Lectures on Dark Matter

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Abstract

These are three lectures on dark matter written for the 21st International Summer Institute on Phenomenology of Elementary Particles and Cosmology (SI2015). The first lecture focuses on the gravitational effects of dark matter. All well-established experimental evidence for dark matter to date relies purely on its gravitational effects. These tie in to interesting questions in cosmology and astrophysics. The second lecture discusses two favored types of particles that could be dark matter: axions and WIMPs. The discussion of WIMPs focuses on particles in $\mathrm{SU}(2)_L$ multiplets, including superpartners. The final lecture discusses prospects for detecting non-gravitational interactions of dark matter, focusing mostly on direct and indirect detection of WIMPs.

Introduction

These lecture notes are incomplete—they don't contain everything I will say in the lectures, and they probably also contain lots of things I *won't* say. I will continue to add to them.

Dark matter is a very active subject of research, perhaps precisely because we know so little about what dark matter is. We know that it is a form of matter with gravitational interactions much like the matter we are made of, but it does not interact with electromagnetism: it might be more accurately, if less poetically, called "transparent matter." We know that it is a large fraction of the energy density of our universe: $\Omega_{\mathrm{DM}} h^2 \approx 0.1196 \pm 0.0031$ (Planck) [1], which translates to about 26% of the energy density. We imagine that it is probably made of some particles, possibly more than one kind of particle, which are likely stable on timescales longer than the age of the Universe (though even that is not necessary [2, 3]). A wide variety of particle physics models of what dark matter might be have been constructed, and a comparably large range of experiments aimed at detecting dark matter are being pursued. In these three lectures I can only introduce a few aspects of this field. The particle physics I will introduce is fairly conservative: I will tell you about axions and WIMPs because I think that they remain very plausible dark matter candidates, with the strongest ties to well-motivated theories beyond the

Standard Model. There are many other particle dark matter candidates, but given limited time I think that these two old ideas remain the best things to explain. In the first lecture I will make an effort to develop a bit more of the cosmology and astrophysics of dark matter's gravitational interactions than you might see in most other sets of dark matter lectures. A number of review articles and sets of lecture notes exist if you want to dig deeper into some of the topics I discuss [4–10], or those that I have omitted entirely [11,12].

I have put some exercises in these notes. Some of them are concrete calculations, and others are vaguer questions aimed to get you thinking about the material.

Conventions and units

I will often work in particle physics units $\hbar = c = 1$. It's useful to recall that $\hbar c \approx 0.2\,\mathrm{GeV\,fm}$.

> Exercise: consider a physical process in which all kinematic invariants are of order the weak scale. What order of magnitude do you expect the cross section to be, in square centimeters? What if the invariants are of order the QCD scale?

Sometimes I may write results in terms of the reduced Planck mass $M_{\mathrm{Planck}} \approx 2.4\times 10^{18}$ GeV. This is related to Newton's constant by $M_{\mathrm{Planck}}^2 = (8\pi G)^{-1}$ (once we have fixed $\hbar$ and c to 1).

The mass of the Sun is about $M_\odot \approx 1.1 \times 10^{57}$ GeV. Distances in astronomy are often measured in parsecs (pc), where $1\ \mathrm{pc} \approx 3.1 \times 10^{18}\ \mathrm{cm}$. This is about 3.3 light-years (recall that the speed of light is $\approx 3 \times 10^{10}\ \mathrm{cm/s}$ and that a year is about $\pi \times 10^7$ s). When measuring the mass density of a galaxy, it is convenient to use units like $M_\odot/\mathrm{pc}^3$, while for direct detection experiments looking for particles with masses at particle physics scales like GeV or TeV in a volume of order a cubic meter, it's more useful to use units like $\mathrm{GeV/cm^3}$. The conversion between these is

$$1\ M_\odot/\mathrm{pc}^3 \approx 35\ \mathrm{GeV/cm^3}. \tag{1}$$

Stars orbiting a galaxy move at velocities of order a few hundred kilometers per second. If we want to ask about long-term properties of their orbits, they are traveling thousands of parsecs over millions of years. Fortuitously, velocities in terms of everyday units and units suitable for galactic orbits are nearly the same:

$$1\ \mathrm{km/s} \approx 1.02\ \mathrm{pc/Myr}. \tag{2}$$

Astronomers and cosmologists tend to use the word "baryons" idiosyncratically to mean anything made out of normal matter—including leptons—and the word "metals" to mean any element heavier than helium.

1 Dark Matter and Gravity

1.1 Rotation curves

Stars are orbiting in the gravitational potential of a galaxy. Galaxies typically consist of a flattened disk and a spherical bulge. We could ask if the observed stars and hydrogen gas (measured, for instance, by its 21 centimeter emission) would give rise to a gravitational potential that accounts self-consistently for their motion. In the case of the Solar System, for example, this works very well, with the Sun providing the dominant potential and Jupiter and other planets small corrections. Let's look at the approximation of a circular orbit in a spherically symmetric mass distribution with mass $M(r)$ enclosed within radius r:

$$\frac{v^2}{r} = \frac{GM(r)}{r^2} \tag{3}$$

If all the mass is contained within a radius $r_{\rm max}$, expect the long-distance orbits to have

$$v_{\rm circ}(r) = \sqrt{\frac{GM_{\rm tot}}{r}} \tag{4}$$

Thus, one expects measured rotational velocities to fall off like $r^{-1/2}$, but what is observed (see Fig. 1) is an approximately constant velocity out to distances much larger than the visible size of a galaxy. (One can modify this reasoning to think about non-circular orbits or the fact that galaxies are not spherically symmetric but have density peaking within a plane; the qualitative conclusion is unchanged.)

Exercise: the Sun is about 8 kpc from the center of the Milky Way galaxy. Suppose the mass interior to our radius were $10^{10}\ M_\odot$. What circular velocity would be expected at our location? Express the answer in kilometers per second and as a fraction of the speed of light. What if the mass interior to our radius were $10^{11}\ M_\odot$?

Exercise: how would you measure a rotation curve? Are three-dimensional velocities of stars in a distant galaxy observable?

1.2 Jeans equations and the Oort limit

Measuring rotation curves is one way to use the velocities of stars to infer the total mass of a galaxy. Because we live inside the Milky Way galaxy, this has been a bit harder to do within our own galaxy than it is for distant galaxies. But a similar idea was used as early as the 1920s by Jan Oort. Oort was a very clever Dutch astronomer. He used the observed orbits of comets

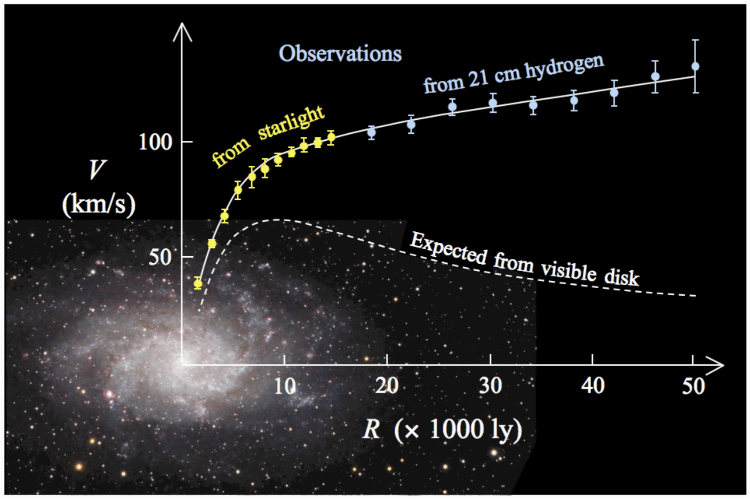

Figure 1: Image via Wikipedia (attributed to Stefania Deluca): rotation curve of M33 (the Triangulum Galaxy). https://en.wikipedia.org/wiki/Galaxy_rotation_curve

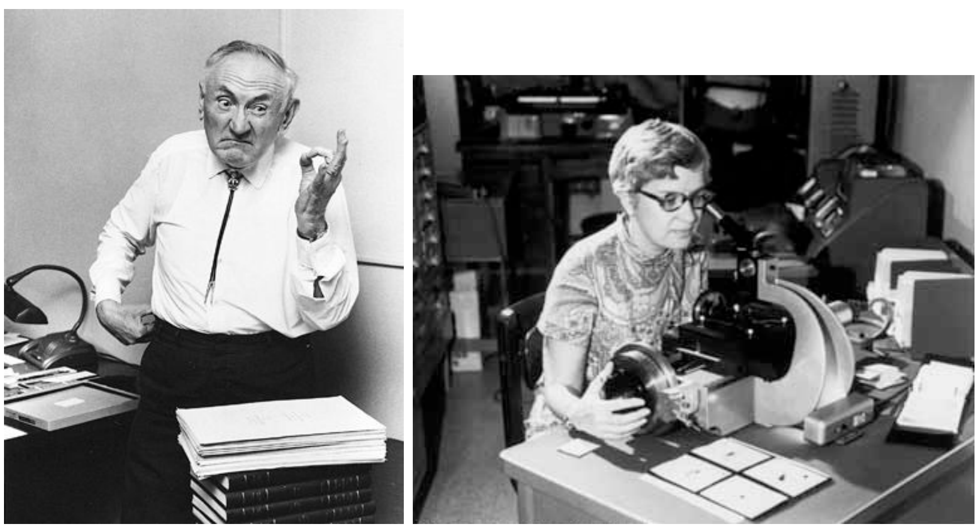

Figure 2: Two pioneers of dark matter physics. Left: the prescient Swiss astronomer Fritz Zwicky used an argument based on the virial theorem ($2\langle T\rangle = -\langle V\rangle$) and observations of the Coma Cluster to conclude that it contained a large amount of invisible matter. Right: these arguments were put on a firmer basis with the work of Vera Rubin (who worked at the Carnegie Institute in Washington) on rotation curves in the early 1970s. (Source: American Institute of Physics, Emilio Segré Visual Archives)

in our Solar System to conclude that there is a vast cloud of comets, now known as the Oort cloud, located tens of thousands of light years from the Sun. He also used measurements of the velocities of stars near the Sun to draw conclusions about the mass density of the galactic disk near our location. The constraint on the mass density obtained in this way is sometimes referred to as the "Oort limit." More refined versions of Oort's estimate have been done in recent years, e.g. references [13–15].

The underlying idea is as follows. Suppose that we make the simplifying assumption that the galaxy is axisymmetric, i.e. that the mass distribution of the galaxy is a function $\rho(R, z)$ in cylindrical coordinates. (This ignores effects like the spiral arms, but let's go with it for now.) We will use Poisson's equation

$$\nabla^2\Phi(R, z) = \partial_z^2\Phi + \frac{1}{R}\partial_R(R\,\partial_R\Phi) = 4\pi G\rho(R, z). \tag{5}$$

Here ρ is the total mass density of the galaxy. It contains a few pieces: the galactic plane, which has a scale height of a few hundred parsecs and scale radius of a few kiloparsecs, falling off exponentially beyond these distances; the galactic bulge, a large, approximately spherical concentration of stars in the inner kiloparsec or so of the galaxy; and the dark matter halo, which extends to much larger distances and again is approximately spherical.

Aside: why is so much matter in a disk?

The Poisson equation alone is not very useful, because we don't measure ρ directly (we can't see dark matter, which is one thing contributing to ρ) and we can't measure accelerations very well either (though they are closely related to to $\Phi(R, z)$). What do we measure? We can see stars and figure out where they are and how fast they are moving. We would like to use that information to draw conclusions about the total mass distribution $\rho(R, z)$, including dark matter. The way to do that is usually via the ***Jeans equation***, which itself is derived from collisionless Boltzmann equations. We first measure the positions and velocities of a population of tracer stars. We then want to use the fact that they're moving in the potential set up by *all* the matter in the galaxy, including dark matter, do learn about that potential. This relies on the fact that stars are collisionless, which is something you should think about:

> Exercise: how often does a star in the Milky Way galaxy pass very close to another star? By "very close" I have in mind that a close encounter is one where a single other star is the major thing determining the acceleration on the first star, rather than the collective gravitational field of all the matter in the galaxy. Convince yourself that star–star encounters can be neglected.

If we neglect collisions, Boltzmann's equations just tell us that, if the galaxy is in approximate equilibrium, the number of stars of a given population in some region of phase space will not change. Specifically, let's define the phase-space density of a set i of stars as $f_i(\vec{x}, \vec{v})$, where

$$\int d^3v\, f_i(\vec{x}, \vec{v}) = n_i(\vec{x}), \tag{6}$$

with n_i the ordinary number density in space. The collisionless Boltzmann equation is then

$$\frac{df_i}{dt} \equiv \frac{\partial f_i}{\partial_t} + \frac{\partial \vec{x}}{\partial t}\cdot\frac{\partial f_i}{\partial \vec{x}} + \frac{\partial \vec{v}}{\partial t}\cdot\frac{\partial f_i}{\partial \vec{v}} = 0. \tag{7}$$

Of course, $\frac{\partial \vec{x}}{\partial t} = \vec{v}$ and $\frac{\partial \vec{v}}{\partial t} = \vec{a} = -\vec{\nabla}\Phi$. The assumption that the galaxy has equilibrated is, mathematically speaking, that $\frac{\partial f_i}{\partial_t} = 0$. If we drop that term, you see that we have an equation relating the phase space distribution f_i (which is observable, if we can locate stars and measure how fast they move) with derivatives of the gravitational potential $\vec{\nabla}\Phi$. In cylindrical coordinates for axisymmetric systems, the collisionless Boltzmann equation takes the form

$$v_R \frac{\partial f_i}{\partial R} + v_z \frac{\partial f_i}{\partial z} + \left(\frac{v_\phi^2}{R} - \frac{\partial \Phi}{\partial R}\right) \frac{\partial f_i}{\partial v_R} - \frac{v_R v_\phi}{R} \frac{\partial f_i}{\partial v_\phi} - \frac{\partial \Phi}{\partial z} \frac{\partial f_i}{\partial v_z} = 0. \tag{8}$$

Let's think a little more about our "equilibrium" assumption. We want to require that the phase-space density of the population of stars we are observing is more or less constant over time. This means that we need to coarse-grain: clearly if we look in a small region, say $1\mathrm{pc}^3$ in size around the Sun, this region currently contains one star (the Sun) and if we wait a few million years it might contain no stars. So we need to sample a big enough volume of space that it's sensible to take an average and assume that any star leaving the region will typically be replaced by a new star entering the region. We also want to be able to measure *all* the stars in the region, ideally; if we're biased toward only being able to see stars in some particular part of phase space, we won't learn about the full distribution $f_i(\vec{x}, \vec{v})$. Obviously, this requirement gets easier as our astronomical observations get better and better. Some existing estimates of the Oort limit rely on data from a satellite called Hipparcos, which measured positions and velocities of $\sim 10^5$ stars. Holmberg and Flynn, for example, estimated the local matter density by focusing on K-type stars within about 100 parsecs of the Sun [14]. But in another decade we should have positions and velocities of $\sim 10^9$ stars in the Milky Way from a European Space Agency satellite called Gaia, so this type of analysis can be carried out more thoroughly and accurately.

You might also wonder in what sense equilibrium is achieved. Often, we think of systems of many particles as attaining equilibrium through many collisions. But we argued that stars don't collide. How, then, do they equilibrate? By collectively moving in the same gravitational potential, it is possible for stars to attain a sort of equilibrium—not thermal equilibrium, what is sometimes called "virialization." Donald Lynden-Bell gave the process by which systems of many gravitationally interacting bodies attain equilibrium the paradoxical name "violent relaxation." This, and more generally the problem of how to do statistical mechanics in systems with long-range interactions, are interesting subjects that I won't say more about both because of lack of time and my own limited expertise.

What is a Jeans equation? The collisionless Boltzmann equation above is a partial differential equation valid at arbitrary points in phase space. We would like to get something easier to work with, which we do by integrating over velocity. The Jeans equation is simply a moment of the collisionless Boltzmann equation, obtained by multiplying by velocities and then integrating d^3v. To derive the vertical Jeans equation, for example, we assume azimuthal symmetry and multiply the Boltzmann equation by v_z, then integrate d^3v. Odds moments drop out of the resulting equation, giving us:

$$\int d^3v \left[v_R v_z \frac{1}{R} \partial_R (R f_i) + v_z^2 \partial_z f_i + f_i \partial_z \Phi \right] = 0. \tag{9}$$

We have integrated by parts to turn $-\int d^3v\, v_z \frac{\partial f_i}{\partial v_z}$ into $+\int d^3v\, f_i$. To declutter our notation, we write this in terms of velocity dispersions

$$\begin{aligned}\left\langle v_z^2\right\rangle_i &\equiv \frac{1}{n_i(R,z)}\int d^3v\, v_z^2 f_i(\vec{x},\vec{v}), &(10)\\ \left\langle v_z v_R\right\rangle_i &\equiv \frac{1}{n_i(R,z)}\int d^3v\, v_z v_R f_i(\vec{x},\vec{v}), &(11)\end{aligned}$$

and more generally it's useful to introduce averages of functions of velocity as angle brackets defined in analogous ways. With this notation the vertical Jeans equation is

$$\frac{1}{R}\partial_R(Rn_i\left\langle v_z v_R\right\rangle_i) + \partial_z(n_i\left\langle v_z^2\right\rangle_i) + n_i\partial_z\Phi = 0. \tag{12}$$

For stellar populations in our vicinity the cross-term $\left\langle v_z v_R\right\rangle_i$ is small, so roughly speaking this equation tells us that we can reconstruct the gravitational potential Φ by measuring the density and vertical velocities of stars and integrating.

The radial Jeans equation can also be useful, and we should be careful about the size of terms like the one involving $\left\langle v_z v_R\right\rangle_i$ that it is tempting to neglect. Estimates based on these techniques can pin down the surface density (i.e. the vertically integrated mass density) of the Milky Way disk in our neighborhood, and the result can be compared to the contributions of stars and gas. What's left is dark matter. A recent estimate [15] using Jeans equation techniques is

$$\rho_{\mathrm{DM}} = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}. \tag{13}$$

This is the dark matter mass density near our location. In the center of the galaxy, it is expected to be much larger, although dust obscures the galactic center and this, together with the large distance, makes it difficult to do similar studies to learn the dark matter density there. As we will see, this leads to an important uncertainty in studies of indirect detection.

1.3 Gravitational lensing and the Bullet cluster

Another form of evidence for dark matter comes from *gravitational lensing*: if we look near a galaxy or galaxy cluster and light coming from more distant galaxies, we can see that the light is distorted. A circle of light will be smeared out into an arc, for instance. Astrophysicists have refined this into a powerful tool for measuring the masses of distant objects, and by comparing this to estimates of the amount of visible mass inferred from the luminosity of the object, it is possible to conclude that there is missing mass.

Mergers of galaxy clusters provide a particularly vivid application of this technique. In the famous case of the Bullet Cluster merger [16, 17], lensing allows us to see a physical separation between ordinary matter and dark matter. When the clusters collide, individual galaxies within the clusters as well as dark matter behave as collisionless objects, streaming through the collision. But most of the ordinary matter in the clusters is in the form of hot gas within and between the galaxies, and the two merging gas clouds can be seen from their hot X-ray emission to be interacting within the collision region (forming a visible shock wave). Gravitational lensing shows that most of the mass of the clusters passed through the collision region unscathed, as we expect from dark matter.

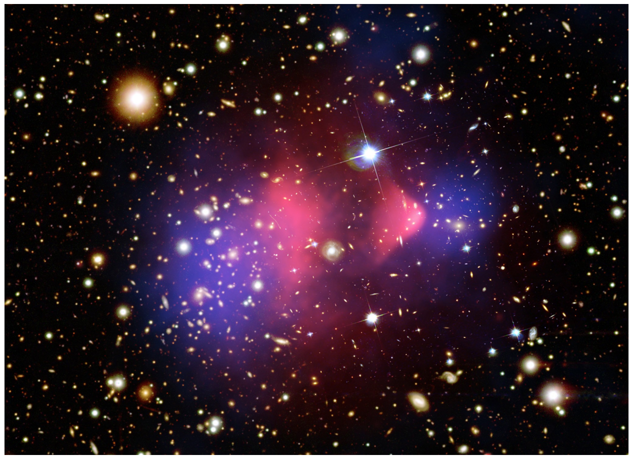

Figure 3: Image from NASA Astronomy Photo of the Day, August 24, 2006. The Bullet Cluster: pink indicates hot gas as measured by X-ray emission. Blue indicates mass distribution measured by gravitational lensing.

If dark matter had very strong self-interactions, then just as the gas of ordinary matter interacts and gets "stuck" in the center of the merging galaxy clusters, dark matter would as well. Thus, the Bullet Cluster not only gives a powerful visual illustration of the existence of dark matter, it also tells us something about the dark matter–dark matter scattering cross section. These observations (as well as others, like halo shapes) are compatible with rather large dark matter self-interactions, imposing a bound [18,19]:

$$\sigma/m \lesssim 1~\mathrm{cm}^2/\mathrm{g} \sim 1~\mathrm{barn}/\mathrm{GeV}. \tag{14}$$

Astronomers tend to say that we know that dark matter interacts very weakly, but by the standards of particle physics, the cross section could still be huge! In fact the situation is even more extreme. Dark matter could be composed of multiple kinds of particles, some of which have much larger self-interaction cross sections (as long as they are a small fraction of all the dark matter by mass) [20]. Such subdominant self-interacting dark matter species could play roles like forming dark disks within galaxies [20] or explaining how supermassive black holes form so early in the cosmological history of the universe [21].

1.4 Clumping of matter: the Jeans instability

When we look out into space, we see stars, clusters of stars, and clouds of gas arranged into large groups called galaxies. A galaxy like the Milky Way or Andromeda has a mass on the order of 10^{11} or $10^{12} M_\odot$, but there are also much smaller galaxies, like the dwarf satellite galaxies of the Milky Way (generally named for the constellations in which they are found, these include classical dwarfs like Draco and Fornax, and more recently discovered ones like Reticulum 2), with masses of order 10^8 or $10^9 M_\odot$, and larger clusters of galaxies, with masses up to around $10^{15} M_\odot$. Our theory of cosmology, which involves scale-invariant initial conditions together with some simple estimates (some of which go under the name of the Press–Schechter formalism) does a pretty good job of predicting the abundance of these different galaxies. Explaining that in detail is beyond the scope of these lectures, but I want to tell you a little bit about what we know about the composition and formation of these large associations of matter.

The first question is why we should find matter grouped in this way at all. You might imagine various ways that matter could be distributed through the universe. It could exist in random agglomerations of every possible scale. Or maybe it could be completely homogeneous and isotropic, a featureless void containing nothing. Instead, what we see is homogeneous and isotropic at sufficiently large scales—if you coarse-grain over distances of order 1 Gpc—but very nonuniform at smaller scales, like those of galaxies. We believe that the initial conditions of the universe were very uniform, and that structure grew as the universe expanded.

The basic lesson is that *matter clumps.* This lesson has a fancier name: the ***Jeans instability***. Beginning with a nearly perfectly homogeneous medium, a small overdensity in one area draws matter in from surrounding areas. This enhances the overdensity there but also depletes the surrounding medium, making the density contrast even bigger. This sort of runaway phenomenon is an instability: in this case, it's what leads galaxies to form from tiny primordial perturbations.

Gravity can be resisted by various other effects, like pressure, that stabilize a system. We can estimate a critical size for an unstable region: in a fluid with sound speed c_s and density $\rho = \bar{\rho} + \delta\rho$, with ρ_0 a constant and $\delta\rho$ a small perturbation, the equation of motion for the perturbation once we take gravity into account is

$$\frac{\partial^2 \delta\rho}{\partial t^2} - c_s^2 \nabla^2 \delta\rho = -4\pi G \bar{\rho} \delta\rho. \tag{15}$$

If we plug in a plane wave perturbation with wave number k and frequency ω, we have a dispersion relation

$$\omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}. \tag{16}$$

This tells us that for small enough $\vec{k}$, the perturbations are unstable—they will grow exponentially with time. The critical wavelength is called the ***Jeans length***,

$$\lambda_J = \frac{2\pi}{k_J} = c_s \sqrt{\frac{\pi}{G\bar{\rho}}}. \tag{17}$$

This tells us that on small scales, gradients can resist gravitational collapse, but large wavelength perturbations are unstable to gravitational collapse. As a result, given the mass density $\bar{\rho}$ of a fluid and its sound speed c_s, we can estimate the *radius* and *mass* of collapsed regions. Cosmologically, this holds as well: regions above a critical size undergo gravitational collapse. Once $\delta\rho/\bar{\rho} \sim 1$, we can't do a linearized analysis anymore: this is the point at which we go from thinking about a homogeneous universe to thinking about dark matter halos or galaxies.

An even simpler version of this estimate follows from dimensional analysis: in a region of size R and uniform density, the gravitational potential energy is $\propto GM^2/R \sim G(\rho R^3)^2/R$ and the kinetic energy goes as $NT \sim (\rho R^3/m)(mc_s^2)$, so at large R the gravitational energy wins and the scaling comes out the same as the more careful estimate above.

Exercise: let's look at a different example where we compare the strength of gravity to the strength of other forces. The mass of the Sun is about $10^{57}\mathrm{GeV}$. Notice that this is about $M_{\mathrm{Pl}}^3/m_{\mathrm{proton}}^2$. Because M_{Pl} is constructed out of G_N, $\hbar$, and c, thinking about the pressure in a classical plasma resisting gravitational collapse isn't enough to see why this relationship might be true. A quantum effect matters. What is this quantum effect? Derive the scaling: the largest star has a mass of order $M_{\mathrm{Pl}}^3/m_{\mathrm{proton}}^2$. (If you're careful enough, you will find additional factors of $(m_p/m_e)^{3/4}\alpha^{3/2}$ that, taken together, are order one.)

You've heard a lot about the hierarchy problem, but here we get a very visceral sense of what it means: if gravity were not ***much*** weaker than the other forces of particle physics, we wouldn't have macroscopic objects like stars and planets! So we need the hierarchy to exist, whether we have a good explanation for it or not.

(You can find more qualitative estimates of the properties of macroscopic objects in terms of fundamental physical constants in refs. [22, 23].)

1.5 Essential cosmology: homogeneous and isotropic

A review of general relativity and FRW cosmology is beyond the scope of these lectures. You can consult textbooks (e.g. by Dodelson and by Weinberg) for more details. I will just quote a few results that you will need to know:

We perturb around a homogeneous, isotropic metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \tag{18}$$

where $a(t)$ is called the scale factor. The rate of change of the scale factor is known as the Hubble constant (even though it isn't constant!):

$$H(t) = \frac{da}{dt}. \tag{19}$$

Notice that $H(t)$ has dimensions of energy (in particle physics units where $\hbar$ and c are set equal to 1). Astronomers usually quote it in units of (km/sec)/Megaparsecs.

Matter and radiation are diluted in an expanding universe. For nonrelativistic particles that are not interacting, the total number of particles is fixed but the volume of space grows larger, so the number density drops:

$$n_{\mathrm{matter}}(t) \propto a(t)^{-3}, \tag{20}$$

and the corresponding energy density scales in the same way:

$$\rho_{\text{matter}}(t) \propto a(t)^{-3}. \tag{21}$$

For radiation (i.e. relativistic particles like photons, with negligible mass), there are two effects; the number density of photons decreases like volume just as for matter:

$$n_{\text{photon}}(t) \propto a(t)^{-3}, \tag{22}$$

but the wavelength of each photon stretches ("redshifts") with the scale factor, so its energy decreases by a factor of $a(t)$. Thus the energy density decreases like the fourth power:

$$\rho_{\text{radiation}}(t) \propto a(t)^{-4}. \tag{23}$$

(You might wonder about mildly relativistic particles, for which the energy is comparable to the mass. In this case the answer lies in between matter and radiation: the wavelength redshifts and the momentum decreases by a factor of $a(t)$, while the energy stored in mass does not redshift. If you have a little familiarity with relativity in curved space, deriving this is a simple exercise that you should do.)

Two other less familiar forms of energy are a cosmological constant, for which $\rho(t)$ is constant over time, and curvature energy (the effect of spatial curvature, which corresponds to replacing δ_{ij} in the FRW metric with the metric on a sphere or hyperbolic space), which decreases like $a(t)^{-2}$.

Einstein's equations relate the energy density stored in matter and radiation over time to the expansion rate of the universe. For an FRW universe the equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}, \tag{24}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{25}$$

Here p is pressure. In the cases we are interested in, p is straightforwardly related to ρ: $p = 0$ for nonrelativistic matter (at leading order; $p \propto (v/c)^2$ at subleading order. For radiation, $\rho + 3p = 0$; this is related to the fact that in conformal field theory the trace of the stress-energy tensor $T^\mu_{\ \mu} = 0$. For a cosmological constant, $p = -\rho$, and $\ddot{a}$ is positive: this drives the accelerating expansion of the universe. Each of these cases can be expressed as $p = w\rho$ where w is known as the "equation of state parameter."

Three important limits are:

- in a radiation-dominated universe, the scale factor increases as $a(t) \sim t^{1/2}$.
- in a matter-dominated universe, the scale factor increases as $a(t) \sim t^{2/3}$.
- in a cosmological constant-dominated universe, the scale factor increases as $a(t) \sim e^{Ht}$. (This is also known as "de Sitter space.")

1.6 Essential cosmology: perturbing around the background

The FRW equations assume that the metric is homogeneous and isotropic. If we start to consider nonuniformities in the matter distribution, like those that started out small in the early universe and eventually became galaxies, we should perturb around this, expanding in a small field $\delta(x,t) \equiv \delta\rho(x,t)/\rho$ fixed by the matter distribution. Then we should also perturb the metric away from the FRW limit:

$$ds^2 = -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2\left((1-2\Psi)\delta_{ij} + E_{ij}\right)dx^i dx^j. \tag{26}$$

The general perturbation of the metric includes scalar, vector, and tensor modes. The tensors are the usual propagating gravitational waves (or gravitons). Discussing the scalar density perturbations fully requires thinking about gauge invariance: we could, for instance, choose our coordinates so that spatial slices have constant density, moving the inhomogeneities out of the matter density $\rho(x,t)$ and into the metric degrees of freedom like Φ. (This is actually a form of the Higgs mechanism—the graviton can "eat" a scalar since we have spontaneously broken time translation symmetry.) There is a lot of interesting physics here with connections to subjects like the EFT of inflation [24], but it's beyond what we can cover in this lecture. So I will just tell you the result: how do density perturbations grow?

In a *matter-dominated* universe, if we consider a spherical region that is overdense, i.e. a region within which there is a constant overdensity $\delta > 0$, it turns out that the solution for this region looks just like the solution for a closed universe: the radius of this region obeys an equation $\ddot{r} = -GM/r^2$ (looking just like Newtonian physics, even if we solve the full equations of GR!), so at first it expands outward, but at a *slightly* slower rate than the overall expansion of the universe, eventually reaching a maximum and turning around to collapse. We say that *matter perturbations grow linearly in a matter-dominated universe*, meaning that as long as the perturbation is small, it grows proportional to the scale factor of the universe, $a(t) \sim t^{2/3}$:

$$\delta(t) \propto t^{2/3} \sim a(t). \tag{27}$$

(There is also a decaying solution $\delta(t) \sim 1/t$, but it is the growing mode that we're interested in.) This equation stops being true once $\delta \sim 1$, at which point the density perturbation goes nonlinear. This leads to the formation of structure, like dark matter subhalos or galaxies.

In a *radiation-dominated* universe, the solution is different:

$$\delta(t) \sim \log a(t). \tag{28}$$

Density perturbations in matter still become larger with time in a radiation-dominated universe, but only in a logarithmic manner. So structure doesn't really form until after the universe becomes matter-dominated.

This leads to a cosmological argument for the existence of dark matter. If perturbations in matter began growing only after recombination decoupled baryons from photons, at redshift $z \approx 1000$, and perturbations grow linearly in a matter-dominated universe, then we would not have nonlinear structure now unless we had begun with density perturbations $\delta\rho/\rho \sim 10^{-3}$. The CMB anisotropies, on the other hand, show us that the initial density perturbations were

$\delta\rho/\rho \sim \text{few} \times 10^{-5}$. The clumping of ordinary matter through the Jeans instability, then, was insufficient to lead to the galaxies that we see today, given the initial conditions that we have learned about through the CMB.

Dark matter resolves this problem: it provides a form of matter that was not coupled to the photon plasma. This allowed the Jeans instability to begin acting on dark matter well before redshift $z \approx 1000$. The details are still somewhat subtle: perturbations only grow linearly after the time of matter–radiation inequality, which was at $z \approx 3000$, so at first glance dark matter has only improved the argument by a factor of 3. The logarithmic growth of dark matter density perturbations in a radiation-dominated universe also helps to explain the discrepancy. Filling in all the numerical factors to see how the observed amount of dark matter is consistent with the size of primordial perturbations observed in the CMB is a fairly complicated exercise. In fact, the CMB and measurements of large-scale structure (e.g. the matter power spectrum as inferred by the distribution of visible galaxies) provides much richer information about how structure formed in our universe, all of which is beautifully compatible with the ΛCDM paradigm (that is, a universe of cold dark matter together with a cosmological constant). I encourage you to go learn more about this, but for now let's return to talking about dark matter in the late-time universe.

1.7 Small-scale problems of dark matter

Core–cusp problem

Missing satellites problem

"Too big to fail" problem

Planes of satellite galaxies

In 2013 we hosted a small workshop at Harvard to bring together people who were thinking about these small-scale puzzles; you can view the slides at this website to get further information.

Exercise. If dark matter consists of very light particles, you might wonder if the effects of quantum degeneracy—for instance, Fermi degeneracy pressure from the Pauli exclusion principle, or perhaps Bose–Einstein condensation—matter in regions with a high dark matter density. Let's work through a simple dimensional analysis estimate for dwarf galaxies. **fill in details**

1.8 MOND phenomenology

You might have heard of "modified Newtonian dynamics," usually called "MOND," an attempt to describe rotation curves in galaxies—and some other astrophysical phenomena—in terms of modified laws of gravity rather than dark matter. MOND was formulated by Mordehai Milgrom in 1983 [25]. It says that, at low accelerations $a \ll a_0$, Newton's law of motion should be changed from $F = ma$ to $F = ma^2/a_0$. Here F is the usual gravitational force, $F = GMm/r^2$. As a result, if we look at the outskirts of the rotation curve, we should set

$$\frac{GM_{\text{tot}}}{r^2} = \frac{1}{a_0}\left(\frac{v^2}{r}\right)^2 \tag{29}$$

This gives rise to constant v and hence a flat rotation curve, in agreement with data.

Notice that MOND is explicitly a non-relativistic theory. Various relativistic generalizations of it have been proposed, but viewed as quantum field theories they usually have pathological features. There is a very good reason for this: there is a unique long-distance relativistic theory of a massless spin-2 particle, and that theory is Einstein's general relativity [26–29]. You could seek other theories of gravity by breaking Lorentz invariance, but we have extremely strong constraints on Lorentz violation [30–34]. (Some versions of "modified gravity" leave GR intact but introduce very light scalar fields that also mediate long-range forces; these are usually plagued with naturalness problems, among other concerns.)

I think that a charitable way to look at MOND is as a phenomenology in search of a theory. MOND fits a lot of data on galactic scales that exhibits surprising regularities. This phenomenology was reviewed by Famaey and McGaugh [35]. For instance, they discuss the baryonic Tully-Fisher relation, which is the small scatter of observed galaxies in the (outer velocity, baryonic mass) plane about a simple power law (with slope 4 in a log–log plot) over five decades in baryonic mass, as depicted in Fig. 4.

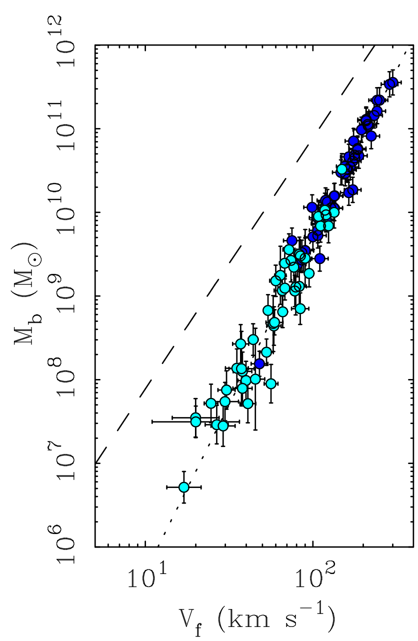

Figure 4: Figure from ref. [35]. The baryonic Tully-Fisher relation.

Dimensional analysis links this observed power law to Milgrom's critical acceleration, $a = V_f^4/(GM_b)$. These regularities could be emergent properties of ordinary dark matter interacting just through gravity. As numerical simulations of galaxy formation become increasingly powerful, we may learn more about how this could be true. Alternatively, these regularities may tell us important things about how dark matter and baryons interact with each other.

I think that MOND phenomenology is worth paying some attention to; it's an interesting set of astrophysical data that is not clearly understood. MOND theory, in its current state, is generally attempting radical modifications of gravity that are unlikely to yield sensible theories. But maybe one of you can find the correct explanation for the observed phenomenology, whether it involves new dark matter–baryon interactions or just a better development of the statistical mechanics of gravitationally interacting particles that can explain the observed simplicity as an emergent phenomenon.

2 Particle Dark Matter: Two Prime Candidates

The general idea of particle dark matter is to consider a large collection of particles that are stable on the timescale of the age of the Universe. By referring to "particle dark matter," I am distinguishing it from dark matter in the form of clumpy objects (whether made of known or novel particles) or black holes. Most theories that are *not* particle dark matter are subject to fairly strong observational constraints, but there is no airtight argument that particle dark matter is the correct idea.

2.1 Axions and strong CP

One compelling theory of dark matter is that it consists of a collection of particles called *axions*, which were proposed for completely independent reasons. The axion was conjectured as a solution to the strong CP problem, which is related to a beautiful set of ideas involving nonabelian gauge theory, anomalies, and instantons. For the purposes of these lectures I'll assume you have some familiarity with these ideas. If you don't, then the general idea that a coherently oscillating scalar field can play the role of cold dark matter is still one that you can understand—but the specifics of why the axion is a good candidate for such a field may elude you.

Summary of strong CP problem and axion solution goes here

The theta term in the Lagrangian, $\bar{\theta}\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$, is constrained by the absence of measured neutron electric dipole moments to satisfy $\left|\bar{\theta}\right| \lesssim 10^{-10}$. An axion can provide a dynamical explanation of this curious experimental fact.

2.2 Coherent states of the harmonic oscillator

You might have learned about coherent states of the harmonic oscillator in a quantum mechanics class. It's useful to review these a little bit, because axion dark matter is a coherently oscillating scalar field, and the connection of this to a collection of nonrelativistic particles is essentially the same story (but with many different oscillator modes for different points in space). Recall that the harmonic oscillator Hamiltonian is

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 = \left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\omega, \tag{30}$$

where the annihilation or lowering operator $\hat{a} = \sqrt{\frac{m\omega}{2}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$. Recall also that the "number operator" $\hat{N} = \hat{a}^\dagger\hat{a}$ has nonnegative integer eigenvalues $0, 1, \ldots, n, \ldots$ which are also energy eigenstates, and can be thought of as exciting n quanta of the underlying oscillator. Number eigenstates are not minimum-uncertainty wavepackets: $\Delta x\,\Delta p = n + 1/2$ for the state $|n\rangle$. Also, as energy eigenstates, they exhibit no interesting time evolution; their phase simply oscillates over time. Thus, these states look very far from classical motion.

Coherent states, on the other hand, are the quantum states that are most closely analogous to classical solutions of the harmonic oscillator equation of motion. They are sometimes introduced as eigenstates of $\hat{a}$: one defines a state $|\alpha\rangle$ by the relation $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$, then solves for such states and finds that they look like Gaussians sloshing back and forth in the potential. These coherent states are minimum-uncertainty wavepackets: they do not spread in either position or momentum space. Let us try to understand this better. Notice that the ground state $|0\rangle$ is a coherent state: it is annihilated by $\hat{a}$. It is also a Gaussian, minimum-uncertainty wave packet (i.e. it has $\Delta x\,\Delta p = 1/2$). Now consider a ***different*** harmonic oscillator Hamiltonian centered at a position x_0 instead of at the origin:

$$\hat{H}' = \frac{1}{2m}\hat{p}_x^2 + \frac{1}{2}m\omega^2\left(\hat{x} - x_0\right)^2. \tag{31}$$

Define shifted raising and lowering operators $\hat{a}'$ and $(\hat{a}')^\dagger$ such that $\hat{H}' = \hbar\omega\left((\hat{a}')^\dagger\hat{a}' + \frac{1}{2}\right)$. Now, the ground state $|0'\rangle$ of the shifted harmonic oscillator Hamiltonian can be obtained by acting on the ground state $|0\rangle$ of the original (unshifted) harmonic oscillator Hamiltonian with the translation operator by a distance x_0, i.e.

$$|0'\rangle = \hat{T}(x_0)\,|0\rangle\,. \tag{32}$$

It turns out that this ***translated*** ground-state wavefunction is another coherent state of our ***original*** harmonic oscillator Hamiltonian. I'll let you work out the details:

Exercise:

- $|0'\rangle$ is a minimum-uncertainty wave packet. Explain why that must be true.
- Argue that $\hat{a}'\,|0'\rangle = 0$. Now express $\hat{a}'$ in terms of the lowering operator $\hat{a}$ of the original Hamiltonian. Use this to compute $\hat{a}\,|0'\rangle$. Is $|0'\rangle$ an eigenstate of the (original, unshifted) lowering operator? If so, what is its eigenvalue?

A coherent state, then, looks a little like a ground-state wavepacket that got accidentally left in the wrong place: because it is displaced from the origin, it feels a restoring force, and executes a very classical-looking simple harmonic motion. At any given time, $\Delta x\, \Delta p$ remains minimal. In the coherent state $|\alpha\rangle$, the number operator has expectation value $\left\langle \alpha \middle| \hat{N} \middle| \alpha \right\rangle = |\alpha|^2$, and also variance $|\alpha|^2$. In fact, in general, the probability of measuring number n is precisely the Poisson distribution with mean $|\alpha|^2$.

Exercise: show that

$$|\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}, \tag{33}$$

i.e. show that coherent states lead to a Poisson distribution.

This is another way in which coherent states are "as classical as possible": they reproduce ordinary counting statistics for finding discrete particles.

2.3 Coherently oscillating scalar fields

The mechanism producing a large abundance of axion dark matter in the late-time universe is the *misalignment mechanism*. It applies not only to the axion but to many light scalar fields like moduli, saxions, or general axion-like particles that may have nothing to do with the strong CP problem. Such fields can have couplings to curvature or to the inflaton that give them large masses in the early universe, so they might have been trapped at a minimum of their potential during inflation. But, in general, there's no reason for the minimum of the potential during inflation to be in the same place as the minimum of the potential in the late-time universe. So the initial condition for the field can be far from its minimum.

The equation of motion for a scalar field in an expanding universe is

$$\ddot{\phi}(x,t) + 3H\dot{\phi}(x,t) - \frac{\partial V}{\partial \phi}(x,t) = 0. \tag{34}$$

The second term, known as Hubble friction, has the effect of damping the time evolution of ϕ until the point when $3H \approx \partial^2 V/\partial\phi^2$, i.e. until the Hubble expansion rate drops below the mass of the particle. After this happens, the particle "realizes" that it is sitting on the slope of a hill and begins to roll down.

Let's assume that the potential is dominated by a mass term, $V(\phi) \approx \frac{1}{2}m^2\phi^2$. Until $3H \sim m_\phi$, the solution resembles an overdamped oscillator and does not evolve. Afterwards, an approximate solution is

$$\phi(t) \quad = \quad \phi_0(t)\cos(m_\phi t), \tag{35}$$

where $\phi_0(t)$ represents the slow loss of energy via Hubble friction: $\phi_0(t) \propto a(t)^{-3/2}$. There is a separation of time scales in the solution: just as for a massive scalar field in flat space, there is a rapid oscillation with frequency set by the mass m_ϕ, which can be averaged over when thinking about how $\phi_0(t)$ changes in a Hubble time. Notice that if we interpret a coherently oscillating scalar quantum field with frequency m_ϕ as being ***the same thing as*** a collection of nonrelativistic particles of mass m_ϕ, this is perfectly consistent with the classical picture of a conserved number of particles sitting still within an expanding background. This interpretation is exactly the right way to think about the problem. The arguments of the previous section help to justify it: the classical solution of the equation of motion of the scalar field should be thought of as a good approximation to a quantum state which is the field theory analogue of the coherent states of quantum mechanics. This quantum state has an expectation value for the number density of particles that is sharply peaked around $\phi(x,t)^2/m_\phi$, with variations determined by Poisson statistics.

Working through the details for the axion leads to

$$\Omega_a h^2 \approx 0.3 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6}. \tag{36}$$

This is a bit more subtle than the discussion above since the axion mass is temperature-dependent; rather than comparing H to a fixed number m_a to see when the axion starts oscillating, we have to compare it to a time-dependent $m_a(H)$. See [36] for an early important paper on axion cosmology and [37] for a more recent and up-to-date review article.

2.4 Boltzmann equation for a thermal relic

The most frequently-studied paradigm for dark matter in the early universe is that it is a ***thermal relic***. This is not the only paradigm—axions can have a large abundance of dark matter even if they are never in thermal equilibrium with the Standard Model fields. Even weak-scale dark matter doesn't ***have*** to be a thermal relic; we don't have any experimental evidence for thermal physics above the temperature at which Big-Bang Nucleosynthesis happened (a few MeV). But the thermal relic is an interesting scenario precisely because it can be nicely consistent with weak-scale dark matter, in a way that I will now explain.

In general, one can study the abundances of various particles in the early universe using a set of Boltzmann equations that keep track of the number of particles in different regions of phase space, solving partial differential equations for the phase space density $f_i(\vec{x},\vec{p})$ of particle species i. We can get away with some simplifications: if we assume thermal equilibrium, the number of particles of a given momentum is simply determined by the total number of particles (their energies are Boltzmann distributed). Furthermore, in the late universe the dark matter particles are nonrelativistic, so we will not ask about their momenta, just about how many total dark matter particles are left. This lets us right a simple ordinary differential equation for the number density n_χ of dark matter particles (which I will call χ), rather than a complicated set of partial differential equations. If we just have noninteracting, non-relativistic dark matter particles, they are diluted according to the equation $n_\chi \propto a^{-3}$; this result is captured by the equation

$$\frac{dn_\chi}{dt} = -3Hn_\chi, \tag{37}$$

recalling that $H = da/dt$. This says that as time passes, the number density of dark matter dilutes because the volume of the universe is expanding.

Now suppose that the dark matter particles are in thermal equilibrium with the Standard Model. Actually, we should distinguish between different kinds of equilibrium. There is ***kinetic equilibrium***, which ensures that the ***momentum*** distribution of different kinds of particles are described by the same temperature. Kinetic equilibrium can be maintained by interactions that don't change the ***number*** of particles of different types, like $\chi\gamma \to \chi\gamma$ (scattering of dark matter and a photon). Another kind of equilibrium is ***chemical equilibrium***, which relates the absolute ***number*** of particles of different types by the corresponding Boltzmann statistics. (Possibly taking into account chemical potentials, which I will neglect for simplicity.) Chemical equilibrium is captured by number-changing interactions that convert between particle types, like annihilation of dark matter ($\chi\chi \to \gamma\gamma$) or the inverse process ($\gamma\gamma \to \chi\chi$). If I say "thermal equilibrium" in this lecture without clarifying, I will generally mean chemical equilibrium.

Dark matter annihilation happens at a rate set by the square of the number density of DM (because we need two χ particles to annihilate) and the thermally-averaged annihilation cross section $\langle \sigma_{\rm ann} v \rangle$ (which is temperature-dependent). There can be multiple annihilation processes with different choices of $\langle \sigma_{\rm ann} v \rangle$, but if we just want to count the dark matter particles, we can add up all the channels without caring what Standard Model particles they annihilate to. Thus annihilation adds a term $-\langle \sigma_{\rm ann} v \rangle n_\chi^2$ to the right-hand side of the Boltzmann equation. On the other hand, the inverse annihilation processes convert Standard Model particles to dark matter particles, adding a term that looks like $+\langle \sigma_{\rm inv.\,ann} v \rangle n_{\rm SM}^2$. (In general, there might be terms for $\gamma\gamma \to \chi\chi$, $q\bar{q} \to \chi\chi$, and many other processes; I am being schematic by writing this as a single term.) However, we can exploit chemical equilibrium to rewrite this term: if n_χ is equal to its chemical equilibrium value $n_{\chi;{\rm eq}}$ at a given temperature, annihilation and inverse annihilation must compensate for each other. This tells us that, at any given temperature,

$$\langle \sigma_{\rm inv.\,ann} v \rangle n_{\rm SM}^2 = \langle \sigma_{\rm ann} v \rangle n_{\chi;{\rm eq}}^2. \tag{38}$$

Using this result leads us to the equation for a thermal relic dark matter particle:

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle \sigma_{\rm ann} v \rangle \left(n_\chi^2 - n_{\chi;{\rm eq}}^2\right). \tag{39}$$

Again, the first term on the right hand side is dilution by the expanding volume of space; the second is loss of χ particles by annihilation to Standard Model states; and the third term is inverse annihilation that produces χ particles out of the Standard Model plasma.

> Exercise: write a computer program that numerically solves this differential equation in the background of a radiation-dominated universe (where $a(t) \propto t^{1/2}$). Try to reproduce the basic phenomenology I will explain below. (One thing you might experiment with is changing the time variable among, e.g., t, a, T, $x = m/T$, or conformal time η to see how the efficiency of your numerical computation behaves in different cases.)

2.5 Thermal freezeout: qualitative solution

We would like to solve the Boltzmann equation (39) to see what the final abundance of χ particles is in the universe today, assuming that we started with χ in chemical equilibrium with the Standard Model. Suppose that the universe was ***not*** expanding at all, i.e. $H = 0$. Then we could solve the equation

$$\frac{dn_\chi}{dt} = -\left\langle \sigma_{\rm ann} v \right\rangle \left(n_\chi^2 - n_{\chi;\rm eq}^2 \right). \tag{40}$$

This is a basic equation of thermal equilibrium; if $n_\chi > n_{\chi;\rm eq}$, annihilation will happen more often than inverse annihilation until the number of χ particles drops to its equilibrium value. If $n_\chi > n_{\chi;\rm eq}$, inverse annihilation happens more often and the number of χ particles increases. Thus the number of χ particles will approach its equilibrium value.

Now let's superimpose the expansion of the universe on this story. The momentum of particles is proportional to $1/a(t)$, as wavelengths are stretched by the expansion of space. So the temperature of the Standard Model plasma is dropping, $T \propto 1/a$. If the expansion is slow enough, we can still ignore the $-3Hn_\chi$ term in the Boltzmann equation, and the main effect of the expansion is to change T, which changes the equilibrium value $n_{\chi;\rm eq}$. In particular, in thermal equilibrium, the number density of χ particles goes as $\exp(-E/T) \approx \exp(-m/T)$ when $T \ll m$ and the particles are nonrelativistic. This says that the number density of particles is dropping exponentially as the temperature falls. What happens is that annihilation processes like $\chi\chi \to \gamma\gamma$ happen easily, but because χ is heavy, inverse annihilation processes like $\gamma\gamma \to \chi\chi$ happen only very rarely (with photons on the tail of the momentum distribution). So thermal equilibrium demands that the number density of dark matter falls to exponentially small values.

At some point, we can no longer ignore the expansion of the universe. As n_χ becomes smaller and smaller, eventually we will hit a point where the annihilation rate $-\left\langle \sigma_{\rm ann} v \right\rangle n_\chi^2$ is no longer negligible compared to the dilution term $-3Hn_\chi$. At that point, the universe is expanding fast enough relative to the $\chi\chi$ annihilation that the χ particles are getting swept away from each other before they can find each other and annihilate. The solution to the equation will then approach a value called the "freeze-out abundance," $n_{\chi;\,\rm f.o.}$, at which point the annihilation process no longer matters and the simple dilution equation (37) is correct.

This freezeout behavior is depicted in Fig. 5. This figure demonstrates that, as $\left\langle \sigma_{\rm ann} v \right\rangle$ becomes larger, the time at which the solution stops tracking the thermal equilibrium value becomes later and as a result the final dark matter abundance is smaller.

2.6 The WIMP miracle

We have discussed the qualitative behavior of the solution to the equation (39), finding that the final dark matter abundance will be smaller if $\left\langle \sigma_{\rm ann} v \right\rangle$ is larger.. Let us now be more quantitative. We have argued that the solution transitions from tracking the thermal equilibrium abundance of χ to a "freeze-out" value that simply dilutes with the expansion of space. What we would like to know is the temperature T at which this transition happens, which will determine the freeze-out abundance.

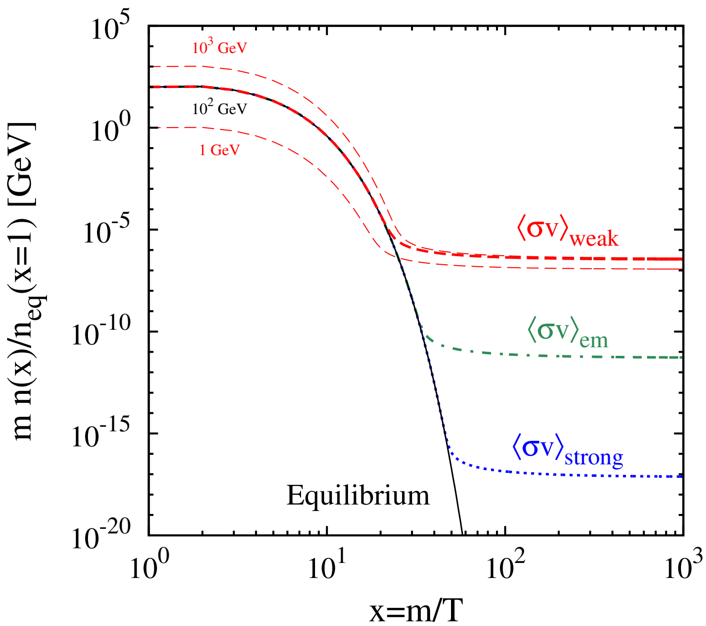

Figure 5: Figure from ref. [38]. Freezeout of a WIMP beginning in thermal equilibrium. Temperature is higher on the left, which corresponds to earlier time.

The first thing to do is pin down the temperature dependence of the terms where we can. For $T \ll m$, the appropriate expression for the equilibrium abundance of χ is

$$n_{\chi;\text{eq}} = g_\chi \left(\frac{mT}{2\pi}\right)^{3/2} \exp(-m/T). \tag{41}$$

Here g_χ counts the number of independent χ states (e.g. spin degrees of freedom). The factor $\exp(-m/T)$ is the previously identified Boltzmann factor. And the final factor $\left(\frac{mT}{2\pi}\right)^{3/2}$ is the density of states for non-relativistic particles. The next fact that we need is the Hubble scale in a radiation-dominated universe at temperature T, which follows from the Friedmann equation (24):

$$H^2 = \frac{\rho}{3M_{\text{Pl}}^2} = \frac{\pi^2 g_* T^4}{90 M_{\text{Pl}}^2}, \tag{42}$$

where g_* counts the number of relativistic degrees of freedom. (It is itself a function of T, since degrees of freedom change from relativistic to nonrelativistic when the temperature drops below their mass.)

We can crudely estimate that the transition we are looking for is when $3Hn_\chi$ dominates over $\langle \sigma_{\text{ann}} v\rangle\, n_\chi^2$ and we can set $n_\chi = n_{\chi;\text{eq}}$ since we expect chemical equilibrium to hold up until this time. This lets us estimate the freezeout abundance as

$$n_{\chi;\text{f.o.}} \approx \left.\frac{3H}{\langle \sigma_{\text{ann}} v\rangle}\right|_{T=T_{\text{f.o.}}}, \tag{43}$$

where we must self-consistently estimate the freezeout temperature $T_{\text{f.o.}}$. Defining $x = m/T$, we would like to solve for $x_{\text{f.o.}}$ in the equation

$$x_{\text{f.o.}}^{1/2} e^{-x_{\text{f.o.}}} \approx \frac{2\pi^{5/2}}{15^{1/2}} \frac{g_*(x_{\text{f.o.}})^{1/2}}{g_\chi} \frac{1}{m_\chi M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}}}. \tag{44}$$

What is this equation telling us? First, we expect $\langle \sigma_{\text{ann}} v \rangle \sim 1/M^2$ where M is a typical mass scale in the annihilation problem or combination of mass and temperature scales. In particular, the denominator contains a dimensionless quantity $m_\chi M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}} \gg 1$, so at a first crude level we expect that the order of magnitude of the right-hand-side is set by this quantity and the left-hand-side is dominated by the factor $e^{-x_{\text{f.o.}}}$, so

$$x_{\text{f.o.}} \sim \log(m_\chi M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}}). \tag{45}$$

The other factors in equation (44) should give subdominant corrections to this leading estimate. Because the Planck scale is roughly e^{30} times larger than typical particle physics scales, we might expect $x_{\text{f.o.}} \sim 30$, and so particles freeze out at temperatures ~ 30 times smaller than their mass. (This tells us that it was consistent to use purely non-relativistic estimates of $n_{\chi;\text{eq}}$.)

Once the dark matter particles freeze out, their number density scales as $a(t)^{-3}$, and the temperature of the Standard Model plasma also drops as $T \propto a(t)^{-1}$. (This isn't ***quite*** true since the number of degrees of freedom of the Standard Model change—e.g., at some point electrons and positrons annihilate to photons, increasing the temperature of the photons relative to this estimate—but it's good enough at an order of magnitude level.) So we can estimate that the number density of dark matter particles today is

$$n_{\chi;\,\text{now}} \approx n_{\chi;\,\text{f.o.}} \left(\frac{T_{\text{now}}}{T_{\text{f.o.}}}\right)^3 \approx \frac{3\pi g_{*;\text{f.o.}}^{1/2} T_{\text{f.o.}}^2}{90^{1/2} M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}}} \left(\frac{T_{\text{now}}}{T_{\text{f.o.}}}\right)^3 = \frac{\pi g_{*;\text{f.o.}}^{1/2} T_{\text{now}}^3 x_{\text{f.o.}}}{10^{1/2} M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}} m_\chi}, \tag{46}$$

where we used equation (43) for $n_{\chi;\,\text{f.o.}}$ and substituted $T_{\text{f.o.}} = m_\chi / x_{\text{f.o.}}$ in the last step. This is a nice result, because it means that the ***energy density*** of the χ particles today is

$$\rho_{\chi;\,\text{now}} = m_\chi n_{\chi;\,\text{now}} = \frac{\pi g_{*;\text{f.o.}}^{1/2} T_{\text{now}}^3 x_{\text{f.o.}}}{10^{1/2} M_{\text{Pl}} \left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}}}, \tag{47}$$

which cancels the factor of m_χ in the denominator. That is, the energy density of the dark matter today depends on:

- The annihilation cross section of the dark matter when it froze out, $\left.\langle \sigma_{\text{ann}} v \rangle\right|_{x=x_{\text{f.o.}}}$. The energy density in dark matter goes ***inversely*** as this quantity, as seen in Fig. 5.
- The number of relativistic degrees of freedom of the Standard Model plasma when the dark matter froze out, $g_{*;\text{f.o.}}^{1/2}$. As long as freezeout happened at reasonably low temperatures, we know this number from our knowledge of the Standard Model.

- The freezeout temperature relative to the dark matter mass, $x_{\rm f.o.}$, which fortutiously depends only logarithmically on the detailed particle physics of dark matter annihilation, and usually turns out to be about 20.

Putting this together, up to mild dependence on some other numbers, we see that *achieving the right thermal relic dark matter abundance is equivalent to finding a model with a particular value of* $\langle \sigma_{\rm ann} v\rangle|_{x=x_{\rm f.o.}}$. What is this value? We need

$$\rho_{\chi;\,{\rm now}} \approx 0.26 \rho_{\rm now} = 0.26 \times 3 H_{\rm now}^2 M_{\rm Pl}^2, \tag{48}$$

so we require

$$\langle \sigma_{\rm ann} v\rangle|_{x=x_{\rm f.o.}} \sim \frac{\pi g_{*;{\rm f.o.}}^{1/2} T_{\rm now}^3 x_{\rm f.o.}}{10^{1/2} 0.26 \times 3 H_{\rm now}^2 M_{\rm Pl}^3}. \tag{49}$$

The temperature of our universe now is $T_{\rm now} = 2.7~{\rm K} \approx 2.3 \times 10^{-4}~{\rm eV} = 2.3 \times 10^{-13}~{\rm GeV}$. The Hubble scale now is $H_{\rm now} \approx 1.4 \times 10^{-42}$ GeV. Putting this together, our estimate is (plugging in $g_{*;{\rm f.o.}} \approx 10$ and $x_{\rm f.o} \approx 20$ as guesses)

$$\langle \sigma_{\rm ann} v\rangle|_{x=x_{\rm f.o.}} \sim \left(\frac{1}{5.3~{\rm TeV}}\right)^2. \tag{50}$$

This shows that the cross sections we want correspond to mass scales in the vicinity of a TeV! This estimate has been a little bit sloppy in a few places. A detailed recent calculation in ref. [38] gives a precise result of

$$\langle \sigma_{\rm ann} v\rangle = 2.2 \times 10^{-26}~{\rm cm}^3/{\rm s} \approx \left(\frac{1}{23~{\rm TeV}}\right)^2 \tag{51}$$

for dark matter masses above 10 GeV.

Exercise: figure out some of the missing details in the above derivation. For instance, refine the estimate for $x_{\rm f.o.}$ and understand why it's not quite true that $T_{\rm now}/T_{\rm f.o.} = a(t_{\rm f.o.})/a(t_{\rm now})$.

What might a typical weak-scale cross section look like? One sensible estimate might be

$$\sigma_{\rm weak} = \left(\frac{g^2}{4\pi m_W}\right)^2 \approx \left(\frac{1}{48~{\rm TeV}}\right)^2. \tag{52}$$

So we're in the same neighborhood. This result is known as the *WIMP miracle*. We merely asked, for a dark matter candidate in thermal equilibrium with the Standard Model that freezes out at temperatures below its mass: what annihilation cross section would match the dark matter abundance in our universe today? The inputs to this calculation were quantities like the temperature of the CMB, the expansion rate of the universe today, and the strength of gravity (via M_{Planck}). No particle physics involving the weak scale went into the calculation at all: every energy scale in the calculation was either many orders of magnitude larger or many orders of magnitude smaller. But the mass scale set by $\langle \sigma v \rangle^{-1/2}$ is near the TeV scale! Notice here that the "W" in WIMP doesn't just mean "weak" as in *a* weak interaction—though many people use the word WIMP in such a broad context—it meant *the* weak interaction, SU(2)$_L$. So let's look at the possibility that dark matter lives in SU(2)$_L$ multiplets.

2.7 Matter in SU(2) multiplets

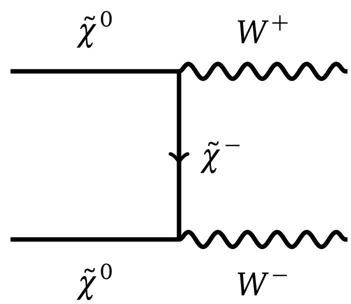

Figure 6: Annihilation of dark matter particles in SU(2)$_L$ multiplets to W^+W^- through t-channel exchange of one of their SU(2)$_L$ partner particles. This process generally has a large rate and leads light SU(2)$_L$-charged dark matter particles to have too small a thermal relic abundance. For wino DM, for instance, thermal relics arise at masses of 3 TeV; for higgsinos, the thermal relic happens at a mass of about 1 TeV.

Minimal dark matter [39]
The well-tempered neutralino [40]

3 Detecting Dark Matter's Non-Gravitational Interactions

3.1 Ways of looking for dark matter

We can classify ways of looking for dark matter interactions with Standard Model particles via three channels:

- $\chi\,\text{SM} \to \chi\,\text{SM}$: in this case, a dark matter particle scatters with a Standard Model particle. This is the approach taken by *direct detection* experiments: a dark matter particle enters the detector and exchanges momentum with an ordinary particle.
- $\chi\,\chi \to \text{SM}\,\text{SM}$: in this case, dark matter particles annihilate to Standard Model particles that we can observe. This can happen very far from the Earth, e.g. in the galactic center or in a dwarf satellite galaxy. As a result, we call this *indirect detection*.

- SM SM $\to \chi\,\chi$: In this case, the χ particles are only in the final state, so we are not really detecting primordial dark matter particles. Instead, we could produce the same particle in a collider experiment.

Crossing symmetry can relate the ***amplitudes*** of these different processes. Importantly, however, the kinematics and phase space of experiments in these different channels can be very different. As a result, a particle that is easily excluded by direct detection experiments might be hard to detect in indirect detection experiments, and vice versa. This leads to a powerful ***complementarity*** between different experiments: we need them all to thoroughly explore the full space of dark matter theories, even within a relatively well-defined context like WIMPs.

3.2 Direct detection

Direct detection experiments rely on large volumes of detector material to increase the chance that a dark matter particle passing through the Earth will scatter in the detector. They are built far underground to try to eliminate backgrounds like cosmic rays. For example, the LUX dark matter experiment uses 370 kilograms of liquid xenon in the Sanford Underground Laboratory in Lead, South Dakota in the US, about 1.5 kilometers underground. Not all of the detector volume is used: events in the outer detector are more likely to be backgrounds like neutrinos. The published LUX result [41] used 85.3 days of data and 118 kg of "fiducial" detector.

The typical velocity of a dark matter particle in the halo is $v \sim 200$ km/s $\sim 10^{-3}c$, so a dark matter particle with a mass of 100 GeV only has a kinetic energy of about 100 keV. As a result, detectors must be sensitive to much smaller energies than, for instance, a collider experiment. The LUX detector observes nuclear recoil events using a combination of ionization and scintillation. Other experiments operate differently: for instance, CDMS used solid-state germanium and silicon detectors and observed a combination of ionization and phonons [42]. If a dark matter particle of mass m scatters on a target nucleus of mass m_T, the reduced mass of the system is $\mu = mm_T/(m+m_T)$. If it scattered on a single proton, the reduced mass would be $\mu_p = mm_p/(m+m_p)$. There are many different operators that can couple dark matter to the Standard Model, and these can lead to different dependence of the cross-section on the dark matter velocity in the lab frame, on the momentum transfer, and on the spin of the dark matter or the nucleus. But the most commonly-plotted quantity is the bound assuming a constant spin-independent dark matter–nucleus scattering cross section σ_p^{SI} with the assumption that the cross section for scattering on the target is (see, for instance, [43]):

$$\sigma_T^{\mathrm{SI}} = \frac{\mu_T^2}{\mu_p^2}\sigma_p^{\mathrm{SI}}\left(Z_T + \frac{f_n}{f_p}(A_T - Z_T)\right)^2 F_{\mathrm{SI},T}^2(E_R). \tag{53}$$

Here Z is the charge of the nucleus, A its atomic number, f_n/f_p is the ratio of the dark matter coupling to neutrons and protons, and $F_{\mathrm{SI},T}^2(E_R)$ is the "standard" spin-independent form factor for the target nucleus, which describes the way that mass is distributed within the nucleus. It reflects the fact that long-wavelength probes see the entire nucleus and can scatter coherently, while short-wavelength probes see individual nucleons. What an experiment ***actually*** probes is the differential cross section as a function of recoil energy, $d\sigma_T/dE_R$, for dark matter scattering on a particular target. Bounds are plotted in terms of $\sigma_p^{\rm SI}$ to allow easy comparison of different experiments, but you should keep in mind that the underlying assumption of equation (53) may not be true.

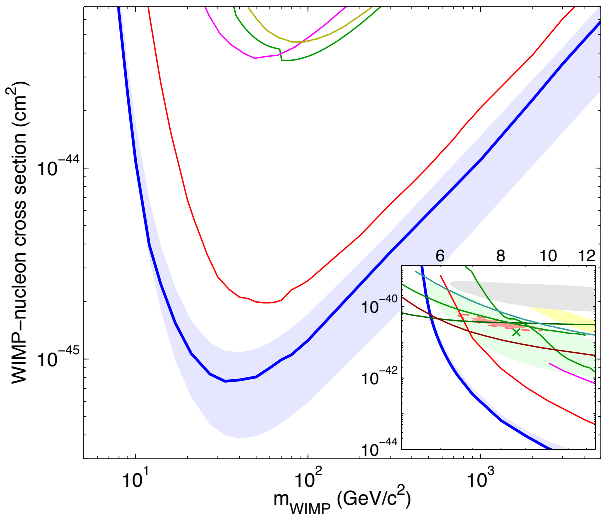

Figure 7: Figure from ref. [41]. Results of the LUX experiment in blue. (The other curves are results of previous experiments.) The "WIMP–nucleon cross section" on the vertical axis is what we have called $\sigma_p^{\rm SI}$ in equation (53). The present constraints are on the order of 10^{-45} cm^2.

Recently there has been a trend toward more model-independent approaches. Because dark matter particles in the galaxy have slow velocities $v \sim 10^{-3}$, their interactions in direct detection experiments are always non-relativistic. This leads to the possibility of classifying dark matter scattering effects in terms of non-relativistic operators, an approach that was first pursued in ref. [44] and subsequently refined by others who studied nuclear responses in great detail [45,46] and even provided a useful Mathematica code for calculation [47]. This framework is more comprehensive than the use of equation (53). But for now, I want to review the standard results relying on the simplest assumptions.

The idea of direct detection has been around since the mid-1980s [48, 49], but there have been two important experimental thresholds reached in the intervening decades. The first was the exclusion of dark matter particles that scatter with nuclei via exchange of a Z boson, like the (left-handed) sneutrino, which was excluded by the mid-1990s [50]; the second is the recent progress toward excluding dark matter particles that scatter with nuclei via Higgs boson exchange. Let's try to understand the characteristic rates of these signals. If dark matter couples to a Z boson or a Higgs boson, it is straightforward to perturbatively compute a high-energy scattering process of dark matter on quarks. The tricky part about direct detection is that the scattering happens at low energies with nuclei, so we have to match operators coupling dark matter to quarks with operators coupling dark matter to nuclei. This means that we require nuclear matrix elements like:

$$\langle N(p+q)|\mathcal{O}(x)|N(p)\rangle\,, \tag{54}$$

where $\mathcal{O}(x)$ is some quark bilinear that the Z or Higgs couples to, like $\bar{q}\gamma^\mu(a+b\gamma^5)q$ or $\bar{q}q$, and $|N(p)\rangle$ is a state of the ***nucleus*** with momentum p. Nuclear form factors like those you can use the code of [47] for do part of this work for us, helping to unravel the dependence on the momentum transfer q. In the case of conserved currents, matrix elements are constrained by Ward identities—e.g. if $\mathcal{O}(x)$ is the electromagnetic current $J^\mu_{\rm EM}(x)$, matrix elements just count the charge of the nucleus. But more generally, knowing a matrix element requires nonperturbative information about QCD. For the scalar operators coupling to the Higgs, lattice calculations give reliable estimates of these matrix elements (see [51] for lattice results and the appendix of [52] for further discussion).

details

Result: for elastic scattering through a Z boson,

$$\sigma_p \sim 10^{-39}\ {\rm cm}^2, \tag{55}$$

several orders of magnitude larger than current limits allow. For elastic scattering through a Higgs boson, e.g. for a scalar with quartic coupling $\lambda\,|S|^2\,|H|^2$, the cross section is

$$\sigma \approx \lambda^2\left(\frac{100\ {\rm GeV}}{m_{\rm DM}}\right)^2 \times 3\times 10^{-44}\ {\rm cm}^2. \tag{56}$$

This is in the range that is currently being probed (a similar result holds for fermions, although in that case the operator is dimension 5 and the analogue of λ is v/Λ).

It is often said that direction detection experiments have imposed very strong constraints on WIMP dark matter. But this is not generically true: they have ruled out particular ***kinds*** of WIMP dark matter. WIMPs that elastically scatter through Z bosons are excluded, but higgsinos, for instance, are SU(2)$_L$ doublets that have large enough inelastic splittings that their scattering through a Z is highly suppressed. Triplets of SU(2)$_L$ (e.g. winos) do not scatter through Z bosons. Particles scattering through the Higgs are rather constrained, but this requires multiple fermion states that are well-mixed. Dark matter in arbitrary SU(2)$_L$ multiplets is actually fairly safe from direct detection bounds.

3.3 Indirect detection

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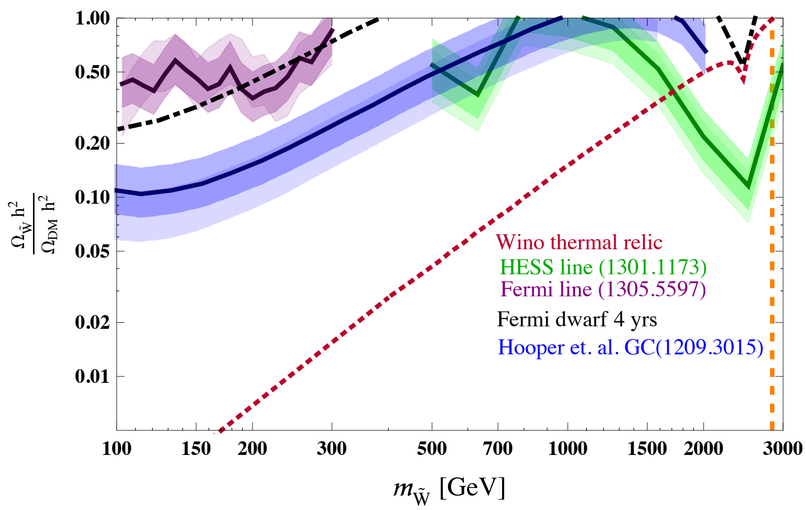

Figure 8: Figure from ref. [53]. (See also [54].) Bounds on wino dark matter from a combination of gamma ray continuum at Fermi-LAT, sensitive to $\tilde{W}^0\tilde{W}^0 \to W^+W^-$, and gamma ray line searches at Fermi-LAT and HESS, sensitive to $\tilde{W}^0\tilde{W}^0 \to \gamma\gamma$.

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