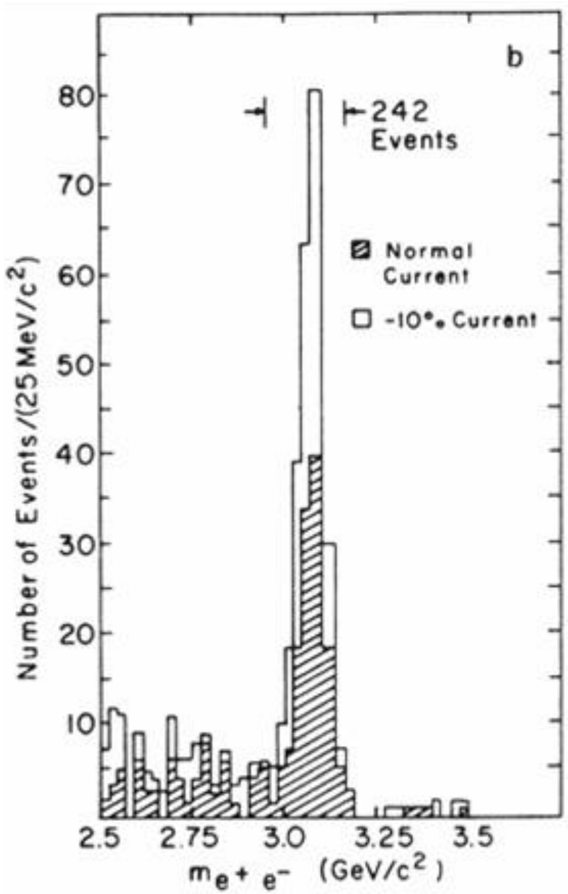


Phys. Rev. Lett. 25, 1523 (1970)

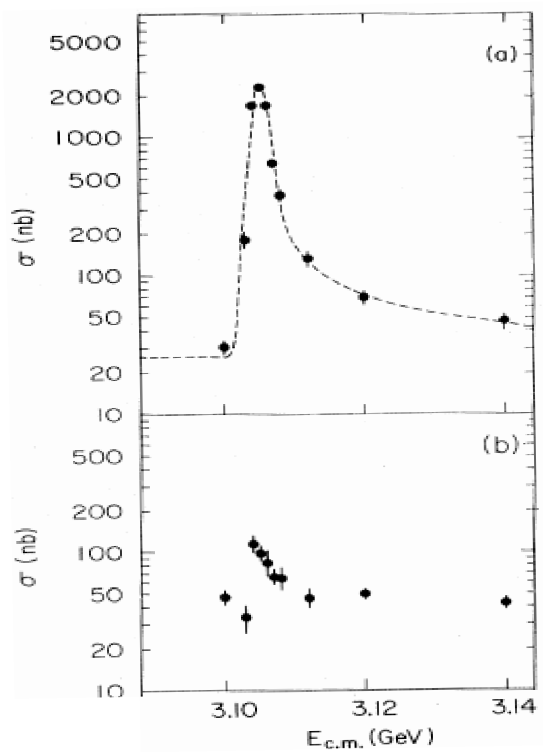
Production of muon pairs at AGS, BNL

$$p(29\,\text{GeV}) + U \Longrightarrow \mu^+\mu^-(M_{\mu\mu}) + X$$

Discovery of the J/ψ - November, 1974



(SLAC)



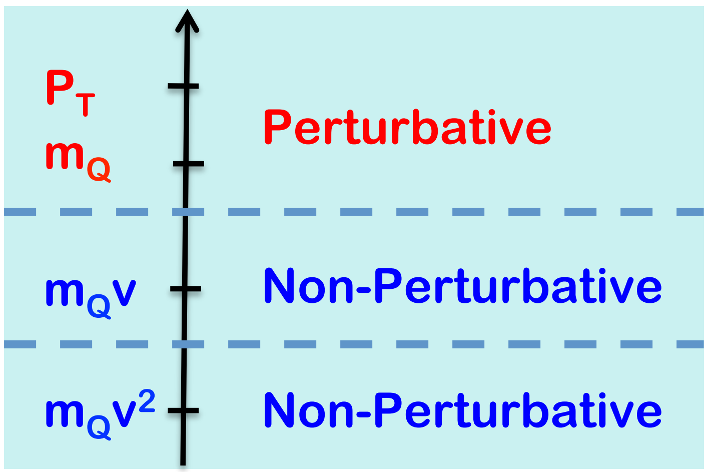
Heavy quarkonium production

- **One of the simplest QCD bound states:**

  Localized color charges (heavy mass), non-relativistic relative motion

  Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

- **Well-separated momentum scales – effective theory:**



Hard — Production of $Q\bar{Q}$ [pQCD]

Soft — Relative Momentum [NRQCD]

$\Lambda_{\rm QCD}$

Ultrasoft — Binding Energy [pNRQCD]

- **Cross sections and observed mass scales:**

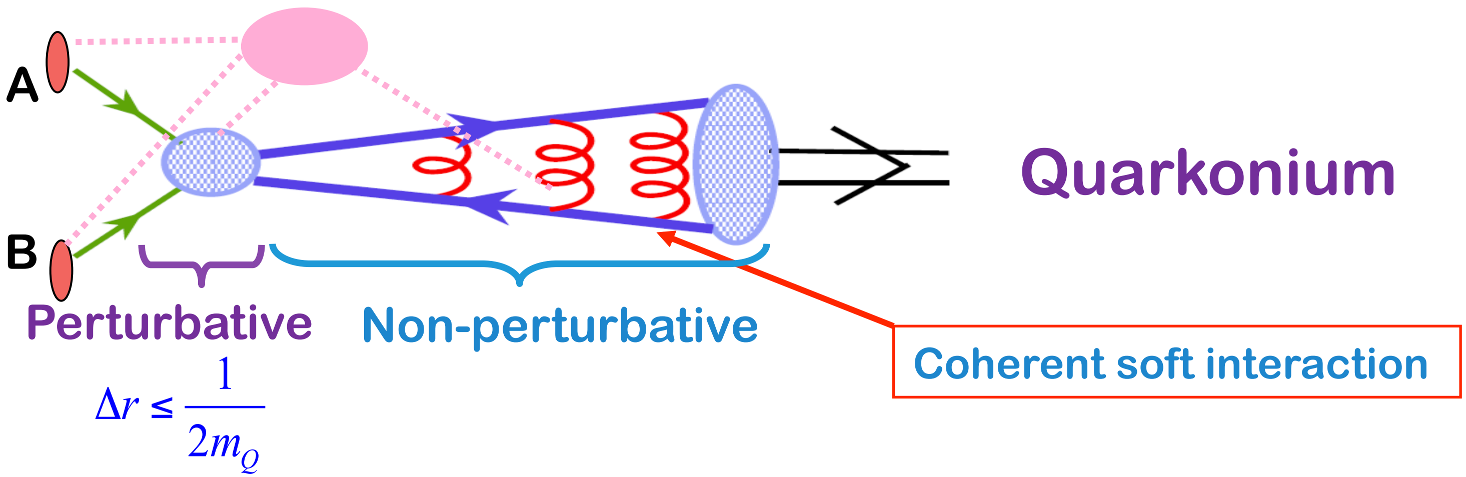
$$\frac{d\sigma_{AB\to H(P)X}}{dy\,dP_T^2} \qquad \sqrt{S}, \qquad P_T, \qquad M_H,$$

PQCD is "expected" to work for the production of heavy quarks

Difficulty: Emergence of a quarkonium from a heavy quark pair?

Basic production mechanism

- QCD factorization is likely to be valid for producing the pairs:
  - ✧ Momentum exchange is much larger than 1/fm
  - ✧ Spectators from colliding beams are "frozen" during the hard collision



- Approximation: on-shell pair + hadronization

$$\sigma_{AB\to J/\psi}(P_{J/\psi}) \approx \sum_n \int dq^2 \left[\sigma_{AB\to[Q\bar{Q}](n)}(q^2)\right] F_{[Q\bar{Q}(n)]\to J/\psi}(P_{J/\psi}, q^2)$$

Models & Debates

⇔ Different assumptions/treatments on $F_{[Q\bar{Q}(n)]\to J/\psi}(P_{J/\psi}, q^2)$ how the heavy quark pair becomes a quarkonium?

A long history for the production

- **Color singlet model: 1975 –**

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- **Color evaporation model: 1977 –**

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- **NRQCD model: 1986 –**

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- **QCD factorization approach: 2005 –**

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …
  Kang, Ma, Qiu, Sterman (2014)

  $P_T \gg M_H$: M_H/P_T power expansion + α_s – expansion

  Unknown, but universal, fragmentation functions – evolution

- **Soft-Collinear Effective Theory + NRQCD: 2012 –**

  Fleming, Leibovich, Mehen, …

NRQCD – most successful so far

Butenschoen and Kniehl, arXiv: 1105.0820

- NRQCD factorization:

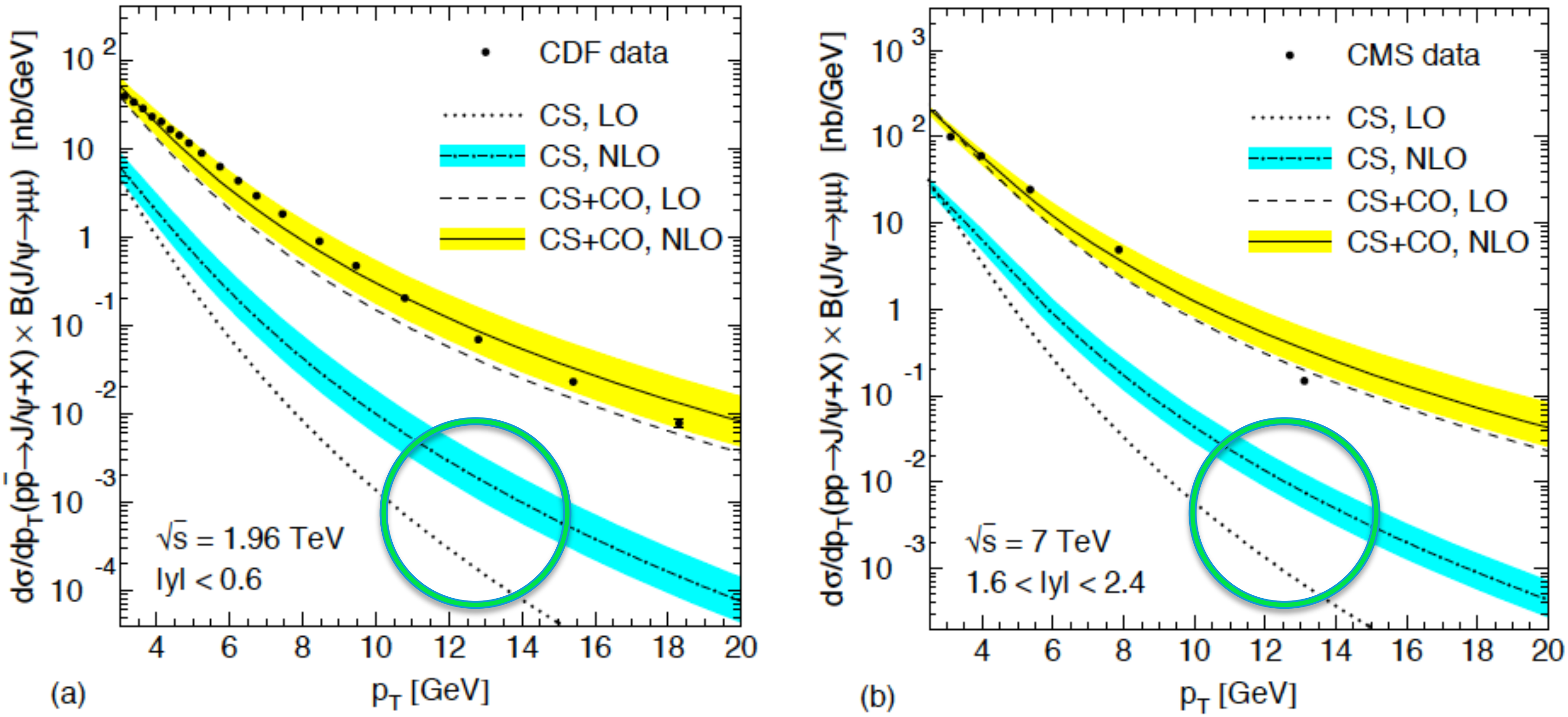
$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle$$

✧ 4 leading channels in v

$$^3S_1^{[1]},\ ^1S_0^{[8]},\ ^3S_1^{[8]},\ ^3P_J^{[8]}$$

✧ Full NLO in α_s

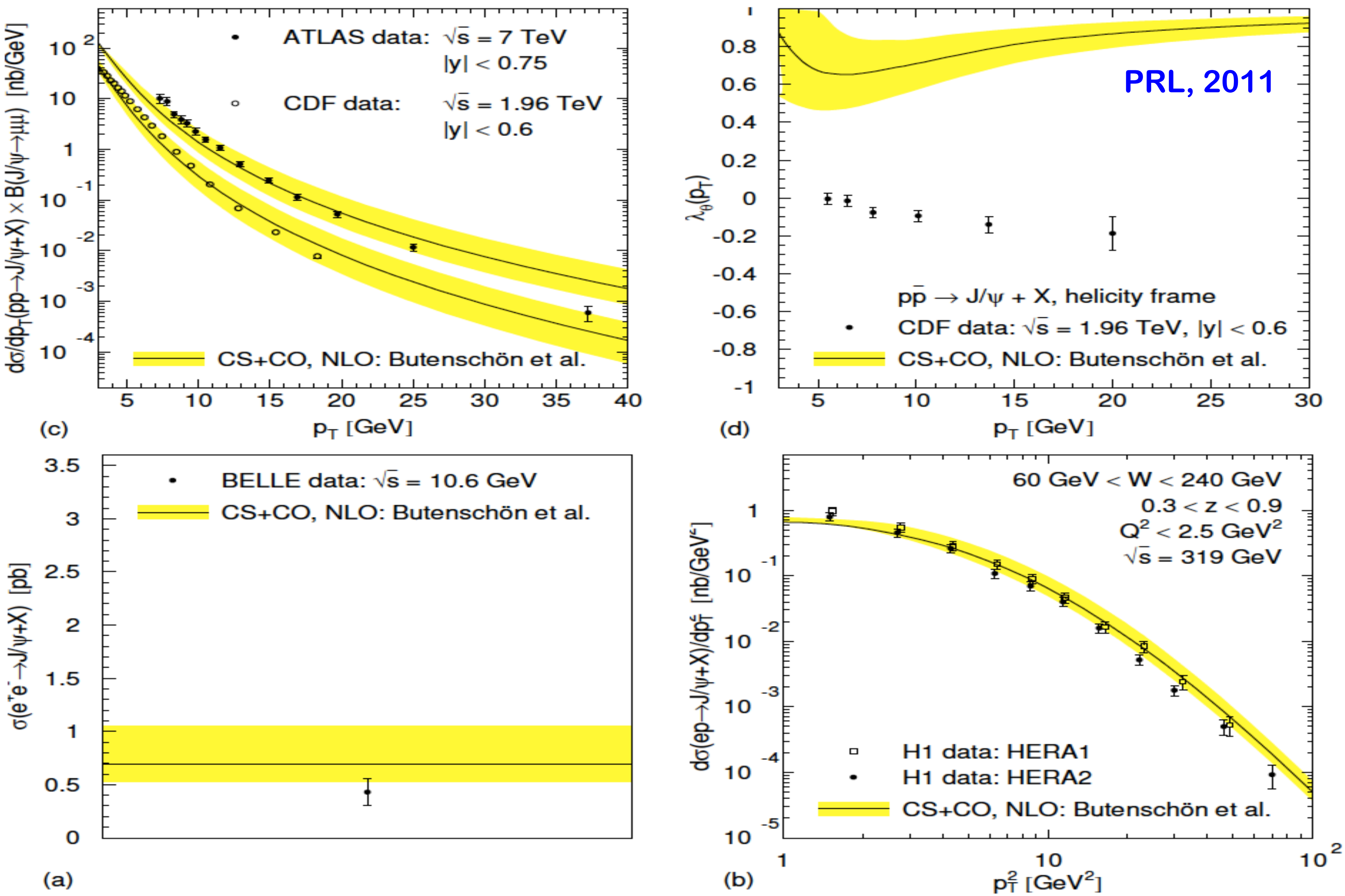
- Phenomenology:



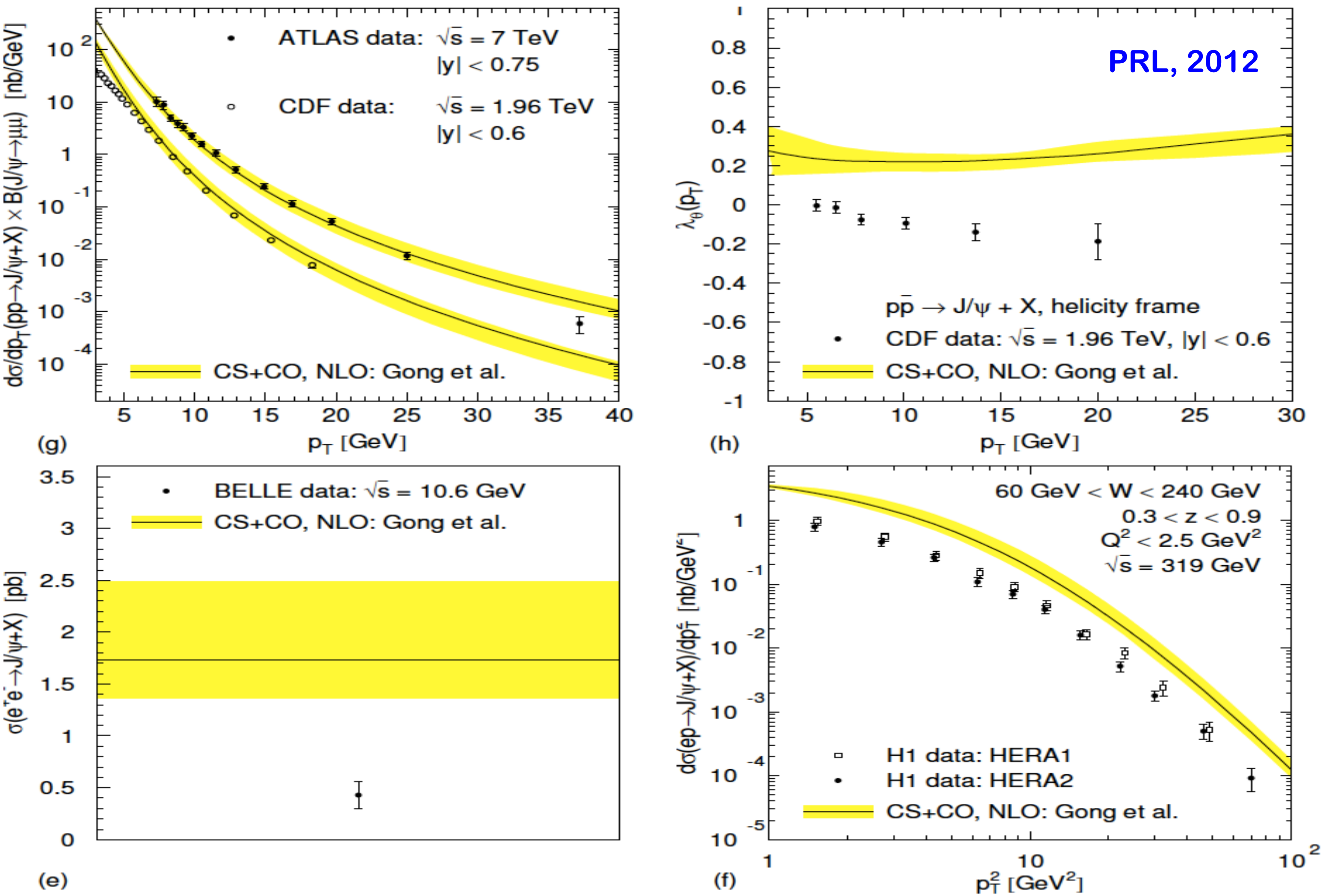
- Why is NLO so large? Polarization puzzle?

PRL 106, 022003 (2011)

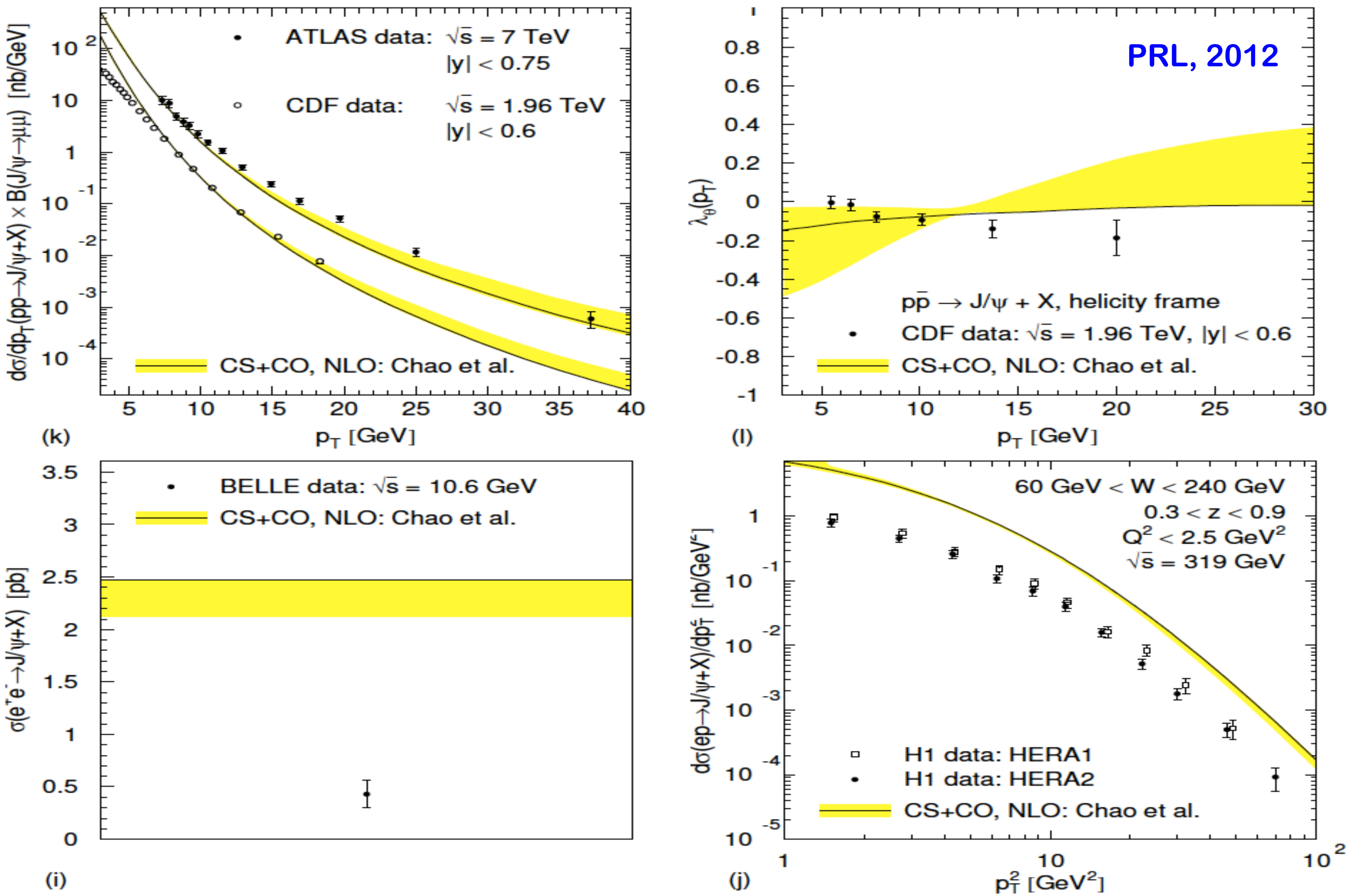
Production (NRQCD) – Butenschoen et al.



Production (NRQCD) – Gong et al.



Production (NRQCD) – Chao et al.

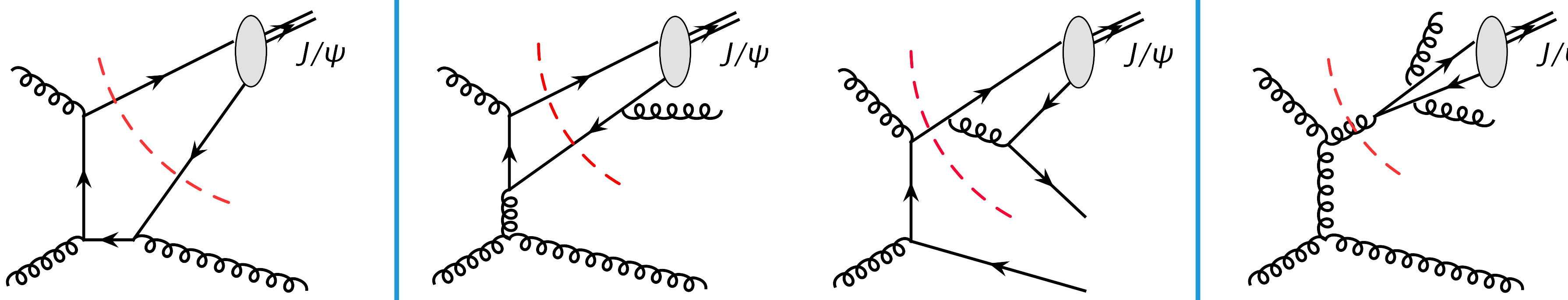


Why high orders in NRQCD are so large?

Kang, Qiu and Sterman, 2011

See also talk by H. Zhang

❑ Consider J/ψ production in CSM:



LO in α_s

NNLP $\propto \alpha_s^3 \frac{m_Q^4}{p_T^8}$

NLO in α_s

NLP in 1/P$_T$ $\propto \alpha_s^4 \frac{m_Q^2}{p_T^6}$

NNLO in α_s

LP: $\propto \alpha_s^5 \frac{1}{p_T^4}$

- ✧ High-order correction receive power enhancement
- ✧ Expect no further power enhancement beyond NNLO
- ✧ $[\alpha_s \ln(p_T^2/m_Q^2)]^n$ ruins the perturbation series at sufficiently large p$_T$

Leading order in α_s-expansion =\= leading power in 1/p$_T$-expansion!

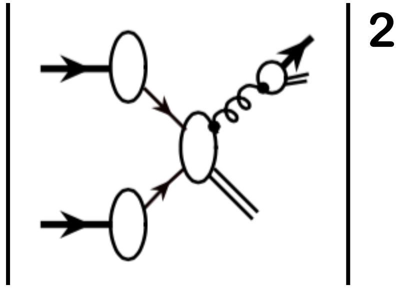
At high p$_T$, fragmentation contribution dominant

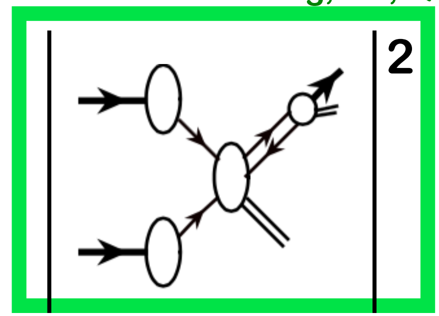
QCD factorization – Kang et al.

Kang, Ma, Qiu and Sterman, 2014

- **Power Expansion:**

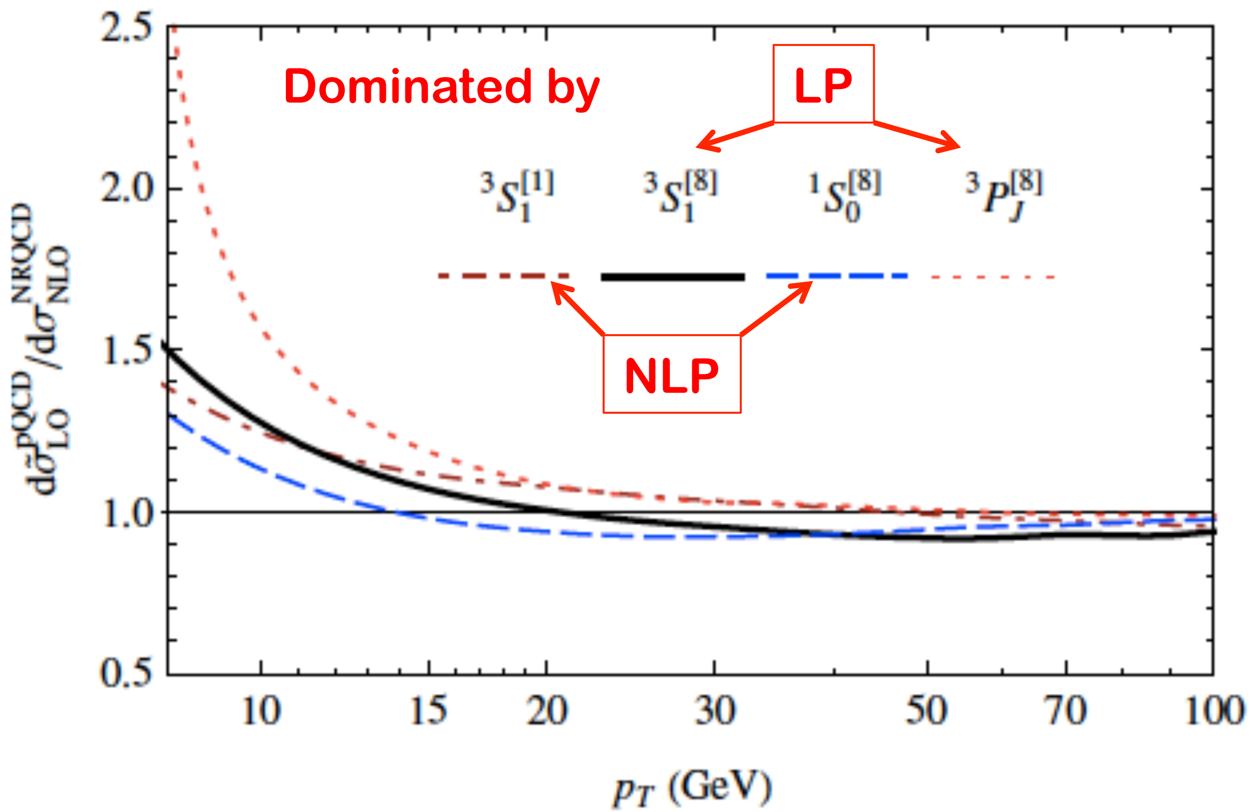
$$\frac{d\sigma_{AB\to H+X}}{dy\,dp_T^2} = \left| \quad \right|^2 + \left| \quad \right|^2 + \ldots$$





NLP

- **Channel-by-channel comparison with NLO NRQCD:**



independent of NRQCD matrix elements

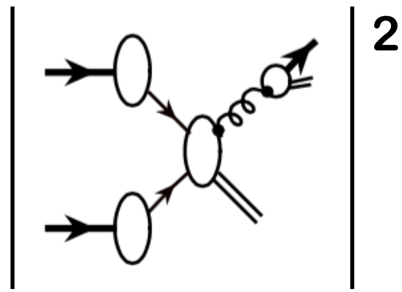
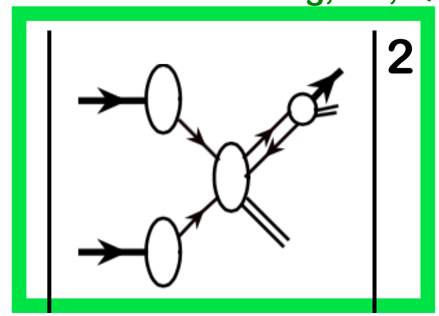
LO QCD analytical results reproduce NLO NRQCD calculations (numerical)

PRL, 2014

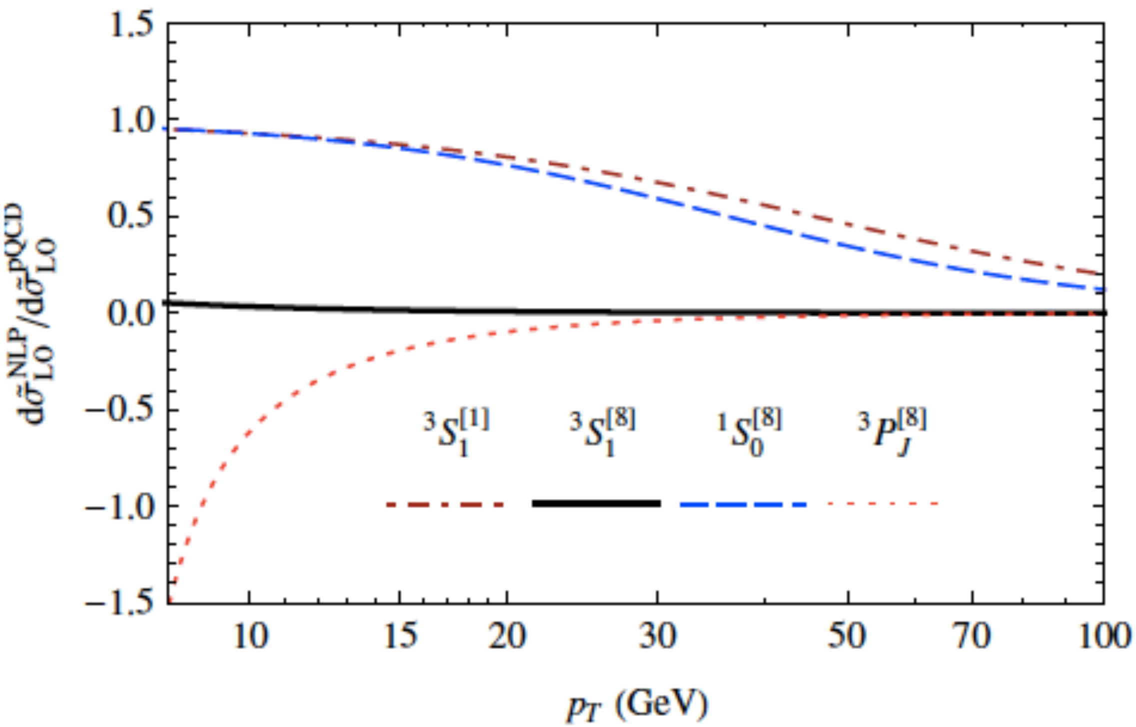
QCD factorization – Kang et al.

Kang, Ma, Qiu and Sterman, 2014

- Power Expansion:

$$\frac{d\sigma_{AB\to H+X}}{dy\,dp_T^2} =$$  $+$  $+\ldots$

- Channel-by-channel, LP vs. NLP (both LO):



LP dominated

$^3S_1^{[8]}$ and $^3P_J^{[8]}$

NLP dominated

$^1S_0^{[8]}$

for wide P_T

P_T distribution is consistent with distribution of $^1S_0^{[8]}$

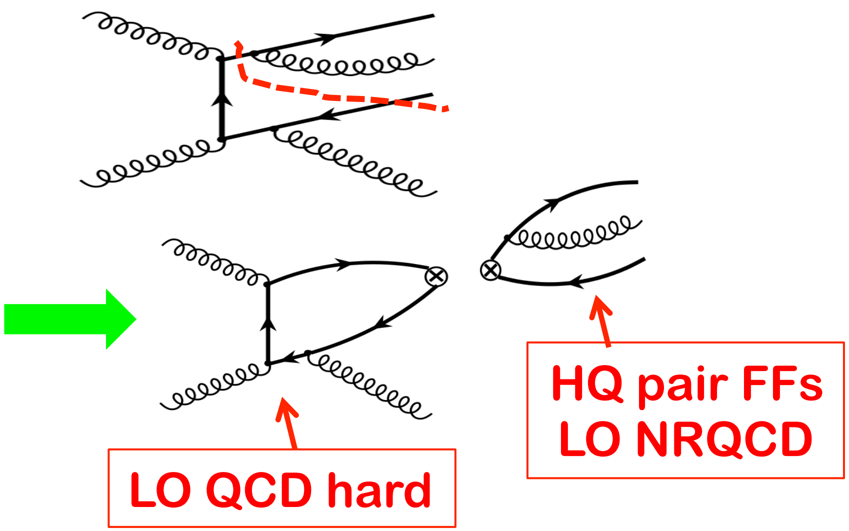
QCD Factorization = better controlled HO corrections!

PRL, 2014

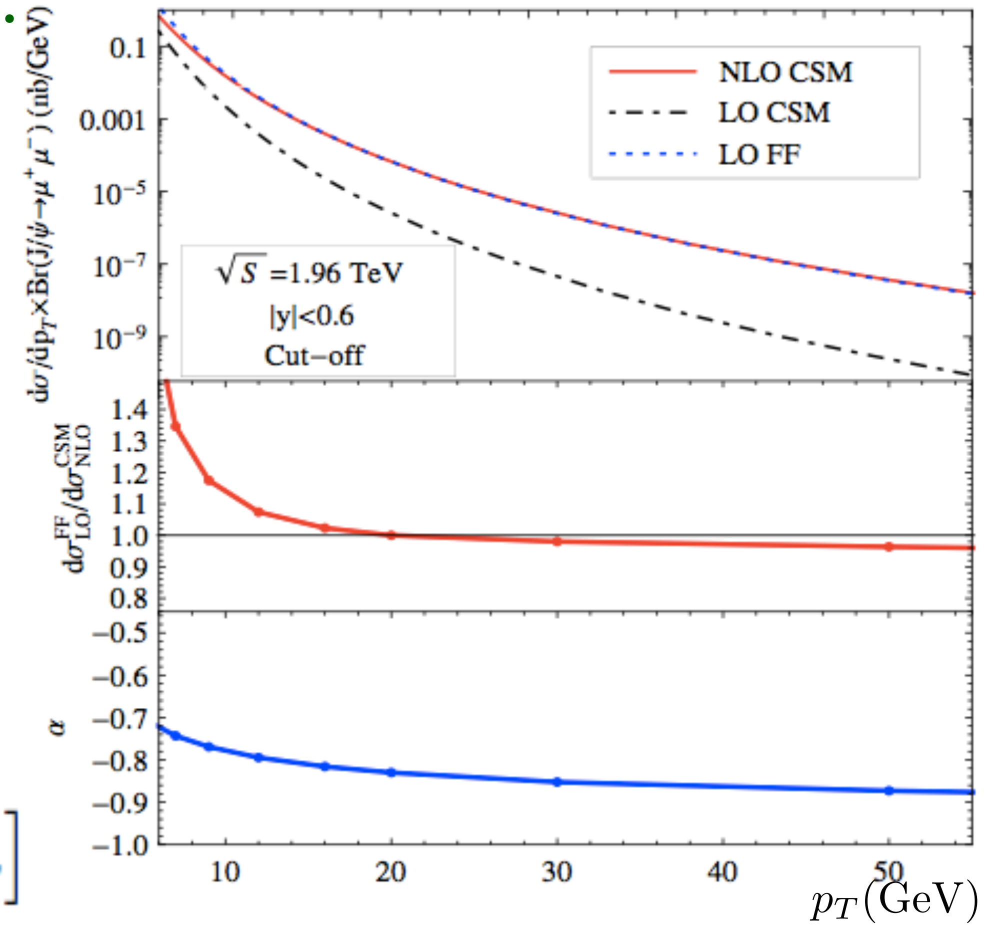
LO QCD factorization vs NLO NRQCD

Kang, Ma, Qiu and Sterman, 2014

❑ Color singlet as an example:



$$\sigma^{(\mathrm{NLO})}_{\mathrm{NRQCD}} \propto \left[d\hat{\sigma}^{A(\mathrm{LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(\mathrm{LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \right]$$



LO pQCD: reproduces NLO CSM rate for p_T > 10 GeV!

NLO pQCD can be done, while NNLO NRQCD is impossible!

QCD Factorization = better controlled HO corrections!

Matching from high p_T to low p_T

- **Matching if both factorizable:**

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0) + E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

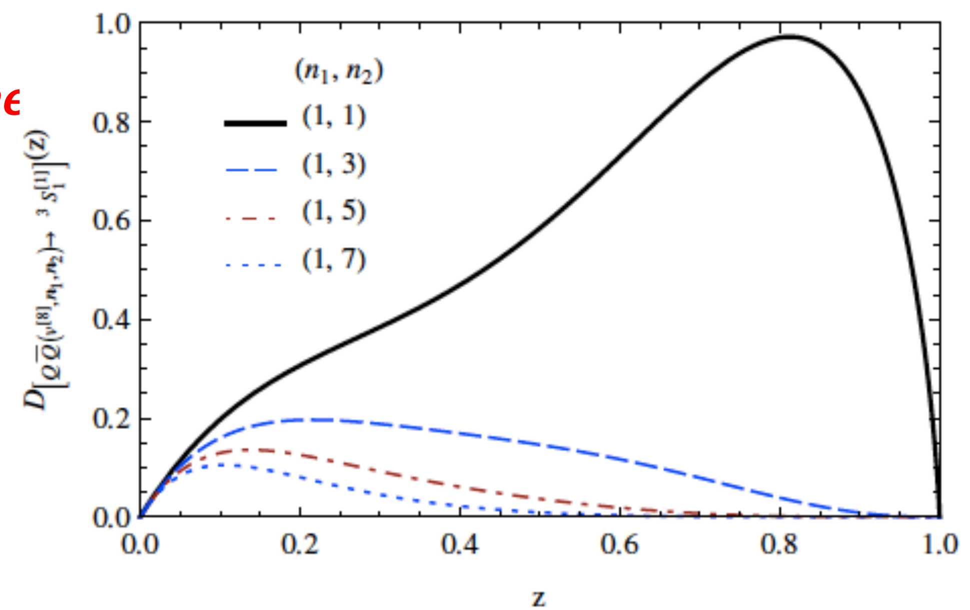
Mass effect + P_T region ($P_T \gtrsim m_Q$)

- **Fragmentation functions – nonperturbative!**

Responsible for "polarization", relative size of production channe

- **Model of FFs:**
  - ✧ **NRQCD factorization of FFs**
  - ✧ **Express all FFs in terms of *a few* NRQCD LDMEs**

$$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z, \zeta_1, \zeta_2)$$

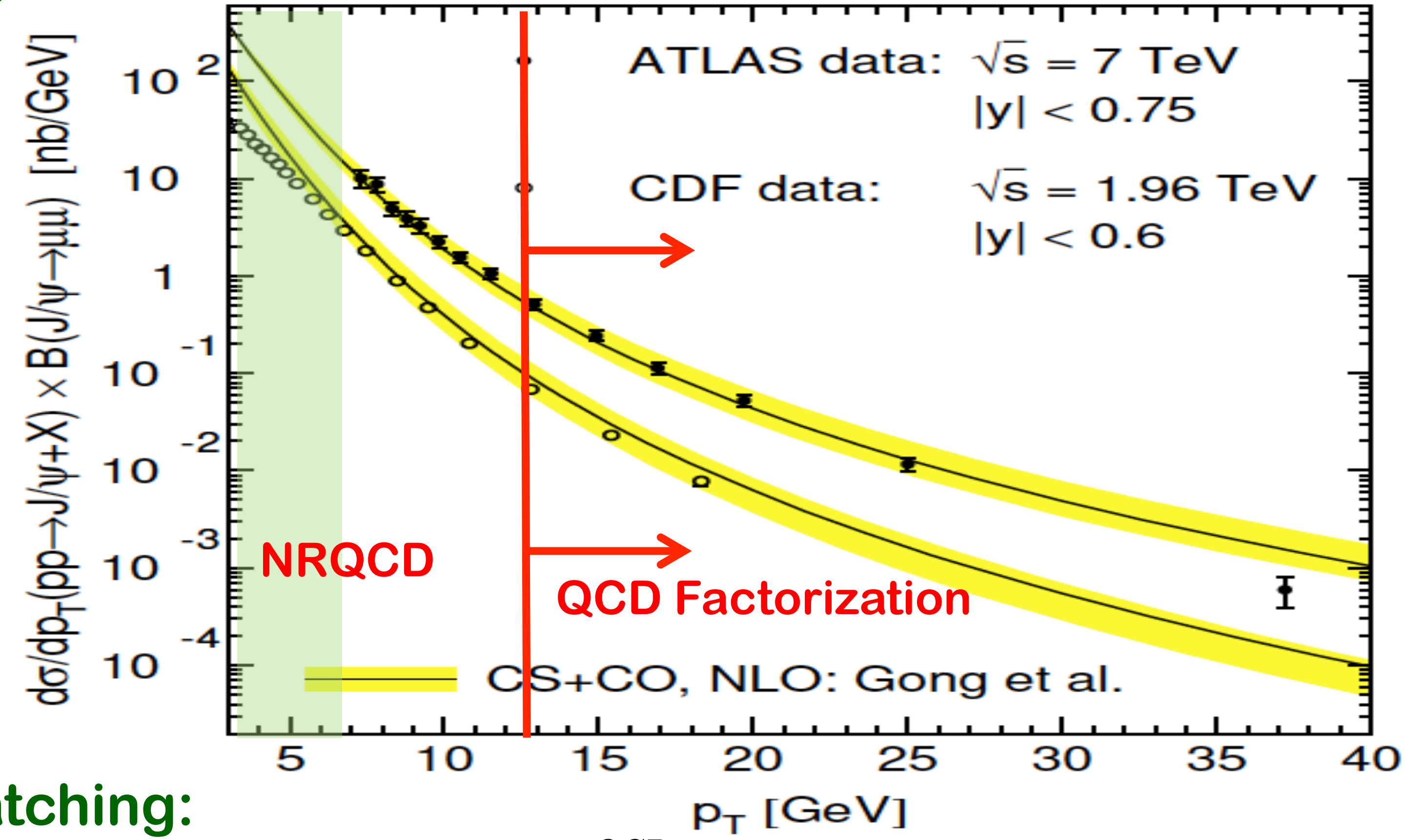


QCD factorization approach is ready to compare with Data

Matching between QCD and NRQCD

- Expectation:

Kang, Ma, Qiu and Sterman, 2014



- Matching:

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

$$+E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

Mass effect + expanded P_T region ($P_T \gtrsim m_Q$)

Summary of lecture three

- Many new techniques have been developed in recent years for NNLO or higher order calculations – not discussed here

- QCD resummation techniques have been well-developed, and have played a key role in improving the precision of theoretical predictions

- Heavy quarkonium production is still a very fascinating subject challenging our understanding of QCD bound states

- Theory had a lot advances in last decade in dealing with observables with multiple observed momentum scales:

  Provide new probes to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion

- Proton spin provides another controllable "knob" to help isolate various physical effects

Backup slides