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Fragmentation Function Study in e^+e^- Annihilation

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In collaboration with Z.T. Liang, Y.K. Song, K.B. Chen & W.H. Yang

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- Collinear Expansion
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Introduction

截面 = 硬部分 $\otimes$ 部分子分布函数/碎裂函数 + $\cdots$

PDFs: 强子中部分子的数密度分布 DIS, Drell-Yan

FFs: 部分子强子化，喷注中强子的数密度分布 SIDIS, e^+e^-

非微扰物理量：
探索 QCD 性质，
包含强子化机制非常丰富的信息，
鉴定强子化模型

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自旋传递因子： $+\lambda_q\lambda_h\Delta D_{1L}(z)$ 纵向极化夸克碎裂产生纵向极化强子的数密度

光锥动量分数 ⟶ 光锥动量分数 + 横动量

横动量分布各向同性

$$D_1(z) + \lambda_q \lambda_h \Delta D_{1L}(z) \xrightarrow{1\text{D} \to 3\text{D}} D_1(z, p_T^2) + \lambda_q \lambda_h \Delta D_{1L}(z, p_T^2)$$

光锥动量分数 → 光锥动量分数 + 横动量

横动量分布各向同性

$$D_1(z) + \lambda_q \lambda_h \Delta D_{1L}(z) \xrightarrow{1D \to 3D} D_1(z, p_T^2) + \lambda_q \lambda_h \Delta D_{1L}(z, p_T^2)$$

强子的横向极化 $S_\perp$

$$+\frac{1}{M}\lambda_q p_T \cdot S_\perp \Delta D_{1T}^\perp(z, p_T^2) + \frac{1}{M}\epsilon_\perp^{p_T S_\perp} D_{1T}^\perp(z, p_T^2)$$

p_T 一次方依赖项

Introduction

光锥动量分数 ⟶ 光锥动量分数 + 横动量

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强子的横向极化 $S_\perp$

$$+\frac{1}{M}\lambda_q p_T \cdot S_\perp \Delta D_{1T}^\perp(z, p_T^2) + \frac{1}{M}\epsilon_\perp^{p_T S_\perp} D_{1T}^\perp(z, p_T^2)$$

p_T 一次方依赖项

夸克的横向极化 s_T

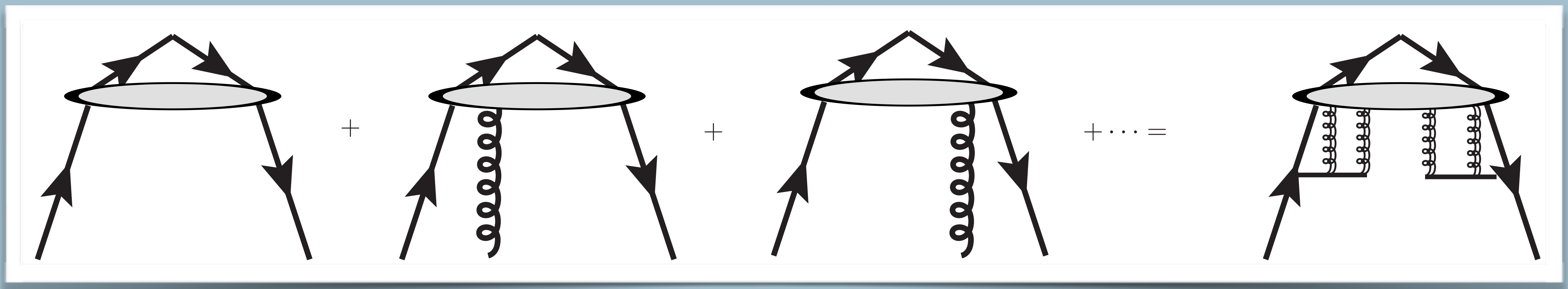
$$+\frac{1}{M}\lambda_h p_T \cdot s_T \Delta H_{1L}^\perp(z, p_T^2) + \frac{1}{M}\epsilon_\perp^{p_T s_T} H_1^\perp(z, p_T^2)$$

$$+ S_\perp \cdot s_T H_{1T}(z, p_T^2) + \frac{1}{M^2} p_T \cdot S_\perp p_T \cdot s_T H_{1T}^\perp(z, p_T^2)$$

横向自旋传递因子

夸克胶子关联矩阵

$$q \to h+X$$



$$\hat{\Xi}^{1\mathrm{D}} = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

$$\hat{\Xi}^{3\mathrm{D}} = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^-} e^{-ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

Introduction

$$\hat{\Xi}^{3D} = \Xi^{(0)}\mathbf{1} + \tilde{\Xi}^{(0)}\gamma_5 + \Xi^{(0)}_{\alpha\beta}\sigma^{\alpha\beta}\gamma_5 \quad \Leftarrow \quad \text{Chiral} - \text{Odd}$$

$$+\Xi^{(0)}_{\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(0)}_{\alpha}\gamma^{\alpha}\gamma_5 \quad \Leftarrow \quad \text{Chiral} - \text{Even}$$

Leading twist / Spin-1/2

$$z\Xi^{(0)}_{\alpha}(z,k_\perp) = p_\alpha[D_1(z,k_\perp) + \frac{1}{M}\epsilon_\perp^{k_\perp S_\perp} D_{1T}^\perp(z,k_\perp)]$$

$$z\tilde{\Xi}^{(0)}_{\alpha}(z,k_\perp) = p_\alpha[\lambda_h \Delta D_{1L}(z,k_\perp) + \frac{1}{M}k_\perp \cdot S_\perp \Delta D_{1T}^\perp(z,k_\perp)]$$

$$z\Xi^{(0)}_{\alpha\beta}(z,k_\perp) = p_{[\alpha}\Big\{\frac{1}{M}\epsilon_{\perp\beta]k_\perp} H_1^\perp(z,k_\perp) + S_{\perp\beta]} H_{1T}(z,k_\perp) + \frac{1}{M}k_{\perp\beta]}\lambda_h H_{1L}^\perp(z,k_\perp) + \frac{1}{M^2}k_{\perp\beta]}k_\perp \cdot S_\perp H_{1T}^\perp(z,k_\perp)\Big\}$$

K.B. Chen, WSY, W.H. Yang, Z.T. Liang, arXiv:1505.02856

$$\hat{\Xi}^{3D} = \Xi^{(0)}\mathbf{1} + \tilde{\Xi}^{(0)}\gamma_5 + \Xi^{(0)}_{\alpha\beta}\sigma^{\alpha\beta}\gamma_5 \qquad \Leftarrow \qquad \mathrm{Chiral-Odd}$$

$$+\Xi^{(0)}_{\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(0)}_{\alpha}\gamma^{\alpha}\gamma_5 \qquad \Leftarrow \qquad \mathrm{Chiral-Even}$$

Leading twist / Spin-1/2

Sivers-type FF

$$z\Xi^{(0)}_{\alpha}(z,k_\perp) = p_\alpha\Big[D_1(z,k_\perp) + \frac{1}{M}\epsilon_\perp^{k_\perp S_\perp} D_{1T}^\perp(z,k_\perp)\Big]$$

$$z\tilde{\Xi}^{(0)}_{\alpha}(z,k_\perp) = p_\alpha\Big[\lambda_h \Delta D_{1L}(z,k_\perp) + \frac{1}{M} k_\perp \cdot S_\perp \Delta D_{1T}^\perp(z,k_\perp)\Big]$$

$$z\Xi^{(0)}_{\alpha\beta}(z,k_\perp) = p_{[\alpha}\Big\{\frac{1}{M}\epsilon_{\perp\beta]k_\perp} H_1^\perp(z,k_\perp) + S_{\perp\beta]} H_{1T}(z,k_\perp) + \frac{1}{M} k_{\perp\beta]}\lambda_h H_{1L}^\perp(z,k_\perp) + \frac{1}{M^2} k_{\perp\beta]} k_\perp \cdot S_\perp H_{1T}^\perp(z,k_\perp)\Big\}$$

- 单自旋不对称
- 方位角不对称

Collins Function

K.B. Chen, WSY, W.H. Yang, Z.T. Liang, arXiv:1505.02856

Spin Density Matrices

Spin $\frac{1}{2}$ $$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + S^i \sigma^i)$$

Spin 1 $$\rho = \begin{pmatrix} \rho_{++} & \rho_{+0} & \rho_{+-} \\ \rho_{0+} & \rho_{00} & \rho_{0-} \\ \rho_{-+} & \rho_{-0} & \rho_{--} \end{pmatrix} = \frac{1}{3} + \frac{1}{2} S^i \sigma^i + T^{ij} \Sigma^{ij}$$

Covariant Decomposition

$$S^\rho = \lambda_{\rm h} \left(\frac{p^+}{M} \bar{n}^\rho - \frac{M}{2p^+} n^\rho \right) + S_\perp^\rho$$

$$T^{\mu\nu} = \frac{1}{2} \Big\{ \frac{4}{3} S_{LL} \Big[(\frac{p^+}{M})^2 \bar{n}^\mu \bar{n}^\nu + (\frac{M}{2p^+})^2 n^\mu n^\nu - \frac{1}{2} (\bar{n}^{\{\mu} n^{\nu\}} - g_\perp^{\mu\nu}) \Big]$$
$$+ [\frac{p^+}{M} \bar{n}^{\{\mu} - \frac{M}{2p^+} n^{\{\mu}] S_{LT}^{\nu\}} + S_{TT}^{\mu\nu} \Big\}$$

Vector Meson (18 leading twist FFs)

quark polarization	hadron polarization	TMD FFs	integrated over $\vec{k}_{F\perp}$	name
U	U	$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T	$D_{1T}^{\perp}(z,k_{F\perp})$	$\times$	
	LL	$D_{1LL}(z,k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT	$D_{1LT}^{\perp}(z,k_{F\perp})$	$\times$	
	TT	$D_{1TT}^{\perp}(z,k_{F\perp})$	$\times$	
L	L	$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T	$G_{1T}^{\perp}(z,k_{F\perp})$	$\times$	
	LT	$G_{1LT}^{\perp}(z,k_{F\perp})$	$\times$	
	TT	$G_{1TT}^{\perp}(z,k_{F\perp})$	$\times$	
T	U	$H_1^{\perp}(z,k_{F\perp})$	$\times$	Collins function
	$T(\parallel)$	$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$	$H_{1T}^{\perp}(z,k_{F\perp})$		
	L	$H_{1L}^{\perp}(z,k_{F\perp})$	$\times$	
	LL	$H_{1LL}^{\perp}(z,k_{F\perp})$	$\times$	
	LT	$H_{1LT}(z,k_{F\perp})$, $H_{1LT}^{\perp}(z,k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z,k_{F\perp})$, $H_{1TT}^{\prime\perp}(z,k_{F\perp})$	$\times$, $\times$	

K.B. Chen, WSY, W.H. Yang, Z.T. Liang, arXiv:1505.02856

Collinear Expansion

- DIS

  R.K. Ellis, W. Furmanski and R. Petronzio, NPB 1982,1983
  J.W. Qiu and G. Sterman NPB, 1991

- SIDIS

  Z.T. Liang and X.N. Wang, PRD, 2007
  Y.K. Song, J.H. Gao, Z.T. Liang, X.N. Wang, PRD, 2011

- Inclusive e^+e^- Annihilation

  S.Y. Wei, Y.K. Song and Z.T. Liang PRD, 2014

- Semi-inclusive e^+e^- Annihilation

  S.Y. Wei, K.B. Chen, Y.K. Song and Z.T. Liang PRD, 2015

$$W_{\mu\nu} = W^{(0)}_{\mu\nu} + W^{(1)}_{\mu\nu} + W^{(2)}_{\mu\nu} + \cdots \quad \Rightarrow \quad W_{\mu\nu} = \tilde{W}^{(0)}_{\mu\nu} + \tilde{W}^{(1)}_{\mu\nu} + \tilde{W}^{(2)}_{\mu\nu} + \cdots$$

Inclusive Process $e^+ + e^- \to h + X$

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\ \hat{\Xi}^{(0)}(z_B, p, S)\right] \qquad \Rightarrow \quad \text{twist 2, twist 3,} \ \cdots$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\ \omega_\rho^{\ \rho'}\hat{\Xi}^{(1)}_{\rho'}(z_B, p, S)\right] \qquad \Rightarrow \quad \text{twist 3,} \ \cdots$$

Semi-inclusive Process $e^+ + e^- \to h + \text{jet} + X$

$$\tilde{W}^{(0,\mathrm{si})}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}(z_B, k'_\perp, p, S)\right] \qquad \Rightarrow \quad \text{twist 2, twist 3,} \ \cdots$$

$$\tilde{W}^{(1,L,\mathrm{si})}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\hat{\Xi}^{(1,L)}_{\rho'}(z_B, k'_\perp, p, S)\right] \qquad \Rightarrow \quad \text{twist 3,} \ \cdots$$

Collinear Expansion

1D Correlation Matrices

$$\hat{\Xi}^{(0)}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

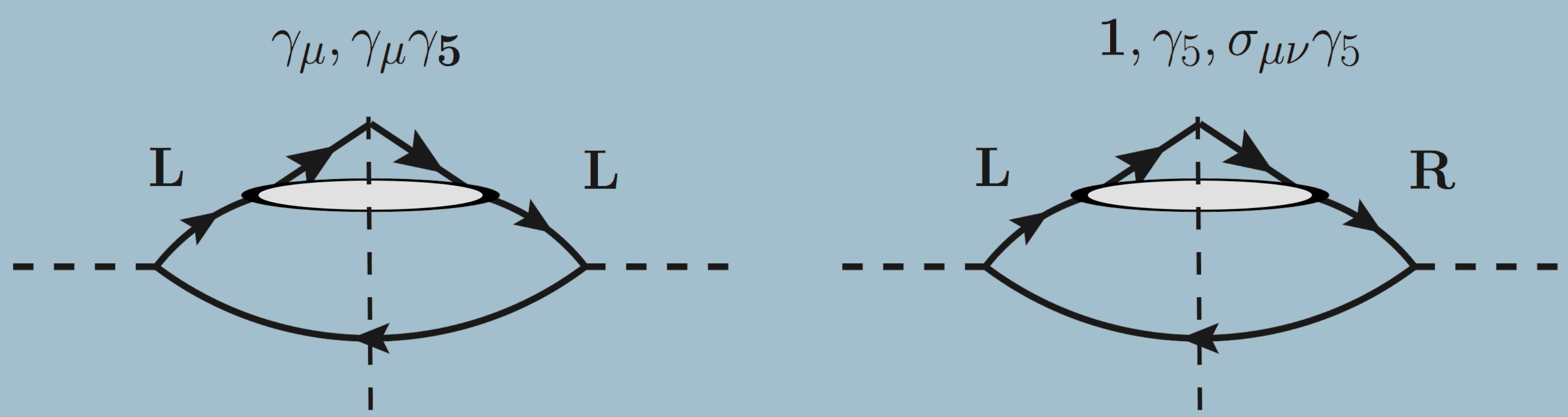
3D Correlation Matrices

$$\hat{\Xi}^{(0)}(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^-+ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^-+ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$\hat{\Xi}^{(0)}(z,k_\perp,p,S) = \Xi^{(0)}\mathbf{1} + \tilde{\Xi}^{(0)}\gamma_5 + \Xi^{(0)}_{\alpha\beta}\sigma^{\alpha\beta}\gamma_5 \qquad \Leftarrow \qquad \text{Chiral} - \text{Odd}$$

$$+\Xi^{(0)}_{\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(0)}_{\alpha}\gamma^{\alpha}\gamma_5 \qquad \Leftarrow \qquad \text{Chiral} - \text{Even}$$



Collinear Expansion

$$\hat{\Xi}^{(0)}(z, k_\perp, p, S) = \Xi^{(0)}\mathbf{1} + \tilde{\Xi}^{(0)}\gamma_5 + \Xi^{(0)}_{\alpha\beta}\sigma^{\alpha\beta}\gamma_5 \qquad \Leftarrow \qquad \text{Chiral} - \text{Odd}$$

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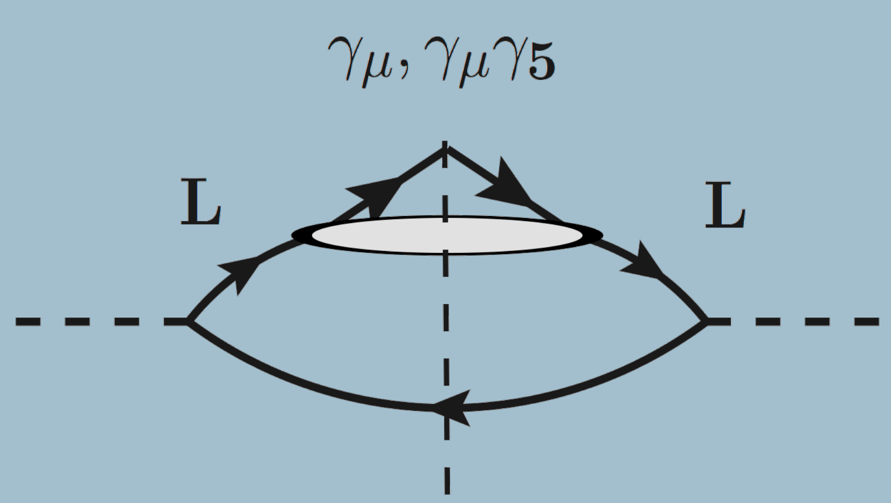
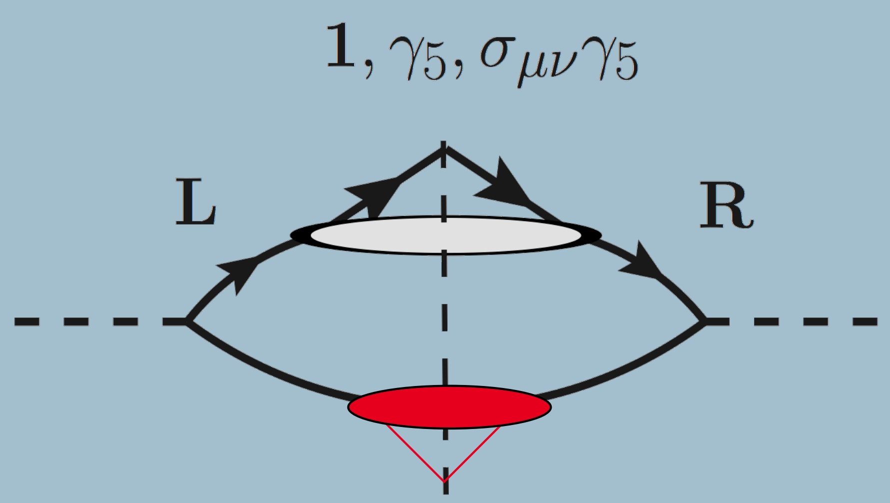




Inclusive Process

WSY, Y.K. Song, Z.T. Liang, PRD89,(2014)014024

Inclusive Process & 1D Correlation Matrices

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B,p,S) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B,p,S)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho{}^{\rho'}\Xi^{(1)\alpha}_{\rho'}(z_B,p,S) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho{}^{\rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B,p,S)$$

$$\Xi^{(0)\alpha}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\tilde{\Xi}^{(0)\alpha}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma_5\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\tilde{\Xi}^{(1)\alpha}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma_5\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

Up to Twist-3

Spin independent	$z\Xi^{(0)\alpha} = p^\alpha D_1(z)$
$S_\perp$ **dependent**	$z\Xi^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{\perp\gamma} D_T(z)$ $z\tilde{\Xi}^{(0)\alpha} = \lambda_{\rm h} p^\alpha \Delta D_{1L}(z) + M S_\perp^\alpha \Delta D_T(z)$ $z\Xi^{(1)\rho\alpha} = M\epsilon_\perp^{\rho\gamma} S_{\perp\gamma} p^\alpha \xi_{\perp S}^{(1)}(z)$ $z\tilde{\Xi}^{(1)\rho\alpha} = iMS_\perp^\rho p^\alpha \tilde{\xi}_{\perp S}^{(1)}(z)$
T **dependent**	$z\Xi^{(0)\alpha} = S_{LL} p^\alpha D_{1LL}(z) + M S_{LT}^\alpha D_{LT}(z)$ $z\tilde{\Xi}^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{LT,\gamma} \Delta D_{LT}(z)$ $z\Xi^{(1)\rho\alpha} = M S_{LT}^\rho \xi_{LTS}^{(1)}(z) p^\alpha$ $z\tilde{\Xi}^{(1)\rho\alpha} = iM\epsilon_\perp^{\rho\gamma} S_{LT,\gamma} \tilde{\xi}_{LTS}^{(1)}(z) p^\alpha$

Inclusive Process $e^+ + e^- \to h + X$

Leading Twist FFs

$D_1(z)$	Unpolarized quark	$\longrightarrow$	Unpolarized hadron
$\Delta D_{1L}(z)$	Polarized quark	$\longrightarrow$	Polarized hadron
$D_{1LL}(z)$	Unpolarized quark	$\longrightarrow$	Spin alignment

Twist-3 FFs

$S_\perp$ dependent	T dependent
$D_T(z),\ \Delta D_T(z)$	$D_{LT}(z),\ \Delta D_{LT}(z)$

Inclusive Process $e^{+}+e^{-} \rightarrow h+X$

Light-cone gauge: $A^{+}(\xi)=0$ ⟶ Gauge Link = 1

PDFs

$$\Phi_{\alpha}^{(0)}(x,p,S) = \mathcal{F}(\xi^{-})\langle p|\bar{\psi}(0)\gamma_{\alpha}\psi(\xi^{-})|p\rangle = f_1 p_{\alpha} + f_T M\epsilon_{\perp\alpha S_{\perp}}$$

Time-reversal invariance: $f_1 = f_1, \quad f_T = -f_T$

FFs

$$\Xi_{\alpha}^{(0)}(x,p,S) = \mathcal{F}(\xi^{-})\mathrm{Tr}\langle 0|\gamma_{\alpha}\psi(0)|hX\rangle\langle hX|\psi(\xi^{-})|0\rangle = D_1 p_{\alpha} + D_T M\epsilon_{\perp\alpha S_{\perp}}$$

Time-reversal invariance: $D_1^{\text{in state}} = D_1^{\text{out state}}, \; D_T^{\text{in state}} = -D_T^{\text{out state}}$

Inclusive Process $e^+ + e^- \to h + X$

Spin-0 hadrons, $e^+ + e^- \to Z^0 \to h + X$

$$\frac{d^2\sigma}{dzdy} = \sum_q \frac{2\pi\alpha^2}{Q^2} \chi T_0^q(y) D_1^{q\to \mathrm{h}}(z)$$

$$D_1^{q\to \mathrm{h}}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^-/z} \mathrm{Tr}\left[\gamma^+ \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle\right]$$

For $e^+ + e^- \to \gamma^* \to h + X$

$$\frac{d\sigma^{\mathrm{em}}}{dz} = \sum_q \frac{4\pi\alpha^2}{3Q^2} e_q^2 D_1^{q\to \mathrm{h}}(z)$$

Inclusive Process $e^+ + e^- \to h + X$

非极化（自旋求和）碎裂函数的参数化

- 1994 年，BKK 组，$\pi^\pm$，$K^\pm$ [73]；
- 1997 年，DSV 组，$\Lambda/\bar{\Lambda}$ [67]；
- 1997 年，BFG/BFGW 组，光子 [74,75]（2000 年更新）；
- 2000 年，KKP/KKKS 组，$\pi^\pm$，$K^\pm$，$p/\bar{p}$ [76]，D^0，D^+，D^{*+} [77]（2007 年更新）；
- 2000 年，Kretzer，$\pi^\pm$，$K^\pm$，$\sum_h h^\pm$ [78]；
- 2005 年，AKK 组，$\pi^\pm$，$K^\pm$，$p/\bar{p}$ [79]，K_S^0，$\Lambda/\bar{\Lambda}$ [80,81]（2008 年更新）；
- 2007 年，DSS，$\pi^\pm$，$K^\pm$，p [82,83]；
- 2007 年，HKNS 组，$\pi^\pm$，$K^\pm$，p [84]；
- 2010 年，AESSS 组，η 介子 [85]。

Inclusive Process $e^{+}+e^{-}\rightarrow h+X$

Spin-1/2 hadrons, $e^{+}+e^{-}\rightarrow Z^{0}\rightarrow h+X$

$$\frac{d^2\sigma}{dzdy}=\chi\frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z)+\lambda_{\rm h}T_1(y)\Delta D_{1L}(z)\big] + \frac{4M}{zQ^2}\big[T_2(y)D_T(z)\epsilon_{\perp}^{l_\perp S_\perp}+T_3(y)\Delta D_T(z)l_\perp\cdot S_\perp\big]\Big\}$$

$\Delta D_{1L}(z)$ 纵向极化

$D_T(z)$ 垂直于轻子面横向极化

$\Delta D_T(z)$ 平行于轻子面横向极化

Spin-1/2 hadrons, $e^{+} + e^{-} \rightarrow Z^{0} \rightarrow h + X$

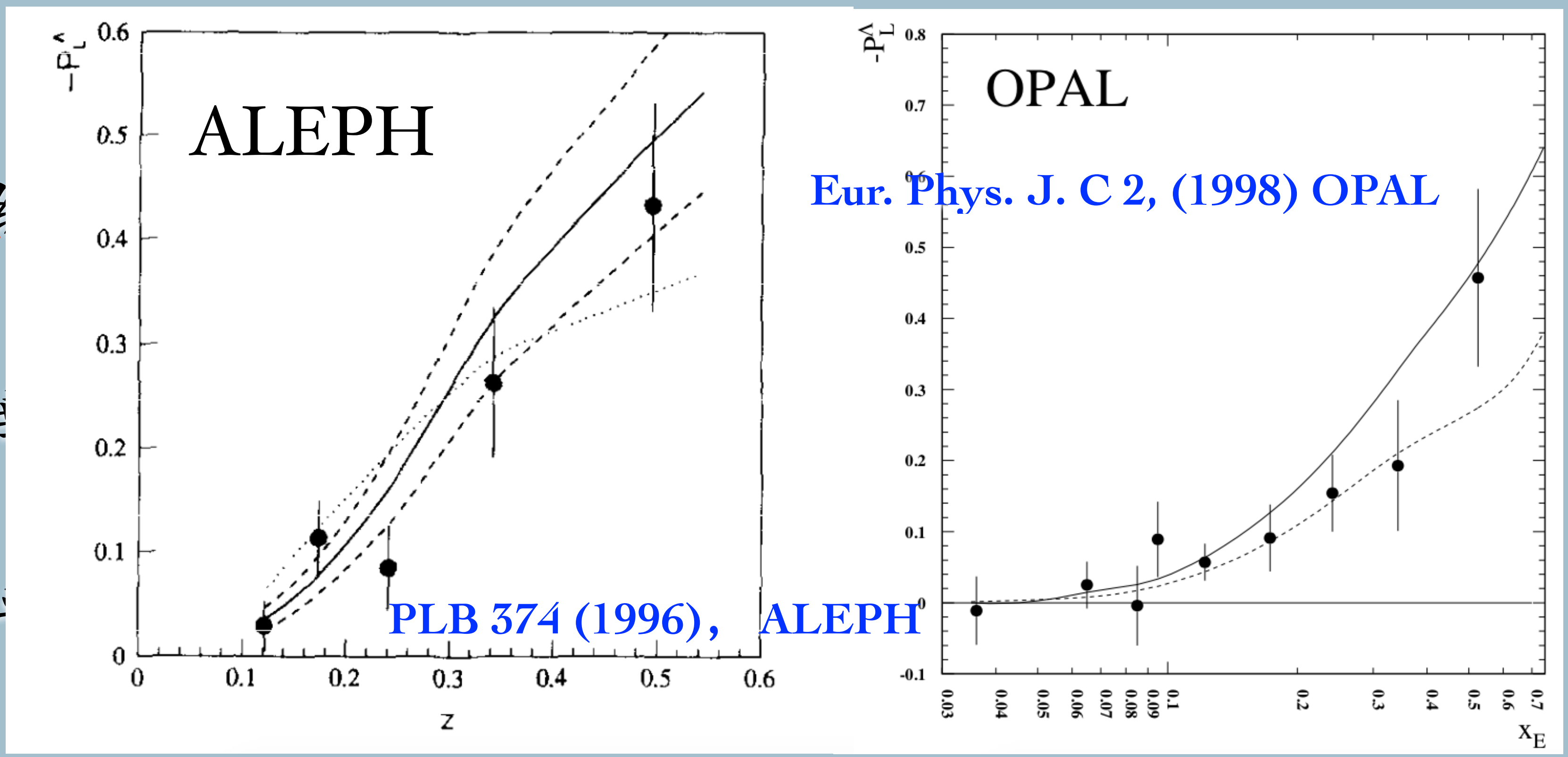
$$\frac{d^2\sigma}{dzdy} = \chi\frac{2\pi\alpha^2}{Q^2}\Big\{\left[T_0(y)D_1(z) + \lambda_{\rm h}T_1(y)\Delta D_{1L}(z)\right] + \frac{4M}{zQ^2}\left[T_2(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp} + T_3(y)\Delta D_T(z) l_\perp\cdot S_\perp\right]\Big\}$$

DSV parameterization for unpolarized & longitudinally polarized Λ (PRD57, 5811 (1998))

$\Delta D_{1L}(z)$

$D_T(z)$

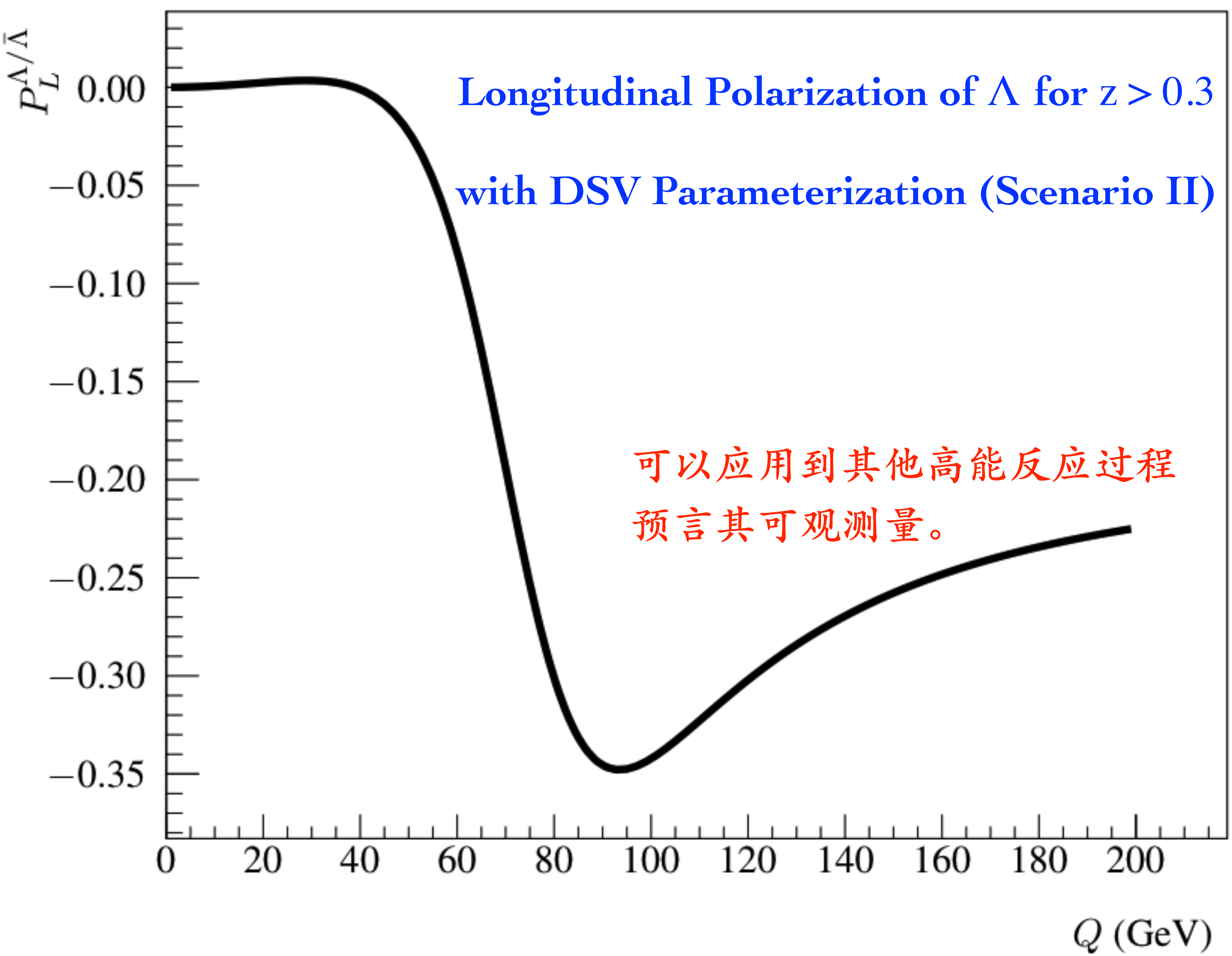
$\Delta D_T(z)$



PLB 374 (1996), ALEPH

Eur. Phys. J. C 2, (1998) OPAL

Inclusive Process $e^{+}+e^{-} \to h+X$



Spin-1/2 hadrons, $e^{+} + e^{-} \to Z^{0} \to h + X$

$$\frac{d^2\sigma}{dzdy} = \chi\frac{2\pi\alpha^2}{Q^2}\Big\{\left[T_0(y)D_1(z) + \lambda_{\rm h}T_1(y)\Delta D_{1L}(z)\right] + \frac{4M}{zQ^2}\left[T_2(y)D_T(z)\epsilon_{\perp}^{l_\perp S_\perp} + T_3(y)\Delta D_T(z)l_\perp\cdot S_\perp\right]\Big\}$$

$\Delta D_{1L}(z)$ 纵向极化

$D_T(z)$ 垂直于轻子面横向极化

$\Delta D_T(z)$ 平行于轻子面横向极化

Power Suppressed by M/Q

Spin-1/2 hadrons, $e^+ + e^- \rightarrow \gamma^* \rightarrow h + X$

$$\frac{d\sigma^{\rm em}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\left\{A(y)D_1(z) + \frac{4M}{zQ^2}B(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp}\right\}$$

$$P_{L\rm h}(z,y) = 0 \qquad\qquad P_{\rm hx}^{\rm em}(z,y) = 0$$

$$P_{\rm hy}^{\rm em}(z,y) = \frac{4M}{zQ}\frac{\sqrt{y(1-y)}(1-2y)}{(1-y)^2+y^2}\frac{\sum_q e_q^2 D_T^{q\to \rm h}(z)}{\sum_q e_q^2 D_1^{q\to \rm h}(z)}$$

naive-T-odd FF

$$MS_\perp^2 D_T(z) = \frac{z}{4}\sum_X\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\epsilon_\perp^{\alpha\gamma}S_{\perp\gamma}{\rm Tr}[\gamma_\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|{\rm hX}\rangle\langle{\rm hX}|\bar\psi(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

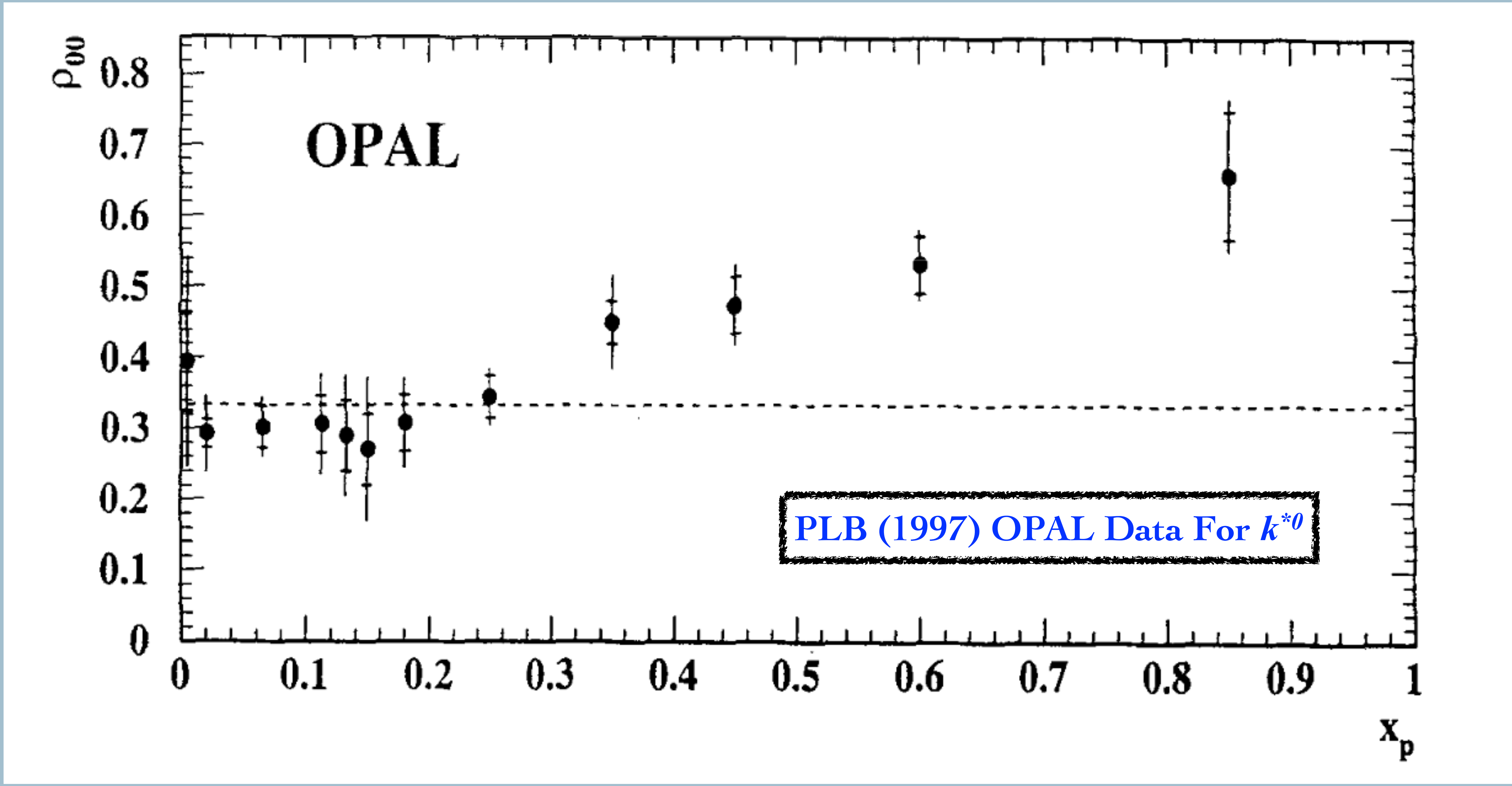
Spin-1 hadrons, $e^{+} + e^{-} \to Z^{0} \to h + X$

$$\frac{d\sigma}{dzdy} = \chi\frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + T_0(y)S_{LL}D_{1LL}(z) + \lambda_{\rm h}T_1(y)\Delta D_{1L}(z)\big]$$
$$+\frac{4M}{zQ^2}\big[T_2(y)\epsilon_\perp^{l_\perp S_\perp}D_T(z) + T_3(y)l_\perp\cdot S_\perp\Delta D_T(z)\big]$$
$$+\frac{4M}{zQ^2}\big[T_2(y)l_\perp\cdot S_{LT}D_{LT}(z) + T_3(y)\epsilon_\perp^{l_\perp S_{LT}}\Delta D_{LT}(z)\big]\Big\}$$

Spin alignment

$$\rho_{00} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q t_0^q D_{1LL}^{q\to {\rm h}}(z)}{\sum_q t_0^q D_1^{q\to {\rm h}}(z)}$$

Inclusive Process $e^{+}+e^{-} \rightarrow h+X$



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Inclusive Process $e^{+}+e^{-} \to h+X$

Spin-1 hadrons, $e^+ + e^- \to \gamma^* \to h + X$

$$\frac{d\sigma^{\rm em}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\Big\{A(y)\big[D_1(z) + S_{LL}D_{1LL}(z)\big] + \frac{M}{zQ}B(y)\big[\epsilon_\perp^{l_\perp S_\perp} D_T(z) + l_\perp \cdot S_{LT}D_{LT}(z)\big]\Big\}$$

Spin alignment

$$\rho_{00}^{\rm em} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q e_q^2 D_{1LL}^{q\to \rm h}(z)}{\sum_q e_q^2 D_1^{q\to \rm h}(z)}$$

在 BES 等正负电子湮灭实验上均可测量到该张量极化

Semi-inclusive Process

WSY, K.B. Chen,Y.K. Song, Z.T. Liang, PRD91,(2015)034015

Semi-inclusive Process & 3D Correlation Matrices

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B, p, S, k'_\perp) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B, p, S, k'_\perp)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho{}^{\rho'}\Xi^{(1)\alpha}_{\rho'}(z_B, p, S, k'_\perp) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho{}^{\rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B, p, S, k'_\perp)$$

$$\Xi^{(0)\alpha}(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\Xi^{(0)\alpha}(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}\mathrm{Tr}[\gamma^\alpha\langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

New Structures up to Twist-3

Spin independent

$$z\Xi^{(0)}_{\alpha}(z,k_\perp,p) = k_{\perp\alpha}\hat{D}^\perp(z,k_\perp)$$
$$z\tilde{\Xi}^{(0)}_{\alpha}(z,k_\perp,p) = \epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^\perp(z,k_\perp)$$
$$z\Xi^{(1)}_{\rho\alpha}(z,k_\perp,p) = p_\alpha k_{\perp\rho}\,\xi^{(1)}_\perp(z,k_\perp)$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha}(z,k_\perp,p) = ip_\alpha\epsilon_{\perp\rho k_\perp}\tilde{\xi}^{(1)}_\perp(z,k_\perp)$$

$S_\perp$ dependent

$$z\Xi^{(0)}_{\alpha} = p_\alpha\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^\perp_{1T} + k_{\perp\alpha}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^\perp_T + \lambda_h\epsilon_{\perp\alpha k_\perp}\hat{D}^\perp_L$$
$$z\tilde{\Xi}^{(0)}_{\alpha} = p_\alpha\frac{k_\perp\cdot S_\perp}{M}\Delta\hat{D}^\perp_{1T} + \frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^\perp_T + \lambda_h k_{\perp\alpha}\Delta\hat{D}^\perp_L$$
$$z\Xi^{(1)}_{\rho\alpha} = p_\alpha\left[k_{\perp\rho}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\xi^{(1)\perp}_T + \lambda_h\epsilon_{\perp\rho k_\perp}\xi^{(1)\perp}_L\right]$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = ip_\alpha\left[\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\rho k_\perp}\tilde{\xi}^{(1)\perp}_T + \lambda_h k_{\perp\rho}\tilde{\xi}^{(1)\perp}_L\right]$$

***T* dependent**

$$z\Xi^{(0)}_{\alpha} = p_\alpha \frac{S_{LT}\cdot k_\perp}{M}\hat{D}^{\perp}_{1LT} + p_\alpha \frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\hat{D}^{\perp}_{1TT}$$
$$+ k_{\perp\alpha}S_{LL}\hat{D}^{\perp}_{LL} + k_{\perp\alpha}\frac{k_\perp\cdot S_{LT}}{M}\hat{D}^{\perp}_{LT} + S_{TT\alpha\beta}k^{\beta}_{\perp}\hat{D}^{\perp A}_{TT} + k_{\perp\alpha}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\hat{D}^{\perp C}_{TT}$$

$$z\tilde{\Xi}^{(0)}_{\alpha} = p_\alpha \frac{\epsilon^{k_\perp S_{LT}}_{\perp}}{M}\Delta\hat{D}^{\perp}_{1LT} + p_\alpha \frac{\epsilon_{\perp k_\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\Delta\hat{D}^{\perp}_{1TT}$$
$$+\epsilon_{\perp\alpha k_\perp}\left[S_{LL}\Delta\hat{D}^{\perp}_{LL} + \frac{k_\perp\cdot S_{LT}}{M}\Delta\hat{D}^{\perp}_{LT} + \frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\Delta\hat{D}^{\perp C}_{TT}\right] + \epsilon_{\perp\alpha\beta}S^{\beta\gamma}_{TT}k_{\perp\gamma}\Delta\hat{D}^{\perp A}_{TT}$$

$$z\Xi^{(1)}_{\rho\alpha} = p_\alpha\left[k_{\perp\rho}S_{LL}\xi^{\perp}_{LL} + k_{\perp\rho}\frac{k_\perp\cdot S_{LT}}{M}\xi^{\perp}_{LT} + S_{TT\rho\beta}k^{\beta}_{\perp}\xi^{\perp A}_{TT} + k_{\perp\rho}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\xi^{\perp C}_{TT}\right]$$

$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = ip^{\alpha}\left[\epsilon_{\perp\rho k_\perp}S_{LL}\tilde{\xi}^{\perp}_{LL} + \epsilon_{\perp\rho k_\perp}k_\perp\cdot S_{LT}\ \tilde{\xi}^{\perp}_{LT} + \epsilon_{\perp\rho\beta}S^{\beta\gamma}_{TT}k_{\perp\gamma}\tilde{\xi}^{\perp A}_{TT} + \epsilon_{\perp\rho k_\perp}\frac{k_{\perp\gamma}k_{\perp\delta}S^{\gamma\delta}_{TT}}{M^2}\tilde{\xi}^{\perp C}_{TT}\right]$$

New Leading Twist FFs

$\hat{D}_{1T}^{\perp}(z, k_\perp)$	Unpolarized quark $\to$ $S_\perp$ hadron
$\Delta\hat{D}_{1T}^{\perp}(z, k_\perp)$	L polarized quark $\to$ $S_\perp$ hadron
$\hat{D}_{1LT}^{\perp}(z, k_\perp)$	Unpolarized quark $\to$ LT hadron
$\Delta\hat{D}_{1LT}^{\perp}(z, k_\perp)$	L polarized quark $\to$ LT hadron
$\hat{D}_{1TT}^{\perp}(z, k_\perp)$	Unpolarized quark $\to$ TT hadron
$\Delta\hat{D}_{1TT}^{\perp}(z, k_\perp)$	L polarized quark $\to$ TT hadron

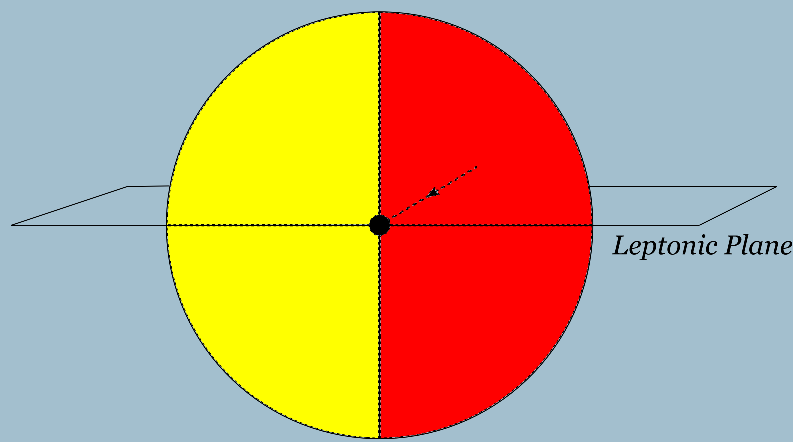
New Twist-3 FFs

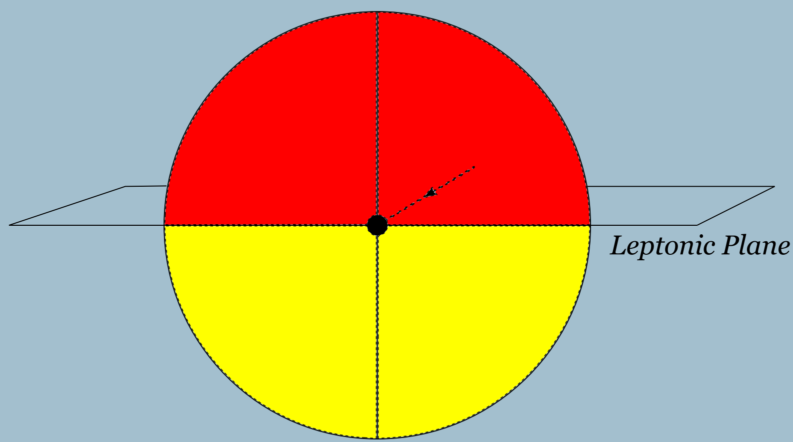
unpolarized: 2 $S_\perp$ dependent: 4 T dependent: 8

Semi-inclusive Process $e^+ + e^- \to h + \text{jet} + X$

Spin-0 hadrons, $e^+ + e^- \to Z^0 \to h + \text{jet} + X$

$$\frac{d\sigma^{(\mathrm{si,unp})}}{dy dz d^2 k'_\perp} = \frac{\alpha^2 \chi}{2\pi Q^2} \Big\{ T_0^q(y) \hat{D}_1(z, k'_\perp) + \frac{4}{zQ^2} \Big[T_2^q(y) l_\perp \cdot k'_\perp \hat{D}^\perp(z, k'_\perp) + T_3^q(y) \epsilon_\perp^{l_\perp k'_\perp} \Delta \hat{D}^\perp(z, k'_\perp) \Big] \Big\}$$





$$A_{\mathrm{unp}}^{\cos\varphi} = -\frac{2|\vec{k}'_\perp|}{zQ} \frac{\sum_q \tilde{T}_2^q(y) \hat{D}^\perp}{\sum_q T_0^q(y) \hat{D}_1}$$

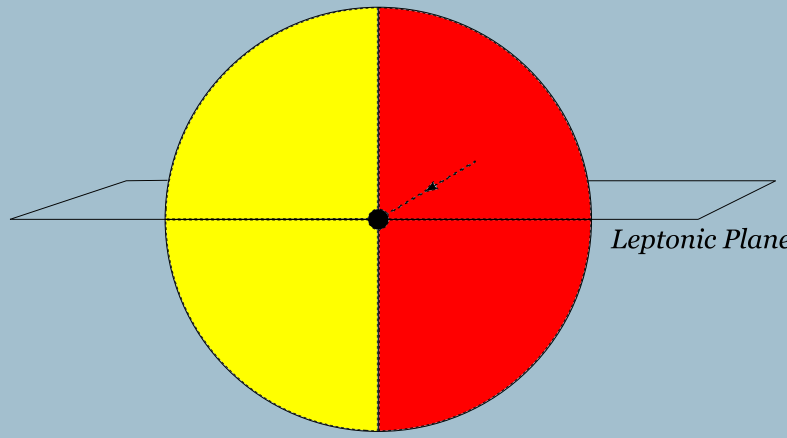
$$A_{\mathrm{unp}}^{\sin\varphi} = \frac{2|\vec{k}'_\perp|}{zQ} \frac{\sum_q \tilde{T}_3^q(y) \Delta\hat{D}^\perp}{\sum_q T_0^q(y) \hat{D}_1}$$

Collinear Frame: Hadron - z, lepton - xoz

Spin-0 hadrons, $e^+ + e^- \to \gamma^* \to h$ +jet+ X

$$\frac{d\sigma^{(\rm si,unp,em)}}{dydzd^2k'_\perp} = \frac{\alpha^2 e_q^2}{2\pi Q^2}\Big\{A(y)\hat{D}_1(z,k'_\perp) + \frac{4l_\perp \cdot k'_\perp}{zQ^2}B(y)\hat{D}^\perp(z,k'_\perp)\Big\}$$

Azimuthal Asymmetries



$$A^{\cos\varphi}_{\rm unp,em}(z,y,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{zQ}\frac{\tilde{B}(y)\sum_q e_q^2 D^{\perp q\to h}(z,k'_\perp)}{A(y)\sum_q e_q^2 D_1^{q\to h}(z,k'_\perp)}$$

$$A^{\sin\varphi}_{\rm unp,em}(z,y,k'_\perp) = 0$$

Semi-inclusive Process $e^+ + e^- \to h$+jet+X

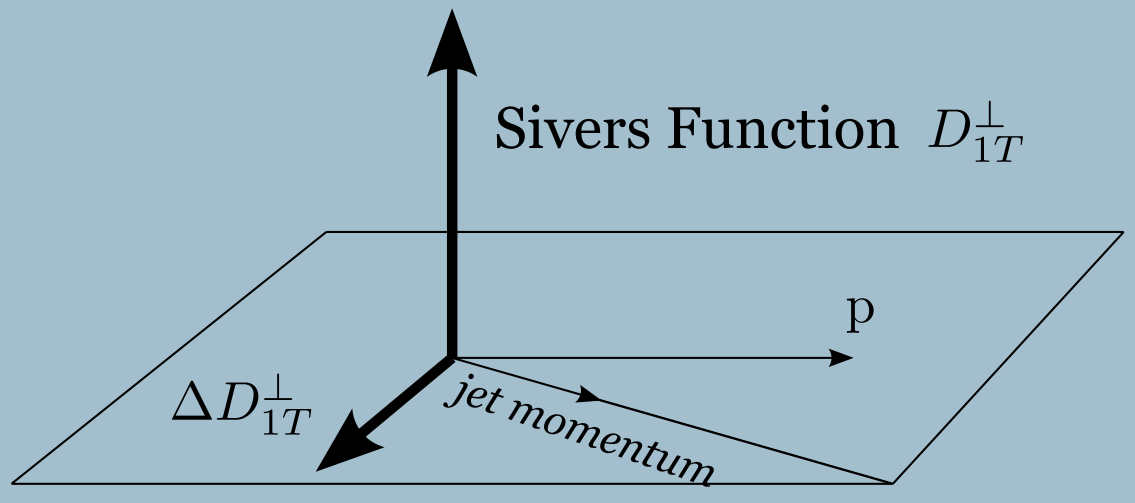
Spin-1/2 hadrons, $e^+ + e^- \to Z^0 \to h$ +jet+ X

平行于强子产生面横向极化

纵向极化

$$\frac{d\sigma^{(\mathrm{si,Vpol})}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp) + T_1^q(y)\Big[\lambda_h\Delta\hat{D}_{1L}(z,k'_\perp) + \frac{k'_\perp\cdot S_\perp}{M}\Delta\hat{D}_{1T}^\perp(z,k'_\perp)\Big]$$
$$+\frac{4\lambda_h}{zQ^2}\Big[T_2^q(y)\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp) + T_3^q(y)l_\perp\cdot k'_\perp\Delta\hat{D}_L^\perp(z,k'_\perp)\Big]$$
$$+\frac{4\epsilon_\perp^{k'_\perp S_\perp}}{zMQ^2}\Big[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}_T^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_T^\perp(z,k'_\perp)\Big]$$
$$+\frac{4M}{zQ^2}\Big[T_2^q(y)\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp) + T_3^q(y)l_\perp\cdot S_\perp\Delta\hat{D}_T(z,k'_\perp)\Big]\Big\}$$

垂直于强子产生面横向极化



Hadron Production Plane: hadron-jet plane

Spin-1 hadrons, $e^{+} + e^{-} \to Z^{0} \to h$ +jet+ X

Leading Twist

Spin alignment

$$\frac{d\sigma^{(\mathrm{si,Tpol})}}{dzdyd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)S_{LL}\hat{D}_{1LL}(z,k'_\perp)$$
$$+T_0^q(y)\frac{k'_\perp\cdot S_{LT}}{M}\hat{D}_{1LT}^\perp(z,k'_\perp)+T_1^q(y)\frac{\epsilon_\perp^{k'_\perp\alpha}S_{LT\alpha}}{M}\Delta\hat{D}_{1LT}^\perp(z,k'_\perp)$$
$$+T_0^q(y)\frac{k'_\perp\cdot S_{TT}\cdot k'_\perp}{M^2}\hat{D}_{1TT}^\perp(z,k'_\perp)+T_1^q(y)\frac{\epsilon_\perp^{k'_\perp\alpha}S_{TT\alpha\beta}k'^\beta_\perp}{M}\Delta\hat{D}_{1TT}^\perp(z,k'_\perp)\Big\}$$

S_{LT}

S_{TT}

Spin-1 hadrons, $e^+ + e^- \to Z^0 \to h$ +jet+ X

Spin alignment

$$S_{LL}^{\text{in}}(y,z) = \frac{\sum_q T_0^q(y) D_{1LL}(z)}{2\sum_q T_0^q(y) D_1(z)} \qquad S_{LL}^{\text{si}}(y,z,k'_\perp) = \frac{\sum_q T_0^q(y) D_{1LL}(z,k'_\perp)}{2\sum_q T_0^q(y) D_1(z,k'_\perp)}$$

Semi-inclusive Process $e^+ + e^- \to h$+jet+X

Spin-1 hadrons, $e^+ + e^- \to Z^0 \to h$ +jet+ X

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$S_{\rm LT}$ and $S_{\rm TT}$

$$S_{LT}^n(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{3M} \frac{\sum_q P_q(y) T_0^q(y) \Delta D_{1LT}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)} \qquad S_{LT}^t(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{3M} \frac{\sum_q T_0^q(y) D_{1LT}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)}$$

$$S_{TT}^{nt}(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|^2}{3M^2} \frac{\sum_q P_q(y) T_0^q(y) \Delta D_{1TT}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)} \qquad S_{TT}^{nn}(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|^2}{3M^2} \frac{\sum_q T_0^q(y) D_{1TT}^\perp(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)}$$

Semi-inclusive Process $e^+ + e^- \to h$+jet+X

Spin-1 hadrons, $e^+ + e^- \to Z^0 \to h$ +jet+ X

Spin alignment

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$S_{\rm LT}$ and $S_{\rm TT}$

P-Odd

$$S_{LT}^{t}(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{3M}\frac{\sum_q T_0^q(y) D_{1LT}^{\perp}(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)}$$

$$S_{TT}^{nn}(y,z,k'_\perp) = -\frac{2|\vec{k}'_\perp|^2}{3M^2}\frac{\sum_q T_0^q(y) D_{1TT}^{\perp}(z,k'_\perp)}{\sum_q T_0^q(y) D_1(z,k'_\perp)}$$

Summary

By applying collinear expansion to both inclusive and semi-inclusive e^+e^- annihilation processes, we have constructed a framework at leading order pQCD to study the leading twist and higher twist contributions for hadrons with different spins systematically.

We made a complete calculation for the azimuthal asymmetries and polarizations of hadrons in terms of corresponding gauge invariant fragmentation functions up to twist-3. Our results provide a basis to study the fragmentation functions in e^+e^- annihilation experiments.

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Thanks for your attention!

THE END