Neutrino Physics

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Lecture A:
Neutrino's history and lepton family

Lecture B:
Neutrino masses and flavor mixing

Lecture C:
Neutrino oscillation phenomenology

Lecture D:
Selected topics on cosmic neutrinos

ν ν ν

@ Weihai High Energy Physics School, 2—10/8/2015

What is mass?

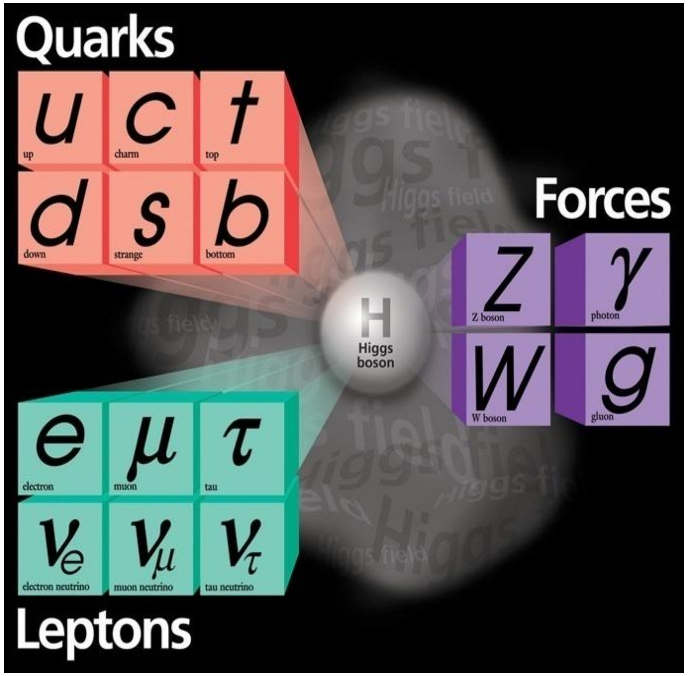
Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of W / Z and fermion masses in the SM.

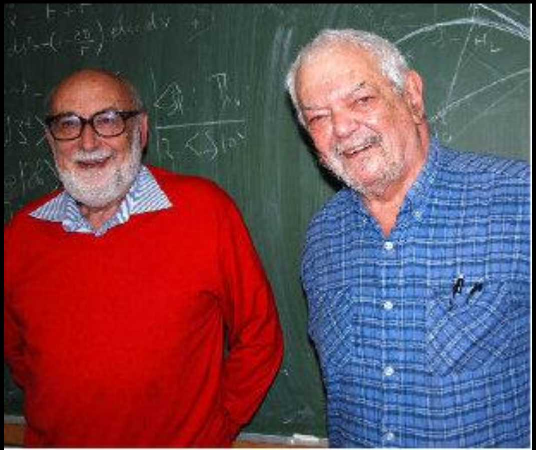
$$L_{SM} = L(f,G) + \underline{L(f,H)} + \underline{L(G,H)} + L(G) - V(H)$$

All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.



1959年，刚刚在比利时获得博士学位的恩格勒（F. Englert）来到美国名校康奈尔大学，成为布劳特（R. Brout）教授的博士后。

两年以后，恩格勒要回国了。布劳特辞了教职，跟随自己的博士后同去同去于是一同去。



刹那间，全世界同行们的眼镜碎了一地……..

那神马，难道两个大男人之间……..的情感？

三年之后，他们发表了去年获得诺奖的论文。

我们向希格斯老师学习什么？

★ 一生在一个领域只发表10篇论文，论文平均引用率700次。

★ 其余时间专心教书，好像也不带学生，然后耐心等待获奖。

Higgs: Yukawa interaction

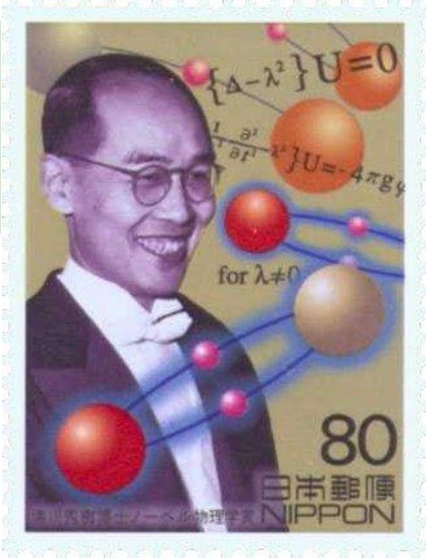
force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	gluon/π	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	photon	$=0$
weak	10^{-6}	$10^{-18}\,\mathrm{m}$	W/Z/H	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	graviton	$=0$

Yukawa relation for the mediator's mass *M* and the force's range *R*:

$$\boxed{M \simeq \frac{200\mathrm{MeV}\times10^{-15}\,\mathrm{m}}{R}}$$

$$\boxed{L_{\mathrm{SM}} = \underline{L(f,G)+L(f,H)}+L(G,H)+L(G)-V(H)}$$

Fermion masses, flavor mixing, CP violation

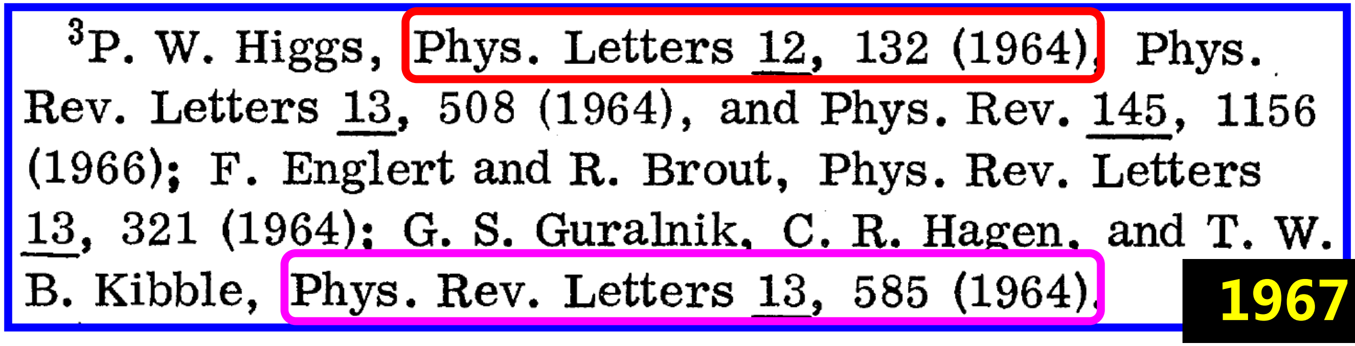


悬疑: Higgs, how are you?

温伯格一失手成千古恨

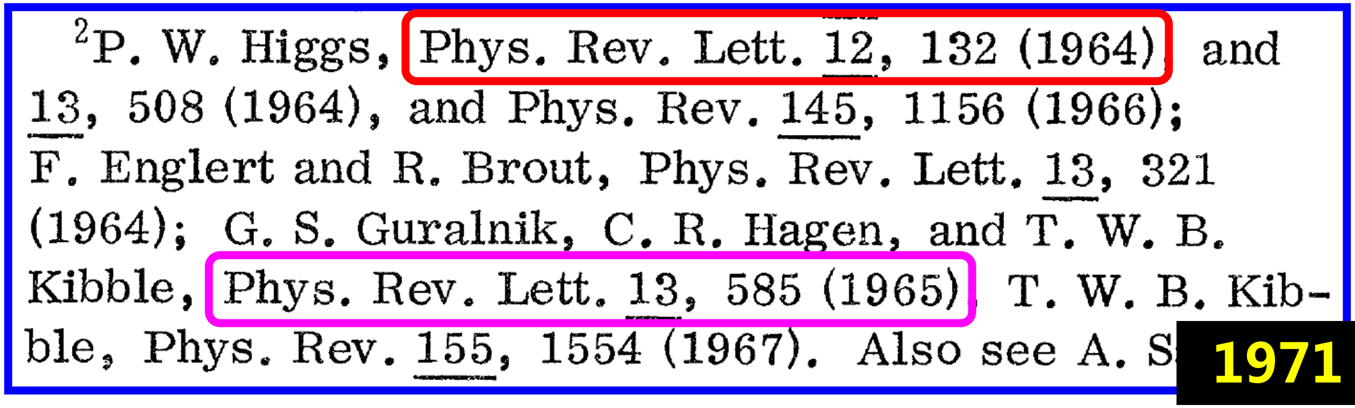
Steven Weinberg

对不起, 都怪我! 2012



[3] P. W. Higgs, Phys. Letters 12, 132 (1964), Phys. Rev. Letters 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

1967

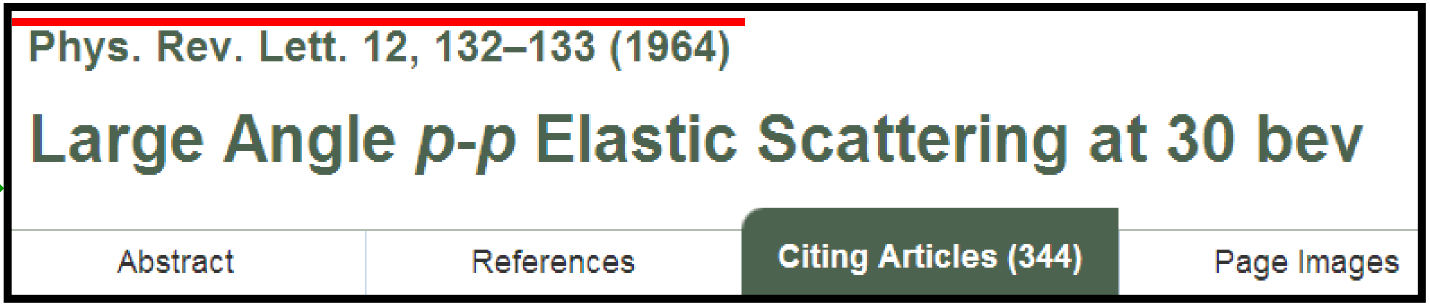


[2] P. W. Higgs, Phys. Rev. Lett. 12, 132 (1964) and 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1965). T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). Also see A. S

1971

莫名其妙的受益者

W. Baker等10人



Phys. Rev. Lett. 12, 132–133 (1964)

Large Angle *p-p* Elastic Scattering at 30 bev

Lecture B

- ★ Dirac and Majorana mass terms
- ★ Seesaw mechanisms
- ★ The lepton flavor mixing matrix

Steven Weinberg (2003):

How could I do anything without knowing everything that had already been done?

I must start doing research and pick up what I needed to know as I went along.

No one knows everything, and you don't have to.

In the SM

All ν's are massless due to the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance:

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: It's tooooooooo light and almost left-handed;
NO: No fundamental symmetry/conservation law.

Some notations

Define the left- and right-handed neutrino fields:

$$\nu_{\rm L} = \begin{pmatrix} \nu_{e\rm L} \\ \nu_{\mu\rm L} \\ \nu_{\tau\rm L} \end{pmatrix}$$

[redacted]

$$N_{\rm R} = \begin{pmatrix} N_{1\rm R} \\ N_{2\rm R} \\ N_{3\rm R} \end{pmatrix}$$

Extend the SM's particle content

$$\begin{aligned} \psi_{\rm L} &\equiv \frac{1-\gamma_5}{2}\psi \\ \psi_{\rm R} &\equiv \frac{1+\gamma_5}{2}\psi \end{aligned}$$

Their charge-conjugate counterparts are defined below and transform as right- and left-handed fields, respectively:

$$(\nu_{\rm L})^c \equiv \mathcal{C}\overline{\nu_{\rm L}}^T \, , \qquad (N_{\rm R})^c \equiv \mathcal{C}\overline{N_{\rm R}}^T$$

$$\overline{(\nu_{\rm L})^c} = (\nu_{\rm L})^T\mathcal{C} \, , \qquad \overline{(N_{\rm R})^c} = (N_{\rm R})^T\mathcal{C}$$

$$(\nu_{\rm L})^c = (\nu^c)_{\rm R} \text{ and } (N_{\rm R})^c = (N^c)_{\rm L} \text{ hold}$$

(can be proved easily)

Properties of the charge-conjugation matrix:

$$\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu \, , \qquad \mathcal{C}\gamma_5^T\mathcal{C}^{-1} = \gamma_5 \, , \qquad \mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T = -\mathcal{C}$$

They are from the requirement that the charge-conjugated field must satisfy the same Dirac equation ($\mathcal{C} = i\gamma^2\gamma^0$ in the Dirac representation)

Dirac mass term

A Dirac neutrino field is a 4-component spinor: $\boxed{\nu = \nu_{\rm L} + N_{\rm R}}$

Step 1: the gauge-invariant Dirac mass term and SSB:

$$\boxed{-\mathcal{L}_{\rm Dirac} = \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.}} \Longrightarrow \boxed{-\mathcal{L}'_{\rm Dirac} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + {\rm h.c.}}$$

$$\boxed{M_{\rm D} = Y_\nu \langle H \rangle \text{ with } \langle H \rangle \simeq 174\ {\rm GeV}}$$

Step 2: basis transformation:

$$\boxed{V^\dagger M_{\rm D} U = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}}$$

$$\boxed{-\mathcal{L}'_{\rm Dirac} = \overline{\nu'_{\rm L}} \widehat{M}_\nu N'_{\rm R} + {\rm h.c.}}$$

Mass states link to flavor states:

$$\boxed{\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{ and } \ N'_{\rm R} = U^\dagger N_{\rm R}}$$

$$\boxed{\nu' = \nu'_{\rm L} + N'_{\rm R} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}$$

Step 3: physical mass term and kinetic term:

$$\boxed{-\mathcal{L}'_{\rm Dirac} = \overline{\nu'} \widehat{M}_\nu \nu' = \sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i}$$

$$\boxed{\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}} \gamma_\mu \partial^\mu \nu_{\rm L} + i\overline{N_{\rm R}} \gamma_\mu \partial^\mu N_{\rm R} = i\overline{\nu'} \gamma_\mu \partial^\mu \nu' = i \sum_{k=1}^{3} \overline{\nu_k} \gamma_\mu \partial^\mu \nu_k}$$

Dirac neutrino mixing

Standard weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^{-}_{\mu} + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^{-}_{\mu} + {\rm h.c.}$$

In the mass basis

Without loss of generality, one may choose mass states=flavor states for charged leptons. So *V* is just the PMNS matrix of neutrino mixing.

Both the mass & CC terms are invariant with respect to a global phase transformation: lepton number (flavor) conservation (violation).

$$l(x) \longrightarrow e^{i\Phi} l(x)$$

$$\nu^{\prime}_{\rm L}(x) \rightarrow e^{i\Phi} \nu^{\prime}_{\rm L}(x)$$

$$N^{\prime}_{\rm R}(x) \rightarrow e^{i\Phi} N^{\prime}_{\rm R}(x)$$

		e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
√	L	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1
×	L_e	$+1$	$+1$	-1	-1	0	0	0	0	0	0	0	0
×	L_μ	0	0	0	0	$+1$	$+1$	-1	-1	0	0	0	0
×	L_τ	0	0	0	0	0	0	0	0	$+1$	$+1$	-1	-1

Majorana mass term (1)

A Majorana mass term can be obtained by introducing a Higgs triplet into the SM, writing out the gauge-invariant Yukawa interactions and Higgs potentials, and then integrating out heavy degrees of freedom (type-II seesaw mechanism):

$$\boxed{-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + {\rm h.c.}}$$

The Majorana mass matrix must be a symmetric matrix. It can be diagonalized by a unitary matrix

$$\boxed{\overline{\nu_L} M_{\rm L} (\nu_L)^c = [\overline{\nu_L} \underline{M_{\rm L}} (\nu_L)^c]^T = -\overline{\nu_L} \mathcal{C}^T M_{\rm L}^T \overline{\nu_L}^T = \overline{\nu_L} \underline{M_{\rm L}^T} (\nu_L)^c}$$

Diagonalization:

$$\boxed{V^\dagger M_{\rm L} V^* = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}}$$

$$\boxed{-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'_{\rm L}} \widehat{M}_\nu (\nu'_{\rm L})^c + {\rm h.c.}}$$

$$\boxed{\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{ and } \ (\nu'_{\rm L})^c = \mathcal{C}\overline{\nu'_{\rm L}}^T}$$

$$\boxed{\nu' = \nu'_{\rm L} + (\nu'_{\rm L})^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}$$

Physical mass term:

$$\boxed{-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'} \widehat{M}_\nu \nu' = \frac{1}{2}\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i}$$

$$\boxed{\text{Majorana condition } (\nu')^c = \nu'}$$

Majorana mass term (2)

Kinetic term (you may prove: $\overline{(\psi_{\rm L})^c}\gamma_\mu\partial^\mu(\psi_{\rm L})^c = \overline{\psi_{\rm L}}\gamma_\mu\partial^\mu\psi_{\rm L}$):

$$\boxed{\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} = i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} = \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k}$$

Question: why there is a factor **1/2** in the **Majorana** mass term?

Answer: it allows us to get the normal **Dirac** equation of motion.

A proof: write out the Lagrangian of free massive **Majorana** neutrinos:

$$\boxed{\begin{aligned}\mathcal{L}_\nu &= i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} - \left[\frac{1}{2}\overline{\nu_{\rm L}}M_{\rm L}(\nu_{\rm L})^c + {\rm h.c.}\right] \\ &= i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} - \left[\frac{1}{2}\overline{\nu'_{\rm L}}\widehat{M}_\nu(\nu'_{\rm L})^c + {\rm h.c.}\right] \\ &= \frac{1}{2}\left(i\overline{\nu'}\gamma_\mu\partial^\mu\nu' - \overline{\nu'}\widehat{M}_\nu\nu'\right) = -\frac{1}{2}\left(i\partial^\mu\overline{\nu'}\gamma_\mu\nu' + \overline{\nu'}\widehat{M}_\nu\nu'\right)\end{aligned}}$$

Euler-Lagrange equation:

$$\boxed{\partial^\mu\frac{\partial\mathcal{L}_\nu}{\partial\left(\partial^\mu\overline{\nu'}\right)} - \frac{\partial\mathcal{L}_\nu}{\partial\overline{\nu'}} = 0} \Longrightarrow \begin{array}{l}\boxed{i\gamma_\mu\partial^\mu\nu' - \widehat{M}_\nu\nu' = 0} \\ \boxed{i\gamma_\mu\partial^\mu\nu_k - m_k\nu_k = 0}\end{array}$$

Majorana neutrino mixing

Standard weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

The PMNS matrix *V* contains 2 extra CP-violating phases.

Mass and CC terms are not simultaneously invariant under a global phase transformation --- Lepton number violation

$$\boxed{l(x) \rightarrow e^{i\Phi} l(x)}$$

$$\boxed{\nu'_{\rm L}(x) \rightarrow e^{i\Phi} \nu'_{\rm L}(x)}$$

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2} \overline{\nu'_{\rm L}} \widehat{M}_\nu (\nu'_{\rm L})^c + {\rm h.c.}$$

$$\overline{\nu'_{\rm L}} \rightarrow e^{-i\Phi} \overline{\nu'_{\rm L}} \text{ and } (\nu'_{\rm L})^c \rightarrow e^{-i\Phi} (\nu'_{\rm L})^c$$

$$e^{-2i\Phi}$$

ββ decay

ββ decay: certain even-even nuclei have a chance to decay into the 2nd nearest neighbor, if two subsequent β decays through an intermediate state can happen.

necessary conditions:

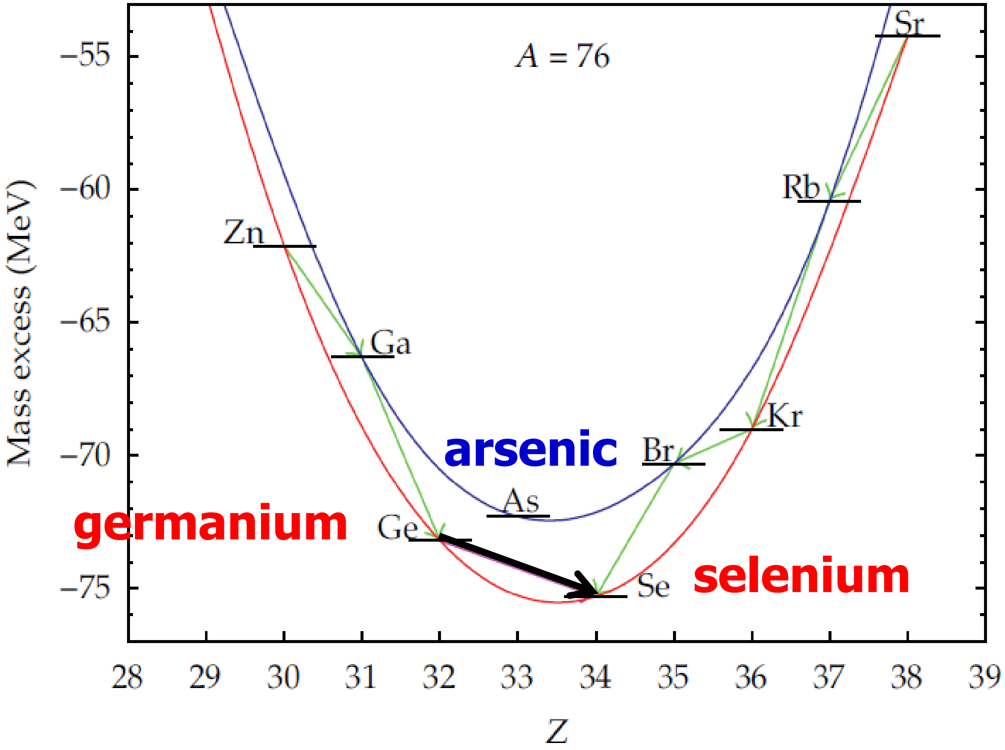
$$m(Z,A) > m(Z+2,A)$$

$$m(Z,A) < m(Z+1,A)$$

$$(Z,A) \to (Z+2,A) + 2e^- + 2\bar{\nu}_e \,.$$

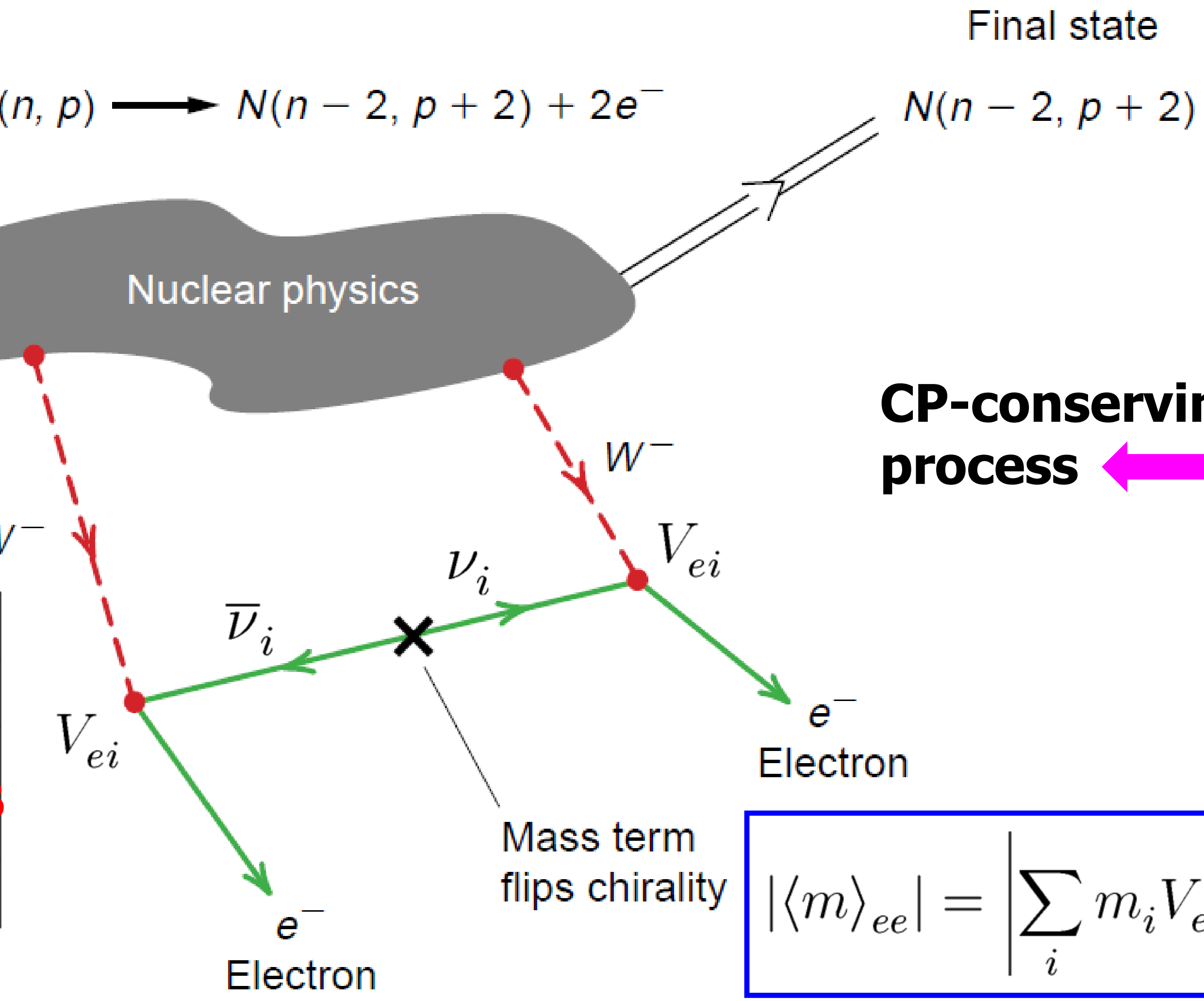
1935

Maria Goeppert Mayer



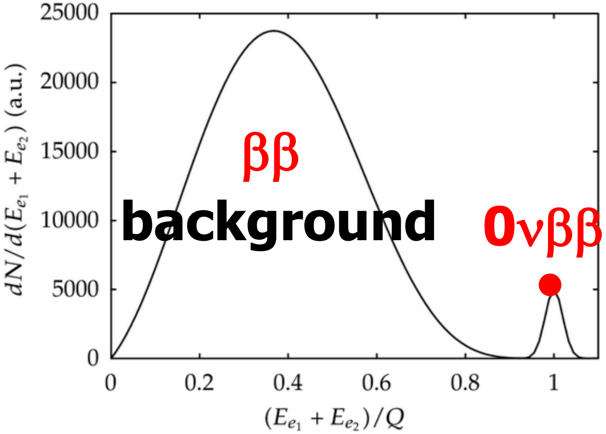
0νββ

The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



Lepton number violation →

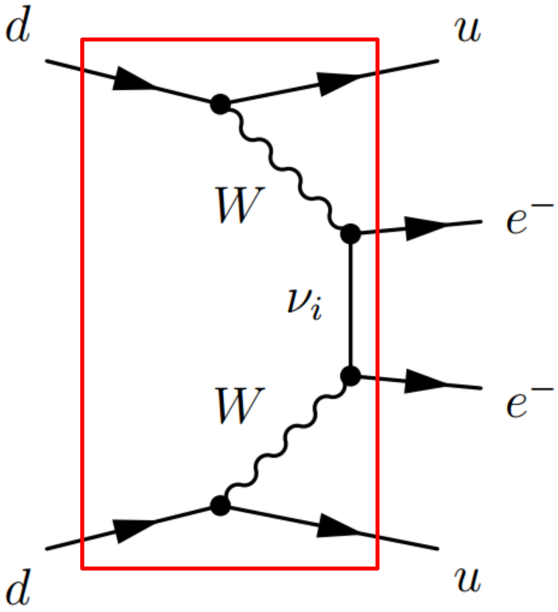
CP-conserving process ←

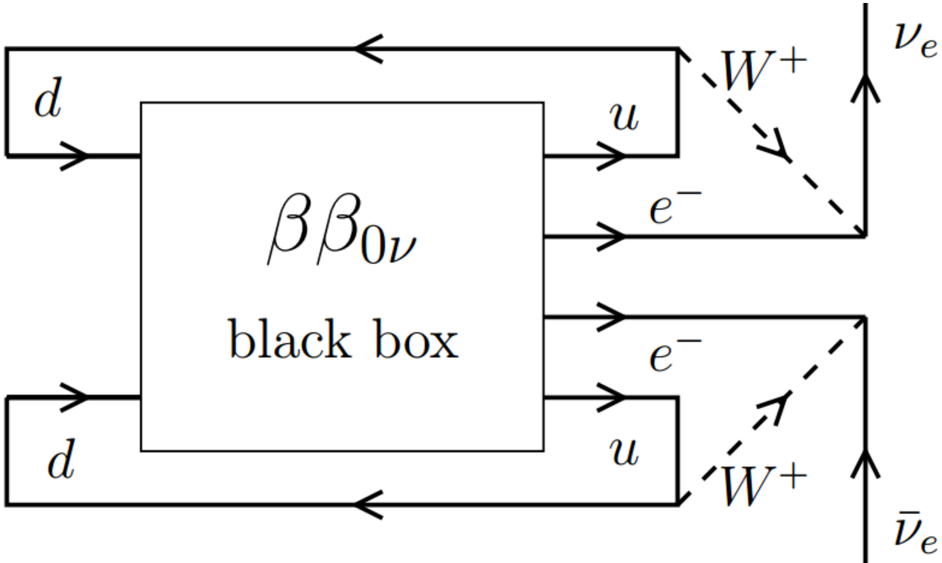


$$\boxed{|\langle m\rangle_{ee}| = \left|\sum_i m_i V_{ei}^2\right|}$$

Schechter-Valle theorem

THEOREM (1982): if a 0νββ decay happens, there must be an effective Majorana mass term.





Bruno Pontecorvo's Prediction

That is why we want to see 0νββ

Four-loop ν mass:

$$\boxed{\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})}$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

GERDA limit

GERDA essentially kills the Heidelberg-Moscow claim.

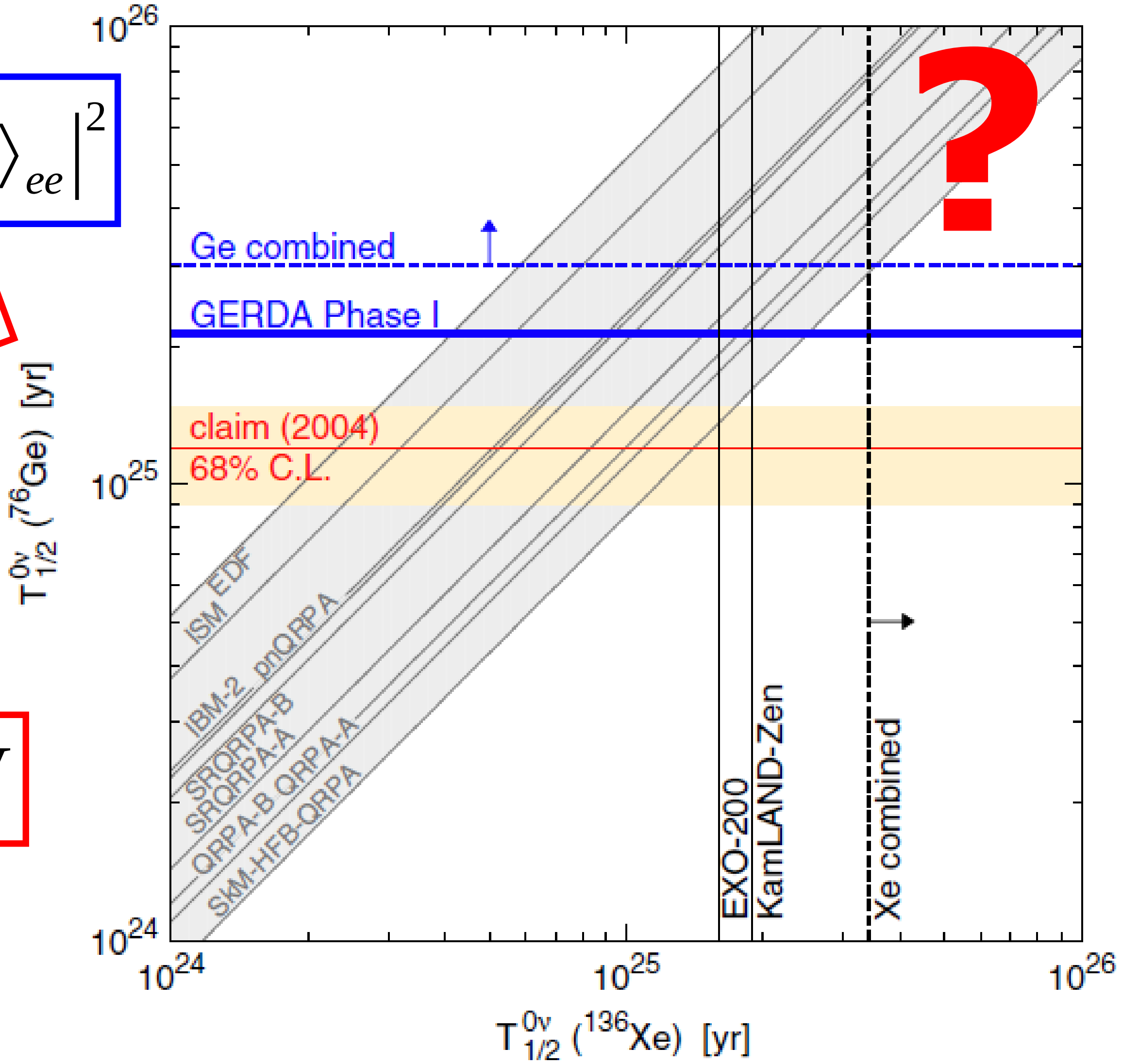
PRL **111,** 122503 (2013)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2\left|\langle m\rangle_{ee}\right|^2$$

$$T^{0\nu}_{1/2} > 3.0\times10^{25}\ \text{yr}\ (90\%\ \text{C.L.}).$$

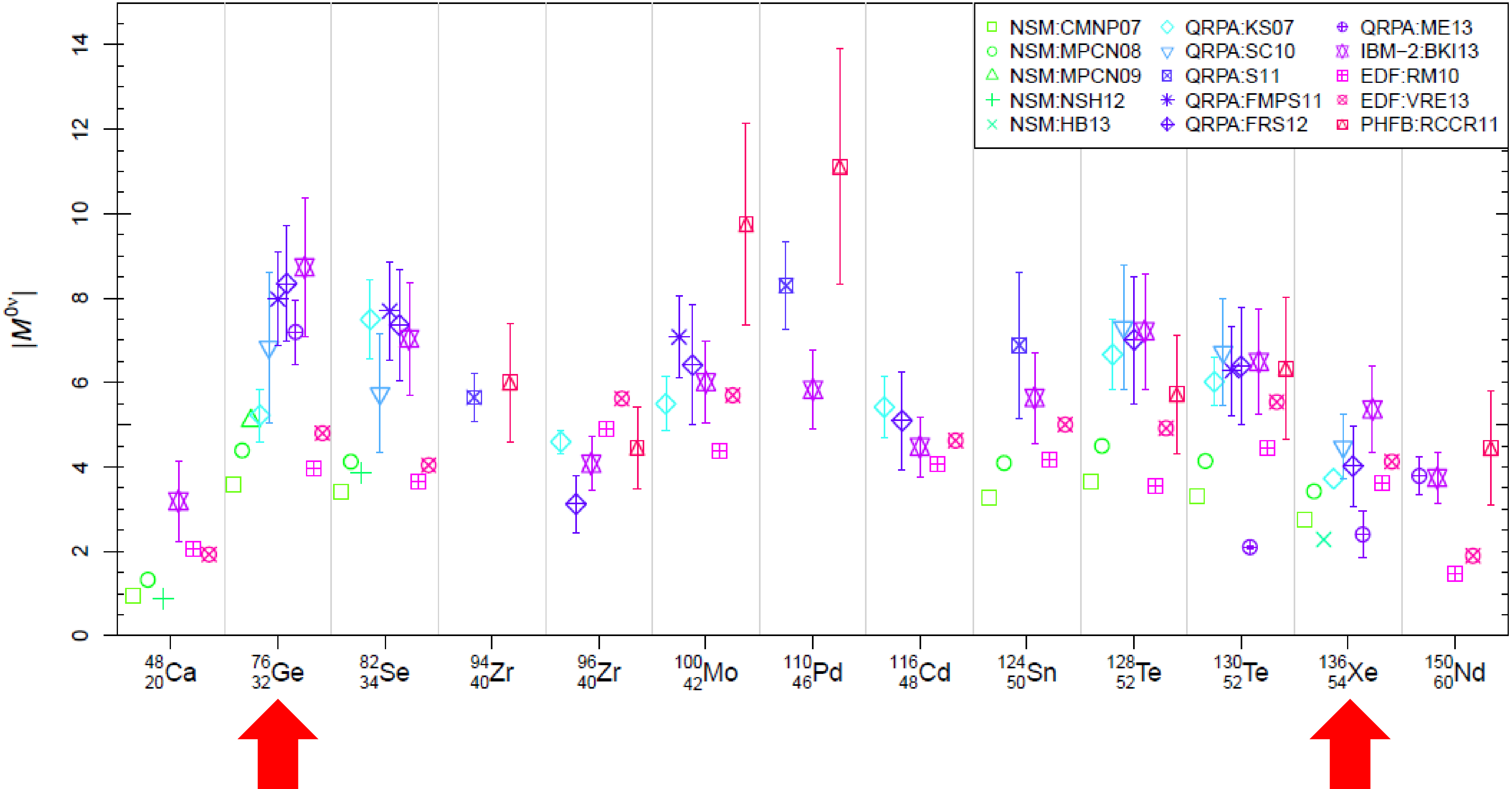
$$\left|\langle m\rangle_{ee}\right| < 0.2 \rightarrow 0.4\ \text{eV}$$

$$|\langle m\rangle_{ee}| = \left|\sum_i m_i V_{ei}^2\right|$$



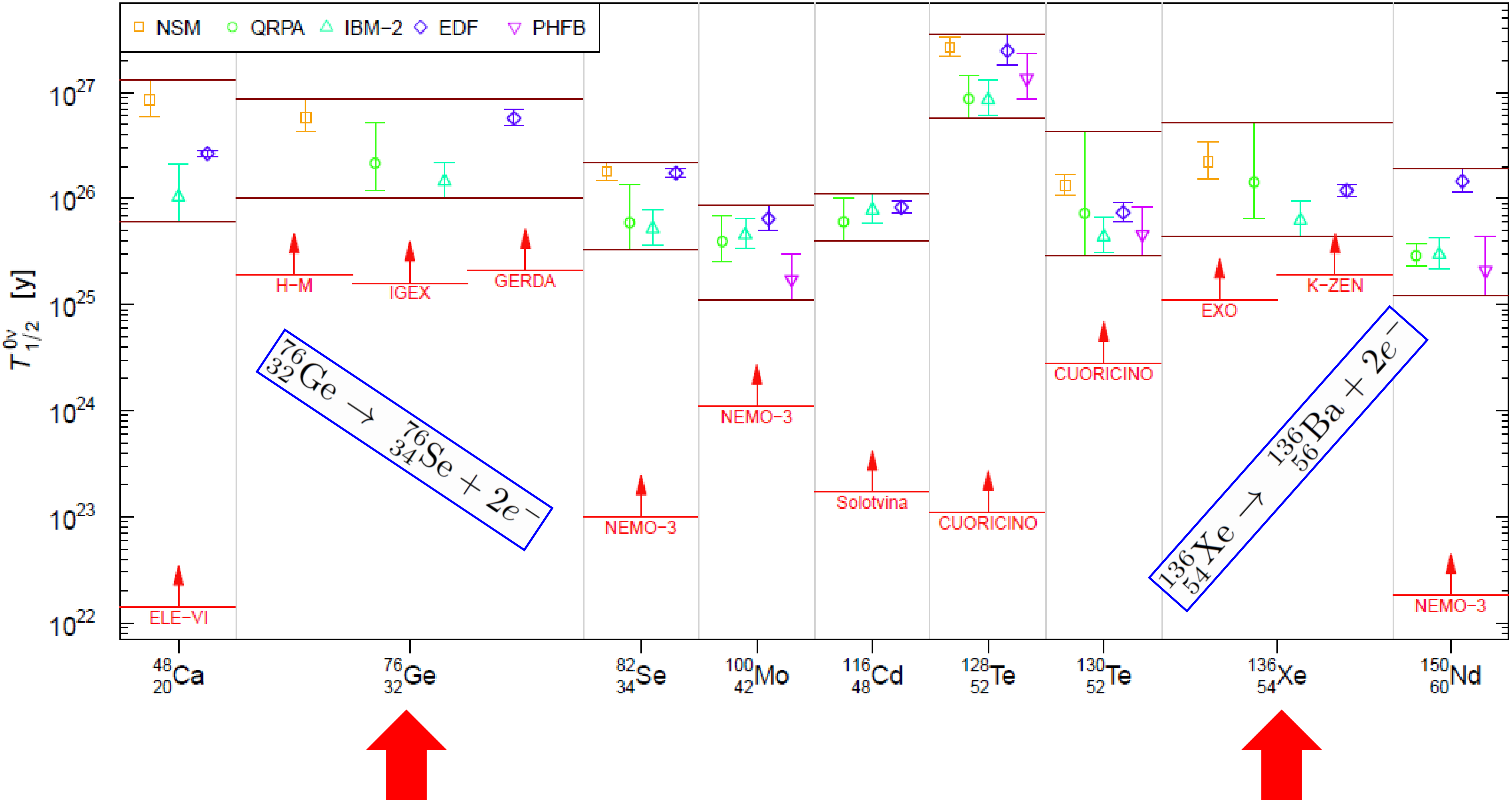
Nuclear matrix elements

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, large uncertainties (a factor of 2 or 3) are unavoidable.

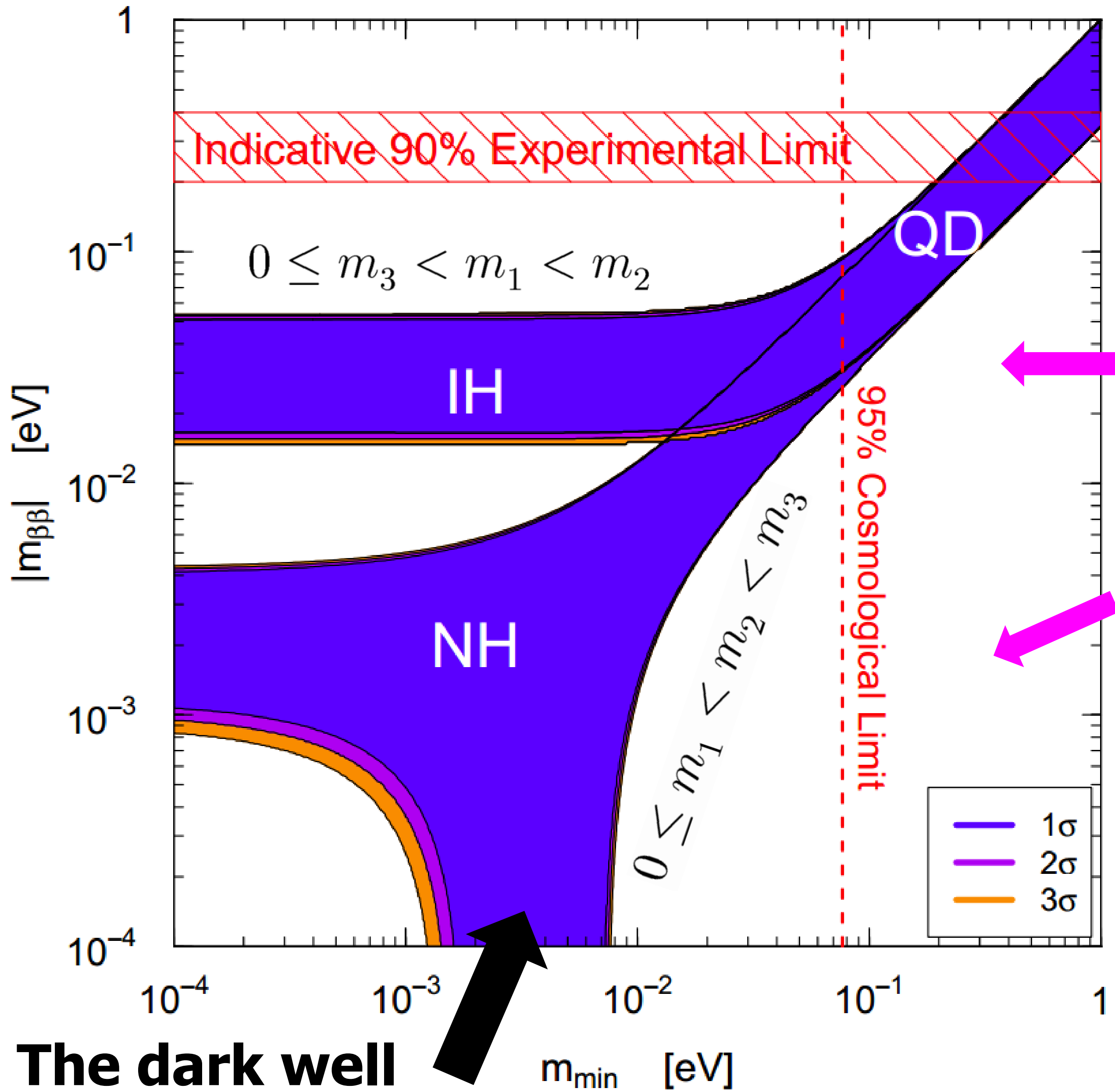


Half-life

Comparing the 90% C.L. experimental lower limits on the half-life of a 0νββ-decaying nuclide with the corresponding range of theoretical prediction, given a value of 0.1 eV for the effective Majorana neutrino mass term (Bilenky and Giunti, 1411.4791).



Effective 0νββ mass



The dark well

The effective mass

$$|\langle m \rangle_{ee}| = \left| \sum_i m_i V_{ei}^2 \right|$$

Maury Goodman asks

An intelligent design?

I asked myself in 2003

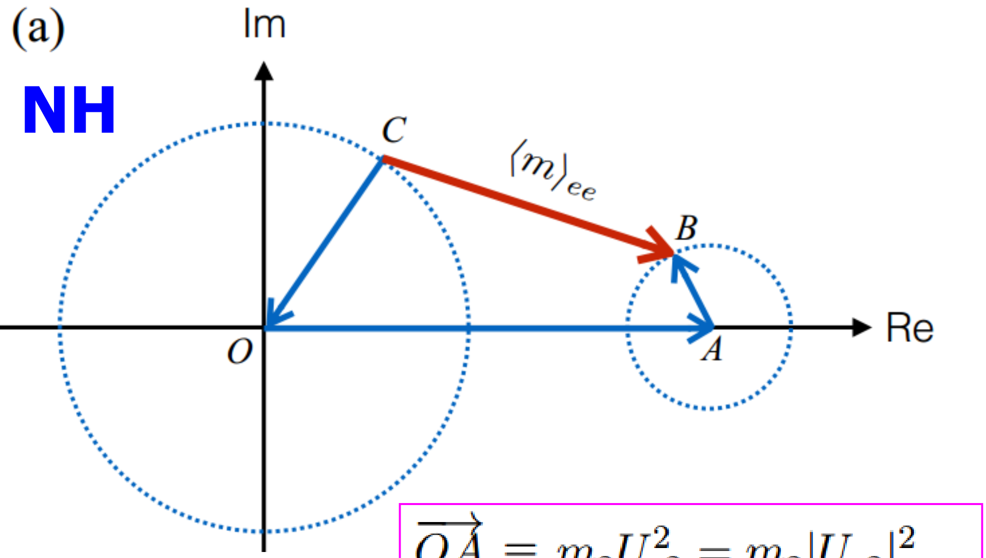
Vanishing 0νββ mass?

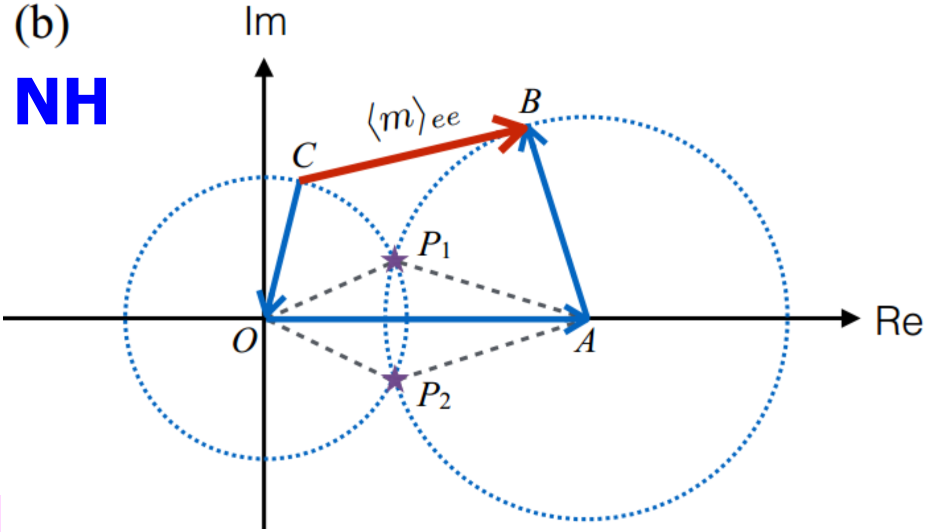
hep-ph/0305195, PRD

CP phases also matter

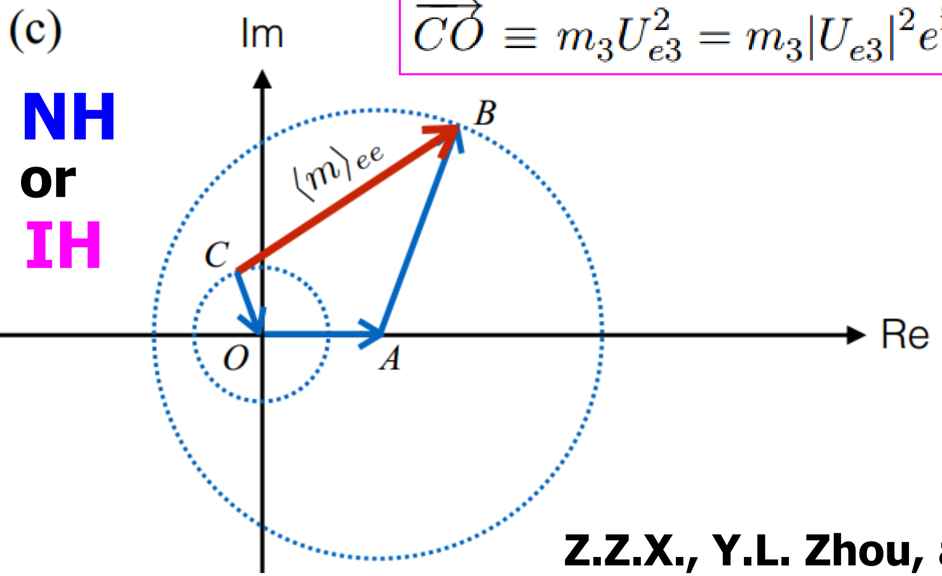
In case of new physics, is it destructive or constructive?

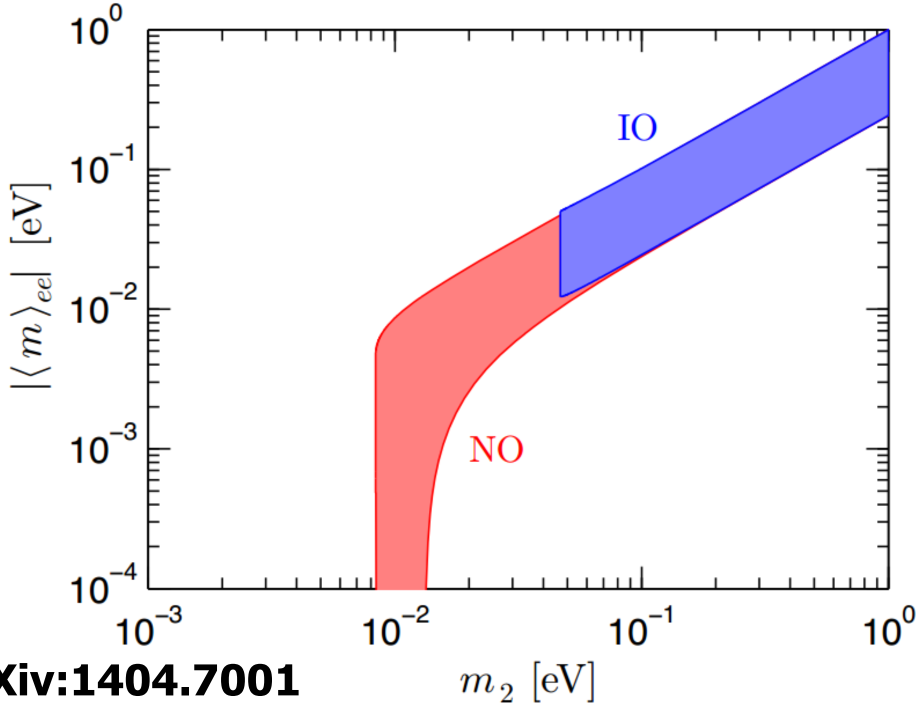
Coupling-rod diagram





$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \ ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{\mathrm{i}\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{\mathrm{i}\sigma}$$





Z.Z.X., Y.L. Zhou, arXiv:1404.7001

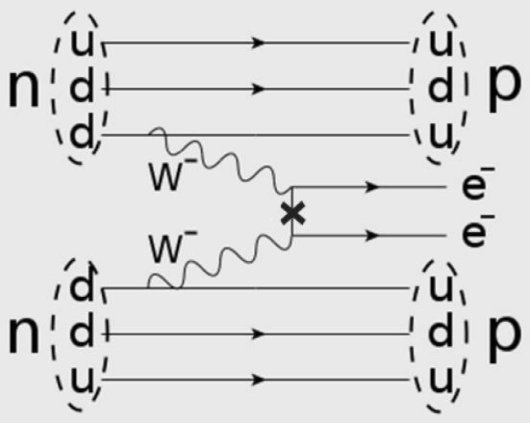
New physics

Type (A): NP directly related to extra species of neutrinos.

Example 1: heavy Majorana neutrinos from type-I seesaw

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}}$$

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^{3} V_{ei}^2 m_i \;-\; \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2$$



In most cases the heavy contribution is negligible

Example 2: light sterile neutrinos from LSND etc

$$\langle m \rangle'_{ee} \equiv \sum_{i=1}^{6} m_i V_{ei}^2 = \underline{\langle m \rangle_{ee}} \left(c_{14} c_{15} c_{16}\right)^2 + \underline{m_4 \left(\hat{s}^*_{14} c_{15} c_{16}\right)^2 + m_5 \left(\hat{s}^*_{15} c_{16}\right)^2 + m_6 \left(\hat{s}^*_{16}\right)^2}$$

In this case the new contribution might be constructive or destructive

Type (B): NP has little to do with the neutrino mass issue.

SUSY, Left-right, and some others that I don’t understand

YES or NO?

QUESTION: are massive neutrinos the **Majorana** particles?

One might be able to answer **YES** through a measurement of the **0νββ** decay or other **LNV** processes someday, but how to answer with **NO**?

YES

or

I don't know!

The same question: how to distinguish between **Dirac** and **Majorana** neutrinos in a realistic experiment?

Answer 1: The **0νββ** decay is currently the only possibility.

Answer 2: In principle their dipole moments are different.

Answer 3: They show different behavior if nonrelativistic.

Hybrid mass term (1)

A hybrid mass term can be written out in terms of the left- and right-handed neutrino fields and their charge-conjugate counterparts:

$$\begin{aligned}-\mathcal{L}'_{\rm hybrid} &= \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}\\ &= \frac{1}{2}\begin{bmatrix}\overline{\nu_{\rm L}} & \overline{(N_{\rm R})^c}\end{bmatrix}\begin{pmatrix} M_{\rm L} & M_{\rm D}\\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix}(\nu_{\rm L})^c\\ N_{\rm R}\end{bmatrix} + {\rm h.c.}\,,\end{aligned}$$

type-(I+II) seesaw

Here we have used

$$\overline{(N_{\rm R})^c} M_{\rm D}^T (\nu_{\rm L})^c = \left[(N_{\rm R})^T \mathcal{C} M_{\rm D}^T \mathcal{C}\,\overline{\nu_{\rm L}}^T\right]^T = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R}$$

Diagonalization by means of a 6×6 unitary matrix:

$$\begin{pmatrix} V & R\\ S & U\end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D}\\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix} V & R\\ S & U\end{pmatrix}^* = \begin{pmatrix}\widehat{M}_\nu & \mathbf{0}\\ \mathbf{0} & \widehat{M}_N\end{pmatrix}$$

$$\widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\},\ \widehat{M}_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

Majorana mass states

$$\nu' = \begin{bmatrix}\nu'_{\rm L}\\ (N'_{\rm R})^c\end{bmatrix} + \begin{bmatrix}(\nu'_{\rm L})^c\\ N'_{\rm R}\end{bmatrix} = \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ N_1\\ N_2\\ N_3\end{pmatrix}$$

$$(\nu')^c = \nu'$$

It is actually a Majorana mass term!

$$-\mathcal{L}'_{\rm hybrid} = \frac{1}{2}\begin{bmatrix}\overline{\nu'_{\rm L}} & \overline{(N'_{\rm R})^c}\end{bmatrix}\begin{pmatrix}\widehat{M}_\nu & \mathbf{0}\\ \mathbf{0} & \widehat{M}_N\end{pmatrix}\begin{bmatrix}(\nu'_{\rm L})^c\\ N'_{\rm R}\end{bmatrix} + {\rm h.c.}$$

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} + S^\dagger (N_{\rm R})^c$$

$$N'_{\rm R} = R^T (\nu_{\rm L})^c + U^T N_{\rm R}$$

Hybrid mass term (2)

Physical mass term:

$$-\mathcal{L}'_{\text{hybrid}} = \frac{1}{2}\overline{\nu'}\begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}\nu' = \frac{1}{2}\sum_{i=1}^{3}\left(m_i\overline{\nu_i}\nu_i + M_i\overline{N_i}N_i\right)$$

Kinetic term:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic}} &= i\overline{\nu_{\mathrm{L}}}\gamma_\mu\partial^\mu\nu_{\mathrm{L}} + i\overline{N_{\mathrm{R}}}\gamma_\mu\partial^\mu N_{\mathrm{R}} \\
&= \frac{i}{2}\left[\overline{\nu_{\mathrm{L}}} \quad \overline{(N_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu_{\mathrm{L}} \\ (N_{\mathrm{R}})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu_{\mathrm{L}})^c} \quad \overline{N_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu_{\mathrm{L}})^c \\ N_{\mathrm{R}}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\mathrm{L}}} \quad \overline{(N'_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^\dagger\begin{pmatrix}V & R \\ S & U\end{pmatrix}\begin{bmatrix}\nu'_{\mathrm{L}} \\ (N'_{\mathrm{R}})^c\end{bmatrix} \\
&\quad + \frac{i}{2}\left[\overline{(\nu'_{\mathrm{L}})^c} \quad \overline{N'_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^T\begin{pmatrix}V & R \\ S & U\end{pmatrix}^*\begin{bmatrix}(\nu'_{\mathrm{L}})^c \\ N'_{\mathrm{R}}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\mathrm{L}}} \quad \overline{(N'_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu'_{\mathrm{L}} \\ (N'_{\mathrm{R}})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu'_{\mathrm{L}})^c} \quad \overline{N'_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu'_{\mathrm{L}})^c \\ N'_{\mathrm{R}}\end{bmatrix} \\
&= i\overline{\nu'_{\mathrm{L}}}\gamma_\mu\partial^\mu\nu'_{\mathrm{L}} + i\overline{N'_{\mathrm{R}}}\gamma_\mu\partial^\mu N'_{\mathrm{R}} \\
&= \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\left(\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k + \overline{N_k}\gamma_\mu\partial^\mu N_k\right)
\end{aligned}$$

Non-unitary flavor mixing

Weak charged-current interactions of leptons:

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

$$\boxed{\nu_{\rm L} = V\nu'_{\rm L} + R(N'_{\rm R})^c}$$

In the mass basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

V = **non-unitary light neutrino mixing (PMNS) matrix** $\boxed{VV^\dagger + RR^\dagger = 1}$

R = **light-heavy neutrino mixing (CC interactions of heavy neutrinos)**

Neutrino oscillations ***V*** ⟺ ***R*** **Collider signatures**

TeV seesaws may bridge the gap between neutrino & collider physics.

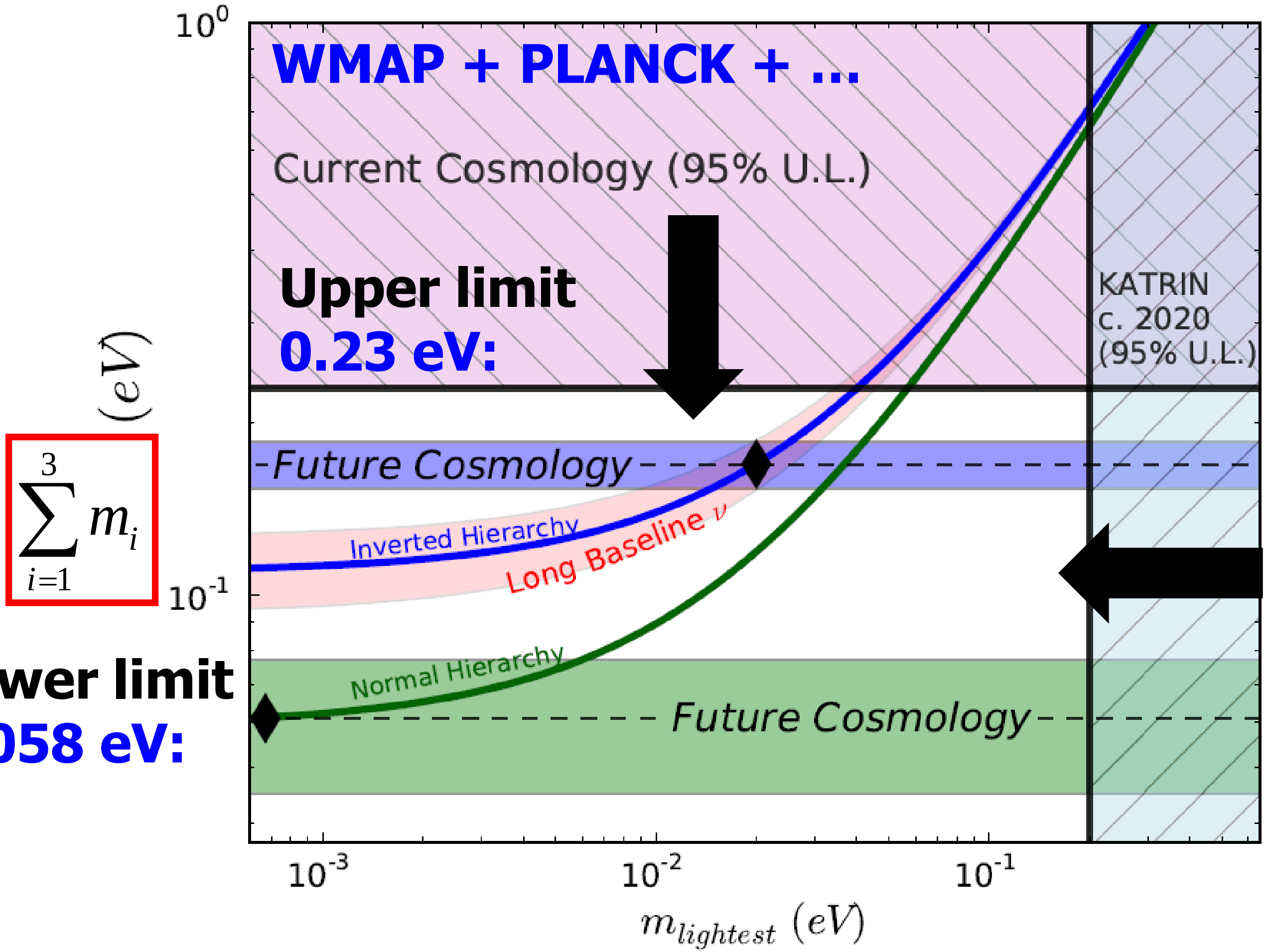
Neutrino mass scale

Three ways: the β decay, the 0νββ decay, and cosmology (CMB + LSS).

$$\langle m \rangle_e^2 = \sum_{i=1}^{3} m_i^2 \left| U_{ei} \right|^2$$

$$\left| \langle m \rangle_{ee} \right| = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|$$

$$\sum_{i=1}^{3} m_i$$



Upper limit
0.23 eV:

Lower limit
0.058 eV:

mass scale
≤ O(0.1) eV

Why so tiny?

arXiv:1309.5383

Stage-4 CMB

$$\sigma\left(\sum m_\nu\right) = 16 \text{ meV}$$
$$\sigma\left(N_{\rm eff}\right) = 0.020\,.$$

Seesaw mechanisms (1)

A hybrid mass term may have three distinct components:

$$
\begin{aligned}
-\mathcal{L}'_{\rm hybrid} &= \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.} \\
&= \frac{1}{2}\begin{bmatrix}\overline{\nu_{\rm L}} & \overline{(N_{\rm R})^c}\end{bmatrix} \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.} \ ,
\end{aligned}
$$

- ♣ **Normal Dirac mass term**, **proportional to the scale of electroweak symmetry breaking (~ 174 GeV);**
- ♣ **Light Majorana mass term**, **violating the SM gauge symmetry and much lower than 174 GeV ('t Hooft's naturalness criterion);**
- ♣ **Heavy Majorana mass term**, **originating from the SU(2)_L singlet and having a scale much higher than 174 GeV.**

A strong hierarchy of 3 mass scales allows us to make approximation

$$
\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}
$$

Seesaw mechanisms (2)

The above unitary transformation leads to the following relationships:

$$R\widehat{M}_N = M_{\rm L} R^* + M_{\rm D} U^* \qquad S\widehat{M}_\nu = M_{\rm D}^T V^* + M_{\rm R} S^*$$

$$M_{\rm R} \gg M_{\rm D} \gg M_{\rm L}$$

$$R \sim S \sim \mathcal{O}(M_{\rm D}/M_{\rm R})$$

$$U\widehat{M}_N = M_{\rm R} U^* + M_{\rm D}^T R^* \qquad V\widehat{M}_\nu = M_{\rm L} V^* + M_{\rm D} S^*$$

$$U\widehat{M}_N U^T = M_{\rm R}(UU^\dagger)^T + M_{\rm D}^T(R^* U^T) \approx M_{\rm R} \, ,$$

$$V\widehat{M}_\nu V^T = M_{\rm L}(VV^\dagger)^T + M_{\rm D}(S^* V^T) \approx M_{\rm L} + M_{\rm D}(S^* V^T)$$

$$S^* V^T = M_{\rm R}^{-1} S\widehat{M}_\nu V^T - M_{\rm R}^{-1} M_{\rm D}^T (VV^\dagger)^T \approx -M_{\rm R}^{-1} M_{\rm D}^T$$

Then we arrive at the type-(I+II) seesaw formula:

$$M_\nu \equiv V\widehat{M}_\nu V^T \approx M_{\rm L} - M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

Type-I seesaw limit: $M_\nu \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ **(Fritzsch, Gell-Mann, Minkowski, 1975; Minkowski, 1977; ...)**

Type-II seesaw limit: $M_\nu = M_{\rm L}$ **(Konetschny, Kummer, 1977; ...)**

History of type-I seesaw

The seesaw idea originally appeared in a paper's footnote.

Seesaw—A Footnote Idea:

H. Fritzsch, M. Gell-Mann,
P. Minkowski, PLB 59 (1975) 256

This idea was very clearly elaborated by Minkowski in Phys. Lett. B 67 (1977) 421 ---- but it was unjustly forgotten until 2004.

The idea was later on embedded into the GUT frameworks in 1979 and 1980:

- T. Yanagida 1979
- M. Gell-Mann, P. Ramond, R. Slansky 1979
- S. Glashow 1979
- R. Mohapatra, G. Senjanovic 1980

It was Yanagida who named this mechanism as "seesaw".

What is History?

History is a set of lies agreed upon

Napoleon Bonaparte

Summary of 3 seesaws

Type-I seesaw: SM + right-handed neutrinos + L violation

(Minkowski 1977; Yanagida 1979; Glashow 1979; Gell-Mann, Ramond, Slansky 1979; Mohapatra, Senjanovic 1980)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}}$$

Type-II seesaw: SM + 1 Higgs triplet + L violation

(Konetschny, Kummer 1977; Magg, Wetterich 1980; Schechter, Valle 1980; Cheng, Li 1980; Lazarides et al 1980)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}}$$

Type-III seesaw: SM + 3 triplet fermions + L violation

(Foot, Lew, He, Joshi 1989)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2}{\rm Tr}\left(\overline{\Sigma} M_\Sigma \Sigma^c\right) + {\rm h.c.}}$$

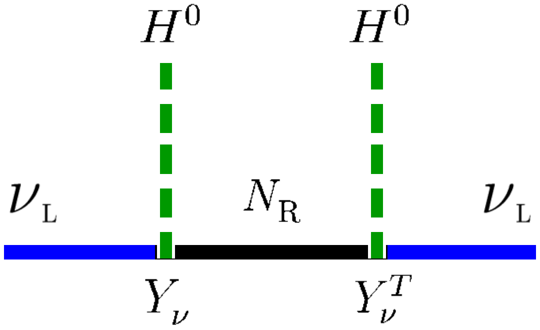
Effective mass term

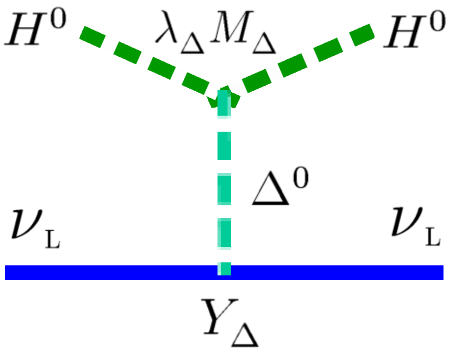
Weinberg (1979): the unique **dimension-five** operator of ν-masses after integrating out heavy degrees of freedom.

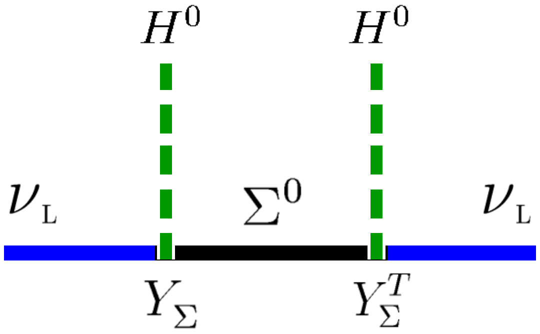
$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} & \text{(Type 1)} \\ -\frac{\lambda_\Delta}{M_\Delta} \left(Y_\Delta\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} & \text{(Type 2)} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} & \text{(Type 3)} \end{cases}$$

$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & \text{(Type 1)} \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & \text{(Type 2)} \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & \text{(Type 3)} \end{cases}$$

After SSB, a Majorana mass term is $-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$ $\quad \langle \tilde{H} \rangle = v/\sqrt{2}$

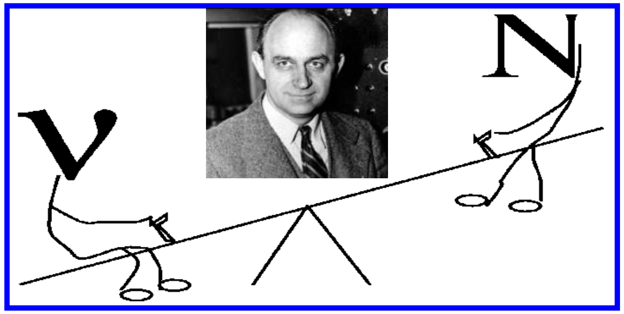






Seesaw scale?

What is the scale at which the seesaw mechanism works?



Planck

GUT to unify strong, weak & electromagnetic forces

Conventional Seesaws: heavy degrees of freedom near GUT

This appears to be rather reasonable, since one often expects **new physics** to appear around a fundamental scale

Fermi

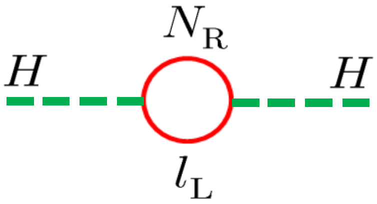
- **Naturalness** √
- **Uniqueness** ×
- **Testability** ×
- **Hierarchy** ×

Hierarchy problem

Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom in seesaw (Vissani 1998; Casas et al 2004; Abada et al 2007)

Type 1:

$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$

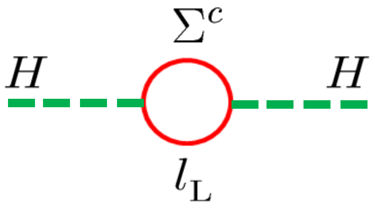


Type 2:

$$\delta m_H^2 = \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right]$$

Type 3:

$$\delta m_H^2 = -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$



here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning:

$$M_i \sim \left[\frac{(2\pi v)^2|\delta m_H^2|}{m_i}\right]^{1/3} \sim \underline{10^7\text{GeV}} \left[\frac{0.2\ \text{eV}}{m_i}\right]^{1/3} \left[\frac{|\delta m_H^2|}{0.1\ \text{TeV}^2}\right]^{1/3}$$

Possible way out: (1) Supersymmetric seesaw? (2) TeV-scale seesaw?

真空稳定

$$\tilde{\lambda} = \lambda - \frac{3}{32\pi^2}\left\{\frac{1}{8}(g'^2+g^2)^2\left[\frac{1}{3}-\ln\left(\frac{g'^2+g^2}{4}\right)\right]+2y_t^4\left[\ln\left(\frac{y_t^2}{2}\right)-1\right]+\frac{1}{4}g^4\left[\frac{1}{3}-\ln\left(\frac{g^2}{4}\right)\right]\right\}$$

Planck scale $\Lambda \sim 10^{19}\,\mathrm{GeV}$

GUT scale? $\Lambda \sim 10^{16}\,\mathrm{GeV}$

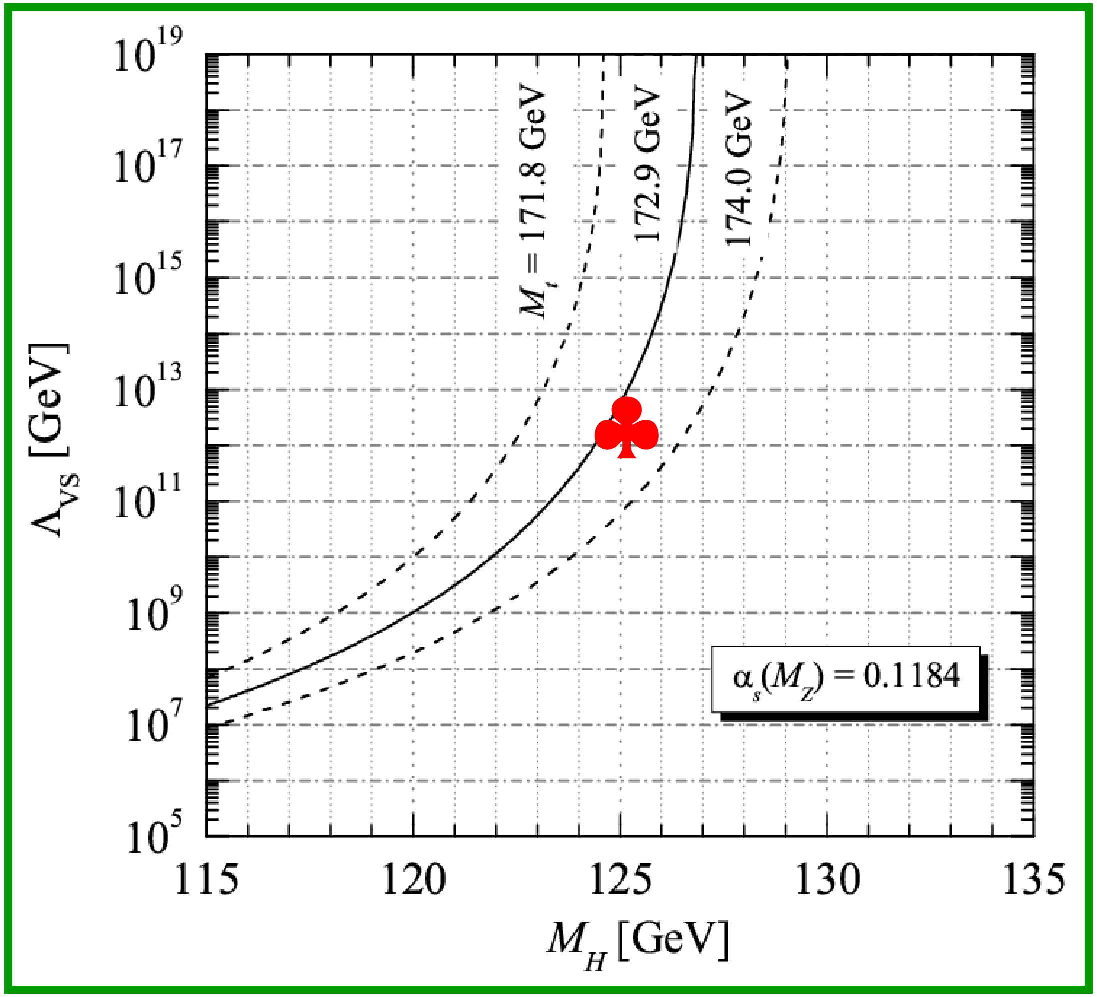
Seesaw scale? $\Lambda \sim 10^{12}\,\mathrm{GeV}$

TeV / SUSY? $\Lambda \sim 10^{3}\,\mathrm{GeV}$

Fermi scale $\Lambda \sim 10^{2}\,\mathrm{GeV}$

QCD scale $\Lambda \sim 10^{2}\,\mathrm{MeV}$

The SM vacuum stability for a light Higgs



Elias-Miro et al., arXiv:1112.3022;
Xing, Zhang, Zhou, arXiv:1112.3112; …

TeV Neutrino Physics?

to discover the SM Higgs boson √

to verify Yukawa interactions √

Why

to pin down heavy seesaw particles

to test seesaw mechanism(s)

Not

to measure low-energy effects

Try

Type-1 seesaw

Type-1 Seesaw: **add 3 *right-handed* Majorana neutrinos into the SM.**

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{e_{\rm L}} M_l E_{\rm R} + \frac{1}{2}\overline{(\nu_{\rm L} \quad N^{\rm c}_{\rm R})} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.}$$

Diagonalization (flavor basis ⇒ mass basis):

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}$$

$$V^\dagger V + \underline{S^\dagger S} = V V^\dagger + \underline{R R^\dagger} = 1$$

Hence *V* is not unitary

Seesaw:

$$M_\nu \;\equiv\; V \widehat{M}_\nu V^T \;\approx\; -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

$$\boldsymbol{R \sim S \sim M_{\rm D} \,/\, M_{\rm R}}$$

Strength of Unitarity Violation

$$V \approx \left(1 - \frac{1}{2} R R^\dagger\right) V_{\rm unitary}$$

Natural or unnatural?

Natural case: no large cancellation in the leading seesaw term.

$$\underbrace{M_\nu}_{0.01\ \mathrm{eV}} \approx M_\mathrm{D} \underbrace{M_\mathrm{R}^{-1}}_{10^{15}\ \mathrm{GeV}} \underbrace{M_\mathrm{D}^{T}}_{100\ \mathrm{GeV}}$$

$$R \sim S \sim M_\mathrm{D} / M_\mathrm{R} \sim 10^{-13}$$

$$\text{Unitarity Violation} \sim 10^{-26}$$

Unnatural case: large cancellation in the leading seesaw term.

$$\underbrace{M_\nu}_{0.01\ \mathrm{eV}} \approx M_\mathrm{D} \underbrace{M_\mathrm{R}^{-1}}_{1\ \mathrm{TeV}} \underbrace{M_\mathrm{D}^{T}}_{100\ \mathrm{GeV}}$$

$$R \sim S \sim M_\mathrm{D} / M_\mathrm{R} \sim 10^{-1}$$

$$\text{Unitarity Violation} \sim 10^{-2}$$

TeV-scale (right-handed) Majorana neutrinos: small masses of 3 light Majorana neutrinos come from sub-leading perturbations.

Structural cancellation

Given diagonal *M_R* with 3 mass igenvalues *M_1*, *M_2* and *M_3*, the leading (i.e., type-I seesaw) term of the active neutrino mass matrix vanishes, if and only if *M_D* has rank 1,

$$\boldsymbol{M}_{\mathrm{D}} = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}$$

and if

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0$$

$$\boldsymbol{M}_{\nu} \approx \boldsymbol{M}_{\mathrm{D}} \boldsymbol{M}_{\mathrm{R}}^{-1} \boldsymbol{M}_{\mathrm{D}}^{T} = 0$$

(Buchmueller, Greub 91; Ingelman, Rathsman 93; Heusch, Minkowski 94; ……; Kersten, Smirnov 07).

Tiny ν-masses can be generated from tiny corrections to this complete "structural cancellation", by deforming *M_D* or *M_R* .

Simple example:

$$M'_{\mathrm{D}} = M_{\mathrm{D}} + \epsilon X_{\mathrm{D}}$$

$$\begin{aligned} M'_{\nu} &= M'_{\mathrm{D}} M_{\mathrm{R}}^{-1} M'^{T}_{\mathrm{D}} \\ &\approx \epsilon \left(M_{\mathrm{D}} M_{\mathrm{R}}^{-1} X_{\mathrm{D}}^{T} + X_{\mathrm{D}} M_{\mathrm{R}}^{-1} M_{\mathrm{D}}^{T} \right) + \mathcal{O}(\epsilon^2) \end{aligned}$$

Fast lessons

Lesson 1: two necessary conditions to test a seesaw model with heavy right-handed Majorana neutrinos at the LHC:

---Masses of heavy Majorana neutrinos must be of *O* (1) TeV or below

---Light-heavy neutrino mixing (i.e. *M*_D/ *M*_R) must be large enough

Lesson 2: A collider signature of the heavy Majorana ν's is essentially decoupled from masses and mixing parameters of light ν's.

Lesson 3: non-unitarity of the light ν flavor mixing matrix might lead to observable effects in ν oscillations and rare processes.

Lesson 4: nontrivial limits on heavy Majorana ν's could be derived at the LHC, if the SM backgrounds are small for a specific final state.

$\Delta L = 2$ like-sign dilepton events

$$\boxed{pp \to W^{\pm}W^{\pm} \to \mu^{\pm}\mu^{\pm}jj \text{ and } pp \to W^{\pm} \to \mu^{\pm}N \to \mu^{\pm}\mu^{\pm}jj}$$

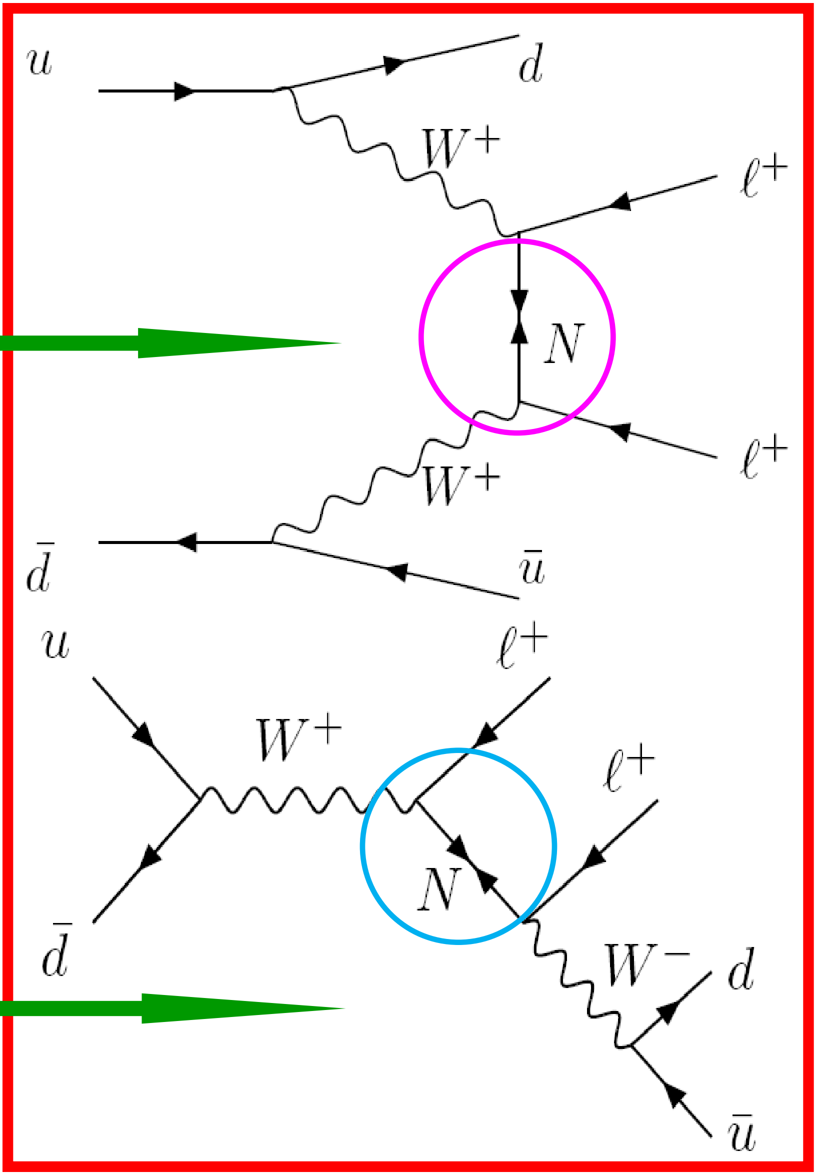
Collider signature

Lepton number violation: like-sign dilepton events at hadron colliders, such as Tevatron (~2 TeV) and LHC (~14 TeV).

collider analogue to 0νββ decay

dominant channel

N can be produced on resonance



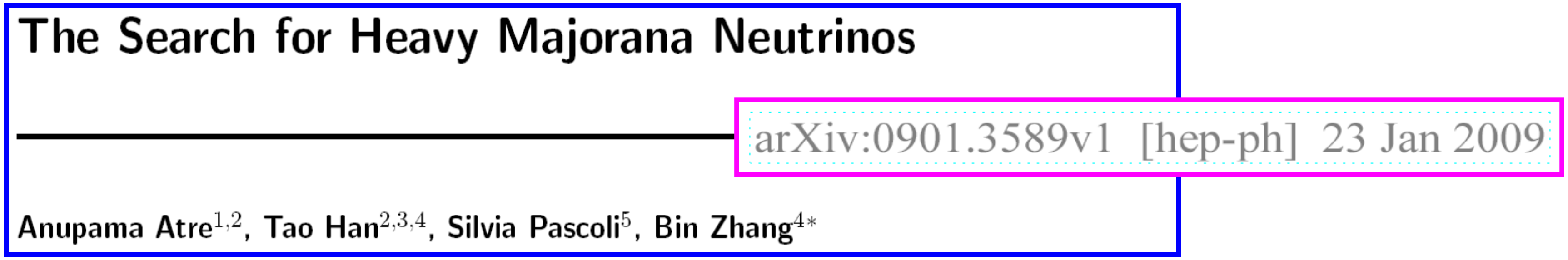
Testability at the LHC

2 recent comprehensive works:

Distinguishing seesaw models at LHC with multi-lepton signals

F. del Aguila, J. A. Aguilar–Saavedra

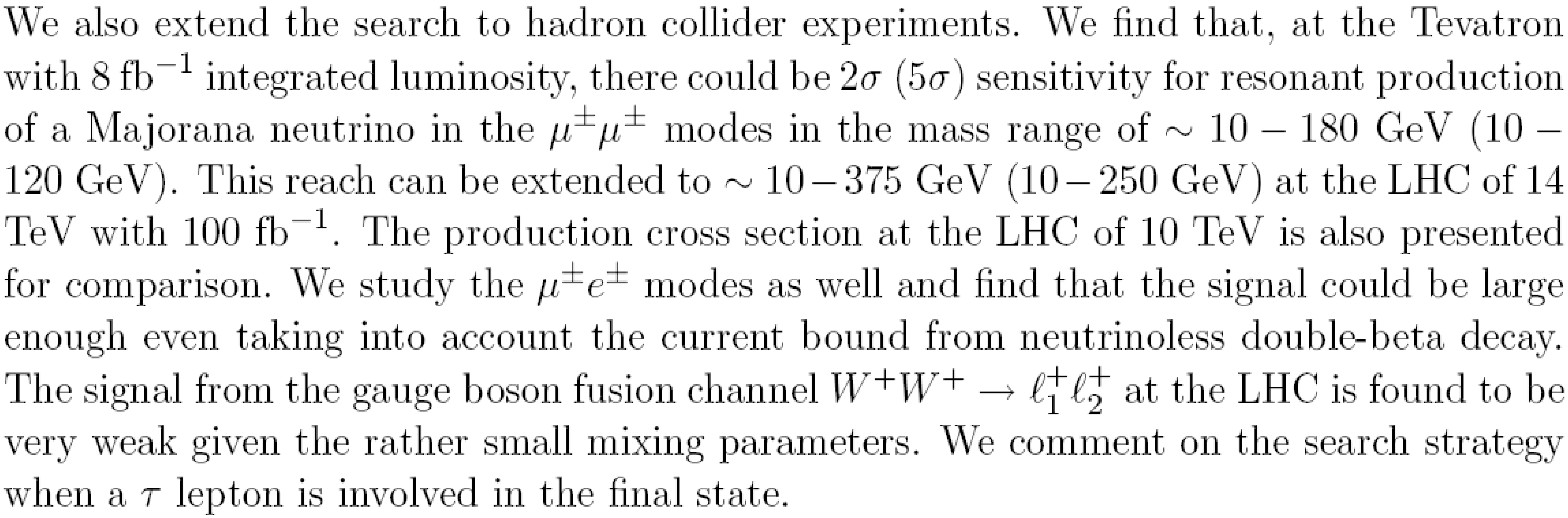
arXiv:0808.2468v2 [hep-ph] 12 Sep 2008



The Search for Heavy Majorana Neutrinos

arXiv:0901.3589v1 [hep-ph] 23 Jan 2009

Anupama Atre[1,2], Tao Han[2,3,4], Silvia Pascoli[5], Bin Zhang[4*]



We also extend the search to hadron collider experiments. We find that, at the Tevatron with 8 fb^{-1} integrated luminosity, there could be 2σ (5σ) sensitivity for resonant production of a Majorana neutrino in the $\mu^\pm\mu^\pm$ modes in the mass range of $\sim 10-180$ GeV ($10-120$ GeV). This reach can be extended to $\sim 10-375$ GeV ($10-250$ GeV) at the LHC of 14 TeV with 100 fb^{-1}. The production cross section at the LHC of 10 TeV is also presented for comparison. We study the $\mu^\pm e^\pm$ modes as well and find that the signal could be large enough even taking into account the current bound from neutrinoless double-beta decay. The signal from the gauge boson fusion channel $W^+W^+ \to \ell_1^+\ell_2^+$ at the LHC is found to be very weak given the rather small mixing parameters. We comment on the search strategy when a τ lepton is involved in the final state.

Non-unitarity

Type-1 seesaw: a typical signature would be the unitarity violation of the 3×3 neutrino mixing matrix *V* in the charged-current interactions

Current experimental constraints at the 90% C.L. (Antusch *et al* 07):

$$|VV^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

$$|V^\dagger V| \approx \begin{pmatrix} 1.00 \pm 0.032 & < \ 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

$\mu \rightarrow e + \gamma$ etc, *W* / *Z* decays, *universality*, ν-oscillation.

accuracy of a few percent!

Extra CP-violating phases exist in a non-unitary ν mixing matrix may lead to observable *CP-violating effects* in short- or medium-baseline ν oscillations (Fernandez-Martinez *et al* 07; Xing 08).

Typical example: non-unitary CP violation in the ν_μ → ν_τ oscillation, an effect probably at the percent level.

Type-2 seesaw

Type-2 (Triplet) **Seesaw:** add **one SU(2)_L** Higgs triplet into the SM.

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} + {\rm h.c.}$$

$$\Delta \equiv \begin{pmatrix} H^- & -\sqrt{2}\,H^0 \\ \sqrt{2}\,H^{--} & -H^- \end{pmatrix}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{e_{\rm L}} M_l E_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} \nu^c_{\rm L} + {\rm h.c.}$$

$$M_{\rm L} \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Potential:

$$V(H,\Delta) = -\mu^2 H^\dagger H + \lambda\left(H^\dagger H\right)^2 + \frac{1}{2} M_\Delta^2 {\rm Tr}\left(\Delta^\dagger \Delta\right) - \underbrace{\left[\lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}\right]}_{L \text{ and } B\text{–}L \text{ violation}}$$

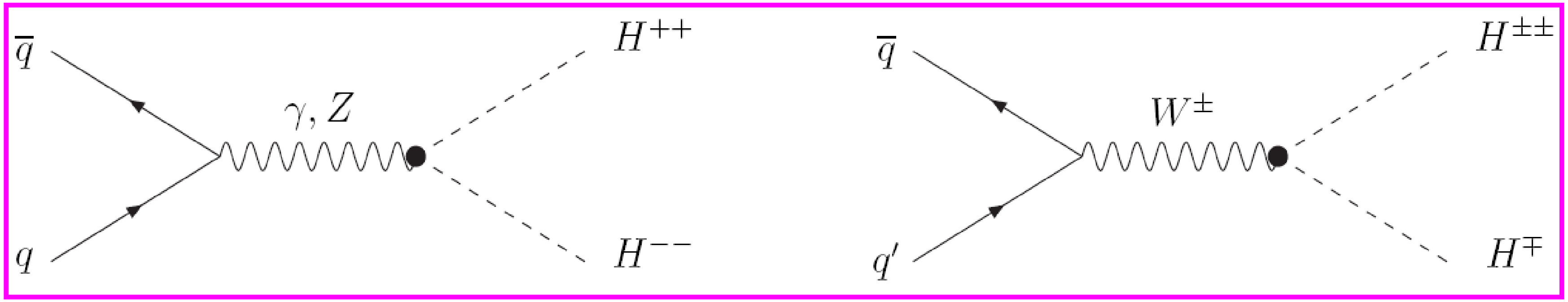
Naturalness? (t' Hooft 79, …, Giudice 08)

(1) *M*_Δ is O(1) TeV or close to the scale of gauge symmetry breaking.
(2) *λ*_Δ must be tiny, and *λ*_Δ =0 enhances the symmetry of the model.

$$\underbrace{M_{\rm L}}_{0.01\ \text{eV}} \approx \lambda_\Delta Y_\Delta \frac{v^2}{\underbrace{M_\Delta}_{1\,\rm TeV}} \quad\Longrightarrow\quad \lambda_\Delta Y_\Delta \sim 10^{-12} \Rightarrow \begin{cases} Y_\Delta \sim 1, \lambda_\Delta \sim 10^{-12} \\ \lambda_\Delta \sim Y_\Delta \sim 10^{-6} \\ \ldots\ldots \end{cases}$$

Collider signature

From a viewpoint of direct tests, the triplet seesaw has an advantage: The SU(2)_L Higgs triplet contains a doubly-charged scalar which can be produced at colliders: it is dependent on its mass but independent of the (small) Yukawa coupling.



Typical LNV signatures: $\boxed{H^{\pm\pm} \longrightarrow l^{\pm}_{\alpha} l^{\pm}_{\beta}}$ $\boxed{H^{+} \longrightarrow l^{+}_{\alpha} \overline{\nu}_{\beta}}$ $\boxed{H^{-} \longrightarrow l^{-}_{\alpha} \nu}$

$$\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta}) = \frac{(2-\delta_{\alpha\beta})|(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2} \,, \qquad \mathcal{B}(H^{+} \to l^{+}_{\alpha} \overline{\nu}) = \frac{\sum\limits_{\beta} |(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2}$$

Testability at the LHC

Lesson one: **the above branching ratios purely depend on 3 neutrino masses, 3 flavor mixing angles and the CP-violating phases.**

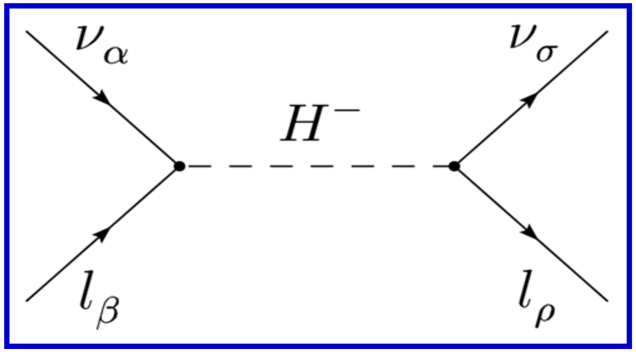
Lesson two: **the Majorana phases may affect LNV** $H^{\pm\pm} \longrightarrow l^{\pm}_{\alpha} l^{\pm}_{\beta}$ **decay modes, but they do not enter** $H^{+} \longrightarrow l^{+}_{\alpha} \bar{\nu}_{\beta}$ **and** $H^{-} \longrightarrow l^{-}_{\alpha} \nu$ **processes.**

$$\left|(M_{\rm L})_{\alpha\beta}\right|^2 = \left|\sum_{i=1}^{3} \left(m_i V_{\alpha i} V_{\beta i}\right)\right|^2 , \qquad \sum_{\beta} \left|(M_{\rm L})_{\alpha\beta}\right|^2 = \sum_{i=1}^{3} \left(m_i^2 \left|V_{\alpha i}\right|^2\right)$$

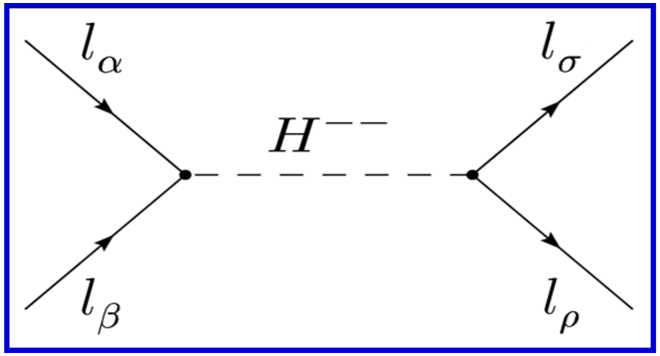
Dimension-6 operator:
(2 low-energy effects)

$$\frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} = -\frac{(Y_\Delta)_{\alpha\beta} (Y_\Delta)^\dagger_{\rho\sigma}}{4M^2_\Delta} \left(\overline{l_{\alpha \rm L}} \gamma^\mu l_{\sigma \rm L}\right) \left(\overline{l_{\beta \rm L}} \gamma_\mu l_{\rho \rm L}\right)$$

1) NSIs of 3 neutrinos



2) LFV of 4 charged leptons



Type-3 seesaw

Type-3 Seesaw: add 3 SU(2)_L triplet fermions (Y = 0) into the SM.

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} {\rm Tr} \left(\overline{\Sigma} M_\Sigma \Sigma^c \right) + {\rm h.c.}$$

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{\begin{pmatrix} e_{\rm L} & \Psi_{\rm L} \end{pmatrix}} \begin{pmatrix} M_l & \sqrt{2} M_{\rm D} \\ \mathbf{0} & M_\Sigma \end{pmatrix} \begin{pmatrix} E_{\rm R} \\ \Psi_{\rm R} \end{pmatrix} + \frac{1}{2} \overline{\begin{pmatrix} \nu_{\rm L} & \Sigma^0 \end{pmatrix}} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_\Sigma \end{pmatrix} \begin{pmatrix} \nu_{\rm L}^c \\ \Sigma^{0c} \end{pmatrix} + {\rm h.c.}$$

$$M_l = Y_l v/\sqrt{2} \, , \quad M_{\rm D} = Y_\Sigma v/\sqrt{2} \, , \quad \Psi = \Sigma^- + \Sigma^{+c}$$

Diagonalization of the neutrino mass matrix:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_\Sigma \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_\Sigma \end{pmatrix}$$

Seesaw formula:

$$M_\nu \equiv V \widehat{M}_\nu V^T \approx -M_{\rm D} M_\Sigma^{-1} M_{\rm D}^T$$

Comparison between type-1 and type-3 seesaws (Abada et al 07):

a) The 3×3 flavor mixing matrix *V* is non-unitary in both cases (CC);

b) The modified couplings between *Z* & neutrinos are different (NC);

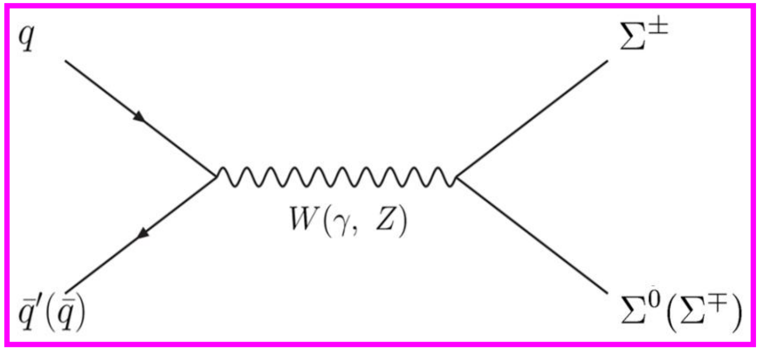
c) Non-unitary flavor mixing is also present in the coupling between *Z* and charged leptons in the type-3 seesaw (NC).

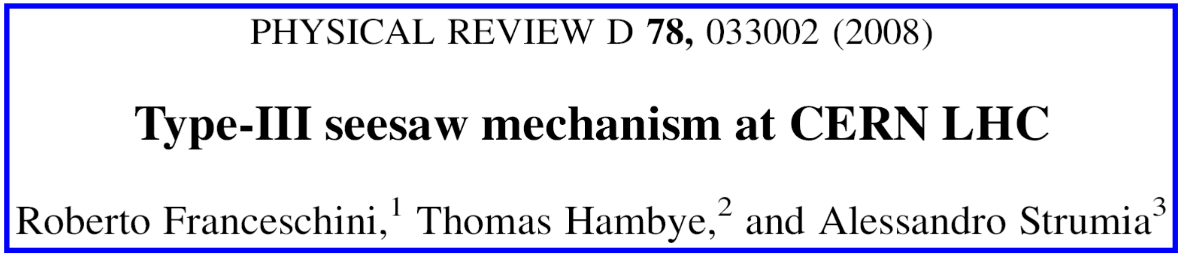
Testability at the LHC

LNV signatures at the **LHC**:

$$pp \to \Sigma^+\Sigma^0 \to l_\alpha^+ l_\beta^+ + Z^0 W^-(\to 4j)$$

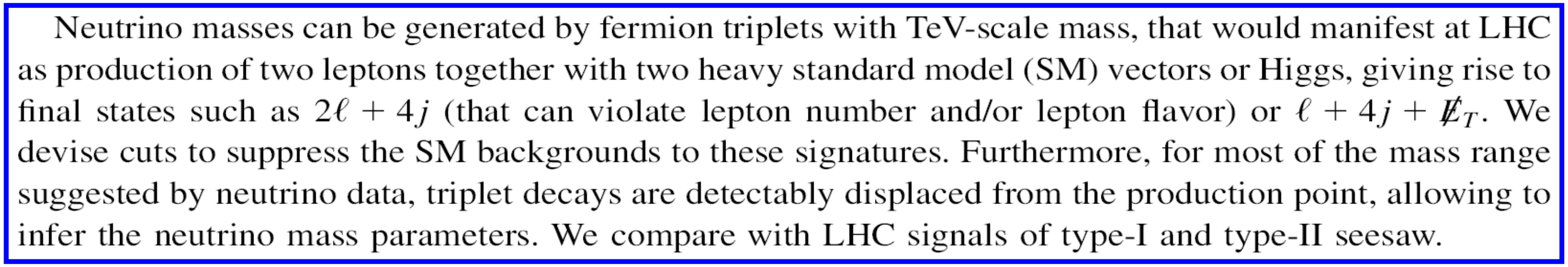
$$pp \to \Sigma^-\Sigma^0 \to l_\alpha^- l_\beta^- + Z^0 W^+(\to 4j)$$




PHYSICAL REVIEW D **78,** 033002 (2008)

Type-III seesaw mechanism at CERN LHC

Roberto Franceschini,[1] Thomas Hambye,[2] and Alessandro Strumia[3]


Neutrino masses can be generated by fermion triplets with TeV-scale mass, that would manifest at LHC as production of two leptons together with two heavy standard model (SM) vectors or Higgs, giving rise to final states such as $2\ell + 4j$ (that can violate lepton number and/or lepton flavor) or $\ell + 4j + \not{E}_T$. We devise cuts to suppress the SM backgrounds to these signatures. Furthermore, for most of the mass range suggested by neutrino data, triplet decays are detectably displaced from the production point, allowing to infer the neutrino mass parameters. We compare with LHC signals of type-I and type-II seesaw.


Distinguishing seesaw models at LHC with multi-lepton signals

F. del Aguila, J. A. Aguilar–Saavedra

2 latest comprehensive works.


arXiv:0808.2468v2 [hep-ph] 12 Sep 2008

Low-energy tests

Type-3 seesaw: a typical signature would be the **non-unitary effects** of the 3×3 lepton flavor mixing matrix ***N*** in both **CC** and **NC** interactions.

Current experimental constraints at the 90% C.L. (Abada *et al* 07):

$$|NN^\dagger| \approx \begin{pmatrix} 1.001 \pm 0.002 & < 1.1 \cdot 10^{-6} & < 1.2 \cdot 10^{-3} \\ < 1.1 \cdot 10^{-6} & 1.002 \pm 0.002 & < 1.2 \cdot 10^{-3} \\ < 1.2 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} & 1.002 \pm 0.002 \end{pmatrix}$$

accuracy at 0.1%.

These bounds are stronger than those obtained in the type-1 seesaw, as the flavor-changing processes with charged leptons are allowed at the tree level in the type-3 seesaw.

Two types of LFV processes:

Radiative decays of charged leptons: $\mu \to e + \gamma$, $\tau \to e + \gamma$, $\tau \to \mu + \gamma$.
Tree-level rare decays of charged leptons: $\mu \to 3e$, $\tau \to 3e$, $\tau \to 3\mu$, $\tau \to e + 2\mu$, $\tau \to 2e + \mu$ **(Abada et al 07, 08; He, Oh 09)**

TeV leptogenesis or muon g-2 problems? (Strumia 08, Blanchet, Chacko, Mohapatra 08, Fischler, Flauger 08; Chao 08, Biggio 08; ……)

Seesaw trivialization

Linear trivialization: use three types of seesaws to make a family tree.

Type 1 + Type 2

Type 1 + Type 3

Type 2 + Type 3

Type 1 + Type 2 + Type 3

Weinberg's *3rd law of progress in theoretical physics* (83): You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry …………………………………… What could be better?

Linearly trivialized seesaws usually work at super-high energies.

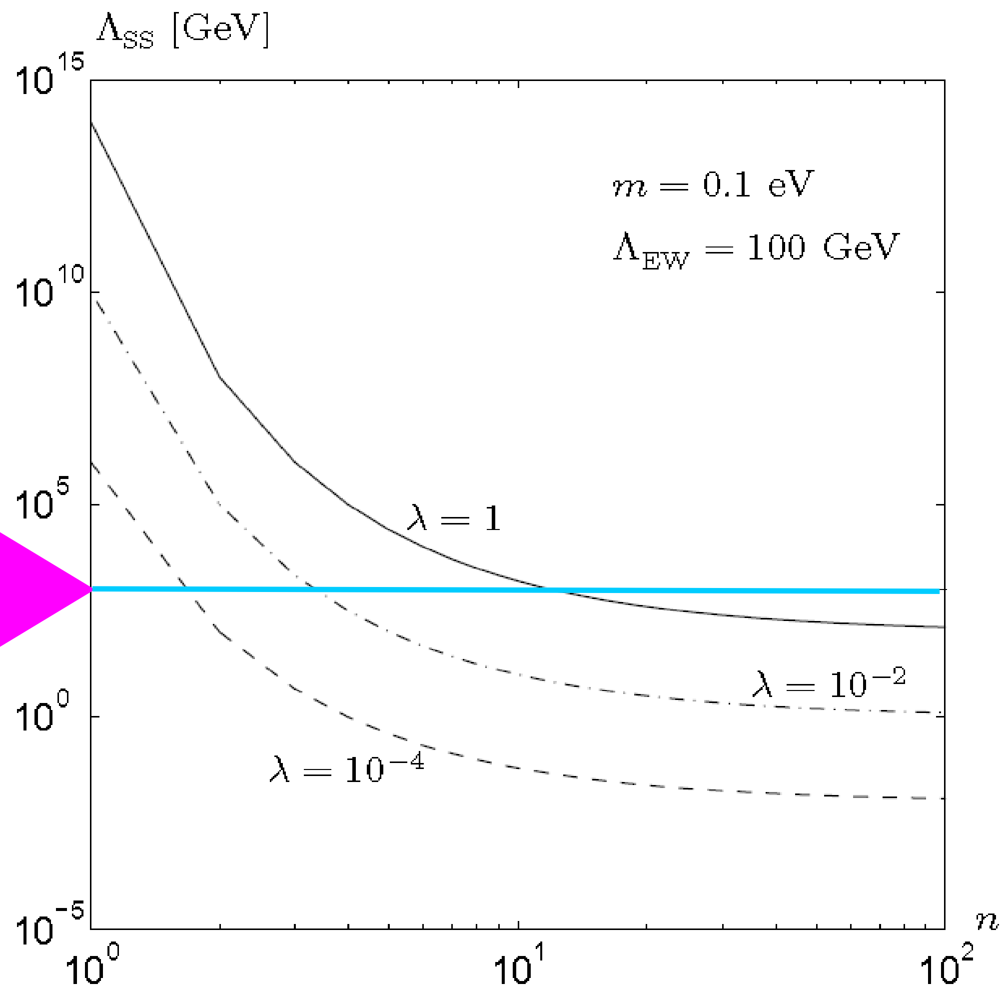
Multiple trivialization: well motivated to lower the seesaw scale.

Illustration

Neutrino mass:

$$m \sim (\lambda \Lambda_{\mathrm{EW}})^{n+1} / \Lambda_{\mathrm{SS}}^{n}$$

$$\Lambda_{\mathrm{SS}} \sim \lambda^{\frac{n+1}{n}} \left[\frac{\Lambda_{\mathrm{EW}}}{100\ \mathrm{GeV}} \right]^{\frac{n+1}{n}} \left[\frac{0.1\ \mathrm{eV}}{m} \right]^{\frac{1}{n}} 10^{\frac{2(n+6)}{n}}\ \mathrm{GeV}$$



TeV scale

Example: inverse seesaw

The Inverse Seesaw: SM + 3 heavy right-handed neutrinos + 3 gauge singlet neutrinos + one Higgs singlet (Wyler, Wolfenstein 83; Mohapatra, Valle 86; Ma 87).

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \overline{N^{\rm c}_{\rm R}} Y_S \Phi S_{\rm R} + \frac{1}{2}\overline{S^{\rm c}_{\rm R}} \mu S_{\rm R} + {\rm h.c.}$$

LNV: tiny

ν-mass matrix:

$$\overline{\left(\nu_{\rm L} \quad N^{\rm c}_{\rm R} \quad S^{\rm c}_{\rm R}\right)} \begin{pmatrix} \mathbf{0} & M_{\rm D} & \mathbf{0} \\ M^T_{\rm D} & \mathbf{0} & M_S \\ \mathbf{0} & M^T_S & \mu \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \\ S_{\rm R} \end{pmatrix}$$

$$\begin{aligned} M_{\rm D} &= Y_\nu \langle H \rangle \\ M_S &= Y_S \langle \Phi \rangle \end{aligned}$$

Effective light ν-mass matrix

$$M_\nu \approx M_{\rm D} \frac{1}{M^T_S} \mu \frac{1}{M_S} M^T_{\rm D} \quad \Longleftrightarrow \quad -\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$$

Merit: more natural tiny ν-masses and appreciable collider signatures; Fault: some new degrees of freedom. **Is Weinberg's 3rd law applicable?**

Multiple seesaw mechanisms: to *naturally* lower seesaw scales to TeV (Babu et al 09; Xing, Zhou 09; Bonnet et al 09, etc).

Appendix

Misguiding principles for a theorist to go beyond the SM
(Schellekens 08: “The Emperor’s Last Clothes?”)

- Agreement with observation
- Consistency
- Uniqueness
- Naturalness
- Simplicity
- Elegance
- Beauty
- ……

?

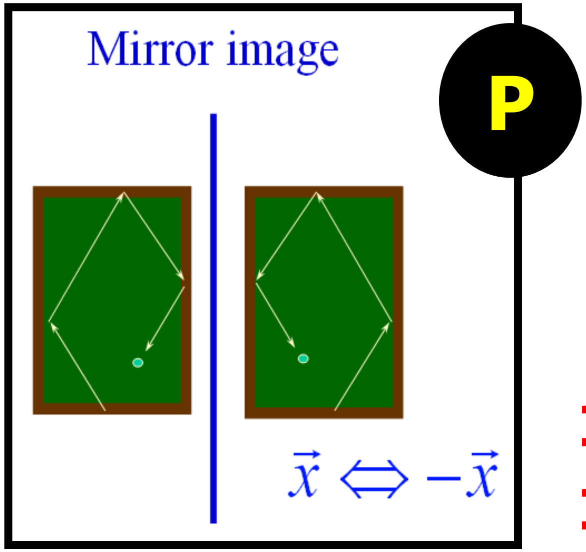
Flavor mixing/CP violation

Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of 2 types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the *W* boson;

Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (Yukawa interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of 2 types of interactions.

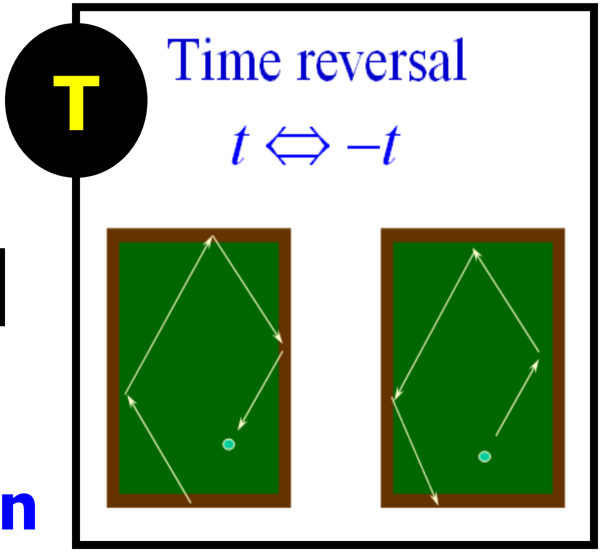


C

Charge-conjugation

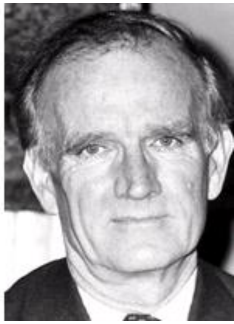
1957: P violation

1964: CP violation



Towards the KM paper

1964: **Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)**



NP 1980

1967: **Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)**

NP 1975

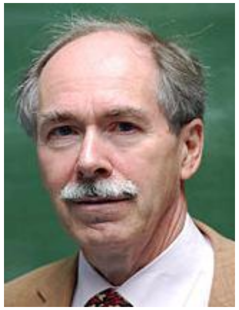
1967: **The birth of the standard electroweak model (S. Weinberg)**

0 citation for the first 4 yrs

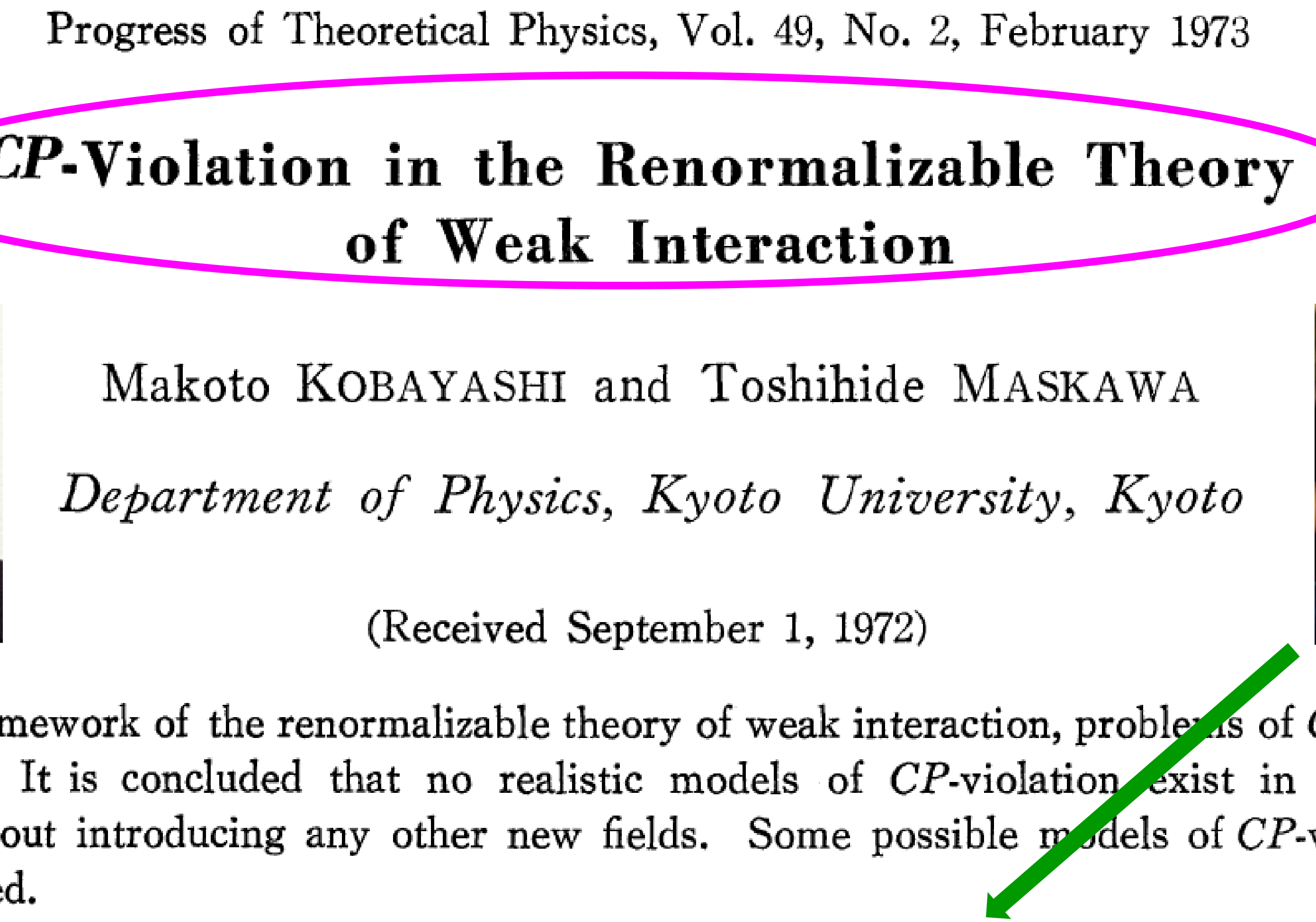
NP 1979

1971: **The first proof of the renormalizability of the standard model (G. ‘t Hooft)**

NP 1999



KM in 1972



Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

Japan's Archimedes

3 families → CP violation: Maskawa's bathtub idea!

Diagnosis of CP violation

In the minimal νSM (SM+3 right-handed ν's) the Kobayashi-Maskawa mechanism is responsible for CP violation.

$$\boxed{\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}}$$

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2}$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q}_{\mathrm{L}} i \not{D} Q_{\mathrm{L}} + \overline{\ell}_{\mathrm{L}} i \not{D} \ell_{\mathrm{L}} + \overline{U}_{\mathrm{R}} i \not{\partial}' U_{\mathrm{R}} + \overline{D}_{\mathrm{R}} i \not{\partial}' D_{\mathrm{R}} + \overline{E}_{\mathrm{R}} i \not{\partial}' E_{\mathrm{R}} + \underline{\overline{N}_{\mathrm{R}} i \not{\partial}' N_{\mathrm{R}}}$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q}_{\mathrm{L}} Y_{\mathrm{u}} \tilde{H} U_{\mathrm{R}} - \overline{Q}_{\mathrm{L}} Y_{\mathrm{d}} H D_{\mathrm{R}} - \overline{\ell}_{\mathrm{L}} Y_{l} H E_{\mathrm{R}} - \underline{\overline{\ell}_{\mathrm{L}} Y_{\nu} \tilde{H} N_{\mathrm{R}}} + \mathrm{h.c.}$$

Nobel Prize 2008

ν's Dirac mass

The strategy of diagnosis:

given proper CP transformations of the gauge, Higgs and fermion fields, one may prove that the 1st, 2nd and 3rd terms are formally invariant, and the 4th term can be invariant only if the corresponding Yukawa coupling matrices are real. (spontaneous symmetry breaking doesn't affect CP.)

CP transformations

Gauge fields:

$$\boxed{\left[B_\mu,\ W^1_\mu,\ W^2_\mu,\ W^3_\mu\right] \xrightarrow{\mathrm{CP}} \left[-B^\mu,\ -W^{1\mu},\ +W^{2\mu},\ -W^{3\mu}\right]}$$

$$\boxed{\left[B_{\mu\nu},\ W^1_{\mu\nu},\ W^2_{\mu\nu},\ W^3_{\mu\nu}\right] \xrightarrow{\mathrm{CP}} \left[-B^{\mu\nu},\ -W^{1\mu\nu},\ +W^{2\mu\nu},\ -W^{3\mu\nu}\right]}$$

Higgs fields:

$$\boxed{H(t,\mathbf{x}) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\mathrm{CP}} H^*(t,-\mathbf{x}) = \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix}}$$

Lepton or quark fields:

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\psi_2 \xrightarrow{\mathrm{CP}} -\overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\psi_1}$$

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\partial^\mu\psi_2 \xrightarrow{\mathrm{CP}} \overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\partial_\mu\psi_1}$$

Spinor bilinears:

	$\overline{\psi_1}\psi_2$	$i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma_\mu\psi_2$	$\overline{\psi_1}\gamma_\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma_{\mu\nu}\psi_2$
C	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma_{\mu\nu}\psi_1$
P	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$-\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
T	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
CP	$\overline{\psi_2}\psi_1$	$-i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma^\mu\psi_1$	$-\overline{\psi_2}\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma^{\mu\nu}\psi_1$
CPT	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$-\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$\overline{\psi_2}\sigma_{\mu\nu}\psi_1$

$\boxed{\mathcal{L}_{\mathrm{G}}}$

$\boxed{\mathcal{L}_{\mathrm{H}}}$

$\boxed{\mathcal{L}_{\mathrm{F}}}$

formally invariant under CP

CP violation

The Yukawa interactions of fermions are formally invariant under CP if and only if

$$Y_{\rm u} = Y_{\rm u}^* , \quad Y_{\rm d} = Y_{\rm d}^*$$
$$Y_l = Y_l^* , \quad Y_\nu = Y_\nu^*$$

If the effective Majorana mass term is added into the SM, then the Yukawa interactions of leptons can be formally invariant under CP if

$$M_{\rm L} = M_{\rm L}^* , \qquad Y_l = Y_l^*$$

If the flavor states are transformed into the mass states, the source of flavor mixing and CP violation will show up in the *CC* interactions:

quarks

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(u \ c \ t)_{\rm L}} \ \gamma^\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + {\rm h.c.}$$

leptons

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

Comment A: **flavor mixing and CP violation can occur since fermions interact with both the gauge bosons and the Higgs boson.**

Comment B: **both the CC and Yukawa interactions have been verified.**

Comment C: **the CKM matrix *U* is unitary, the PMNS matrix *V* is too?**

Parameter counting

The 3×3 unitary matrix V can always be parametrized as a product of 3 unitary rotation matrices in the complex planes:

$$O_1(\theta_1,\alpha_1,\beta_1,\gamma_1) = \begin{pmatrix} c_1e^{i\alpha_1} & s_1e^{-i\beta_1} & 0 \\ -s_1e^{i\beta_1} & c_1e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$O_2(\theta_2,\alpha_2,\beta_2,\gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2e^{i\alpha_2} & s_2e^{-i\beta_2} \\ 0 & -s_2e^{i\beta_2} & c_2e^{-i\alpha_2} \end{pmatrix}$$

$$O_3(\theta_3,\alpha_3,\beta_3,\gamma_3) = \begin{pmatrix} c_3e^{i\alpha_3} & 0 & s_3e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3e^{i\beta_3} & 0 & c_3e^{-i\alpha_3} \end{pmatrix}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ (for $i = 1, 2, 3$)

Category A: 3 possibilities

$$V = O_iO_jO_i \quad (i \neq j)$$

Category B: 6 possibilities

$$V = O_iO_jO_k \quad (i \neq j \neq k)$$

Rephasing

For instance, the standard parametrization is given below:

V

$$= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3 \ , \quad b = -\beta_2 - \alpha_3 \ , \quad c = -\alpha_2 - \alpha_3 \ ;$$

$$x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \ , \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \ , \quad z = \gamma_1 \ .$$

Physical phases

If neutrinos are Dirac particles, the phases *x*, *y* and *z* can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

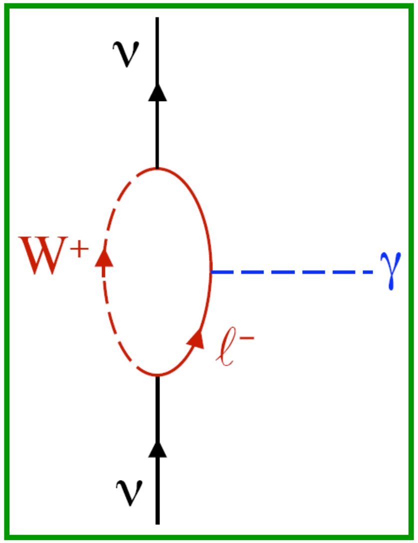
If neutrinos are Majorana particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., *z* = 0). Then

Majorana neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electromagnetic properties

A neutrino does not have electric charges, but it has electromagnetic interactions with the photon via quantum loops.



Given the SM interactions, a massive Dirac neutrino can only have a tiny magnetic dipole moment:

$$\boxed{\mu_\nu \sim \frac{3eG_{\rm F}}{8\sqrt{2}\pi^2} m_\nu = 3\times 10^{-20} \frac{m_\nu}{0.1\,{\rm eV}} \mu_{\rm B}}$$

A massive Majorana neutrino can not have magnetic & electric dipole moments, as its antiparticle is itself.

Proof: Dirac neutrino's electromagnetic vertex can be parametrized as

$$\boxed{\Gamma_\mu(p,p') = f_{\rm Q}(q^2)\gamma_\mu + f_{\rm M}(q^2) i\sigma_{\mu\nu}q^\nu + f_{\rm E}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_{\rm A}(q^2)\left(q^2\gamma_\mu - q_\mu q^\nu\gamma_\nu\right)\gamma_5}$$

Majorana neutrinos

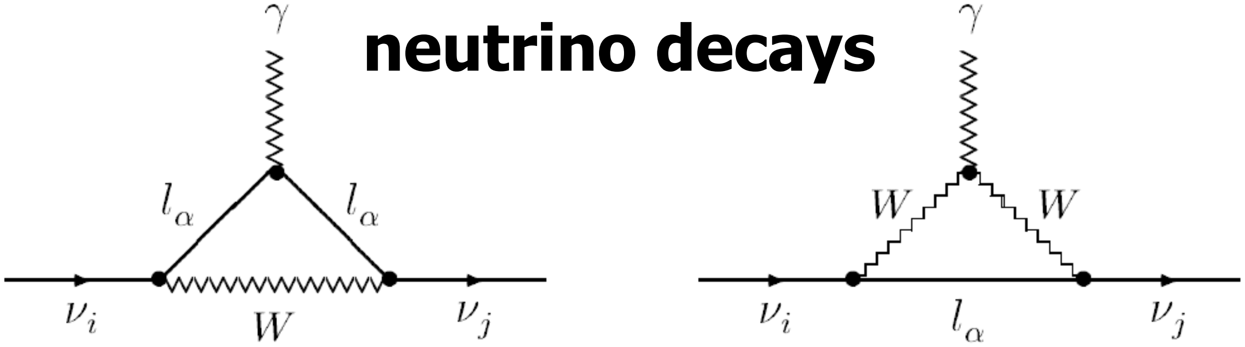
$$\boxed{\underline{\overline{\psi}\Gamma_\mu\psi} = \overline{\psi^c}\Gamma_\mu\psi^c = \psi^T \mathcal{C}\Gamma_\mu \mathcal{C}\overline{\psi}^T = \left(\psi^T \mathcal{C}\Gamma_\mu \mathcal{C}\overline{\psi}^T\right)^T = -\overline{\psi}\mathcal{C}^T\Gamma_\mu^T \mathcal{C}^T\psi = \underline{\overline{\psi}\mathcal{C}\Gamma_\mu^T\mathcal{C}^{-1}\psi}}$$

→ $\boxed{f_{\rm Q}(q^2) = f_{\rm M}(q^2) = f_{\rm E}(q^2) = 0}$ **intrinsic property of Majorana ν's.**

Transition dipole moments

Both Dirac & Majorana neutrinos can have *transition* dipole moments (of a size comparable with μ_ν) that may give rise to neutrino decays, scattering with electrons, interactions with external magnetic field & contributions to ν masses. (Data: < a few × 10^-11 Bohr magneton).

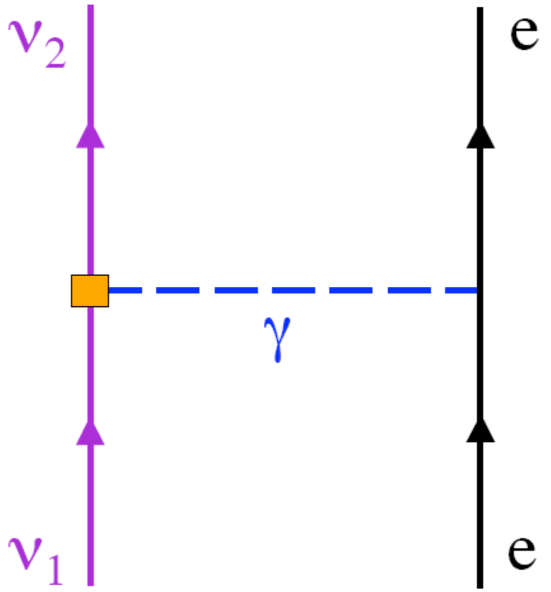
neutrino decays



$$\mu_{\rm eff} \equiv \sqrt{\left|\mu_{ij}\right|^2 + \left|\epsilon_{ij}\right|^2}$$

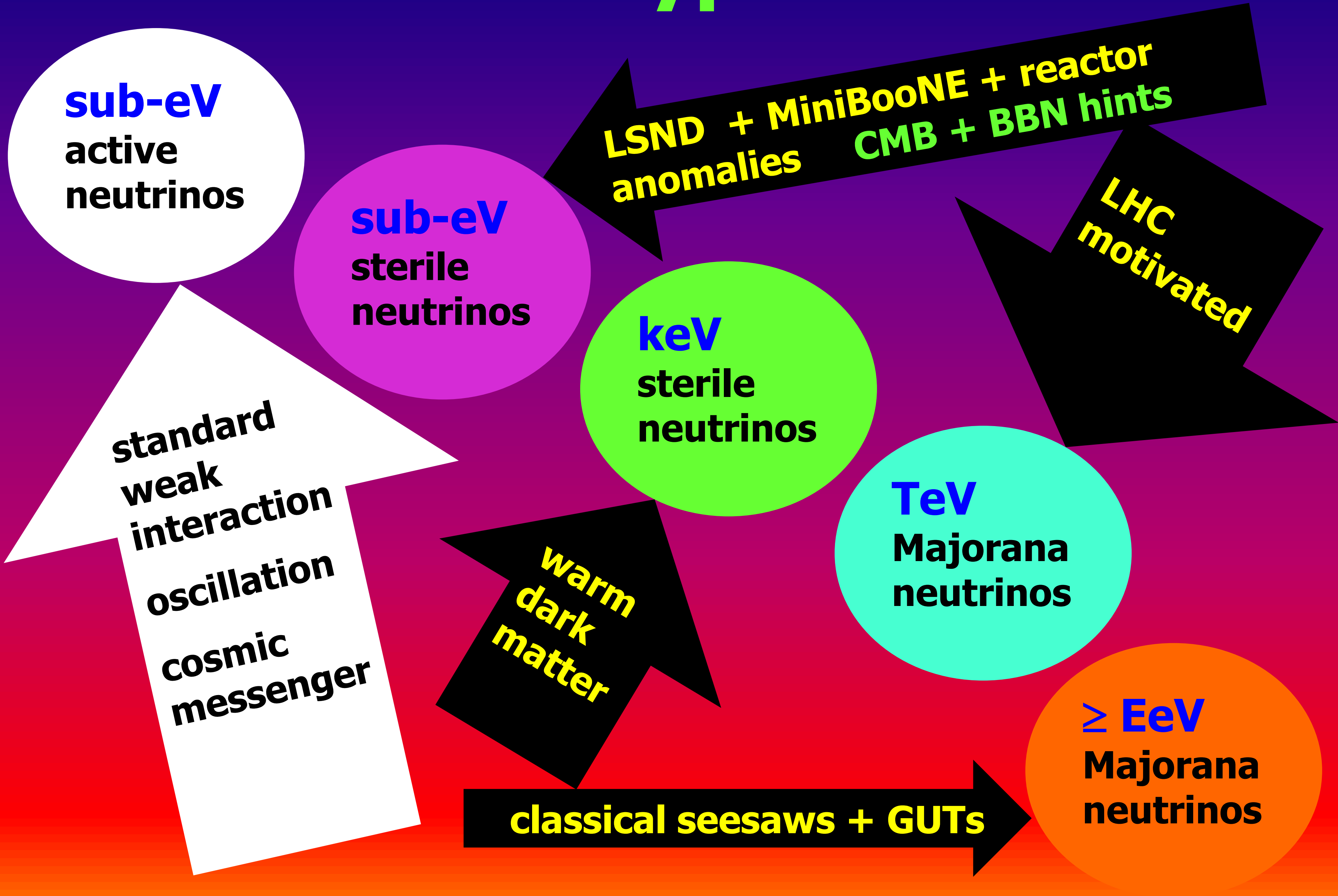
$$\Gamma_{\nu_i \to \nu_j + \gamma} = 5.3 \times \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ {\rm eV}}\right)^3 \left(\frac{\mu_{\rm eff}}{\mu_{\rm B}}\right)^2 {\rm s}^{-1}$$

scattering



$$\frac{{\rm d}\sigma'_\mu}{{\rm d}T} = \frac{\alpha^2 \pi}{m_e^2} \sum_{k=1}^{3} \left| \sum_{j=1}^{3} e^{i q_j L} V_{ej} \left(i \frac{\mu_{jk}}{\mu_{\rm B}} + \frac{\epsilon_{jk}}{\mu_{\rm B}} \right) \right|^2 \left(\frac{1}{T} - \frac{1}{E_\nu} \right)$$

Real + Hypothetical ν's



sub-eV
active
neutrinos

sub-eV
sterile
neutrinos

keV
sterile
neutrinos

TeV
Majorana
neutrinos

≥ EeV
Majorana
neutrinos

standard
weak
interaction

oscillation

cosmic
messenger

LSND + MiniBooNE + reactor
anomalies

CMB + BBN hints

LHC
motivated

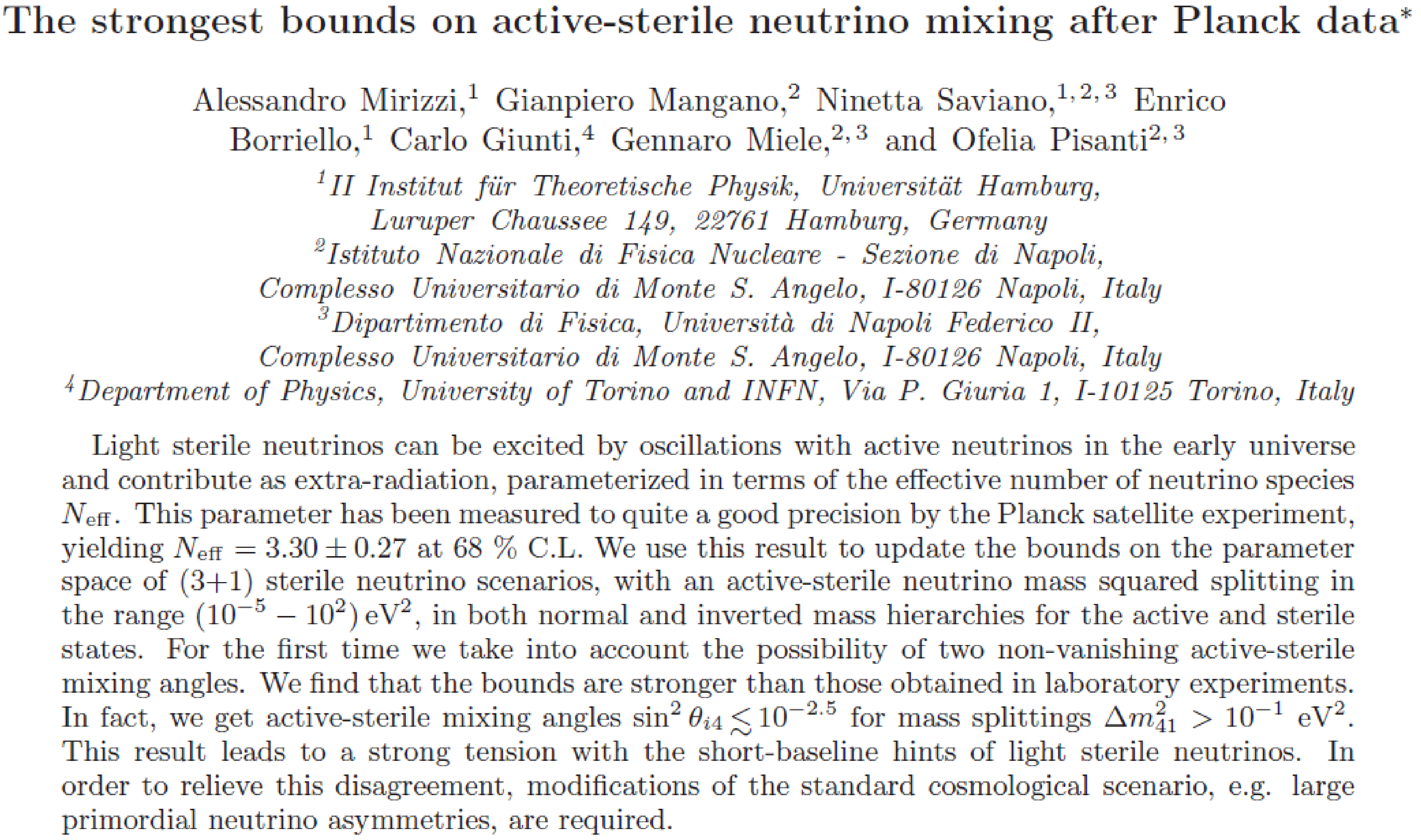
warm
dark
matter

classical seesaws + GUTs

Planck constraints

arXiv:1303.5368 (21 March)

$$\boxed{N_{\rm eff} = 3.30 \pm 0.27 \text{ at } 68\ \% \text{ C.L.}}$$



The strongest bounds on active-sterile neutrino mixing after Planck data*

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Light sterile neutrinos can be excited by oscillations with active neutrinos in the early universe and contribute as extra-radiation, parameterized in terms of the effective number of neutrino species $N_{\rm eff}$. This parameter has been measured to quite a good precision by the Planck satellite experiment, yielding $N_{\rm eff} = 3.30 \pm 0.27$ at 68 % C.L. We use this result to update the bounds on the parameter space of (3+1) sterile neutrino scenarios, with an active-sterile neutrino mass squared splitting in the range $(10^{-5} - 10^{2})\,{\rm eV}^2$, in both normal and inverted mass hierarchies for the active and sterile states. For the first time we take into account the possibility of two non-vanishing active-sterile mixing angles. We find that the bounds are stronger than those obtained in laboratory experiments. In fact, we get active-sterile mixing angles $\sin^2\theta_{i4} \lesssim 10^{-2.5}$ for mass splittings $\Delta m^2_{41} > 10^{-1}$ eV^2. This result leads to a strong tension with the short-baseline hints of light sterile neutrinos. In order to relieve this disagreement, modifications of the standard cosmological scenario, e.g. large primordial neutrino asymmetries, are required.

(3+3) flavor mixing

active flavor

sterile flavor

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}$$

mass state

A full parametrization

$$\mathcal{U} = \begin{pmatrix} 1 & 0 \\ 0 & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix}$$

sterile part **interplay** **active part**

$$\begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix} = O_{23} O_{13} O_{12} \; ,$$

$$\begin{pmatrix} 1 & 0 \\ 0 & U_0 \end{pmatrix} = O_{56} O_{46} O_{45} \; ,$$

$$\begin{pmatrix} A & R \\ S & B \end{pmatrix} = O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14}$$

Full parametrization:
15 rotation angles
15 phase phases

Xing, arXiv:1110.0083

Questions

1) Do you feel happy / painful / sorry to introduce sterile neutrinos into the SM (remember Weinberg's theorem)?

2) How many species of sterile neutrinos should be taken into account for your this or that purpose? 1? 2? 3? ….?

3) If all the current experimental and observational hints disappear, will the sterile neutrino physics still survive?

4) Do you like to throw many stones to only kill few birds or just the opposite? And is this a very stupid question?

Weinberg's *3rd law of progress in theoretical physics* (83):

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry ………………………………… What could be better?