Introduction of Accelerator

Physics and Technology

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Technologies used in modern accelerators

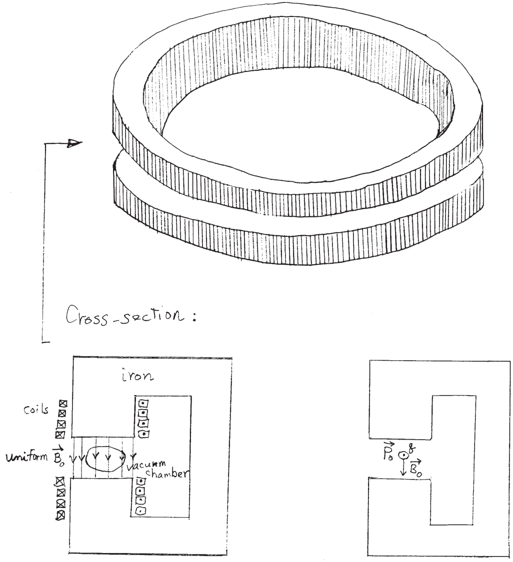
- large scale vacuum
- high power microwave
- superconducting (magnets, microwave) technology
- computer control
- very strong/very high precision magnets
- large scale scientific project management (very important)
- accelerator physics (beam dynamics) (beam physics)
- ...

How to design a storage ring

- charged particles
- Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

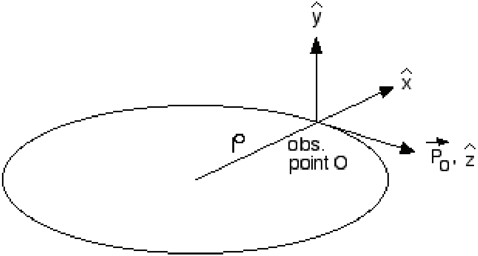
- why magnetic field, not electric field



$$\vec{F} = q\vec{v} \times \vec{B}$$

Stability Principle

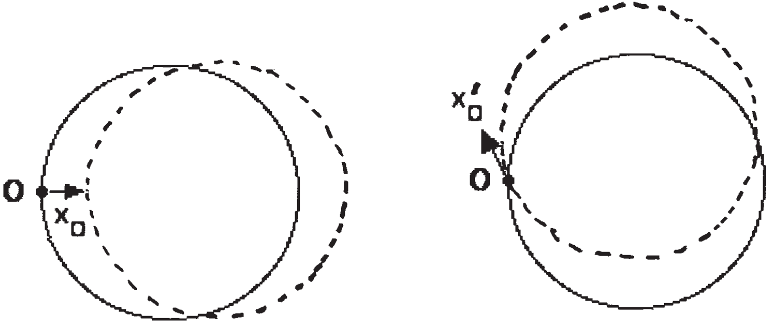
A stable storage ring must also store nonideal particles with “slight deviations” from the ideal conditions, i.e. the accelerator must have a finite acceptance around the ideal condition. Otherwise it is not a stable accelerator.

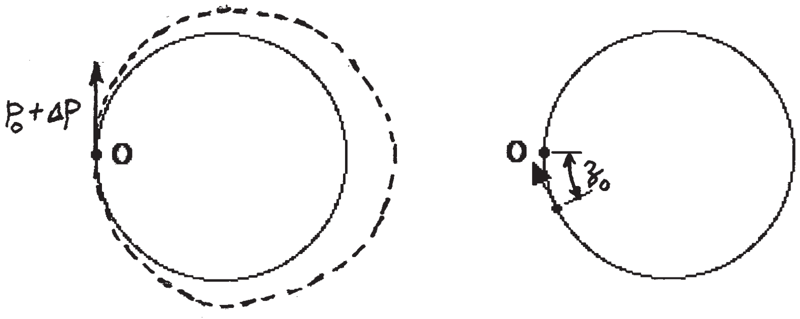


Living in an n-D world, we need to consider $2n$ kinds of deviations. Why twice of n? Because of the fundamental property of natural laws that all dynamics involve second order differential equations, led by Newton's equation $m\ddot{\vec{x}} = \vec{F}$ (or Hamilton equations if you so prefer). Had the dynamics required third order differential equations, then we will need to consider 3n deviations.

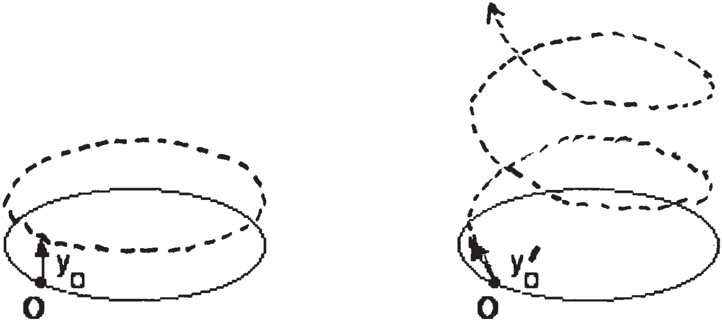
Motion must be stable for particles with all these six kinds of initial deviations: $x_0, x'_0, \Delta P_0, z_0, y_0, y'_0$.

If there exist deviation in x_0 and x_0'

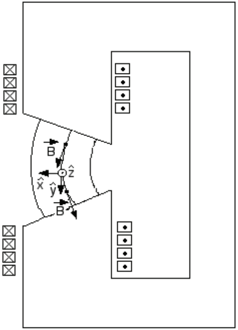


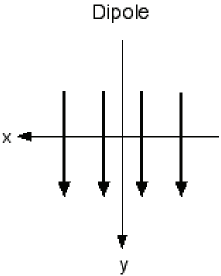

P0+ΔP
O
O
z0

If there exist deviation in y_0 and y_0'



Weak focusing Magnet





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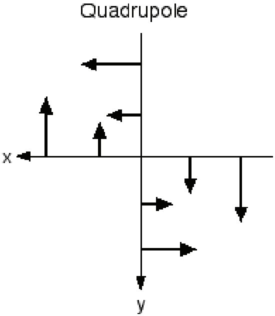
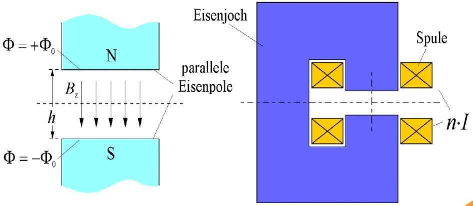


$$\vec{B} = B_0\hat{y} + G(y\hat{x} + x\hat{y})$$

2-D Magnetostatics

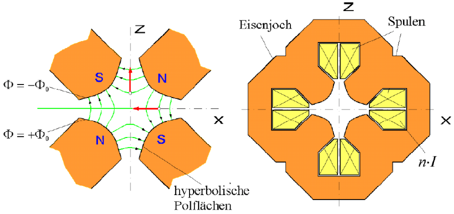
- iron-dominated, uses iron pole face to shape the magnetic field. Because the iron typically saturates when the magnetic field reaches beyond 2 Tesla or so, iron-dominated magnets typically has maximum pole tip field less than 2 Tesla.
- current-dominated, uses little iron and is most likely using superconducting wires to carry the large currents. The superconducting current-dominated magnets typically reach 4-10 Tesla.

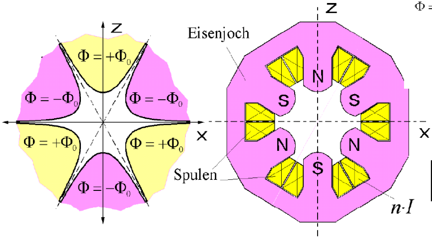
Iron-dominated magnets



Dipole

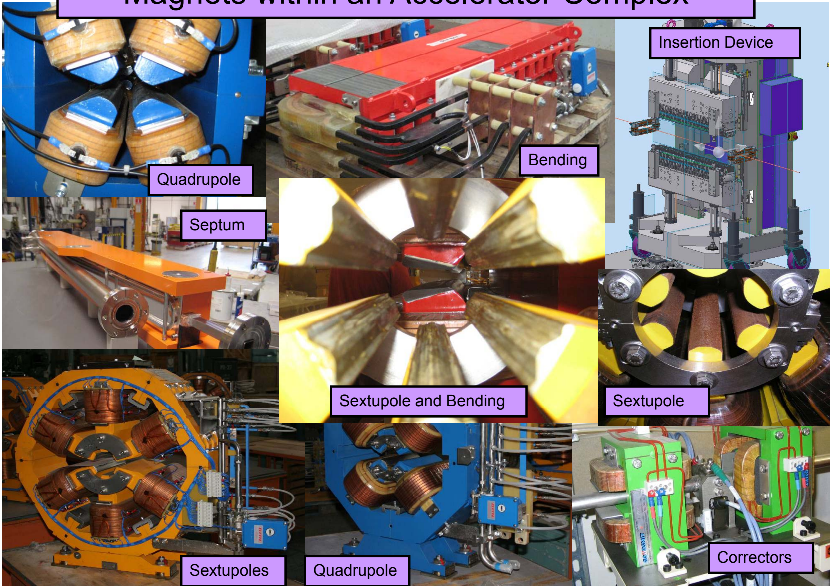
Quadrupole





Sextupole

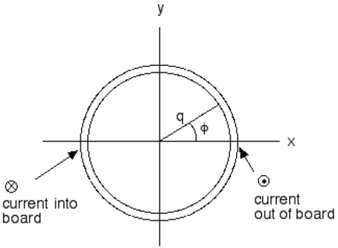
Magnets within an Accelerator Complex



$\cos\theta$ Magnet

Consider a cylindrical infinitely-thin sheet of current distribution

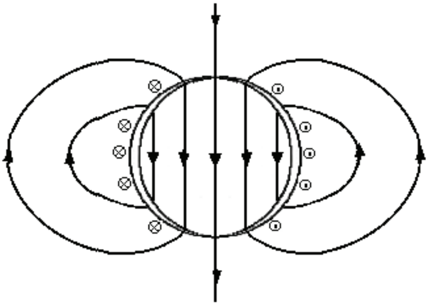
$$J(q,\phi) = \frac{I_0}{2a}\delta(q-a)\cos\phi$$



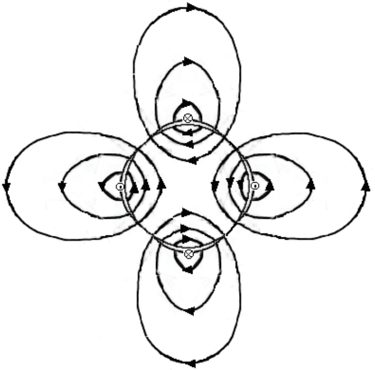
where a is the current-carrying cylinder radius. The right half of the sheet ($\cos\phi > 0$) carries current out of the board. There are no currents at the north and the south poles.

$$B_y + iB_x = \frac{\mu_0 I_0}{4}\begin{cases} -\frac{1}{a} & , \sqrt{x^2+y^2} < a \\ \frac{a}{z^2} & , \sqrt{x^2+y^2} > a \end{cases}$$

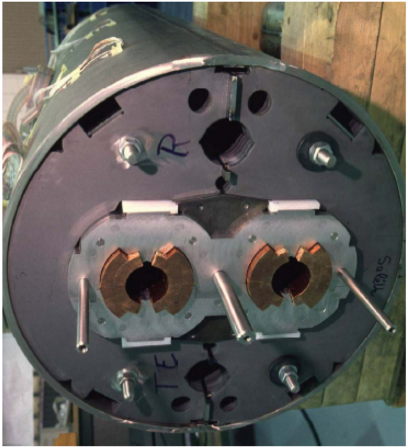
$\cos\theta$ and $\cos 2\theta$ magnet field

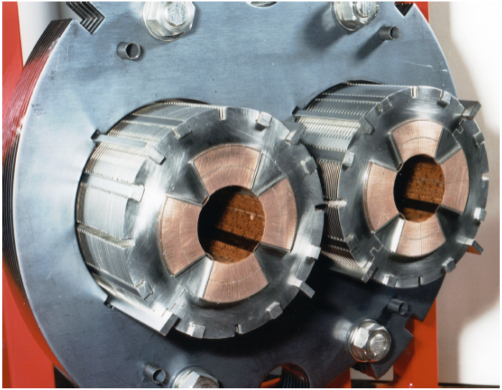


cos θ dipole



cos 2θ quadrupole

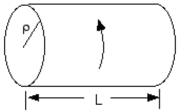


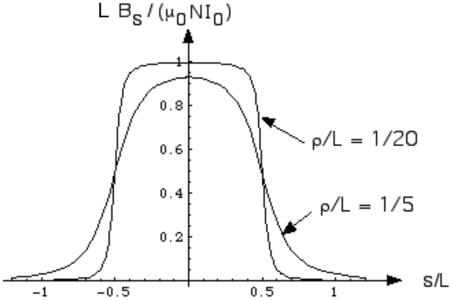


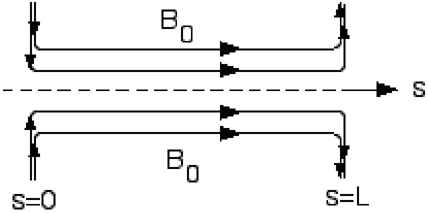
Solenoid

Another common magnet not of a multipole type is the solenoid. It is no longer a 2-D system.

Solenoid

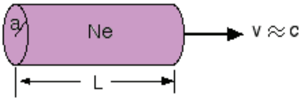


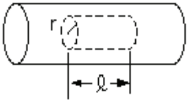




Beam Field & Space Charge

Consider a cylindrically shaped beam with uniform distribution moving in the z-direction as shown below:





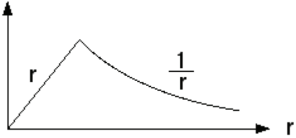
- Appling the Gauss's law,

$$E_r = \begin{cases} \frac{Ne}{2\pi\epsilon_0 a^2 L} r, & (r < a) \\ \frac{Ne}{2\pi\epsilon_0 L}\frac{1}{r}, & (r > a) \end{cases}$$

- Applying Ampere's law,

$$B_\theta = \begin{cases} \frac{\mu_0 vNe}{2\pi a^2 L} r, & (r < a) \\ \frac{\mu_0 vNe}{2\pi L}\frac{1}{r}, & (r > a) \end{cases}$$

$$B_\theta = \frac{v}{c^2} E_r$$



The Lorentz force experienced by a particle in the beam due to the space charge fields it sees,

$$\vec{F} = \frac{Ne^2}{2\pi\epsilon_0 a^2 L\gamma^2} r\hat{r}$$

This almost-perfect cancellation between the electric and the magnetic forces is very important for relativistic particles, without which most accelerators will not work.

Design of an accelerator

To design an accelerator, one first considers the motion of a single charged particle in the environment of magnets and RF cavities. The motion of this single particle in this environment must be stable.

For example, in a circular accelerator, the particle must stay inside of the accelerator vacuum chamber for many many revolutions, typically $\ll 10^{10}$ revolutions — and much more than that of the lifetime of earth around the sun!

Accelerator physicists design accelerators with three basic elements

Element	Function	Field	Focusing
Dipoles	Guide particle trajectory	Magnetic	weak focusing in x
Quadrupoles	Confine particle motion near the design trajectory	Magnetic	x,y
RF cavities	Keep particle energy near the design energy	Electric	z

All these are just to make sure that single-particle motion is stable. With these three elements arranged, the basic layout of an accelerator is determined.

Design of an accelerator (2)

Having provided a design trajectory, and made sure that there are focusing in x,y, and z, there seems to be nothing left to do. But that is not true. We still have to examine the stability of the beam particles in much more detail.

Single-particle stability. This is one very important area of accelerator physics, i.e. single-particle nonlinear dynamics

Multi-particle stability. There is a second significant part of accelerator physics. It is called multi-particle collective beam instability effects, sometimes also called collective beam instabilities, coherent beam instabilities, beam instabilities, or simply instabilities.

Linear Betatron Motion

- Hill's Equation

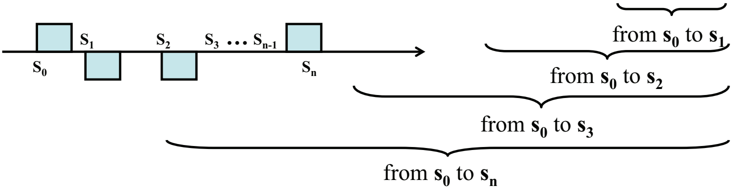
$$u'' + K_u(s)u = 0$$

- Matrix Form

$$\begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix} \equiv M \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

- a beamline with n elements

$$\mathcal{M}(s_n|s_0) = \mathcal{M}(s_n|s_{n-1}) \ldots \mathcal{M}(s_3|s_2) \cdot \mathcal{M}(s_2|s_1) \cdot \mathcal{M}(s_1|s0)$$



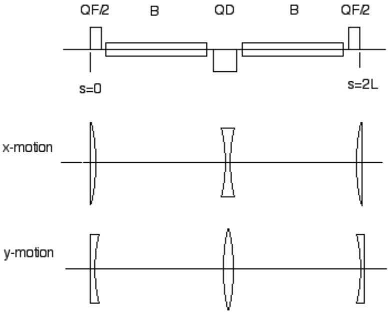
$\Longrightarrow$

$$\begin{pmatrix} u_n \\ u'_n \end{pmatrix} = M(s_n|s_0) \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

On matrix formalism

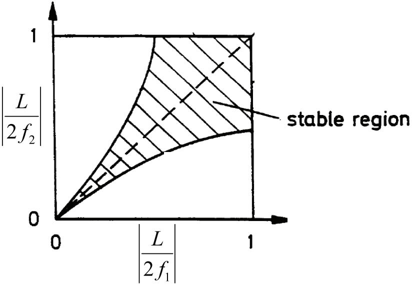
Mathematics	Accelerator physics
linear system	• vectors for phase space coordinate and transfer matrices • separability of beam properties (vector) and accelerator properties (transfer matrix)
matrix multiplication	beamlines
non-commutative	can't switch magents around
similarity transformation	observation of beam dynamics at different locations
eigenvalues	tunes (i.e. natural frequencies)
eigenvalues and trace are invariant under similiarity transformations	• tunes don't change with observation location • stability/instability of beam dynamics doesn't change with observation location
symplecticity	• Hamiltonian dynamics • conservation of phase space
normal form	• Courant-Snyder analysis • β function

FODO



$$M_x = \begin{pmatrix} 1 - 2\frac{L}{f^*} & 2L\left[1 - \frac{L}{2f_2}\right] \\ -\frac{2}{f^*}\left[1 - \frac{L}{2f_1}\right] & 1 - 2\frac{L}{f^*} \end{pmatrix}$$

where $\frac{1}{f^*} = \frac{1}{2f_1} + \frac{1}{2f_2} - \frac{L}{4f_1 f_2}$



Trying too hard to speed up only slows you down!

Stability Criterion

- Stability of a linear system has nothing to do with the observation point.
- Stability of a linear system has nothing to do with the initial condition $\begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$.
- Stability of a non-linear system is related not only to the observation point, but also to the initial condition. There exist a "Dynamic Aperture".

Twiss Functions (Courant-Snyder Parameters)

The solution of Hill's Equation

$$u'' + K(s)u = 0, \qquad K(s+L) = K(s)$$

can be represented in Courant-Snyder formalism

$$u(s) = \sqrt{2J\beta(s)}\cos(\psi(s) + \psi_0)$$

We define α function,

$$\alpha(s) = -\frac{1}{2}\beta'(s)$$

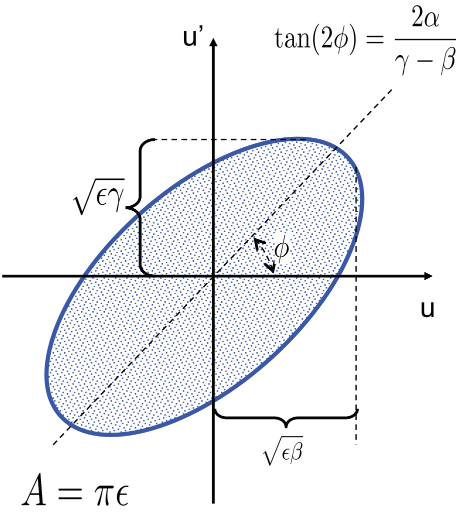
The unit of β is meter, and α is dimensionless.

Twiss parameters, emittance and beam sizes

- Twiss parameters and emittance determine the size and shape of the beam at some observation point

$$\langle x^2 \rangle = \beta_x \langle J_x \rangle$$
$$\langle x x' \rangle = -\alpha_x \langle J_x \rangle$$
$$\langle x'^2 \rangle = \gamma_x \langle J_x \rangle$$

$$\epsilon_x = \langle J_x \rangle$$



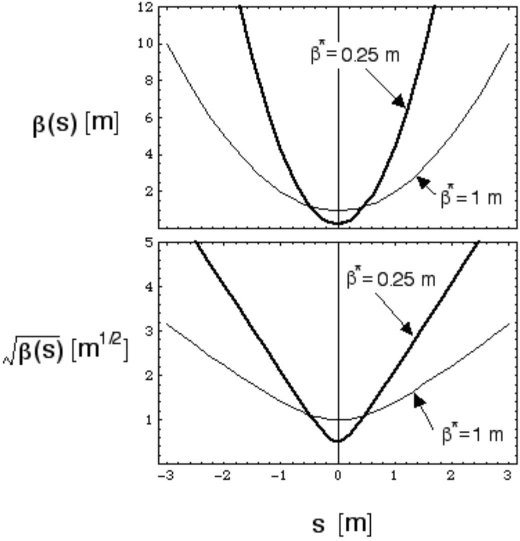
Betatron tune

The quantity Φ is related to another important quantity *betatron tune* per period,

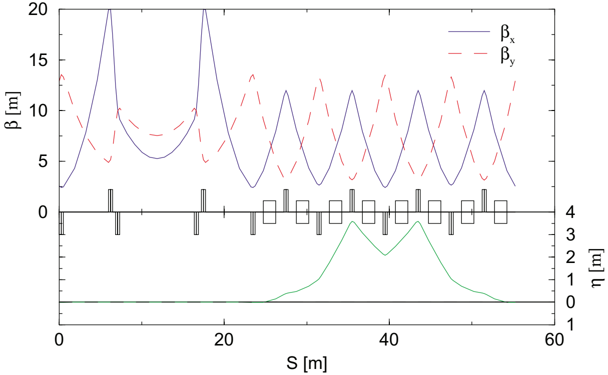
$$\boxed{\nu = \frac{\Phi}{2\pi} = \frac{1}{2\pi}\int_{s}^{s+L}\frac{dt}{\beta(t)} = \frac{1}{2\pi}\oint\frac{ds}{\beta(s)}}$$

β function in a drift space

$$\beta(s) = \beta(0) + \frac{s^2}{\beta(0)}$$



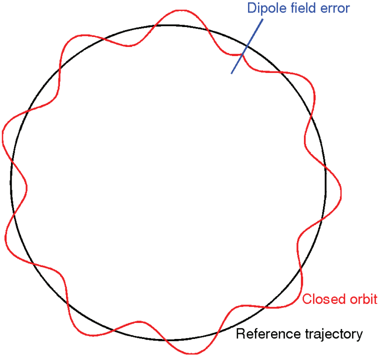
β functions in one period



- The β-functions are necessarily positive, and they are periodic with the lattice period, as evidenced by the fact that their values are equal at the two end-points.

Closed orbit distortion

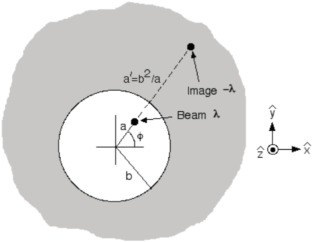
$$u(s) = \theta \frac{\sqrt{\beta_0 \beta(s)}}{2 \sin \pi\nu} \cos(\pi\nu - |\psi(s) - \psi(s_0)|)$$



A dipole field error causes a distortion of the closed orbit. There is a “kink” in the closed orbit at the location of the dipole field error.

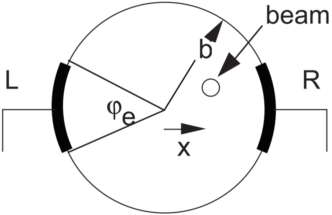
Image Current & Beam Position Monitor

Consider a beam moving inside a perfectly conducting metal pipe in the z-direction. The pipe has a circular transverse cross-section with radius b. Let the beam be represented as an infinitely long moving line charge with linear density λ. The beam is displaced transversely by $\vec{a} = (a\cos\phi, a\sin\phi)$ relative to the axis of the pipe.



We can calculate the surface charge Σ on the conducting pipe wall,

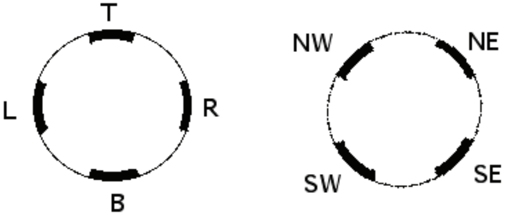
$$\Sigma(\theta) = -\frac{\lambda}{2\pi b}\frac{b^2 - a^2}{a^2 + b^2 - 2ab\cos(\phi - \theta)}$$

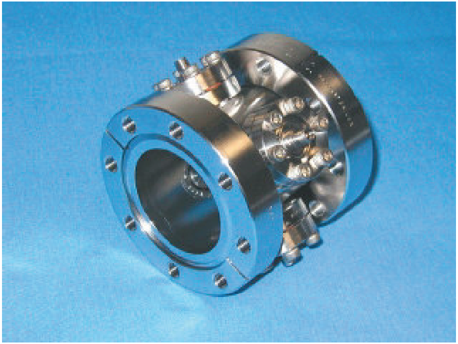


The signal seen by the stripline is obtained by integrating the wall current it carries. One then combines the signals L and R to extract the horizontal beam position,

$$\frac{R - L}{R + L} = \frac{2x}{b}\frac{\sin(\psi_e/2)}{(\psi_e/2)}$$

Image Current & Beam Position Monitor (Cont.)





Dispersion Function

A closed orbit solution of x for the off-momentum particle can be written as

$$x(s) = D(s)\delta$$

where $D(s)$ is called the dispersion function. In other words, we have defined

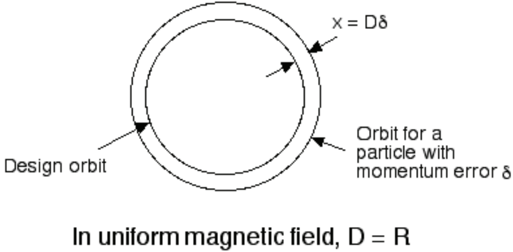
$$\begin{pmatrix}\text{dispersion}\\ \text{function}\end{pmatrix} = \frac{\begin{pmatrix}\text{closed orbit distortion for}\\ \text{a particle with momentum error } \delta\end{pmatrix}}{\delta}$$

The general solution for x of an off-momentum particle is given by

$$x(s) = x_\beta(s) + D(s)\delta$$

Dispersion Function (Cont.)

- Dispersion in a uniform magnetic field



- Dispersion in FODO cells

$$\boxed{\langle D \rangle \approx \frac{R}{\nu_x^2}}$$

Example

A storage ring with $R = 100$ m and $\nu_x \approx 10$, we will have $\beta_x \approx 10$ m and $D \approx 1$m. A particle with momentum error of $\delta = 1\%$ has a dispersive orbit of $D\delta = 1$ cm.
To appreciate the strong suppression effect on dispersion, one should consider a particle moving in a uniform magnetic field. Recalling that the dispersion function $D = R$ in that case, a particle with 1% momentum error will have an orbit as large as 1 m.

Momentum compaction factor

Dispersion function describes the horizontal closed orbit distortion of an off-momentum particle. In the ideal case, the momentum error does not cause any vertical orbit effect. However, it does cause a longitudinal effect because the total circumference of an off-momentum particle will no longer be given by the design circumference C.

A momentum compaction factor α_c is defined by

$$\boxed{\frac{\Delta C}{C} = \alpha_c \delta, \qquad \text{or} \qquad \alpha_c = \frac{1}{\delta}\frac{\Delta C}{C} = \frac{1}{C}\oint \frac{D(S)}{\rho(s)}\, ds}$$

Momentum compaction factor (Cont.)

$$\alpha_c \approx \frac{\langle D \rangle}{R} \approx \frac{1}{\nu_x^2}$$

As discussed earlier, in a uniform magnetic field, we have $D = R$, and then we have a large momentum compaction factor $\alpha_c = 1$. Strong focusing has very much suppressed the value of α_c.

The momentum compaction factor is a fundamental parameter in a lattice design. A large α_c means the path length varies by a large amount with a small momentum error. In such a storage ring, a stored bunched beam will tend to have a long bunch length because a large α_c makes it easy for particles to spread out longitudinally. Therefore, large α_c means long bunch length, i.e. the beam bunch is less compact.

Chromaticity

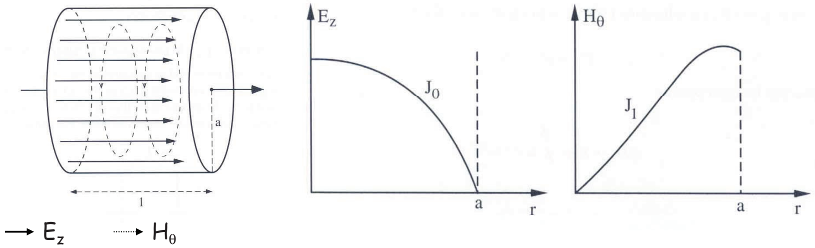
For an off-momentum particle, its momentum deviation δ induces **dipole perturbations** that gives rise to a closed orbit distortion, which we have now discussed in terms of a **dispersion function**. We have been calling such beam dynamical effects caused by momentum deviation *chromatic effects*. Now we discuss another important chromatic effect related to δ-dependent quadrupole perturbations. Basically what happens is the following. Higher momentum particles ($\delta > 0$) have higher rigidity, and therefore experience weaker effect due to magnetic fields. Dispersion comes from the weakened dipoles. The weakened quadrupoles will introduce chromaticities, i.e. the betatron tunes will depend on δ,

$$\nu_{x,y}(\delta) = \nu_{x,y}(0) + \xi_{x,y}\delta$$

where the parameters $\xi_{x,y}$ are the chromaticities.

Pill box cavity

A simplified model of the RF cavity is a pill box cavity with length L and radius R.



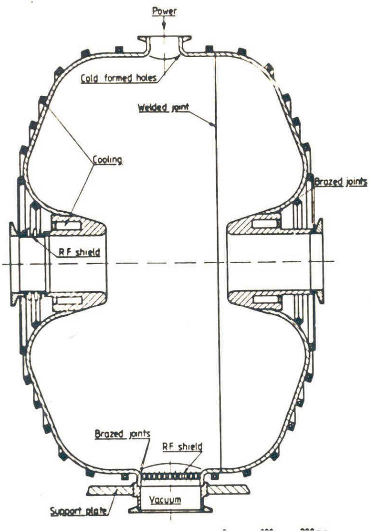
$$E_z(r) = E_0 J_0(\frac{\omega}{c} r) \cos \omega t$$

$$B_\theta(r) = -\frac{E_0}{c} J_1(\frac{\omega}{c} r) \sin \omega t$$

The mode frequency,

$$\omega = 2.405 \frac{c}{R} \qquad [example : R = 30cm, f = 400MHz]$$

A more realistic pill-box cavity



The design of a pill-box cavity can be sophisticated in order to improve its performances:

-A nose cone can be introduced in order to concentrate the electric field around the axis,

-Round shaping of the corners allows a better distribution of the magnetic field on the surface and a reduction of the Joule losses. It also prevent from multipactoring effects.

A good cavity is a cavity which efficiently transforms the RF power into accelerating voltage.

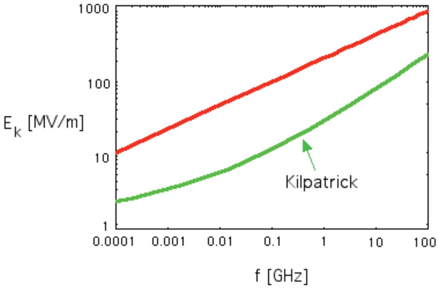
Break Down

The peak field in a cavity in vacuum is limited by breakdown. One often uses the *Kilpatrick limit* (1953) to determine where the breakdown might occur. It is an empirical relation derived from data taken before the era of ultra-highvacuum technology. The maximum field E_k [MV/m] at any frequency f [GHz] according to this criterion is determined by the following equation:

$$f = 0.00164 E_k^2 \exp(-8.5/E_k)$$

The breakdown limit increases as the RF frequency is increased. This is one reason why linear colliders tend to push for technologies of higher frequency RF systems.
Today, with ultra-high-vacuum technology, much higher fields are often achieved. Indeed a more recent fit (although more studies are being carried out in this active research area) gives

$$E_k[\text{MV/m}] = 220(f[\text{GHz}])^{1/3}$$



Principle of phase stability

We assume the longitudinal voltage across an RF cavity is

$$V = V_0 \sin(\omega_{\text{rf}} t + \phi_s)$$

where ϕ_s is the RF phase angle relative to the synchronous particle. The RF frequency ω_{rf} is an integral multiple of the revolution frequency ω_0, i.e.

$$\omega_{\text{rf}} = h\omega_0$$

where h is the harmonic number. Note that we have ignored the r-depedentce of V here because we consider on-axis field with r = 0.
As mentioned, h has to be exactly an integer. Otherwise we will lose the synchronism and lose the ability to accelerate the beam. One might ask how exactly does this condition have to be fulfilled? What if, for example, $\frac{\omega_{\text{rf}}}{\omega_0} = 200.000001$? If this were the case, then after $\frac{1}{2} \times 10^6$ turns, the RF voltage will get out of phase with the beam's arrival time, and we will be decelerating the beam! Since a beam is to be stored much longer than $\frac{1}{2} \times 10^6$ turns, any tiny mismatch of frequencies must not be allowed.

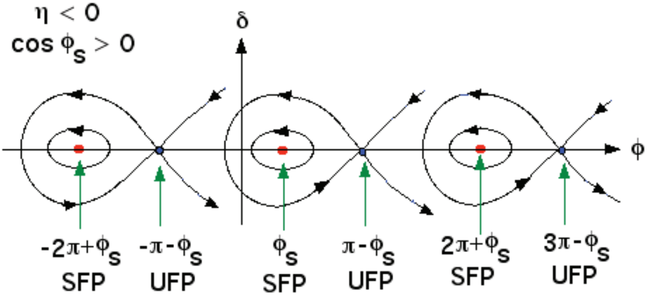
Principle of phase stability (2)

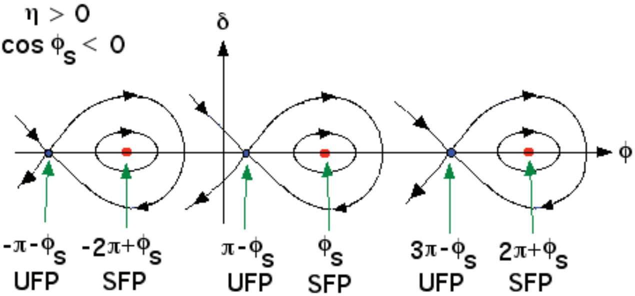
This difficulty was resolved by the important **phase stability principle** of McMillan and Veksler in 1945. What happens is that under some condition of stability, the beam will settle this problem by itself! In particular, the phase stability principle states the following two statements:

1. You first choose your ω_{rf} . Once ω_{rf} is chosen, the beam —at least its synchronous particle —will adjust its revolution frequency ω_0 in such a way that it becomes exactly equal to $\omega_{\text{rf}}/200$ even though its initial ω_0 is slightly off.
2. A particle with slight deviations in z, δ from the synchronous particle will oscillate around the synchronous particle, and these deviations will not grow indefinitely with time.

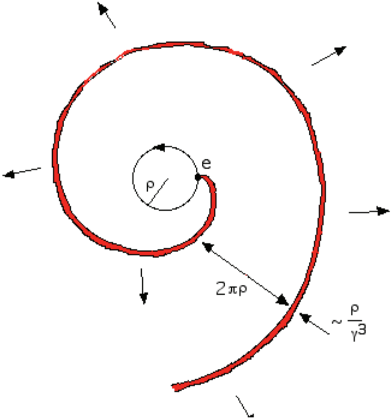
The phase stability is an extremely important principle in accelerator physics. Together with the strong focusing principle, they provide the two foundations for all modern accelerators, phase stability addressing the longitudinal dynamics while strong focusing addressing the transverse dynamics of the particle motion.

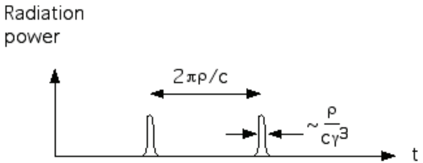
Longitudinal phase space topology & RF bucket





A snapshot of the synchrotron radiation





Incidentally, the arrows in the left figure were not drawn carelessly. Each arrow can be traced back to a tangential point off the electron's circular orbit.

Bending Magnet Radiation Power

The total power of radiation by the point charge, designated as P_γ,is

$$P_\gamma = \frac{dW}{d\psi}\frac{c}{\rho} = \frac{2r_0 mc^3\gamma^4}{3\rho^2}$$

The total energy radiated per revolution, designated as U_0, is

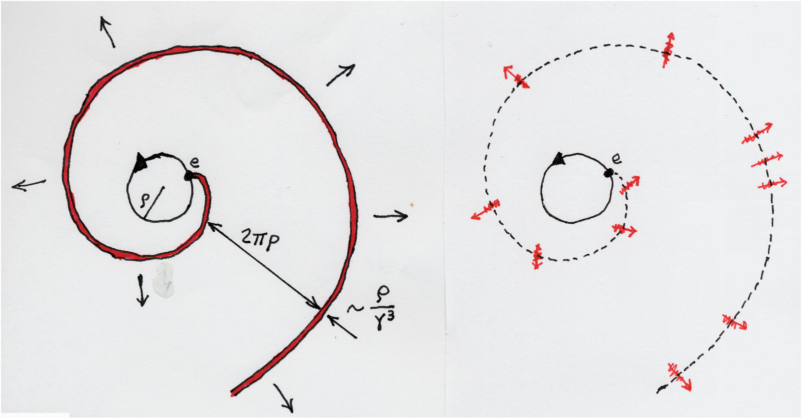
$$U_0 = 2\pi\frac{dW}{d\psi} = \frac{4\pi r_0 mc^2\gamma^4}{3\rho}$$

In practical units, we have

$$U_0 = \begin{cases} 0.0885[\text{MeV}]\dfrac{(E[\text{GeV}])^4}{\rho[\text{m}]} & \text{for electrons} \\ 0.00778[\text{MeV}]\dfrac{(E[\text{TeV}])^4}{\rho[\text{m}]} & \text{for protons} \end{cases}$$

Quantum fluctuations

In the classical picture, synchrotron radiation is described as a continuous emission of electromagnetic waves. In quantum mechanics, however, we understand that the radiation consists really of a large number of discrete photons, each carrying an energy of $u = \hbar\omega$.



A few quantum quantities

The number of photons emitted per revolution

$$\mathcal{N}_0 = \frac{5\pi\alpha\gamma}{\sqrt{3}} \approx \frac{\gamma}{15}$$

Each photon emission takes place over a small piece of arc as the electron (or proton) is being bent. The arc subtends an angle

$$\sim \frac{2}{\gamma}$$

The total bending angle over which the electron is executing an emission is

$$\sim \frac{\gamma}{15} \times \frac{2}{\gamma} = \frac{2}{15} \text{ rad}$$

A particle is radiating a photon over about $\frac{2/15}{2\pi} \approx 2\%$ of its bending trajectory, and is doing nothing over the remaining 98% of the time. Radiation events are rather sparse.

Radiation Damping & Robinson Sum Rule

The radiation damping time $\tau_{x,y,z}$,

$$\frac{1}{\tau_x} + \frac{1}{\tau_y} + \frac{1}{\tau_z} = 2\frac{U_0}{E_s T_0}$$

With radiation damping, then . . .

According to the picture so far, a beam of particles, once injected into a storage ring, will damp to a zero size. This is of course absurd and we must ask what effects will emerge as the beam size gets smaller. A long list follows, e.g.

- space charge effects
- intrabeam scattering
- collective instabilities for high intensity beams
- magnet power supply ripples
- continuous random motion of ground
- quantum excitation (discussed below)

The surprise is that what comes first to limit the shrinking beam size is synchrotron radiation itself!

Equilibrium Beam Parameters

$$\text{equilibrium beam emittance} = \frac{\text{quantum excitation}}{\text{radiation damping}}$$

- energy spread σ_δ

$$\sigma_\delta^2 = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \frac{\gamma^2}{(2+\mathcal{D})\rho}$$

- bunch length σ_z

$$\sigma_z = \frac{c|\eta|}{\omega_s} \sigma_\delta$$

- horizontal emittance

$$\epsilon_x = \frac{\sigma_{x\beta}^2}{\beta_x} \approx \frac{2R}{\nu_x^3} \sigma_\delta^2$$

- horizontal beam size

$$\sigma_{x\beta} \approx \sqrt{2} \frac{R}{\nu_x^2} \sigma_\delta$$

Example

With $R = 30$ m and $E_s = 5$ GeV, we had $\sigma_\delta = 0.8 \times 10^{-3}$. If $\nu_x \approx 5$, then $\sigma_{x\beta} \approx 1.3$ mm

If you don't plan to become a career accelerator physicist, and someone asks you what you know about accelerator physics, you should quote

$$\text{rms beam size} = \sqrt{\beta(s)\epsilon_{\mathsf{rms}}}$$

References

- A. Chao, Accelerator Physics, USPAS 2007
- ......