# Quantum Chromodynamics
# (QCD)

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**Stony Brook University**

**Weihai High Energy Physics School (WHEPS)**

**Shandong University – Weihai, Weihai, Shandong, China, August 1-11, 2015**

# The plan for my four lectures

- ☐ The Goal:

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- ☐ The Plan (approximately):

  Fundamentals of QCD, factorization, evolution,

  and elementary hard processes

  Two lectures

  Role of QCD in high energy collider phenomenology

  One lecture

  QCD and hadron structure and properties

  One lecture

# Summary of lecture three

- ☐ **Many new techniques have been developed in recent years for NNLO or higher order calculations – not discussed here**

- ☐ **QCD resummation techniques have been well-developed, and have played a key role in improving the precision of theoretical predictions**

- ☐ **Heavy quarkonium production is still a very fascinating subject challenging our understanding of QCD bound states**

- ☐ **Theory had a lot advances in last decade in dealing with observables with multiple observed momentum scales:**

  ***Provide new probes to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion***

- ☐ **Proton spin provides another controllable "knob" to help isolate various physical effects**

# Nucleon is not elementary!

## 1933: Proton's magnetic moment

$g \neq 2$

**Otto Stern**

**Nobel Prize 1943**

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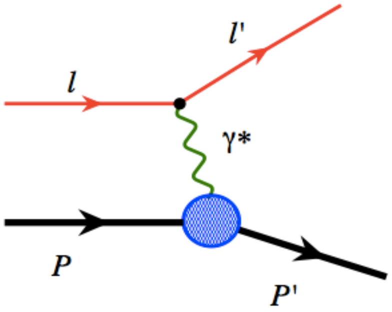

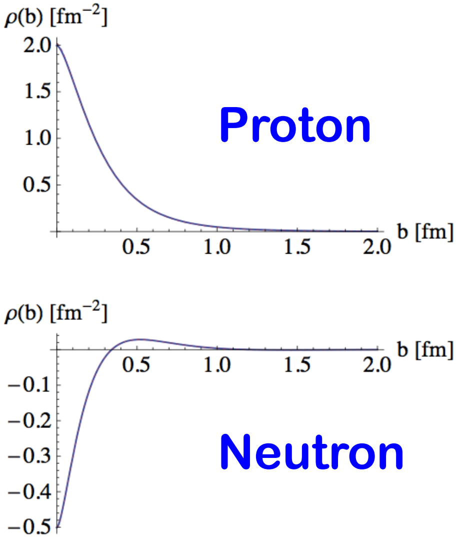

***Form factors***

→ ***Electric charge distribution***

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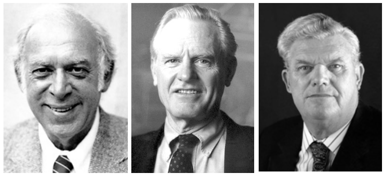

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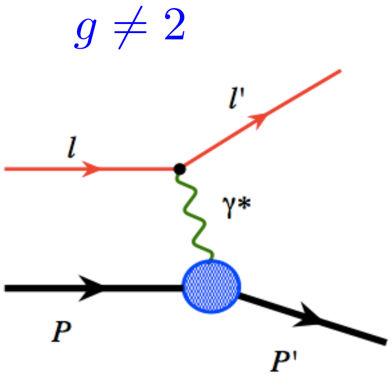
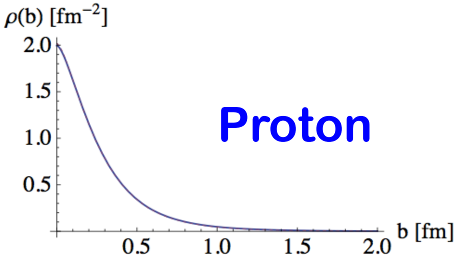
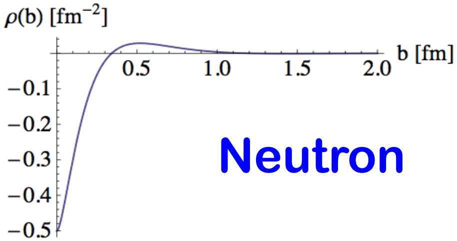


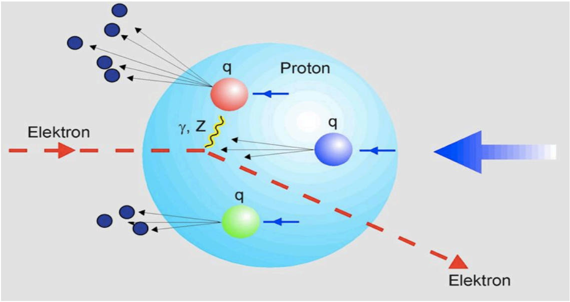




*Form factors*

→ *Electric charge distribution*

*Modern “Rutherford’s experiment”*



*Point-like partons*

→ *Discovery of QCD*

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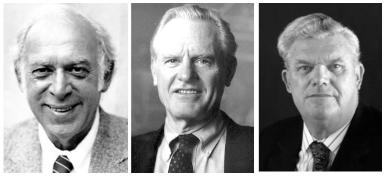

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## 1974: QCD Asymptotic Freedom

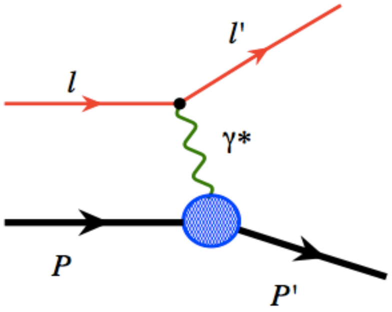
**David J. Gross**

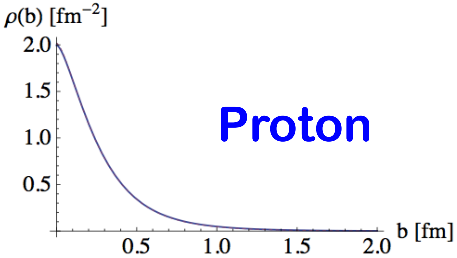
**H. David Politzer**

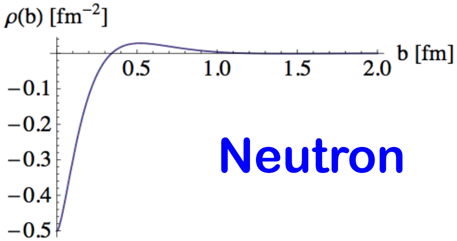
**Frank Wilczek**

**Nobel Prize 2004**

$g \neq 2$



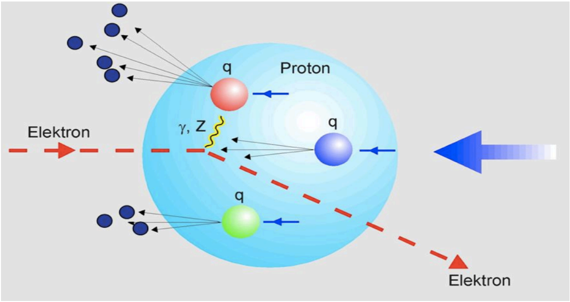




*Form factors*

→ *Electric charge distribution*

*Modern "Rutherford's experiment"*



*Point-like partons*

→ *Discovery of QCD*

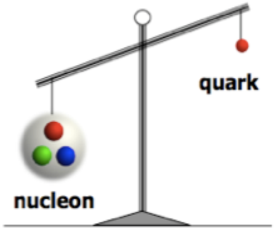
*Scaling violation*

→ *Perturbative QCD – theory tool*

*Factorization - PDFs*

# Hadron properties

❑ **How does QCD generate energy for the proton's mass?**

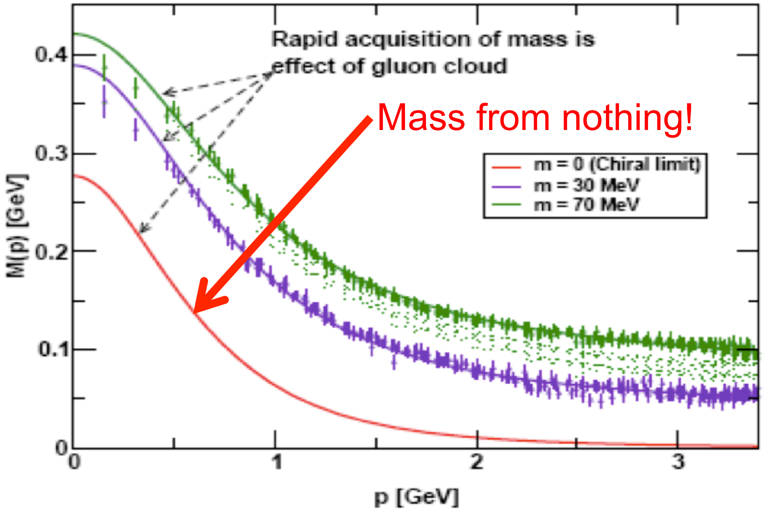


$m_q \sim 10$ MeV

$m_N \sim 1000$ MeV

**Quark mass $\sim 1\%$ proton's mass**

***Higgs mechanism is not enough!!!***



Mass from nothing!

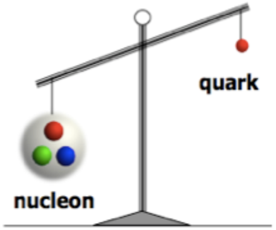
*C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50*
*M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227*

❑ **Generation of mass:**
***from QCD dynamics?***

- ✧ **BSE calculation results confirmed by lattice simulation**
- ✧ **Light-quark mass comes from a cloud of soft gluons**

# Hadron properties

## ❑ How does QCD generate energy for the proton's mass?



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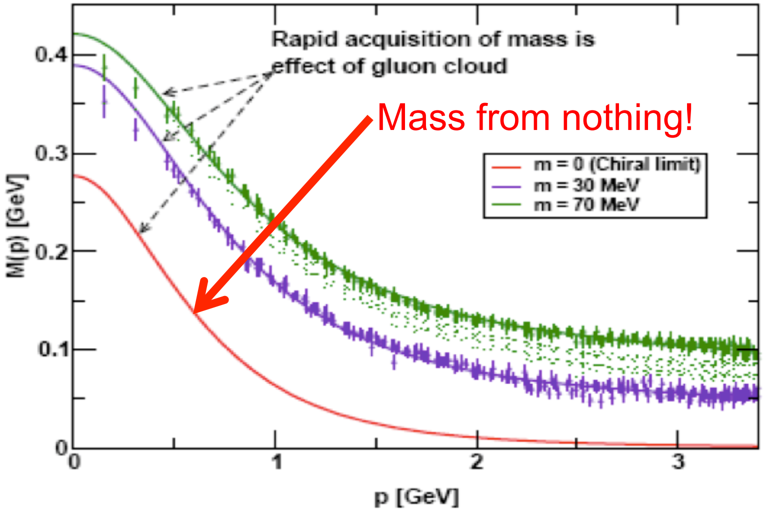
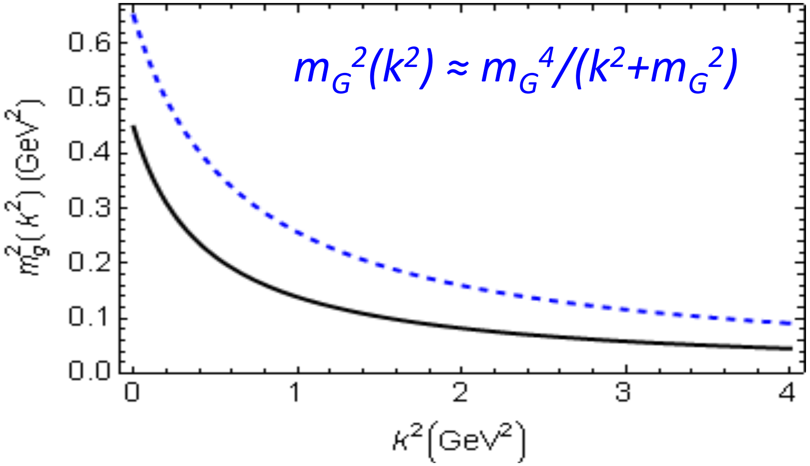


*C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50*
*M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227*

## ❑ Generation of mass: *from QCD dynamics?*

- ✧ **BSE calculation results confirmed by lattice simulation**
- ✧ **Light-quark mass comes from a cloud of soft gluons**
- ✧ **Gluon is massless in UV, but "massive" in IR**



*Qin et al., Phys. Rev. C 84 042202 (Rapid Comm.)*

# Hadron mass sum rule

Ji, 1994

❑ **QCD definition:**

$$M = \frac{\langle P| \int d^3x\, T^{00}(0,\mathbf{x}) |P\rangle}{\langle P|P\rangle} \equiv \langle T^{00} \rangle$$

QCD energy-momentum tensor:

$$T^{\mu\nu} = \frac{1}{2}\, \overline{\psi} i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} \psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha} \Longrightarrow H_{\rm QCD} = \int d^3x\, T^{00}(0,\mathbf{x})$$

❑ **Decomposition:**

$$H_{\rm QCD} = H_q + H_m + H_g + H_a$$

| Mass type | $H_i$ | $M_i$ | $m_s \to 0$ (MeV) | $m_s \to \infty$ (MeV) |
|---|---|---|---|---|
| Quark energy | $\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi$ | $3(a-b)/4$ | 270 | 300 |
| Quark mass | $\overline{\psi} m \psi$ | $b$ | 160 | 110 |
| Gluon energy | $\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$ | $3(1-a)/4$ | 320 | 320 |
| Trace anomaly | $\frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2)$ | $(1-b)/4$ | 190 | 210 |

$$a(\mu^2) = \sum_f \int_0^1 x[q_f(x,\mu^2) + \overline{q}_f(x,\mu^2)]dx$$

$$bM = \langle P| m_u \overline{u} u + m_d \overline{d} d |P\rangle + \langle P| m_s \overline{s} s |P\rangle$$

✧ None of these terms is a "direct" physical measurable (e.g. cross section)!

*Can we "measure" them with controllable approximation?*

*Can we "measure" them by lattice calculation, or other approaches?*

# Lattice QCD

❑ Formulated in the discretized Euclidean space:

$$S^f = a^4 \sum_x \left[ \frac{1}{2a} \sum_\mu [\bar{\psi}(x)\gamma_\mu U_\mu(x)\psi(x+a\hat{\mu}) - \bar{\psi}(x+a\hat{\mu})\gamma_\mu U_\mu^\dagger(x)\psi(x)] + m\bar{\psi}(x)\psi(x) \right]$$

$$S^g = \frac{1}{g_0^2} a^4 \sum_{x,\mu\nu} \left[ N_c - \mathrm{ReTr}[U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)] \right]$$

$$U_\mu(x) = e^{-igaT^a A_\mu^a(x+\frac{1}{2})}$$

❑ Boundary condition is imposed on each field in finite volume:

Momentum space is restricted in finite Brillouin zone: $\{-\frac{\pi}{a}, \frac{\pi}{a}\}$

Lattice QCD is an Ultra-Violet (UV) finite theory

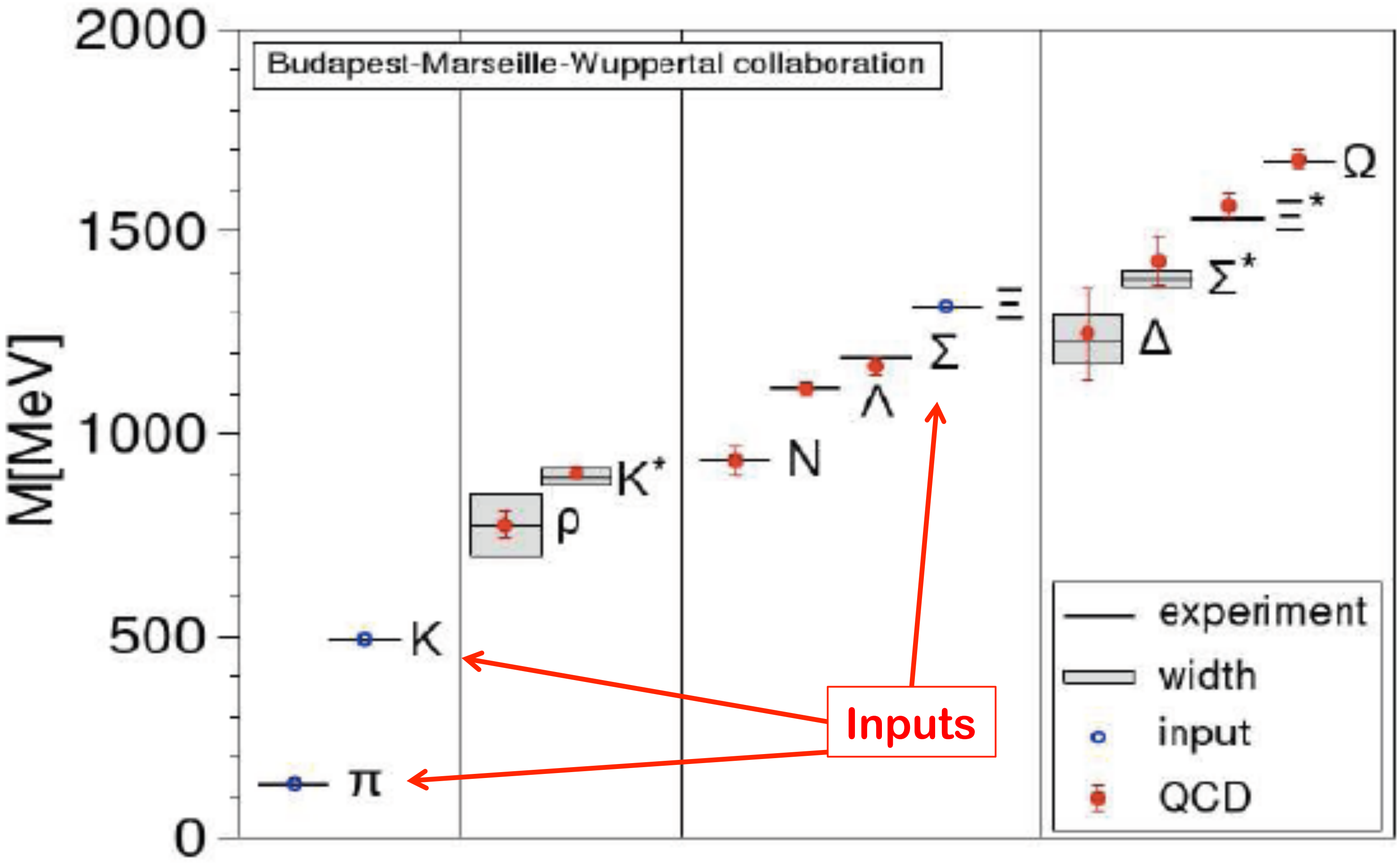
Lattice action is not unique, above action is the simplest one!

Many implementations were proposed to reduce the discretization error

# Hadron properties from Lattice QCD

- ❑ **Low-lying hadron mass spectrum:**

S. Durr et al. Science 322, 1124 2008

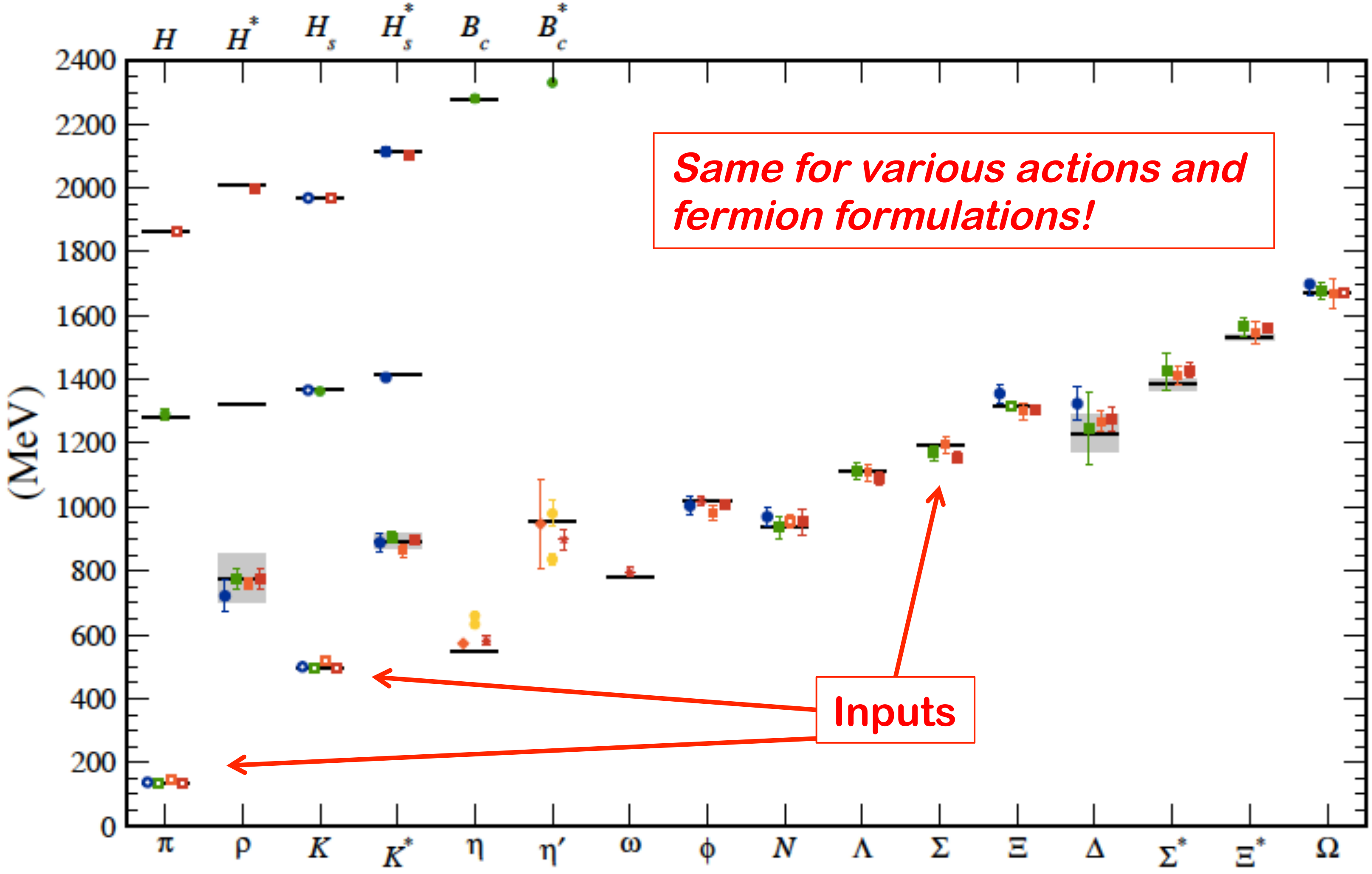


Predictions with limited inputs

# Hadron properties from Lattice QCD

❑ **Low-lying hadron mass spectrum:**

**A. Kronfeld, 1209.3468**



*Same for various actions and fermion formulations!*

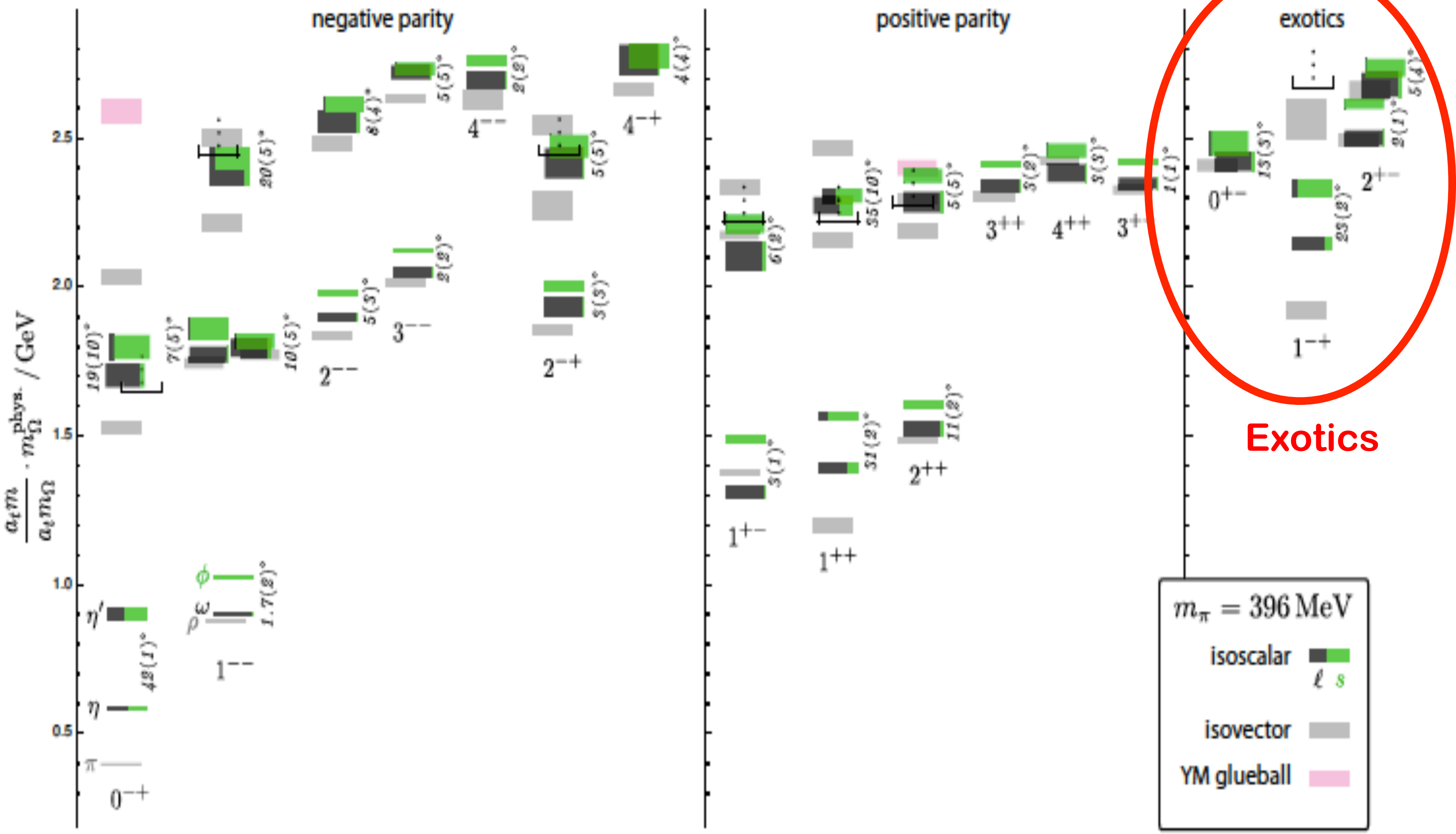
Inputs

Predictions with limited inputs

# Hadron properties from Lattice QCD

## ❑ Meson resonances:

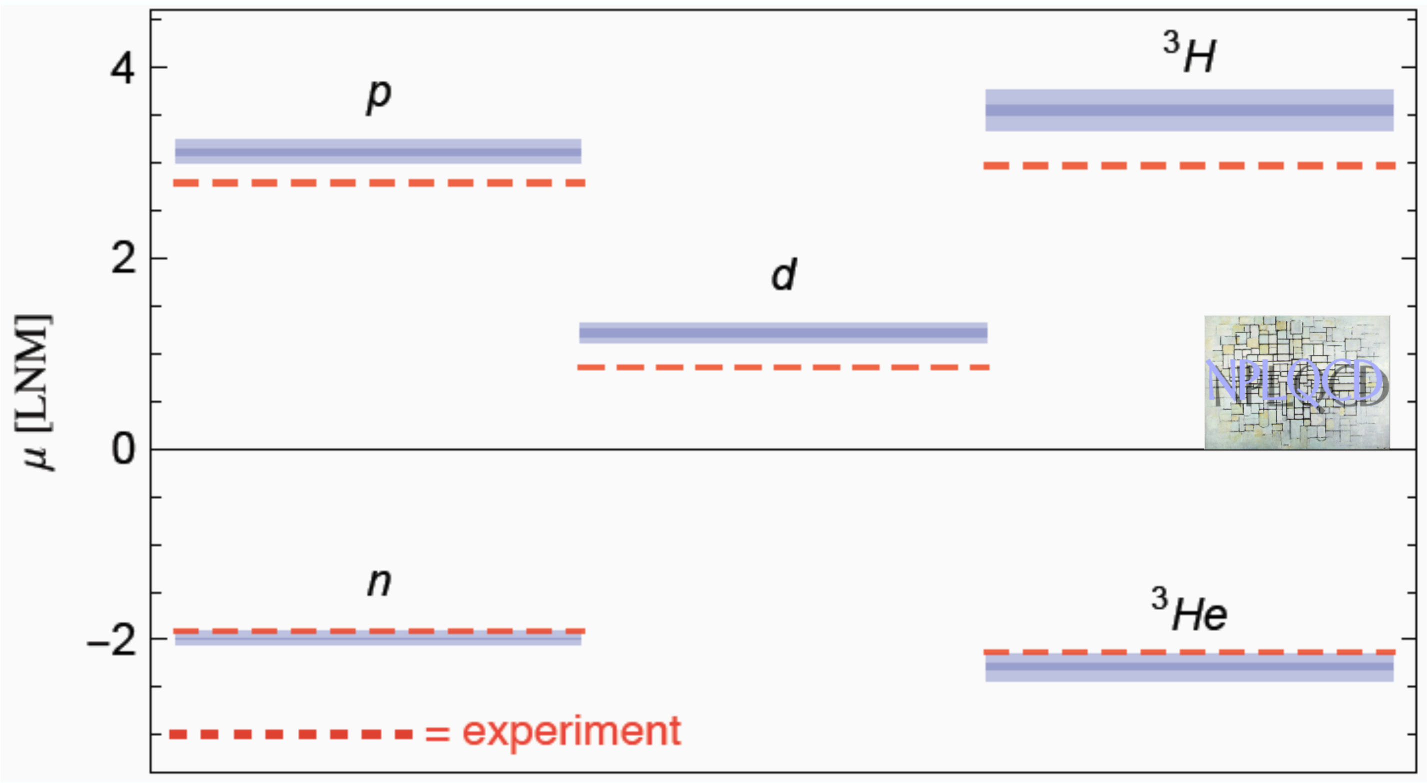
Dudek et al, Phys.Rev. D88 (2013) 094505

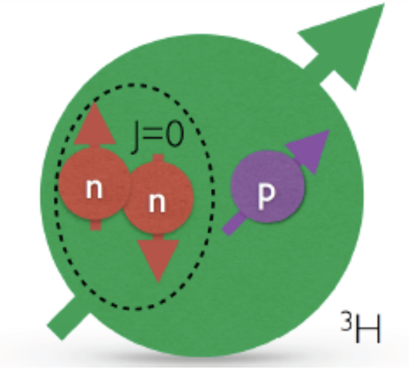


# Hadron properties from Lattice QCD

- ☐ **Magnetic moments:**

S.R. Beane et al., Phys.Rev.Lett. 113 (2014) 252001





Theory at $m_\pi$ = 806 MeV vs. the nature!

*Nuclei are (nearly) collections of nucleons – shell model phenomenology!*

# Proton spin

- ❑ **Proton is NOT elementary, but, a composite particle:**
  - ✧ Proton-spin = Proton’s angular momentum when it is at rest
  - ✧ Proton-spin = One number touches every part of the quantum world

    from the quantum mechanics to the quantum field theory and QCD

$$S(\mu) = \frac{1}{2}$$

$$\mu \Rightarrow \infty$$

  - ✧ Proton-spin = One number carries every secrets of QCD dynamics

    from the “unknown” confinement to the “well-known” asymptotic freedom

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**from the "unknown" confinement to the "well-known" asymptotic freedom**

## ❑ Quark Model:

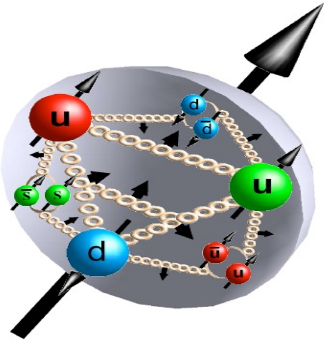
✧ **Expectation:** $S_p \equiv \langle p\uparrow | S | p\uparrow \rangle = \frac{1}{2}, \qquad S = \sum_i S_i$

✧ **Wave function:** $|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$

**Skyrmion Model, MIT Bag Model, Chiral Bag Model, …**

# Proton spin in QCD

## ☐ Complexity of the proton in QCD:



**Known from QCD**

$$S(\mu) = \sum_f \langle P,S|\hat{J}_f^z(\mu)|P,S\rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$

**From QCD, But, unknown**

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5\psi_q + \psi_q^\dagger(\vec{x}\times(-i\vec{D}))\,\psi_q\right] \qquad \vec{J}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

## ☐ Asymptotic limit:

Ji, 2005

$$J_q(\mu\to\infty) \Rightarrow \frac{1}{2}\frac{3N_f}{16+3N_f} \sim \frac{1}{4} \qquad J_g(\mu\to\infty) \Rightarrow \frac{1}{2}\frac{16}{16+3N_f} \sim \frac{1}{4}$$

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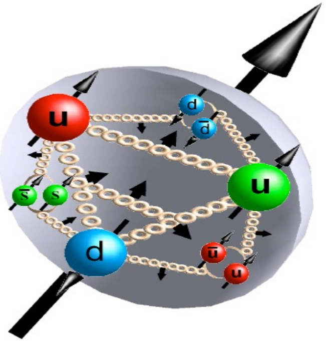

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## ❑ Spin sum rule – not unique!

$$S(\mu) = \frac{1}{2}\Sigma(\mu) + L_q(\mu) + \Delta G(\mu) + [\overbrace{J_g(\mu) - \Delta G(\mu)}^{L_g(Q^2)}]$$

Intrinsic parton's spin: $\Sigma(Q^2) = \sum_q \left[\Delta q(Q^2) + \Delta\bar{q}(Q^2)\right], \quad \Delta G(Q^2)$

dynamical parton motion: $L_q(Q^2), \quad L_g(Q^2)$

- Matrix elements of quark and gluon fields are NOT physical observables!
- Infinite possibilities of decompositions – connection to observables?

# Parton helicity distributions

## Quark helicity distribution:

$$\Delta q(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{1}{2} \left[ \langle p, s_\parallel | \overline{\psi}_q(0) \gamma^+ \frac{1+\gamma^5}{2} \psi_q(y^-) | p, s_\parallel \rangle \right.$$
$$\left. - \langle p, -s_\parallel | \overline{\psi}_q(0) \gamma^+ \frac{1-\gamma^5}{2} \psi_q(y^-) | p, -s_\parallel \rangle \right]$$

$$\xrightarrow{\text{P + T}} \quad \Delta q(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p, s_\parallel | \left[ \overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \psi_q(y^-) \right] | p, s_\parallel \rangle$$

- **Fourier Transform of light-cone matrix element:** $\langle p, s_\parallel | \mathcal{O}_q(y^-) | p, s_\parallel \rangle$

$$\mathcal{O}_q(y^-) = \overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \psi(y^-)$$

**The $\gamma^5$ flips the quark helicity at the cut-vertex**

- **Necessary condition for nonvanish asymmetries – P + T:**

$$\langle p, s_\parallel | \mathcal{O}_q(y^-) | p, s_\parallel \rangle \Longleftrightarrow -\langle p, -s_\parallel | \mathcal{O}_q(y^-) | p, -s_\parallel \rangle$$

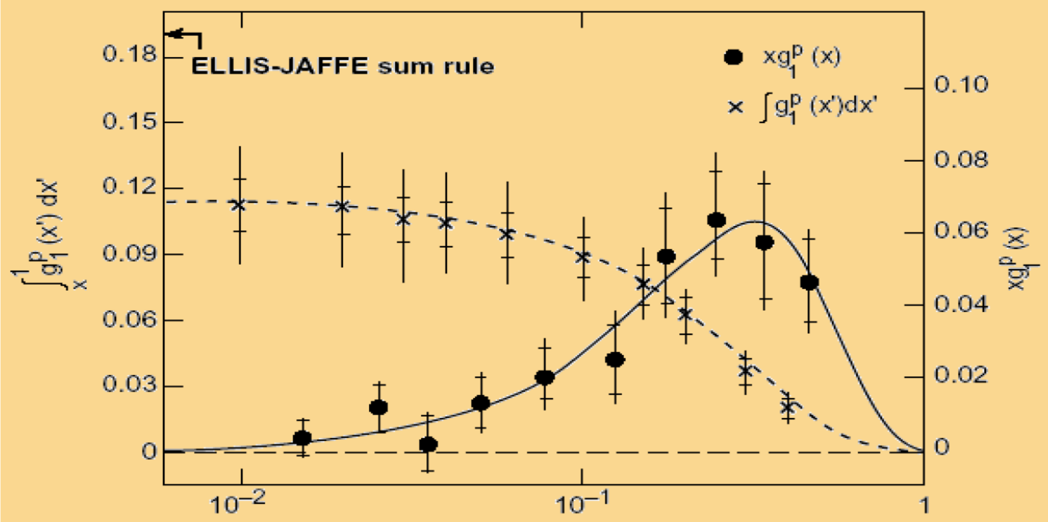
## Gluon helicity distribution:

$$\mathcal{O}_g(y^-) = \frac{1}{xp^+} F^{+\alpha}(0) \left[ -i\, \varepsilon_{\alpha\beta} \right] F^{+\beta}(y^-)$$

**The $i\varepsilon_{\alpha\beta}$ flips gluon helicity at the cut-vertex**

# Proton "spin crisis" – excited the field

## ❑ EMC (European Muon Collaboration '87) – "the Plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}\left(1/Q\right)$$

- ✧ **Combined with earlier SLAC data:** $\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$
- ✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$ **from low energy neutron & hyperon $\beta$ decay**

➡ $\Delta\Sigma = \sum_q\left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$

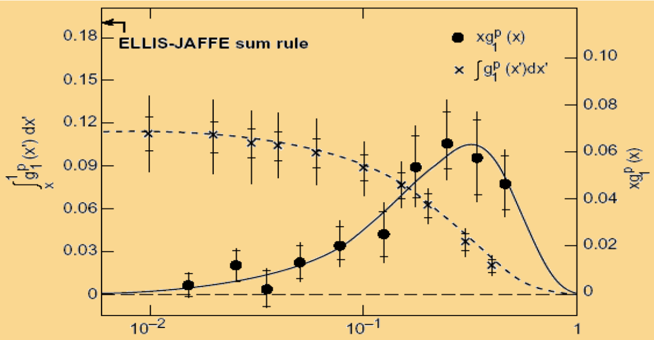
## ❑ "Spin crisis" or puzzle:

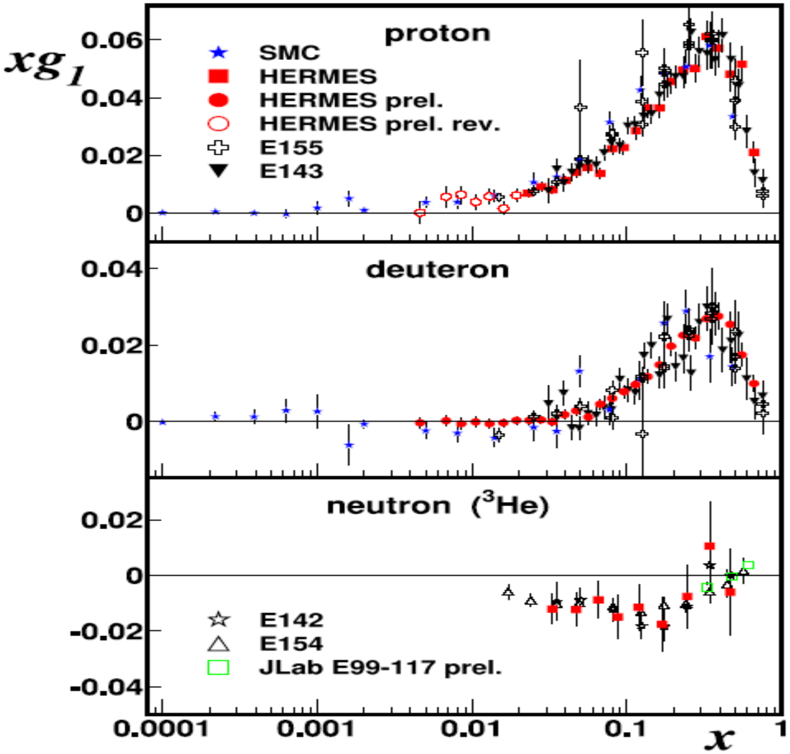
- ✧ **Strange sea polarization is sizable & negative**
- ✧ **Very little of the proton spin is carried by quarks**

➡ **New era of spin physics**

# Inclusive DIS data – over 20 years

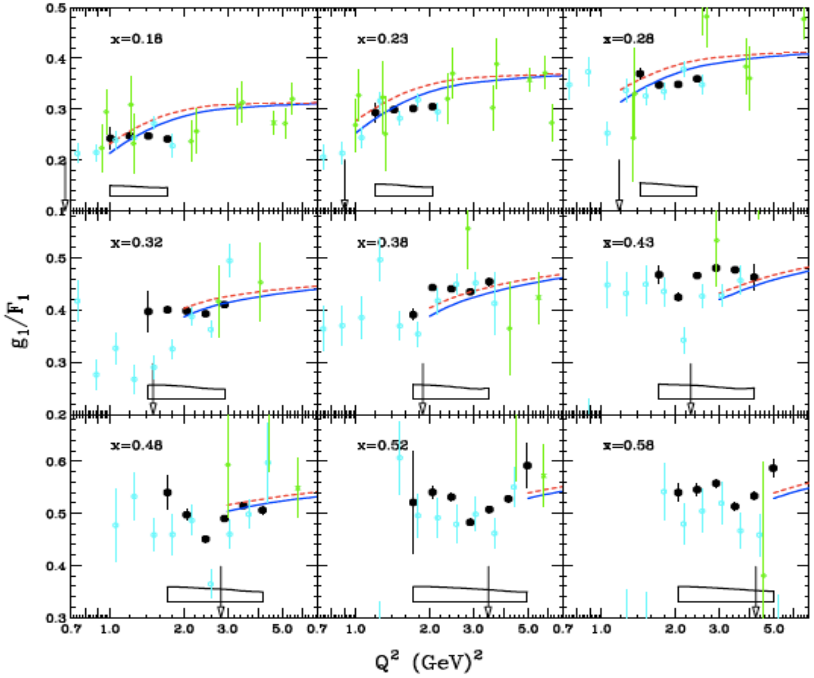
- ☐ **The "Plot" is greatly improved:**



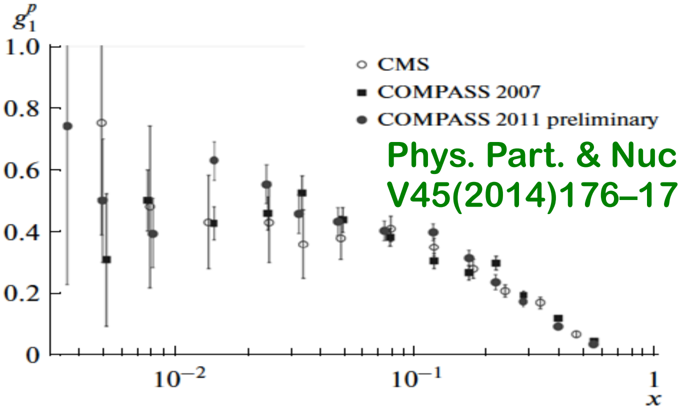


**JLab/CLAS**

arXiv:1404.6231

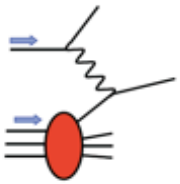
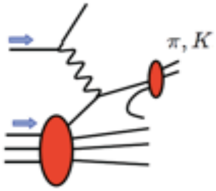
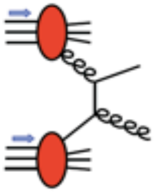


**Lower Q²**
**HT's**



Phys. Part. & Nucl, V45(2014)176–178

# Recent helicity PDF fits @ NLO

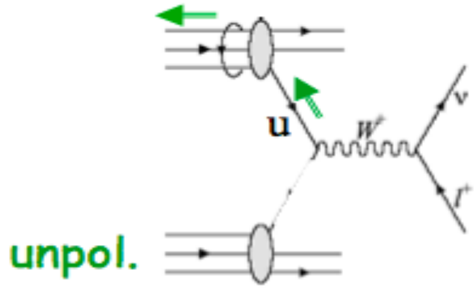
| |  |  |  | uncertainties | last update |
|---|---|---|---|---|---|
| **NNPDF**<br>Ball, Forte, Guffanti, Nocera, Rodolfi, Rojo | ✓ | ✗ | ✗ | 100 replicas<br>stat. approach | 1303.7236 |
| **DSSV**<br>de Florian, Sassot, MS, Vogelsang | ✓ | ✓ | ✓ | L.M. $\Delta\chi^2 = 8$ (1)<br>(Hessian $\Delta\chi^2 = 1$) | 0904.3821<br>[DSSV+/++: 1112.0904<br>1304.0079] |
| **LSS**<br>Leader, Sidorov, Stamenov | ✓ | ✓ | ✗ | Hessian $\Delta\chi^2 = 1$ | 1010.0574 |
| **BB**<br>Blumlein, Bottcher | ✓ | ✗ | ✗ | Hessian $\Delta\chi^2 = 1$ | 1010.3113 |
| ⋮ | | ⋮ | | | ⋮ |
| **GRSV**<br>Gluck, Reya, MS, Vogelsang | ✓ | ✗ | ✗ | ✗ | 1st NLO analysis<br>9508347 |

# Sea quark polarization – RHIC W program

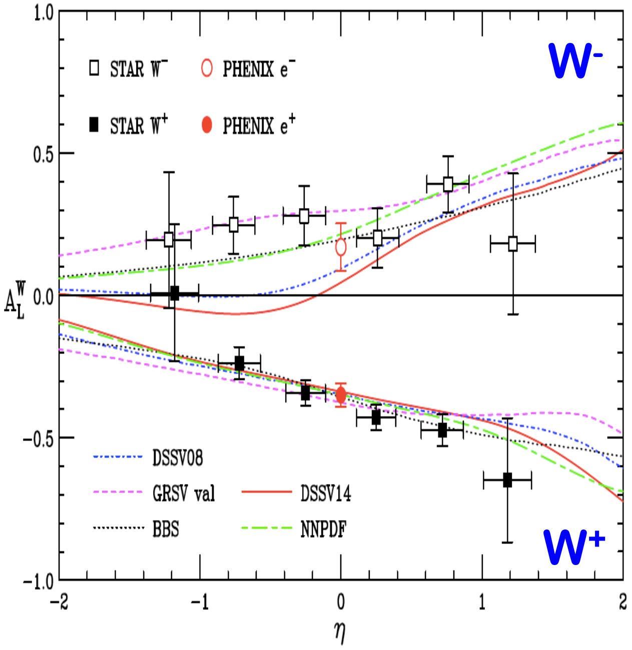
## ☐ Single longitudinal spin asymmetries:

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s)$$

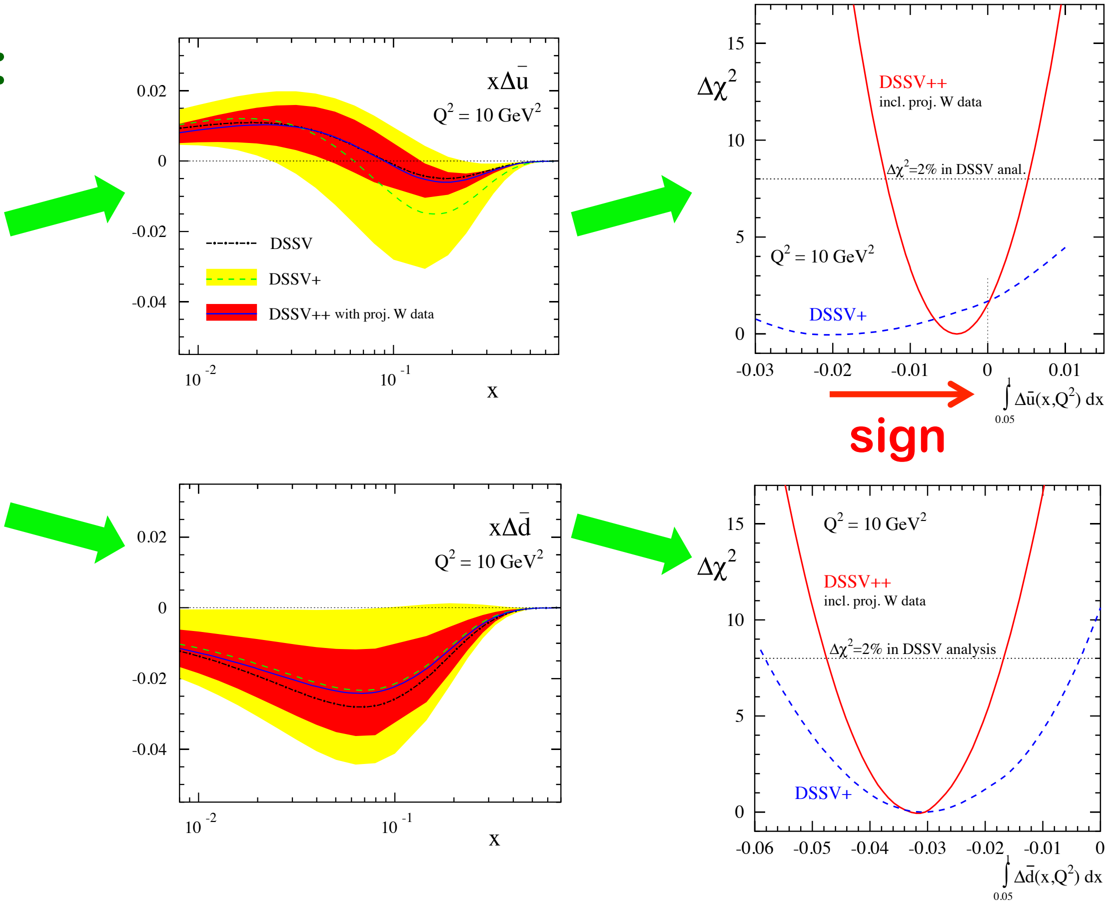
**Parity violating weak interaction**



## ☐ From 2013 RHIC data:



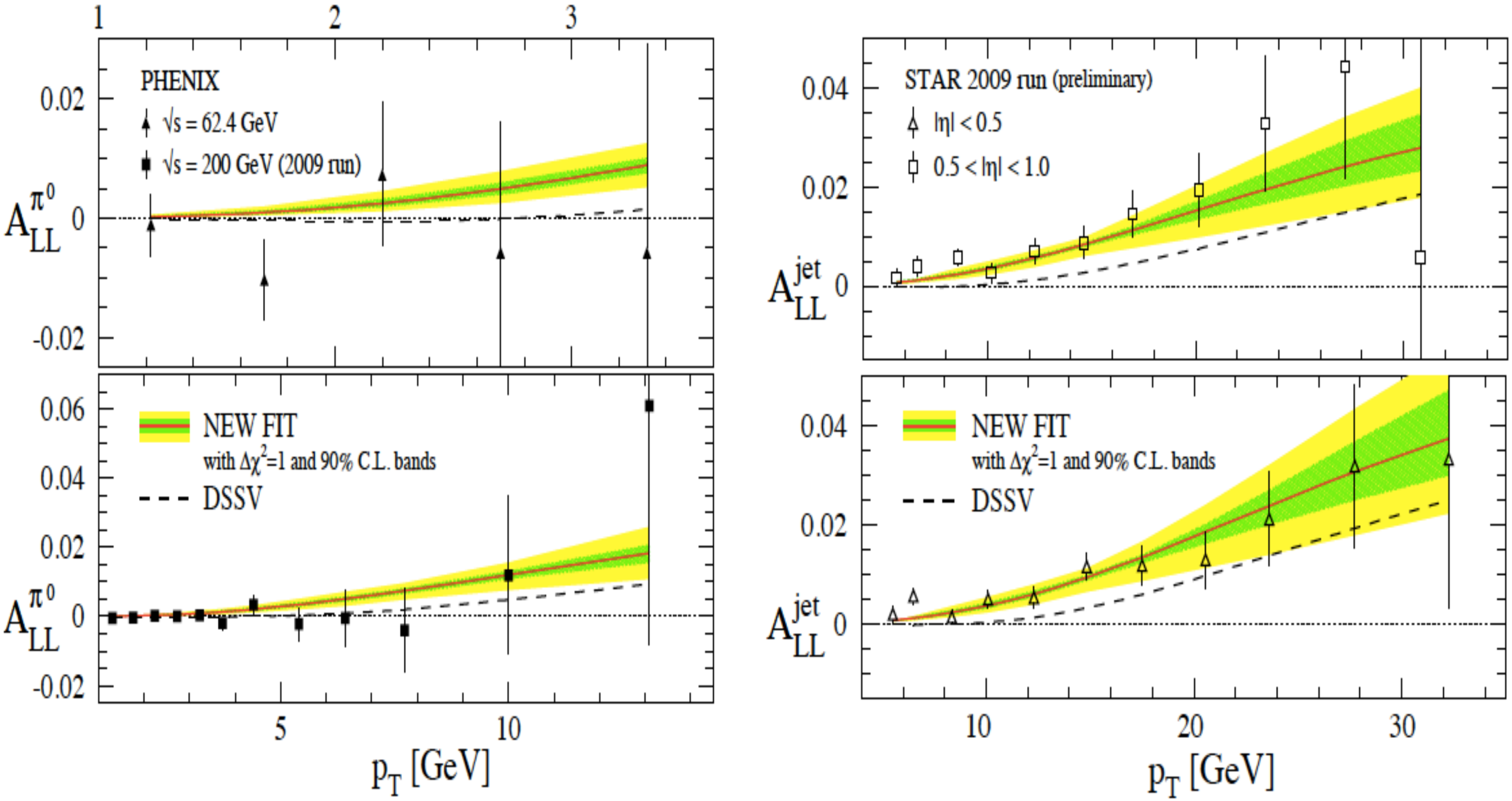
**W-production at RHIC**



# Gluon helicity contribution – RHIC data

- ❑ **RHIC 2009 data:**

**Jet/pion production at RHIC – gluon helicity:**

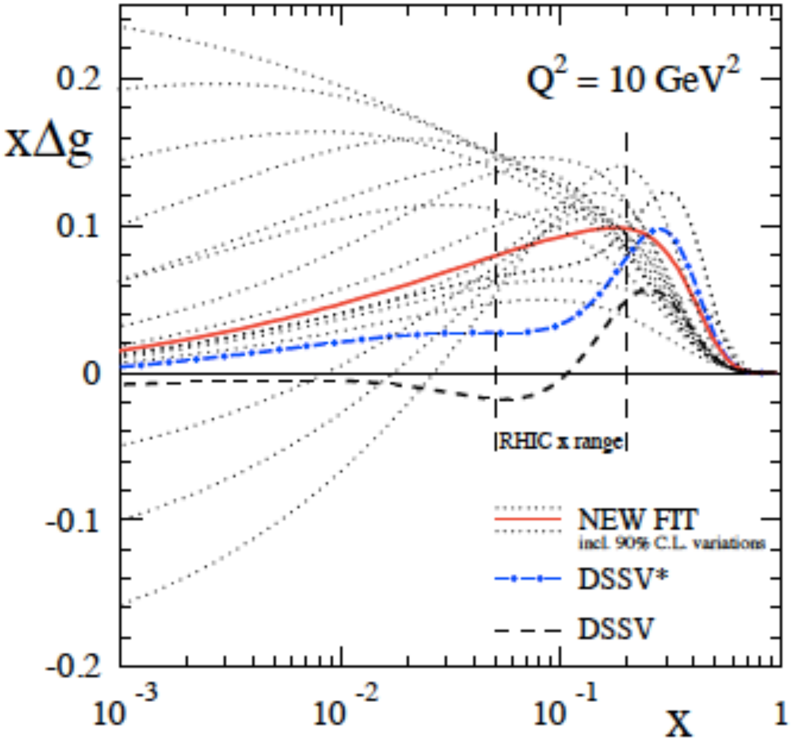


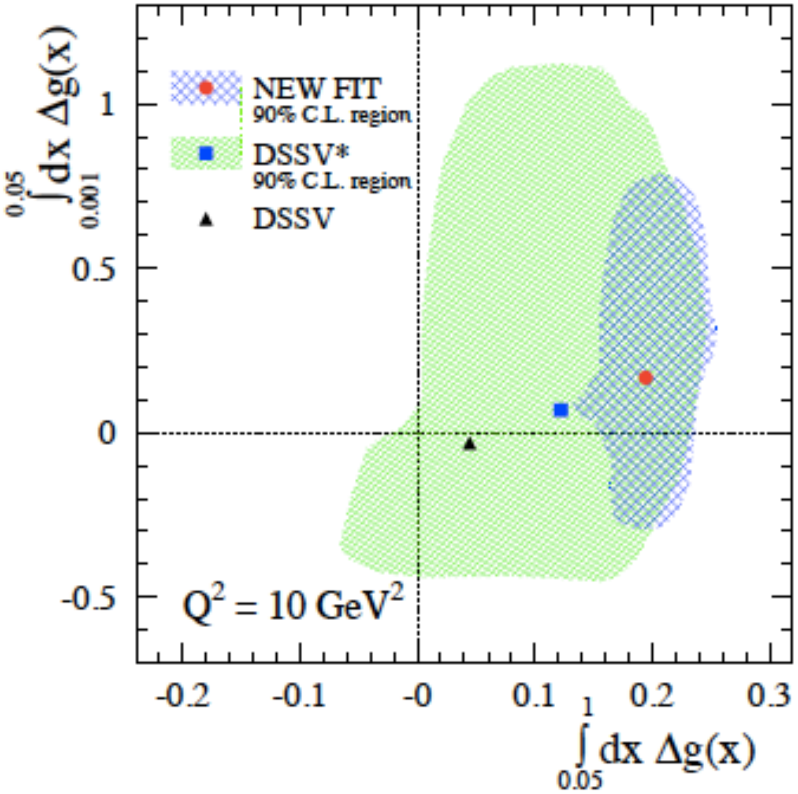
# Global QCD analysis of helicity PDFs

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014) 012001

results featured in Sci. Am., Phys. World, ...

## ❑ Impact on gluon helicity:



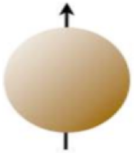


- ✧ Red line is the new fit
- ✧ Dotted lines = other fits with 90% C.L.

- ✧ 90% C.L. areas
- ✧ Leads $\Delta G$ to a positive #

# Current understanding for Proton Spin

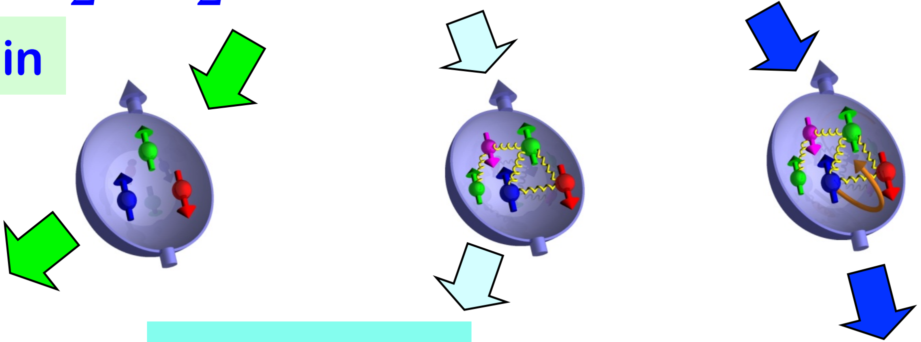
- ☐ **The sum rule:** $$S(\mu) = \sum_f \langle P,S|\hat{J}_f^z(\mu)|P,S\rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$
  - Infinite possibilities of decompositions – connection to observables?
  - Intrinsic properties + dynamical motion and interactions
- ☐ **An incomplete story:**



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …

**Proton Spin**



**Quark helicity**
**Best known**

$$\frac{1}{2}\int dx\left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

**Sea quarks?**

**Gluon helicity**
**Start to know**

$$\Delta G = \int dx \Delta g(x)$$

$$\sim 20\%(\text{with RHIC data})$$

**Orbital Angular Momentum of quarks and gluons**
**Little known**

**Net effect of partons' transverse motion?**

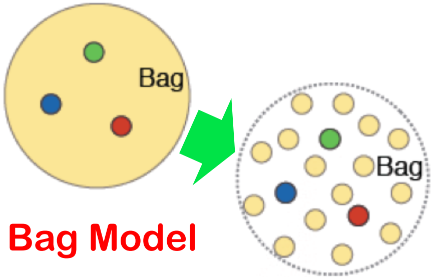
# Hadron structures

- ☐ **What does the proton look like?**

**Gluon radius?**

**Static:**

**Hard probe:**



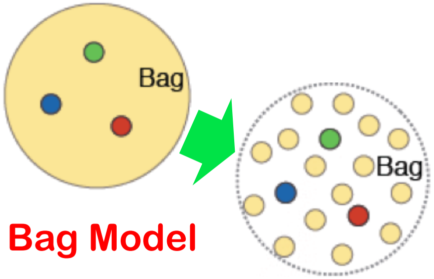
Bag Model

# Hadron structures

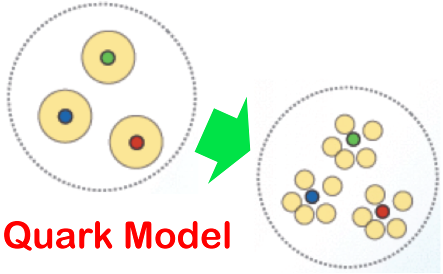
❑ **What does the proton look like?** Gluon radius?

Static:

Hard probe:



Bag Model



Quark Model

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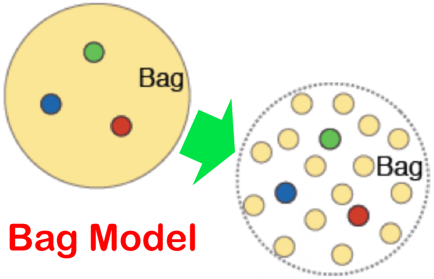
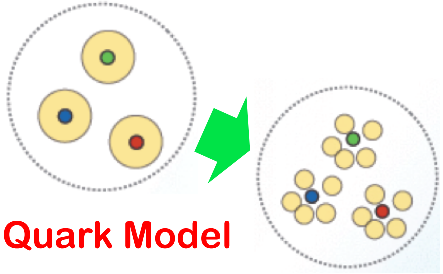
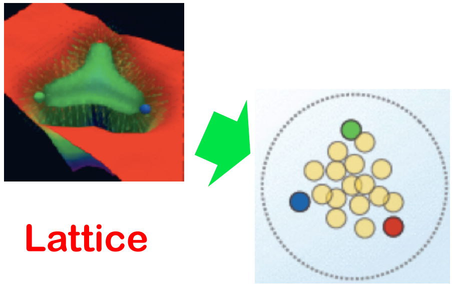


Bag Model



Quark Model



Lattice

# Hadron structures

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**Gluon radius?**

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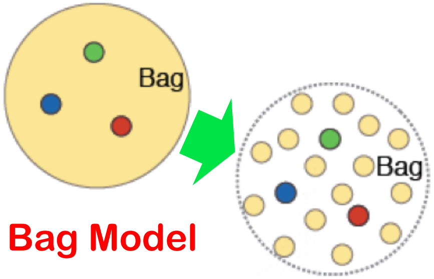
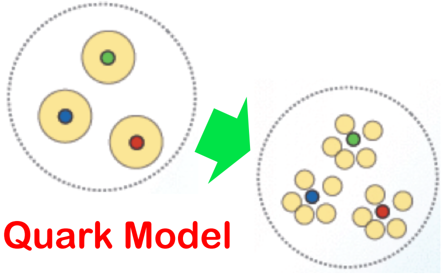
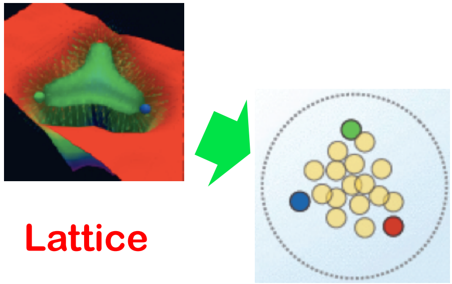
**Hard probe:**



**Bag Model**

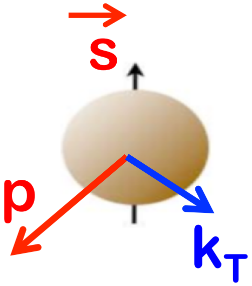


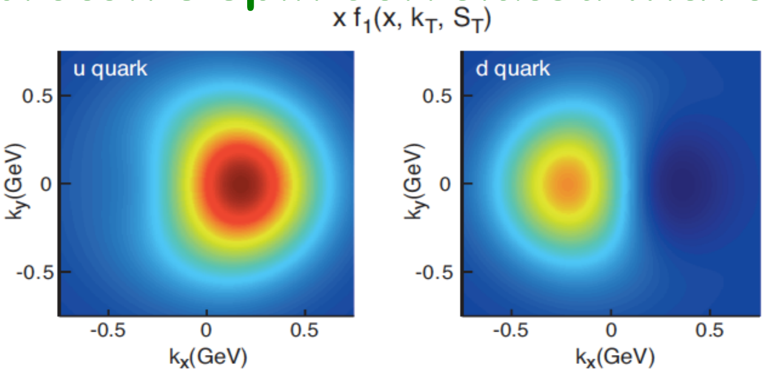
**Quark Model**



**Lattice**

☐ **How is proton's spin correlated with the motion of quarks/gluons?**





**Deformation of parton's**
***confined motion***
**when hadron is polarized?**

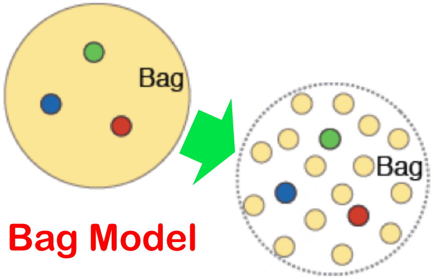
**TMDs!**

# Hadron structures

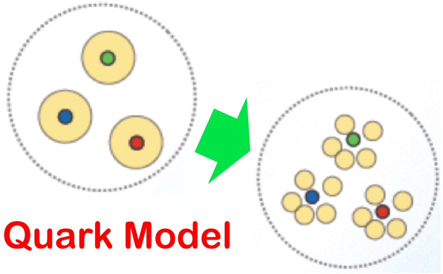
- **What does the proton look like?** **Gluon radius?**

**Static:**

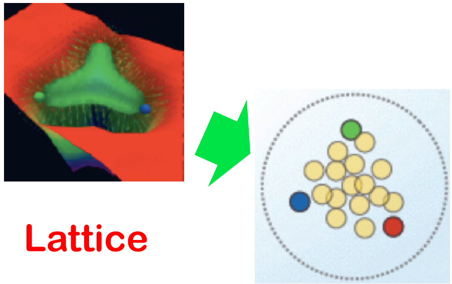
**Hard probe:**



**Bag Model**



**Quark Model**



**Lattice**

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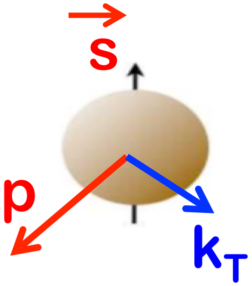
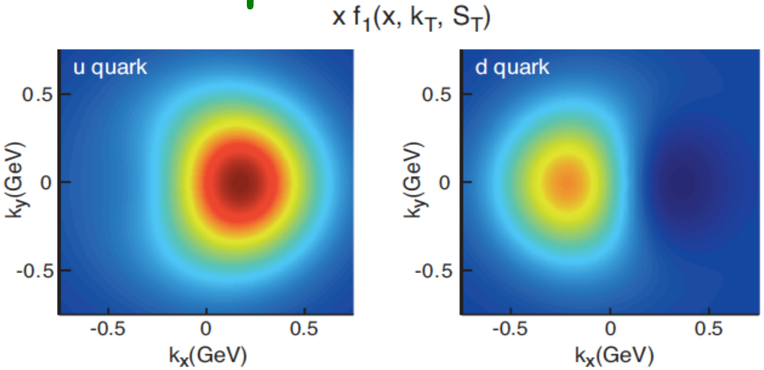
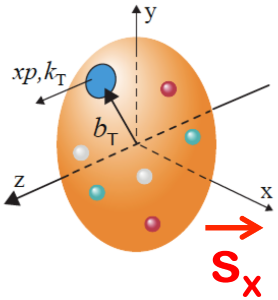


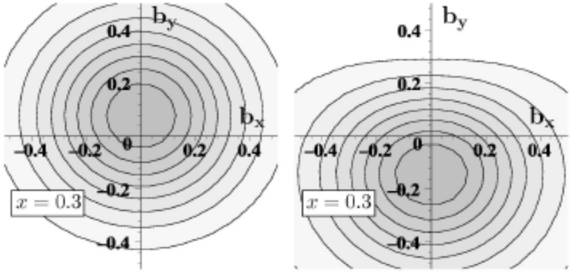


**Deformation of parton's**
***confined motion***
**when hadron is polarized?**

**TMDs!**

- **How does proton's spin influence the spatial distribution of partons?**



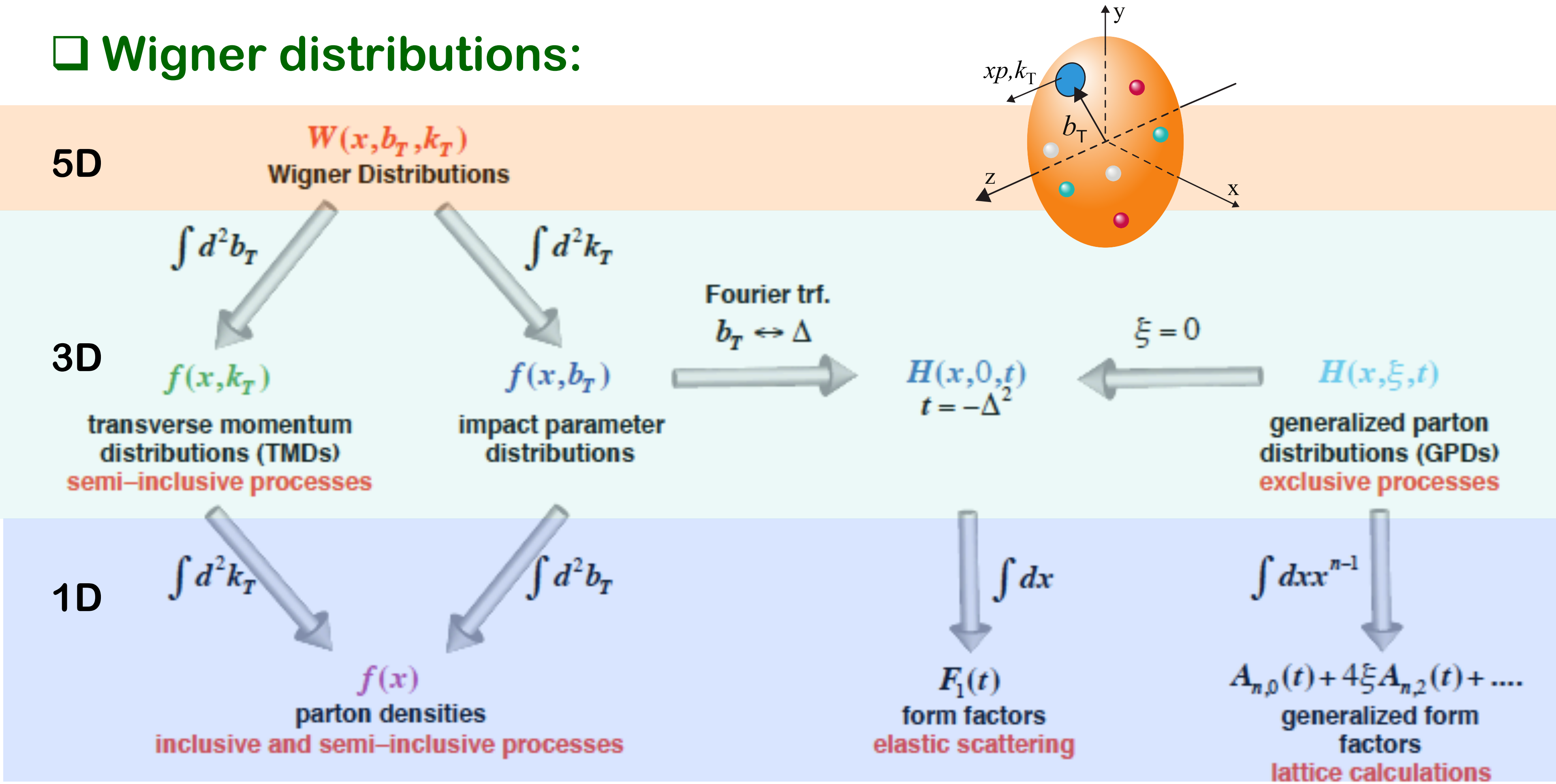


**Deformation of parton's**
***spatial distribution***
**when hadron is polarized?**

**GPDs!**

# Unified view of nucleon structure

- **Wigner distributions:**



5D

$W(x, b_T, k_T)$

Wigner Distributions

$\int d^2 b_T$

$\int d^2 k_T$

3D

$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

Fourier trf.

$b_T \leftrightarrow \Delta$

$H(x, 0, t)$

$t = -\Delta^2$

$\xi = 0$

$H(x, \xi, t)$

generalized parton distributions (GPDs)

exclusive processes

1D

$\int d^2 k_T$

$\int d^2 b_T$

$\int dx$

$\int dx x^{n-1}$

$f(x)$

parton densities

inclusive and semi–inclusive processes

$F_1(t)$

form factors

elastic scattering

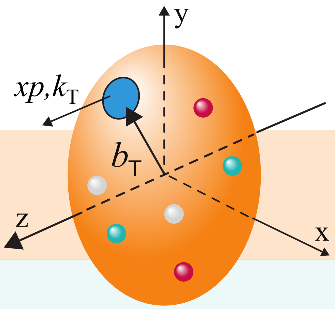
$A_{n,0}(t) + 4\xi A_{n,2}(t) + \ldots.$

generalized form factors

lattice calculations

# Unified view of nucleon structure

- ❑ **Wigner distributions:**



**5D**

$W(x,b_T,k_T)$

Wigner Distributions

$\int d^2b_T$

$\int d^2k_T$

**3D**

$f(x,k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

**HERMES**
**JLab12**
**COMPASS**

$f(x,b_T)$

impact parameter distributions

Fourier trf.

$b_T \leftrightarrow \Delta$

$H(x,0,t)$

$t=-\Delta^2$

$\xi=0$

$H(x,\xi,t)$

generalized parton distributions (GPDs)

exclusive processes

**JLab12**
**COMPASS**
**for**
**Valence**

**1D**

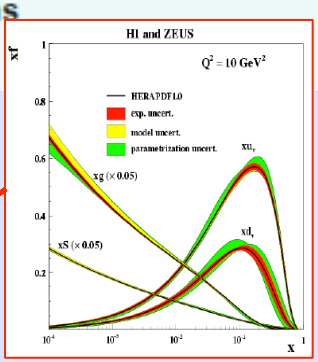
$\int d^2k_T$

$\int d^2b_T$

$f(x)$

parton densities

inclusive and semi–inclusive processes



$\int dx$

$F_1(t)$

form factors

elastic scattering

$\int dx x^{n-1}$

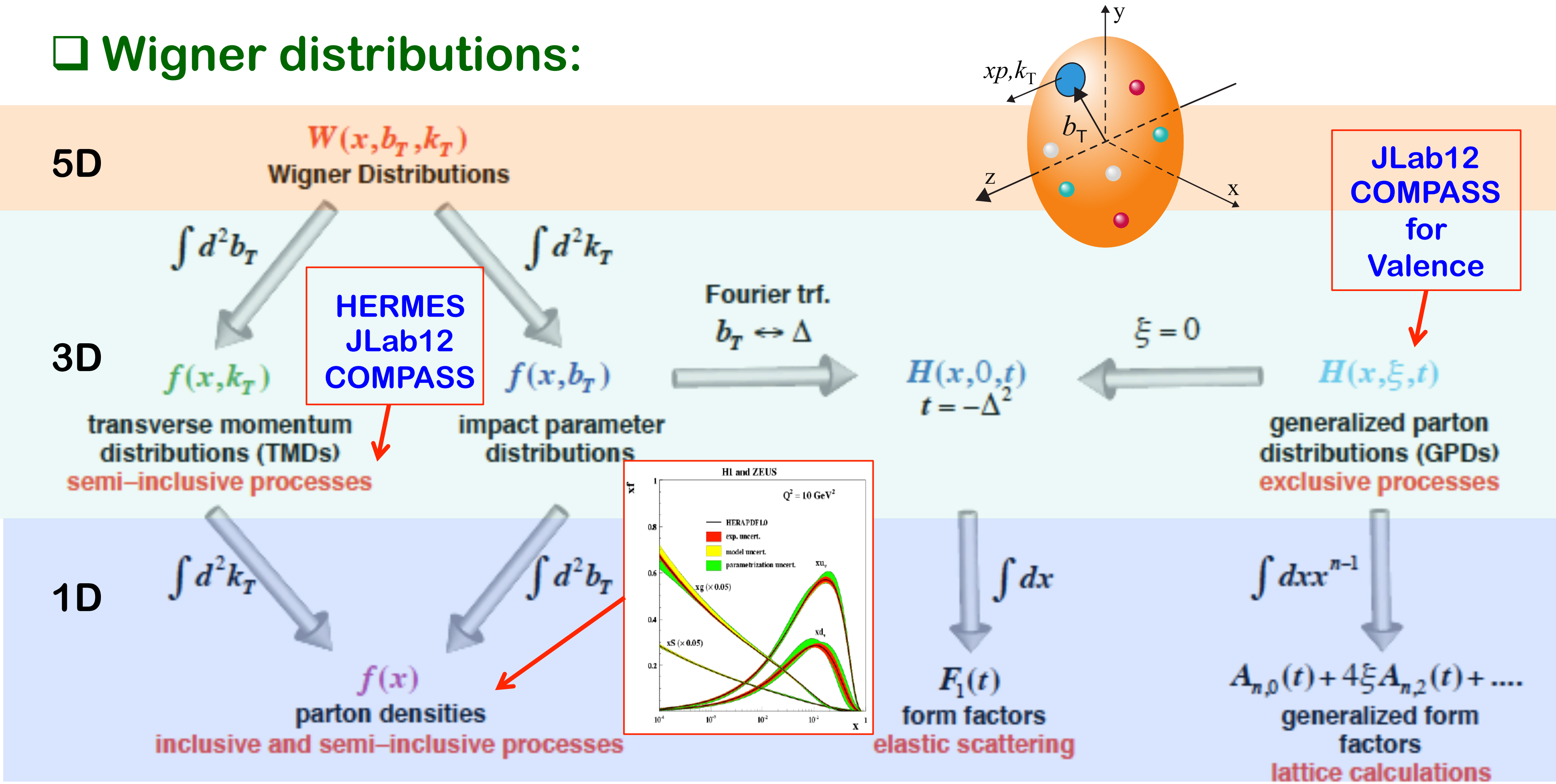
$A_{n,0}(t)+4\xi A_{n,2}(t)+....$

generalized form factors

lattice calculations

# Unified view of nucleon structure

- ❑ Wigner distributions:



5D

$W(x,b_T,k_T)$

Wigner Distributions

$\int d^2b_T$

$\int d^2k_T$

**HERMES**
**JLab12**
**COMPASS**

3D

$f(x,k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x,b_T)$

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**JLab12**
**COMPASS**
**for**
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generalized parton distributions (GPDs)

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$\int dx$

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$\int dx x^{n-1}$

$A_{n,0}(t)+4\xi A_{n,2}(t)+....$

generalized form factors

lattice calculations

- ❑ 3D imaging of sea and gluons:
  - ✧ TMDs – confined motion in a nucleon (semi-inclusive DIS)
  - ✧ GPDs – Spatial imaging of quarks and gluons (exclusive DIS)

# Polarization and spin asymmetry

**Explore new QCD dynamics – vary the spin orientation:**

- ☐ **Cross section:**

**Scattering amplitude square – Probability – Positive definite**

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- ☐ **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ **– Positive definite**

- ☐ **Asymmetries or difference of cross sections:**

  - **both beams polarized** $A_{LL}, A_{TT}, A_{LT}$

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

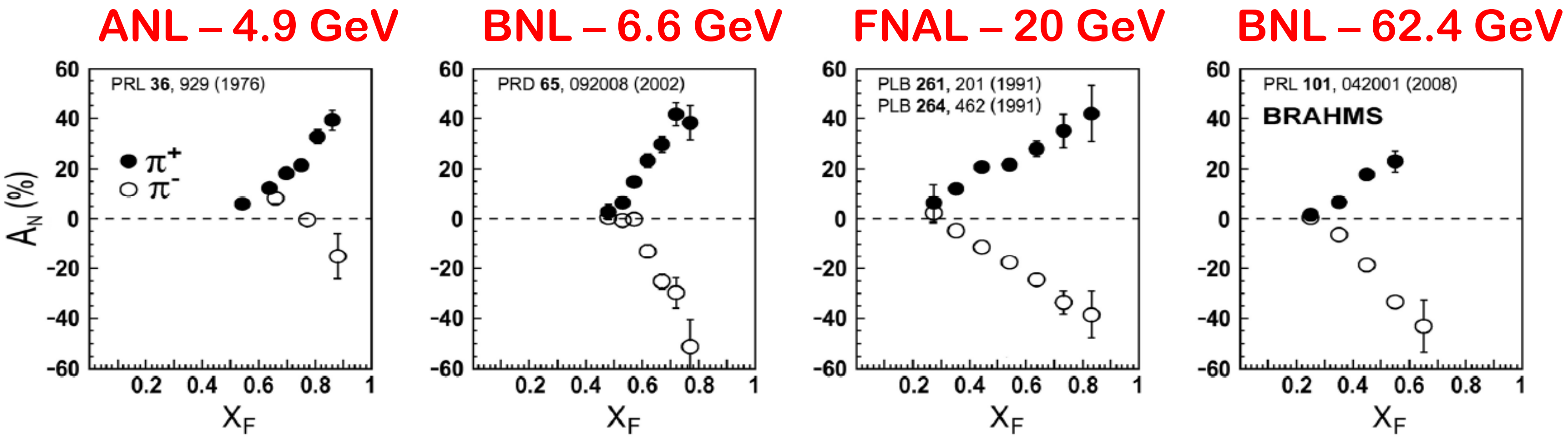
  - **one beam polarized** $A_L, A_N$ **– Not necessary positive!**

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

**Chance to see quantum interference directly**

# Transverse single-spin asymmetry (SSAs)

- $A_N$ - consistently observed for over 35 years!

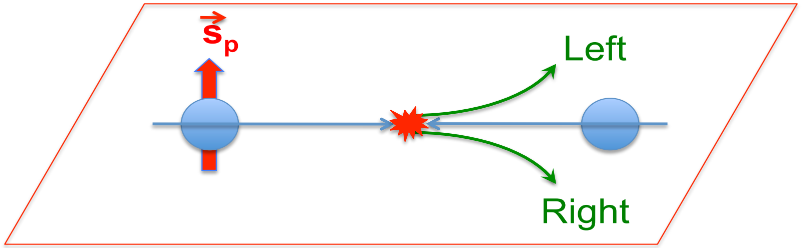

ANL – 4.9 GeV

BNL – 6.6 GeV

FNAL – 20 GeV

BNL – 62.4 GeV

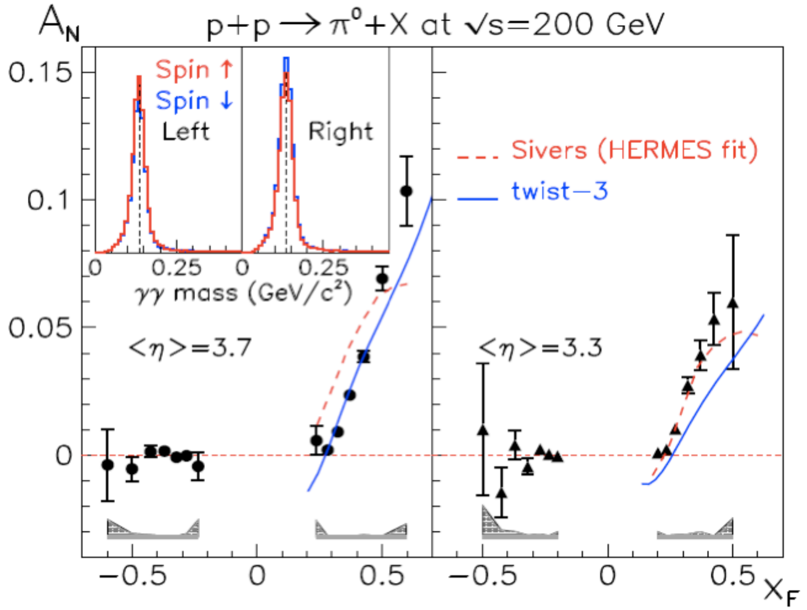
- Definition:



$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$$

Vanish if active parton has no kT!!!

BNL – 200 GeV

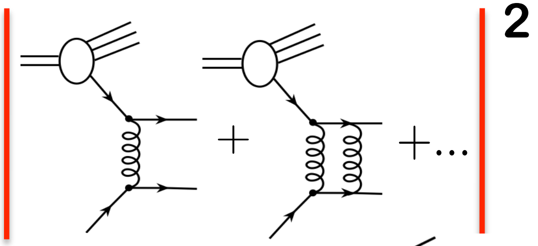


# Do we understand it?

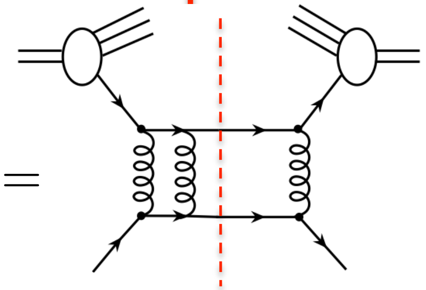
Kane, Pumplin, Repko, PRL, 1978

## ☐ Early attempt:

**Cross section:** $\sigma_{AB}(p_T,\vec{s}) \propto$



**Asymmetry:** $\sigma_{AB}(p_T,\vec{s}) - \sigma_{AB}(p_T,-\vec{s}) \;=$



$\propto\ \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

## ☐ What do we need?

$$A_N \ \propto\ i\vec{s}_p\cdot(\vec{p}_h\times\vec{p}_T)\ \Rightarrow\ i\epsilon^{\mu\nu\alpha\beta}p_{h\mu}s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

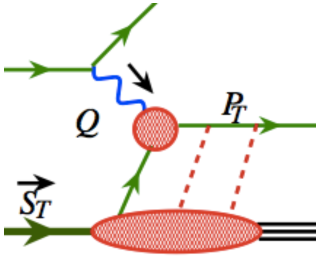
## ☐ Vanish without parton’s transverse motion:

➡ A direct probe for parton’s transverse motion,

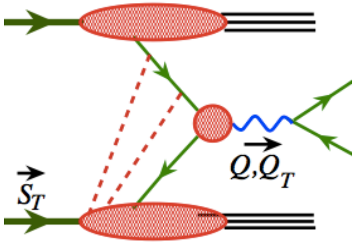
Spin-orbital correlation, QCD quantum interference

# Current understanding of SSAs

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q>>P_T$

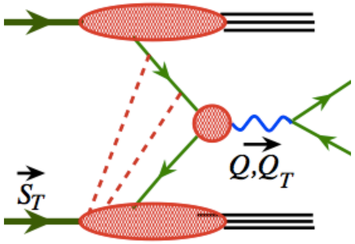


DY: $Q>>Q_T$

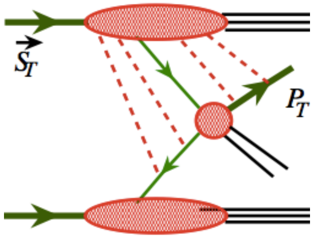
**TMD factorization**
**TMD distributions**

*Direct information on parton $k_T$*

- One scale observables – $Q >> \Lambda_{QCD}$:



DY: $Q \sim Q_T$

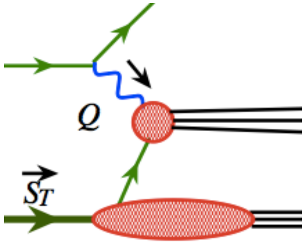


Jet, Particle: $P_T$

**Collinear factorization**
**Twist-3 distributions**

*Information on moments of parton $k_T$*

- Symmetry plays important role:



Inclusive DIS
Single scale
Q

**Parity**
**Time-reversal**

$A_N = 0$

# Factorized Drell-Yan cross section – Lec. 2

- **TMD factorization ( $q_\perp \ll Q$ ):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$ , is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ( $q_\perp \sim Q$ ):**

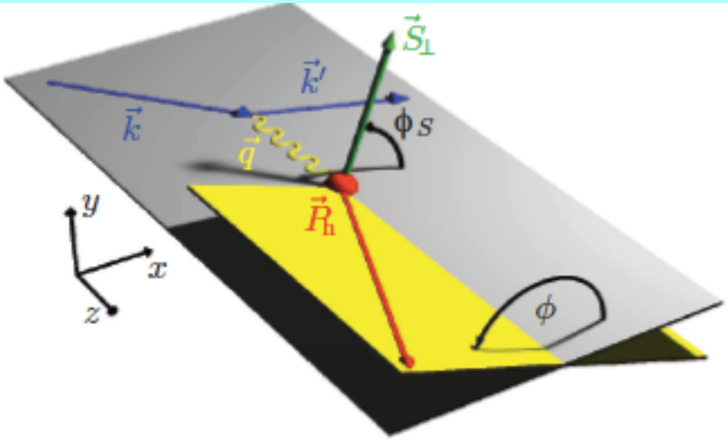
$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

→ same formula with polarized PDFs for γ*,W/Z, $H^0$...

# Semi-inclusive DIS (SIDIS)



**❑ Process:**

$$e(k) + N(p) \longrightarrow e'(k') + h(P_h) + X$$

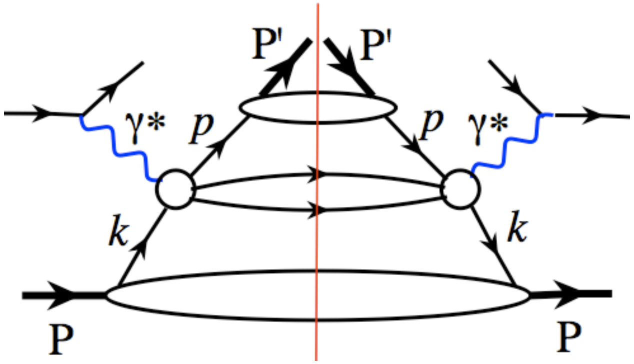
**❑ Natural event structure:**

**In the photon-hadron frame:** $P_{h_T} \approx 0$

***Semi-Inclusive DIS is a natural observable with TWO very different scales***

$Q \gg P_{h_T} \gtrsim \Lambda_{\rm QCD}$ **Localized probe sensitive to parton's transverse motion**

**❑ Collinear QCD factorization holds if $P_{hT}$ integrated:**



$$d\sigma_{\gamma^* h \to h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f \to f'} \otimes D_{f' \to h'}(z)$$

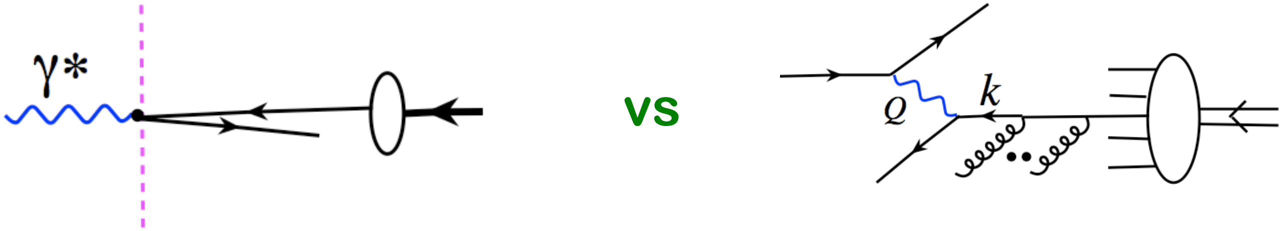
$$z = \frac{P_h \cdot p}{q \cdot p} \qquad y = \frac{q \cdot p}{k \cdot p}$$

**❑ "Total c.m. energy":** $s_{\gamma^* p} = (p+q)^2 \approx Q^2 \left[\frac{1-x_B}{x_B}\right] \approx \frac{Q^2}{x_B}$

# Single hadron production at low $p_T$

- ☐ Unique kinematics - unique event structure:

Briet frame:  Large $Q^2$ virtual photon acts like a "wall"



High energy low $p_T$ jet (or hadron) - ideal probe for parton's transverse motion!

- ☐ Need for TMDs, if we observe $p_T \sim 1/\text{fm}$:

$$\int d^4k_a \, \mathcal{H}(Q, p_T, k_a, k_b) \left(\frac{1}{k_a^2 + i\varepsilon}\right) \left(\frac{1}{k_a^2 - i\varepsilon}\right) \mathcal{T}(k_a, 1/r_0)$$

$$\approx \int \frac{dx}{x} d^2k_{a\perp} \; \mathcal{H}(Q, p_T, k_a^2 = 0, k_b) \left[\int dk_a^2 \left(\frac{1}{k_a^2 + i\varepsilon}\right) \left(\frac{1}{k_a^2 - i\varepsilon}\right) \mathcal{T}(k_a, 1/r_0)\right]$$

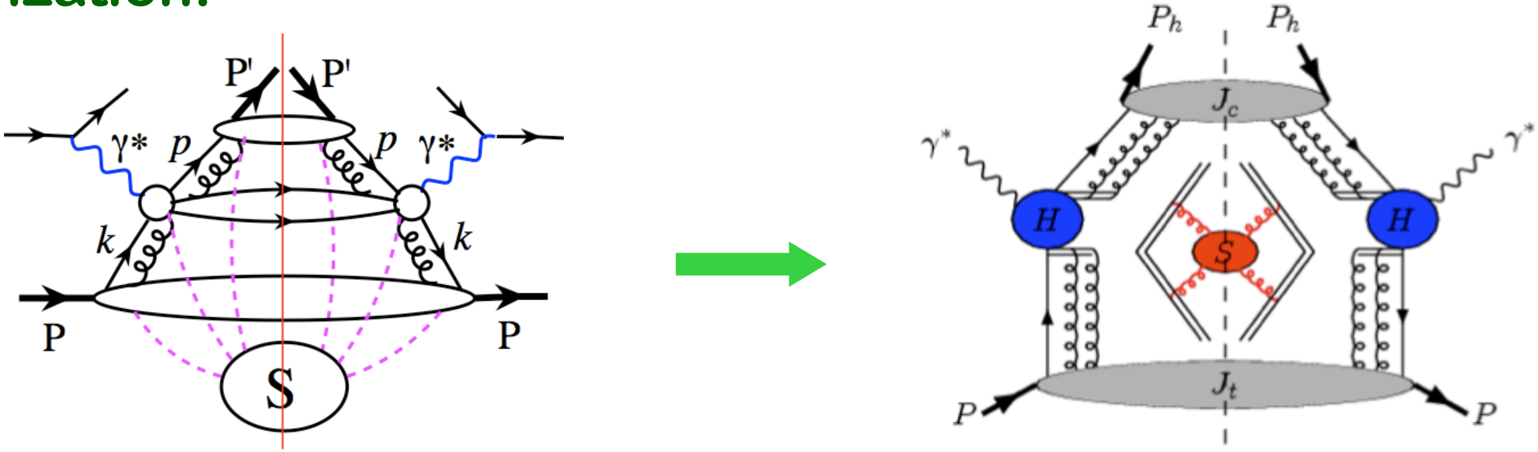
Can't set $k_T \sim 0$, since $k_T \sim p_T$

TMD distribution

# QCD factorization for SIDIS

**❑ Factorization:**

**Ji, Ma, Yuan**



**❑ Low $P_{hT}$ – TMD factorization:**

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f \otimes \mathcal{D}_{f\to h} \otimes \mathcal{S} + \mathcal{O}\left(\frac{P_{h\perp}}{Q}\right)$$

**❑ High $P_{hT}$ – Collinear factorization:**

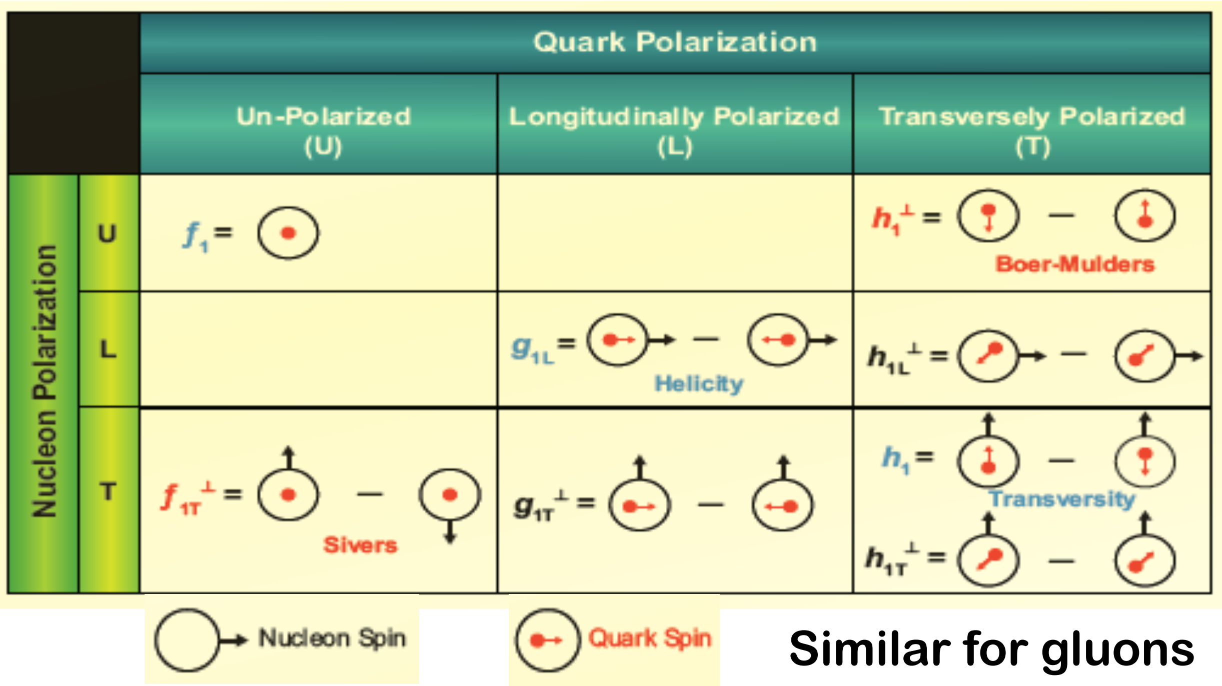
$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

**❑ $P_{hT}$ Integrated - Collinear factorization:**

$$\sigma_{\rm SIDIS}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

# TMD parton distributions (TMDs)

- ☐ Power of spin – many more correlations:

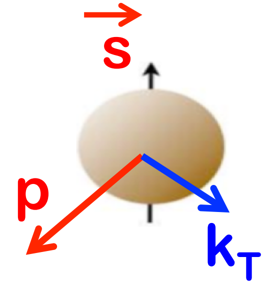


| | | Quark Polarization | | |
|---|---|---|---|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1$ = | | $h_1^\perp$ = − <br>Boer-Mulders |
| | L | | $g_{1L}$ = − <br>Helicity | $h_{1L}^\perp$ = − |
| | T | $f_{1T}^\perp$ = − <br>Sivers | $g_{1T}^\perp$ = − | $h_1$ = − <br>Transversity<br>$h_{1T}^\perp$ = − |

Nucleon Spin

Quark Spin

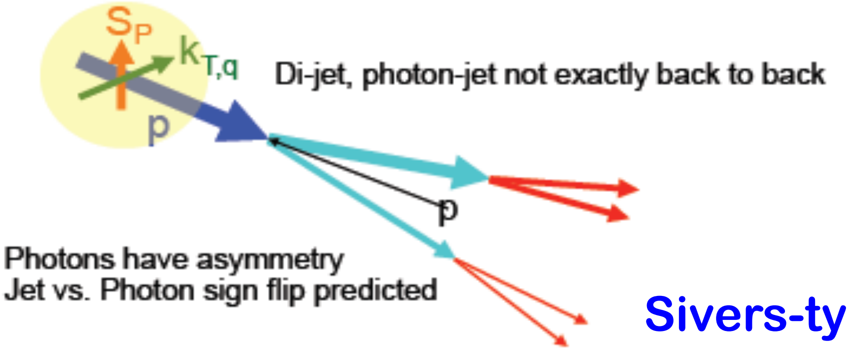
Similar for gluons



**Require two Physical scales**

**More than one TMD contribute to the same observable!**

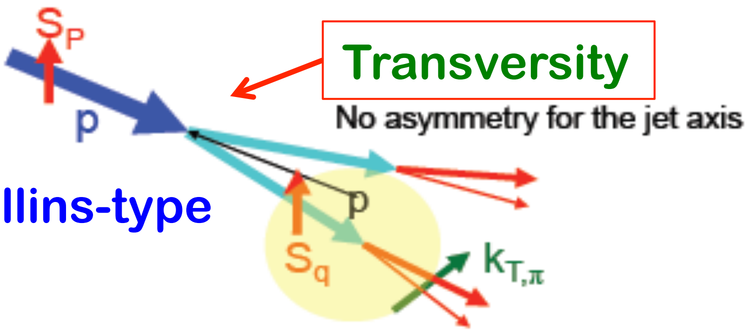
- ☐ $A_N$ – single hadron production:



Di-jet, photon-jet not exactly back to back

Photons have asymmetry
Jet vs. Photon sign flip predicted

**Sivers-type**



**Transversity**

No asymmetry for the jet axis

**Collins-type**

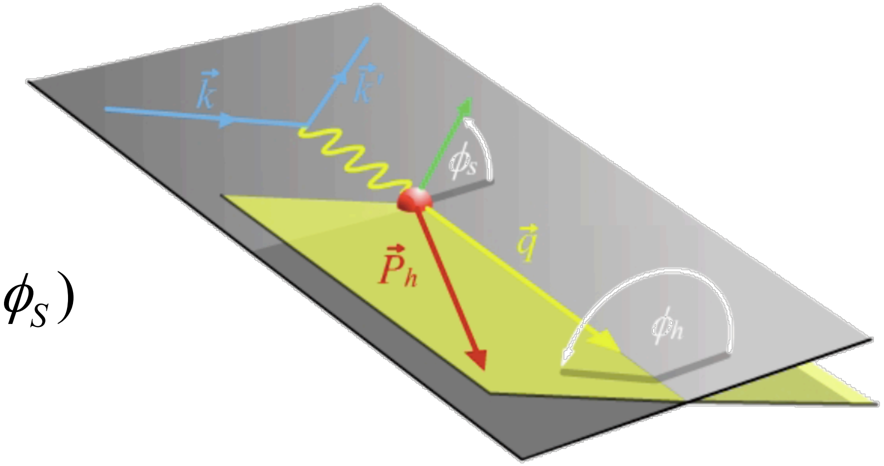
# SIDIS is the best for probing TMDs

## ❑ Naturally, two planes:

$$A_{UT}(\varphi_h^l, \varphi_S^l) = \frac{1}{P}\frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}$$

$$= A_{UT}^{Collins}\sin(\phi_h + \phi_S) + A_{UT}^{Sivers}\sin(\phi_h - \phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h - \phi_S)$$



## ❑ Separation of TMDs:

$$A_{UT}^{Collins} \propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

**Collins frag. Func.**
**from $e^+e^-$ collisions**

***Hard, if not impossible, to separate TMDs in hadronic collisions***

**Using a combination of different observables (not the same observable):**
**jet, identified hadron, photon, …**

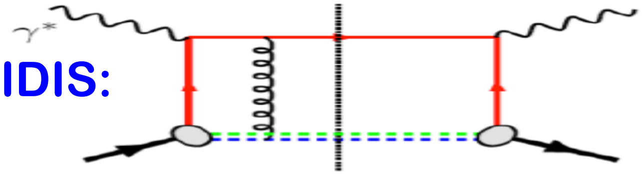
# Broken universality for TMDs

## ☐ Definition:

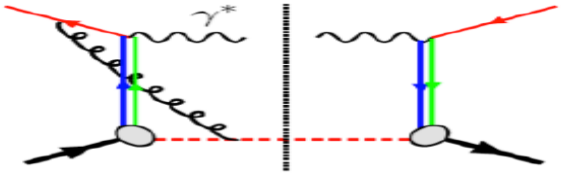
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\boxed{\text{Gauge link}}\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

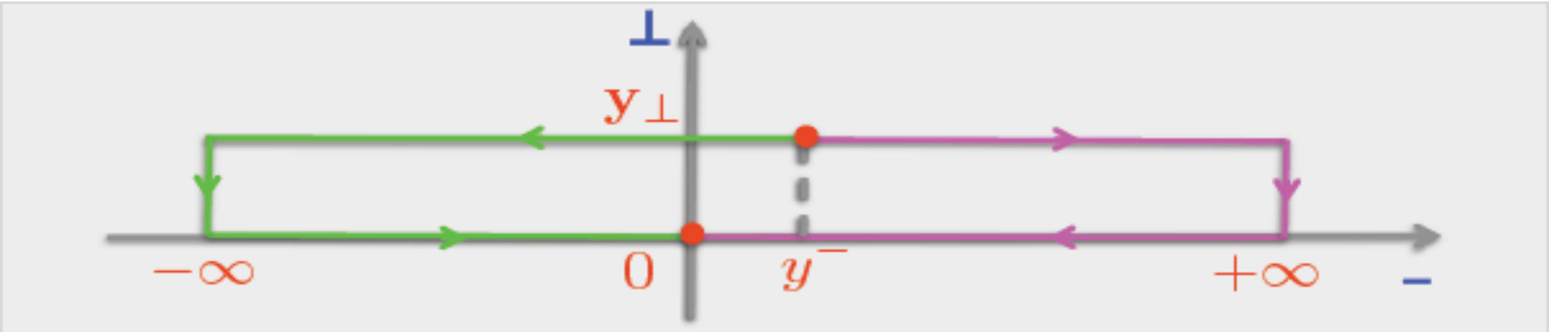
## ☐ Gauge links:

**SIDIS:**



**DY:**





## ☐ Process dependence:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

**Collinear factorized PDFs are process independent**

# Modified universality

- ☐ **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- ☐ **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- ☐ **Modified universality:**

$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

Same function, but, opposite sign!

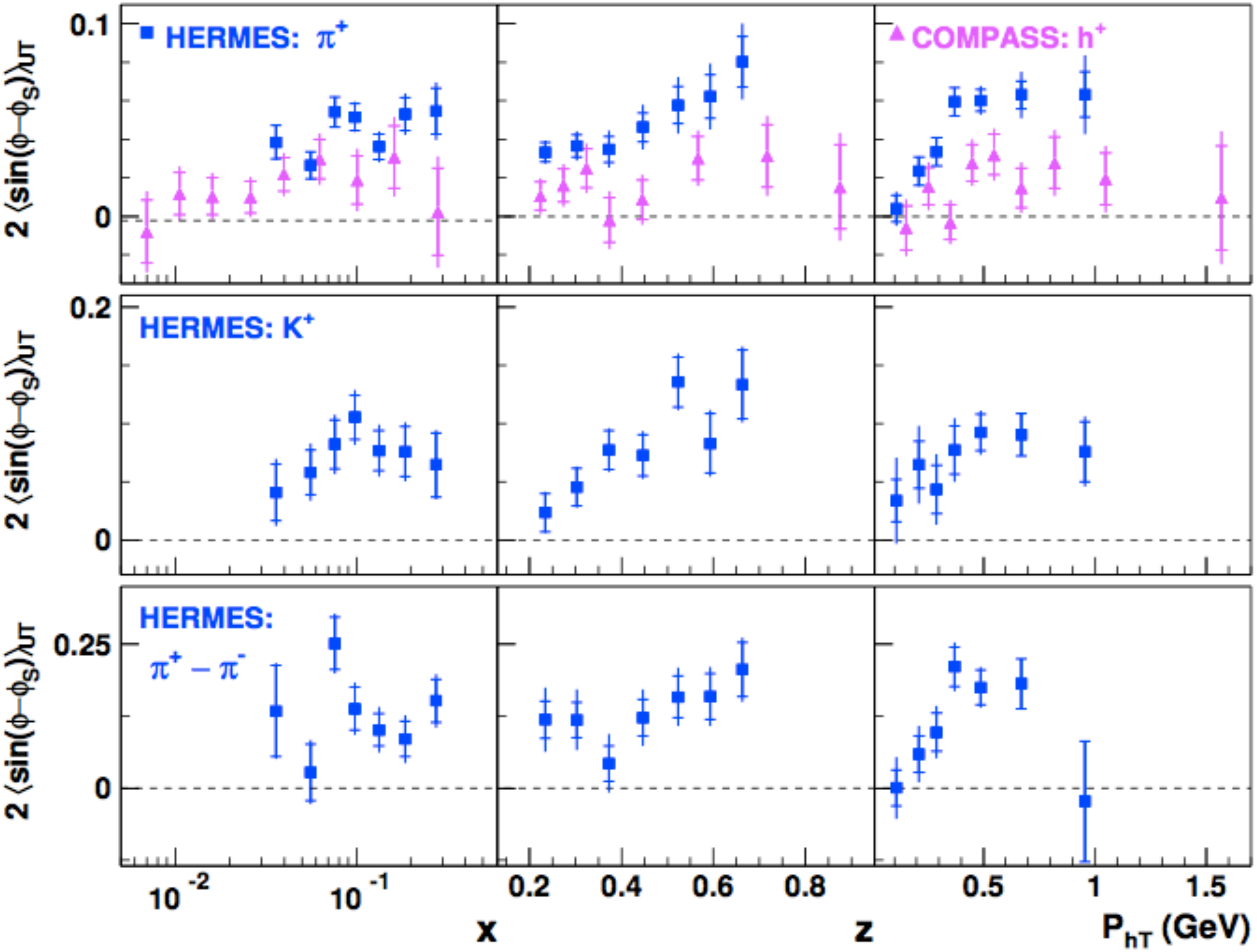
- ☐ **The sign change = Critical test of TMD factorization!**

Same applies to TMD gluon distribution

Spin-averaged TMD is process independent

# Sivers asymmetries from SIDIS

☐ From SIDIS (HERMES and COMPASS) – low Q:



Non-zero Sivers effects Observed in SIDIS!

Visible $Q^2$ dependence

Major theory development in last few years

Drell-Yan $A_N$: COMPASS, RHIC run 17th, Fermilab Drell-Yan, …

# Evolution equations for TMDs

Boer, 2001, 2009, JI, Ma, Yuan, 2004
Idilbi, et al, 2004, Kang, Xiao, Yuan, 2011
Aybat, Collins, Qiu, Rogers, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012, Sun, Yuan 2013, …

## ❑ Collins-Soper equation:

– b-space quark TMD with $\gamma^+$

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x,\mathrm{b}_\mathrm{T},S;\mu;\zeta_F)}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\tilde{F}_{f/P^\uparrow}(x,\mathrm{b}_\mathrm{T},S;\mu;\zeta_F) \qquad \tilde{K}(b_T;\mu) = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T;y_s,-\infty)}{\tilde{S}(b_T;+\infty,y_s)}\right)$$

## ❑ RG equations:

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \qquad \frac{d\tilde{F}_{f/P^\uparrow}(x,\mathrm{b}_\mathrm{T},S;\mu;\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x,\mathrm{b}_\mathrm{T},S;\mu;\zeta_F).$$

## ❑ Evolution equations for Sivers function:

$$F_{f/P^\uparrow}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^iS^j}{M_p}$$

**CS:**

$$\boxed{\frac{\partial\ln\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{\partial\ln\sqrt{\zeta_F}}} = \tilde{K}(b_T;\mu) \qquad \tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial\tilde{F}_{1T}^{\perp f}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

**RGs:**

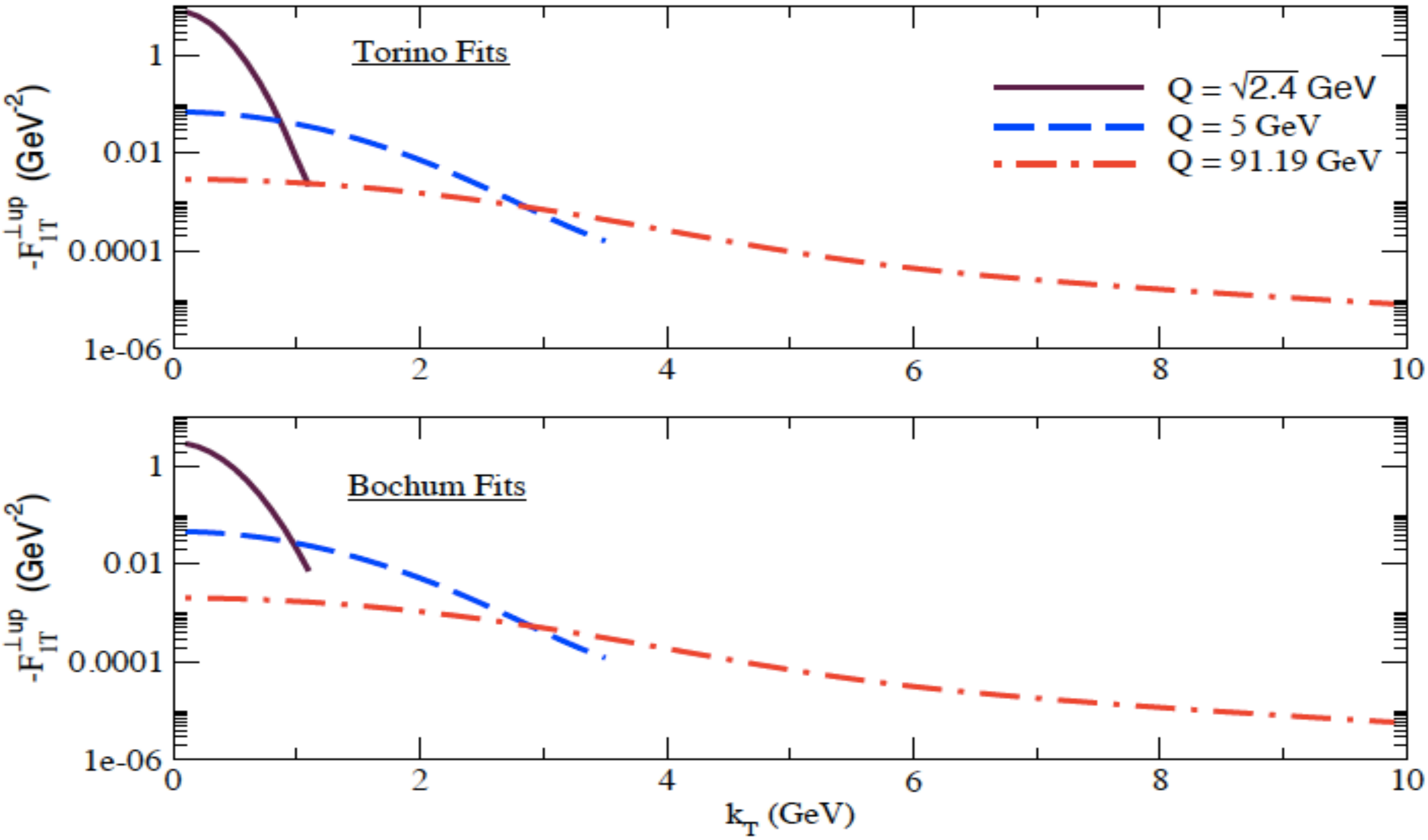
$$\frac{d\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

# Scale dependence of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

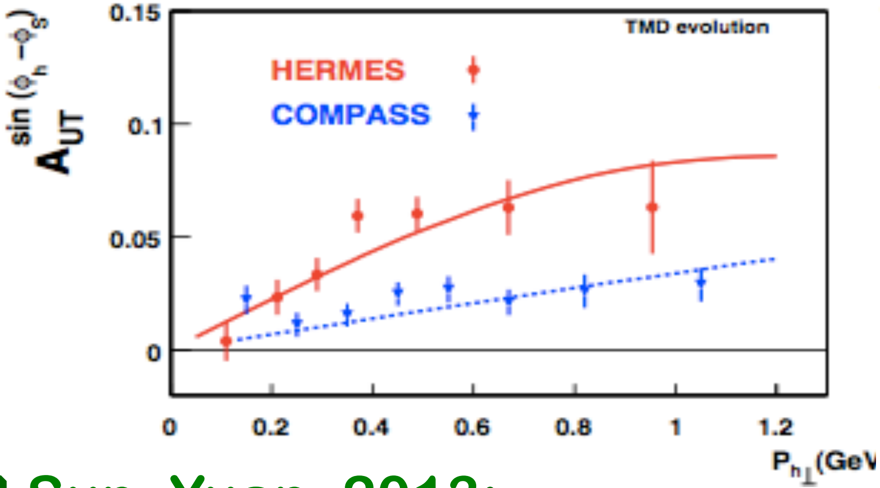
- ☐ **Up quark Sivers function:**

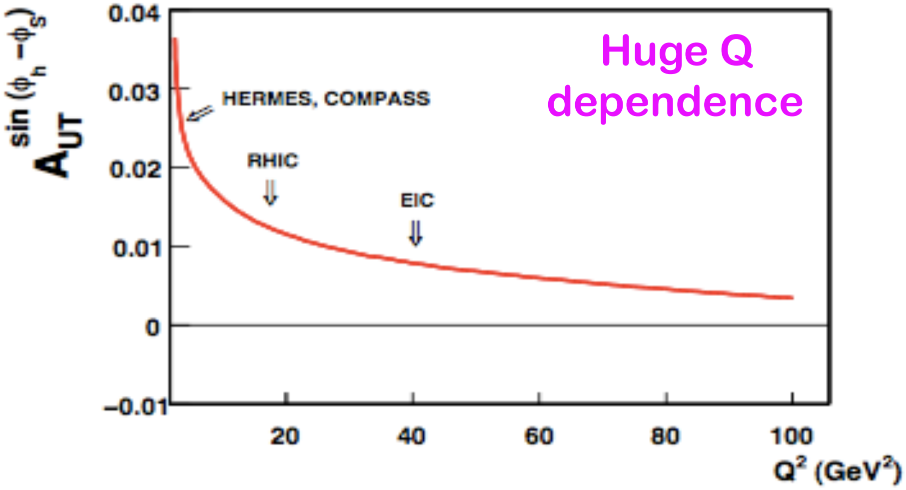


Very significant growth in the width of transverse momentum

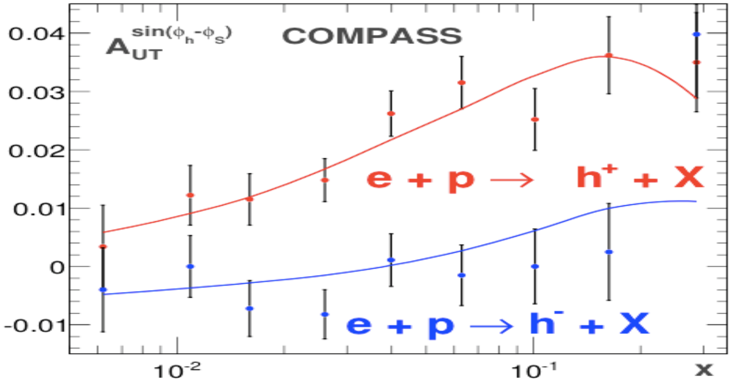
# Nonperturbative input to Sivers function

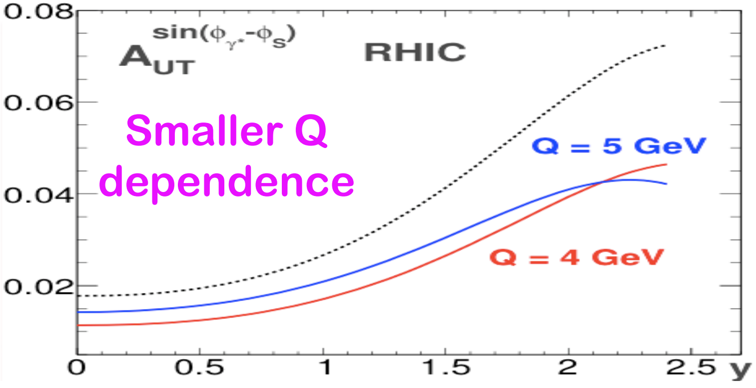
- ☐ Aybat, Prokudin, Rogers, 2012:




Huge Q dependence

- ☐ Sun, Yuan, 2013:



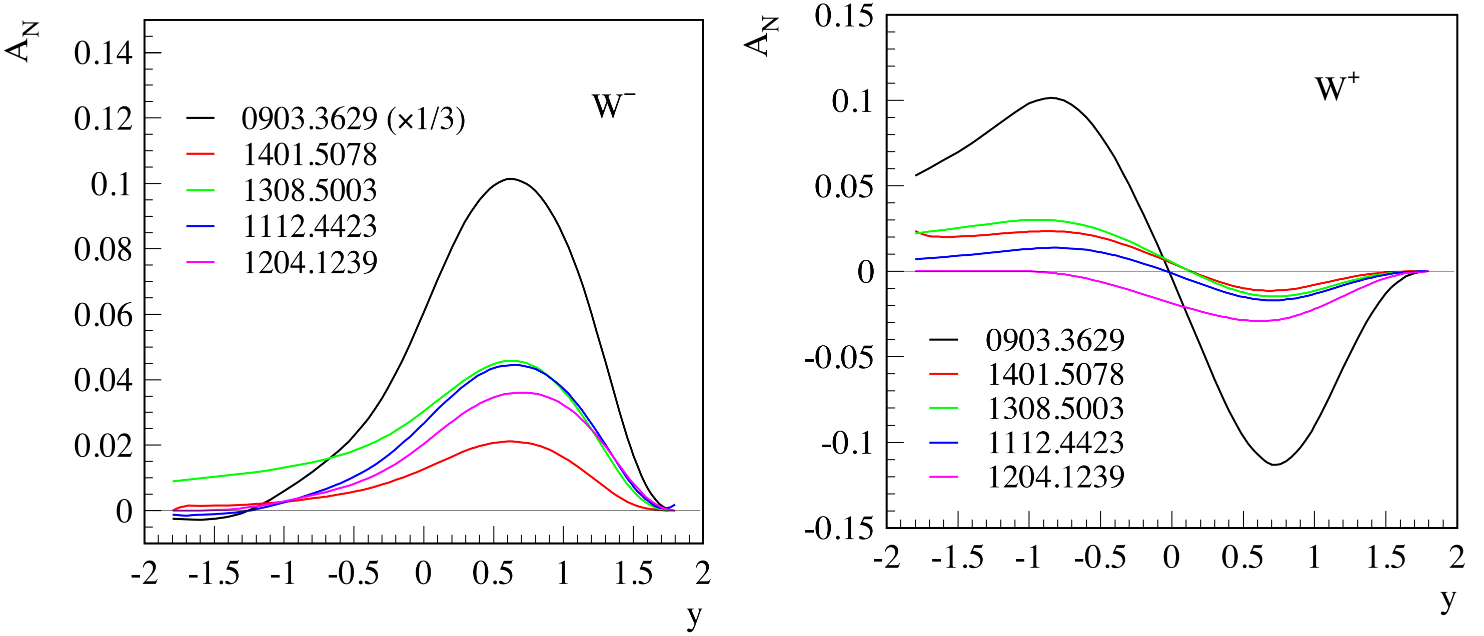

Smaller Q dependence

*No disagreement on evolution equations!*

**Issues:** Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

# “Predictions” for $A_N$ of W-production at RHIC?

- ☐ **Sivers Effect:**
  - ✧ Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
  - ✧ QCD Prediction: Sign change of Sivers function from SIDIS and DY
- ☐ **Current “prediction” and uncertainty of QCD evolution:**

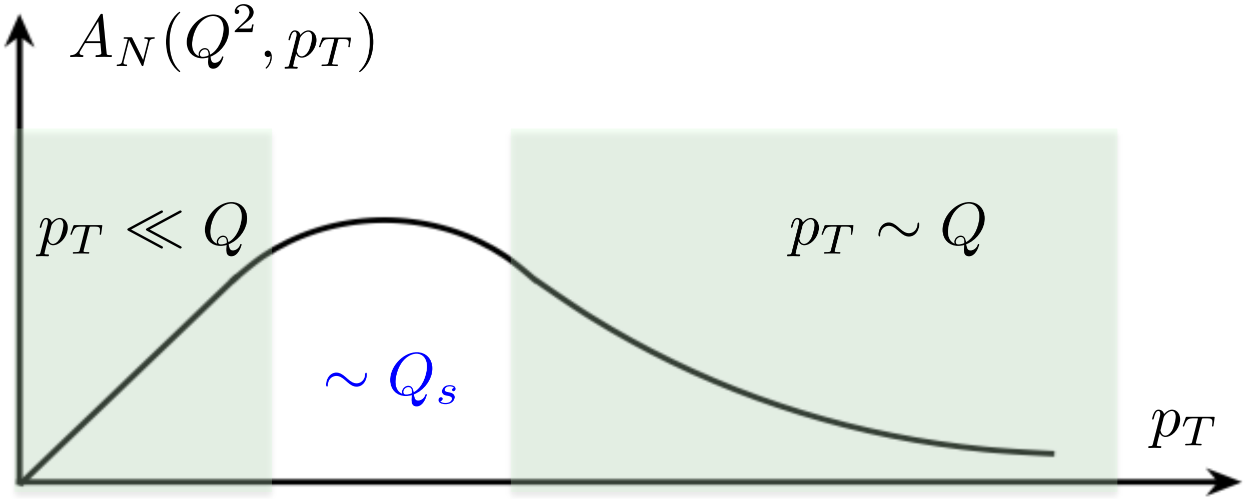


**TMD collaboration proposal: Lattice, theory & Phenomenology**

**RHIC is the excellent and unique facility to test this (W/Z – DY)!**

# Drell-Yan (or SIDIS) from low $p_T$ to high $p_T$

- ☐ Covers both double-scale and single-scale cases:



**TMD** **Collinear Factorization**

- ☐ TMD factorization to collinear factorization:

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorizations are consistent in the overlap region: $\Lambda_{\text{QCD}} \ll p_T \ll Q$

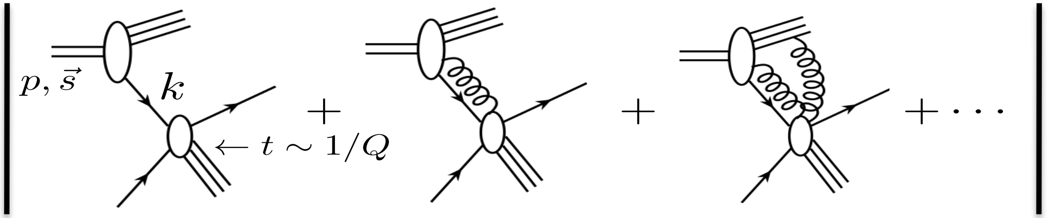
$A_N$ finite – requires correlation of multiple collinear partons

No probability interpretation! New opportunities!

# How collinear factorization generates SSA?

## ❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$



$\Big|^2 \left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – **Expansion**

$$\sigma(Q,s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

**Too large to compete!**

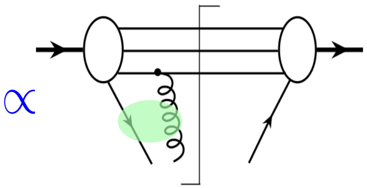
**Three-parton correlation**

## ❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

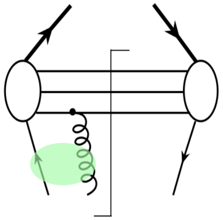
$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$



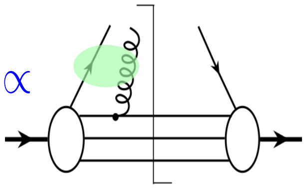
Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$



Kang, Yuan, Zhou, 2010

$T^{(3\sigma)}(x,x) \propto$



Kanazawa, Koike, 2000

**Integrated information on parton's transverse motion!**

***Quantum interference between a single and a composite state***

# Inclusive single hadron production

- ☐ **One large scale:** $A(p_A, S_\perp) + B(p_B) \rightarrow h(p) + X$ **with $p_T$ >> $\Lambda_{QCD}$**

  **Three identified hadrons:** $A(p_A, S_\perp),\ B(p_B),\ h(p)$

- ☐ **QCD collinear factorization:** Qiu, Sterman, 1991, 1998, …

$$
\begin{aligned}
A_N \propto\ & \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp) \\
&= T^{(3)}_{a/A}(x, x, S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^{T}_{ab\to c} \otimes D_{h/c}(z) \\
&+ \delta q_{a/A}(x, S_\perp) \otimes T^{(3\sigma)}_{b/B}(x', x') \otimes \hat{\sigma}^{\phi}_{ab\to c} \otimes D_{h/c}(z) \\
&+ \delta q_{a/A}(x, S_\perp) \otimes \phi_{b/B}(x', x') \otimes \hat{\sigma}^{D}_{ab\to c} \otimes D^{(3)}_{h/c}(z, z)
\end{aligned}
$$

  **Leading power contribution to cross section cancels!**
  **Only one twist-3 distribution at each term!**

- ☐ **Three-type contributions:**

  **Spin-flip: Twist-3 correlation functions, transversity distributions**

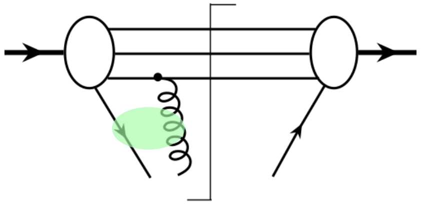
  **Phase: Interference between the real part and imaginary part of the scattering amplitude**

# Twist-3 correlation functions

## ☐ Twist-3 polarized correlation functions:

Efremov, Teryaev, 1982, …
Qiu, Sterman, 1991, …

$T^{(3)}(x, x, S_\perp) \;\propto$

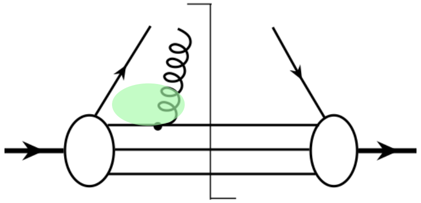


**Moment of Sivers function**

## ☐ Twist-3 unpolarized correlation functions:

Kanazawa, Koike 2000, …

$T^{(3\sigma)}(x', x') \;\propto$

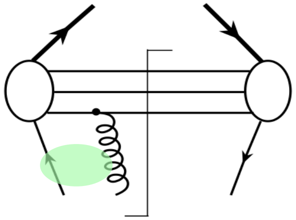


**Moment of Boer-Mulders function**

## ☐ Twist-3 fragmentation functions:

Kang, Yuan, Zhou, 2010

$D^{(3)}(z, z) \;\propto$



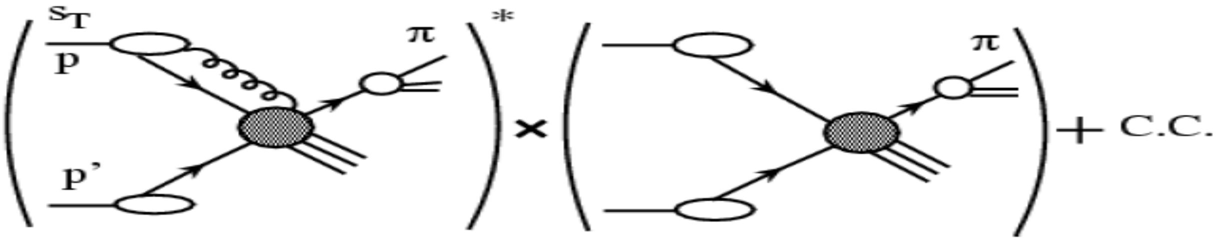
**Moment of Collins function?**

**All these correlation functions have No probability interpretation!**

**Quantum interference between a single and a composite state**

# SSAs generated by twist-3 PDFs

❑ **First non-vanish contribution – interference:**



❑ **Dominated by the derivative term – forward region:**

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right]$$
$$\otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

Qiu, Sterman, 1998, …

$$\boxed{A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if } T_F(x,x) \propto q(x) \propto (1-x)^n}$$

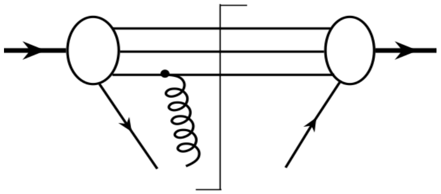
❑ **Complete leading order contribution:**

Kouvaris, Qiu, Vogelsang, Yuan, 2006

$$E_\ell\frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1\frac{dz}{z^2}D_{c\to h}(z)\int_{x'_{\min}}^1\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_{b/B}(x')\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)$$
$$\times\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right]H_{ab\to c}(\hat{s},\hat{t},\hat{u}).$$

# Twist-3 distributions relevant to $A_N$

## ❑ Two-sets Twist-3 correlation functions:



**No probability interpretation!**

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

**Kang, Qiu, 2009**

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

**Role of color magnetic force!**

## ❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

## ❑ Twist-3 fragmentation functions:

**See Kang, Yuan, Zhou, 2010, Kang 2010**

# Test QCD evolution at twist-3 level

Kang, Qiu, 2009; Yuan, Zhou, 2009
Vogelsang, Yuan, 2009, Braun et al, 2009

## ❑ Scaling violation – "DGLAP" evolution:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \otimes \underbrace{\begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix}}_{\int d\xi \int d\xi_2}$$

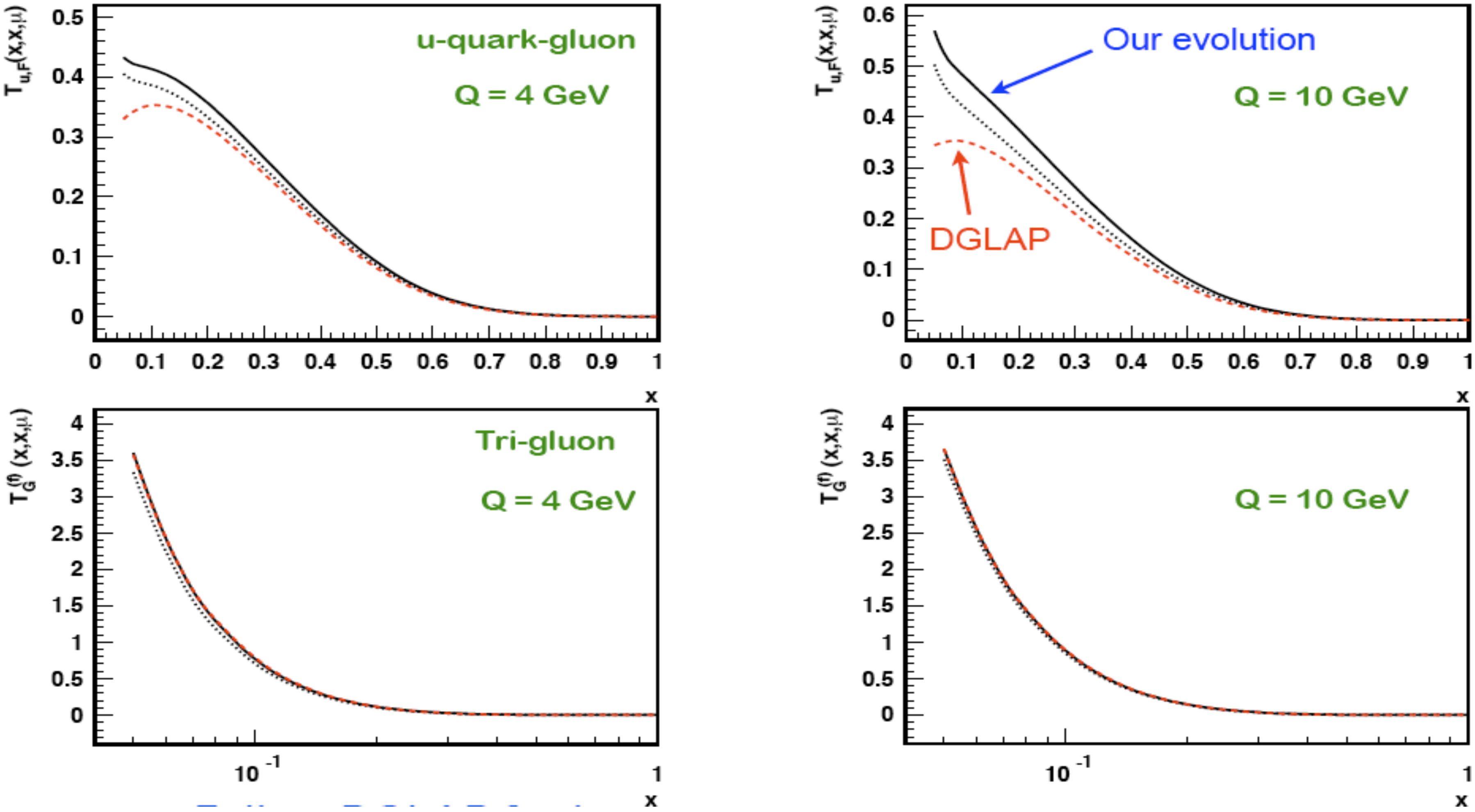
## ❑ Evolution equation – consequence of factorization:

**Factorization:** $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

**DGLAP for f$_2$:** $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

**Evolution for f$_3$:** $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left( \dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

# Scaling violation of twist-3 correlations?



- ✧ **Follow DGLAP at large x**
- ✧ **Large deviation at low x (stronger correlation)**

Kang, Qiu, PRD, 2009

# Twist-3 fragmentation contribution

Metz, Pitonyak, PLB723 (2013)

- ❑ **Leading order results:**

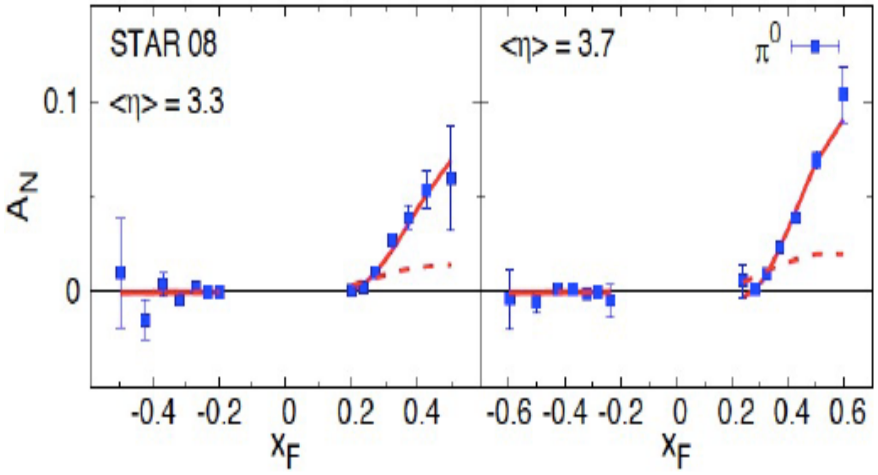
$$\frac{P_h^0\, d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\, S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x'S+T/z} \frac{1}{-x\hat{u}-x'\hat{t}}$$

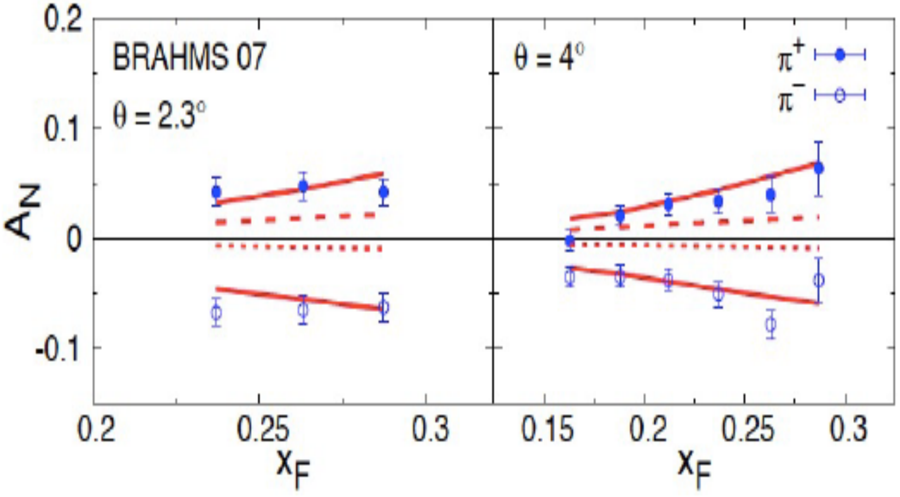
$$\times \frac{1}{x} h_1^a(x)\, f_1^b(x') \left\{ \left( \hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z} H^{C/c}(z)\, S_H^i \right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\, PV \frac{1}{\frac{1}{z}-\frac{1}{z_1}}\, \hat{H}_{FU}^{C/c,\Im}(z,z_1)\, \frac{1}{\xi}\, S_{\hat{H}_{FU}}^i \right\}$$

- ❑ **New fitting results:**

Kanazawa, Koike, Metz, Pitonyak, PRC89, 2014

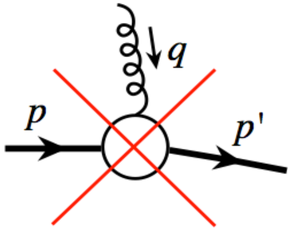




- - - - - **Without FF contribution**

# Spatial imaging of quarks and gluons

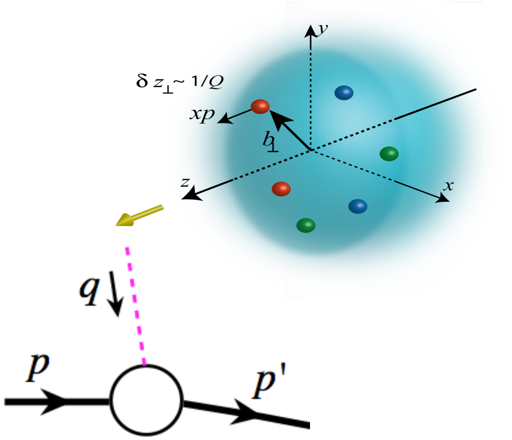
❑ **NO exclusive color form factor:**



✧ Exchange of a colorless "object"

✧ "Localized" probe

✧ Control of exchanging momentum



# Spatial imaging of quarks and gluons

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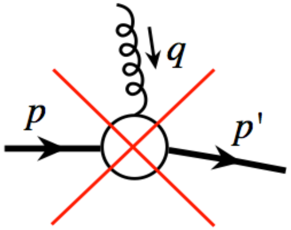

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- ✧ “Localized” probe
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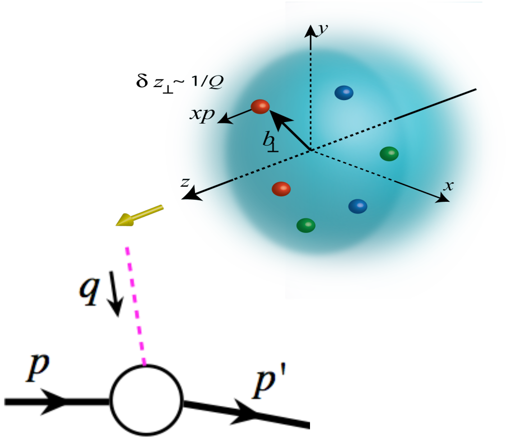
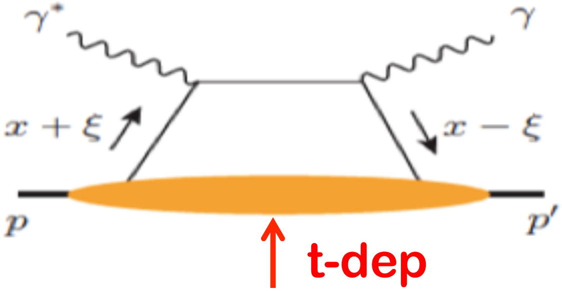


## ❑ Exclusive processes – DVCS:



$$\frac{d\sigma}{dx_B dQ^2 dt}$$ ➡ **GPDs** $H_q(x,\xi,t,Q), E_q(x,\xi,t,Q), \ldots$

$$t = (p' - p)^2$$ **F.T. of t-dep**

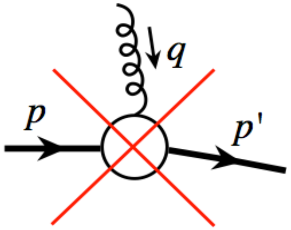
$$\xi = (P' - P) \cdot n/2$$ ➡ **Spatial distributions**

**JLab 12: Valence quarks**

**EIC: Sea quarks**

# Spatial imaging of quarks and gluons

## ❑ NO exclusive color form factor:



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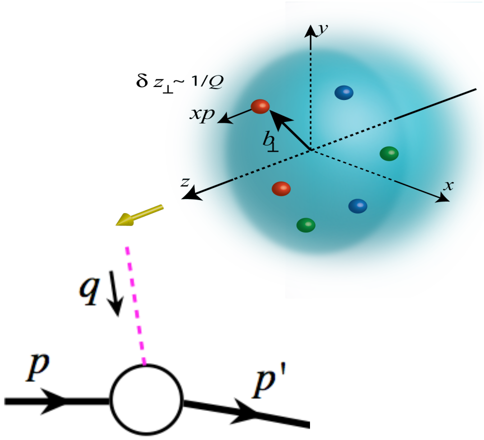

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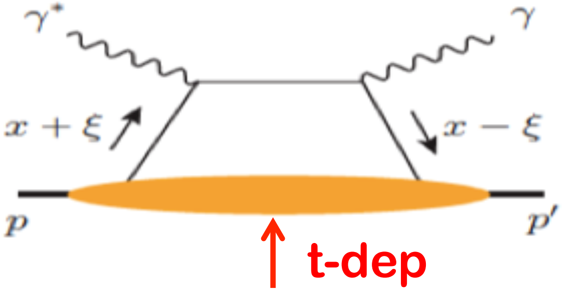


**GPDs**

$$\frac{d\sigma}{dx_B dQ^2 dt} \Longrightarrow H_q(x,\xi,t,Q), E_q(x,\xi,t,Q), \ldots$$

$$t = (p'-p)^2$$

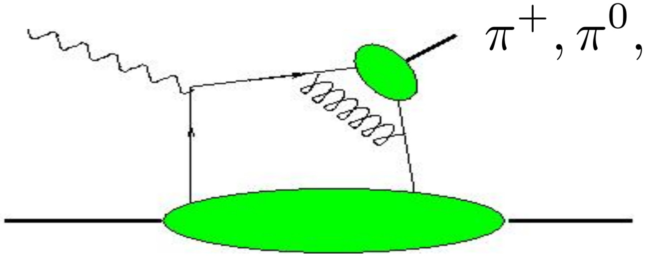
F.T. of t-dep

$$\xi = (P'-P)\cdot n/2 \Longrightarrow$$ Spatial distributions

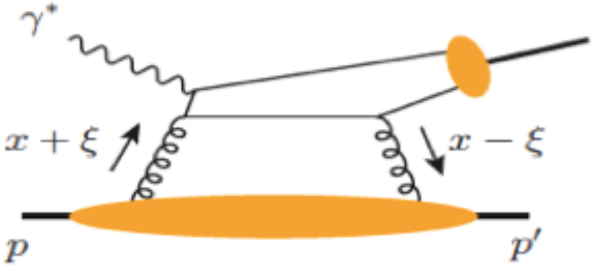
JLab 12: Valence quarks

EIC: Sea quarks

## ❑ Exclusive meson production:

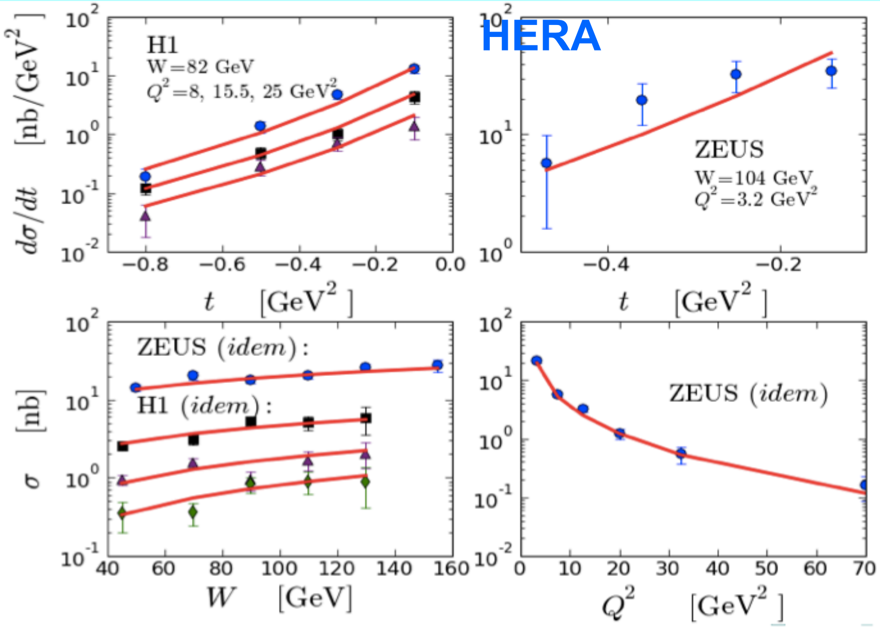


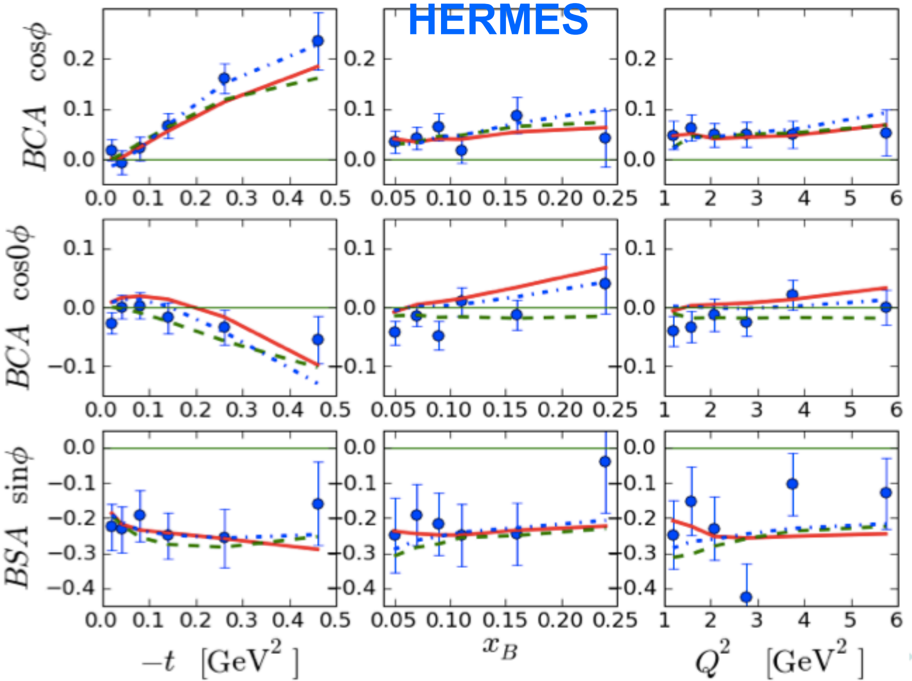
$\pi^+, \pi^0, \ldots$

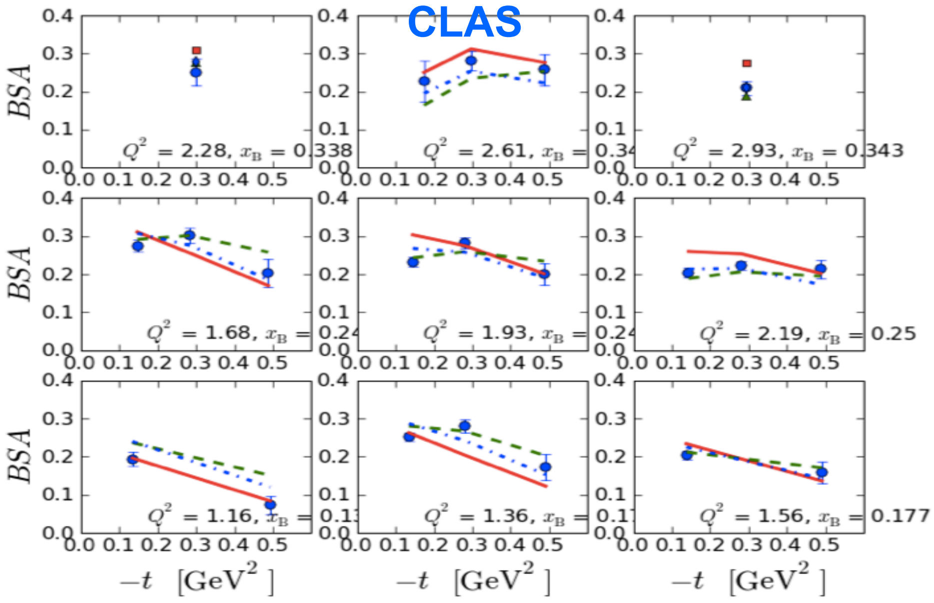


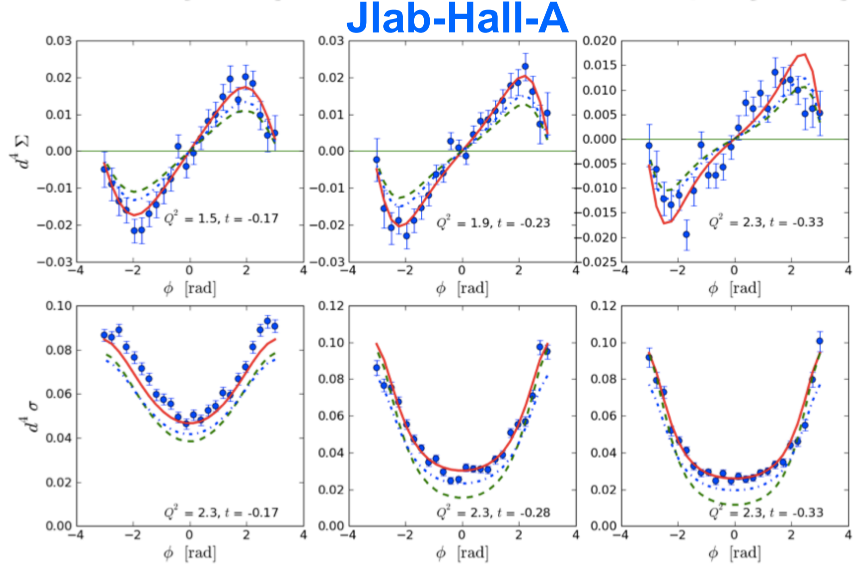
J/Ψ, Φ, ...

# Exclusive DIS – measureable



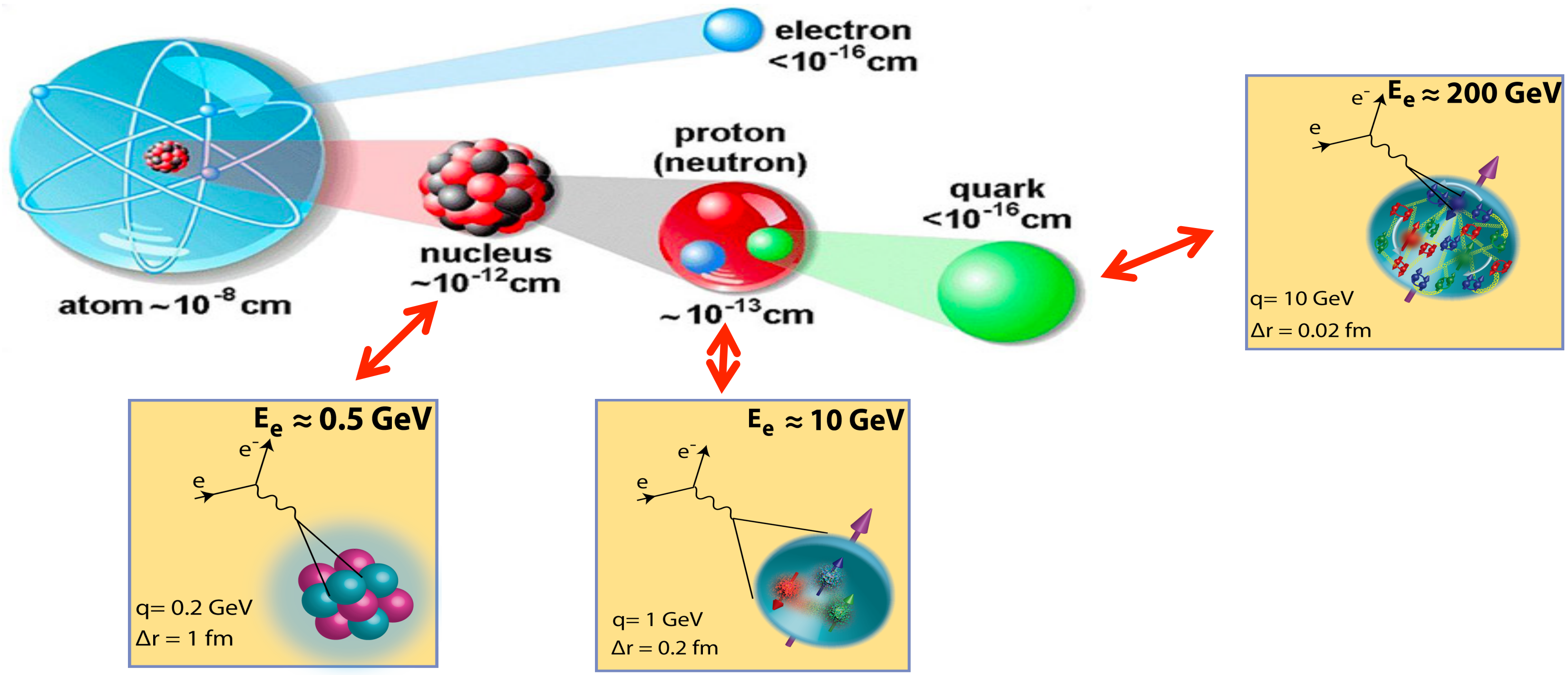






# The future: Electron-Ion Collider (EIC)

- **A giant "Microscope"** – "see" quarks and gluons by breaking the hadron



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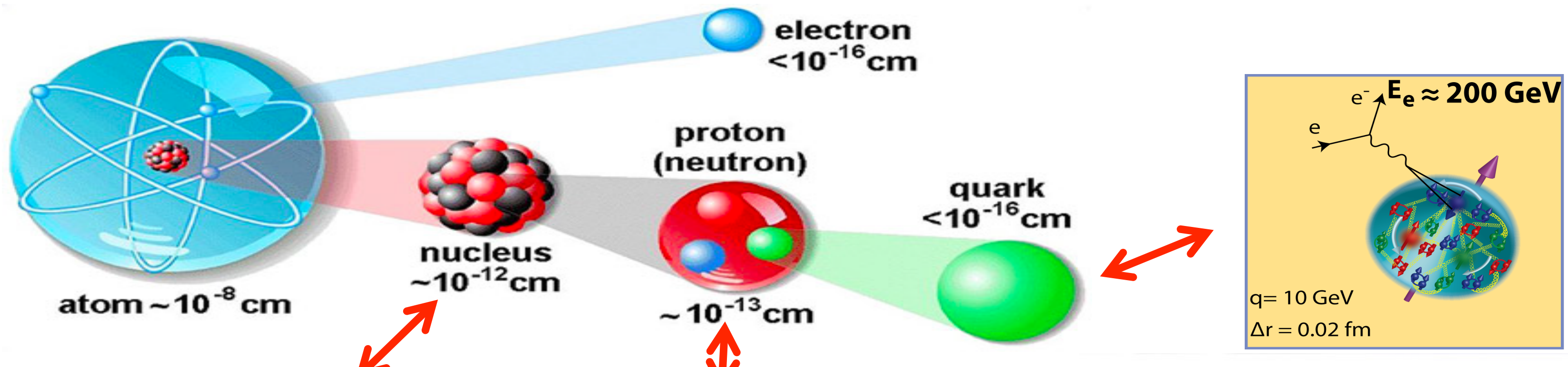
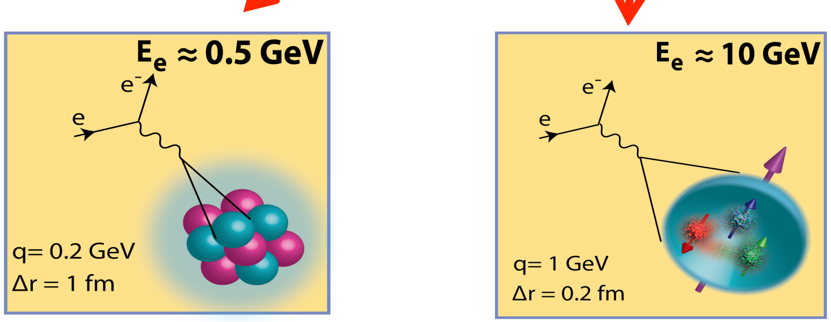
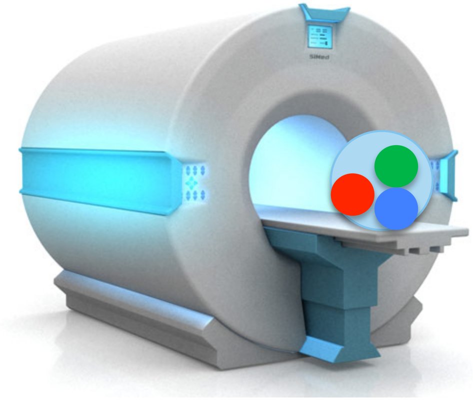


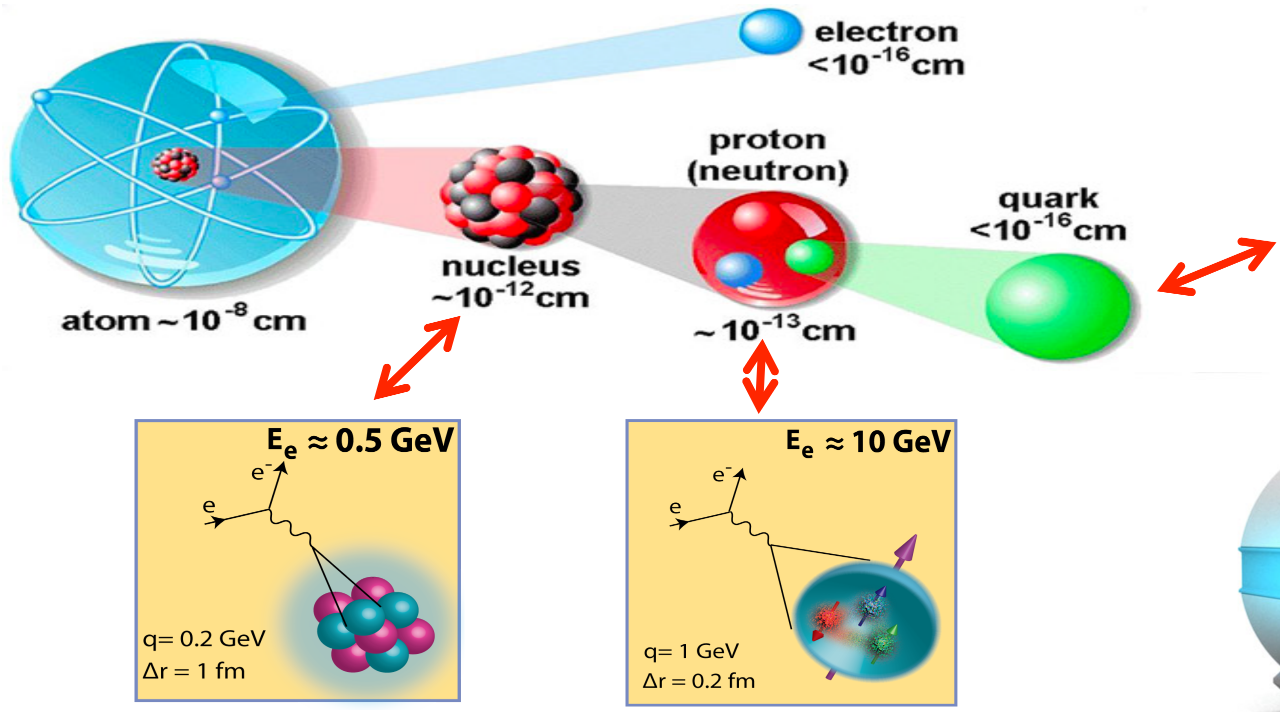


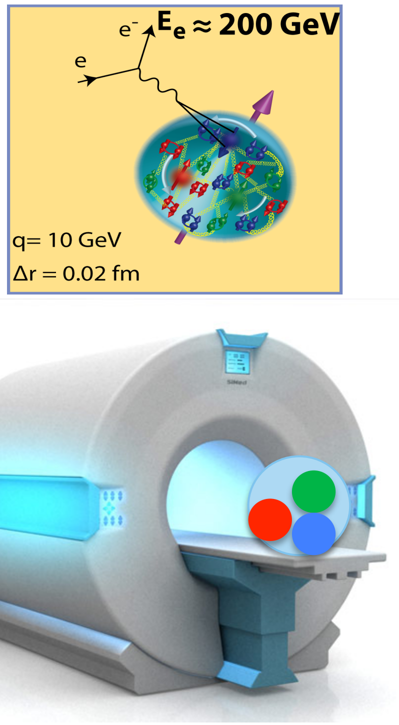


- **A sharpest "CT" (better than 1/10 fm resolution)**
  - "imagine (cat-scan)" nucleon and nuclei without breaking them

# The future: Electron-Ion Collider (EIC)

❑ A giant “Microscope” – “see” quarks and gluons by breaking the hadron





❑ A sharpest “CT” (better than 1/10 fm resolution)

– “imagine (cat-scan)” nucleon and nuclei without breaking them

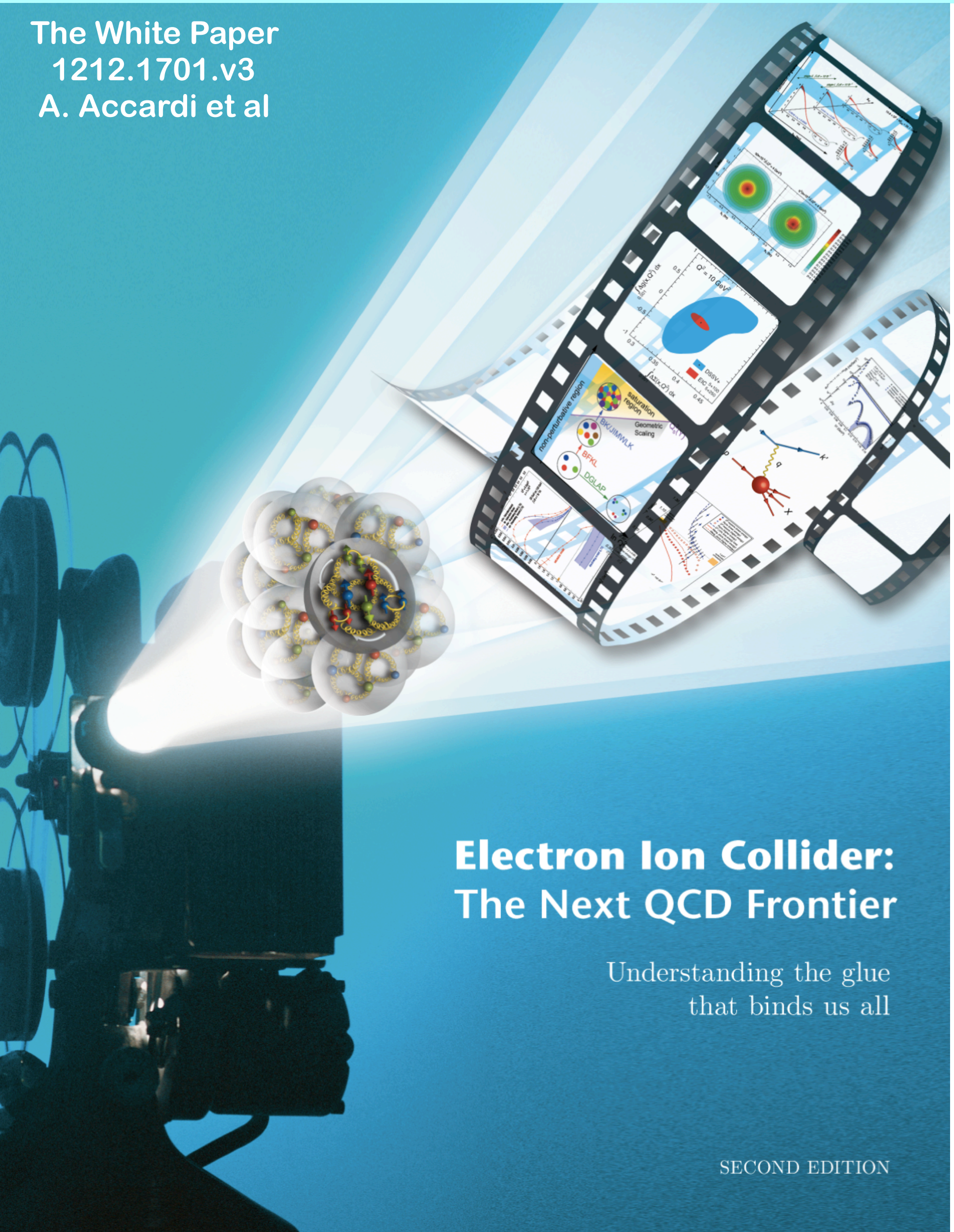
❑ Why now?

Exp – advances in luminosity, energy reach, detection capability, …

Thy – breakthrough in factorization – “see” confined quarks and gluons, …

# US EIC – Science & Machine designs

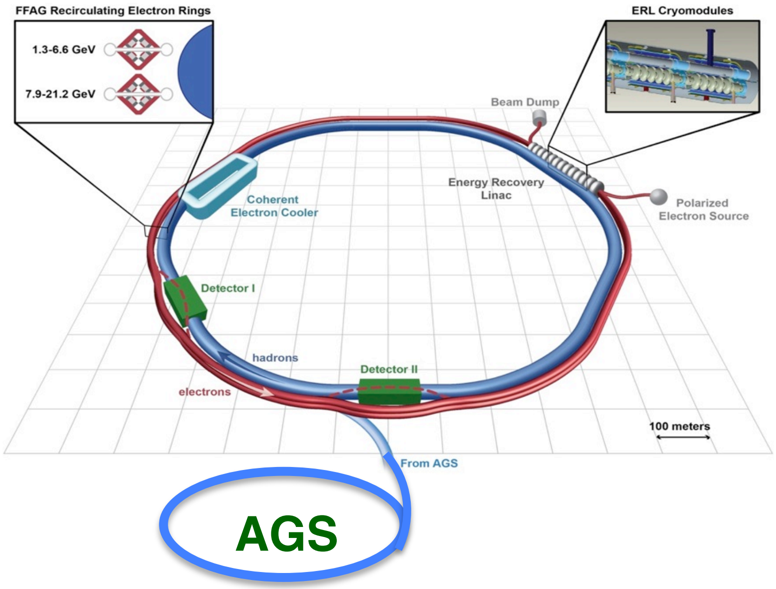
The White Paper
1212.1701.v3
A. Accardi et al

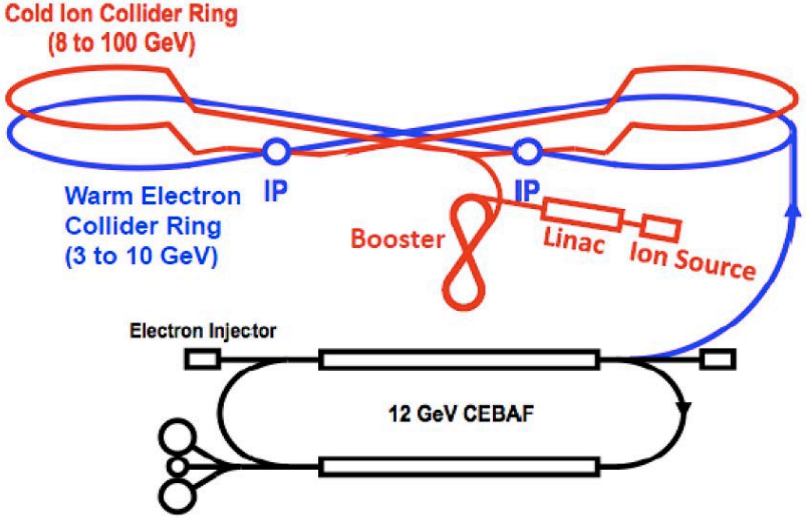


**Electron Ion Collider:**
**The Next QCD Frontier**

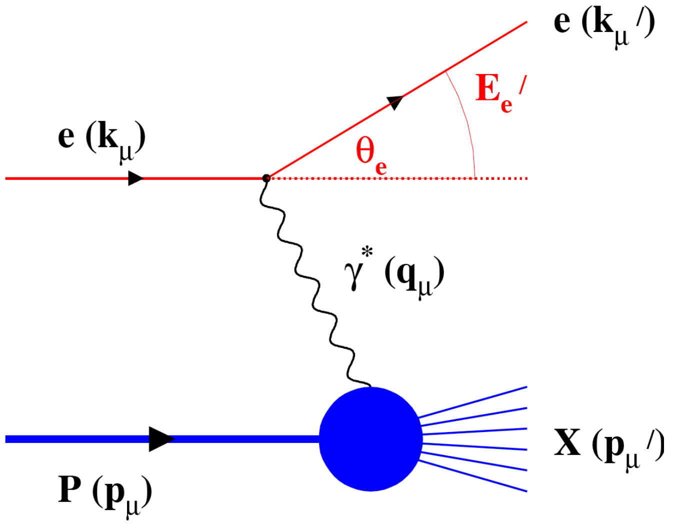
Understanding the glue
that binds us all

SECOND EDITION





# US EIC: Microscope with superfine control



$Q^2$ → Measure of resolution

$y$ → Measure of inelasticity

$x$ → Measure of momentum fraction of the struck quark in a proton

$$Q^2 = S\ x\ y$$

**Inclusive events**: e+p/A → e'+X

Detect only the scattered lepton in the detector

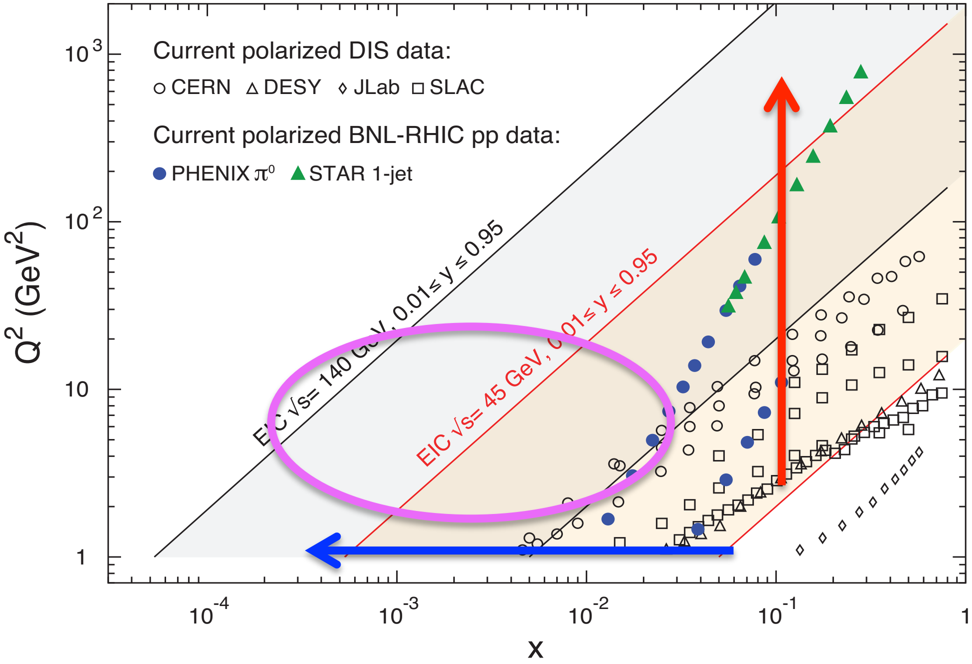
**Semi-Inclusive events**: e+p/A → e'+h($\pi$,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets in the detector

**Exclusive events:** e+p/A → e'+ p'/A'+ h($\pi$,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

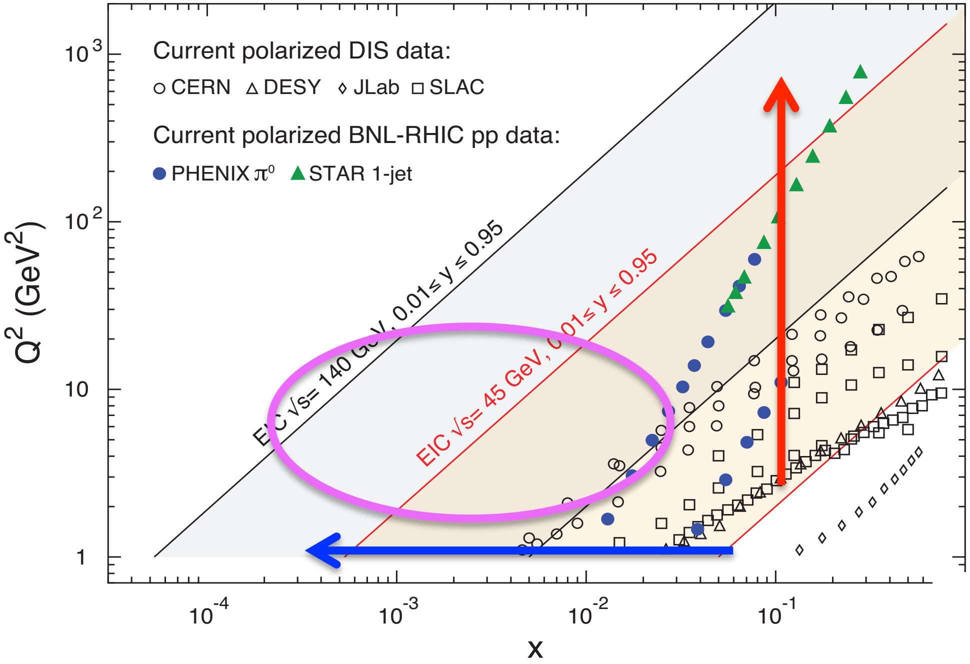
# US EIC – Kinematic reach & properties



**For e-N collisions at the EIC:**

- ✓ Polarized beams: e, p, d/$^3$He
- ✓ Variable center of mass energy
- ✓ Wide $Q^2$ range → evolution
- ✓ Wide x range → spanning from valence to low-x physics
- ✓ 100-1K times of HERA Luminosity

# US EIC – Kinematic reach & properties



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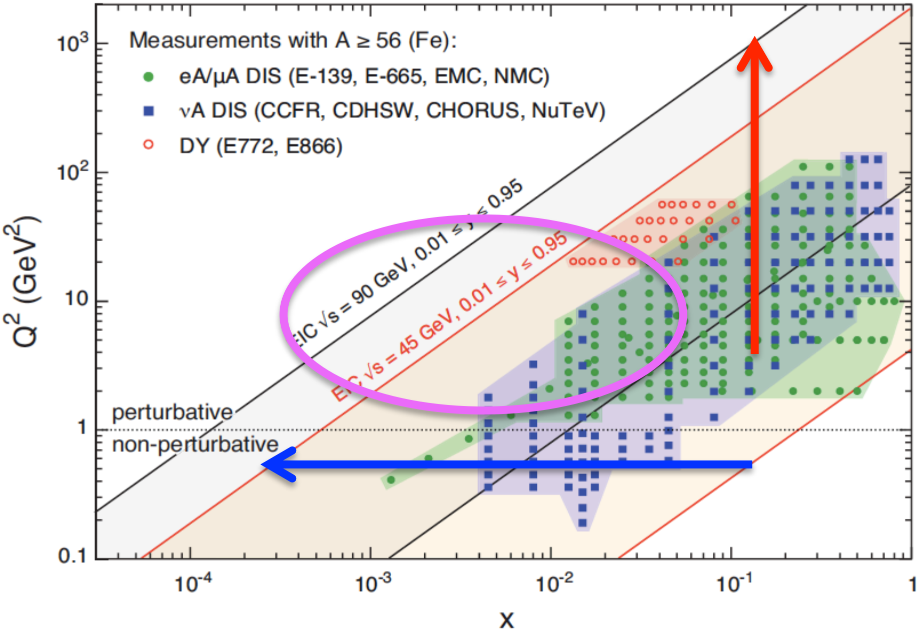
## For e-A collisions at the EIC:

- ✓ Wide range in nuclei
- ✓ Variable center of mass energy
- ✓ Wide $Q^2$ range (evolution)
- ✓ Wide x region (high gluon densities)



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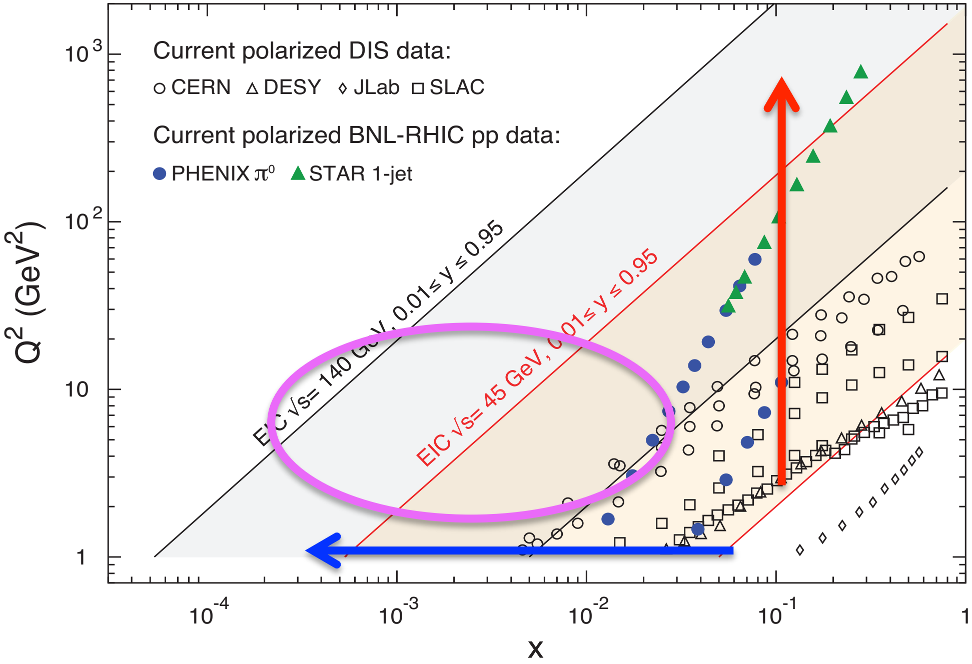

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- ✓ Variable center of mass energy
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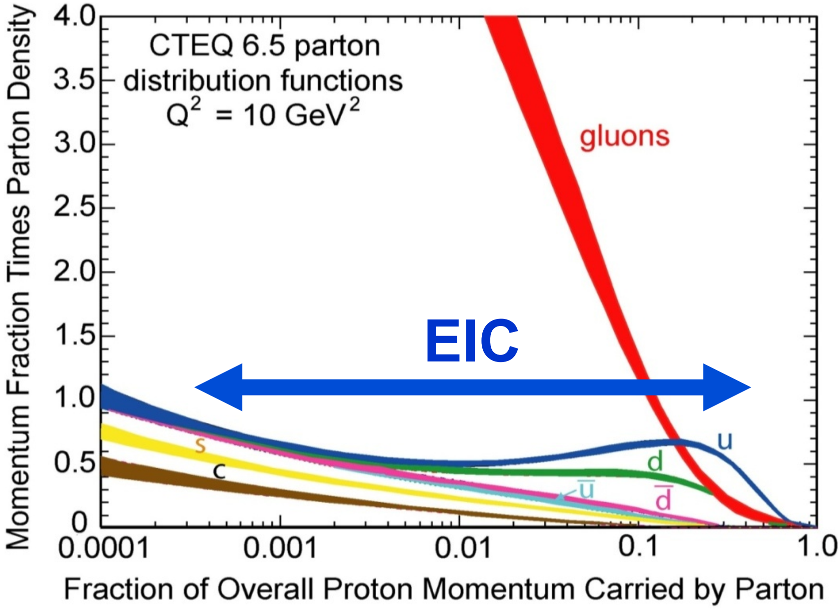
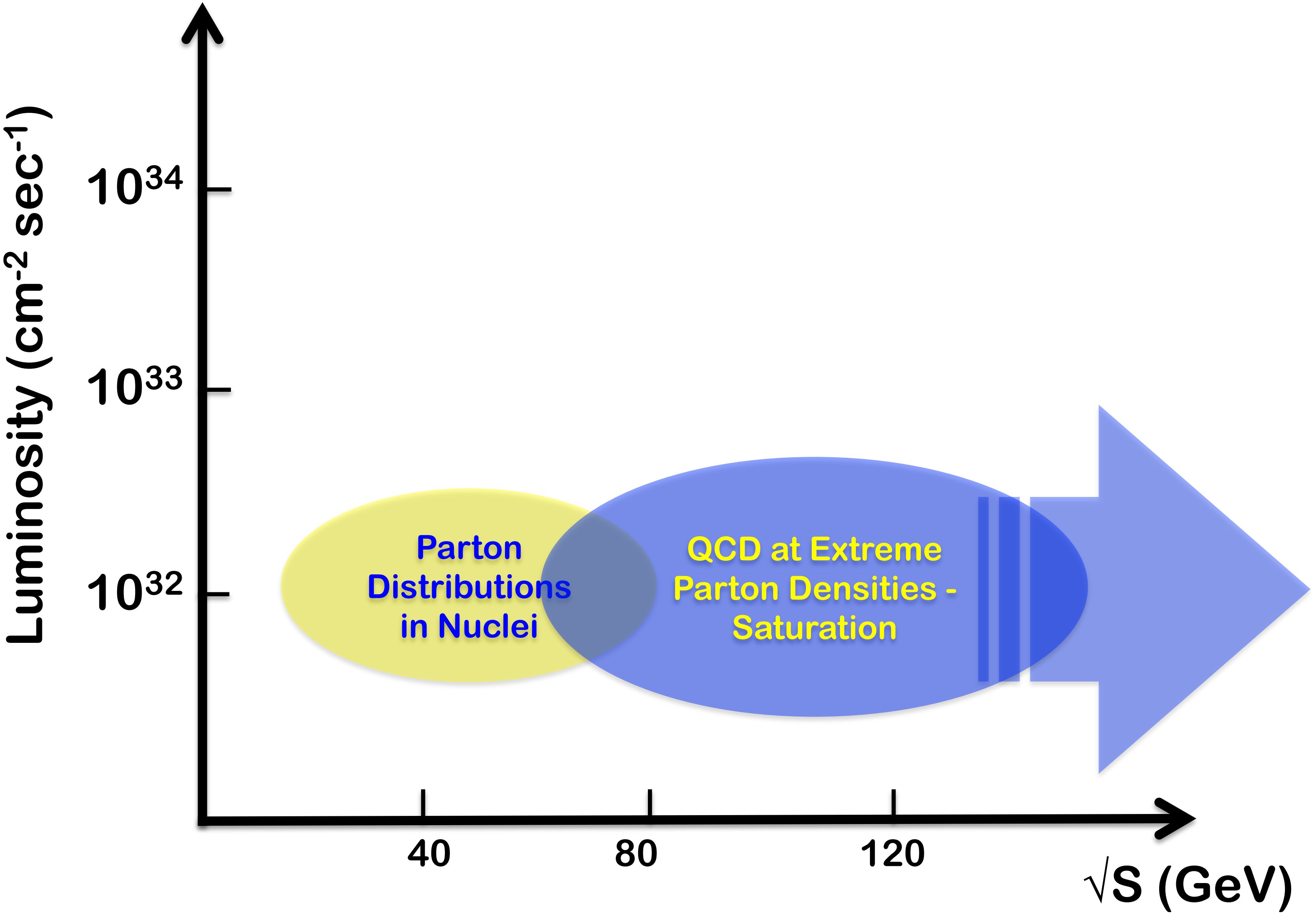
- ✓ Wide x region (high gluon densities)

*EIC explores the "sea" and the "glue", the "valence" with a huge level arm*



# US EIC – Physics vs. Luminosity & Energies



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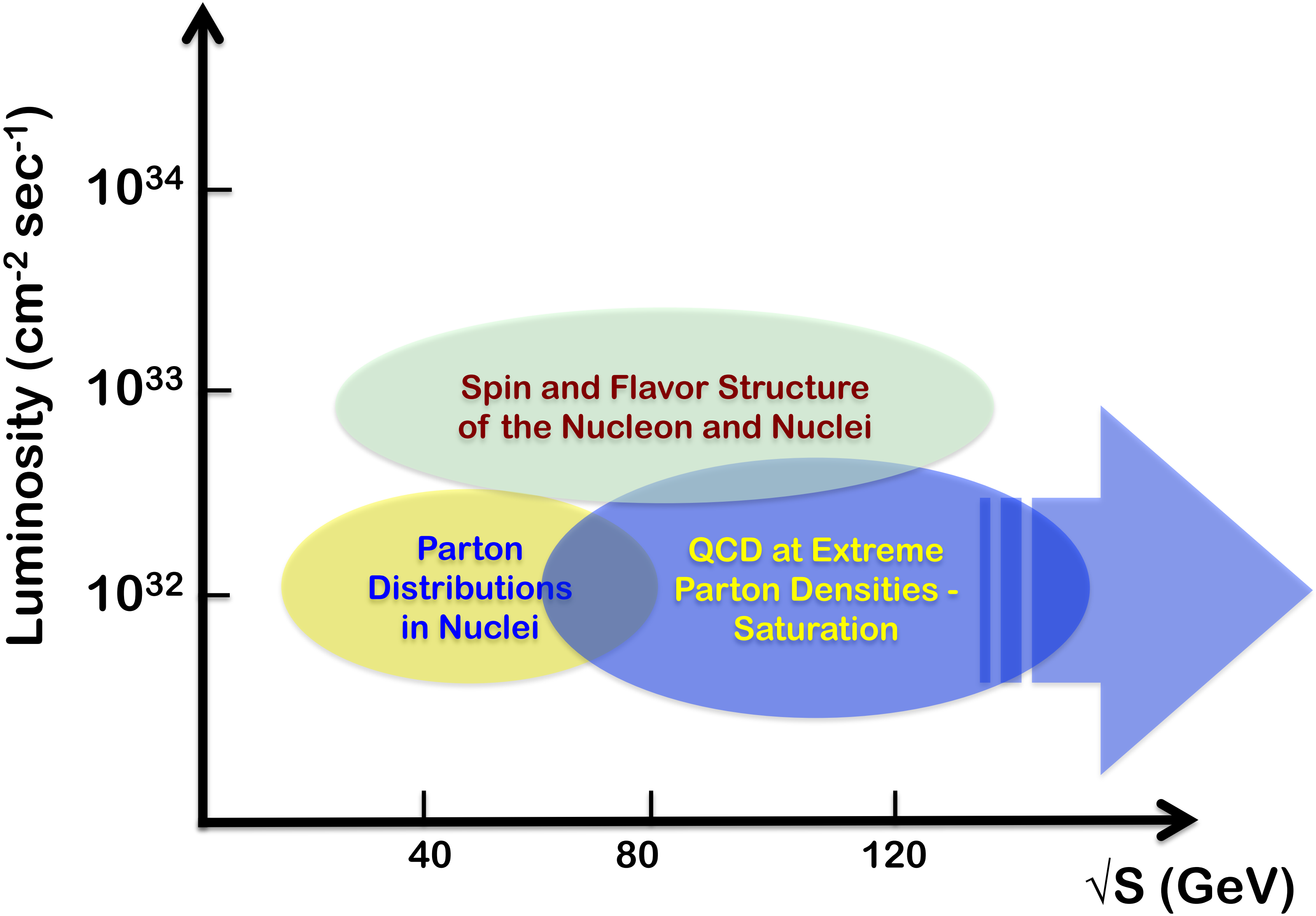

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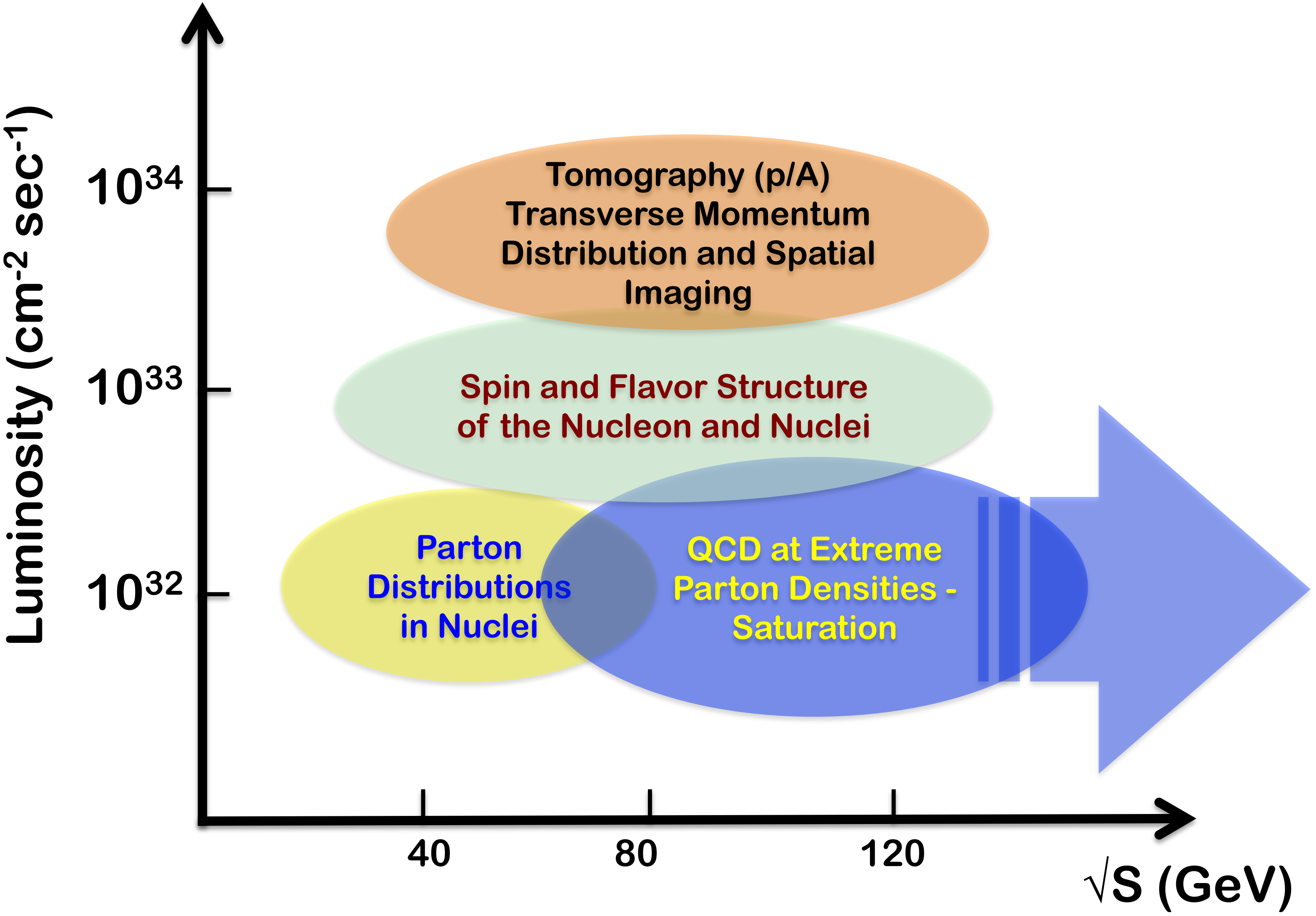


Tomography (p/A)
Transverse Momentum
Distribution and Spatial
Imaging

Spin and Flavor Structure
of the Nucleon and Nuclei

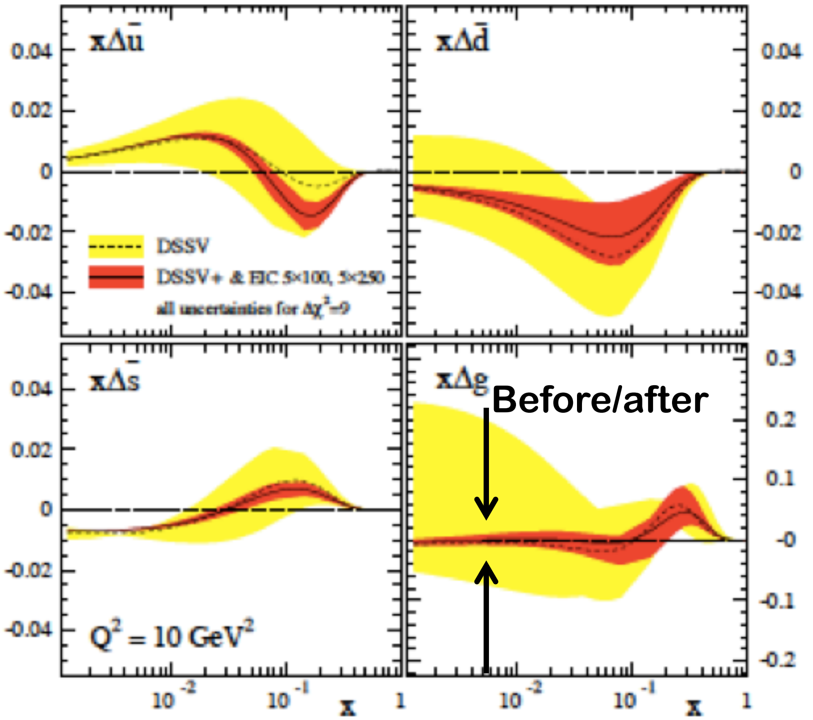
Parton
Distributions
in Nuclei

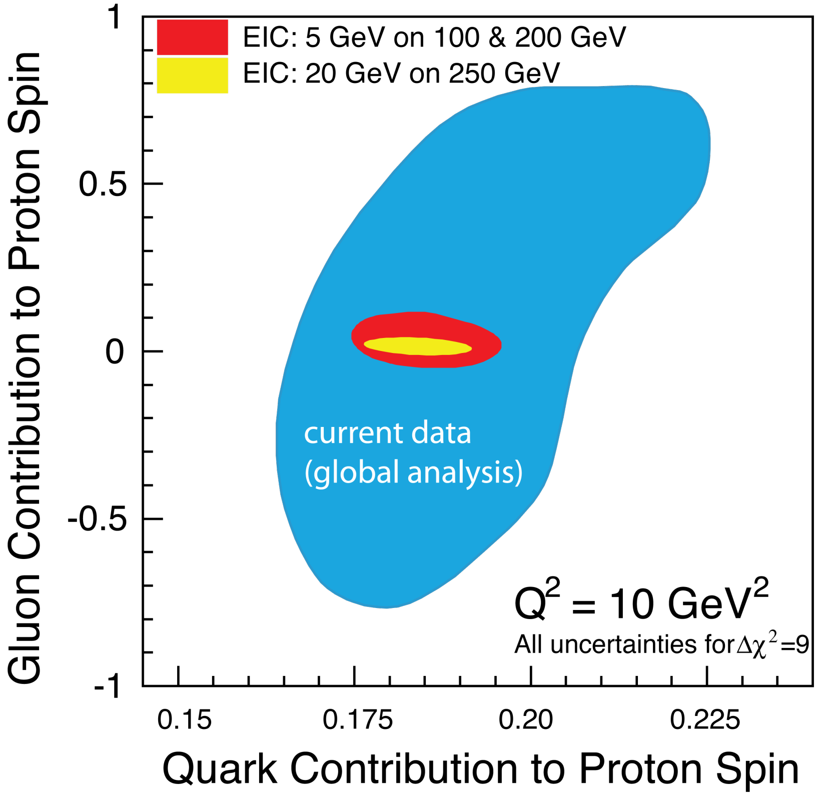
QCD at Extreme
Parton Densities -
Saturation

# Our Understanding of Nucleon Spin

❑ **EIC@US – the decisive measurement (1st year of running):**

**(Low x and wide x range at EIC)**



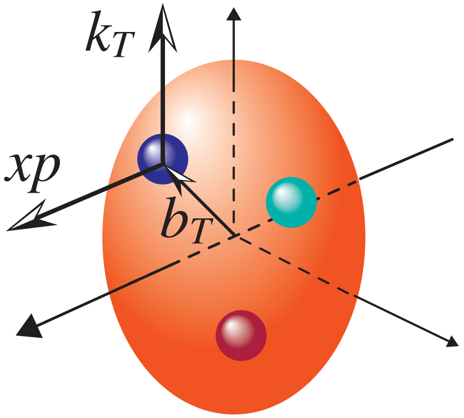


**Precision in $\Delta\Sigma$ and $\Delta g$ ➔ A clear idea of the magnitude of $L_Q+L_G$**

***No other machine in the world can achieve this!***

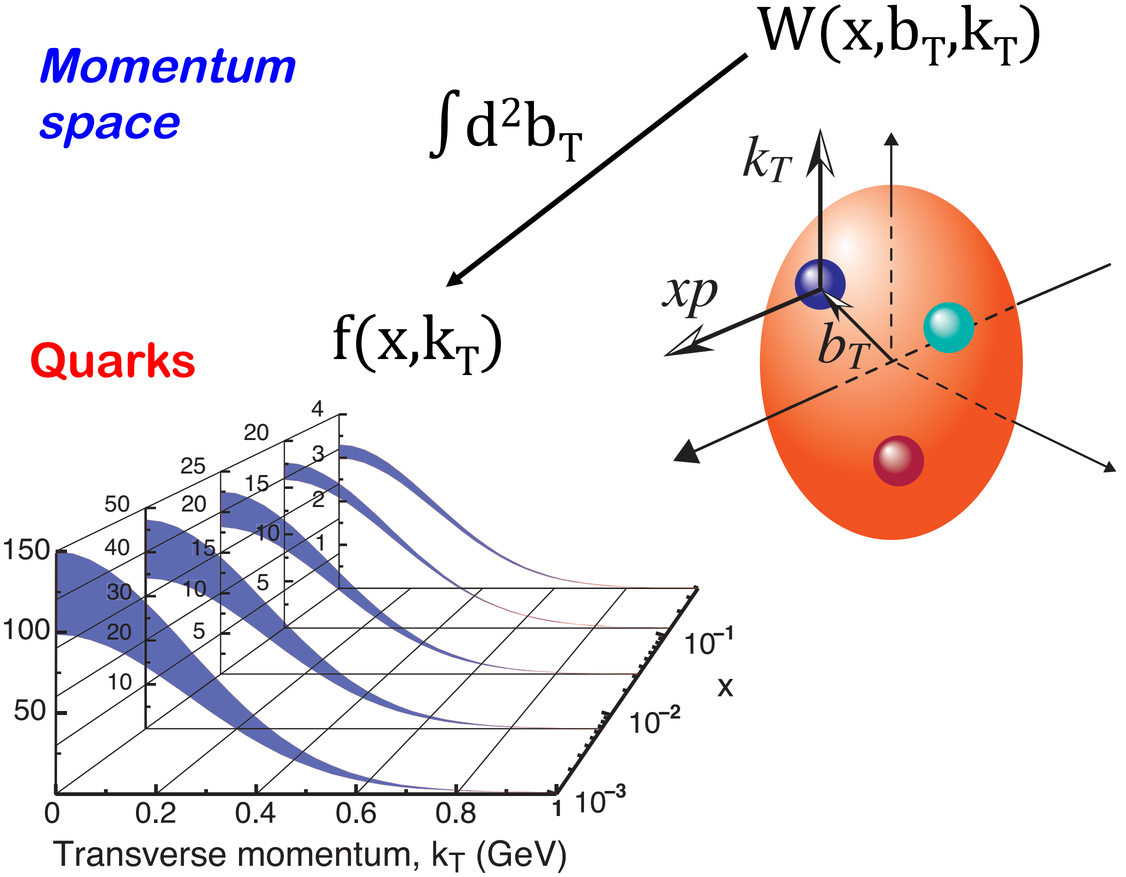
# 3-Dimensional Imaging Quarks and Gluons

$W(x, b_T, k_T)$



# 3-Dimensional Imaging Quarks and Gluons

***Momentum space***



$W(x,b_T,k_T)$

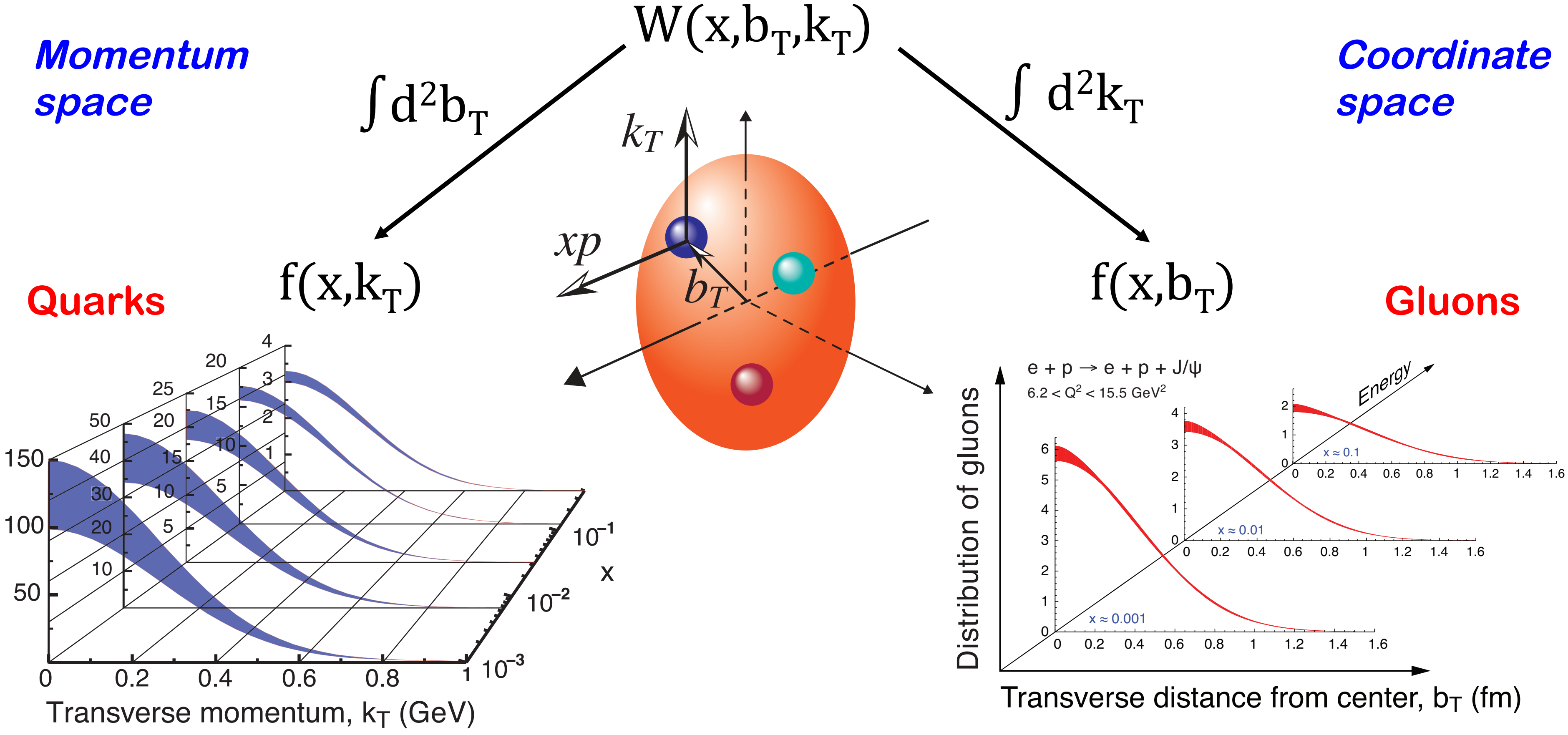
$\int d^2b_T$

$f(x,k_T)$

**Quarks**

**Spin-dependent 3D momentum space images from semi-inclusive scattering**

# 3-Dimensional Imaging Quarks and Gluons



$W(x,b_T,k_T)$

***Momentum space***

$\int d^2b_T$

$f(x,k_T)$

**Quarks**

***Coordinate space***

$\int d^2k_T$

$f(x,b_T)$

**Gluons**

**Spin-dependent 3D momentum space images from semi-inclusive scattering**

**Spin-dependent 2D (transverse spatial) + 1D (longitudinal momentum) coordinate space images from exclusive scattering**

# 3-Dimensional Imaging Quarks and Gluons

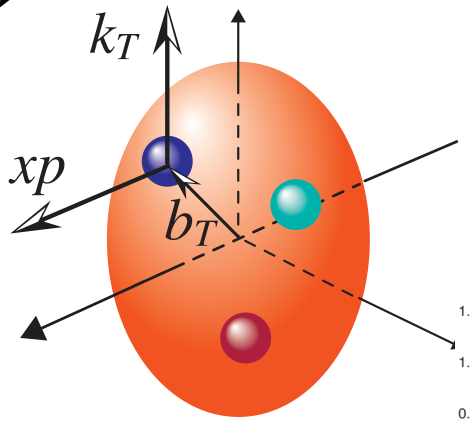
***Momentum space***

$W(x,b_T,k_T)$

***Coordinate space***

$\int d^2b_T$

$\int d^2k_T$

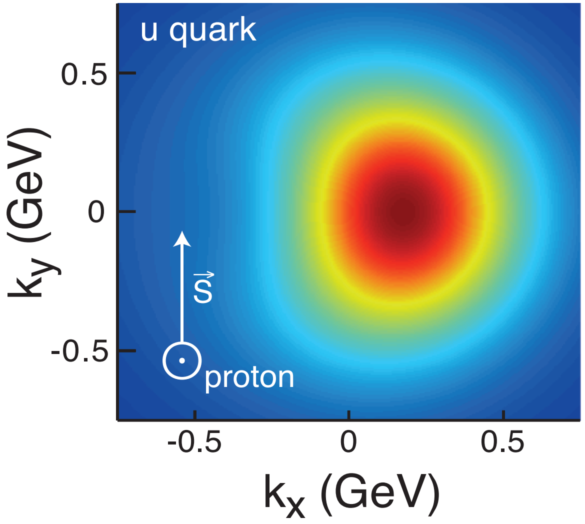


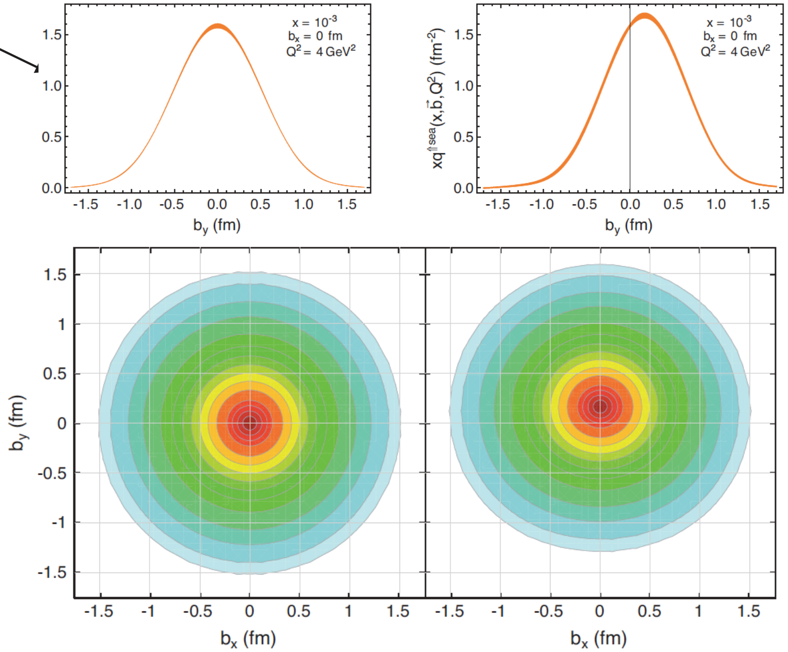
$f(x,k_T)$

$f(x,b_T)$

**Quarks**

**Gluons**





# 3-Dimensional Imaging Quarks and Gluons

***Momentum space***

***Coordinate space***

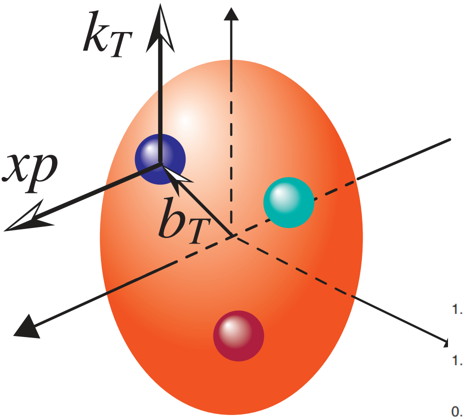
$W(x,b_T,k_T)$

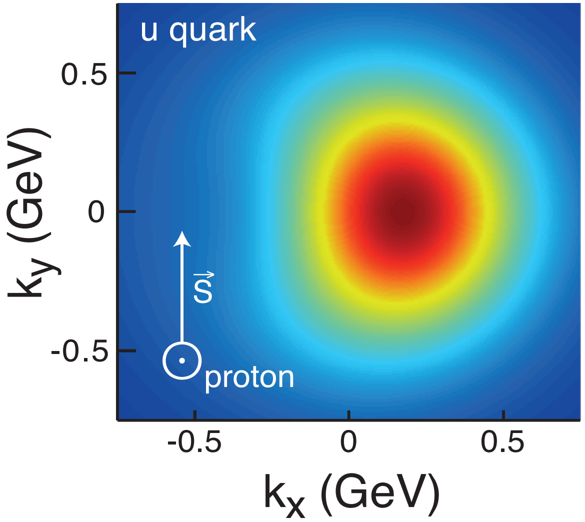
$\int d^2b_T$ $\int d^2k_T$

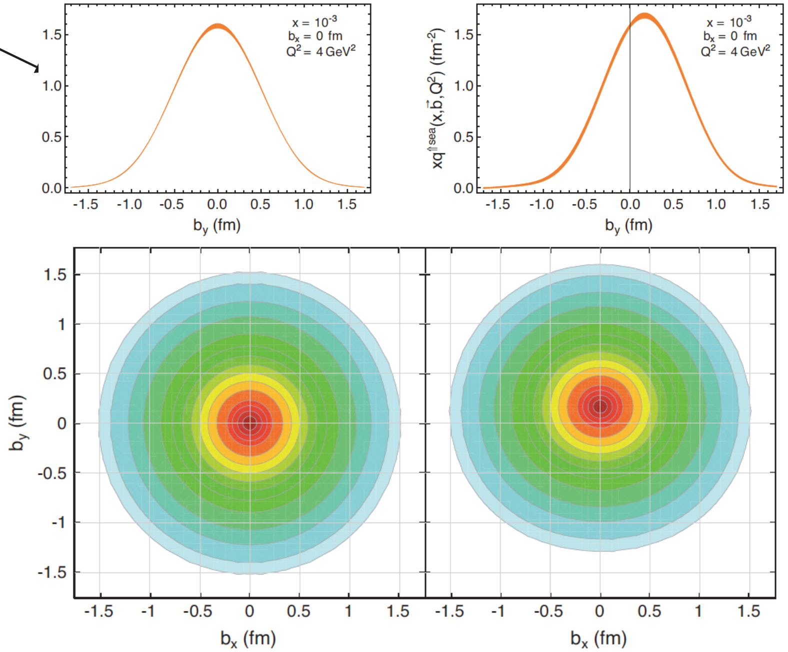
$f(x,k_T)$ $f(x,b_T)$

**Quarks**

**Gluons**







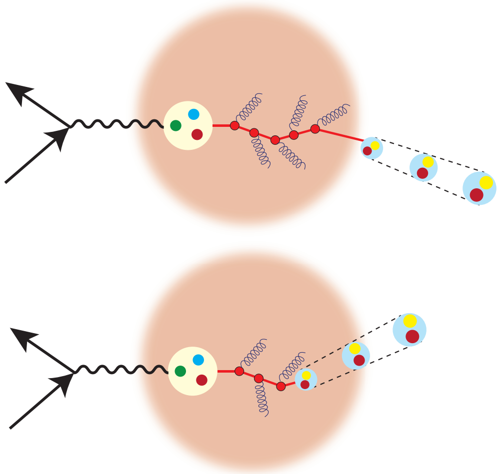
**Position *r* x Momentum *p* → Orbital Motion of Partons**

# Emergence of hadrons from partons

## Nucleus as a Femtometer sized filter

☐ **Unprecedented ν range at EIC:**

*precision & control*



$$\nu = \frac{Q^2}{2mx}$$

**Control of ν by selecting kinematics;**
**Also under control the nuclear size.**

*Colored quark emerges as*
*color neutral hadron*

➔ ***What is nature telling us about confinement?***

# Emergence of hadrons from partons

## Nucleus as a Femtometer sized filter

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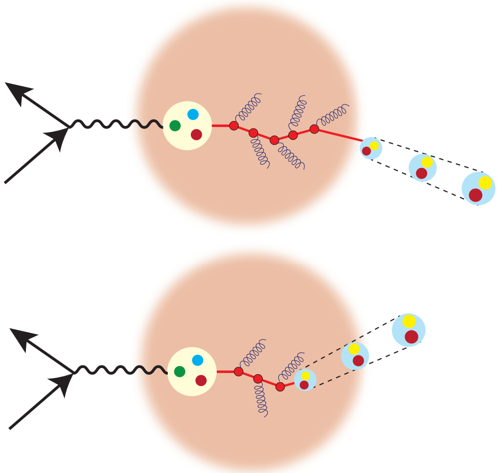

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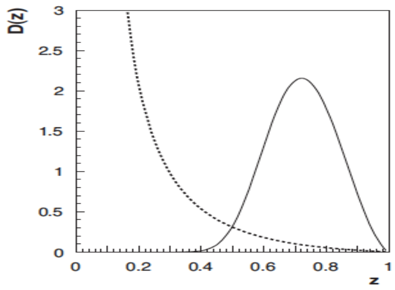
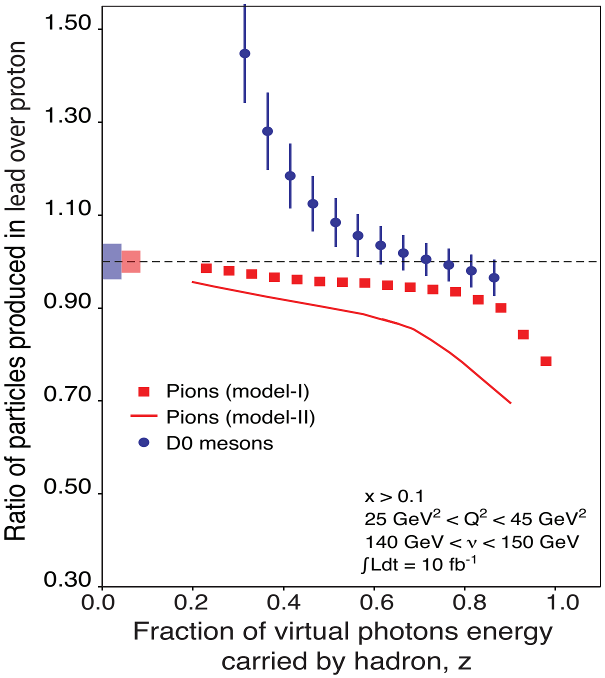


Control of ν by selecting kinematics;
Also under control the nuclear size.

*Colored quark emerges as color neutral hadron*

➔ *What is nature telling us about confinement?*

☐ **Energy loss by light vs. heavy quarks:**



Identify π vs. $D^0$ (charm) mesons in e-A collisions: Understand energy loss of light vs. heavy quarks traversing the cold nuclear matter:

*Connect to energy loss in Hot QCD*

***Need the collider energy of EIC and its control on parton kinematics***

# Hadron structure at large x

❑ **Testing ground for hadron structure at x →1:**

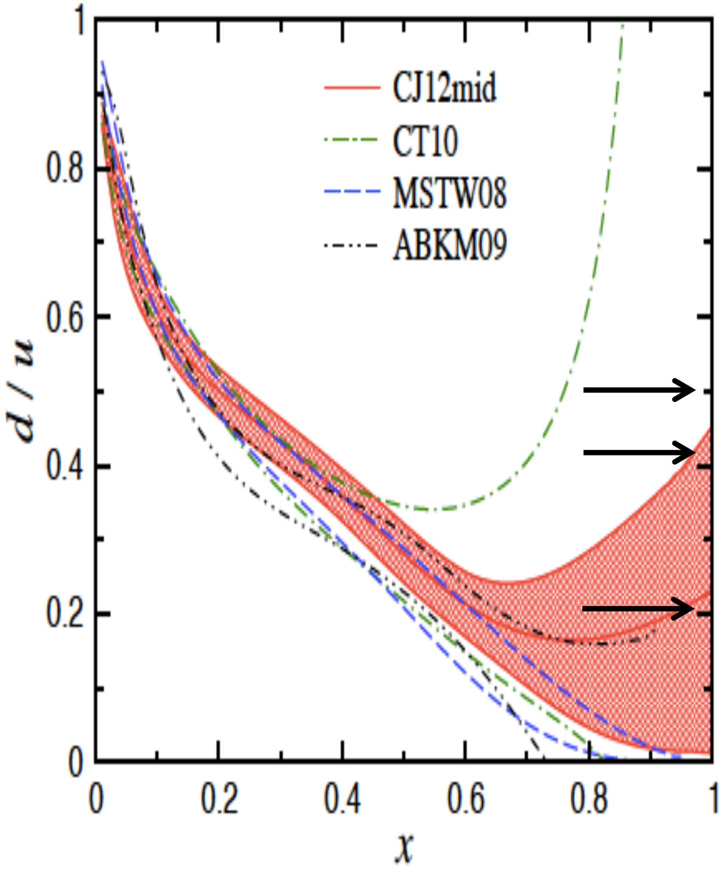
✧ $d/u \to 1/2$ **SU(6) Spin-flavor symmetry**

✧ $d/u \to 0$ **Scalar diquark dominance**

✧ $d/u \to 1/5$ **pQCD power counting**

✧ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2}$ **Local quark-hadrc duality**

$\approx\ 0.42$



# Hadron structure at large x

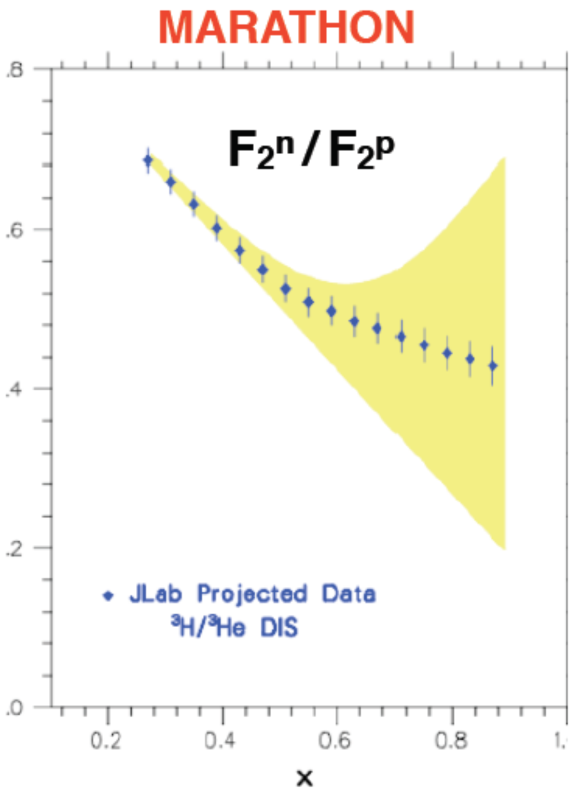
☐ **Testing ground for hadron structure at x → 1:**

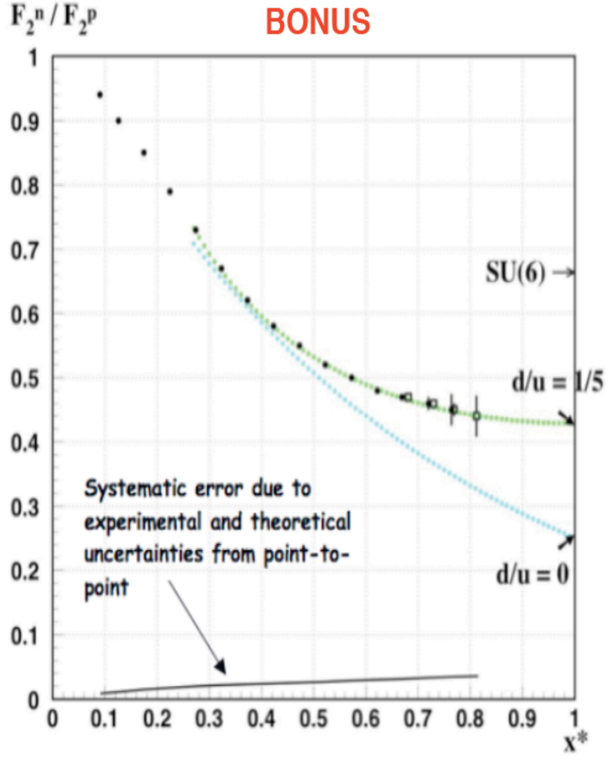
| | | |
|---|---|---|
| ✧ $d/u \to 1/2$ | **SU(6) Spin-flavor symmetry** | ✧ $\Delta u/u \to 2/3$<br>$\Delta d/d \to -1/3$ |
| ✧ $d/u \to 0$ | **Scalar diquark dominance** | ✧ $\Delta u/u \to 1$<br>$\Delta d/d \to -1/3$ |
| ✧ $d/u \to 1/5$ | **pQCD power counting** | ✧ $\Delta u/u \to 1$<br>$\Delta d/d \to 1$ |
| ✧ $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ | **Local quark-hadron duality** | ✧ $\Delta u/u \to 1$<br>$\Delta d/d \to 1$ |

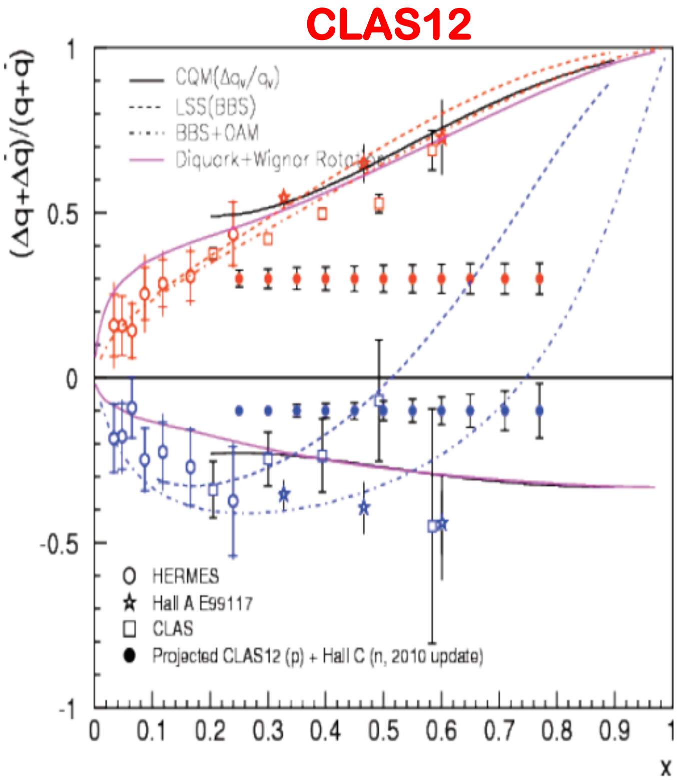
***Can lattice QCD help?***

# Upcoming experiments – JLab12

- ☐ **NSAC milestone HP14 (2018):**


MARATHON


BONUS


CLAS12

**Plus many more JLab experiments:**

**E12-06-110 (Hall C on $^3$He), E12-06-122 (Hall A on $^3$He),**

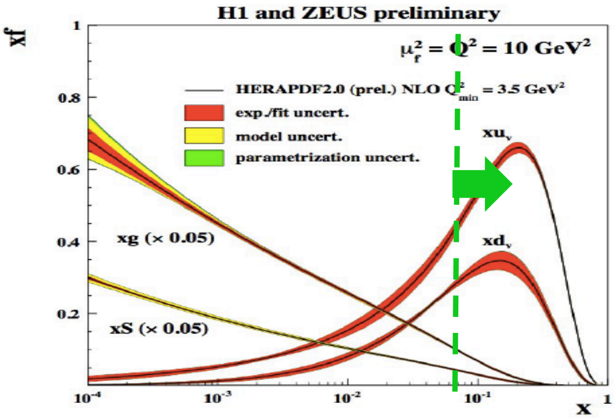
**E12-06-109 (CLAS on $NH_3$, $ND_3$), …**

**and Fermilab E906, …**

**Plus complementary Lattice QCD effort**

# Lattice calculations of hadron structure

**Lattice QCD**



**X-dep distributions**

- ☐ **New ideas – from quasi-PDFs (lattice calculable) to PDFs:**
  - ✧ **High $P_z$ effective field theory approach:**

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Ji, et al., arXiv:1305.1539
1404.6680

  - ✧ **QCD collinear factorization approach:**

$$\tilde{q}(x,\mu^2,P_z) = \sum_f \int_0^1 \frac{dy}{y}\, \mathcal{C}_f\left(\frac{x}{y},\frac{\mu^2}{\bar{\mu}^2},P_z\right) f(y,\bar{\mu}^2) + \mathcal{O}\left(\frac{1}{\mu^2}\right)$$

Ma and Qiu, arXiv:1404.6860
1412.2688
Ishikawa, Qiu, Yoshida,

Parameter like $\sqrt{s}$

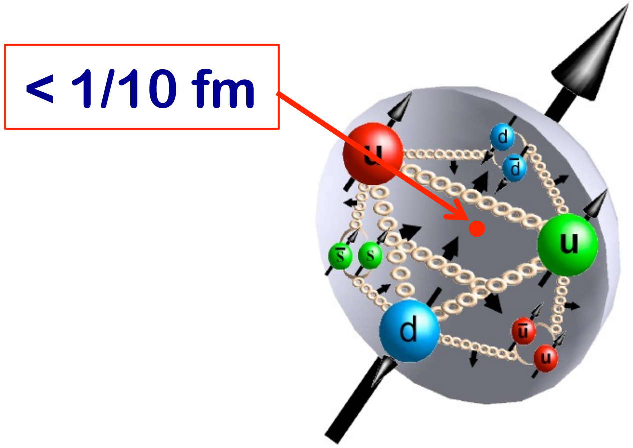
Factorization scale

High twist Power corrections

***Unmatched potential:*** ***PDFs of proton, neutron, pion, …, and TMDs and GPDs, ..***

# Summary

- ❑ Since the “spin crisis” in the 80th, we have learned a lot about proton spin – there is a need for orbital contribution
- ❑ Single transverse-spin asymmetry in real, and is a unique probe for hadron’s internal dynamics – Sivers, Collins, twist-3, … effects
- ❑ Lattice QCD has made a lot of progress, and is ready to make real impact on hadron properties and structure
- ❑ QCD has been extremely successful in interpreting and predicting high energy experimental data!
- ❑ But, we still do not know much about hadron structure – a lot of work to do!



Thank you!

# Backup slides

# Basics for spin observables

- ❑ **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

- ❑ **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- ❑ **IF:** $\langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

**or** $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

**Operators lead to the “+” sign ⟹ spin-averaged cross sections**

**Operators lead to the “-” sign ⟹ spin asymmetries**

- ❑ **Example:**

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \ \Rightarrow\ q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \ \Rightarrow\ \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \ \Rightarrow\ \delta q(x) \to h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow\ \Delta g(x)$$

# Spin decomposition

- ❑ **The "big" question:**

  If there are infinite possibilities, why bother and what do we learn?

- ❑ **The "origin" of the difficulty/confusion:**

  QCD is a gauge theory: a pure quark field in one gauge is a superposition of quarks and gluons in another gauge

- ❑ **The fact:**

  None of the items in all spin decompositions are direct physical observables, unlike cross sections, asymmetries, …

- ❑ **Ambiguity in interpretation – two old examples:**

  ✧ Factorization scheme:

$$F_2(x,Q^2) = \sum_{q,\bar{q}} C_q^{\mathrm{DIS}}(x,Q^2/\mu^2) \otimes q^{\mathrm{DIS}}(x,\mu^2)$$

*No glue contribution to $F_2$?*

  ✧ Anomaly contribution to longitudinal polarization:

$$g_1(x,Q^2) = \sum_{q,\bar{q}} \widetilde{C}_q^{\mathrm{ANO}} \otimes \Delta q^{\mathrm{ANO}} + \widetilde{C}_g^{\mathrm{ANO}} \otimes \Delta G^{\mathrm{ANO}}$$

$$\Longrightarrow \quad \Delta\Sigma \longrightarrow \Delta\Sigma^{\mathrm{ANO}} - \frac{n_f \alpha_s}{2\pi} \Delta G^{\mathrm{ANO}}$$

*Larger quark helicity?*

# Spin decomposition

- ❑ **Key for a good decomposition – sum rule:**
  - ✧ **Every term can be related to a physical observable with controllable approximation – "independently measurable"**

    ***DIS scheme is ok for F2, but, less effective for other observables***

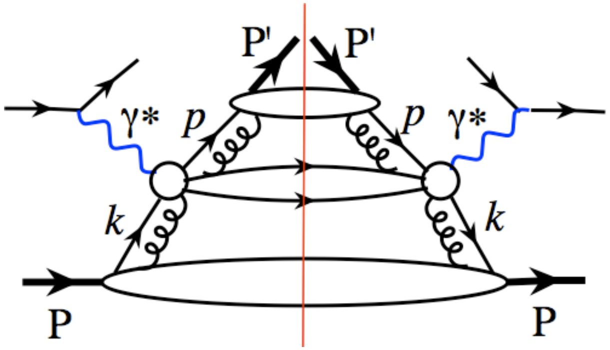
    ***Additional symmetry constraints, leading to "better" decomposition?***
  - ✧ **Natural physical interpretation for each term – "hadron structure"**
  - ✧ **Hopefully, calculable in lattice QCD – "numbers w/o distributions"**

**The most important task is,**

**Finding the connection to physical observables!**

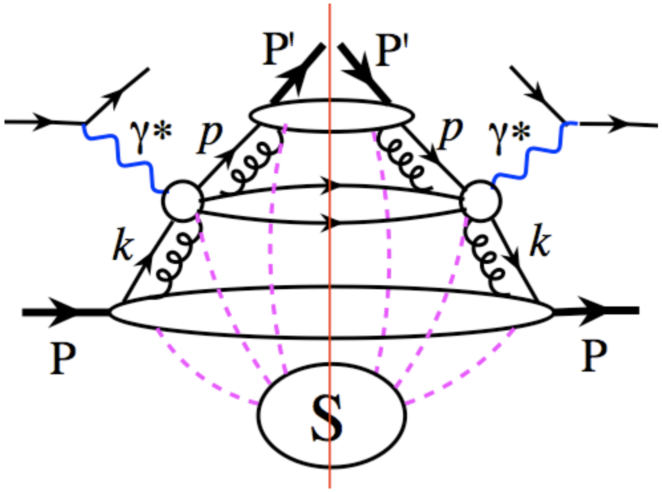
# QCD factorization for SIDIS

## ☐ Collinear gluons:



Collinear longitudinally polarized gluons do not change the collinear collision kinematics

## ☐ Soft interaction:



If the interaction between two jet functions can resolve the “details” of the jet functions, the jet functions could be altered before hard collision – break of the factorization

# Most notable TMD parton distributions (TMDs)

- ❑ **Sivers function – transverse polarized hadron:**

Sivers function

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

- ❑ **Boer-Mulder function – transverse polarized quark:**

$$f_{q,s_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Boer-Mulder function

*Affect angular distribution of Drell-Yan lepton pair*

# Most notable TMD fragmentation functions (FFs)

- **Collins function – FF of a transversely polarized parton:**

$$\begin{aligned} D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) &= D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\ &= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h} H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \end{aligned}$$

Collins function

- **Fragmentation function to a polarized hadron:**

$$\begin{aligned} D_{\Lambda,S_\Lambda/q}(z,\boldsymbol{p}_\perp) &= \frac{1}{2}D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{\Lambda^\uparrow/q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\ &= \frac{1}{2}D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_\Lambda} D_{1T}^{\perp q}(z,p_\perp)\,\boldsymbol{S}_\Lambda\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \end{aligned}$$

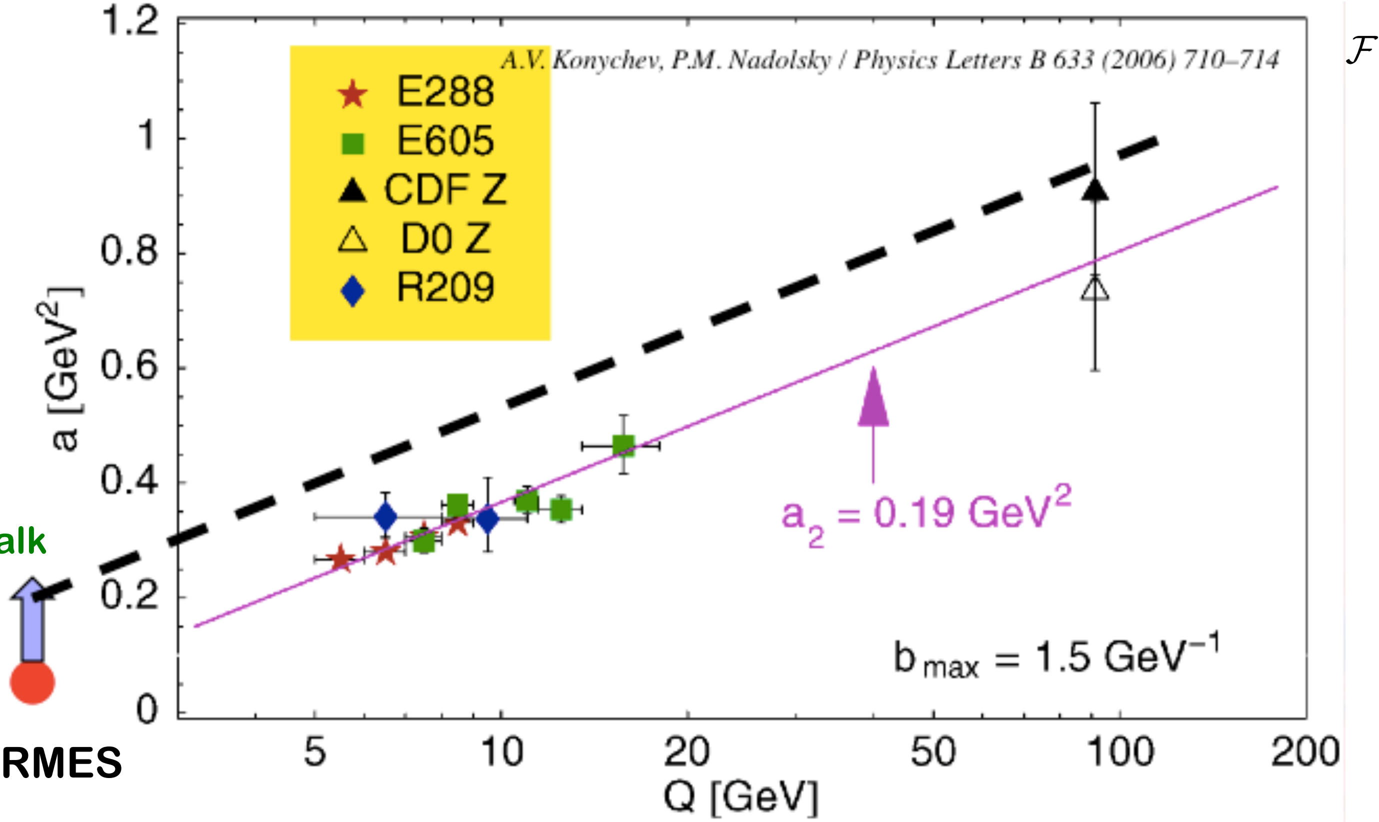
Unpolarized parton fragments into a polarized hadron - $\Lambda$

# Importance of the evolution - II

❑ **Q-dependence of the “form factor” :**

**Konychev, Nadolsky, 2006**

**Yuan’s talk**


A.V. Konychev, P.M. Nadolsky / Physics Letters B 633 (2006) 710–714

**HERMES**

$$\mathcal{F}^{\rm NP}(b,Q) = a(Q^2)\, b^2$$

**At Q ~ 1 GeV, ln(Q/Q$_0$) term may not be the dominant one!**

$$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

**Power correction? (Q$_0$/Q)$^n$-term?** ***Better fits for HERMES data?***