Neutrino Physics

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E. Witten (2000): for neutrino masses, the considerations have always been qualitative, and, despite some interesting attempts, there has never been a convincing quantitative model of the neutrino masses.

Part A: Neutrino concept and beta decays

Part B: Neutrino masses from seesaws

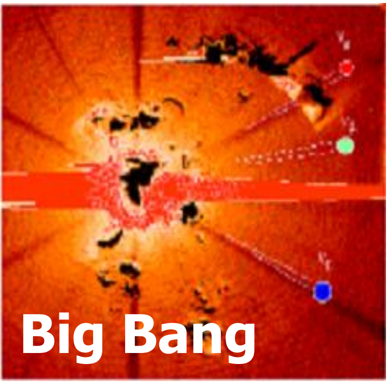
Part C: Flavor mixing & oscillations

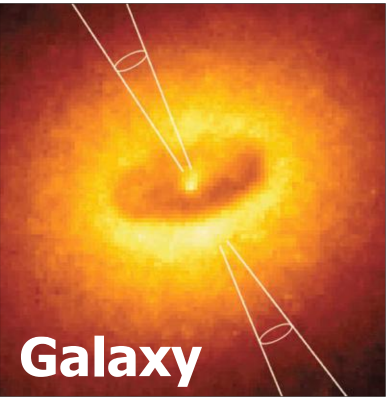
Part D: Experimental discoveries

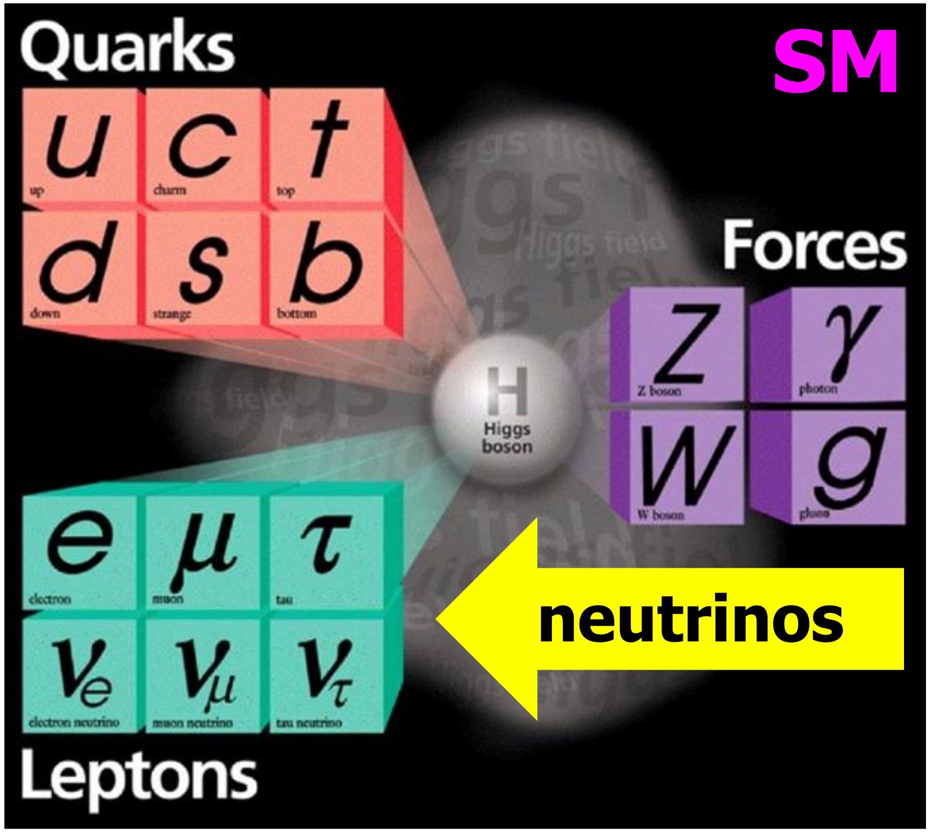
ν ν ν

@Lectures at the TeV summer school, Shandong University, 13/8/15

Neutrinos everywhere

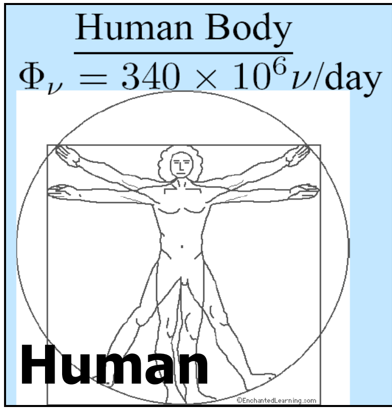


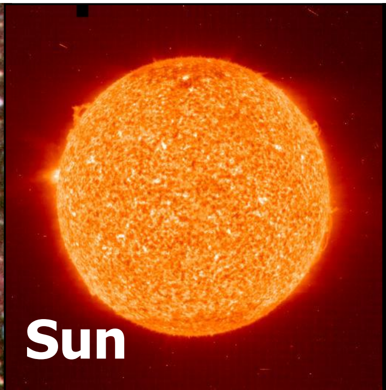




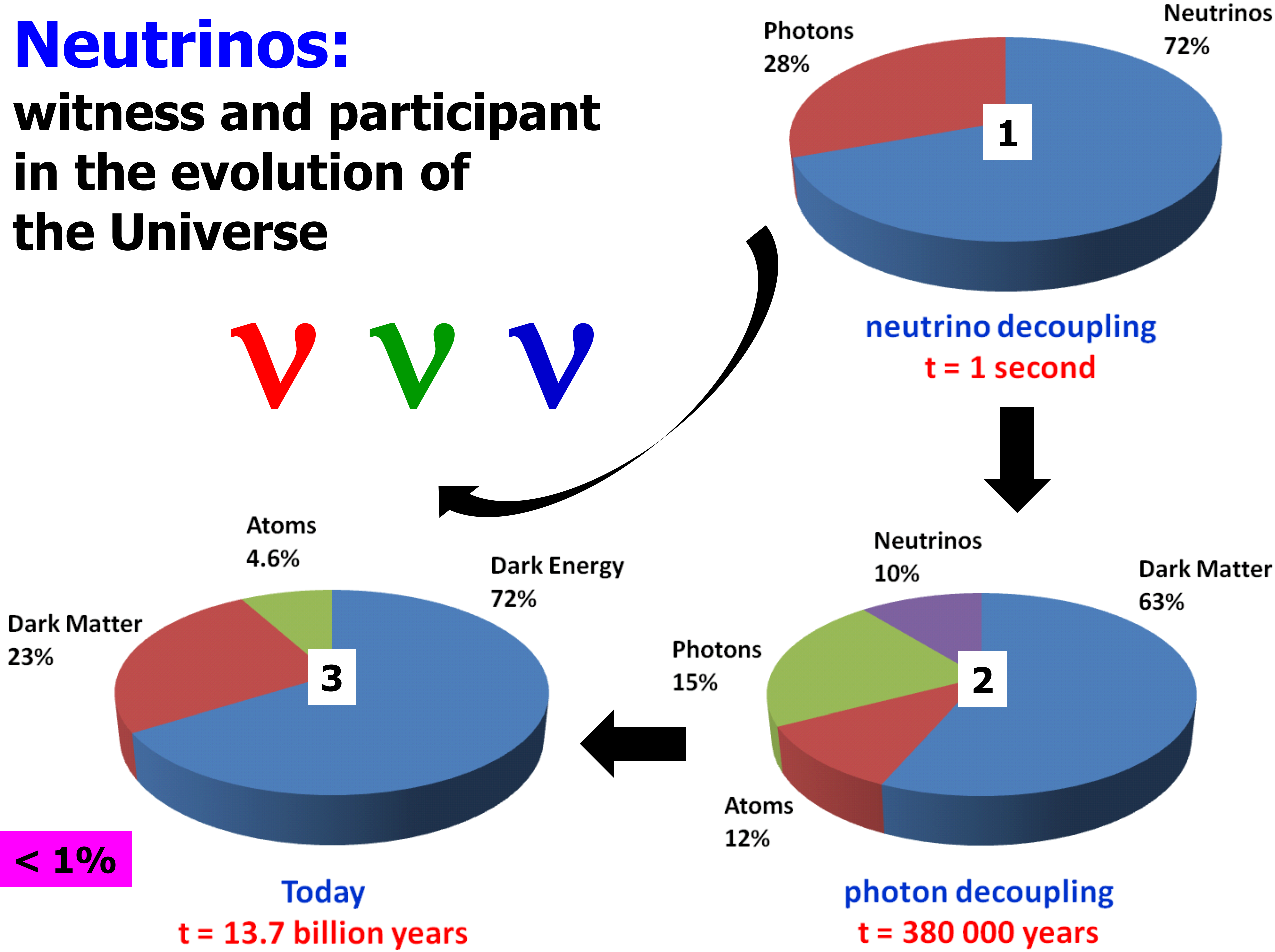
Properties:

charge = 0
spin = ½
mass = 0
speed = c





Neutrinos:
witness and participant in the evolution of the Universe



1998 —— 2015

We've learnt a lot from ν oscillations:

$$\boxed{\Delta m^2_{21}\ ,\ \left|\Delta m^2_{31}\right|\ ,\ \theta_{12}\ ,\ \theta_{13}\ ,\ \theta_{23}}$$

It's more exciting that the SM is incomplete, although the Higgs has been discovered.

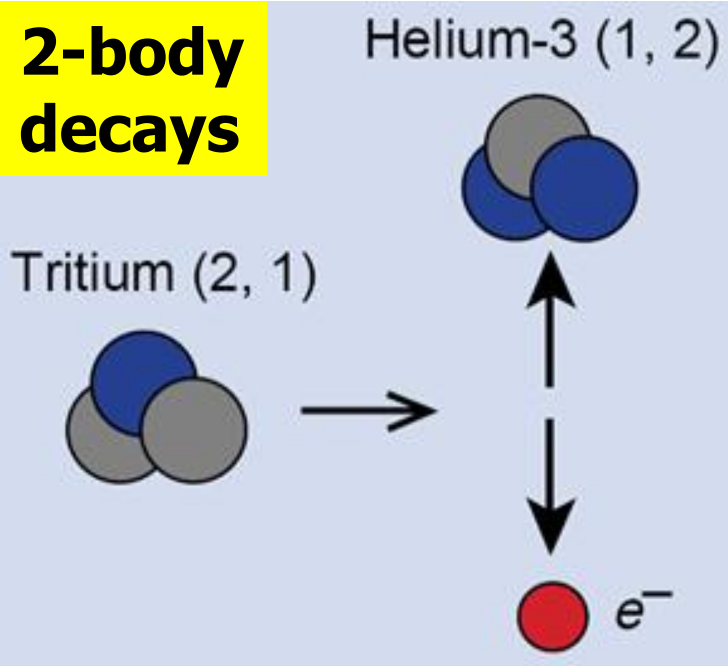
But a number of burning questions:

- ♣ **the Majorana nature?**
- ♣ **the absolute ν mass scale?**
- ♣ **the ν mass hierarchy?**
- ♣ **the octant of θ_{23}?**
- ♣ **the Dirac phase δ?**
- ♣ **the Majorana phases? ……**

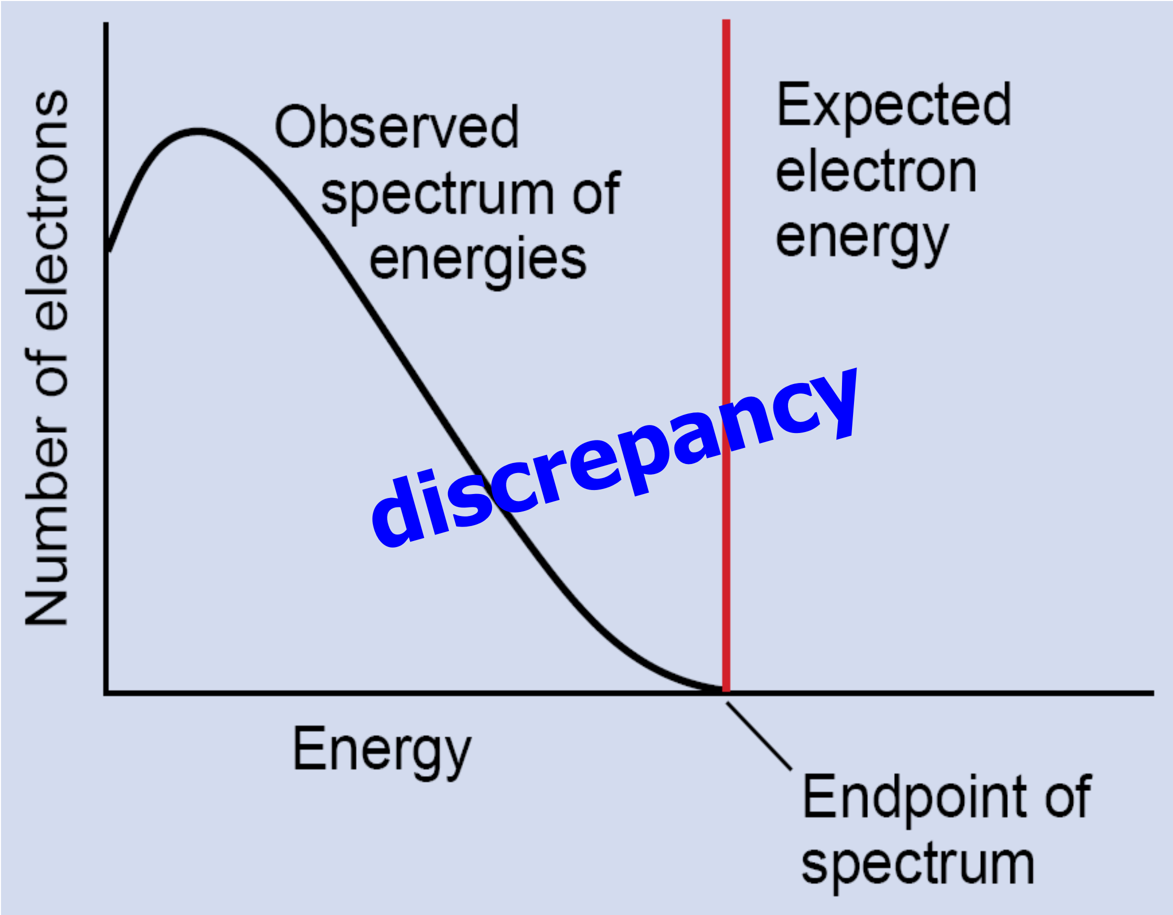
There are many other open questions about ν's in particular physics, cosmology, astrophysics …..

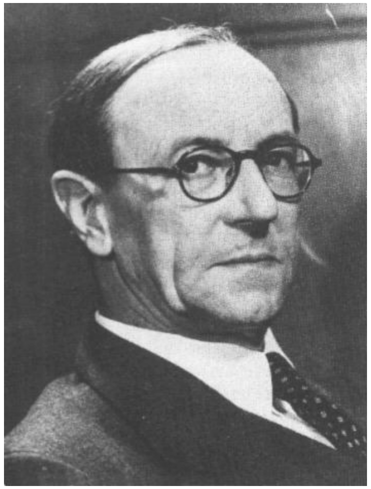
Part A

Beta decays in 1930



Energy crisis = New physics ?





J. Chadwick 1914/C. Ellis 1920-1927

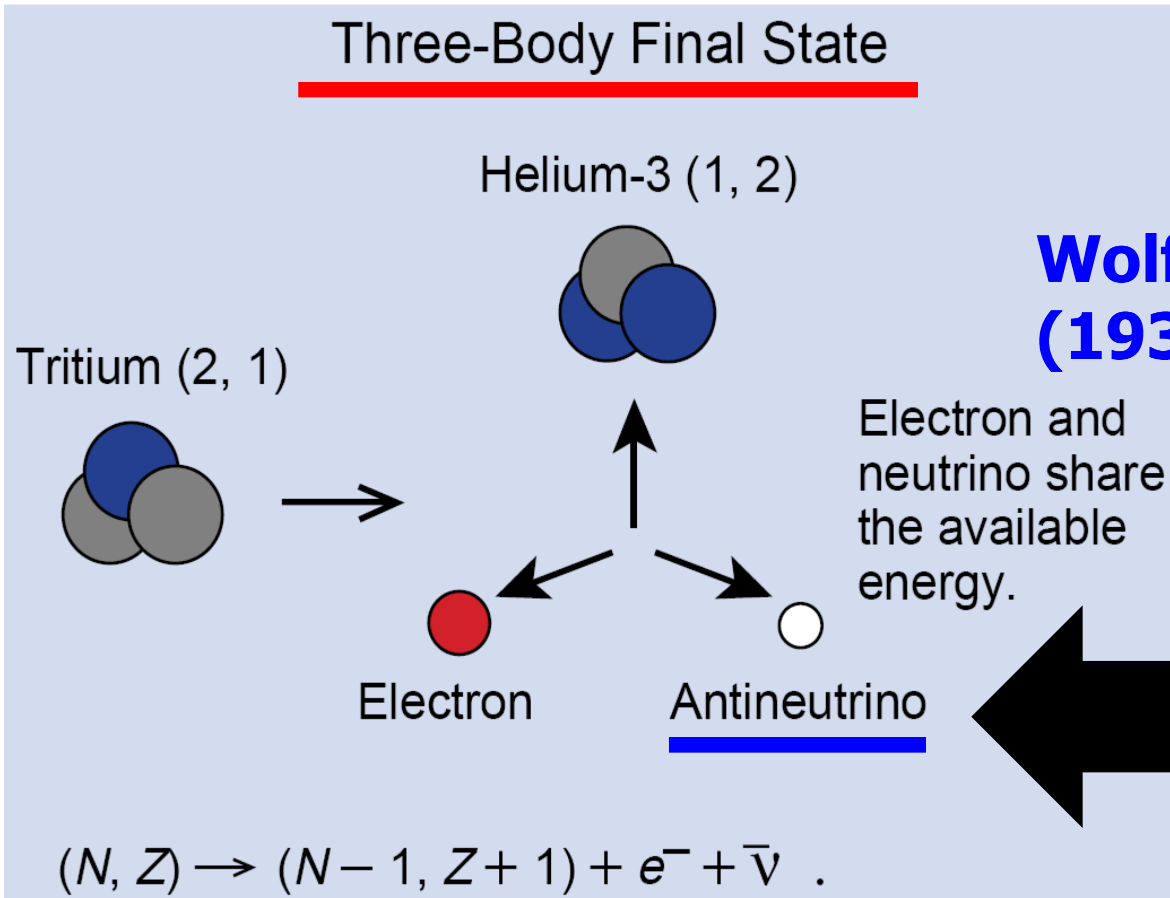
What to do?

Two ways out?

Niels Bohr

- ♣ giving up sth
- ♣ adding in sth

Wolfgang Pauli (1930)



Pauli put forward this idea in a letter instead of a paper.....

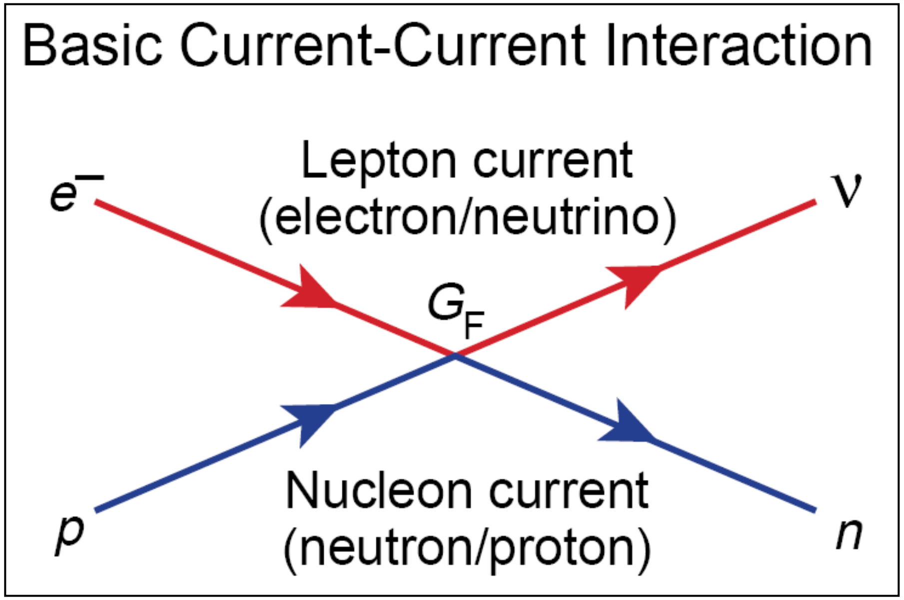
Fermi's theory

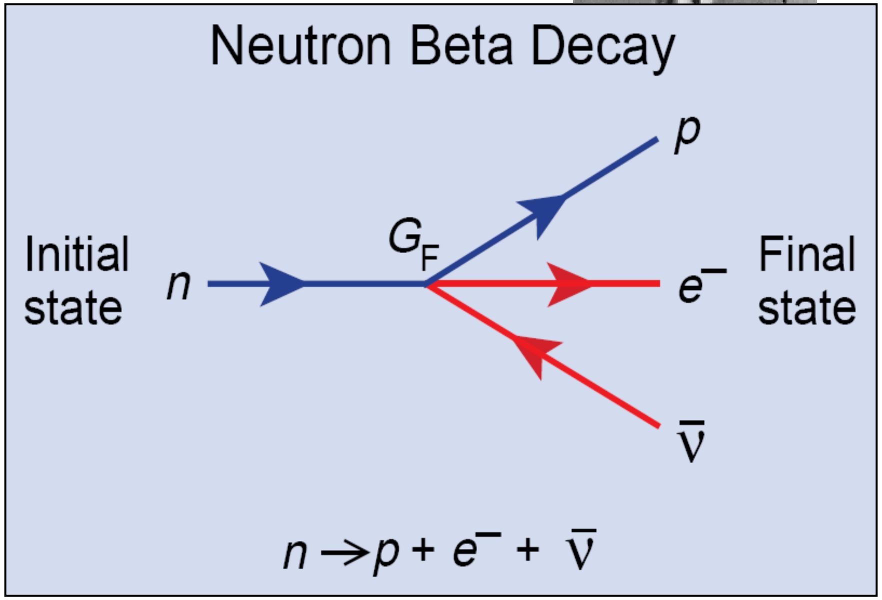
Enrico Fermi assumed a new force for β decay by combining 3 new concepts:

- ★ **Pauli's idea: neutrinos**
- ★ **Dirac's idea: creation of particles**
- ★ **Heisenberg's idea: isospin symmetry**

I will be remembered for this paper.

------ Fermi in Italian Alps, Christmas 1933



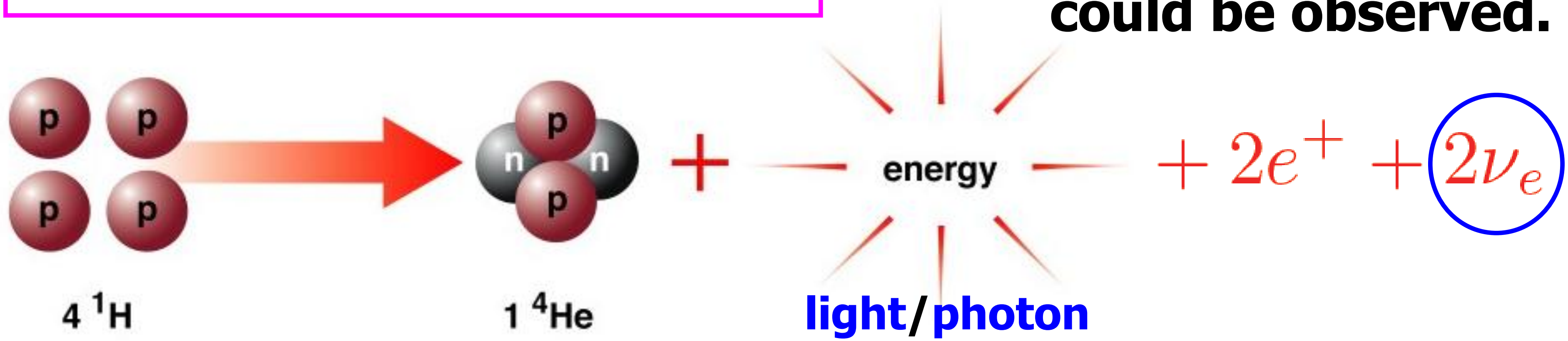


Part A

Why the sun shines?

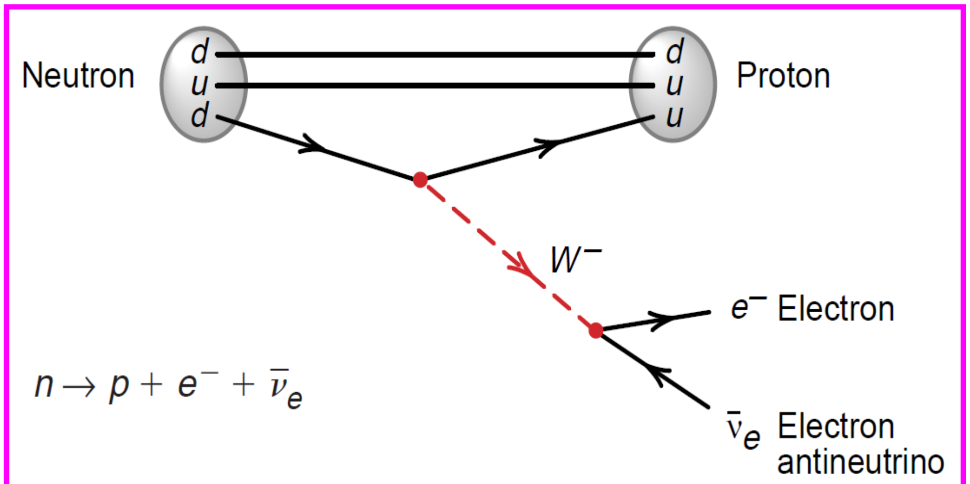
$$4p \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2\nu_e + 26.73\ \mathrm{MeV}$$

Only the neutrinos could be observed.

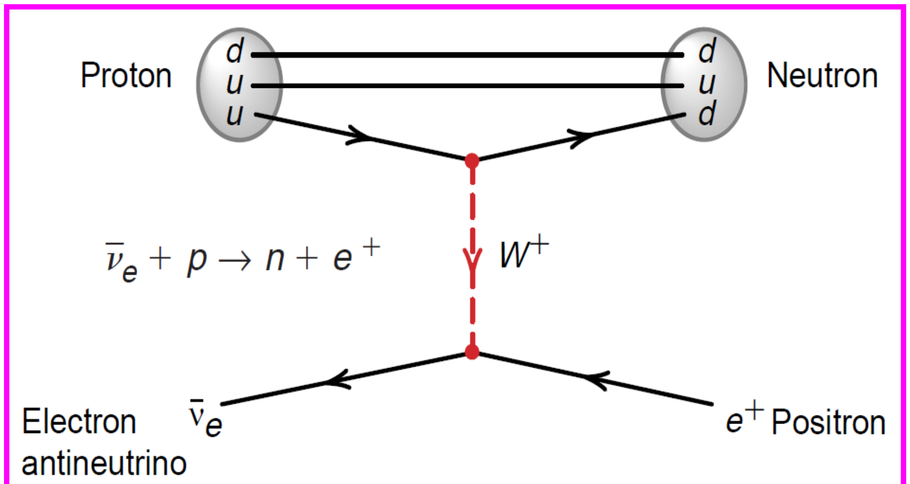


Hans Bethe (1939), George Gamow & Mario Schoenberg (1940, 1941)

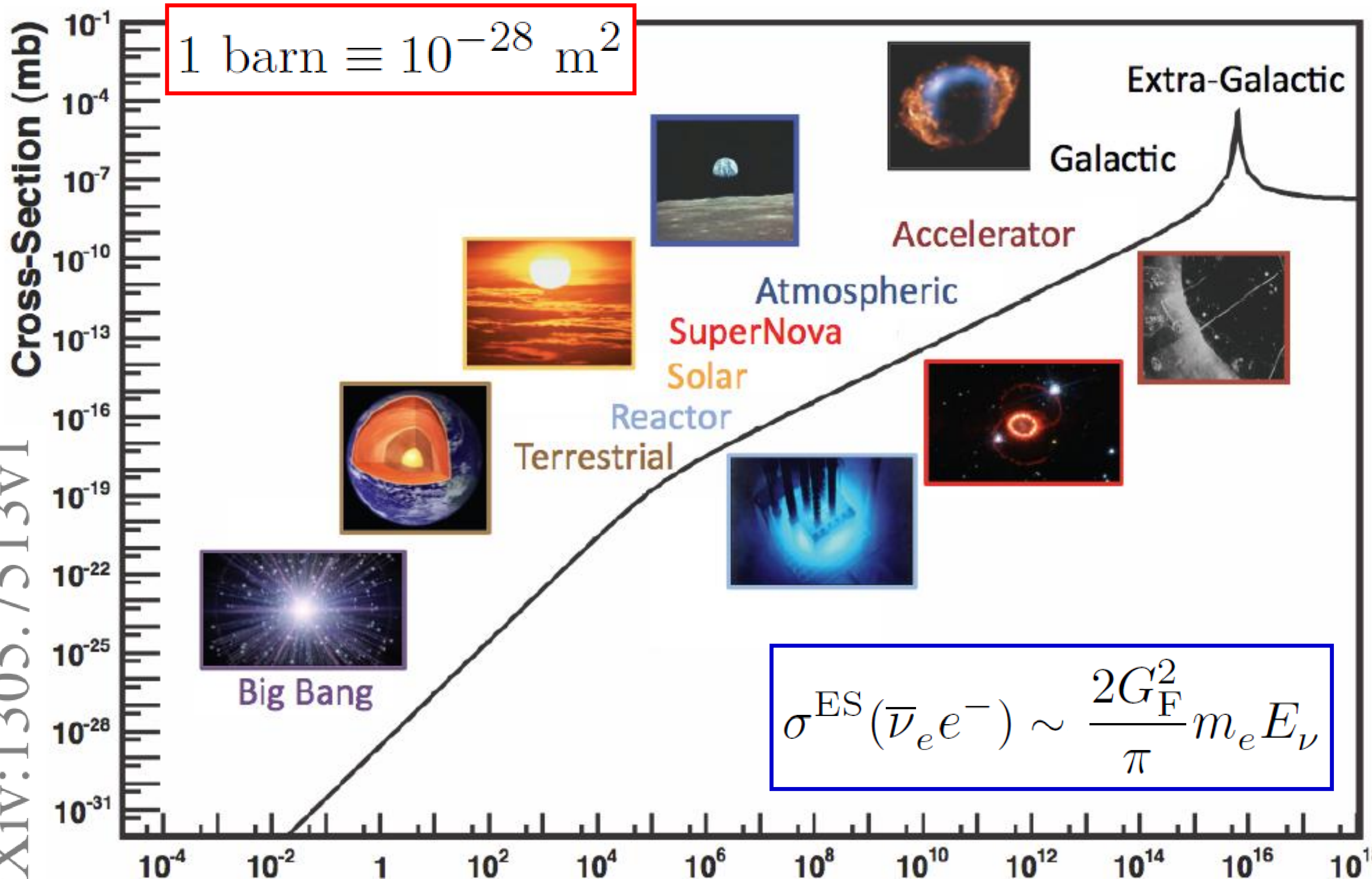
The beta decay



The inverse beta decay



Too shy to be seen?

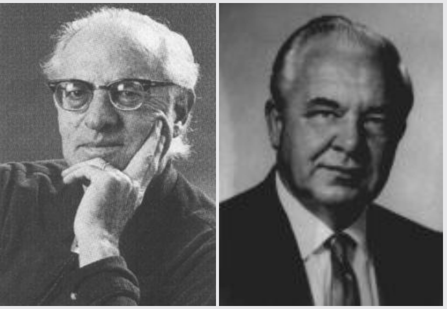


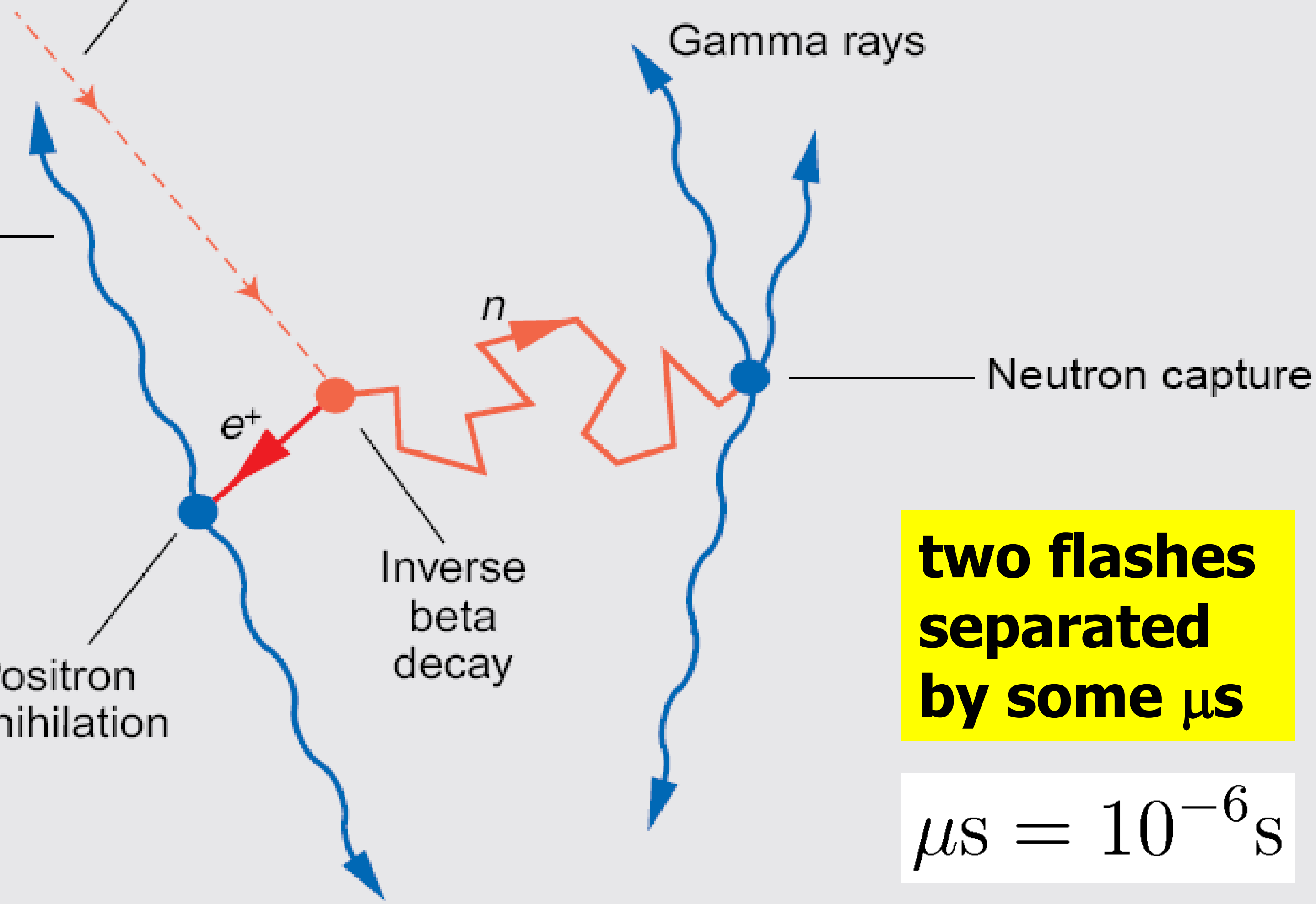
J.A. Formaggic and G.P. Zeller: a review.

Neutrinos in 1956

F. Reines and C. Cowan detected reactor antineutrinos via

$$\overline{\nu}_e + p \to e^+ + n$$





two flashes separated by some μs

$$\mu\mathrm{s} = 10^{-6}\mathrm{s}$$

Many things oscillate

1956: Discovery of electron antineutrino (C.L. Cowan *et al*)

1957: Postulation of neutrino-antineutrino oscillation (B. Pontecorvo)

1962: Discovery of muon neutrino (G. Danby *et al*)

1962: Postulation of neutrino conversion (Z. Maki *et al*)

1968: Discovery of solar neutrino oscillation (R. Davis *et al*)

1987: Discovery of supernova neutrinos (K. Hirata *et al*)

2000: Discovery of tau neutrino (K. Kodama *et al*)

1998

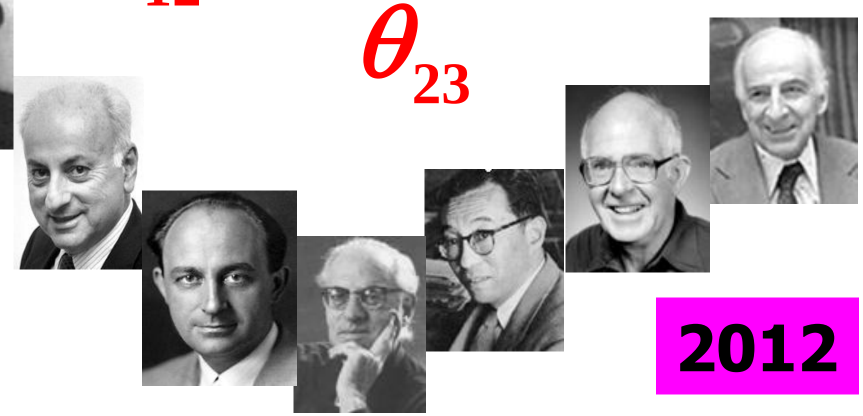
$|\Delta m^2_{31}|$

Δm^2_{21}

θ_{12}

θ_{23}

θ_{13}



2012

What is mass?

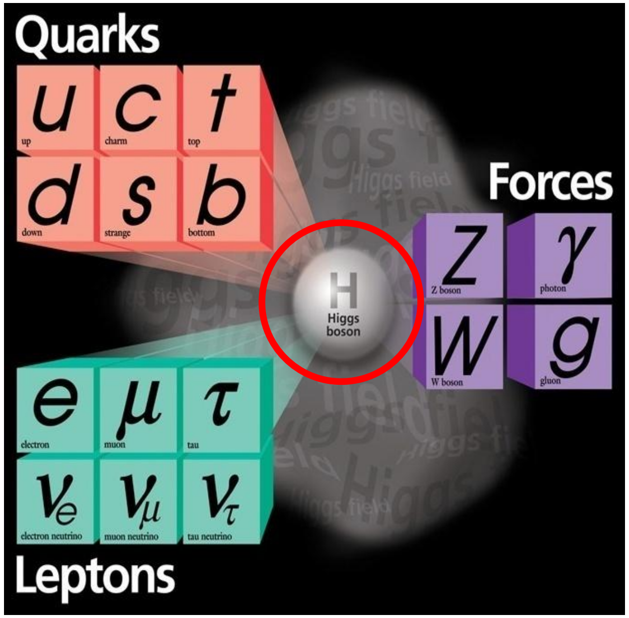
Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of W / Z and fermion masses in the SM.

$$L_{\text{SM}} = L(f,G) + \underline{L(f,H)} + \underline{L(G,H)} + L(G) - V(H)$$

All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.

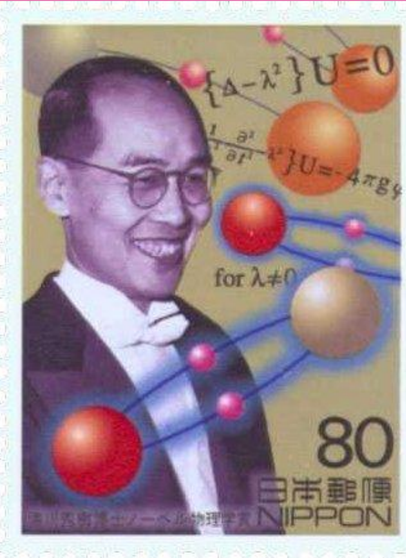


Higgs: Yukawa interaction

force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	gluon/π	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	photon	$=0$
weak	10^{-6}	$10^{-18}\,\mathrm{m}$	W/Z/H	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	graviton	$=0$

Yukawa relation for the mediator's mass *M* and the force's range *R*:

$$\boxed{M \approx \frac{200\,\mathrm{MeV}\times 10^{-15}\,\mathrm{m}}{R}}$$



$$\boxed{L_{\mathrm{SM}} = \underline{L(f,G) + L(f,H)} + L(G,H) + L(G) - V(H)}$$

Fermion masses, flavor mixing, CP violation

In the SM

All ν's are massless because the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance:

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: the neutrinos are all tooooooooo light and apparentlyleft-handed;
NO: no fundamental symmetry/conservation law to forbid ν's masses.

Possible way out: 1) the particle content can be enlarged; 2) the renormalizability can be abandoned.

Beyond the SM (1)

Way 1: to relax the requirement of renormalizability (S. Weinberg 79)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{\mathcal{L}_{\rm d=5}}{\Lambda} + \frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} + \cdots$$

Given the standard-model fields, the lowest-dimension operators that violate lepton and baryon numbers at the tree level are

$$\frac{1}{M} HHLL$$

neutrino mass

Seesaw: $m_{1,2,3} \sim \langle H \rangle^2 / M$

$$m_{1,2,3} < 1\,\text{eV} \;\Rightarrow\; M > 10^{13}\,\text{GeV}$$

$$\frac{1}{M^2} QQQL$$

proton decay

Example : $p \to \pi^0 + e^+$

$$\tau_p > 10^{33}\,\text{years} \;\Rightarrow\; M > 10^{15}\,\text{GeV}$$

Neutrino masses and proton decays at the intensity frontier offer new windows onto physics at super-high energy scales.

Beyond the SM (2)

Way 2: to add **3 right-handed** neutrinos and demand the ***L*** symmetry.

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.}}$$

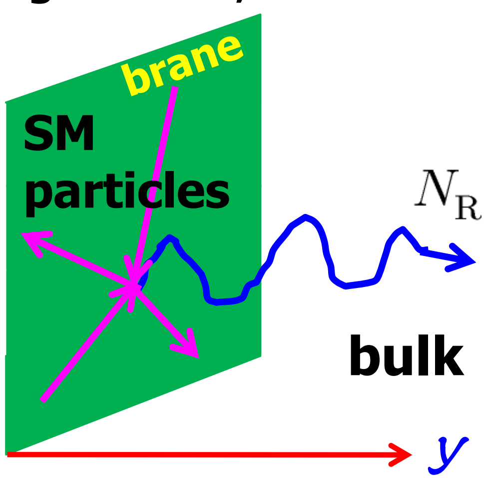
$$\boxed{M_l = Y_l v/\sqrt{2}\ , \quad M_\nu = Y_\nu v/\sqrt{2}}$$

But, such a pure Dirac mass term and lepton number conservation are not convincing, because non-perturbative quantum effects break both *L* and *B* symmetries and only preserve *B* – *L* (G. 't Hooft, 1976).

The flavor hierarchy puzzle:

$$\boxed{y_i/y_e = m_i/m_e \lesssim 0.5~{\rm eV}/0.5~{\rm MeV} \sim 10^{-6}}$$

A very speculative way out: **the smallness of Dirac masses is ascribed to the assumption that *N*_R have access to an extra spatial dimension (Dienes, Dudas, Gherghetta 98; Arkani-Hamed, Dimopoulos, Dvali, March-Russell 98) :**



The wavefunction of *N*_R spreads out over the extra dimension *y*, giving rise to a suppressed Yukawa interaction at *y* = 0.

$$\boxed{\left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=0} \sim \boxed{\frac{1}{\sqrt{L}}} \left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=L}}$$

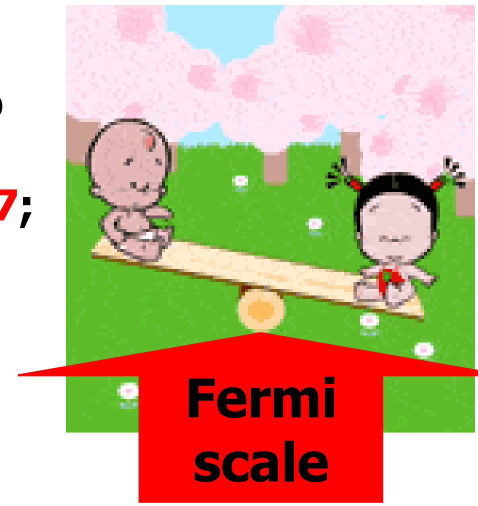
$\Lambda_{\rm String}/\Lambda_{\rm Planck} \sim 10^{-12}$

(e.g., King 08)

Beyond the SM (3)

Way 3: add **new heavy degrees of freedom** and allow the L violation.

Seesaw—A Footnote Idea:
H. Fritzsch, M. Gell-Mann,
P. Minkowski, PLB 59 (1975) 256



Type (1): SM + 3 right-handed neutrinos (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 80)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}}$$

Type (2): SM + 1 Higgs triplet (Konetschny, Kummer 77; Magg, Wetterich 80; Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80; Mohapatra, Senjanovic 80)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2} \overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}}$$

variations

Type (3): SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

combinations

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} {\rm Tr}\left(\overline{\Sigma} M_\Sigma \Sigma^c\right) + {\rm h.c.}}$$

Seesaw mechanisms

Weinberg operator: the unique dimension-five operator of ν-masses after integrating out heavy degrees of freedom.

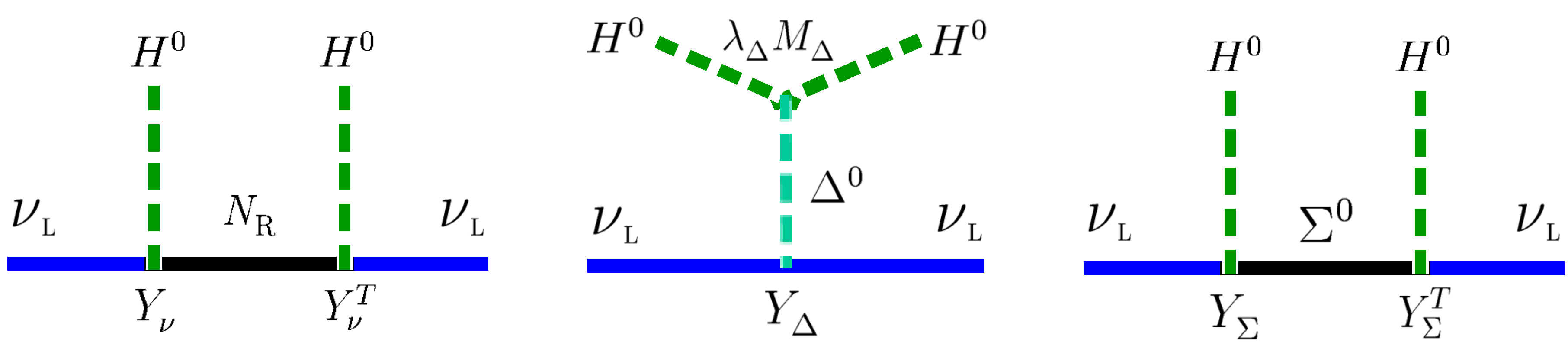
$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ -\frac{\lambda_\Delta}{M_\Delta} (Y_\Delta)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \end{cases}$$

$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & (\text{Type 1}) \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & (\text{Type 2}) \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & (\text{Type 3}) \end{cases}$$

After SSB, a Majorana neutrino mass term is

$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$$

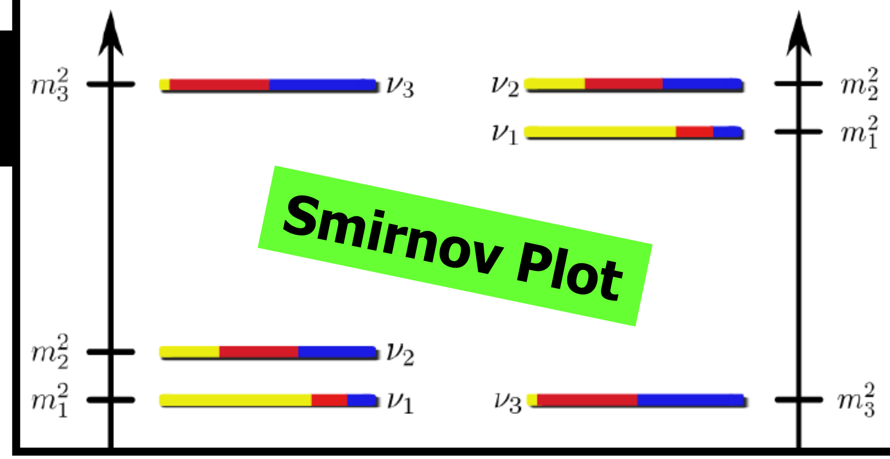
$$\langle \tilde{H} \rangle = v/\sqrt{2}$$

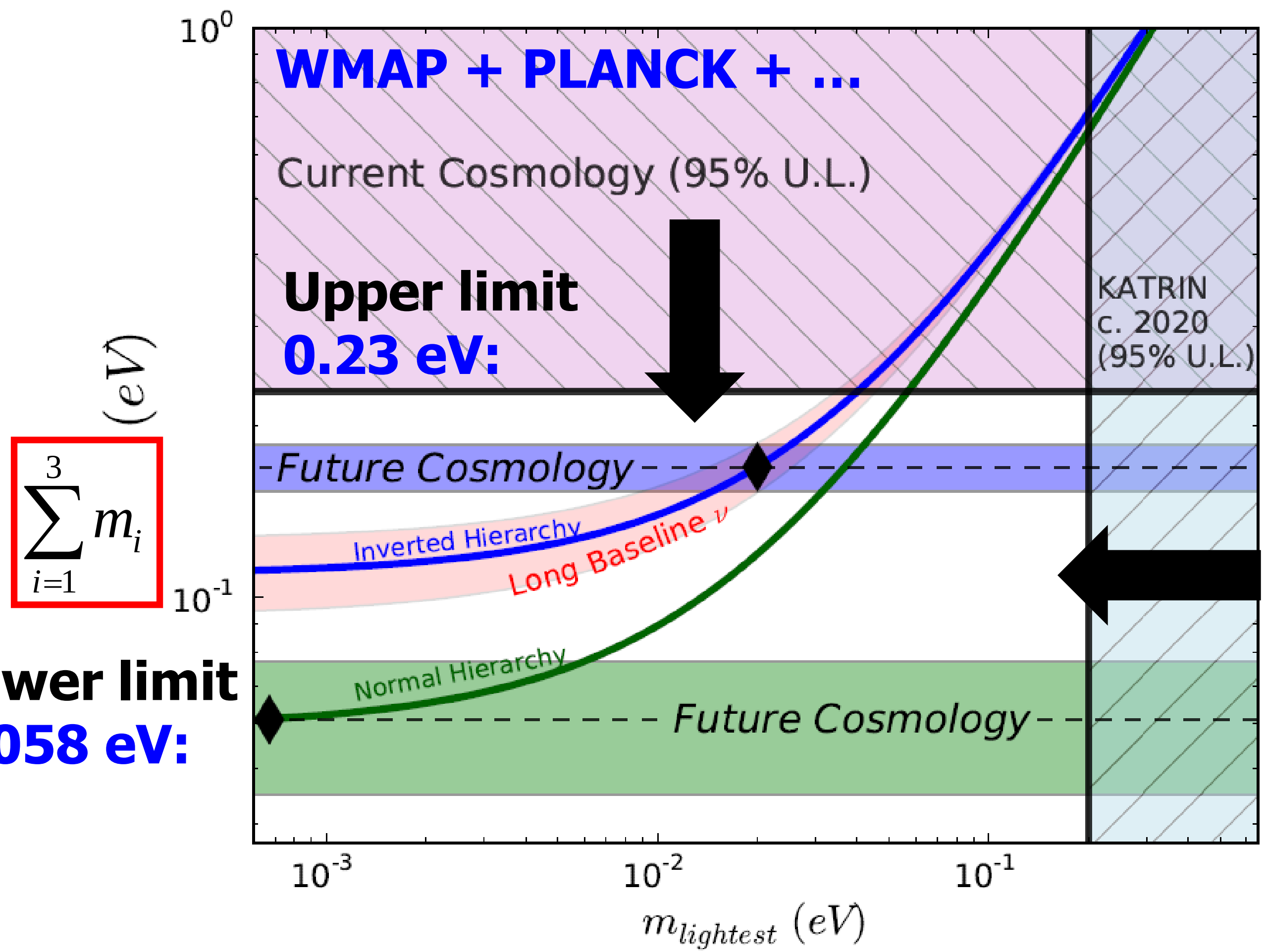


Light neutrino masses

Three ways to probe absolute ν mass:

- ★ **the β decay,**
- ★ **the 0νββ decay,**
- ★ **cosmology (CMB + LSS).**





mass scale ≤ O(0.1) eV

Why so tiny?

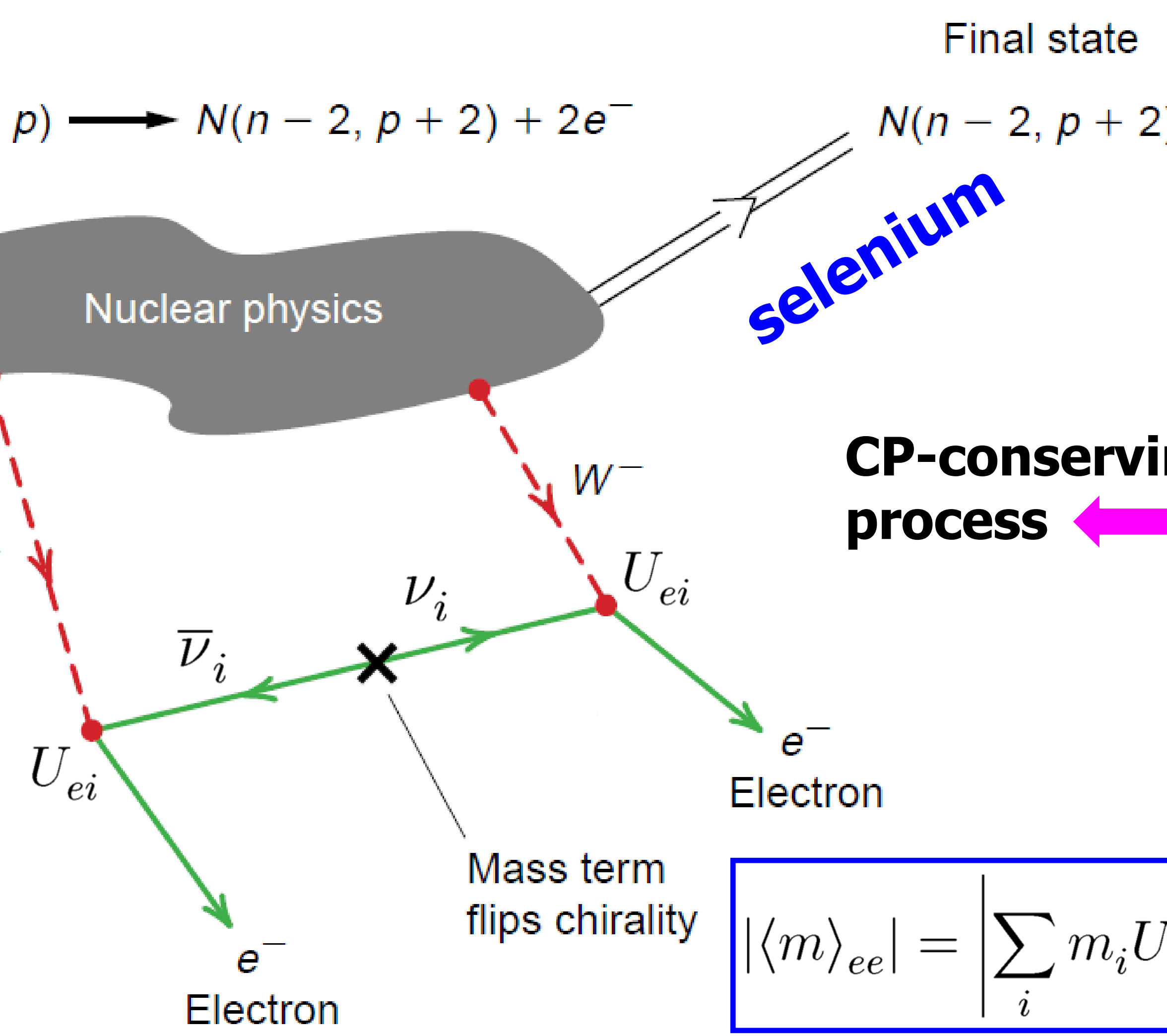
arXiv:1309.5383

Stage-4 CMB

$$\sigma\left(\sum m_\nu\right) = 16\ \text{meV}$$
$$\sigma\left(N_{\text{eff}}\right) = 0.020\,.$$

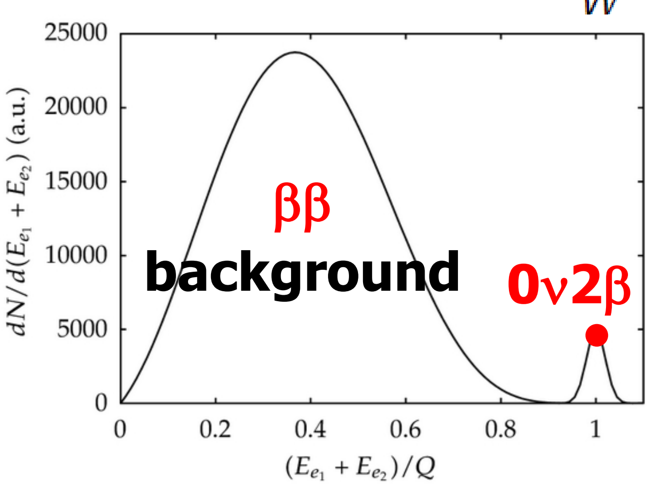
0ν2β decays

The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



Lepton number violation

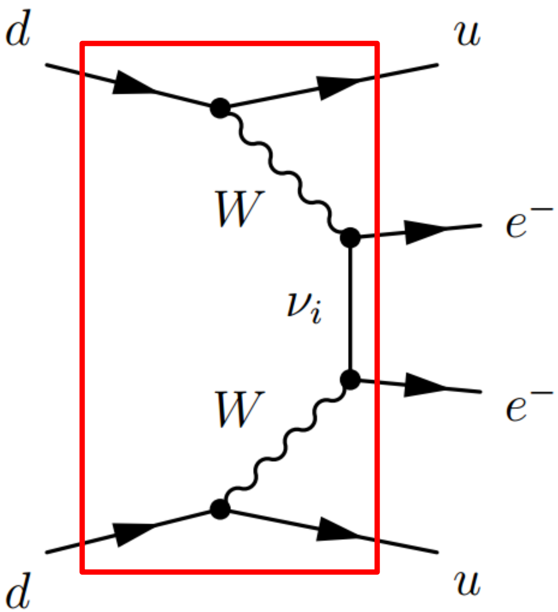
CP-conserving process

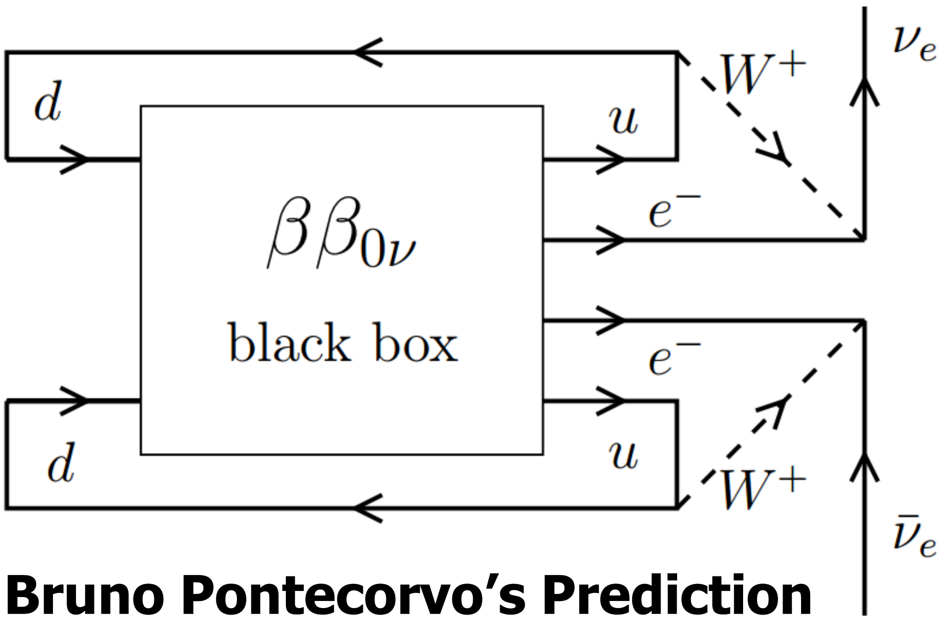


$$|\langle m\rangle_{ee}| = \left|\sum_i m_i U_{ei}^2\right|$$

Schechter-Valle theorem

THEOREM (1982): if a 0ν2β decay happens, there must be an effective Majorana mass term.





Bruno Pontecorvo's Prediction

That is why we want to see 0ν2β

Four-loop ν mass:

$$\boxed{\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})}$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

Ge + Xe

GERDA has killed the Heidelberg-Moscow's claim on 0ν2β.

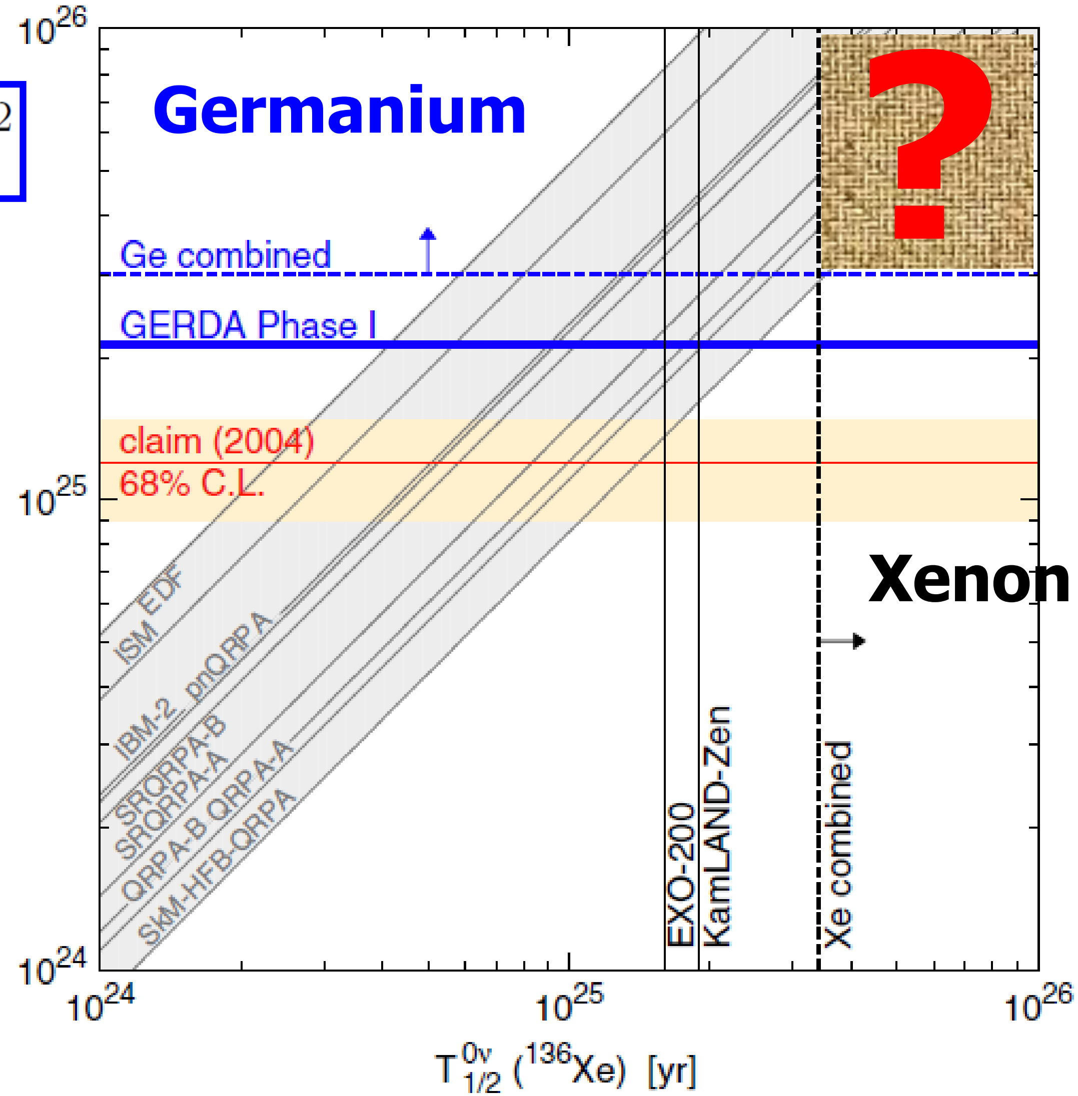
PRL **111**, 122503 (2013)

$$T_{1/2}^{0\nu} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|\langle m\rangle_{ee}\right|^{-2}$$

$$T_{1/2}^{0\nu} > 3.0 \times 10^{25} \text{ yr (90\% C.L.)}.$$

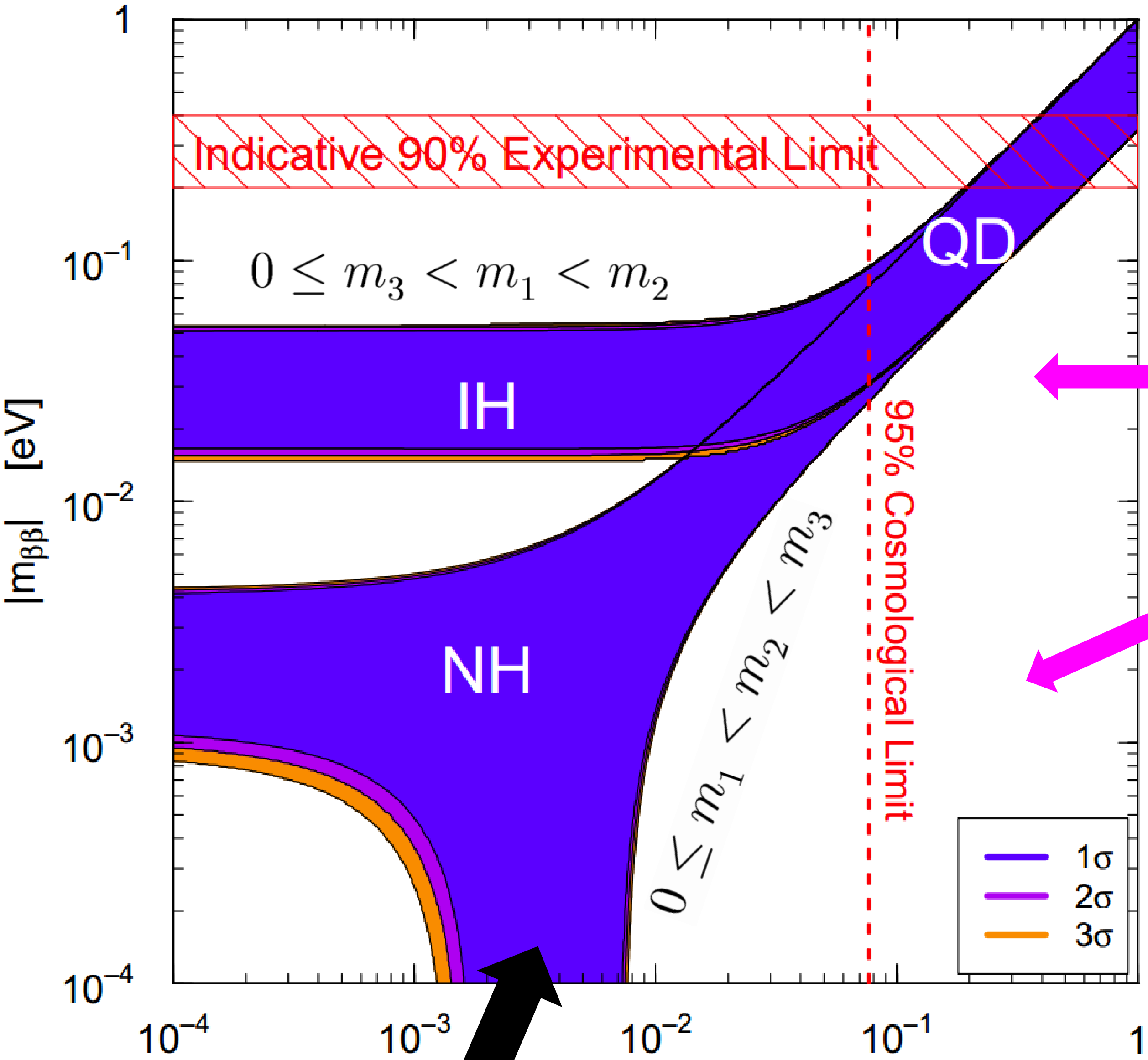
$$\left|\langle m\rangle_{ee}\right| < 0.2 \rightarrow 0.4 \text{ eV}$$

$$\left|\langle m\rangle_{ee}\right| = \left|\sum_i m_i U_{ei}^2\right|$$



Part B

Vissani graph



The dark well

The effective mass

$$|\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right|$$

Maury Goodman asks

An intelligent design?

Vanishing $0\nu2\beta$ mass?

Xing, hep-ph/0305195

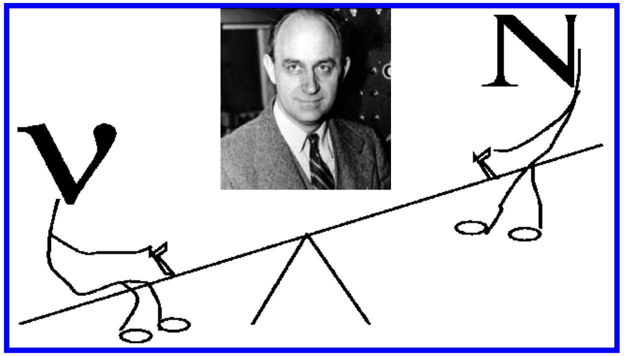
CP phases also matter

Burning Question:

how to interpret a discovery or a null result of $0\nu2\beta$?

The seesaw scale (1)

What is the energy scale at which the seesaw mechanism works and new physics come in?



Planck

GUT to unify strong, weak & electromagnetic forces

Conventional Seesaws: heavy degrees of freedom near GUT

This appears to be rather reasonable, since one often expects new physics to appear around a fundamental scale

Fermi

- Naturalness √
- Testability ×
- Uniqueness ×
- Hierarchy ×

The seesaw scale (2)

Planck scale $\Lambda \sim 10^{19}\,\mathrm{GeV}$

GUT scale? $\Lambda \sim 10^{16}\,\mathrm{GeV}$

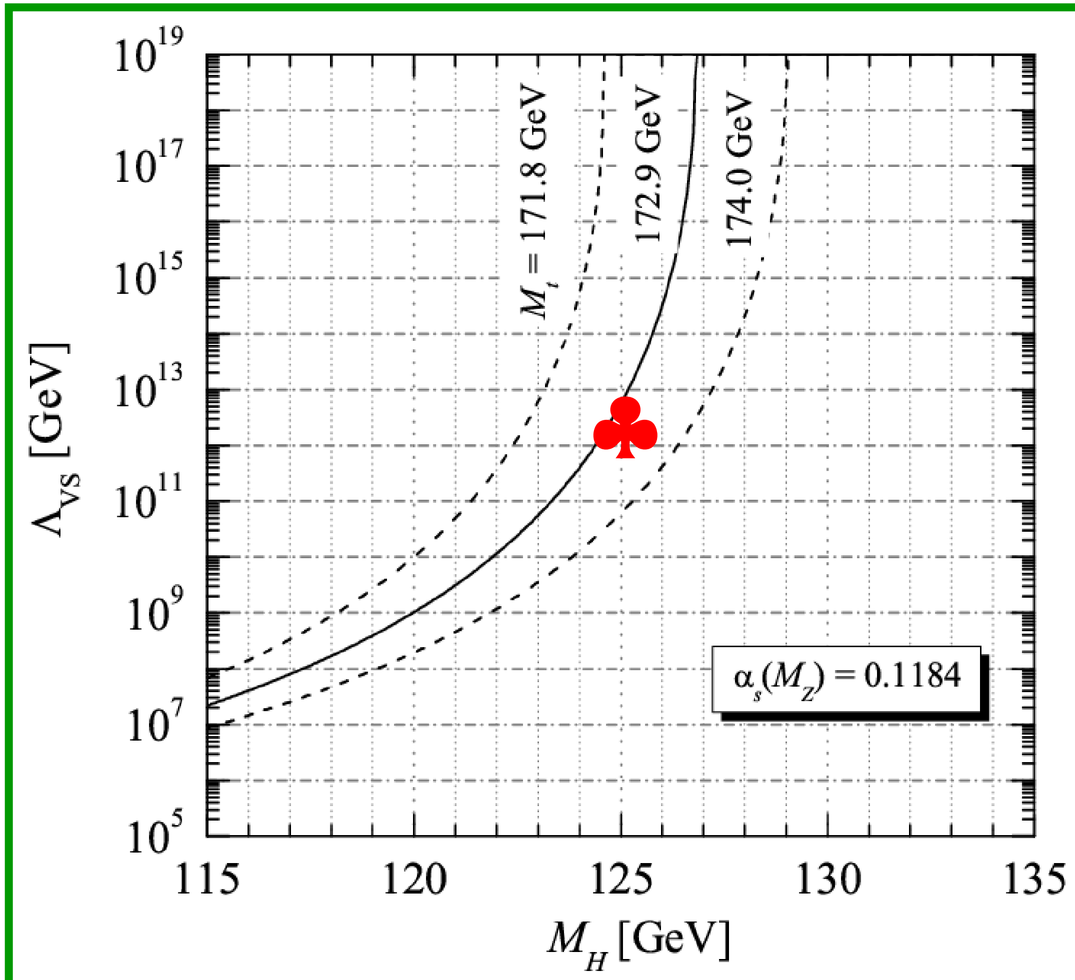
Seesaw scale? $\Lambda \sim 10^{12}\,\mathrm{GeV}$

TeV / SUSY? $\Lambda \sim 10^{3}\,\mathrm{GeV}$

Fermi scale $\Lambda \sim 10^{2}\,\mathrm{GeV}$

QCD scale $\Lambda \sim 10^{2}\,\mathrm{MeV}$

The SM vacuum stability for a light Higgs



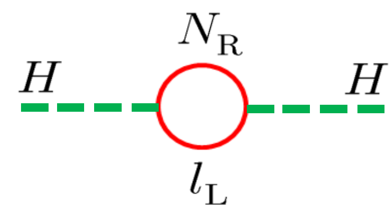
Seesaws could make life easier?

Elias-Miro et al., arXiv:1112.3022; Xing, Zhang, Zhou, arXiv:1112.3112; …….

New hierarchy problem

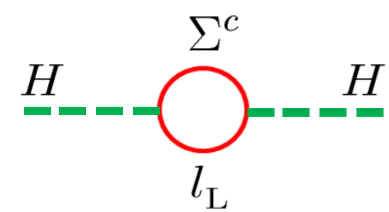
Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom induced in the seesaw mechanisms (Vissani 98; Casas et al 04; Abada et al 07)

Type 1:
$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$



Type 2:
$$\delta m_H^2 = \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right]$$

Type 3:
$$\delta m_H^2 = -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$



here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning

$$M_i \sim \left[\frac{(2\pi v)^2|\delta m_H^2|}{m_i}\right]^{1/3} \sim \underline{10^7 \mathrm{GeV}} \left[\frac{0.2\ \mathrm{eV}}{m_i}\right]^{1/3}\left[\frac{|\delta m_H^2|}{0.1\ \mathrm{TeV}^2}\right]^{1/3}$$

Possible way out: (1) Supersymmetric seesaw? (2) TeV-scale seesaw?

TeV neutrino physics?

to discover the SM Higgs boson **OK**

to verify Yukawa interactions **OK**

to pin down heavy seesaw particles

to single out a seesaw mechanism

to measure all low-energy effects

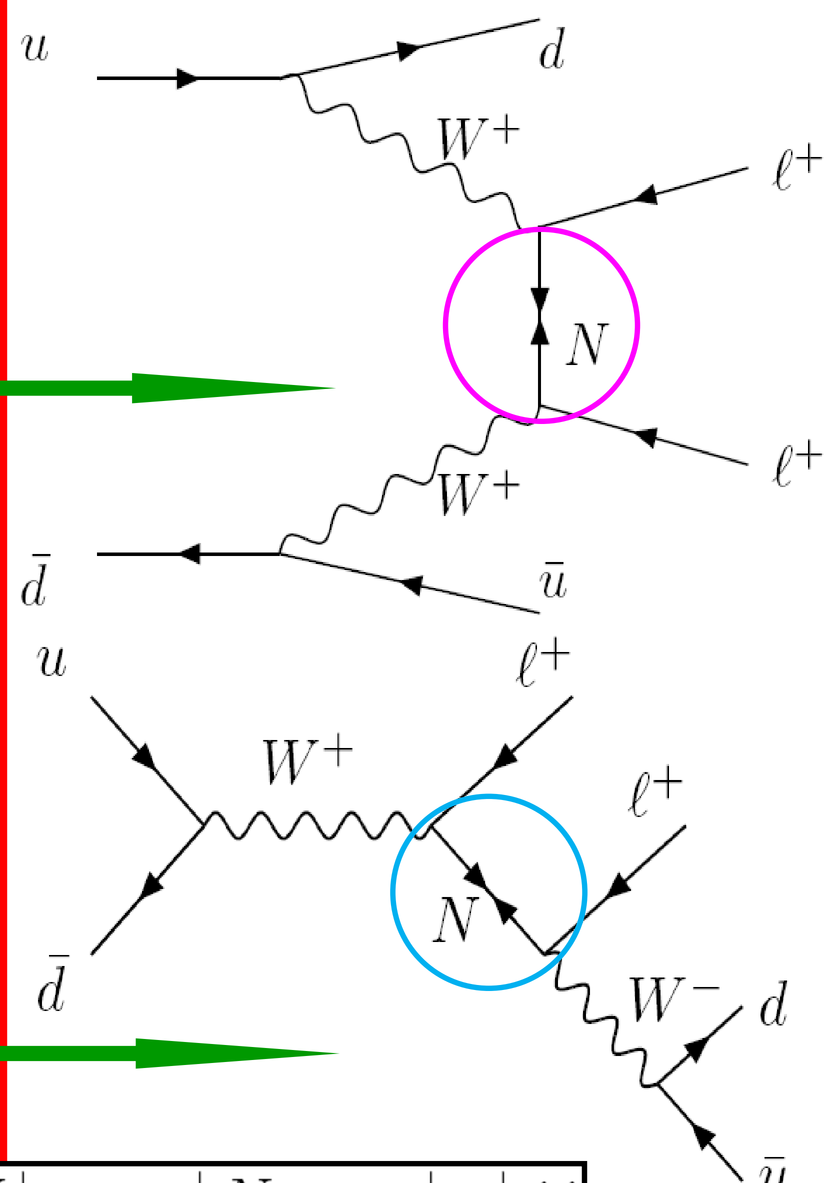
Let's **H**ave **C**offee!

Collider signature (1)

Lepton number violation: **like-sign dilepton events at hadron colliders, such as Tevatron (~2 TeV) and LHC (~14 TeV).**

T. Han, Z.G. Si, K. Wang, B. Zhang, ...

collider analogue to 0νββ decay



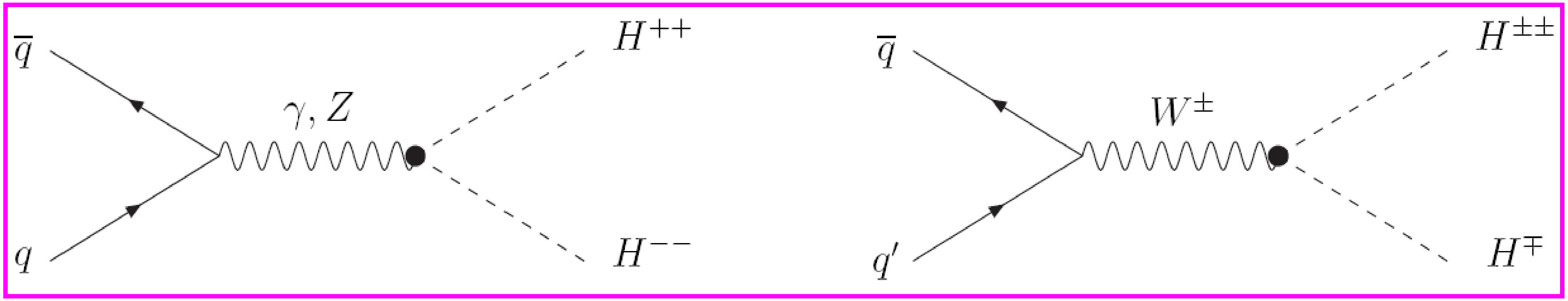
dominant channel

N **can be produced on resonance**

$$pp \to W^{\pm}W^{\pm} \to \mu^{\pm}\mu^{\pm}jj \text{ and } pp \to W^{\pm} \to \mu^{\pm}N \to \mu^{\pm}\mu^{\pm}jj$$

Collider signature (2)

From a viewpoint of direct tests, the triplet seesaw has an advantage: The SU(2)_L Higgs triplet contains a doubly-charged scalar which can be produced at colliders: it is dependent on its mass but independent of the (small) Yukawa coupling.



Typical LNV signatures: $\boxed{H^{\pm\pm} \longrightarrow l_\alpha^{\pm} l_\beta^{\pm}}$ $\boxed{H^{+} \longrightarrow l_\alpha^{+} \bar{\nu}_\beta}$ $\boxed{H^{-} \longrightarrow l_\alpha^{-} \nu}$

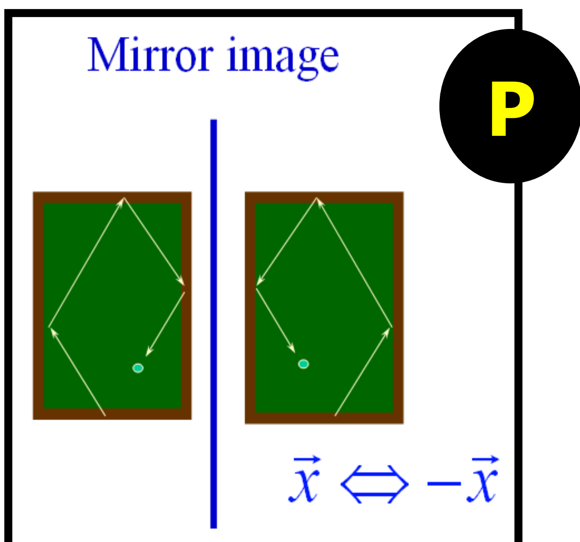
$$\mathcal{B}(H^{\pm\pm} \to l_\alpha^{\pm} l_\beta^{\pm}) = \frac{(2-\delta_{\alpha\beta})|(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2} , \qquad \mathcal{B}(H^{+} \to l_\alpha^{+} \overline{\nu}) = \frac{\sum\limits_{\beta} |(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2}$$

Flavor mixing

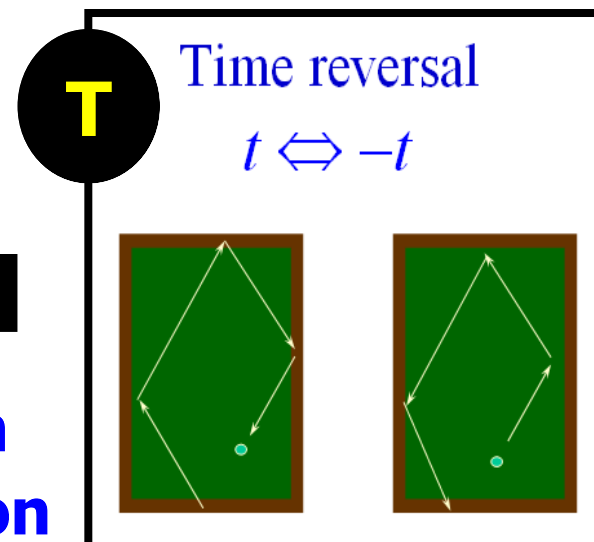
Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of 2 types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the ***W*** boson;
Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of 2 types of interactions.



1957: P violation
1964: CP violation



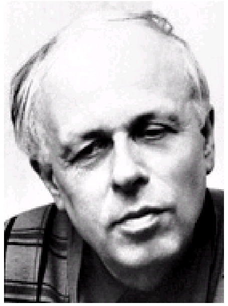
Towards the KM paper

1964: **Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)**

NP 1980

1967: **Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)**

NP 1975



1967: **The standard model of electromagnetic and weak interactions without quarks (S. Weinberg)**

0 citation for the first 4 yrs

NP 1979

1971: **The first proof of the renormalizability of the standard model (G. ‘t Hooft)**

NP 1999

KM in 1972

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

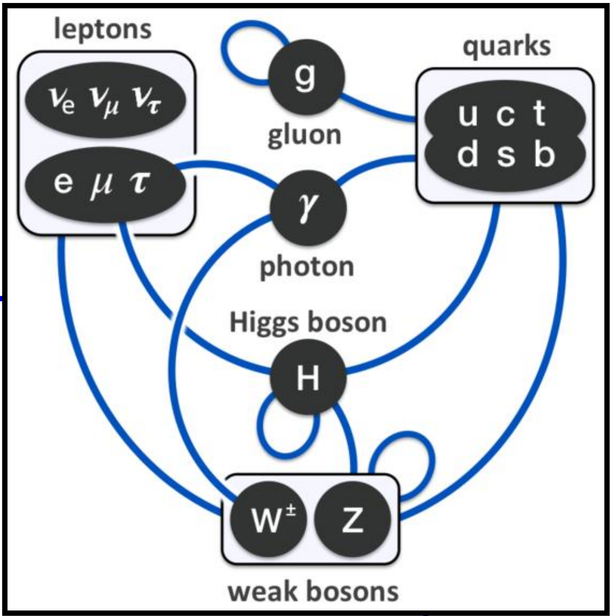
In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

3 families allow for CP violation: Maskawa's bathtub idea!

"as I was getting out of the bathtub, an idea came to me"

Diagnosis of CP violation

In the SM (+ 3 right-handed ν's) , the KM mechanism is responsible for CPV.



$$\boxed{\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}}$$

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}}i\not{D}Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}}i\not{D}\ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}}i\not{\partial}'U_{\mathrm{R}} + \overline{D_{\mathrm{R}}}i\not{\partial}'D_{\mathrm{R}} + \overline{E_{\mathrm{R}}}i\not{\partial}'E_{\mathrm{R}} + \underline{\overline{N_{\mathrm{R}}}i\not{\partial}'N_{\mathrm{R}}}$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}}Y_{\mathrm{u}}\tilde{H}U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}}Y_{\mathrm{d}}HD_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}}Y_{l}HE_{\mathrm{R}} - \underline{\overline{\ell_{\mathrm{L}}}Y_{\nu}\tilde{H}N_{\mathrm{R}}} + \mathrm{h.c.}$$

ν's Dirac mass

The strategy of diagnosis: **given proper CP transformations of gauge, Higgs and fermion fields, we may prove that the 1st, 2nd and 3rd terms are formally invariant, and hence the 4th term can be invariant only if provided the corresponding Yukawa coupling matrices are real. (Note that the SM spontaneous symmetry breaking itself doesn't affect CP.)**

CP violation

The Yukawa interactions of fermions are formally invariant under CP if and only if

$$Y_{\rm u} = Y_{\rm u}^* \; , \qquad Y_{\rm d} = Y_{\rm d}^*$$
$$Y_l = Y_l^* \; , \qquad Y_\nu = Y_\nu^*$$

If the effective Majorana mass term is added into the SM, then the Yukawa interactions of leptons can be formally invariant under CP if

$$M_{\rm L} = M_{\rm L}^* \; , \qquad Y_l = Y_l^*$$

If the flavor states are transformed into the mass states, the source of flavor mixing and CP violation will show up in the *CC* interactions:

quarks

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(u \;\; c \;\; t)_{\rm L}} \; \gamma^\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + {\rm h.c.}$$

leptons

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\; \mu \;\; \tau)_{\rm L}} \; \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

Comment A: CP violation exists since fermions interact with both the gauge bosons and the Higgs boson.

Comment B: both the CC and Yukawa interactions have been verified.

Comment C: the CKM matrix ***U*** is unitary, the MNSP matrix ***V*** is too?

Physical phases

If neutrinos are **Dirac** particles, the phases ***x***, ***y*** and ***z*** can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are **Majorana** particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., $z = 0$). Then

Majorana neutrino mixing matrix

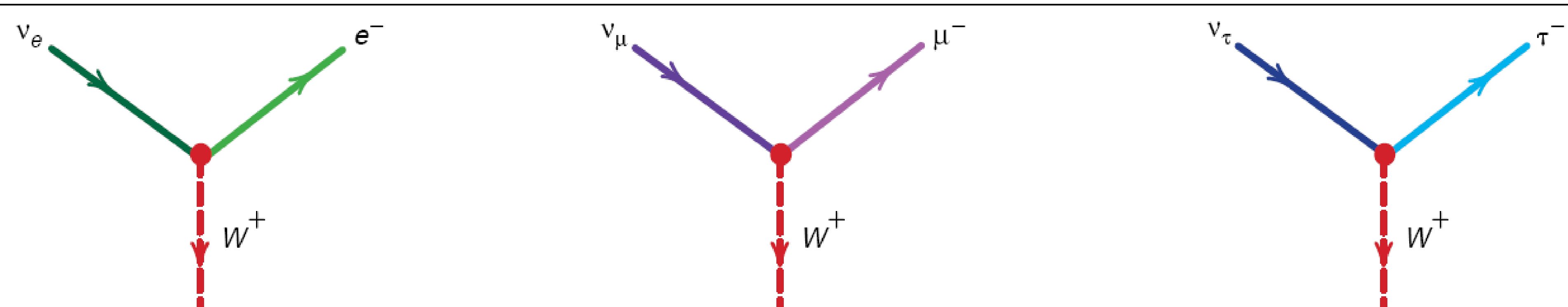
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What is oscillation?

Oscillation — a spontaneous periodic change from one neutrino flavor state to another, is a spectacular quantum phenomenon. It can occur as a natural consequence of neutrino mixing.

In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak charged-current interactions.

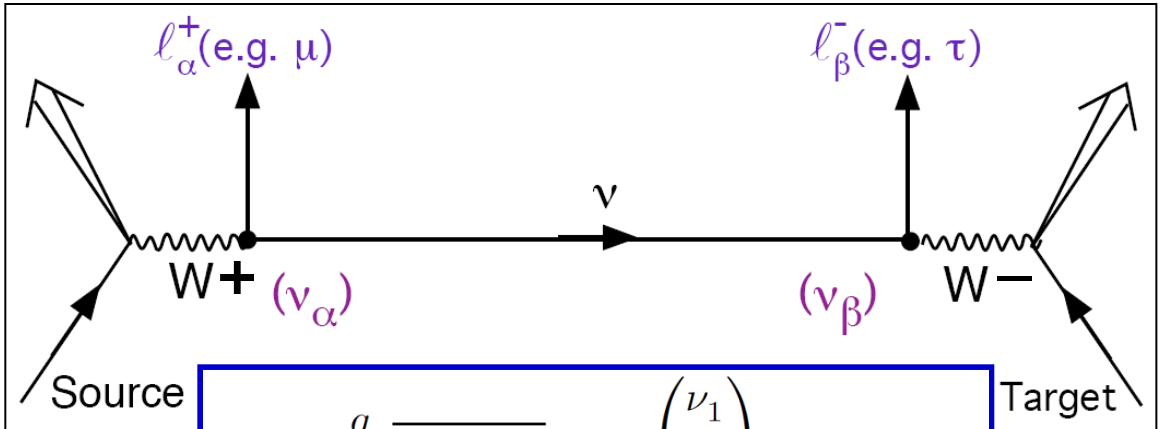
Pure weak state ν_α ← $\nu_{1,2,3}$ **three distinct neutrino mass states (matter waves) travel & interfere with one another** → **Pure weak state** ν_β



For example: $\overline{\nu_e}$ beam : β decay; ν_μ beam : π decay; ν_τ beam : D decay

3-flavor oscillation

Production and detection of a neutrino beam by CC weak interactions:



$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_L}\ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \text{h.c.}$$

$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} V_{\alpha i}^* |\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle$$

$\alpha, \beta, \gamma = e, \mu, \tau$

$i, j, k = 1,2,3$

The amplitude and probability of neutrino oscillations:

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t)\rangle = \left(\sum_{j=1}^{3} V_{\beta j} \langle \nu_j|\right)\left(\sum_{i=1}^{3} V_{\alpha i}^* e^{-iE_i t}|\nu_i\rangle\right) = \sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t}$$

$$P(\nu_\alpha \to \nu_\beta) = \left|\langle \nu_\beta | \nu_\alpha(t)\rangle\right|^2 = \left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t}\right|^2$$

$$= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left[V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^* e^{i(E_j - E_i)t}\right]$$

Part C

CP violation

The final formula of 3-flavor oscillation probabilities with CP violation:

$$
\begin{aligned}
P\left(\nu_\alpha \to \nu_\beta\right) &= \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}\right) \sin^2 \frac{\Delta m^2_{ji} L}{4E} \\
&\quad + 8\mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sin\frac{\Delta m^2_{21} L}{4E} \sin\frac{\Delta m^2_{31} L}{4E} \sin\frac{\Delta m^2_{32} L}{4E}
\end{aligned}
$$

Under CPT invariance, CP- and T-violating asymmetries are identical:

$$
\begin{aligned}
P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) &= P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right) \\
&= 16\mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sin\frac{\Delta m^2_{21} L}{4E} \sin\frac{\Delta m^2_{31} L}{4E} \sin\frac{\Delta m^2_{32} L}{4E}
\end{aligned}
$$

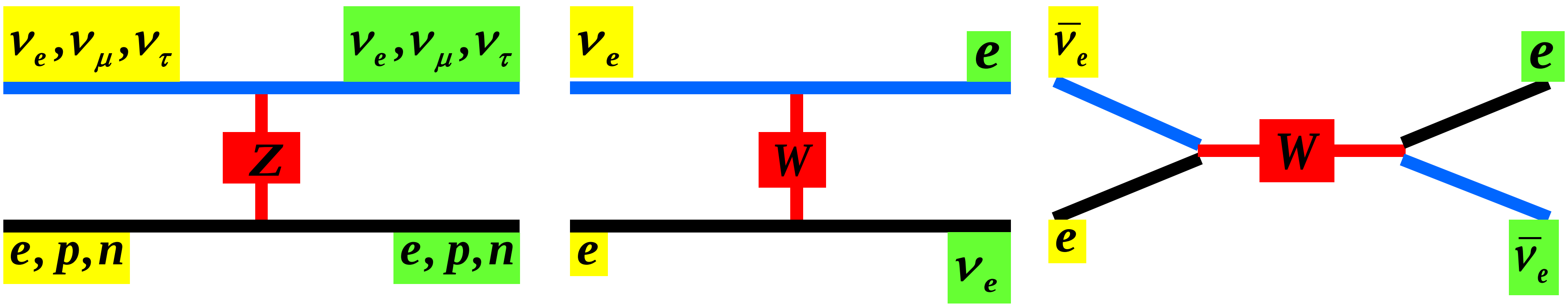
Jarlskog invariant, a rephasing-invariant measure of CP / T violation:

$$
J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta \le 1/6\sqrt{3} \approx 9.6\%
$$

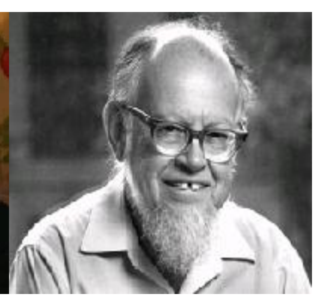
Matter effect

When light travels through a medium, it sees a refractive index due to coherent forward scattering from the constituents of the medium.

A similar phenomenon applies to neutrino flavor states as they travel through matter. All flavor states see a common refractive index from NC forward scattering, and the electron (anti) neutrino sees an extra refractive index due to CC forward scattering in matter.



Consequence of Mikheyev-Smirnov-Wolfenstein (MSW) matter effect:



- Matter-modified oscillation behavior:

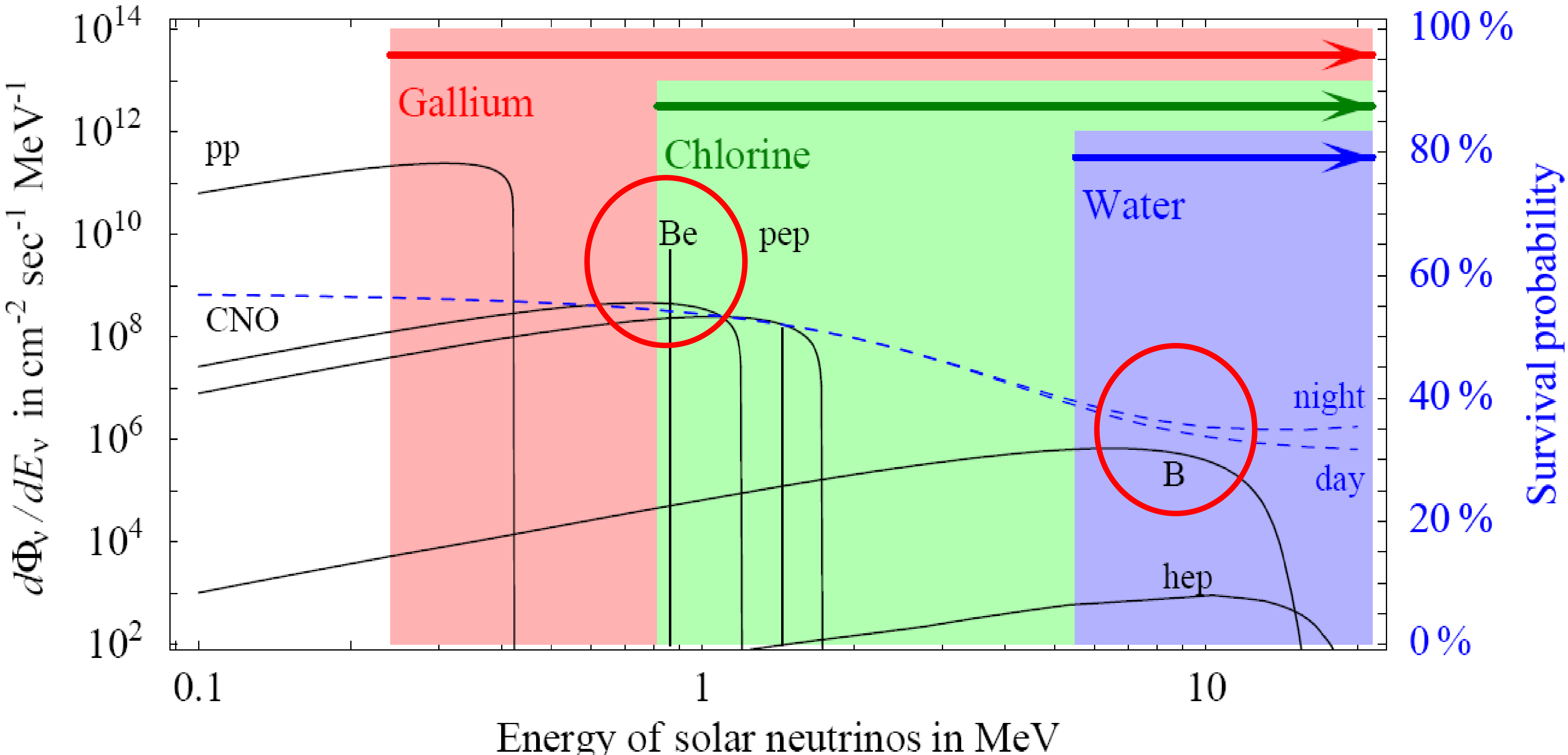
$$\boxed{\Delta m_{ij}^2 \pm 2\sqrt{2} G_F N_e E}$$

- Fake CP-violating effect in oscillation.

Part D

Solar neutrinos

R. Davis observed a solar neutrino deficit, compared with J. Bahcall's prediction for the ν-flux, at the Homestake Mine in 1968.



DATA

Strumia & Vissani, hep-ph/0606054.

Examples: Boron (硼) ν's ~ 32%, Beryllium (铍) ν's ~ 56%

MSW solution

In the two-flavor approximation, solar neutrinos are governed by

$$N_e(0) \approx 6 \times 10^{25}\ \mathrm{cm}^{-3}$$

$$\mathcal{H}_{\text{eff}} = \frac{\Delta m_{21}^2}{4E}\begin{bmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{bmatrix} + \begin{bmatrix} \sqrt{2}G_{\mathrm{F}}N_e(r) & 0 \\ 0 & 0 \end{bmatrix}$$

$7.6 \times 10^{-5}\ \mathrm{eV}^2$

$0.75 \times 10^{-5}\ \mathrm{eV}^2\ /\ \mathrm{MeV}$ (at $r = 0$)

Be-7 ν's: *E* ~ 0.862 MeV. The vacuum term is dominant. The survival probability on the earth is (for theta_12 ~ 34°):

$$\begin{aligned} P(\nu_e \to \nu_e) &\approx 1 - \frac{1}{2}\sin^2 2\theta_{12} \\ &\sim 0.56 \end{aligned}$$

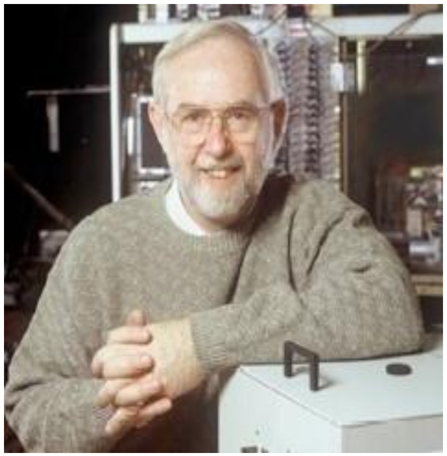
B-8 ν's: *E* ~ 6 to 7 MeV. The matter term is dominant. The produced ν is roughly ν_e ~ ν_2 (for *V* >0). The ν-propagation from the center to the outer edge of the Sun is approximately adiabatic. That is why it keeps to be ν_2 on the way to the surface (for theta_12 ~ 34°):

$$|\nu_2\rangle \approx \sin\theta_{12}|\nu_e\rangle + \cos\theta_{12}|\nu_\mu\rangle$$

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\nu_2\rangle|^2 = \sin^2\theta_{12} \approx 0.32$$

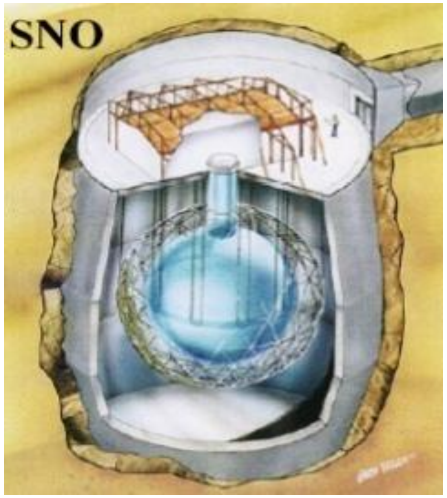
SNO in 2001

The heavy water Cherenkov detector at SNO confirmed the solar neutrino flavor conversion (A.B. McDonald 2001)

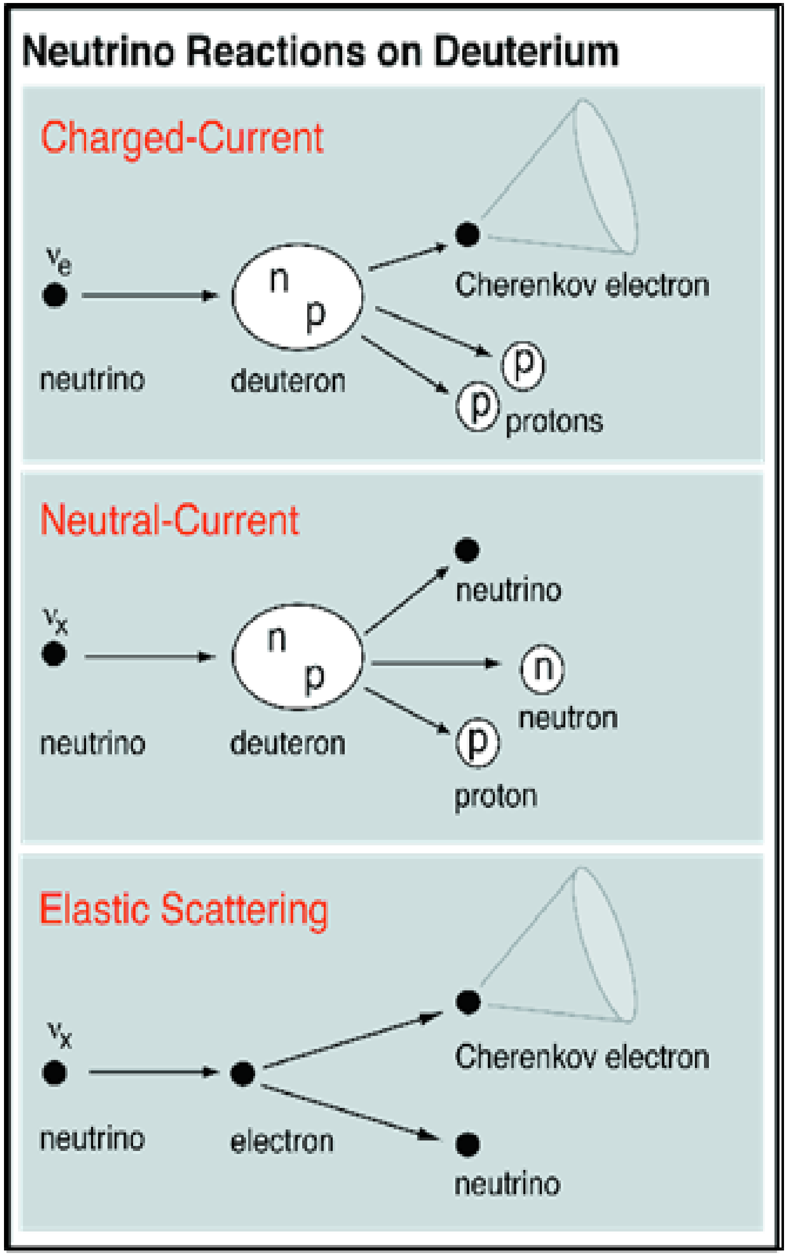


The Salient features:

Boron-8 *e*-neutrinos
- **Flux and spectrum**
- **Deuteron as target**
- **3 types of processes**
- **Model-independent**



At Super-Kamiokande only elastic scattering can happen between solar neutrinos & the ordinary water.

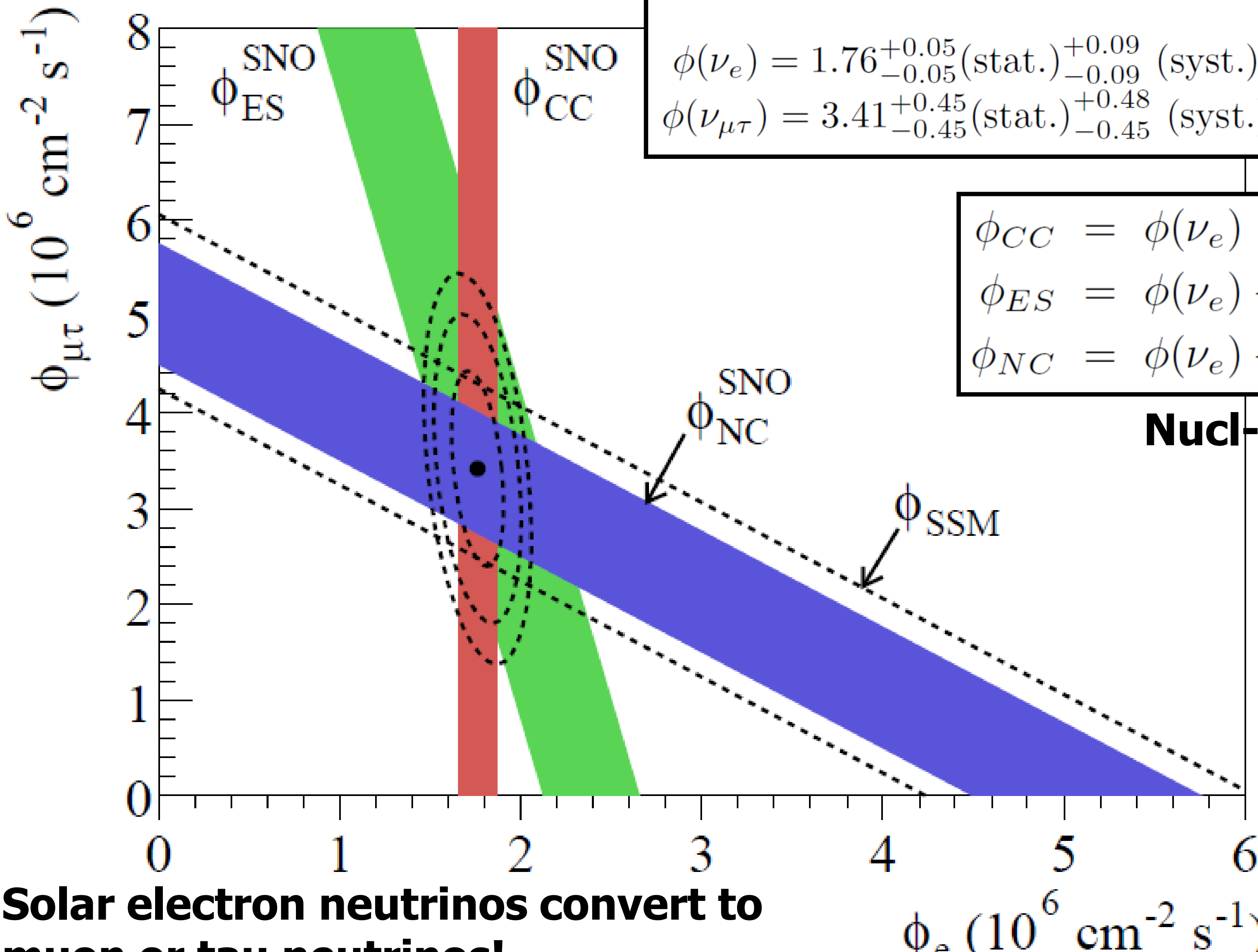


The SNO result

$$\phi_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat.})^{+0.12}_{-0.12}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\phi(\nu_e) = 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi(\nu_{\mu\tau}) = 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\begin{aligned}\phi_{CC} &= \phi(\nu_e)\\ \phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau})\\ \phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau})\end{aligned}$$



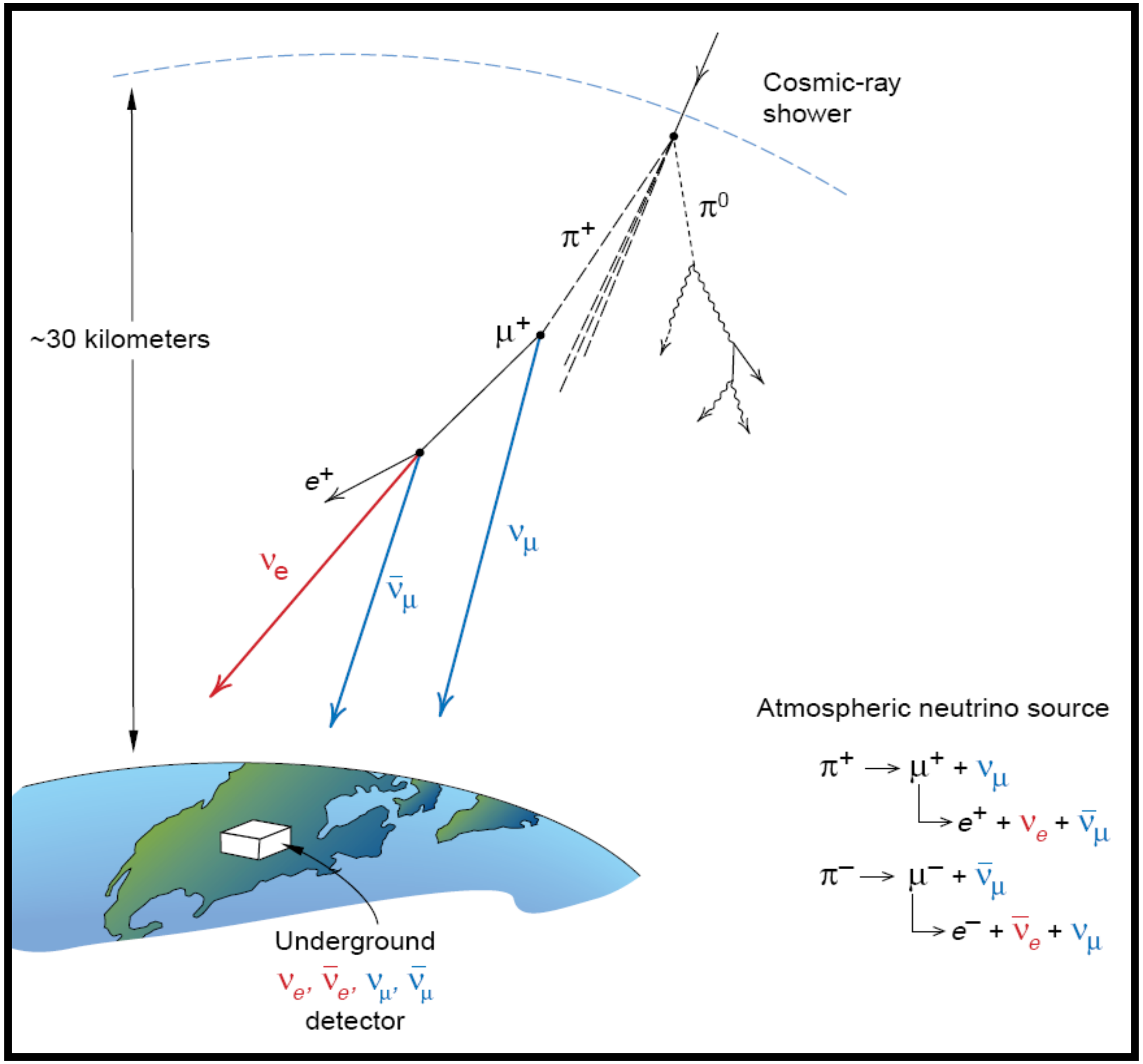
Nucl-ex/0610020

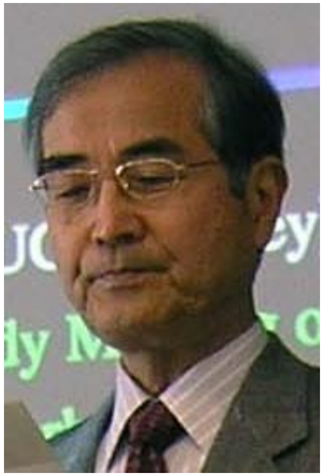
John Bahcall

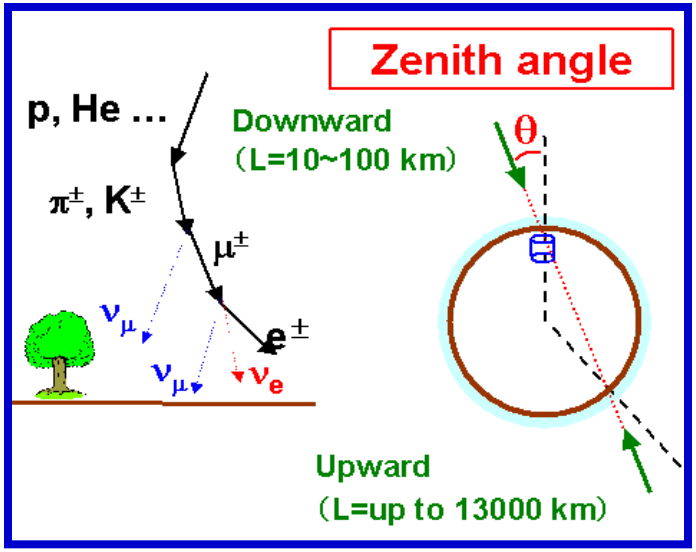
Solar electron neutrinos convert to muon or tau neutrinos!

Atmospheric neutrinos

Atmospheric muon neutrino deficit was firmly established at Super-Kamiokande (Y. Totsuka & T. Kajita 1998).







Zenith angle distributions

C. Sagi/ICHEP04

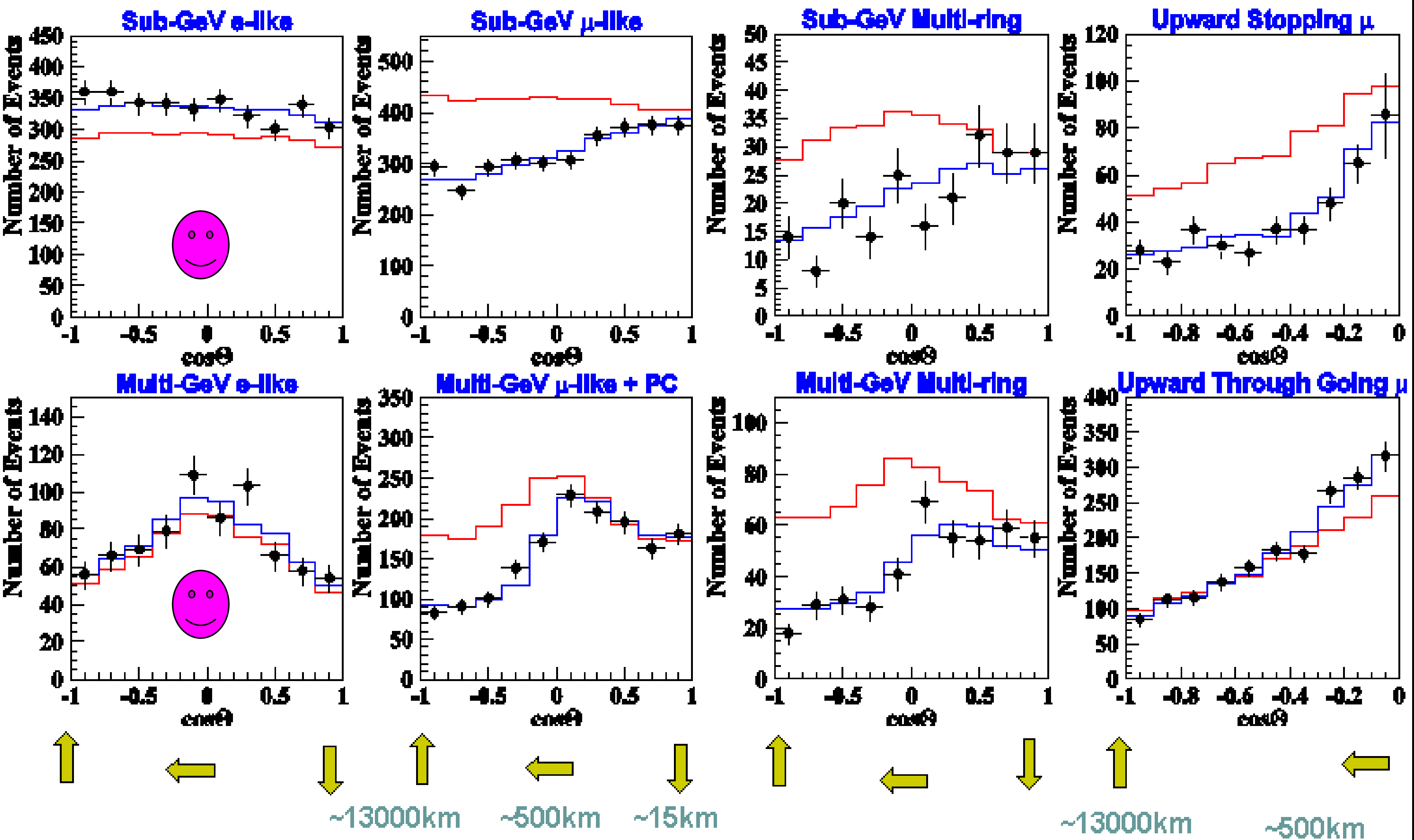
$\nu_\mu \leftrightarrow \nu_\tau$

2-flavor oscillations

Best fit

$\sin^2 2\theta = 1.0$, $\Delta m^2 = 2.1 \times 10^{-3}$ eV^2

Null oscillation



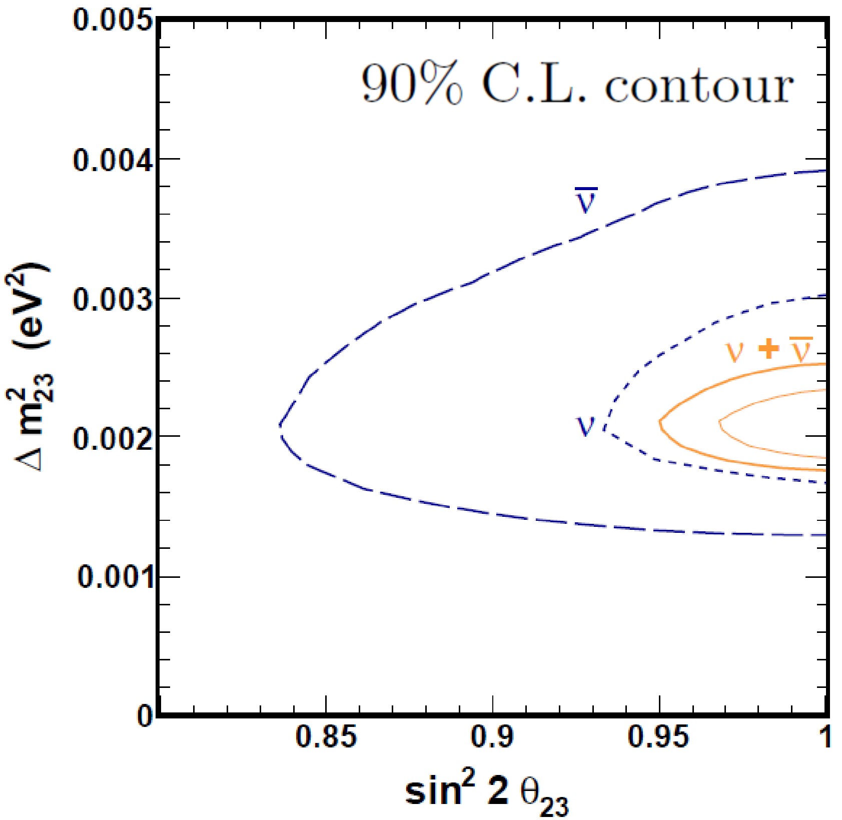
L/E Analysis: SK-I + SK-II

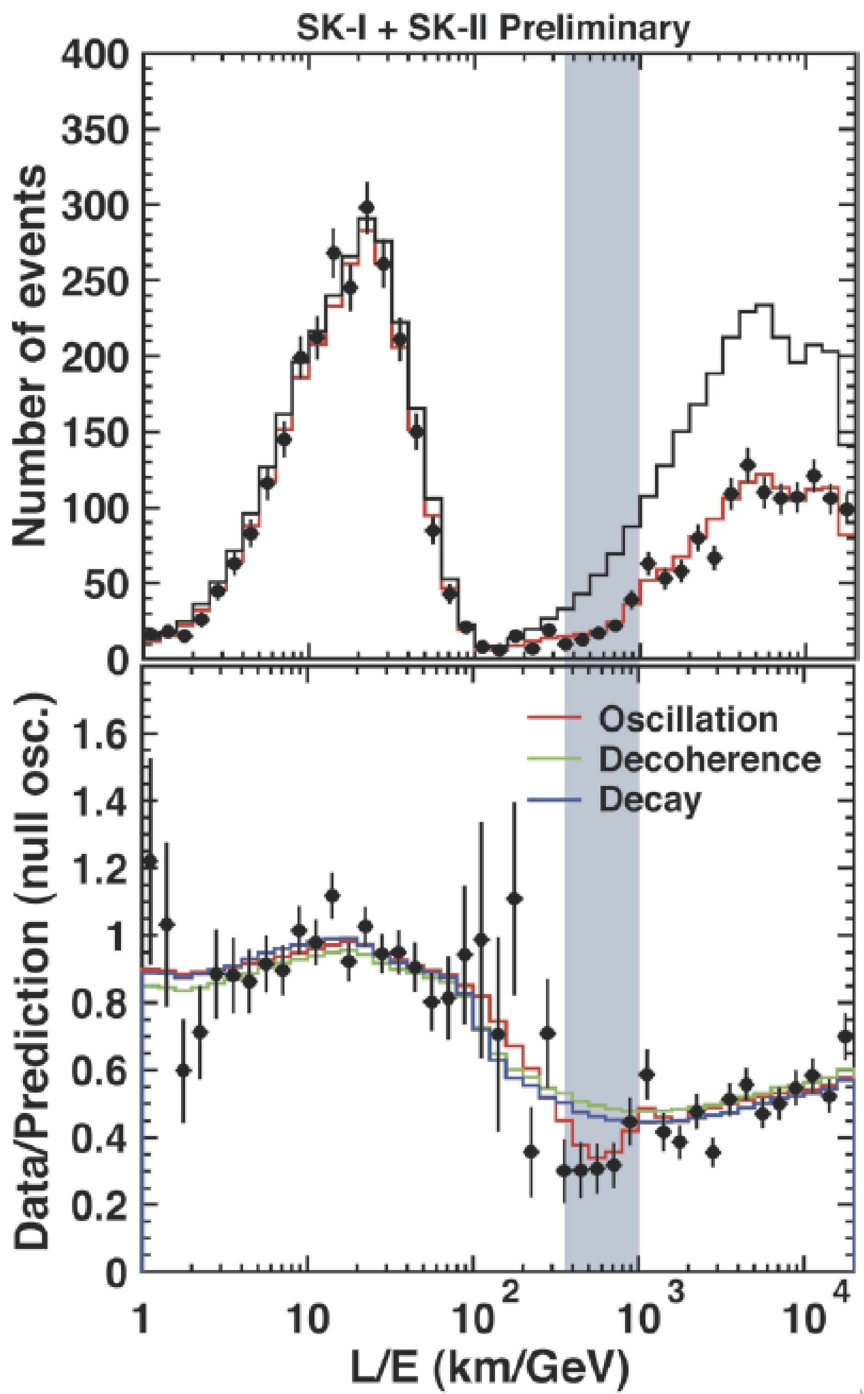
J. Raaf/Neutrino08

Datasets
SK-I FC/PC μ-like: 1489 days
SK-II FC/PC μ-like: 799 days

Phys. Rev. Lett. 107, 241801 (2011)

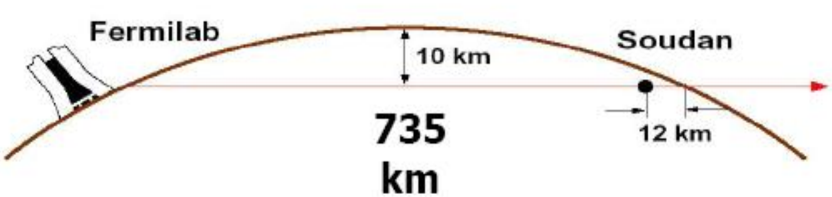
SK-I+II+III data set

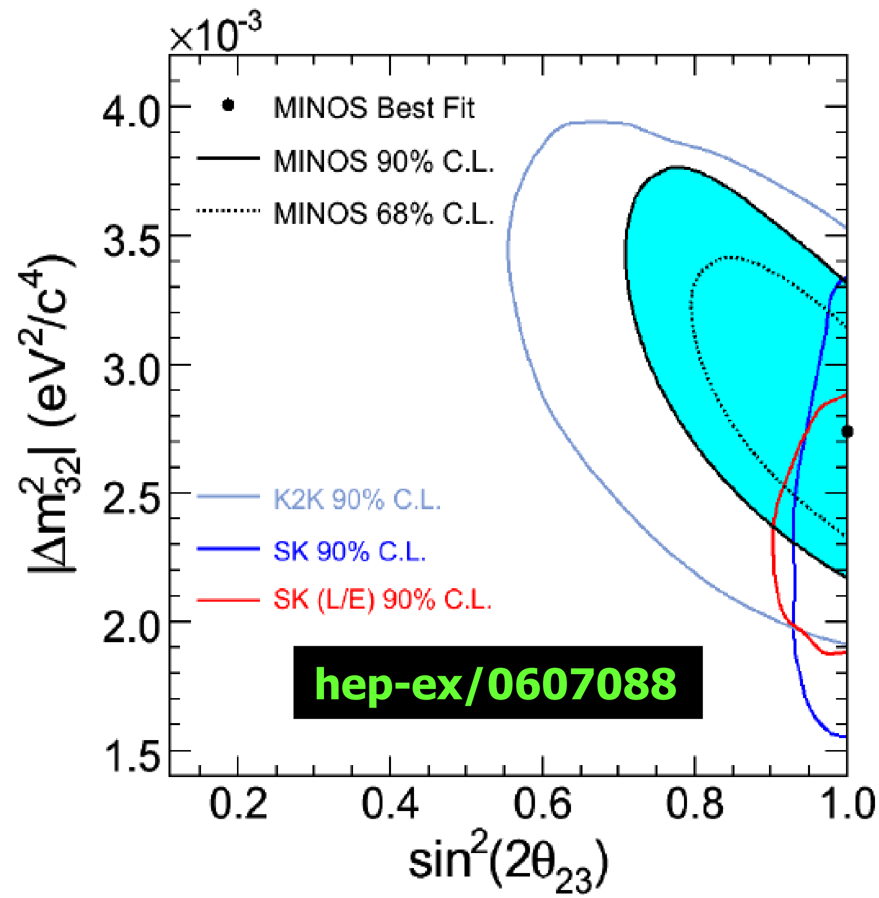




Accelerator neutrinos

The MINOS supports Super-K & K2K data





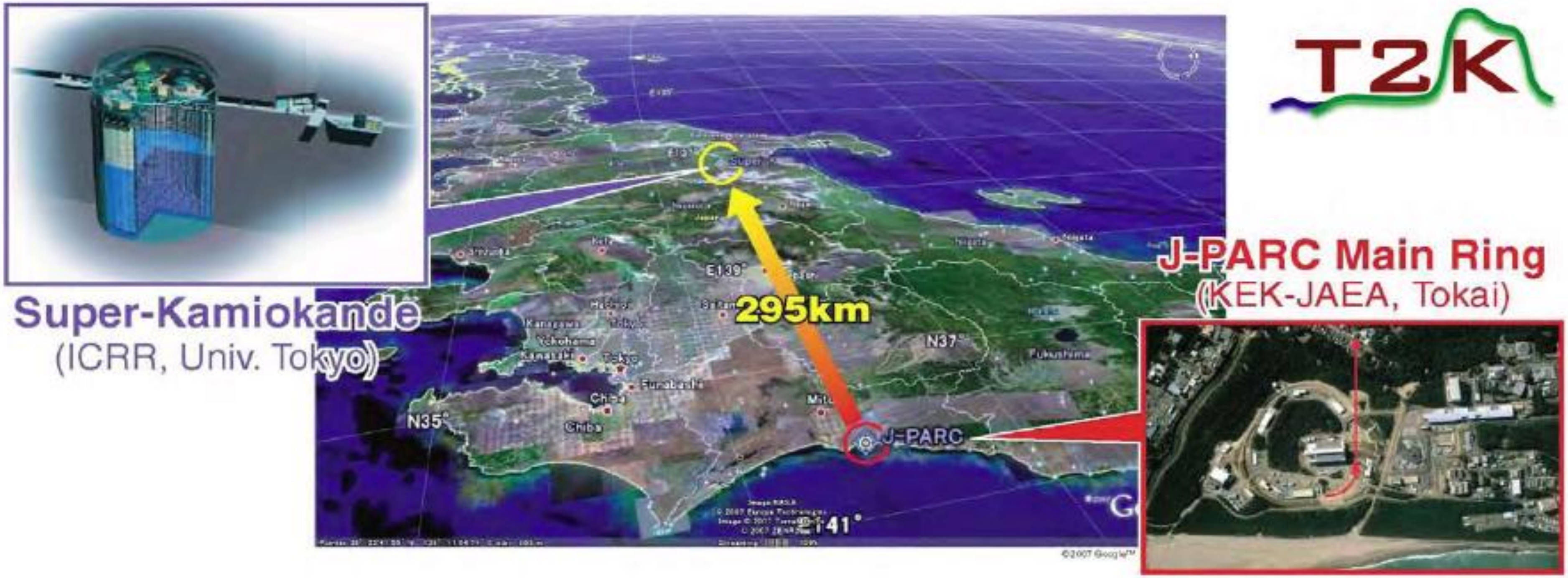
$$\left|\Delta m^2_{32}\right| = 2.74^{+0.44}_{-0.26}\,(\text{stat}+\text{syst})\times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\theta_{23} = 1.00\ _{-0.13}(\text{stat}+\text{syst})$$

Constrained to $\sin^2(2\theta_{23}) \le 1$

T2K in 2011

T2K (Tokai-to-Kamioka) experiment



•arXiv:1106.2822 [hep-ex] 14 June 2011
Hint for unsuppressed theta(13) !

T2K Main Goals:

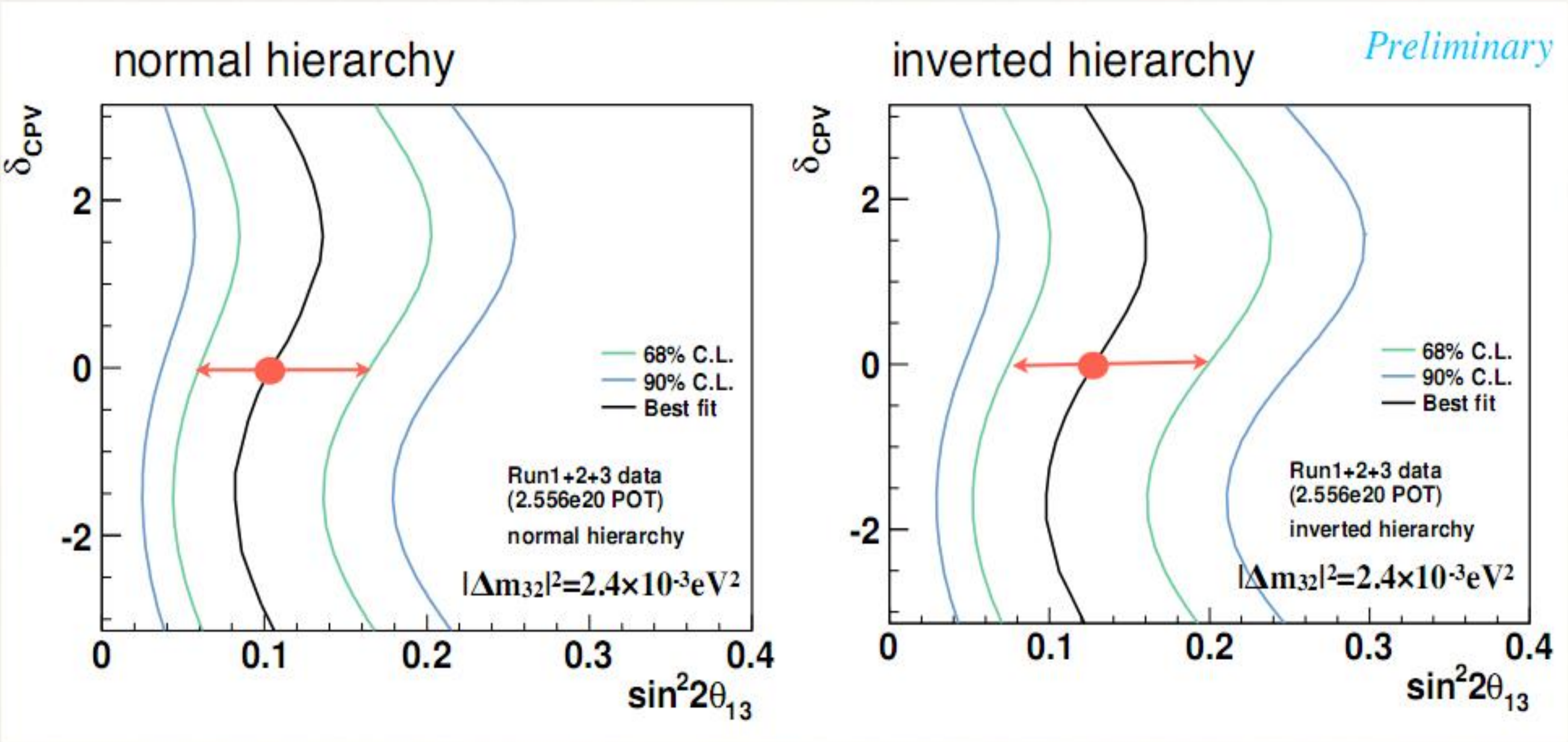
★ Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)

★ Precision measurement of ν_μ disappearance

T. Nakaya (Neutrino 2012)

Allowed Region (constant χ^2 method)

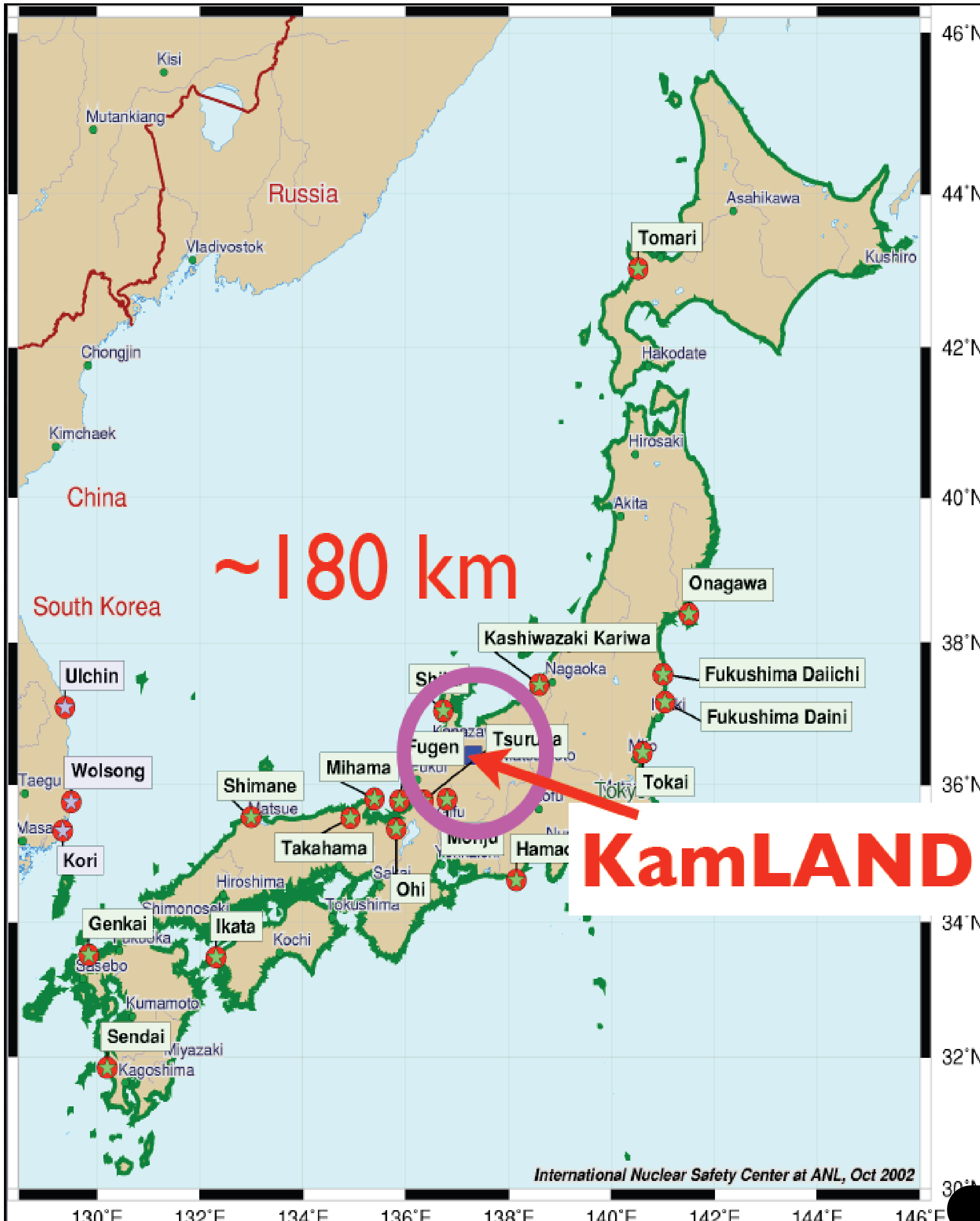
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2(1.27\Delta m_{32}^2 L/E) + CPV + matter\ effect. + ...$$



$\sin^2 2\theta_{13} = 0.104\ ^{+0.060}_{-0.045}$ @ $\delta_{CP} = 0$

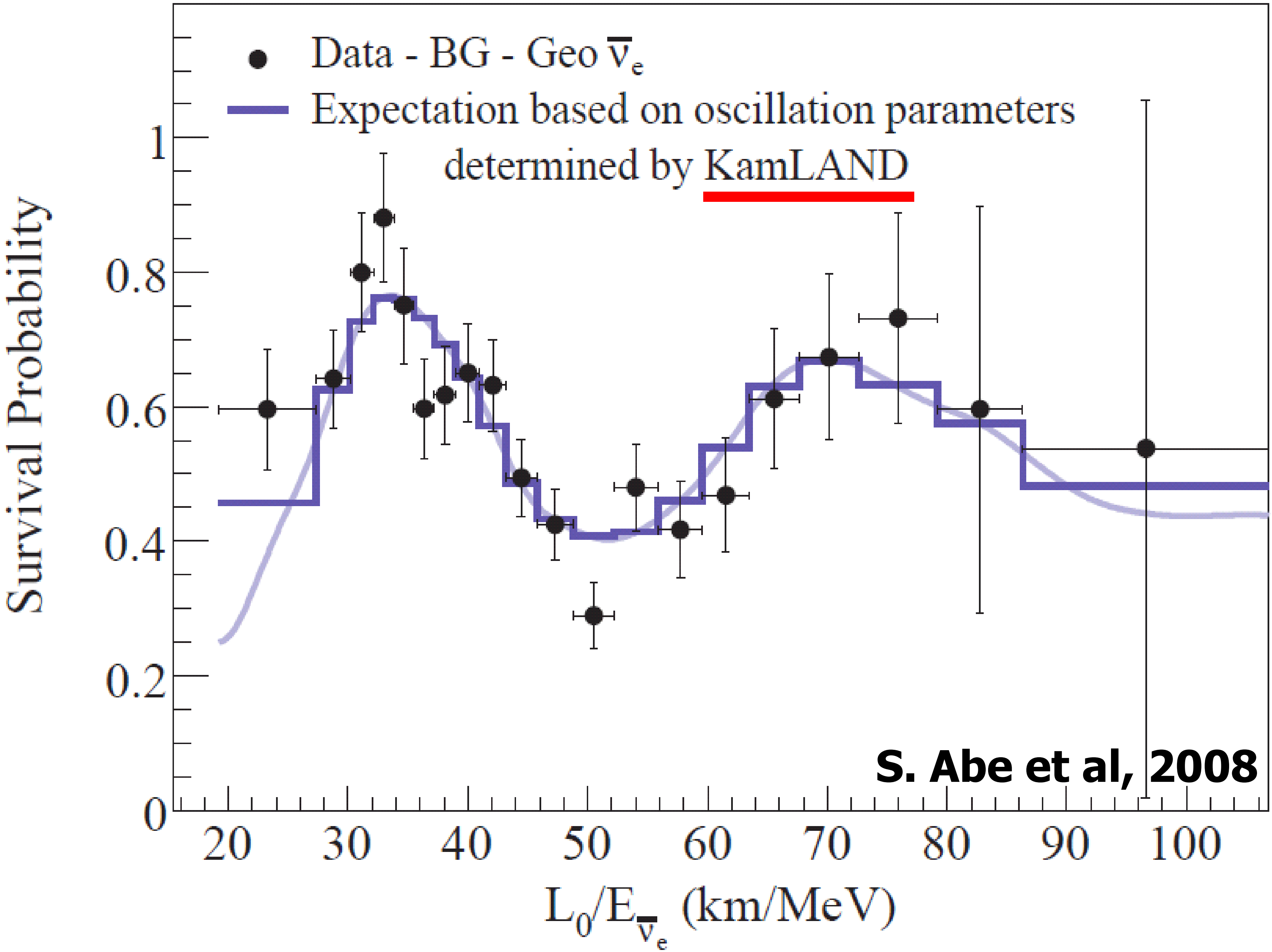
$\sin^2 2\theta_{13} = 0.128\ ^{+0.070}_{-0.055}$ @ $\delta_{CP} = 0$

Reactor antineutrinos

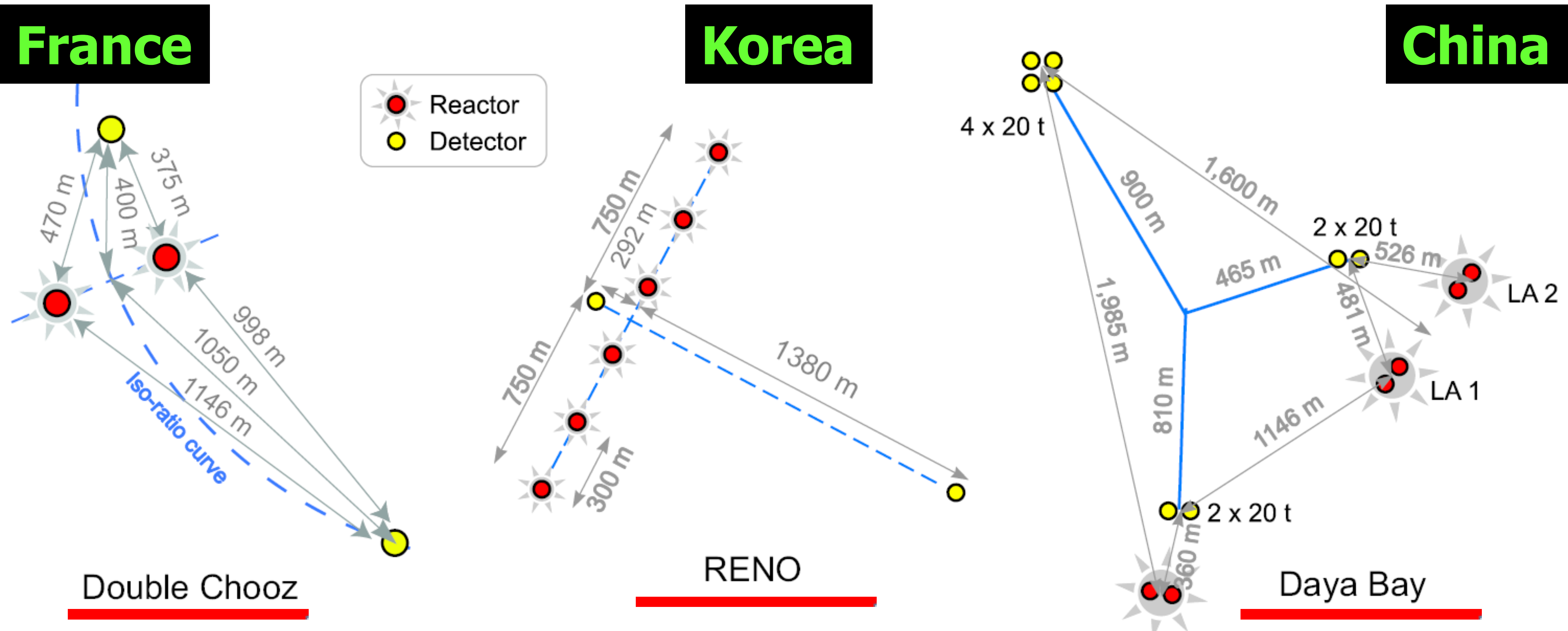


Verify the large angle MSW solution to the solar neutrino Problem

Atsuto Suzuki
Director General


Data - BG - Geo $\bar{\nu}_e$
Expectation based on oscillation parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\bar{\nu}_e}$ (km/MeV)
S. Abe et al, 2008

Hunting for θ_{13}



Setup	**Thermal power** P_{Th} (GW)	**Baseline** L (m)	**Detector mass** m_{Det} (t)	Events/year	Backgrounds/day
Daya Bay [20]	17.4	1700	80	10×10^4	0.4
Double CHOOZ [21]	8.6	1050	8.3	1.5×10^4	3.6
RENO [22]	16.4	1400	15.4	3×10^4	2.6

Daya Bay in 2012

The Daya Bay Experiment

Adjacent mountains with horizontal access provide 860 (250) m.w.e cosmic shielding.

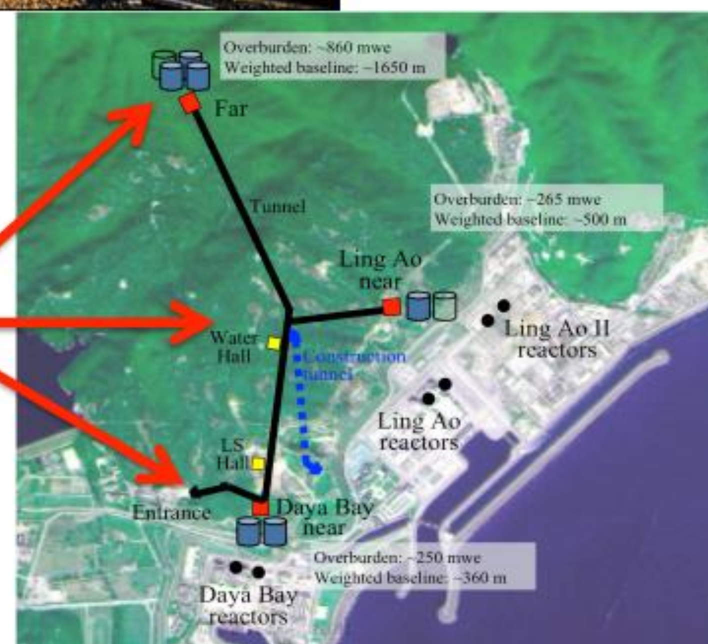
Daya Bay

Ling Ao I + II

6 commercial reactor cores with 17.4 GW_{th} total power.

6 Antineutrino Detectors (ADs) give 120 tons total target mass.

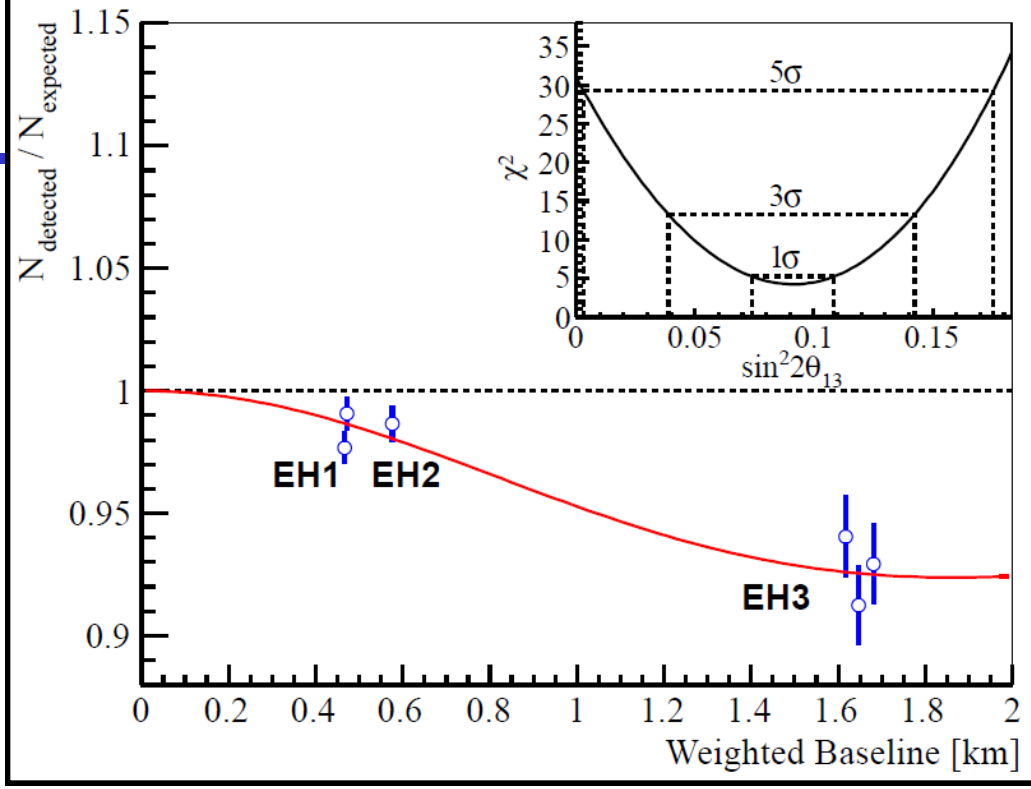
Via GPS and modern theodolites, relative detector-core positions known to 3 cm.



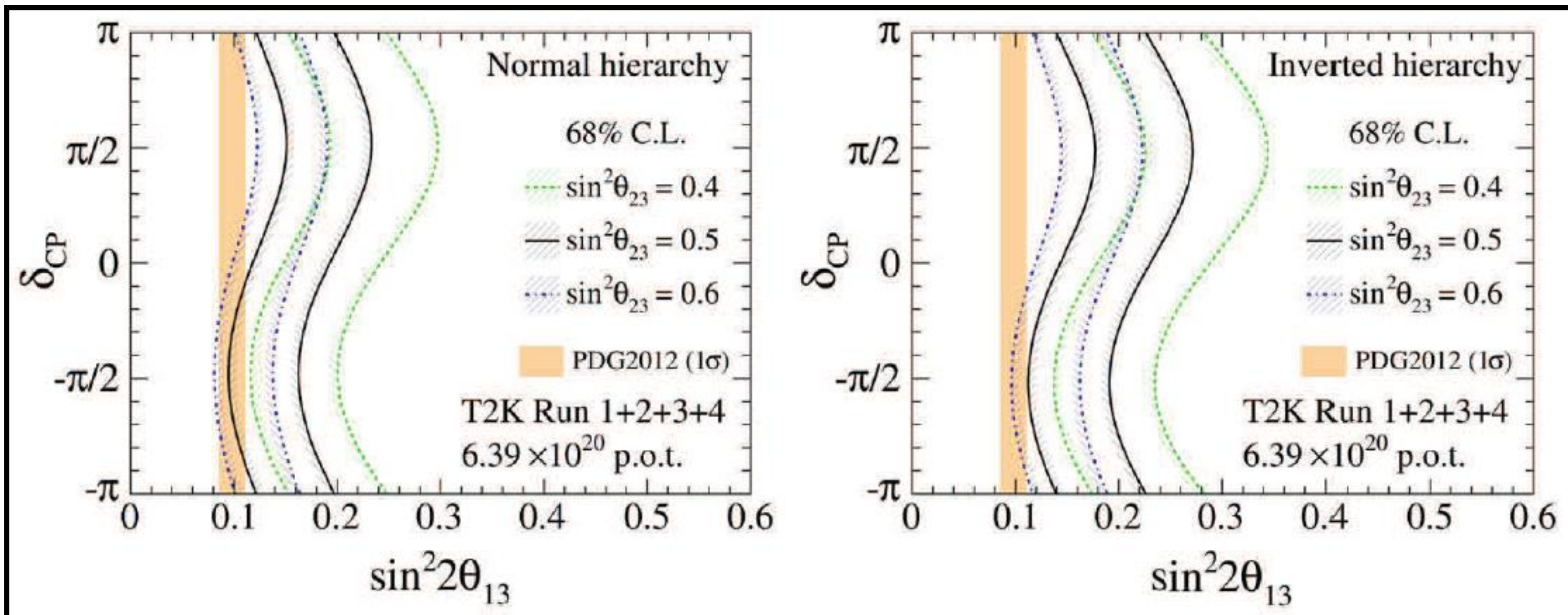
Part D

Milestone

$\theta_{13} \approx 8.8^\circ \pm 0.8^\circ$



T2K + DYB



3-flavor global fit

M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, e-Print: arXiv:1409.5439

	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$31.29 \to 35.91$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \to 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \to 53.3$	$38.3 \to 53.3$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \to 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$7.87 \to 9.11$
$\delta_{CP}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$	$0 \to 360$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.02 \to 8.09$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$	$\begin{bmatrix} +2.325 \to +2.599 \\ -2.590 \to -2.307 \end{bmatrix}$

Quark mixing: $\theta_{12} \simeq 13^\circ\ ,\ \theta_{23} \simeq 2^\circ\ ,\ \theta_{13} \simeq 0.2^\circ\ ,\ \delta \simeq 65^\circ$

Lepton mixing: $\theta_{12} \simeq 33^\circ\ ,\ \theta_{23} \sim 45^\circ\ ,\ \theta_{13} \simeq 8.5^\circ\ ,\ \delta \sim 270^\circ$

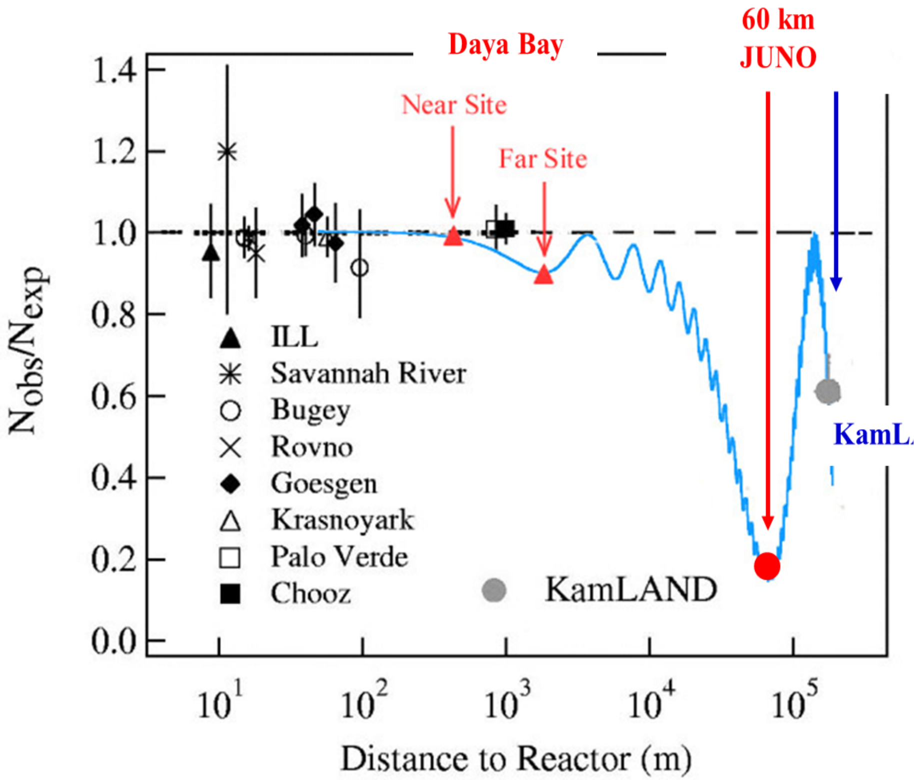
ν mass ordering

Accelerator/atmospheric: terrestrial matter effects play crucial roles.

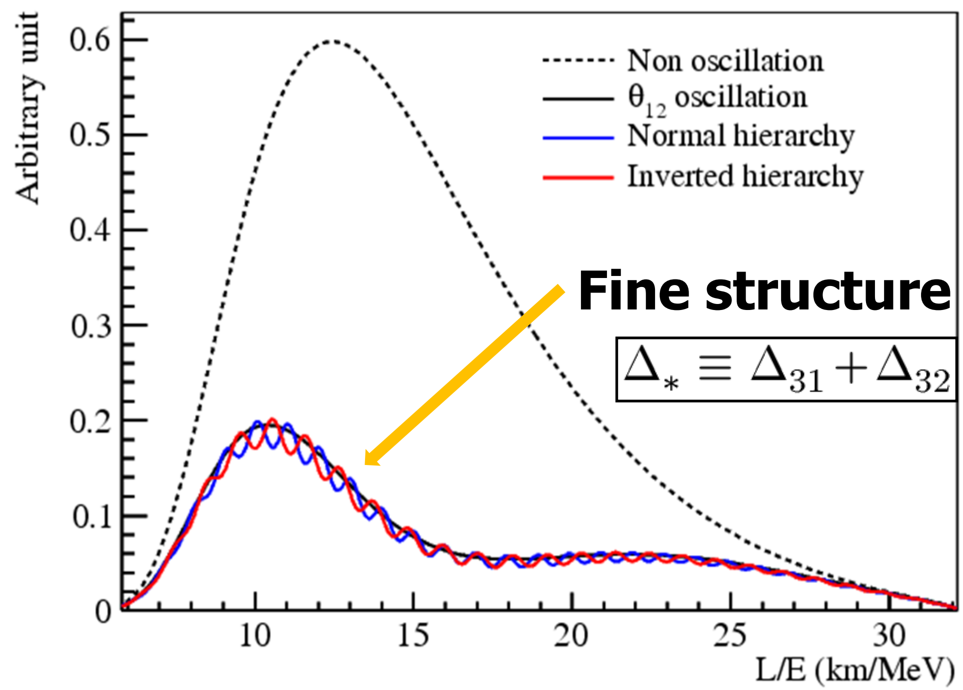
$$\Delta m^2_{31} \mp 2\sqrt{2} G_F N_e E \qquad \theta_{23}$$

its octant matters a lot

Reactor (JUNO): Optimum baseline at the valley of Δm^2_{21} oscillations, corrected by the fine structure of Δm^2_{31} oscillations.



JUNO's idea $\Delta_{ji} \equiv \Delta m^2_{ji} L/(4E)$



$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21} - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \cos\Delta_* \cos\Delta_{21} + \cos 2\theta_{12} \sin\Delta_* \sin\Delta_{21}\right]$$

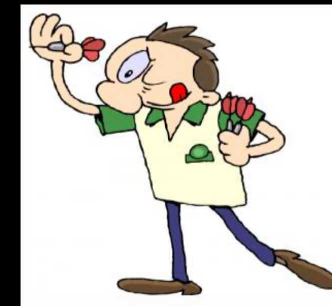
JUNO in progress

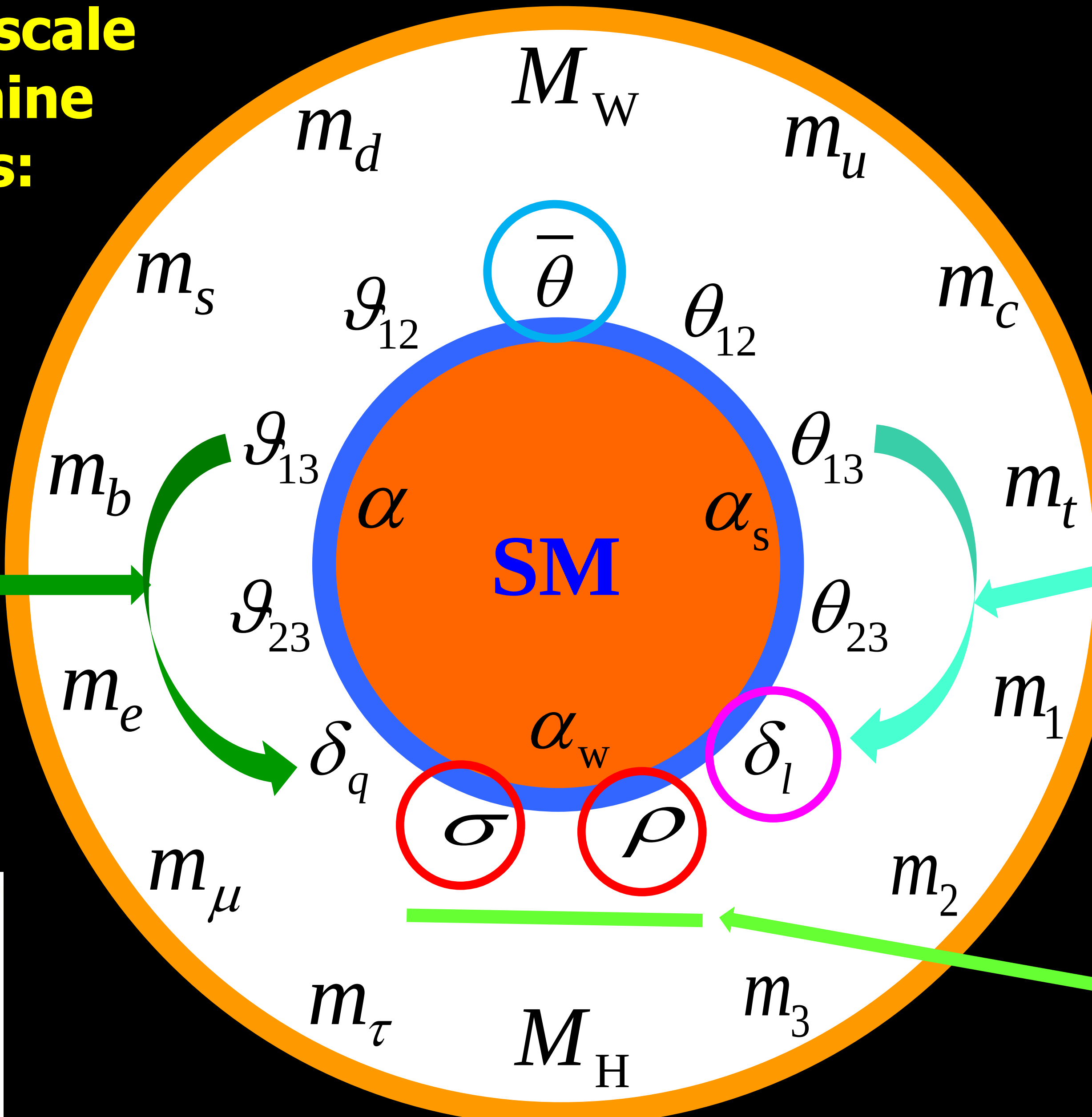
	Daya Bay	Yangjiang	Taishan
Status	running	construction	construction
Power/GW	17.4	17.4	18.4

- Idea 2008
- 20 kton LS detector
  3% E-resolution
- Approved 2/2013
- Groundbreaking 1/2015

SM + neutrinos are left with CP-violating phases

The time scale to determine CP phases:





Real + Hypothetical ν's