



# Radiative Neutrino Mass in Extended Zee Model with Dark Matter

Dart-yin Soh  
with T. C. Yuan

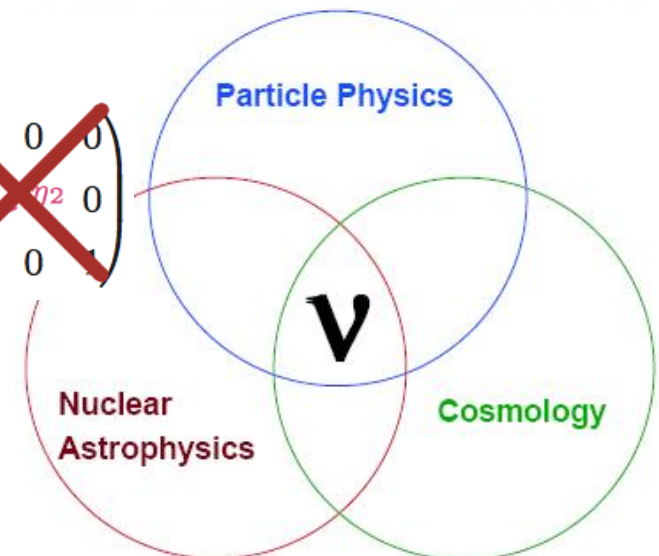
Institute of Physics, Academia Sinica

- Introduction
- Radiative Neutrino Mass
- Is Zee Model Ruled Out by Current Data?
- A New Extended Zee Model
- Outlook

# Neutrino Oscillations and Mixing

- ❖ Standard model is too good with data before neutrino oscillations
- ❖ But neutrino oscillations  $\Rightarrow$  neutrino mixing  $\Rightarrow$  neutrino masses!
- ❖ How to extend the Standard model to generate neutrino mass?
- ❖ Much small masses and much larger mixing comparing with quarks: not natural if only Dirac masses with  $\nu_R$
- ❖ Mixing matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



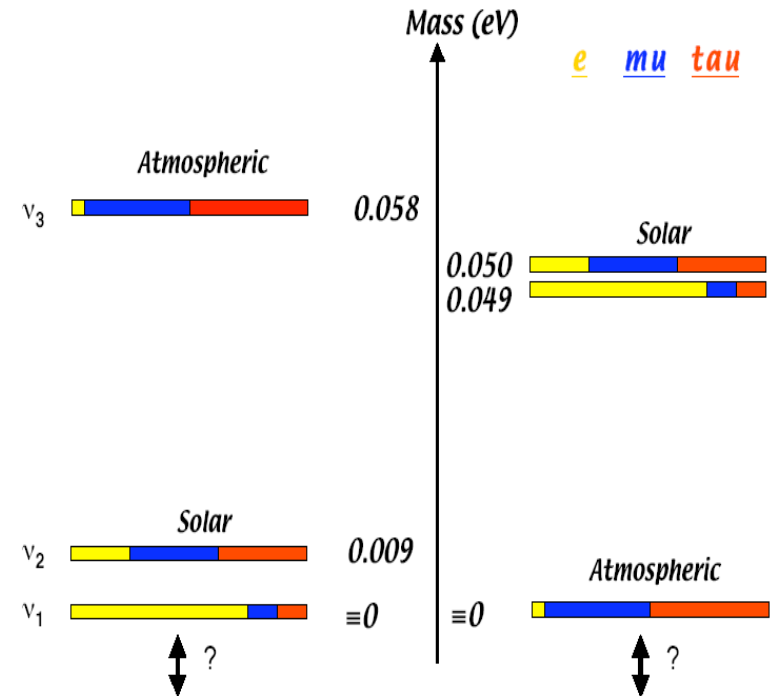
- ❖ Anti-neutrinos are the same as neutrinos?  
Majorana or Dirac?

# Neutrino Oscillations and Mixing

## More precise neutrino data

parameter	best fit	$1\sigma$ range	$2\sigma$ range
$\Delta m_{21}^2$ [ $10^{-5} \text{eV}^2$ ]	7.60	7.42–7.79	7.26–7.99
		Reactor LBL (KamLAND)	
$ \Delta m_{31}^2 $ [ $10^{-3} \text{eV}^2$ ] (NH)	2.48	2.41–2.53	2.35–2.59
$ \Delta m_{31}^2 $ [ $10^{-3} \text{eV}^2$ ] (IH)	2.38	2.32–2.43	2.26–2.48
$\sin^2 \theta_{12}/10^{-1}$	3.23	Accelerator LBL $\nu_\mu$ Disapp (Minos)	
		3.07–3.39	2.92–3.57
		Solar Experiments	
$\sin^2 \theta_{23}/10^{-1}$ (NH)	5.67 (4.67) <sup>a</sup>	4.39–5.99	4.13–6.23
$\sin^2 \theta_{23}/10^{-1}$ (IH)	5.73	5.30–5.98	4.32–6.21
		Atmospheric Experiments	
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.34	2.14–2.54	1.95–2.74
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.40	2.21–2.59	2.02–2.78
		Reactor MBL (Daya-Bay, Reno)	
$\delta/\pi$ (NH)	1.34	0.96–1.98	0.0–2.0
$\delta/\pi$ (IH)	1.48	1.16–1.82	0.0–0.14 & 0.81–2.0

## Normal ordering or inverted ordering?



Predictive models?  
May be ruled out by  
experiments!

# Why Neutrinos Have Masses?

- ❖ God should use natural way to generate neutrinos masses
- ❖ Sea-saw: tree level Majorana masses of  $\nu_R$  & no fine-tuning Dirac

$$\mathcal{L} = -\phi^\dagger \bar{\ell}_L y_\nu N_R - \frac{1}{2} \bar{N}_R^c M N_R + \text{h.c.}$$
$$\rightarrow -\bar{\nu}_L m_D N_R - \frac{1}{2} \bar{N}_R^c M N_R + \text{h.c.} \quad m_D = y_\nu \langle \phi \rangle \quad \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \rightarrow \begin{matrix} m_\nu = -m_D M^{-1} m_D^T + \dots \\ \text{(if } m_D \ll M) \end{matrix}$$

very heavy neutrinos

- ❖ Radiative neutrino masses: naturally small due to the loop corrections, less parameters and thus predictive
  - ◆ Simple and clean: only Majorana Masses of  $\nu_L$
  - ◆ Renormalizable, no counter-term and thus calculable

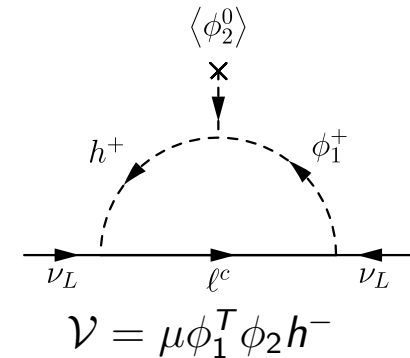
# Radiative Neutrino Masses

## ❖ Zee model (1980)

- ◆ 2HDM+charged singlet
- ◆ Majorana Yukawa couplings
- ◆ Was studied extensively

	$SU(2)_L$	$U(1)_Y$
$\phi_2$	2	1
$h^+$	1	1

· neutrino mass (1-loop level)

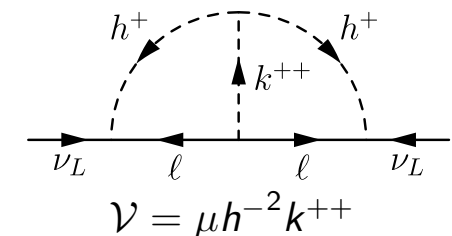


## ❖ Zee-Babu model

- ◆ Majorana Yukawa couplings of both L & R leptons
- ◆ Still compatible with data

	$SU(2)_L$	$U(1)_Y$
$h^+$	1	1
$k^{++}$	1	2

· neutrino mass (2-loop level)

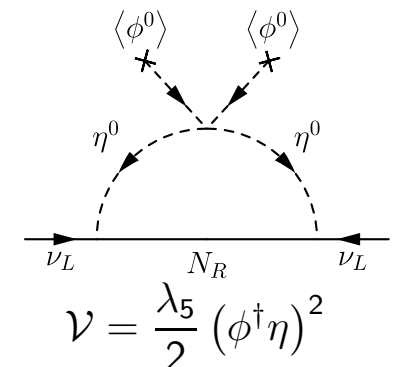


## ❖ Ma model

- ◆ R neutrinos are odd under  $Z_2$
- ◆ Inert doublet scalar
- ◆ Both can be dark matter candidates

	$SU(2)_L$	$U(1)_Y$	$Z_2$
$N_i$	1	0	-1
$\eta$	2	1/2	-1

· neutrino mass (1-loop level)  
DM candidates ( $N_1$  or  $\eta^0$ )



❖ The general Zee model: both doublets have Yukawa: the Yukawa couplings matrix cannot be diagonalized

❖ Neutrino mass matrix

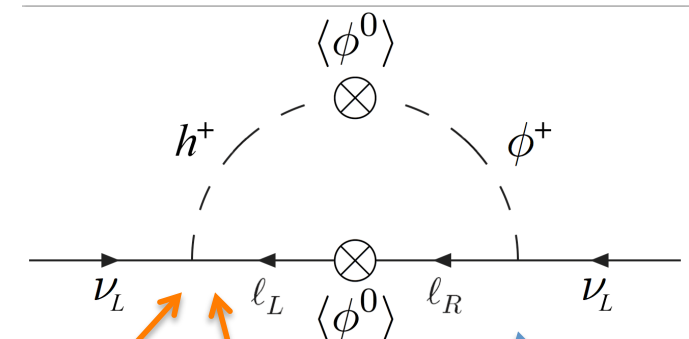
$$M_\nu = \kappa \left( \hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

❖ There are non-zero diagonal elements

❖ But tree level FCNC: Wolfenstein Suggest a  $Z_2$  to prevent the second Yukawa

❖ Mass matrix with vanishing diagonal elements

❖ Phases of  $f_{ab}$  are absorbed to



$$+ 2f_{e\mu} [\overline{\nu_{eL}} (\mu_L)^c - \bar{e}_L (\nu_{\mu L})^c] h^- + 2f_{e\tau} [\overline{\nu_{eL}} (\tau_L)^c - \bar{e}_L (\nu_{\tau L})^c] h^- + 2f_{\mu\tau} [\overline{\nu_{\mu L}} (\tau_L)^c - \bar{\mu}_L (\nu_{\tau L})^c] h^- + \mu (\Phi_1^+ \Phi_2^0 - \Phi_1^0 \Phi_2^+) h^- + \text{h.c.},$$

$$\begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}$$

$$m_{ab} = f_{ab} (m_b^2 - m_a^2) \frac{\kappa v_2}{v_1} F(M_1^2, M_2^2), \quad F(M_1^2, M_2^2) = \frac{1}{16\pi^2} \frac{1}{M_1^2 - M_2^2} \ln \frac{M_1^2}{M_2^2}$$

couplings  $f_{ab}$  are anti-symmetric

- ❖ Zee-Wolfenstein model was ruled out by data
- ❖ Even when  $f_{ab}$  are complex, the mass matrix predicts bimaximal mixing, thus is not compatible with  $\theta_{12} = 33.5^\circ \begin{matrix} +0.8 \\ -0.7 \end{matrix} \begin{matrix} (+2.5) \\ (-2.1) \end{matrix}$
- ❖ Symmetric mass matrix  $U_\nu^T M_\nu U_\nu = D_\nu \equiv \text{diag}(m_1, m_2, m_3)$  Koide (2001)  
X.G. He (2004)  
 $H_\nu = M_\nu^\dagger M_\nu \quad U_\nu^\dagger H_\nu U_\nu = D_\nu^* D_\nu = \text{diag}(|m_1|^2, |m_2|^2, |m_3|^2)$
- ❖ Only solution of inverted ordering is possible, but it give large  $\sin^2 2\theta_{solar} < \approx 1$ , far from the current data
- ❖ Babu and Julio imposed a family-dependent  $Z_4$  symmetry acting on the leptons
 

$\begin{pmatrix} X & a & b \\ a & Y & c \\ b & c & \varepsilon \end{pmatrix}$	$L_i : (-i, i, i);$	$e_i^c : (-i, -i, -i);$	<b>Babu, Lulio (2014)</b>
	$H_1 : +1;$	$H_2 : -1;$	$h^+ : -1.$
- ❖ Non-zero diagonal but avoid FCNC, somehow save Zee model



# Extension of Zee Model without Flavor Sym?

❖ Can we extended the Zee model without imposing a flavor symmetry?

❖ 2-loop corrections can generate diagonal masses

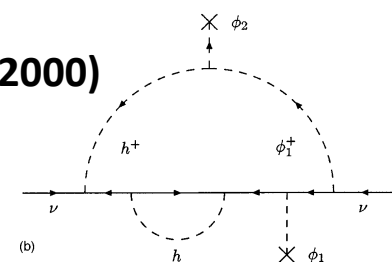
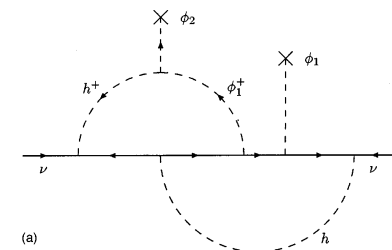
❖ To be compatible with data,  $|f_{e\mu}|m_\mu^2 \simeq |f_{e\tau}|m_\tau^2 \gg |f_{\mu\tau}|m_\tau^2$

❖ 2-loop correction  $(m_\nu^{(2)})_{ab} = \gamma \sum_{c,d} f_{ac} f_{cd}^* f_{db} (m_c^2 - m_d^2) = \gamma (f[m^2, f^*]f)_{ab}$

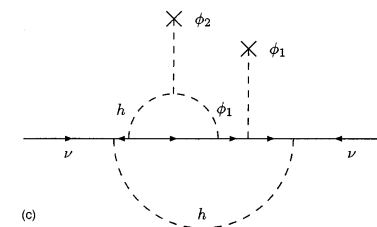
❖ Correction to  $\sin^2 2\theta_{solar}$ :

$$\left| \frac{M_{\nu 11}}{M_{\nu 23}} \right| \sim \frac{|f_{e\mu}| |f_{\mu\tau}| |f_{\tau e}|}{16\pi^2 |f_{\mu\tau}|} \simeq \frac{|f_{e\mu}|^2}{16\pi^2} \left( \frac{m_\mu}{m_\tau} \right)^2 < 10^{-5}$$

❖ Even 2-loop corrections can't help to save the model



Chang, Zee (2000)



# A New Extended Zee Model

- ❖ Type-I 2HDM with 2 extra charged and neutral singlets as the 2 Higgs doublets

$$\Phi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix} = \begin{pmatrix} \varphi_1^+ \\ \frac{(v_1 + \eta_1 + i\phi_1)}{\sqrt{2}} \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} \varphi_2^+ \\ \frac{(v_2 + \eta_2 + i\phi_2)}{\sqrt{2}} \end{pmatrix} \quad \chi_1^0 = \frac{\chi_1^+}{\sqrt{2}} \quad \chi_2^0 = \frac{\chi_2^+}{\sqrt{2}}$$

$$\mathcal{L}_{H,kin} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) + D_\mu^\chi \chi_1^- D^{\chi,\mu} \chi_1^+ + \partial_\mu (\chi_1^0)^* \partial^\mu \chi_1^0 + D_\mu^\chi \chi_2^- D^{\chi,\mu} \chi_2^+ + \partial_\mu (\chi_2^0)^* \partial^\mu \chi_2^0$$

- ❖ Both doublets & neutral singlets can get VEV, and no mixing of  $\chi_1^\pm, \chi_2^\pm$

$$\tan \beta = v_2/v_1 \quad \alpha = u_2/u_1 \quad \tilde{\chi} = \cos \alpha \chi_1^0 + \sin \alpha \chi_2^0 \quad \rho = \sin \alpha \chi_1^0 + \cos \alpha \chi_2^0$$

- ❖ Symmetry

	$l_L$	$l_R$	$Q_L$	$u_R$	$d_R$	$\Phi_1$	$\Phi_2$	$\chi_1^\pm$	$\chi_2^\pm$	$\tilde{\chi}$	$\rho$
$Z_4$	$g_3(-1)$	$g_3(-1)$	$g_3(-1)$	$g_3(-1)$	$g_3(-1)$	$g_2(+1)$	$e(-2)$	$g_2(+1)$	$g_2(+1)$	$g_2(+1)$	$g_2(+1)$
$SU(2)_L$	2	1	2	1	1	2	2	1	1	1	1
Lepton $L$	1	1	0	0	0	0	0	$\pm 2$	$\pm 2$	0	0
$Z_2$						1	1	1	1	1	-1

- ❖  $\mathcal{L}_{lept} = f_{ab}^1 L_{iL}^{aT} C L_{jL}^b \epsilon^{ij} \chi_1^+ + f_{ab}^1 L_{iL}^{b\dagger} C L_{jL}^{a*} \epsilon^{ij} \chi_1^- + f_{ab}^2 L_{iL}^{aT} C L_{jL}^b \epsilon^{ij} \chi_2^+ + f_{ab}^2 L_{iL}^{b\dagger} C L_{jL}^{a*} \epsilon^{ij} \chi_2^-$

$$\mathcal{L}_{12\chi} = \kappa_1 (\Phi_2^{c\dagger} \Phi_1 - \Phi_1^{c\dagger} \Phi_2) \chi_1^- \tilde{\chi} + \kappa_2 (\Phi_2^{c\dagger} \Phi_1 - \Phi_1^{c\dagger} \Phi_2) \chi_2^- \tilde{\chi} + h.c.$$

$$= 2\kappa_1 u (\varphi_1^+ \varphi_2^0 - \varphi_2^+ \varphi_1^0) \chi_1^- + 2\kappa_2 u (\varphi_1^+ \varphi_2^0 - \varphi_2^+ \varphi_1^0) \chi_2^- + h.c. + \dots$$

# A New Extended Zee Model

- ❖ Mass Matrix is complex, but...

$$m_{ab} = \frac{(m_b^2 - m_a^2)}{16\pi^2} u \tan \beta [f_{ab}^1 \kappa_1 F(M_{11}^2, M_{12}^2) + f_{ab}^2 \kappa_2 F(M_{21}^2, M_{22}^2)]$$

- ❖ 2-loop correction to the diagonal

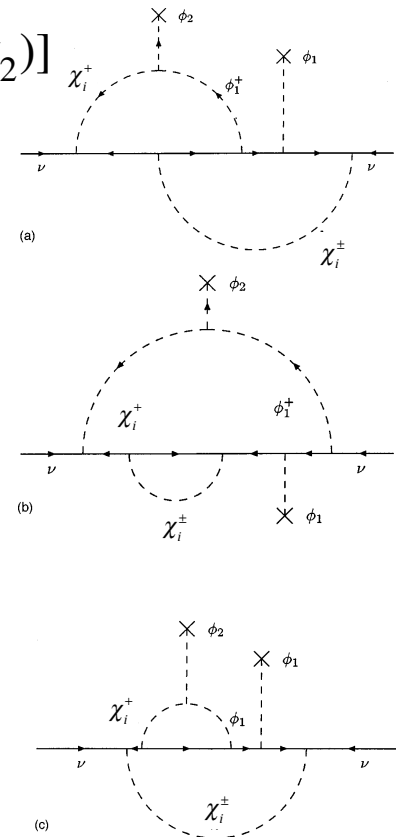
$$m_{ab}^{(2)} = \frac{O(1)}{(16\pi^2)^2} u \sum_{i,j=1,2} \sum_{c,d=e,\mu,\tau} [f_{ac}^i f_{cd}^{j*} f_{db}^j (m_c^2 - m_d^2) \kappa_i F(M_{\chi_1}^2, M_{\chi_2}^2)]$$

- ❖ The ratio contributing to  $\sin^2 2\theta_{solar}$  becomes

$$\frac{|M_{\nu 11}|}{|M_{\nu 23}|} = \frac{1}{16\pi^2} \frac{|f_{\mu\tau}^{1*} f_{e\tau}^1 + f_{\mu\tau}^{2*} f_{e\tau}^2|}{|f_{\mu\tau}^1 \kappa_1 + f_{\mu\tau}^2 \kappa_2|} \frac{|f_{e\mu}^1 \kappa_1 + f_{e\mu}^2 \kappa_2|}{|f_{e\tau}^1 + f_{e\mu}^2|} \neq \frac{1}{16\pi^2} \frac{m_{\tau^2}}{m_{\mu^2}} |f_{e\tau}^1 + f_{e\mu}^2|$$

- ❖ There's room to make our prediction consistent with the solar mixing angle and there's CP phase!

- ❖ Fit the parameters  $f_{ab}^i, \kappa_i$  and constraint from FCNF and  $0\nu\beta\beta$  decay



# A New Extended Zee Model

- ❖ There's also scalar dark matter candidate!

$$\rho = \sin\alpha\chi_1^0 + \cos\alpha\chi_2^0, \quad \langle \rho \rangle = 0$$

- ❖ It's constrained by the neutrino part directly
- ❖ But  $\rho\rho \rightarrow \chi^+\chi^- \rightarrow l^+l^- \nu\nu$  can be sensitive to the neutrino parameters
- ❖ Maybe we can also consider a  $SU_d(2)$  extension of Ma model

- ❖ We will fit the parameters to constrain our model with the current neutrino data
- ❖ Interesting LHC phenomenology, e.g. 2 TeV heavy resonance
- ❖ Relic density for the dark matter candidate



Thank you!