The Standard Model of Electroweak & Strong Interactions

Theory and Practice

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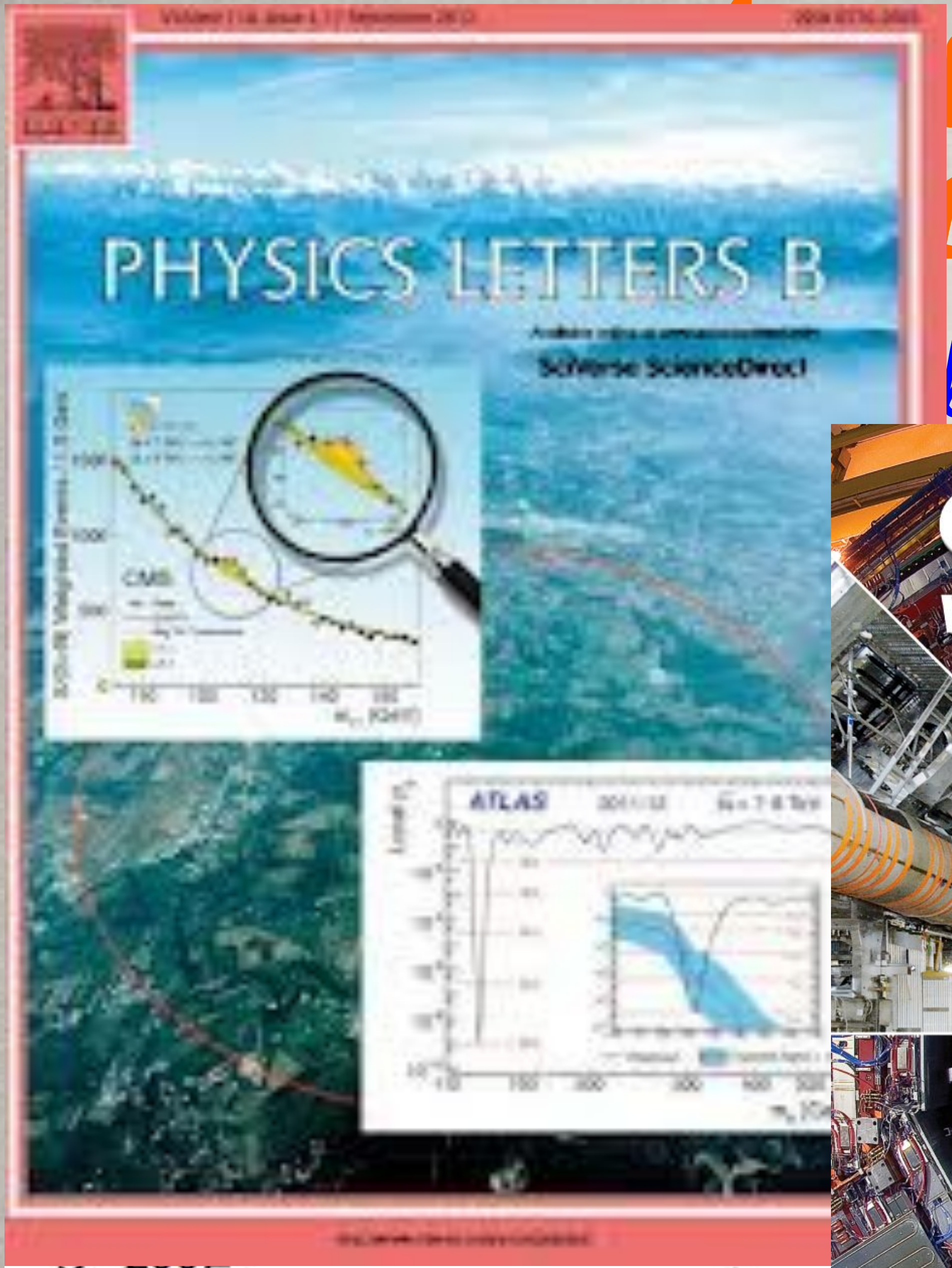
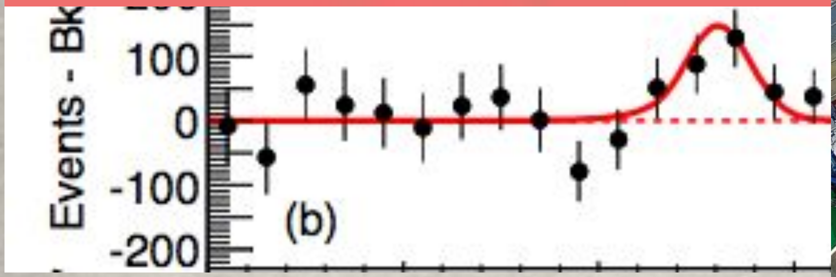
iSTEP 2015, ShanDong Univ., Aug. 12,13, 2015

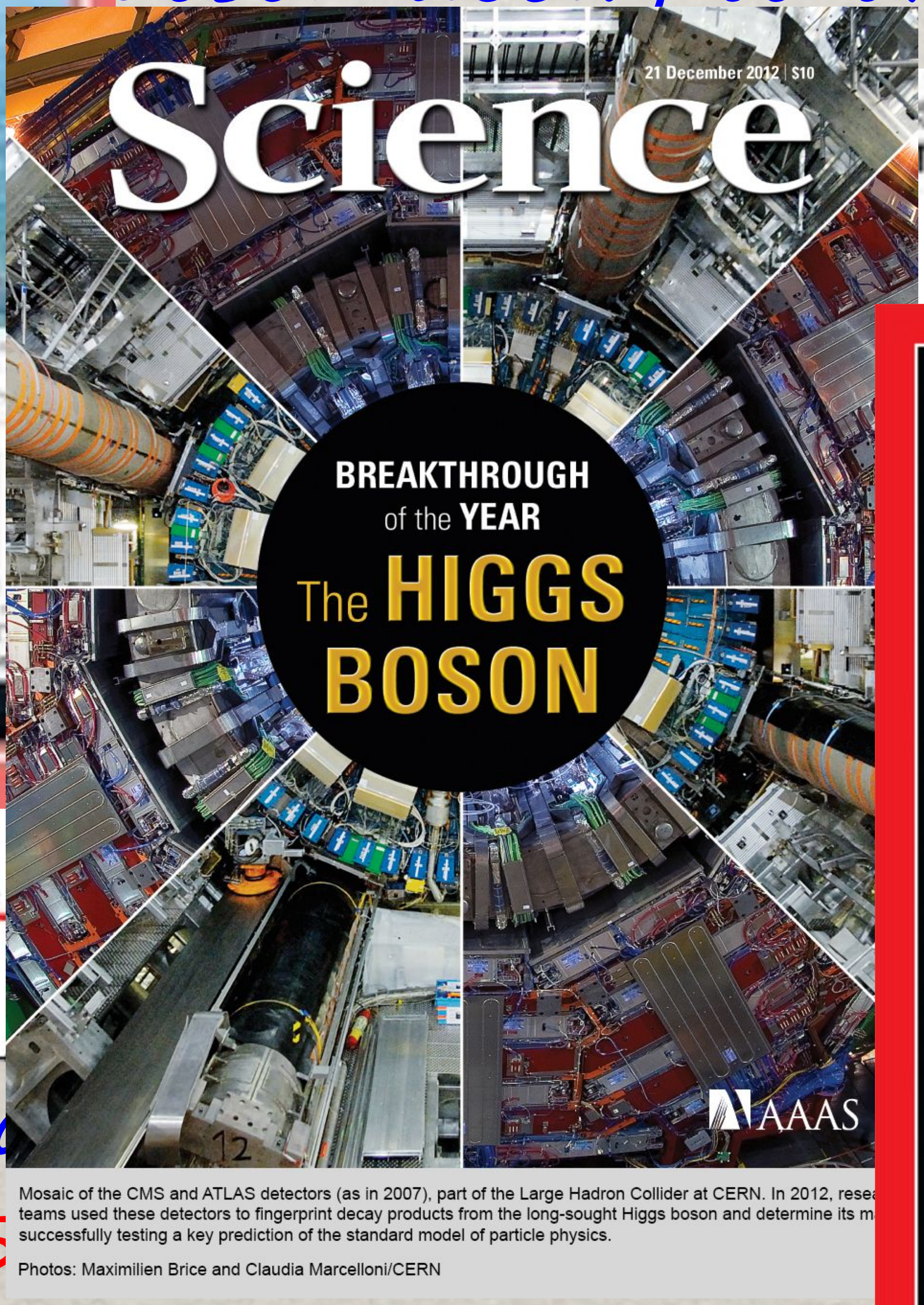
nouncement

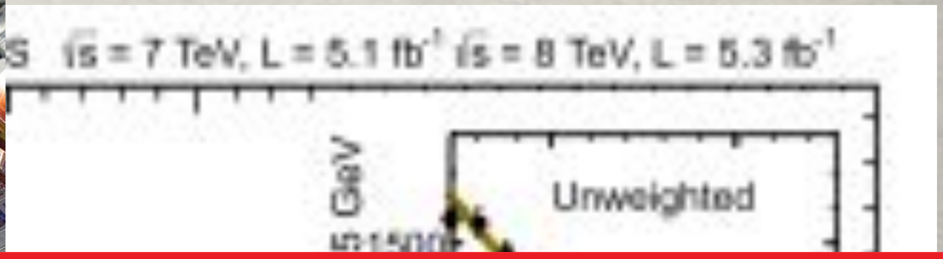
4th of July, 2012:

boson decay to two photons







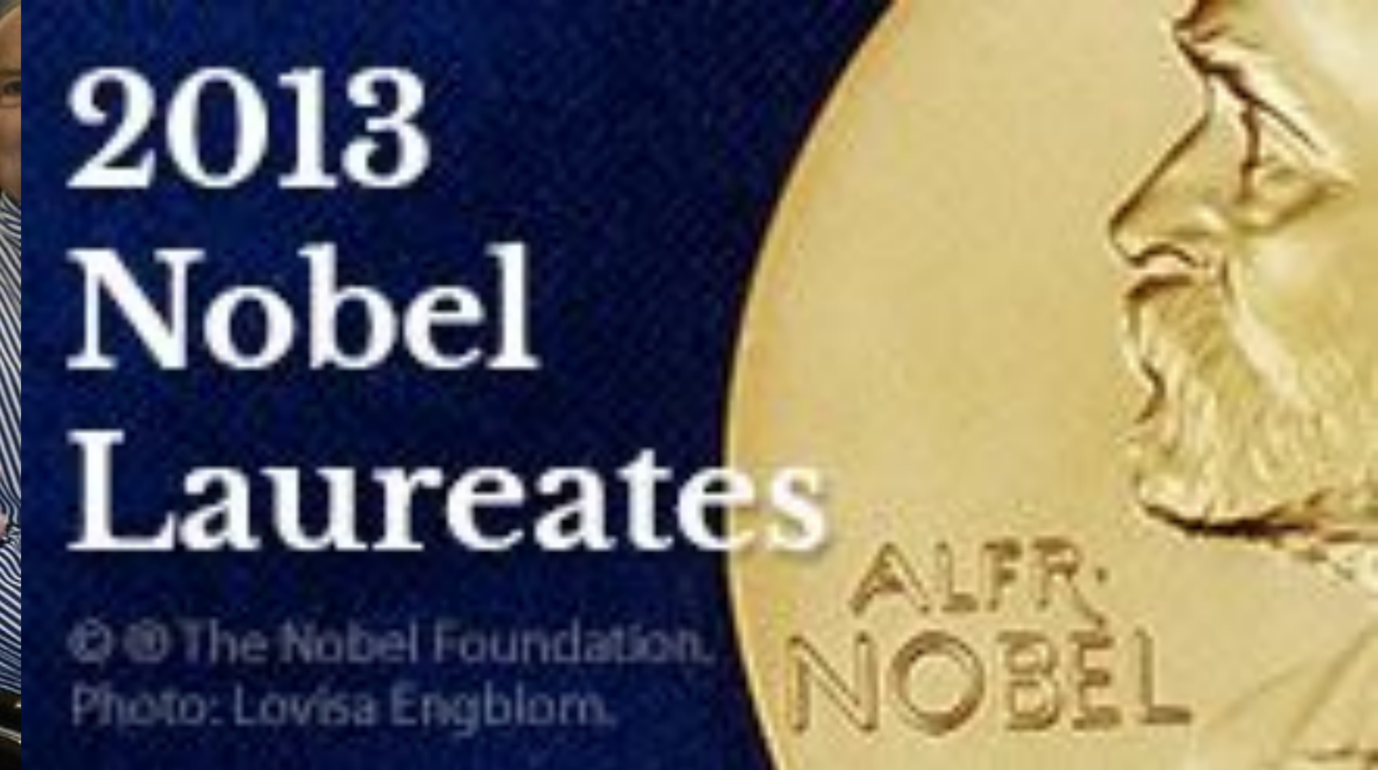


The combined signa

ATLAS: 5

Phys. Lett. B716, 1 (2012)

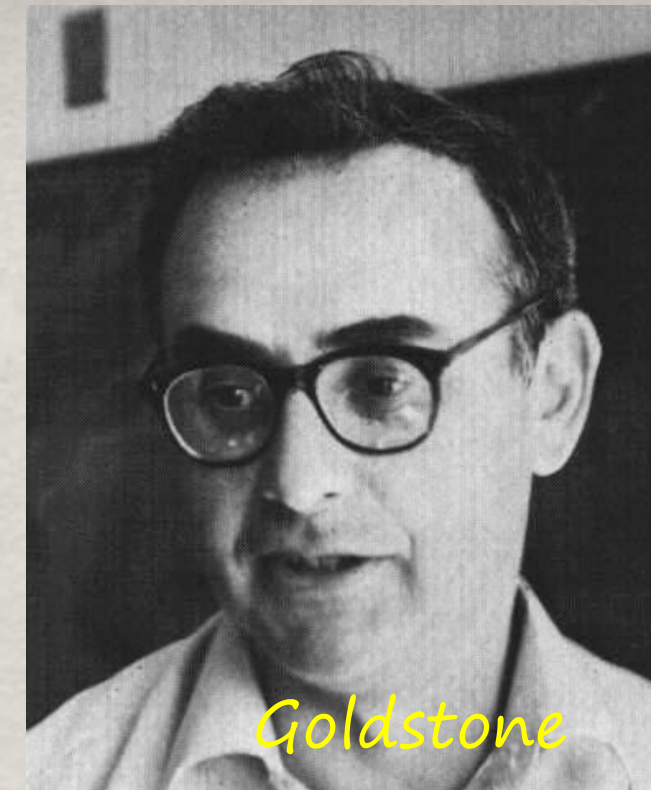
Phys. Let

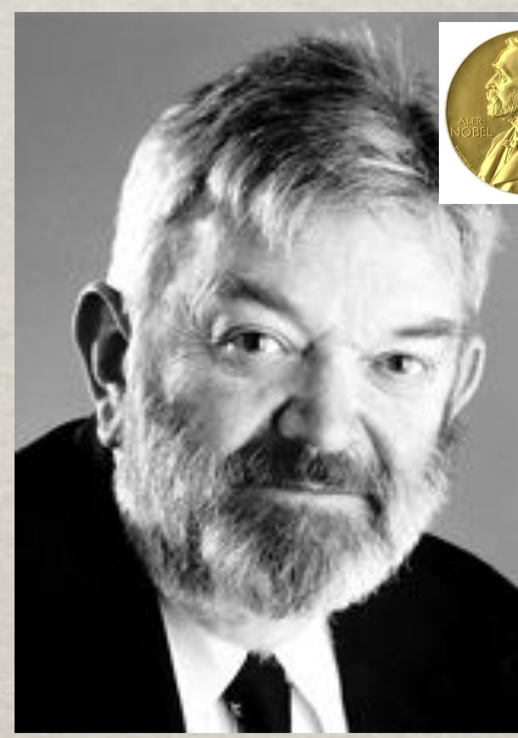
François Englert and Peter W. Higgs

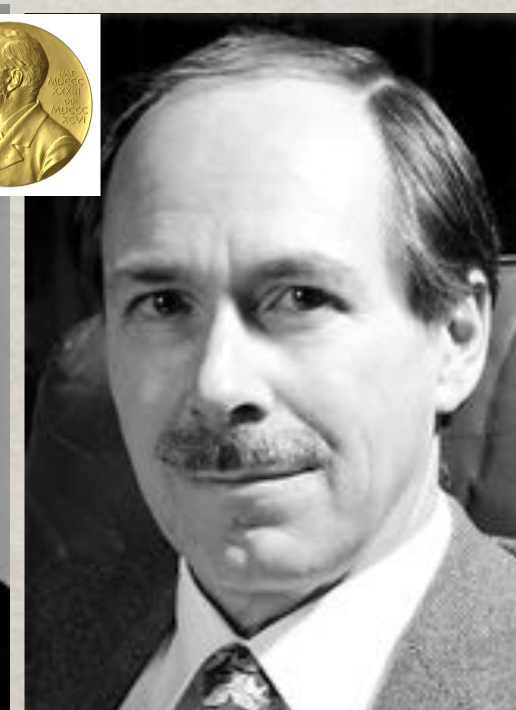
"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

3

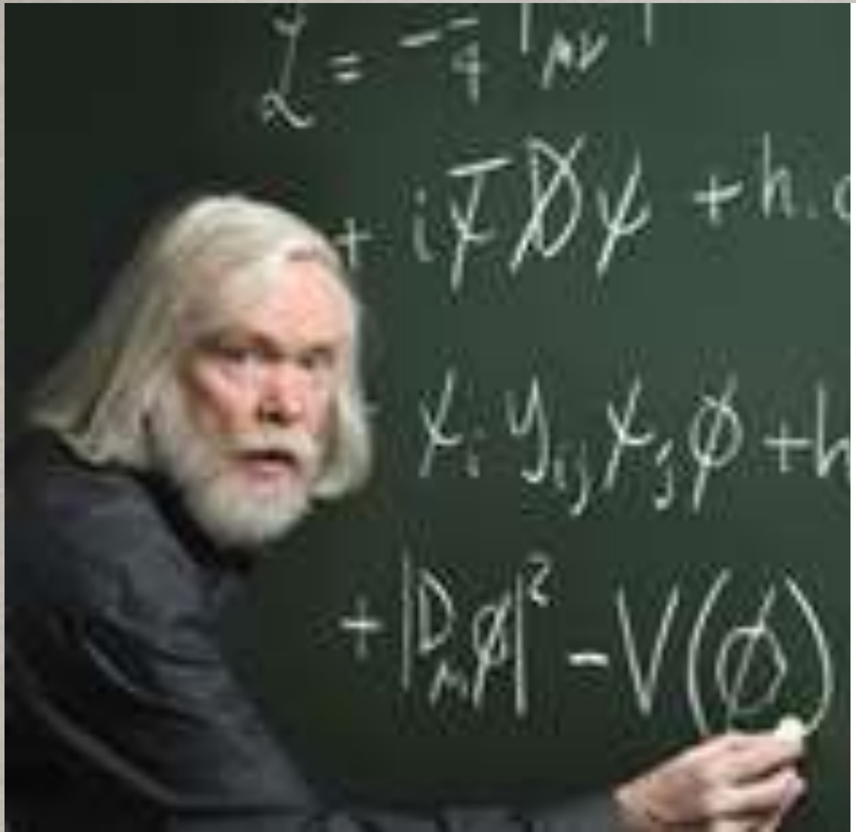


The Higgs mechanism (1964)





The Standard Model (1960-1967, 1972)



A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

Received 7 November 1975

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of the Higgs boson, we give a speculative cosmological argument for a small mass. If its mass is similar to that of the pion, the Higgs boson may be visible in the reactions $\pi^- p \to Hn$ or $\gamma p \to Hp$ near threshold. If its mass is $\lesssim 300$ MeV, the Higgs boson may be present in the decays of kaons with a branching ratio $O(10^{-7})$, or in the decays of one of the new particles: $3.7 \to 3.1 + H$ with a branching ratio $O(10^{-4})$. If its mass is $\leqslant 4$ GeV, the Higgs boson may be visible in the reaction $pp \to H + X$, $H \to \mu^+\mu^-$. If the Higgs boson has a mass $\leqslant 2m_\mu$, the decays $H \to e^+e^-$ and $H \to \gamma\gamma$ dominate, and the lifetime is $O(6 \times 10^{-4}$ to $2 \times 10^{-12})$ seconds. As thresholds for heavier particles (pions, strange particles, new particles) are crossed, decays into them become dominant, and the lifetime decreases rapidly to $O(10^{-20})$ sec for a Higgs boson of mass 10 GeV. Decay branching ratios in principle enable the quark masses to be determined.

Higgs Phenomenology (70's 50 years theory work

Fermi National Accelerator Laboratory

FERMILAB-Pub-84/17-T
LBL-16875
DOE/ER/01545-345
February, 1984

The "EHLQ" (80's)

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FRONTIERS IN PHYSICS

COLLIDER PHYSICS

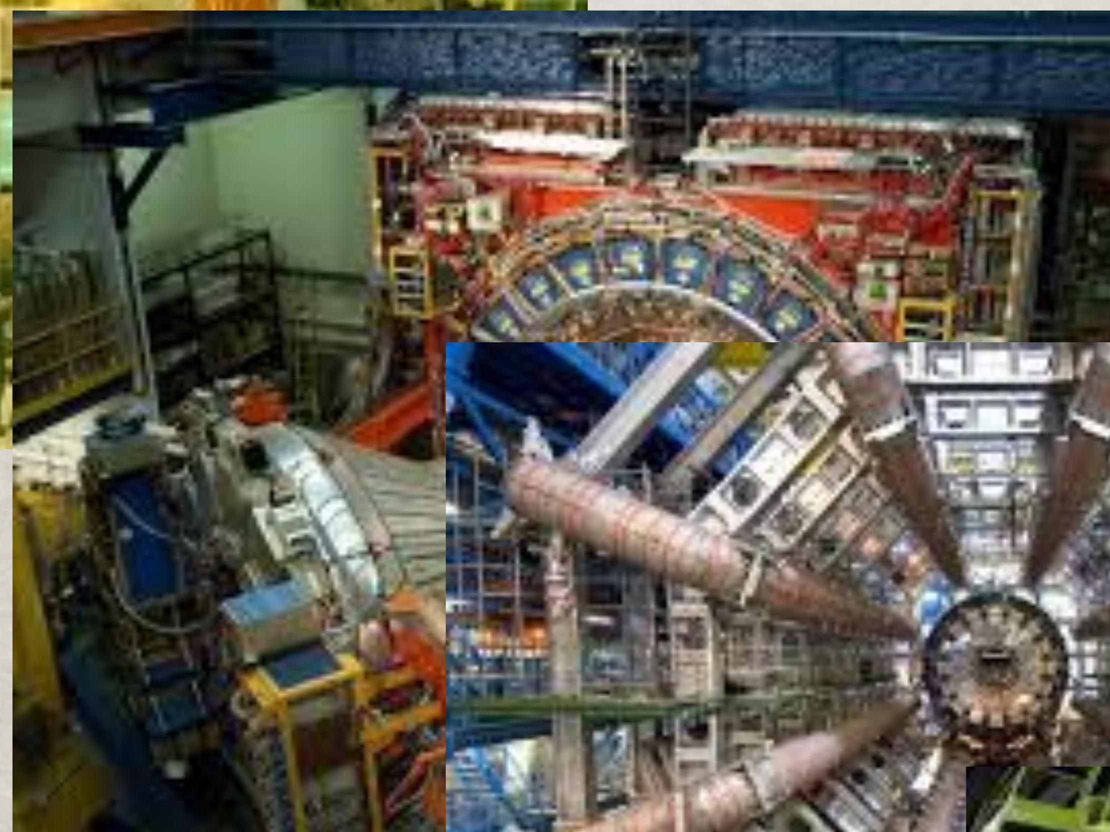
FRONTIERS IN PHYSICS

THE HIGGS HUNTER'S GUIDE

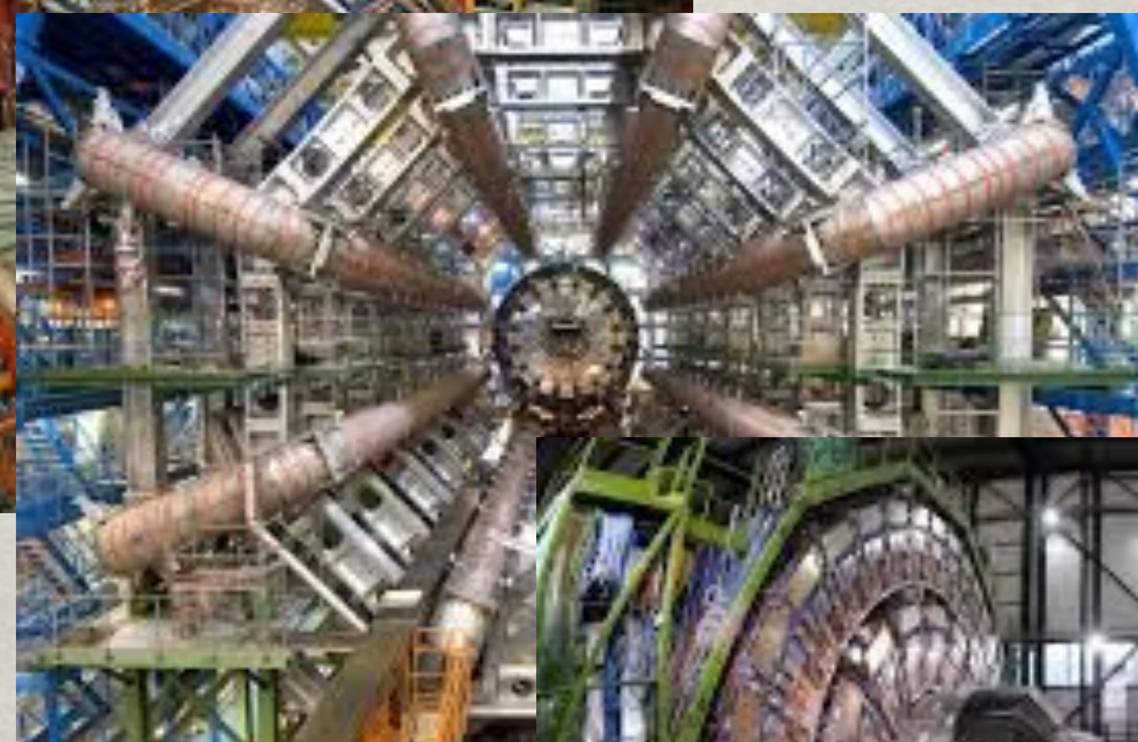
25 year's work by thousands experimenters

We made it !

ALEPH@LEP



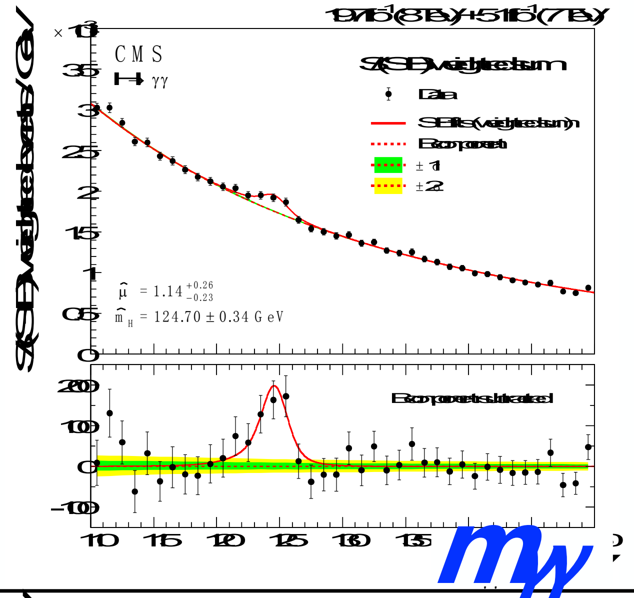
CDF@Tevatron



ATLAS

CMS

LHC Run I Update:

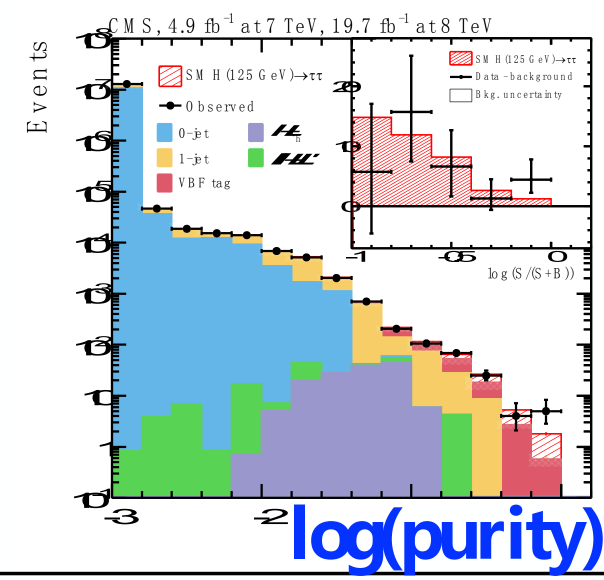


$\gamma\gamma$

$N_{sig} \approx 170$

$N_{bkg} \approx 5000$

5 - 6σ obs.

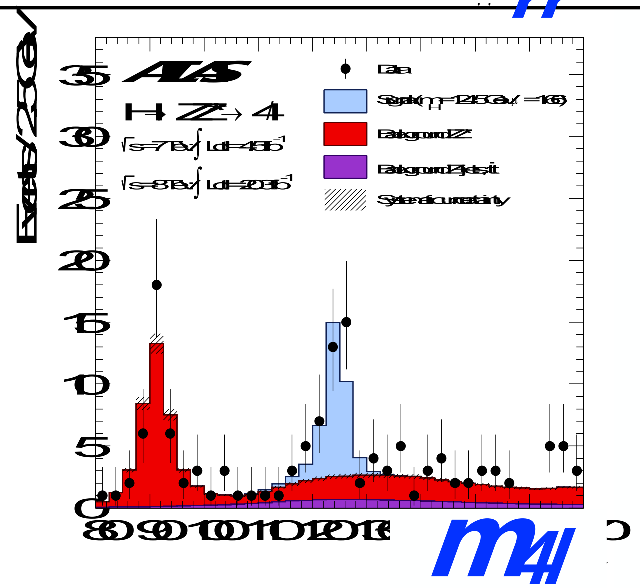


$\tau\tau$

$N_{sig} \approx 650$

$N_{bkg} \approx$ huge

3 - 4σ observed

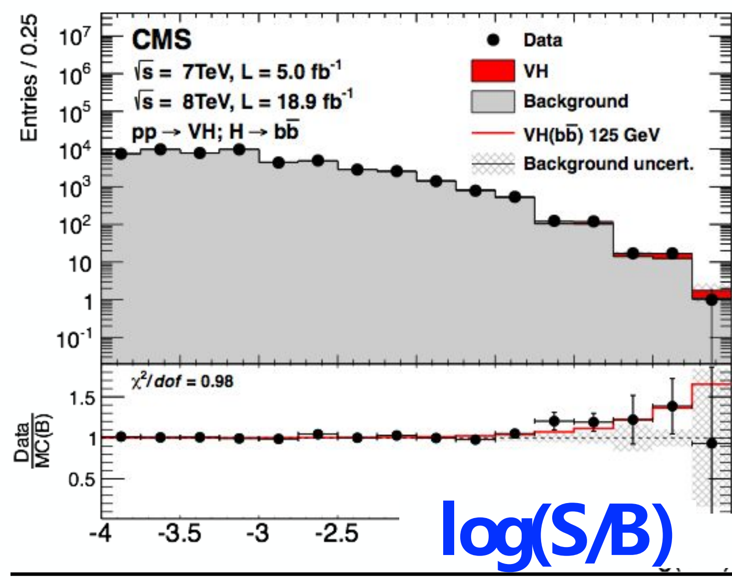


ZZ

$N_{sig} \approx 16$

$N_{bkg} \approx 10$

6 - 7σ obs.

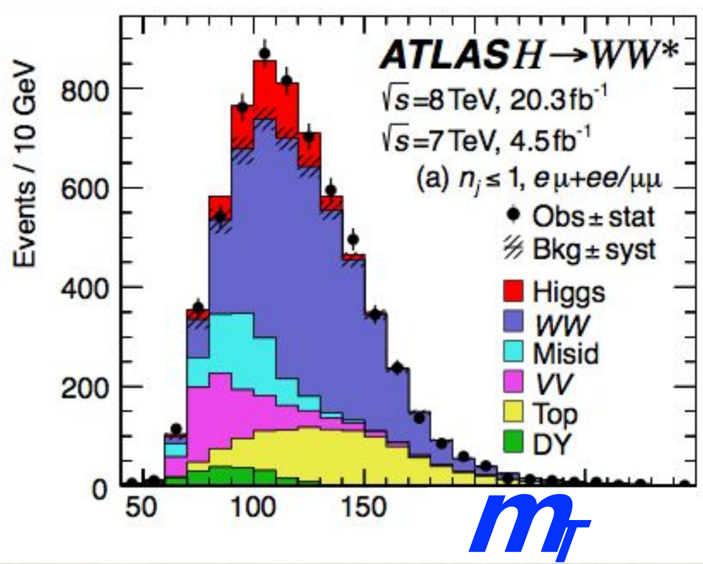


bb

$N_{sig} \approx 60$

$N_{bkg} \approx$ huge

1 - 2σ observed

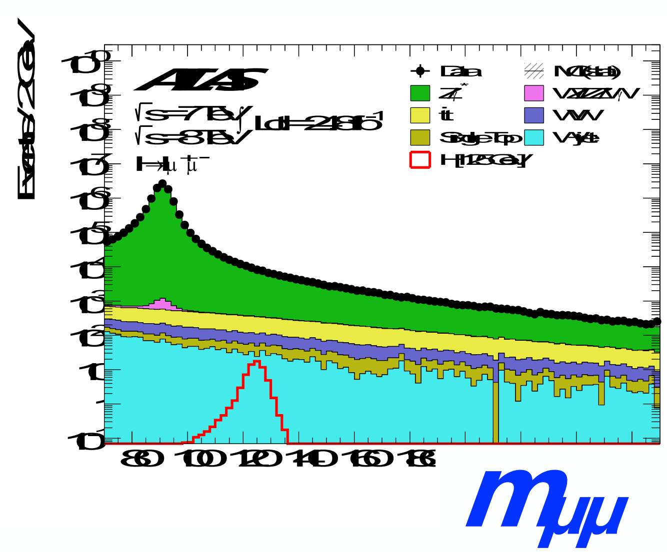


WW

$N_{sig} \approx 500$

$N_{bkg} \approx 7000$

5 - 6σ obs.



$\mu\mu$

$N_{sig} \approx 30$

$N_{bkg} \approx$ huger

< 1σ expected

What We Know

1. X   γγ :

- it's neutral, a boson $\frac{f_s}{\Lambda} H A^{\mu\nu} A_{\mu\nu}$
- can be spin-0
- cannot be spin-1 (Landau-Yang's theorem)
- can be spin-2 $\frac{f_T}{\Lambda} T_{\mu\mu'}\, g_{\nu\nu'} A^{\mu\nu} A^{\mu'\nu'}$
  unlikely/disfavored

2. X   ZZ, W+W- :

- Vacuum Q#: EWSB $(v+H)^2\, g^2 V^{\mu} V_{\mu}$
- CP-odd part of gauge interaction must be small $\frac{f_A}{\Lambda} A\, \tilde{V}^{\mu\nu} V_{\mu\nu}$

3. X not to $\mu^+\mu^-$, e^+e^-, but $\tau^+\tau^-$ seen!

- Non-universal leptonic couplings unlike the gauge couplings

$$(1 + H/v)\, m_f \bar{\psi}_f \psi_f$$

4. XEt needed for gluon fusion

X $\bar{b}b$ seen (vaguely)

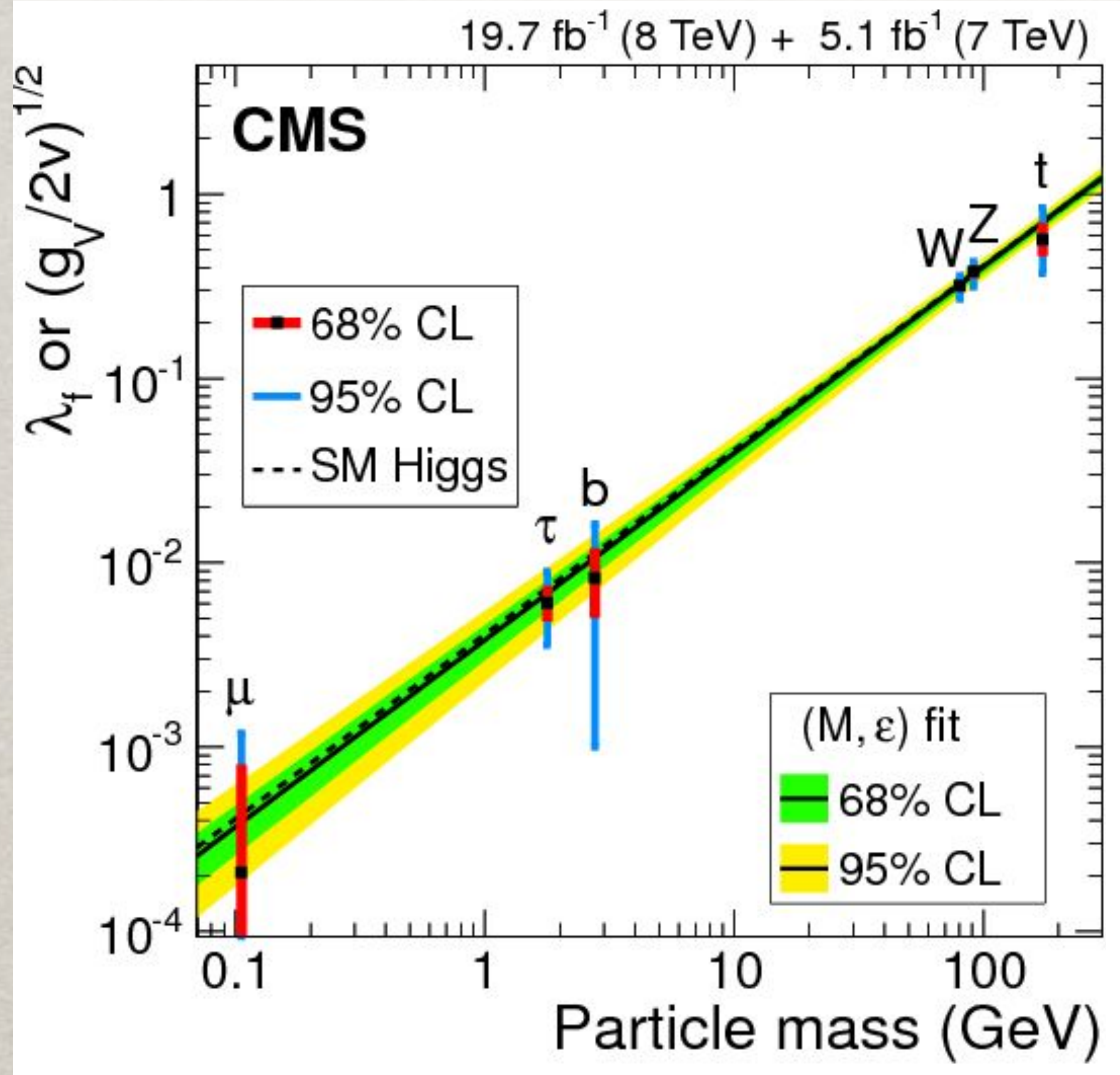
- Non-universal quark couplings

It couples to mass, it is a new class

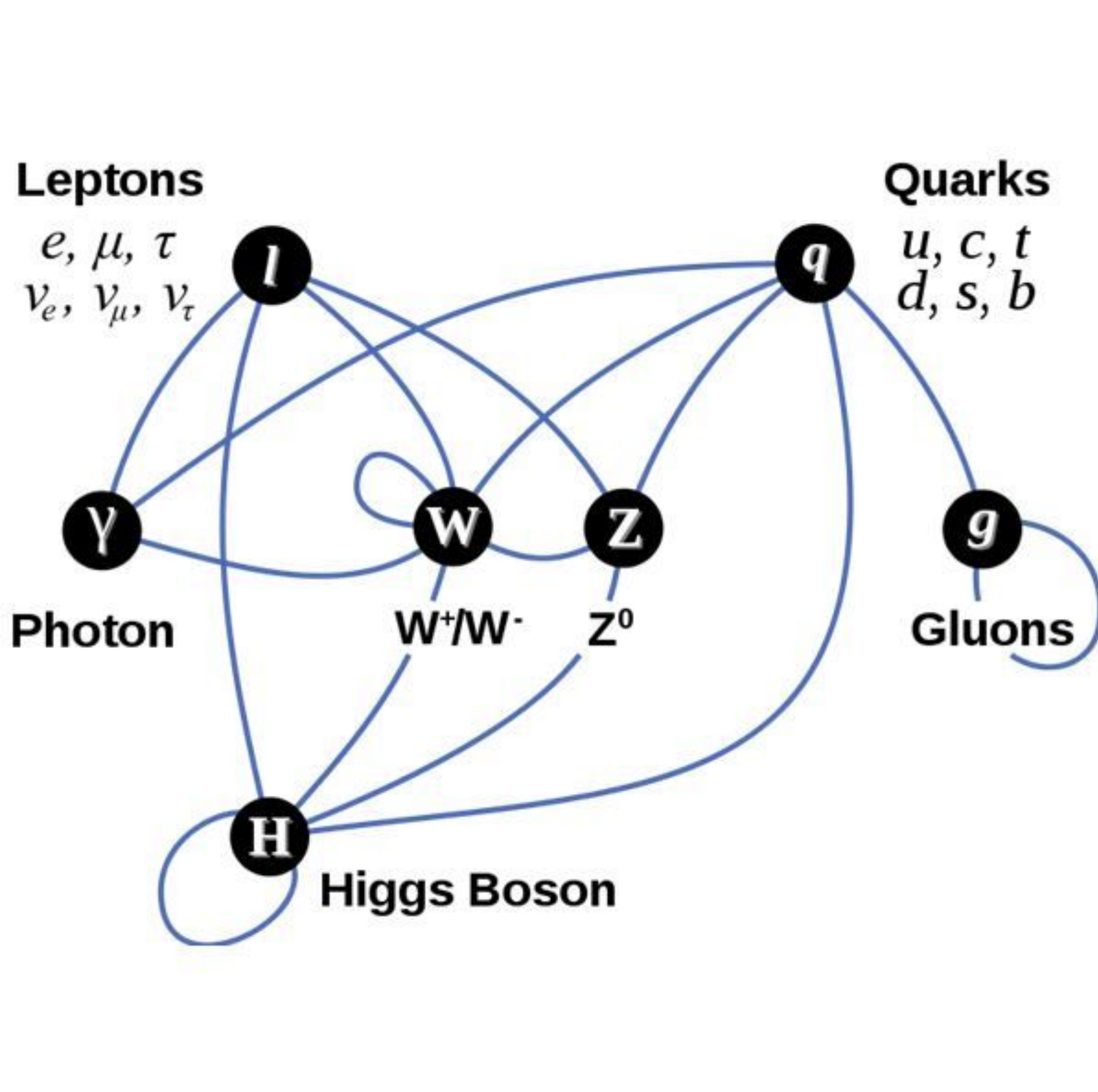
It IS a Higgs!

The SM (like) ?

Need further quantitative verification:



If no more than a few% deviations, I'd DEFINE it the SM Higgs!



Completion of the SM:
A perturbative, renormalizable theory, valid up to a scale
TeV ? ..., M_{Pl} ?

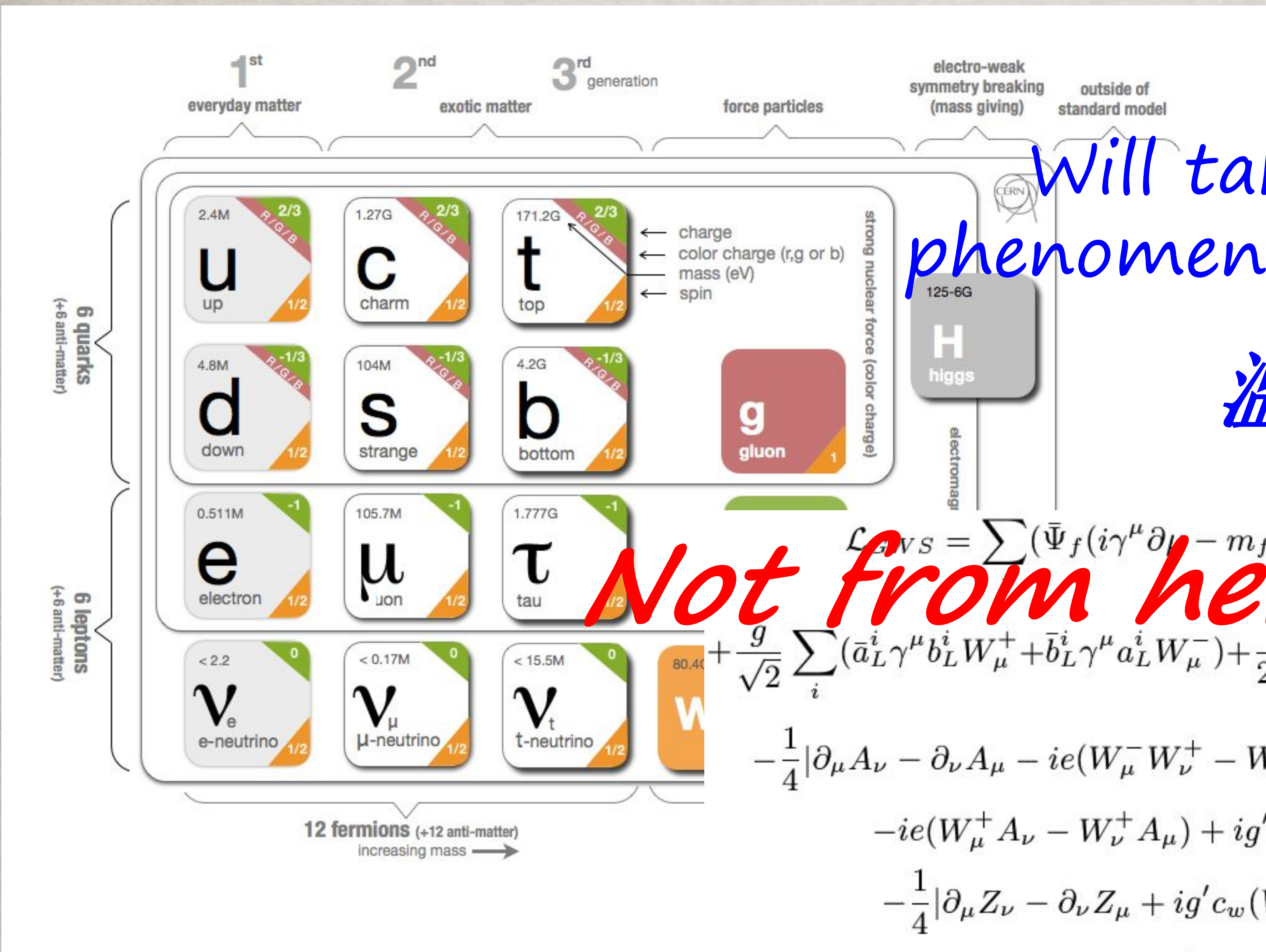
All known physics

$$W = \int_{k<\Lambda} [\mathcal{D}g \ldots] \, \exp\left\{\frac{i}{\hbar}\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{4}F^2 + \bar{\psi} i \not{D}\psi - \lambda\phi\bar{\psi}\psi + |D\phi|^2 - V(\phi)\right]\right\}$$

amplitude — current understanding — quantum mechanics — spacetime — gravity — strong & electroweak — matter — Higgs

These Lectures:

The Standard Model



Will take a historical, phenomenological approach

温故知新

$$\mathcal{L}_{GWS} = \sum_f (\bar{\Psi}_f(i\gamma^\mu\partial_\mu - m_f)\Psi_f - eQ_f\bar{\Psi}_f\gamma^\mu\Psi_f A_\mu)+$$

$$+\frac{g}{\sqrt{2}}\sum_i(\bar{a}^i_L\gamma^\mu b^i_L W^+_\mu + \bar{b}^i_L\gamma^\mu a^i_L W^-_\mu) + \frac{g}{2c_w}\sum_f \bar{\Psi}_f\gamma^\mu(I^3_f - 2s^2_w Q_f - I^3_f\gamma_5)\Psi_f Z_\mu +$$

$$-\frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 - \frac{1}{2}|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu +$$

$$-ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + ig'c_w(W^+_\mu Z_\nu - W^+_\nu Z_\mu|^2 +$$

$$-\frac{1}{4}|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig'c_w(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 +$$

$$-\frac{1}{2}M^2_\eta\eta^2 - \frac{gM^2_\eta}{8M_W}\eta^3 - \frac{g'^2M^2_\eta}{32M_W}\eta^4 + |M_W W^+_\mu + \frac{g}{2}\eta W^+_\mu|^2 +$$

$$+\frac{1}{2}|\partial_\mu\eta + iM_Z Z_\mu + \frac{ig}{2c_w}\eta Z_\mu|^2 - \sum_f \frac{g}{2}\frac{m_f}{M_W}\bar{\Psi}_f\Psi_f\eta$$

Not from here!

Outline

Lecture I: The Making of the SM

A. Deep Root in QED
B. The Strong Nuclear Force QCD
C. The Weak Nuclear Force
D. Electro-Weak Unification: the SM

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking

B. The Nambu-Goldstone Theorem

C. The Higgs Mechanism

D. The Higgs Boson Interactions

Lecture III: Higgs Physics and Beyond

A. What Does THIS Higgs Tell us?
B. SM Higgs Sector at Higher Energies
   & the Need for New Physics
C. Higgs Boson Decays
D. Higgs Physics at the LHC Colliders
E. Higgs Physics at an e+e- Collider

Lecture I: The Making of the SM

A. Deep Root in QED

B. The Strong Nuclear Force QCD till here 90 min!

C. The Weak Nuclear Force

D. Electro-Weak Unification: the SM

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking

B. The Nambu-Goldstone Theorem + 90 min

C. The Higgs Mechanism

D. The Higgs Boson Interactions

Lecture III:

A. What Does THIS Higgs Tell us?

B. SM Higgs Sector at Higher Energies & the Need for New Physics + 120 min

Skipe below …

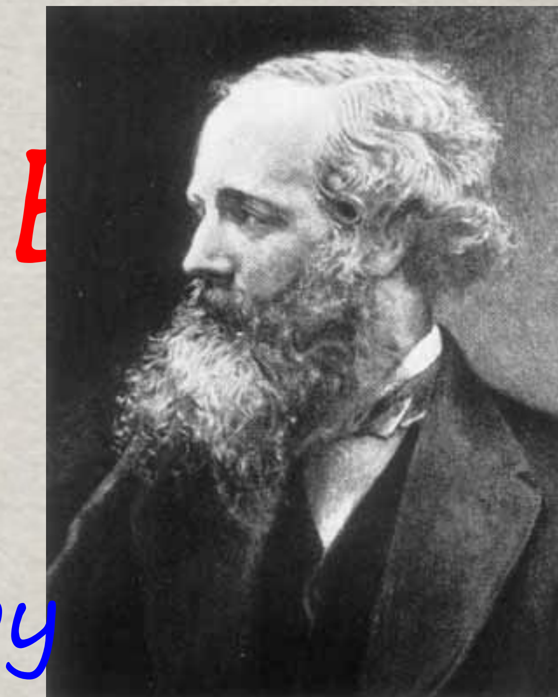
C. Higgs Boson Decays

D. Higgs Physics at the LHC Colliders

E. Higgs Physics at an e+e- Collider

Lect 1. The Making of the SM

A. Deep Root in QE



Maxwell Equations → Lorentz invariance, U(1) Gauge Invariance

Electromagnetic fields can be treated by **E**(x,t), **B**(x,t)
the introduction of co-variant vector potential $A_\mu(x,t)$
makes the symmetries manifest (but redundant)

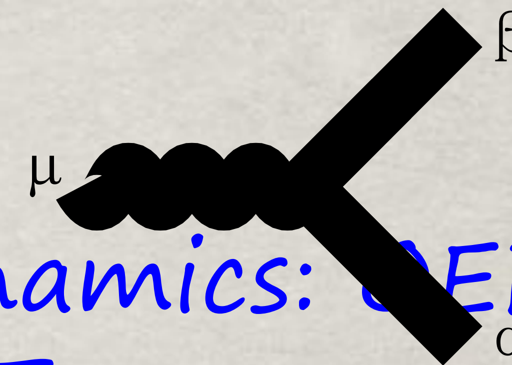
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

1). Lorentz/Local Gauge invariance manifest.
2). Classically, geometrical interpretation: fiber bundles...
3). Quantum-mechanically, wave function for the EM field.

Dirac's relativistic theory:
Lorentz/Local gauge invariant ➔
antiparticle e^+

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

$$D_\mu = \partial_\mu + ieA_\mu$$



$= (+\, ie\, \gamma^\mu)_{\beta\alpha}$

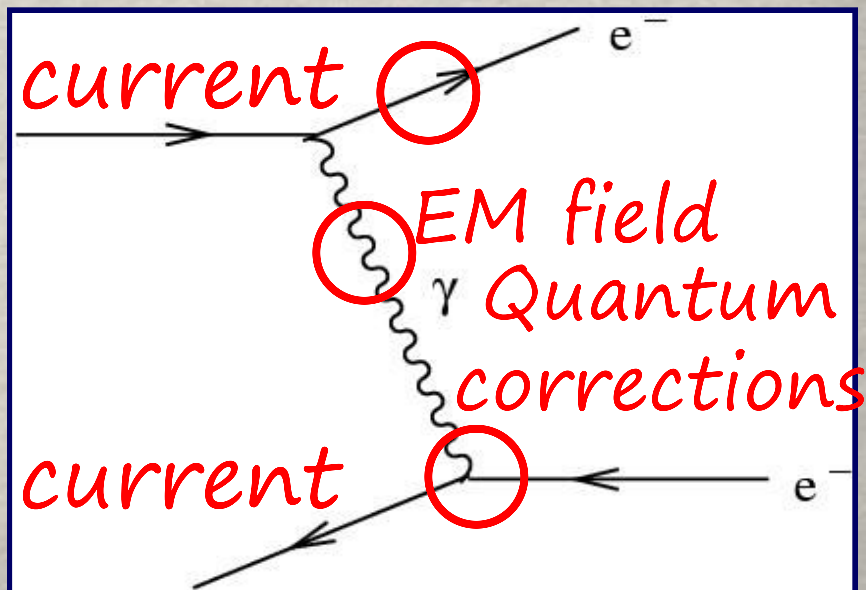
Quantum Electro-Dynamics: QED
Feynman/Schwinger/Tomonaga ➔
Renormalization

$$a_e(\text{Schwinger}) \approx \frac{\alpha}{2\pi} \approx 0.0011614$$

$$a_e^{theo} = 0.001159652181643(763)$$

$$a_e^{exp} = 0.00115965218073(28)$$

QED becomes the most accurate theory in science!



Warmup Exercise 1:

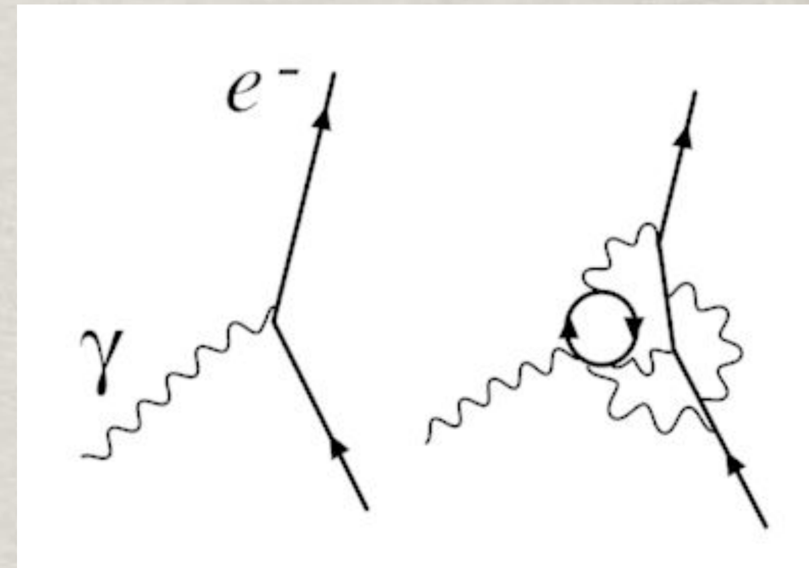
For charge scalar field $\phi^{\pm}$, construct the locally $U(1)_{em}$ gauge invariant Lagranian and derive the Feynman rules for its EM interactions.

Sketch a calculation for the differential and total cross section for the process:

$$e^+ e^- \qquad \phi^+ \phi^-$$

renormalization: Running of coupling with energies

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = \frac{\partial g}{\partial \ln \mu},$$



$$\beta(e) = \frac{e^3}{12\pi^2}$$

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\alpha(Q_0^2)}{3\pi} \ln(Q^2/Q_0^2)}$$

$$\alpha_{QED}(keV) = 1/137$$

$$\alpha_{QED}(M_Z) = 1/128$$

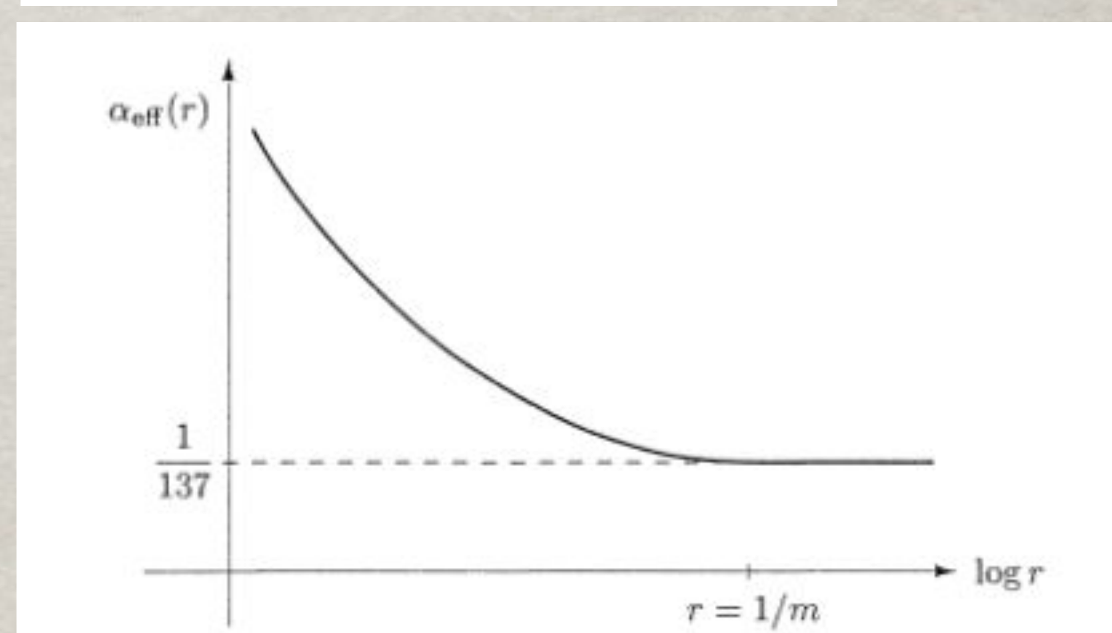


Figure 7.10. A qualitative sketch of the effective electromagnetic coupling constant generated by the one-loop vacuum polarization diagram, as a function of distance. The horizontal scale covers many orders of magnitude.

The Landau pole:
It blows up at high energies!
Must be modified at UV.

B. There Is a strong force!

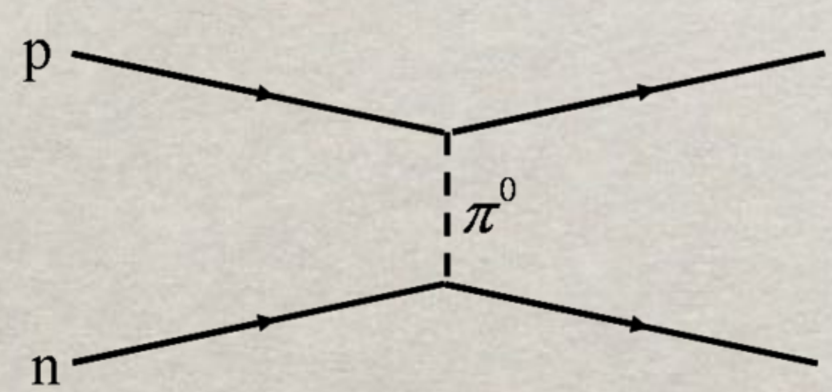
Ever since Rutherford established the atomic nuclear model → a new force to bound p^+ to a nucleus.

The discovery neutron (1932) → a charge-independent force:

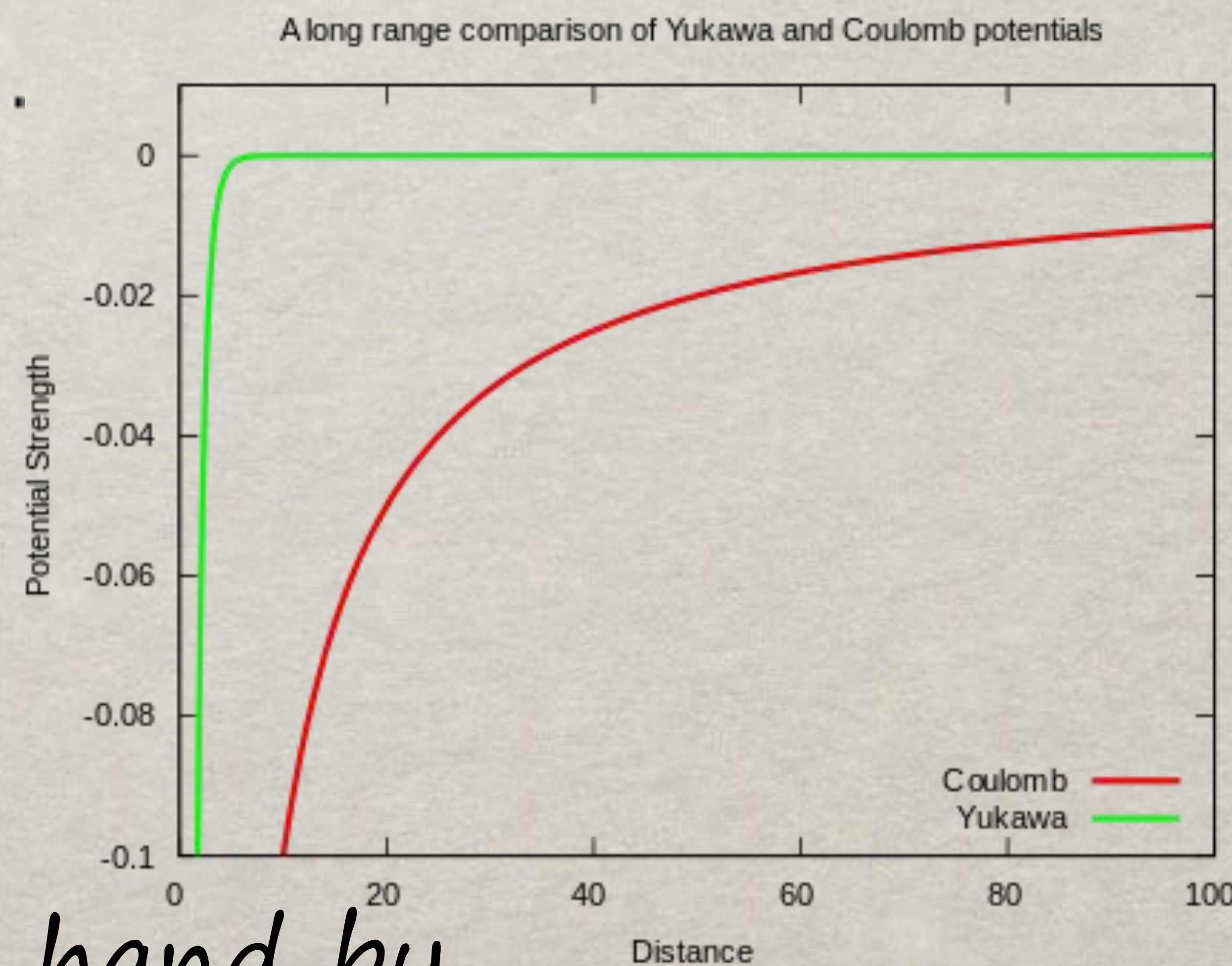
Heisenberg → (p^+, n^0) "iso-spin" doublet

Yukawa (1935) → a universal attractive force via pions

$$\mathcal{L}_{\text{int}}(x) = g\bar{\psi}(x)\phi(x)\psi(x).$$



$$V_{\text{Yukawa}}(r) = -g^2\,\frac{e^{-mr}}{r}$$



Discoveries & theory hand by hand!

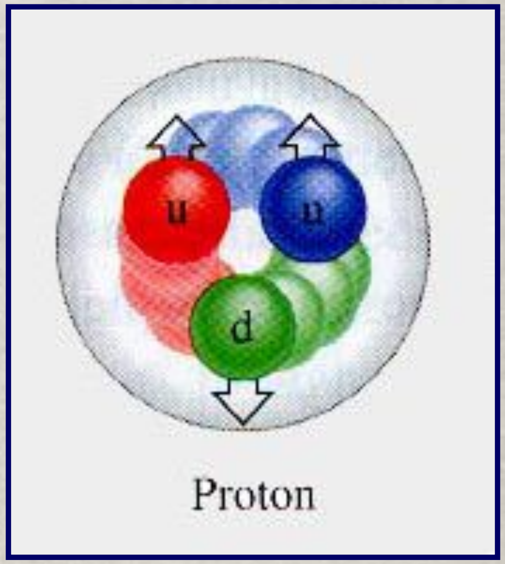
What IS the strong force?

Numerous "hadrons" discovered (50's):

Mesons: π, η, ρ, ω … Baryons: p, n, Δ, Λ, Σ …

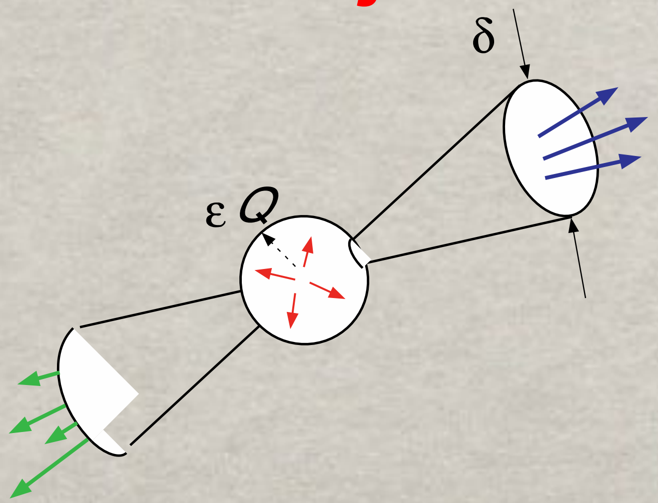
How to understand/describe them?

Hadronic string theory developed.

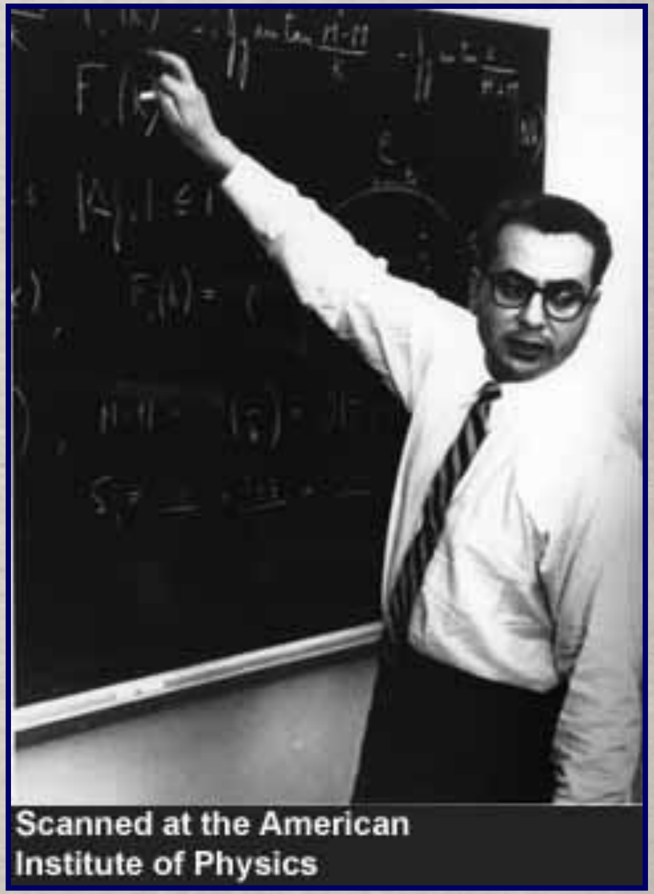


Not until:

- Gell-Mann – Zweig's "quarks" (1963)
- π ϒϒ 3 colors (1964)
- Proton structure by DIS (1969)
- 2 or 3-jet structure (q: 1975, g: 1979)



SU(3)$_C$ gauge theory established (1973)



Quantum-Chromo-Dynamics (QCD)

H. Fritzsch, M. Gell-Mann, H. Leutwyler (1973)

$$\mathcal{L} = \sum_f^{n_f} \bar{q}_f (i\gamma^\mu \partial_\mu - g_s \gamma^\mu A_\mu + m_f) q_f - \frac{1}{2} \mathrm{Tr} F_{\mu\nu}^2$$

Direct analogue of QED

$$F_{\mu\nu} = \partial\mu A_\nu - \partial\mu A_\nu + i g_s [A_\mu, A_\nu]$$

← Non-Abelian

$$A^\mu(x) = \sum_1^8 A(x)_a^\mu \, T^a, \quad [T^a, T^b] = i f_{abc} T^c.$$

QED analogue:

- Similar gauge principles;
- Tempting for perturbative renormalization calcu

Non-Abelian gauge theory: Yang-Mills

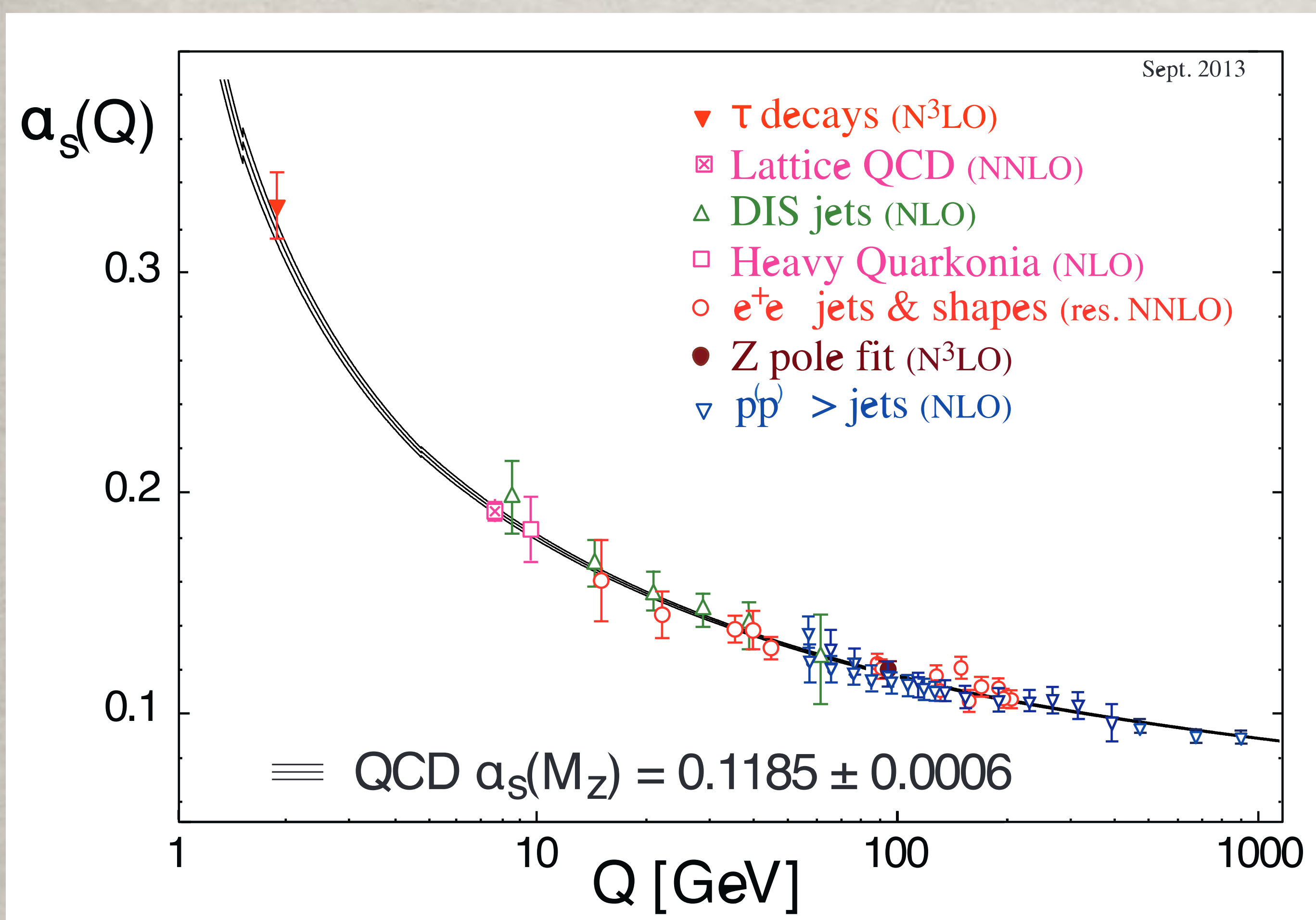
- Self gauge interactions among 8 gluons;
- Coupling rather strong, may invalidate perturbatic

Remarkable Features:

IR confinement & UV asymptotic freedom

Interaction strength changes fast with energy/distance scale:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi}\ln(Q^2/Q_0^2)}$$

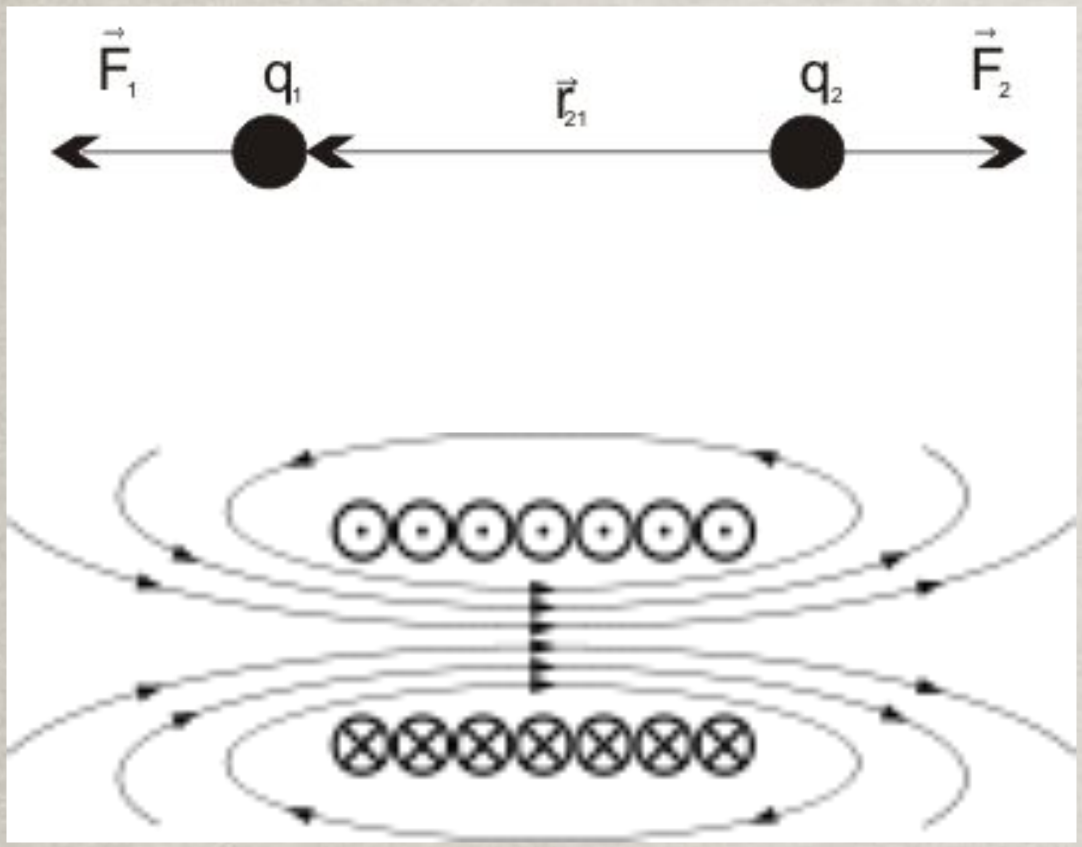


D. Gross,
F. Wilczek,
D. Politzer
(2004)

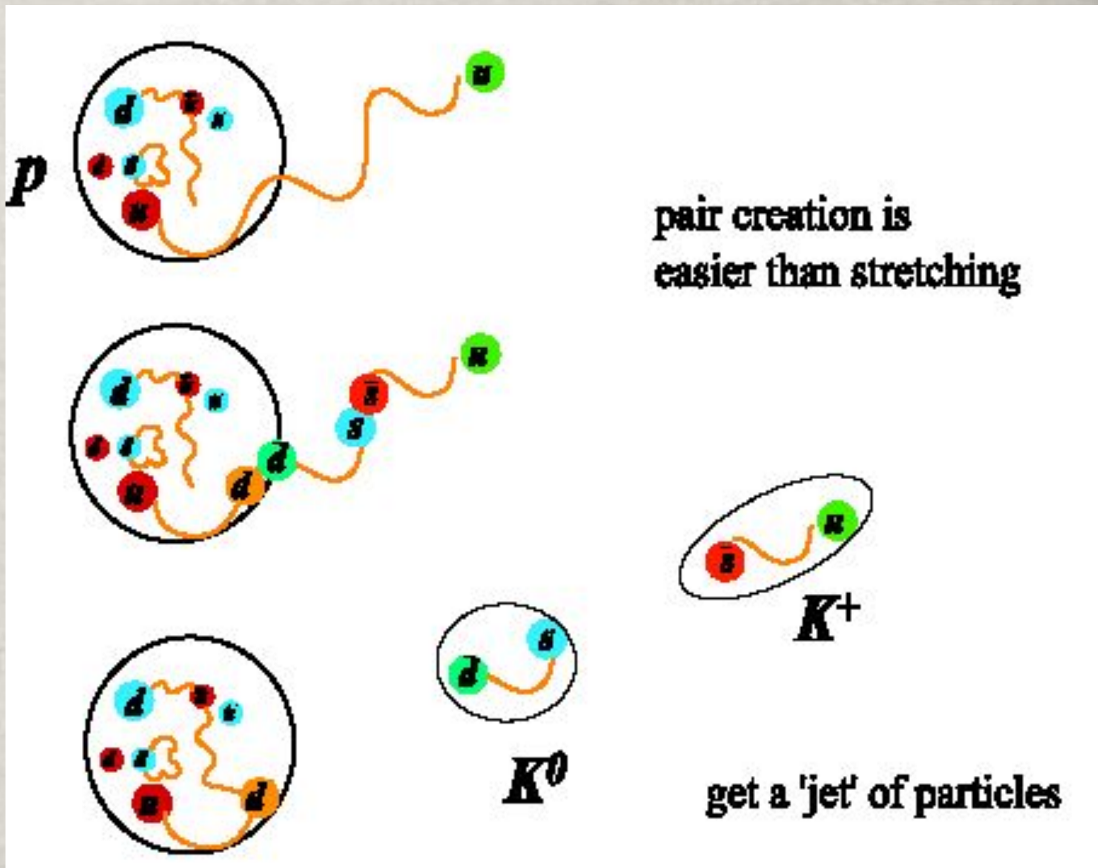
T. Han

QED versus QCD

Electromagnetism vs. Strong force



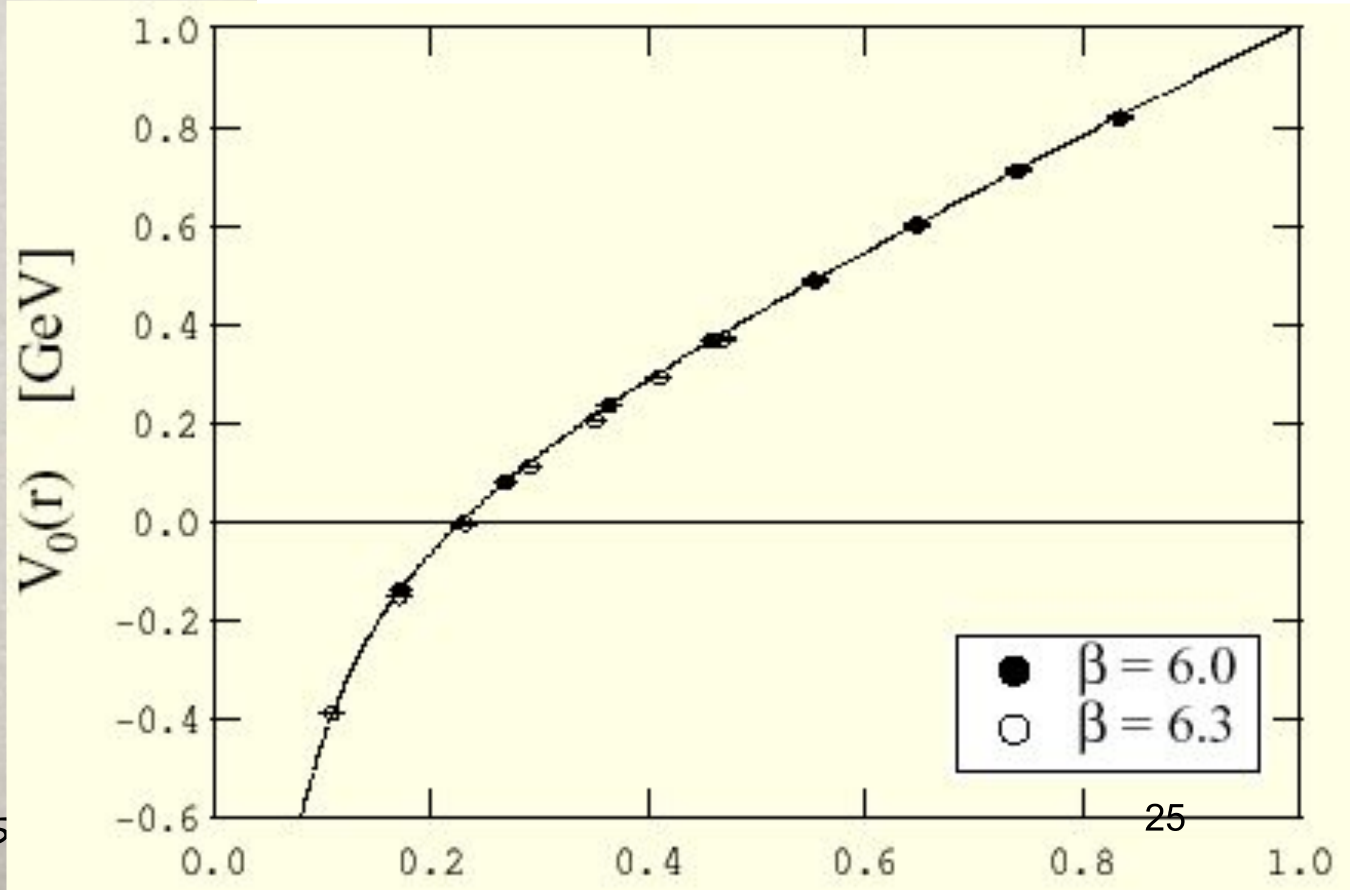
Photons
vs.
gluons



QED: $V(r) = -\alpha_{em}/r$

QCD: $V(r) = -\alpha_s/r + kr$

In long distances, we see charged particles, but not colored particles!



QCD at Low Energies: Quark condensation

Consider the two-flavor massless QCD

$$-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} - \sum_{u,d}(\bar{q}_L\gamma^\mu D_\mu q_L + \bar{q}_R\gamma^\mu D_\mu q_R)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \left(U_L\,\gamma_L + U_R\,\gamma_R\right)\begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow SU(2)_L \otimes SU(2)_R$$

QCD exhibits a L-R chiral symmetry.

Below Λ_{QCD}, QCD becomes strongly interacting and forms condensate:

$$\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \sim v^3$$

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_V, \text{thus } U_L = U_R.$$

Chiral symmetry is broken to iso-spin.

The Spontaneous Symmetry Breaking

" The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry."

The concept of SSB: profound, common.

Known Example: Ferromagnetism

Domains Before Magnetization

Above a critical temperature, the system is symmetric, magnetic dipoles randomly oriented. Below a critical temperature, the ground state is a completely ordered configuration in which all dipoles are ordered in some arbitrary direction,

Domains After Magnetization

SO(3) SO(2)

$(3+3) - 3 = 3$ Nambu–Goldstone bosons:

π^+, π^-, π^0 (u, d bound states)

In the non-linear formulation of the Chiral Lagrangian for the Goldstone bosons:

$$\phi = \frac{v}{\sqrt{2}} \exp(i\vec{\tau} \cdot \vec{\pi}/v) \equiv \frac{v}{\sqrt{2}} U, \quad \mathcal{L} = \frac{v^2}{4} Tr(\partial^\mu U \partial_\mu U)$$

necessarily derivative coupling.

Exercise 5: Linearize the Chiral Lagrangian for ππ interaction and calculate one scattering amplitude.¶

¶ J. Donoghue et al., Dynamics of the SM.

Exercise continued: The pion-pion scattering:

$$\pi_i + \pi_j \to \pi_k + \pi_l \quad (i,j,k,l = 1,2,3)$$

$$T(s,t,u) = A\ \delta_{ij}\delta_{kl} + B\ \delta_{ik}\delta_{jl} + C\ \delta_{il}\delta_{kj}.$$

Crossing symmetry, Bose symmetry

A(s,t) Low Energy theorem:

$$\mathcal{L} \sim \frac{1}{v^2}(\pi_i \partial_\mu \pi_i)^2 \quad \Rightarrow \quad A(s,t) = \frac{s}{v^2}$$

Chiral perturbation theory agrees well with the pion-pion scattering data,¶ supporting the Goldstone nature.

Y. Nambu was the first one to have formulated
the spontaneous symmetry breaking in a relativistic
quantum field theory (1960).

He is the one to propose the understanding of the
nucleon mass by dynamical chiral symmetry breaking: The Nambu-Jona-Lasinio Model.

2008 Nobel Prize in physics: "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Be aware of the difference between the dynamical mass for baryons (you and me) and that of elementary particles by the Higgs mechanism.

“Pseudo-Nambu-Goldstone Bosons”

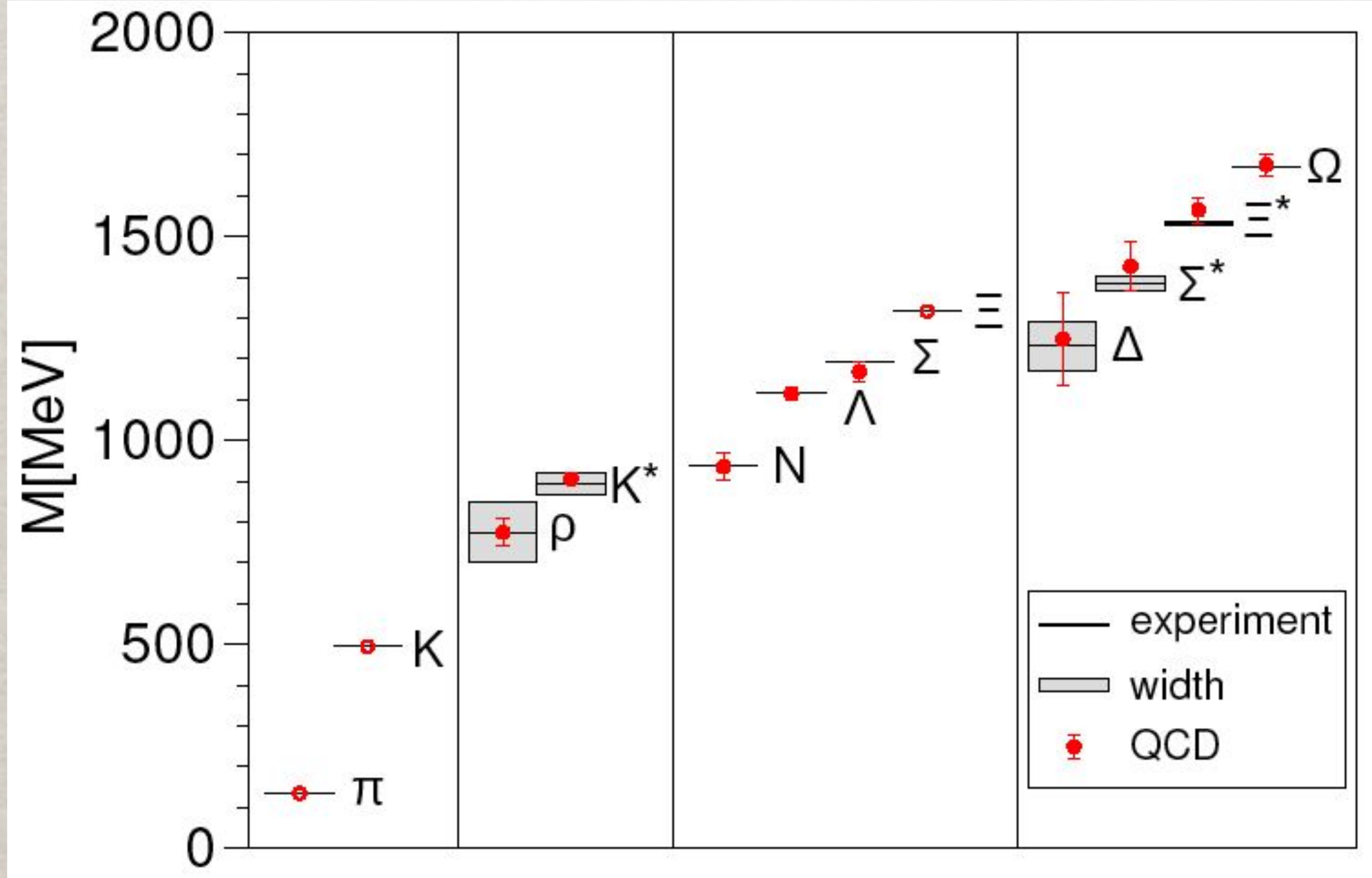
When a continuous symmetry is broken both explicitly AND spontaneously, and if the effect of the explicit breaking is much smaller than the SSB, then the Goldstone are massive, governed by the explicit breaking, thus called:
“Pseudo-Nambu-Goldstone bosons”.

The pions are NOT massless, due to explicit symmetry breaking. They are “Pseudo-Nambu-Goldstone bosons”.

Except the photon, no massless boson
(a long-range force carrier) has been seen
in particle physics!

Most Mass due to QCD:

From quark constituents to hadrons:
(From PDG, based on lattice QCD)



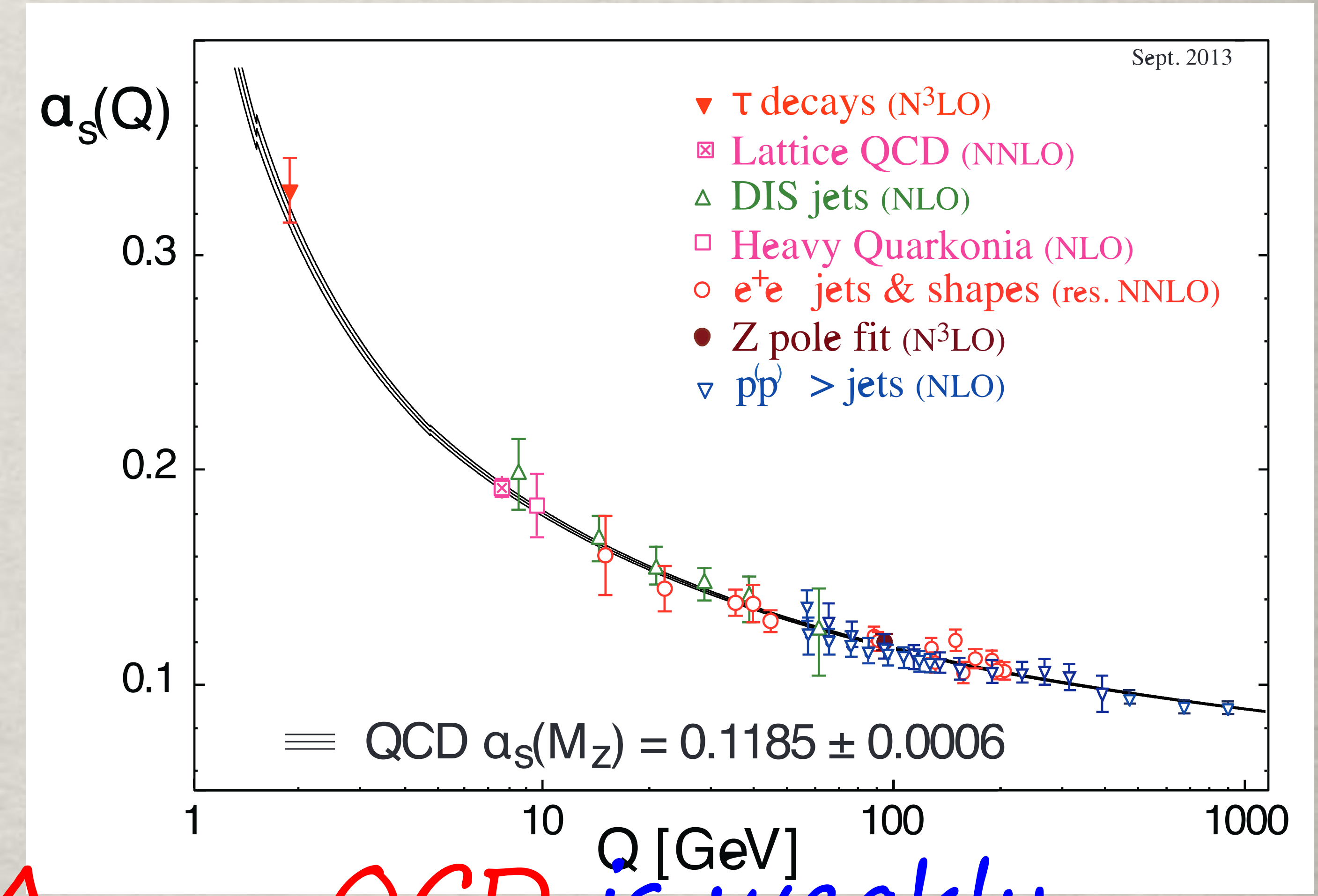
Majority of the (luminous) mass around us is of dynamical origin, from strong interactions (u, d quarks + gluons).
It is not from the Higgs mechanism!.

QCD at High Energies

Interaction strength changes fast with energy/distance scale:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi}\ln(Q^2/Q_0^2)}$$

$$\alpha_s(\mu') = \frac{\alpha_s(\mu)}{1 + b_0 \frac{\alpha_s(\mu)}{4\pi} \ln(\mu'^2/\mu^2)} \equiv \frac{4\pi}{b_0 \ln\left[\frac{\mu'}{\Lambda_{QCD}}\right]^2}.$$



At high energies, $E \gg \Lambda_{QCD}$, QCD is weakly interacting!

- Perturbative prediction for high energy experiments (ee, ep, pp etc. LHC …)
- Think about higher energy physics at M_{GUT}, M_{PL}
- Early universe cosmology at high T.

QCD Factorization Theorem:

J. Collins, D. Soper, G. Sterman (1985)

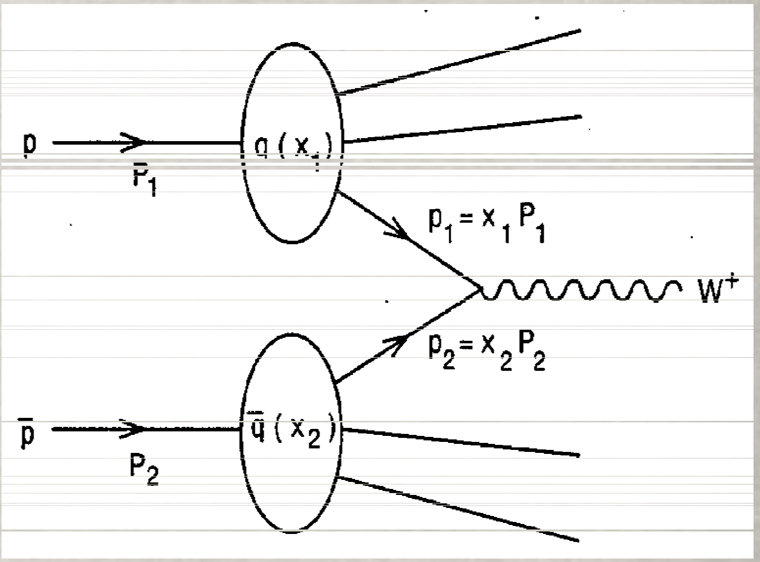
In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

(1). hard subprocess of parton scattering with a large scale $\mu^2 \gg \Lambda^2_{QCD}$;

(2). "parton distribution functions" (hadronic structure with $Q^2 < \mu^2$.)

Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$

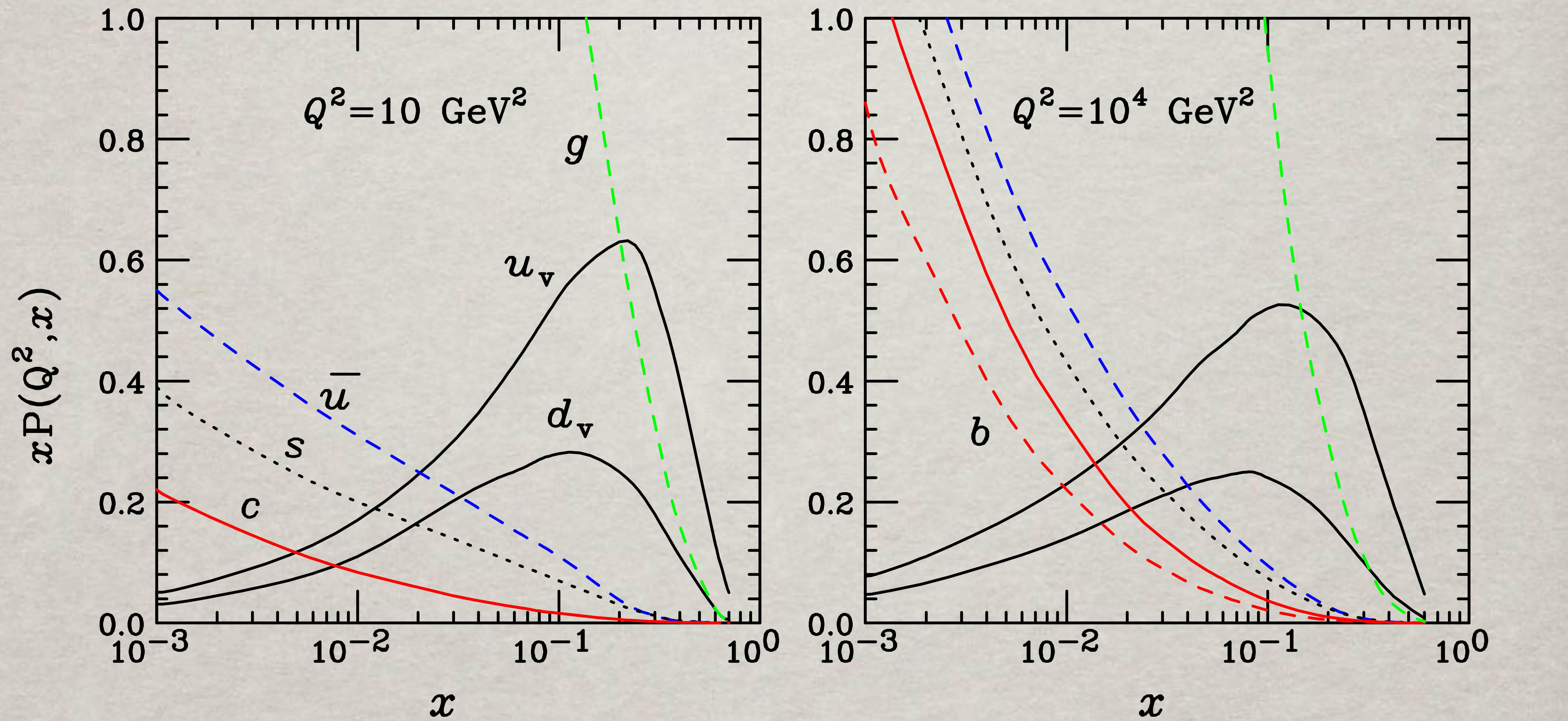


† $\hat{\sigma}_{parton}(s)$ is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;
Infra-red (IR) divergence is cancelled by soft gluon emissions;
Co-linear divergence (massless) is factorized into PDF

PDF's: *q(x, Q²), g(x, Q²), …*

Typical quark/gluon parton distribution functions:



(CTEQ-5)

Quarks carry ½ momentum; gluons carry the other ½

 CTEQ, MRS (Durham), NLOPDF etc.

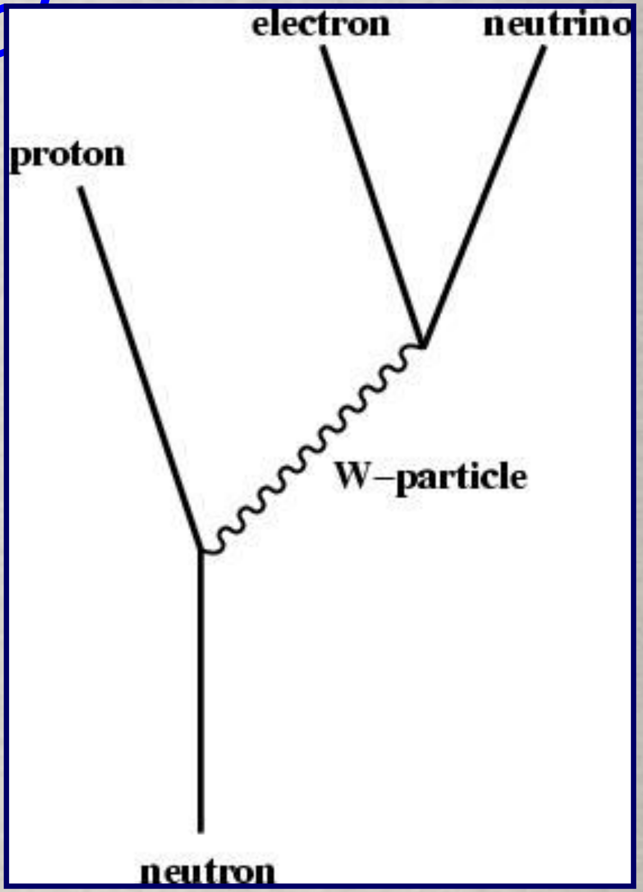
C. The Weak Nuclear Force

beta decay $n \rightarrow p^+ e^- \bar{\nu}$ ➔ Charged current interaction: $W^\pm$

$\nu N \rightarrow \nu N$ ➔ Neutral current interaction via: Z^0 (1973)

$$-\mathcal{L}^{cc}_{eff} = \frac{G_F}{\sqrt{2}} J^\mu_W J^\dagger_{W\mu}, \quad -\mathcal{L}^{NC}_{eff} = \frac{G_F}{\sqrt{2}} J^\mu_Z J_{Z\mu}.$$

$$J^{(\pm)}_\lambda = \sum \bar{\psi}_i \tau_\pm \gamma_\lambda (1-\gamma_5)\psi_i,$$



- Beyond E&M, Fermi was inspired by the current-current interactions to construct the weak interaction (1934).
- parity violation ➔ V-A interactions (1957).

The fact $G_F = (300\ \mathrm{GeV})^{-2}$ implies that:

1. A new mass scale to show up at O(100 GeV).
2. Partial-wave Unitarity requires new physics below

E < 300 GeV

Exercise:

Assume that the ν e ν e scattering amplitude to be

$$M = G_F E_{cm}^2$$

estimate the unitarity bound on the c.m. energy.

Partial wave expansion:

$$a_{I\ell}(s) = \frac{1}{64\pi}\int_{-1}^{1} d\cos\theta \; P_\ell(\cos\theta) \; \mathcal{M}^I(s,t)$$

Partial wave unitarity:

$$Im\,(a_{I\ell}) = |a_{I\ell}|^2 < 1, \quad Re\,(a_{I\ell}) < \frac{1}{2}$$

Unification:

Within a frame work of relativistic, quantum, gauge field theory

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

The birth of the Standard Model:

[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.
[12]M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282

bra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta\to\pi^+\pi^-\gamma)/$
) calculated in Refs. 12 and 14.
. M. Brown and P. Singer, Phys. Rev. Letters 8,
(1962).

A MODEL OF LEPTONS*

Steven Weinberg†

Physics Department,
ambridge, Massachusetts
1967)

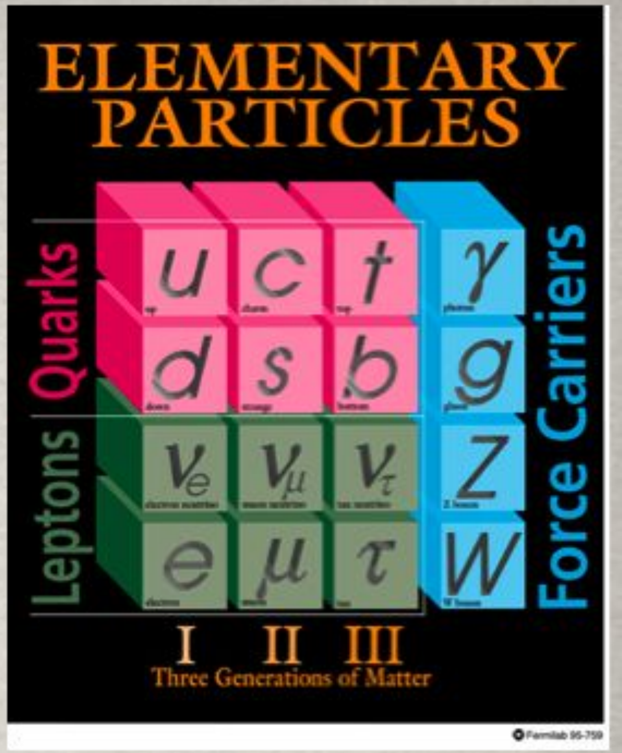
on a right-handed singlet

$$R=[\tfrac{1}{2}(1-\gamma_5)]e.$$

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences

Unification I: Particle representation

- Simple structure and particle contents:



Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad e_R, \; \mu_R, \; \tau_R, \quad (\nu_R' s \; ?)$$

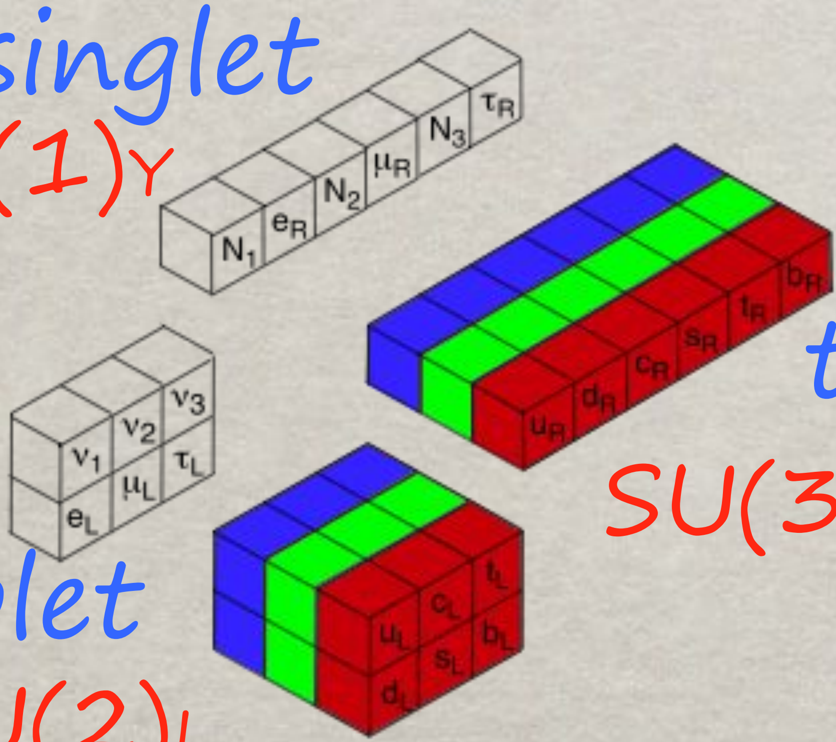
Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_R, \; d_R, \; c_R, \; s_R, \; t_R, \; b_R$$

(1979 Nobel)

singlet

$U(1)_Y$



triplet

$SU(3)_c$

doublet

$SU(2)_L$

The EW Unification II:
The Interactions

$$-\frac{g}{2\sqrt{2}}\sum_i \overline{\Psi}_i\,\gamma^\mu\,(1-\gamma^5)\,(T^+\,W_\mu^+ + T^-\,W_\mu^-)\,\Psi_i$$

$$-e\sum_i q_i\,\overline{\psi}_i\,\gamma^\mu\,\psi_i\,A_\mu\quad -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$-\frac{g}{2\cos\theta_W}\sum_i \overline{\psi}_i\,\gamma^\mu\,(g_V^i - g_A^i\,\gamma^5)\,\psi_i\,Z_\mu\,.$$

$$M_W = \frac{1}{2}g\,v = \frac{e\,v}{2\sin\theta_W},$$

$$M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{e\,v}{2\sin\theta_W\cos\theta_W} = \frac{M_W}{\cos\theta_W},$$

$$M_\gamma = 0.$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu$$

$SU(2)_L$: Non-Abelian gauge theory, asymptotical

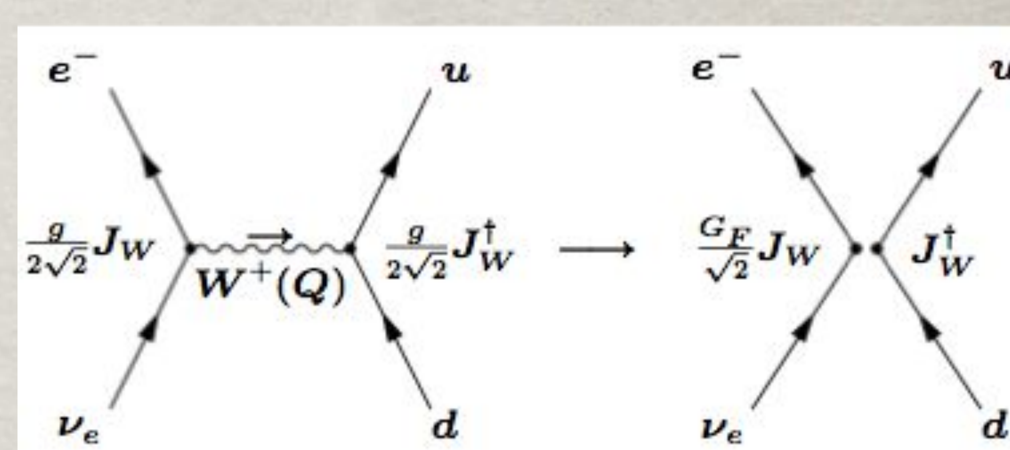
$U(1)_Y$: Non-asymptotically free → Landau pole!

Weak force "NOT Weak!"

$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ interactions.

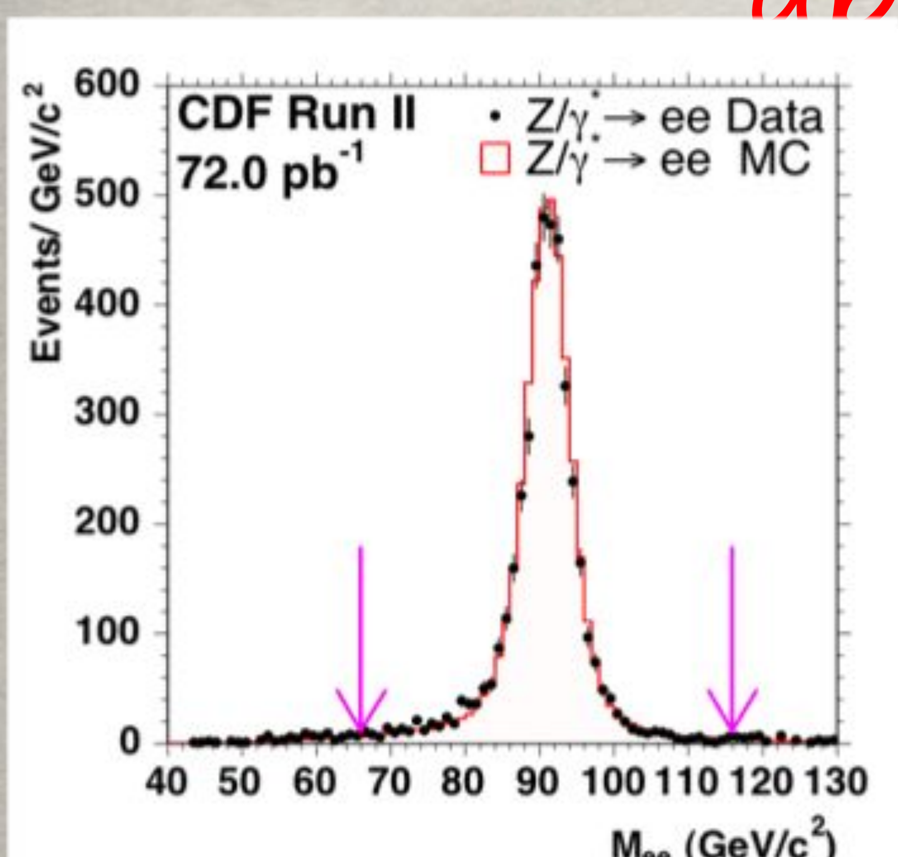
$e = g \sin\theta_W$ coupling unification

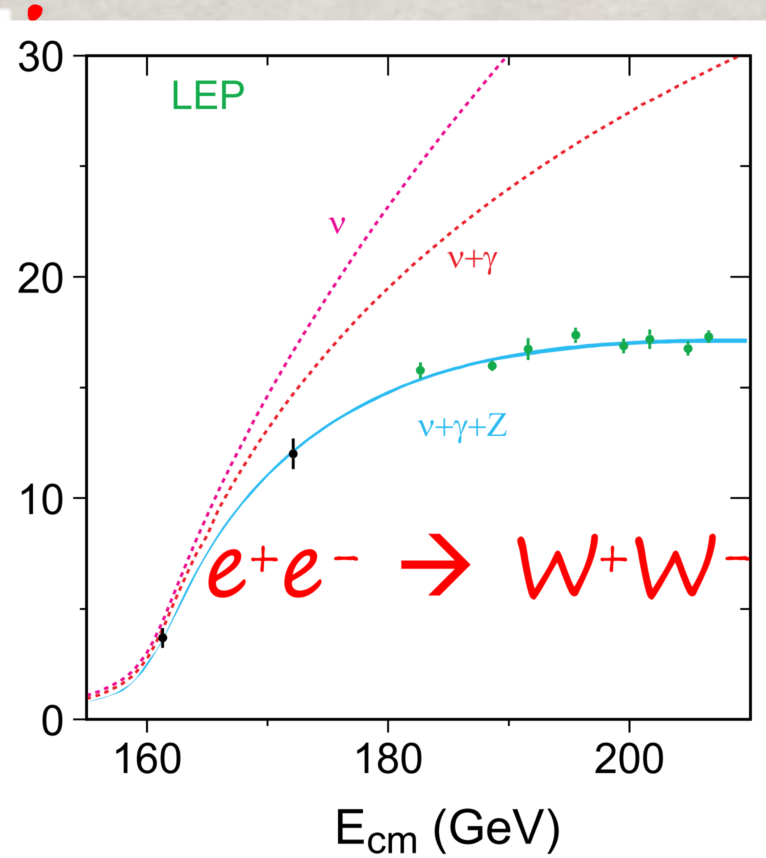
$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ short – range scale.

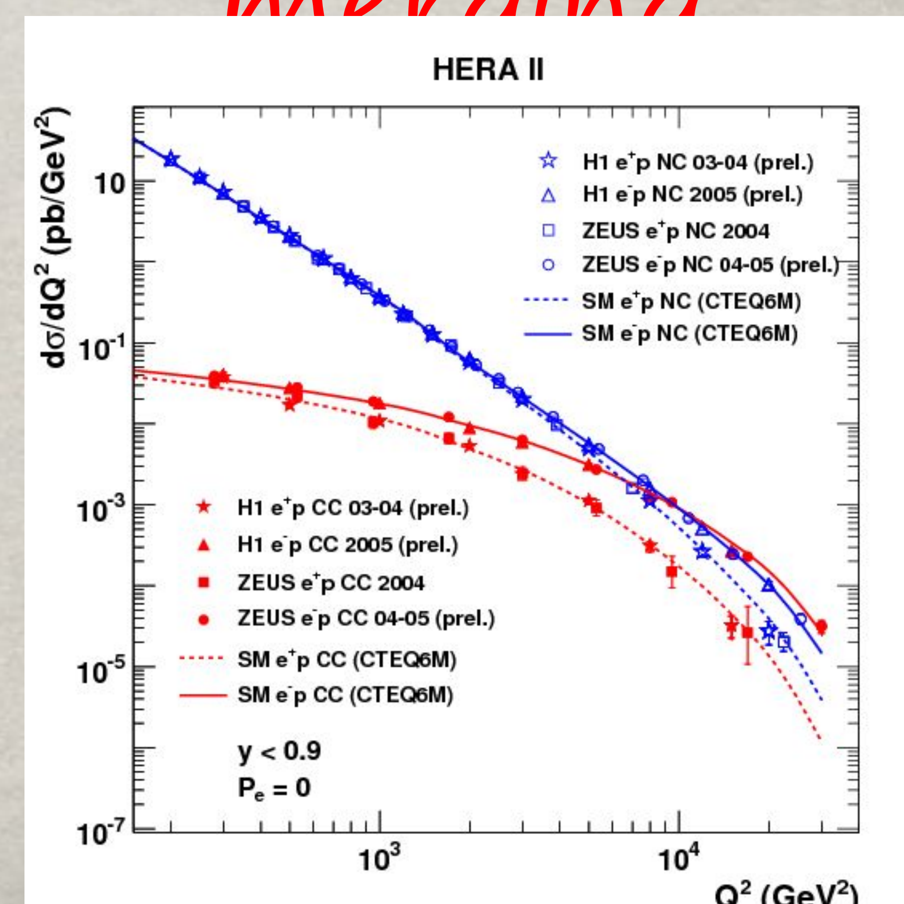


The EW scale is fully open up:

The EW couplings merging:







Quantity	Value	Standard Model	Pull	Dev.
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	0.0
Γ_Z [GeV]	2.4952 ± 0.0023	2.4961 ± 0.0010	−0.4	−0.2
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7426 ± 0.0010	—	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.69 ± 0.06	—	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	84.005 ± 0.015	—	—
σ_{had} [nb]	41.541 ± 0.037	41.477 ± 0.009	1.7	1.7
R_e	20.804 ± 0.050	20.744 ± 0.011	1.2	1.3
R_μ	20.785 ± 0.033	20.744 ± 0.011	1.2	1.3
R_τ	20.764 ± 0.045	20.789 ± 0.011	−0.6	−0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00004	0.8	0.8
R_c	0.1721 ± 0.0030	0.17227 ± 0.00004	−0.1	−0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01633 ± 0.00021	−0.7	−0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.4	0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1034 ± 0.0007	−2.6	−2.3
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0739 ± 0.0005	−0.9	−0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0007	−0.5	−0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23146 ± 0.00012	0.8	0.7
	0.23200 ± 0.00076		0.7	0.6
	0.2287 ± 0.0032		−0.9	−0.9
A_e	0.15138 ± 0.00216	0.1475 ± 0.0010	1.8	2.1
	0.1544 ± 0.0060		1.1	1.3
	0.1498 ± 0.0049		0.5	0.6
A_μ	0.142 ± 0.015		−0.4	−0.3
A_τ	0.136 ± 0.015		−0.8	−0.7
	0.1439 ± 0.0043		−0.8	−0.7
A_b	0.923 ± 0.020	0.9348 ± 0.0001	−0.6	−0.6
A_c	0.670 ± 0.027	0.6680 ± 0.0004	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	−0.4	−0.4

(nearly) perfect agreement between SM theory & expts!

Gauge coupling universality

Some tension!

Only: 3σ discrepancy!

$$a_\mu^{exp} = (1165920.80 \pm 0.63) \times 10^{-9}$$

$$a_\mu^{SM} = (1165918.41 \pm 0.48) \times 10^{-9}$$

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking
B. The Nambu-Goldstone Theorem
C. The Higgs Mechanism
D. The Higgs Boson Interactions

Criticism:

An Anecdote by Yang: SU(2) gauge symmetry

Wolfgang Pauli (1900-1958) was spending the year in Princeton, and was deeply interested in symmetries and interactions.... Soon after my seminar began, when I had written on the blackboard,

Wolfgang Pauli and C. N. Yang

$(\partial_\mu - i\epsilon \mathbf{B}_\mu)\psi$

Pauli asked, "What is the mass of this field $\mathbf{B}_\mu$?" I said we did not know. Then I resumed my presentation but soon Pauli asked the same question again. I said something to the effect that it was a very complicated problem, we had worked on it and had come to no definite conclusions. I still remember his repartee: "That is not sufficient excuse". I was so taken aback that I decided, after a few moments' hesitation, to sit down. There was general embarrassment. Finally Oppenheimer, who was chairman of the seminar, said "We should let Frank proceed". I then resumed and Pauli did not ask any more questions during the seminar.

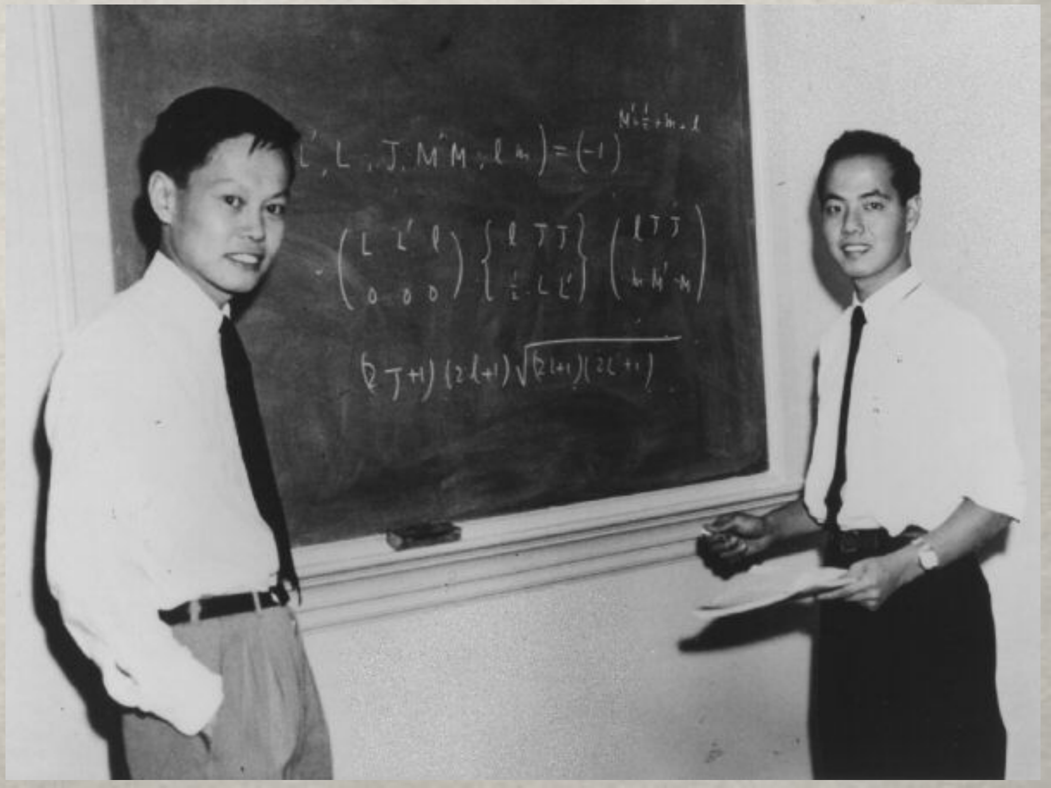
The local gauge symmetry prevents
gauge bosons from acquiring masses!

$$\frac{1}{2}M_A^2 A_\mu A^\mu \to \frac{1}{2}M_A^2 (A_\mu - \frac{1}{e}\partial_\mu \alpha)(A^\mu - \frac{1}{e}\partial^\mu \alpha) \neq \frac{1}{2}M_A^2 A_\mu A^\mu$$

Worse, chiral fermion masses also forbidden
by gauge symmetry!

$$-m_e \bar{e} e = -m_e \bar{e}\Big(\frac{1}{2}(1-\gamma_5) + \frac{1}{2}(1+\gamma_5)\Big) e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$$

``The Left- and right-chiral electrons carry different Weak charges"
(1957 Noble Prize)



Breaking

-- Nature May Not be THAT Symmetric:

"The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry."

Exercise 3:

Find (or make up) other examples for spontaneous symmetry breaking.

Also, think about the relations between the fundamental theoretical formalisms (Newton's Law; Maxwell Equations; Einstein Equation; Lagrangians...) and specific states for a given system (initial and boundary conditions of a system).

B. The Nambu-Goldstone Theorem

-- A show stopper or helper?

"If a continuous symmetry of the system is spontaneously broken, then there will appear a massless degree of freedom, called the Nambu-Goldstone boson."

Symmetry: [Q, H] = QH - HQ = 0

Vacuum state: H |0> = Emin |0>

But: Q |0> ≠ 0 = |0'>

(QH - HQ)|0> = 0 = (Emin - H)|0'>,

thus: H |0'> = Emin |0'>

There is a new, non-symmetric state |0'>,
that has a degenerate energy with vacuum |0>,
thus massless: the Nambu-Goldstone boson.

The Goldstone Theorem (continued)

Broken Symmetries*

JEFFREY GOLDSTONE
Trinity College, Cambridge University, Cambridge, England

AND

ABDUS SALAM AND STEVEN WEINBERG†
Imperial College of Science and Technology, London, England

(Received March 16, 1962)

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

Properties of the Nambu-Goldstone boson:

1. Massless, gapless in spectrum
3. Decouple at low energies:

$\langle G | Q | 0 \rangle \neq 0, \quad \langle G(p) | j^{\mu}(x) | 0 \rangle \sim e^{-ipx} p^{\mu} v$

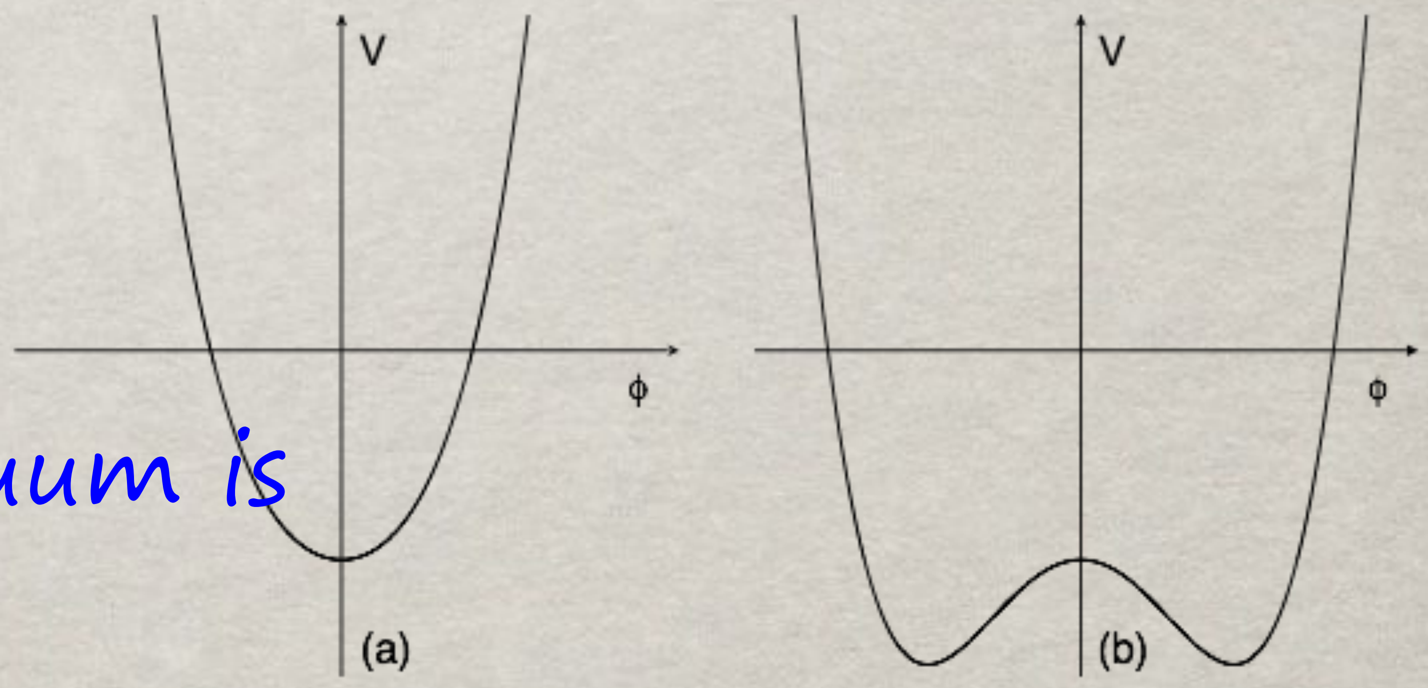
An illustrative (Goldstone's original) Model:

(a). Background complex scalar field Φ:

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \qquad V = \frac{\lambda}{4}\left(\phi^*\phi - \frac{\mu^2}{\lambda}\right)^2$$

Invariant under a U(1) global transformation:

$$\phi \rightarrow e^{i\alpha} \ \phi$$



For $\mu^2 > 0$, the vacuum is shifted, and thus spontaneous symmetry breaking.

$$v = \langle 0|\phi|0\rangle = \mu/\sqrt{\lambda}.$$

Particle spectrum:[§]

Shift: $R = \sqrt{2} Re(\phi - v), \quad I = \sqrt{2}\ Im\phi,$

Then:
$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial^\mu R \partial_\mu R - \frac{\lambda v^2}{2} R^2 - \frac{\lambda \mu}{2\sqrt{2}} R^3 - \frac{\lambda}{16} R^4 \\
&+ \frac{1}{2}\partial^\mu I \partial_\mu I \qquad - \frac{\lambda \mu}{2\sqrt{2}} R I^2 - \frac{\lambda}{16}(R^2 I^2 + I^4)
\end{aligned}
$$

* R is a massive scalar: $M_R = \sqrt{\lambda}\, v$.
* I is massless, interacting.
* Though not transparent, it can be verified:[§]

$$\mathcal{M}(RI \to RI)|_{p\to 0} \to 0!$$

I does decouple at low energies!

Exercise 4: Show this result by an explicit calculation.

[§] C. Burgges, hep-ph/9812468

(b). Field Φ Re-definition:

+ C. Burgges, hep-ph/9812468

Weinberg's 1st Law of Theoretical Physics+:

"You can use whatever variables you like. But if you used the wrong one, you'd be sorry."

Define:

$$\phi(x) = \chi(x)\, e^{i\theta(x)},$$

$$\mathcal{L} = -\partial_\mu \chi \partial^\mu \chi - \chi^2 \partial_\mu \theta \partial^\mu \theta - V(\chi^2).$$

(this is like from the rectangular *form* to the *polar form.)*

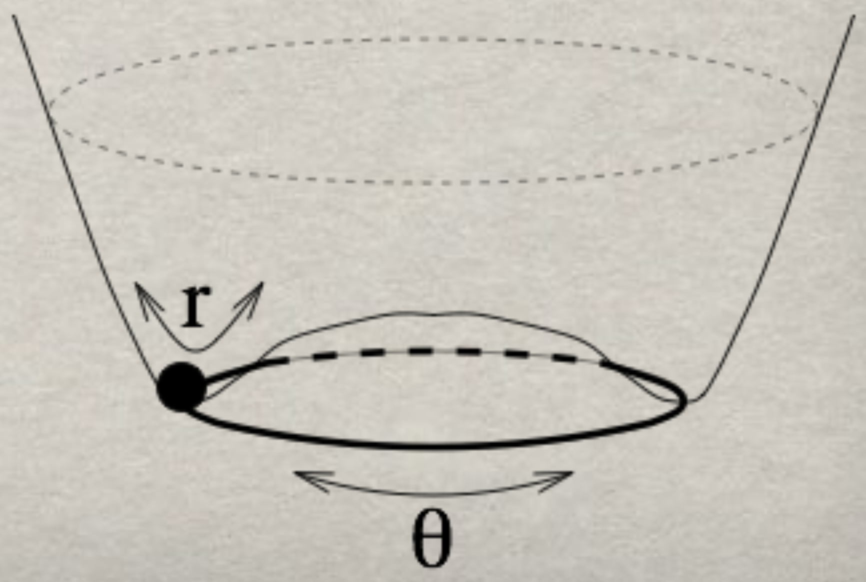
We then see that:

* the θ field is only derivatively coupled, and thus decoupled at low energies
* the θ field respects an inhomogeneous transformation

$$\theta \to \theta + \alpha, \qquad \phi = v e^{i\theta(x)}$$

a phase rotation from the vacuum;

* the χ(x) is massive radial excitation.



"Nambu-Goldstone Bosons"

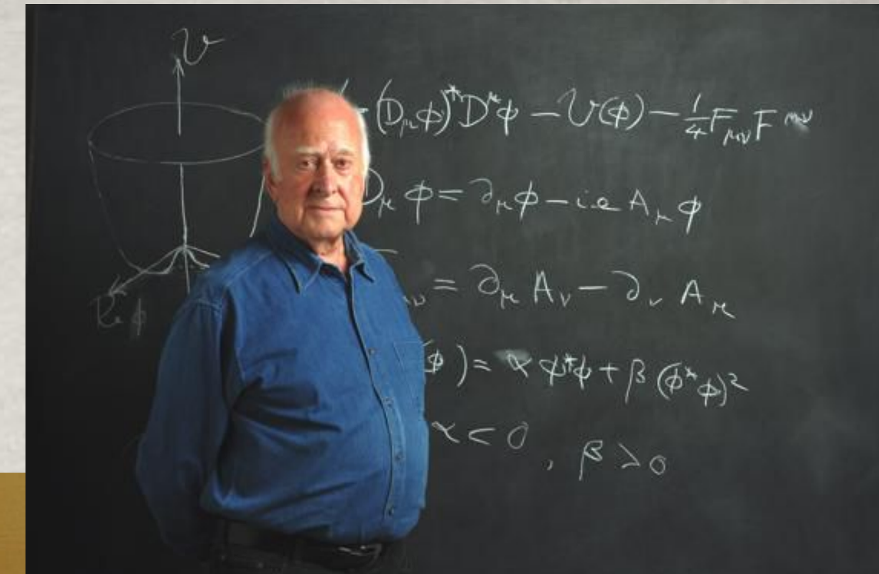
Except the photon, no massless boson
(a long-range force carrier) has been seen
in particle physics!

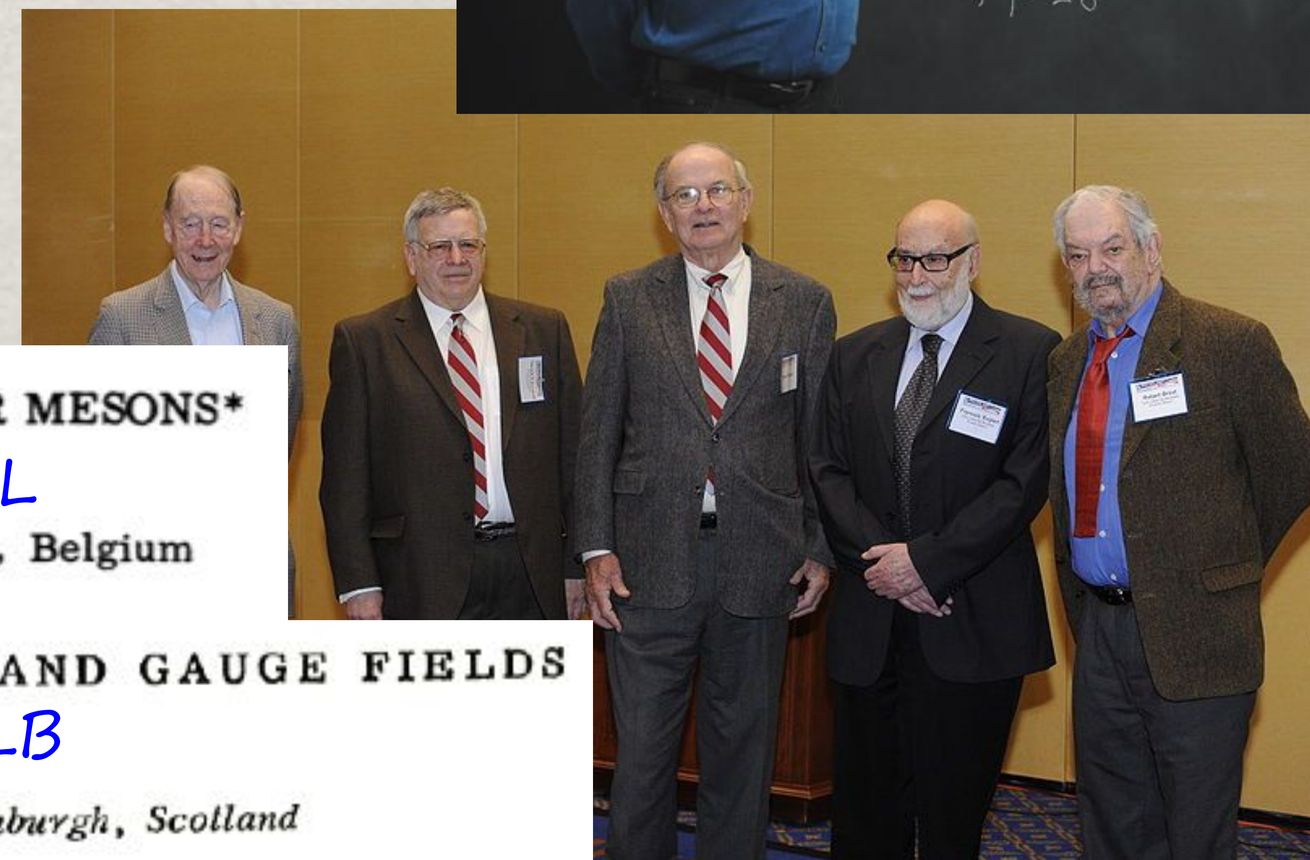
(Recall Pauli's criticism)

The Spontaneous Symmetry Breaking:
Brilliant idea & common phenomena,
confronts
the Nambu-Goldstone theorem!

C: The Magic in 1964: The "Higgs Mechanism"

"If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the





BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

PRL

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

PLB

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

PRL

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England

PRL

(Received 12 October 1964)

An illustrative (original) Model:¶

$$\mathscr{L} = |\mathscr{D}^\mu \phi|^2 - \mu^2 |\phi|^2 - |\lambda| (\phi^* \phi)^2 - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

is a complex scalar field[4] and as usual

$$\mathscr{D}_\mu \equiv \partial_\mu + iqA_\mu$$

and

$$F_{\mu\nu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu.$$

The Lagrangian (5.3.1) is invariant under U(1) rotations

$$\phi \to \phi' = e^{i\theta} \phi$$

and under the local gauge transformations

$$\phi(x) \to \phi'(x) = e^{iq\alpha(x)} \phi(x),$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x).$$

¶ C. Quigg, Gauge Theories of the Strong ...

An illustrative (original) Model:¶

After the EWSB,

parameterized in terms of

$$\langle\phi\rangle_0 = v/\sqrt{2}, \qquad \phi = e^{i\zeta/v}(v+\eta)/\sqrt{2}$$
$$\approx (v+\eta+i\zeta)/\sqrt{2}.$$

Then the Lagrangian appropriate for the study of small oscillations is

$$\mathscr{L}_{\text{so}} = \tfrac{1}{2}[(\partial_\mu\eta)(\partial^\mu\eta) + 2\mu^2\eta^2] + \tfrac{1}{2}[(\partial_\mu\zeta)(\partial^\mu\zeta)]$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + qvA_\mu(\partial^\mu\zeta) + \frac{q^2v^2}{2}A_\mu A^\mu + \cdots$$

The gauge field acquires a mass, mixes with the Goldstone boson.

Upon diagonalization:

$$\frac{q^2v^2}{2}\left(A_\mu + \frac{1}{qv}\partial_\mu\zeta\right)\left(A^\mu + \frac{1}{qv}\partial^\mu\zeta\right),$$

a form that pleads for the gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial^\mu\zeta,$$

which corresponds to the phase rotation on the scalar field

$$\phi \to \phi' = e^{-i\zeta(x)/v}\phi(x) = (v+\eta)/\sqrt{2}.$$

the resultant Lagrangian is then:

$$\mathscr{L}_{so} = \tfrac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2] - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'^\mu$$

- an η-field, with $(\text{mass})^2 = -2\mu^2 > 0$; the Higgs boson!
- a massive vector field A'_μ, with mass $= qv$
- no ζ-field.

- By virtue of a gauge choice - the unitary gauge,

  the ζ-field disappears in the spectrum: a massless

  photon "swallowed" the massless NG boson!

Degrees of freedom count:

Before EWSB: After:

2 (scalar)+2 (gauge pol.); 1 (scalar)+3 (gauge pol.)

- Two problems provide cure for each other!

  massless gauge boson + massless NG boson

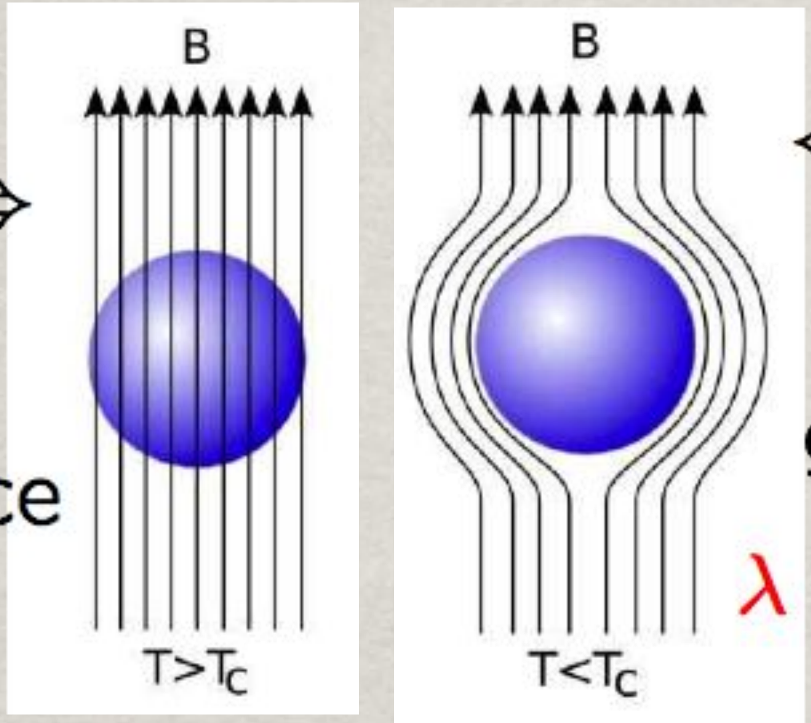
  ⇒ massive gauge boson + no NG boson

Known example: Superconductivity

Normal phase$\Rightarrow$

$E^2 = p^2c^2$

Long-range force



$\Leftarrow$Superconducting phase

$E^2 = p^2c^2 + m^2c^4$

gap leads to $\sim exp(-r/\lambda)$

$\lambda \sim m^{-1}$ penetration depth

In "conventional" electro-magnetic superconductivity:

$m_\gamma \sim m_e/1000$, $T_c^{em} \sim \mathcal{O}(\text{few } K)$. BCS theory.

In "electro-weak superconductivity":

$m_w \sim G_F^{-\frac{1}{2}} \sim 100$ GeV, $T_c^w \sim 10^{15} K!$

True understanding was the work of many hands, most notably:†

chiral fermions to dynamically generate the nucleon mass

(Nambu–Jona-Lasinio model)

- 1961,1962: Goldstone theorem challenged the implementation of spontaneous symmetry breaking for gauge symmetry: No experimental observation for a massless Goldstone boson.
- 1963: Anderson conjectured a non-relativistic version of a massive Goldstone mode, the "plasmon" in superconductor.
- 1964: Englert+Brout; Higgs; Guralnik+Hagen+Kibble showed the U(1) photon mass generation m… the Goldstone theorem in locally gauge invariant

† Univ. of Edinburgh, Peter Higgs and the Higgs Boson.

§ Sidney Coleman:

(in 1989) that they "had been looking forward to tearing apart this idiot who thought he could get around the Goldstone theorem".

Evading the Goldstone Theorem continues †

- 1964: Higgs (PRL) first commented on the spin-zero boson, in the revised version (upon Nambu's request to compare with the other's works)¶
- 1966: Higgs (PRD) laid out the scalar scattering/decay in an Abelian U(1) model.‡

¶ It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

PHYSICAL REVIEW VOLUME 145, NUMBER 4 27 MAY 1966

‡ **Spontaneous Symmetry Breakdown without Massless Bosons***

PETER W. HIGGS†
Department of Physics, University of North Carolina, Chapel Hill, North Carolina
(Received 27 December 1965)

- 1967: Weinberg (PRL) laid out the fermion mass generation, formulated the $SU(2)_L \times U(1)_Y$ SM.

† Peter Higgs: My Life as a Boson.

† Univ. of Edinburgh, Peter Higgs and the Higgs Boson.

As for the name ...

1972: Ben Lee (Rochester Conf. at FNAL) named "Higgs boson" and the "Higgs mechanism".§

§ Peter Higgs: My Life as a Boson.

Font Size:

The New York Review of Books

The Crisis of Big Science

MAY 10, 2012

Steven Weinberg

As to my responsibility for the name "Higgs boson," because of a mistake in reading the dates on these three earlier papers, I thought that the earliest was the one by Higgs, so in my 1967 paper I cited Higgs first, and have done so since then. Other physicists apparently have followed my lead. But as Close points out, the earliest paper of the three I cited was actually the one by Robert Brout and François Englert. In extenuation of my mistake, I should note that Higgs and Brout and Englert did their work independently and at about the same time, as also did the third group (Gerald Guralnik, C.R. Hagen, and Tom Kibble). But the name "Higgs boson" seems to have stuck. ↩

D. Higgs Boson Interactions

1. The SM Lagrangian:

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}.$$

The gauge part is

Pure gauge sector:

$$\mathcal{L}_{gauge} = -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

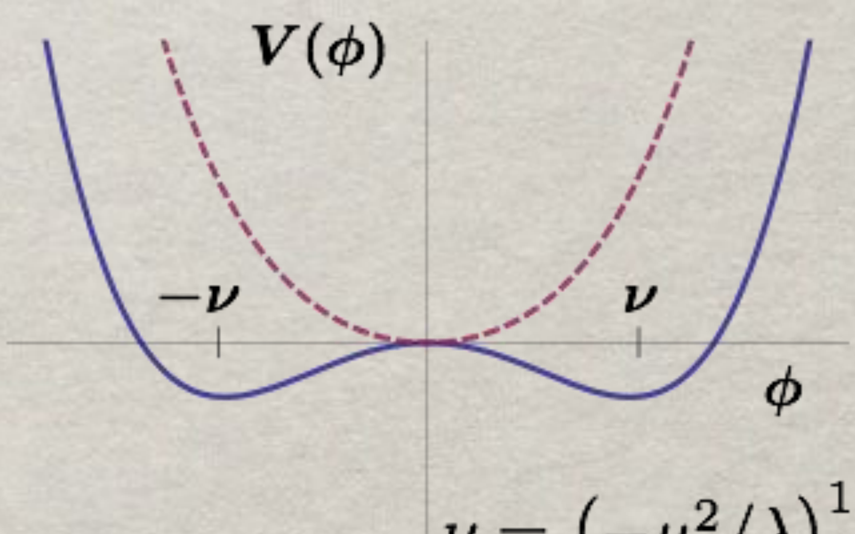
The scalar part of the Lagrangian is

The Higgs: $\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$ $\quad D_\mu\phi = \left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + \frac{ig'}{2}B_\mu\right)\phi,$

$$V(\phi) = +\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

$$\phi = \frac{1}{\sqrt{2}}e^{i\sum\xi^i L^i}\begin{pmatrix}0\\ \nu+H\end{pmatrix}$$

$$\phi \to \phi' = e^{-i\sum\xi^i L^i}\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu+H\end{pmatrix}$$



$$\nu = (-\mu^2/\lambda)^{1/2}$$

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$$

$$= \underline{M_W^2 W^{\mu+}W^-_\mu}\left(1+\frac{H}{\nu}\right)^2 + \underline{\frac{1}{2}M_Z^2 Z^\mu Z_\mu}\left(1+\frac{H}{\nu}\right)^2 \qquad M_H^2 = -2\mu^2 = 2\lambda v^2$$

$$+\frac{1}{2}(\partial_\mu H)^2 - V(\phi). \qquad V(\phi) = -\frac{\mu^4}{4\lambda} - \underline{\mu^2 H^2} + \lambda\nu H^3 + \frac{\lambda}{4}H^4.$$

The Fermions:[§]

$$\mathcal{L}_f = \sum_{m=1}^{F} \left(\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL} + \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR} \right)$$

$$D_\mu q^0_{mL} = \left(\partial_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu + \frac{ig'}{6}B_\mu\right) q^0_{mL} \qquad D_\mu u^0_{mR} = \left(\partial_\mu + \frac{2ig'}{3}B_\mu\right) u^0_{mR}$$

$$D_\mu l^0_{mL} = \left(\partial_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu - \frac{ig'}{2}B_\mu\right) l^0_{mL} \qquad D_\mu d^0_{mR} = \left(\partial_\mu - \frac{ig'}{3}B_\mu\right) d^0_{mR}$$

$$D_\mu e^0_{mR} = \left(\partial_\mu - ig' B_\mu\right) e^0_{mR}$$

$$D_\mu \nu^0_{mR} = \partial_\mu \nu^0_{mR},$$

Gauge invariant, massless.

However, a fermion mass must flip chirality:

$$m_f(\bar{f}_L f_R + \bar{f}_R f_L)$$

and thus not SM gauge invariant $L \neq R$

Need something like a doublet:

$$y_f(\bar{f}_1, f_2)_L \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_L f_R$$

that's the Higgs doublet!

[§] P. Langacker: TASI Lectures 2007.

The gauge invariant Yukawa interactions:

Need a doublet with a flip Y: $\tilde{\phi} = i\sigma_2 \phi^*$

$$\mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F} \Big[\Gamma^u_{mn} \bar{q}^{\,0}_{mL} \tilde{\phi} u^0_{nR} + \Gamma^d_{mn} \bar{q}^{\,0}_{mL} \phi d^0_{nR}$$

$$+ \Gamma^e_{mn} \bar{l}^{\,0}_{mn} \phi e^0_{nR} + \Gamma^\nu_{mn} \bar{l}^{\,0}_{mL} \tilde{\phi} \nu^0_{nR} \Big] + h.c.,$$

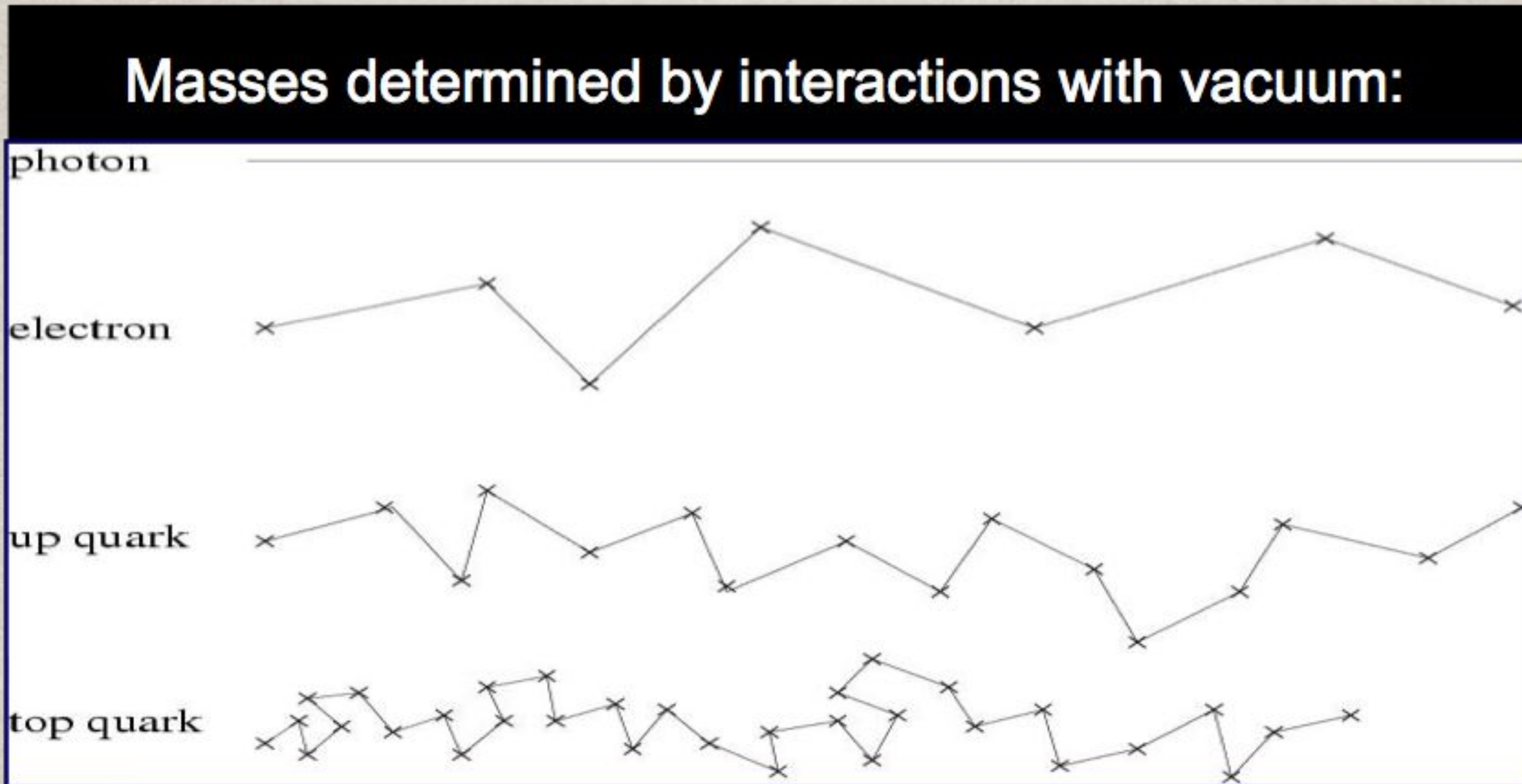
After the EWSB,

$$-\mathcal{L}_{Yuk} \to \sum_{m,n=1}^{F} \bar{u}^0_{mL} \Gamma^u_{mn} \left(\frac{\nu + H}{\sqrt{2}} \right) u^0_{mR} + (d, e, \nu) \text{ terms}$$

$$= \bar{u}^0_L \left(M^u + h^u H \right) u^0_R + (d, e, \nu) \text{ terms } + h.c.,$$

$$-\mathcal{L}_{Yuk} = \sum_i m_i \bar{\psi}_i \psi_i \left(1 + \frac{g}{2M_W} H \right) = \sum_i \underline{m_i \bar{\psi}_i \psi_i \left(1 + \frac{H}{\nu} \right)}$$

Higgs Boson Couplings:

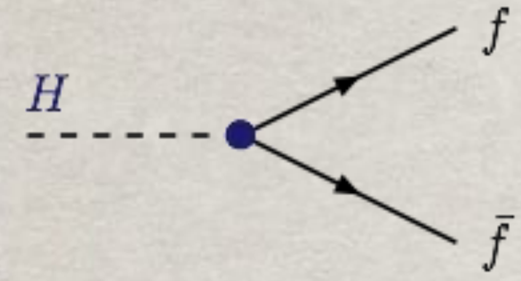


$$M_{W,Z} = \frac{1}{2} g_V v, \quad m_f = \frac{g_f}{\sqrt{2}} v, \quad v^{-2} = \sqrt{2}\, G_F.$$

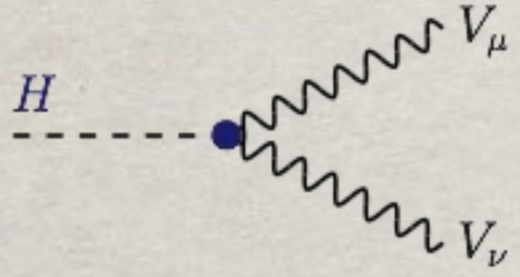
Thus, where ever is mass, there will be H!

The Low-Energy-theorem: $m_i \to m_i\left(1 + \frac{H}{v}\right)$ for $p_H < v$.

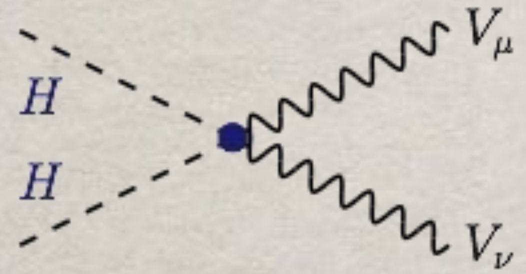
Feynman rules:



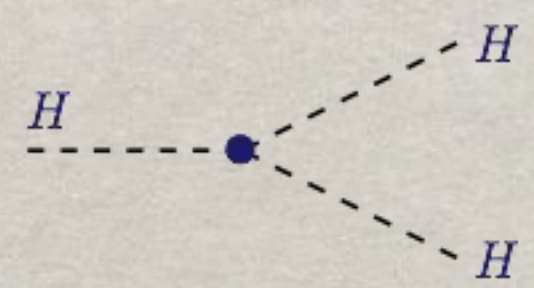
$$g_{Hff} = m_f/v = (\sqrt{2}G_\mu)^{1/2} m_f \qquad \times (i)$$



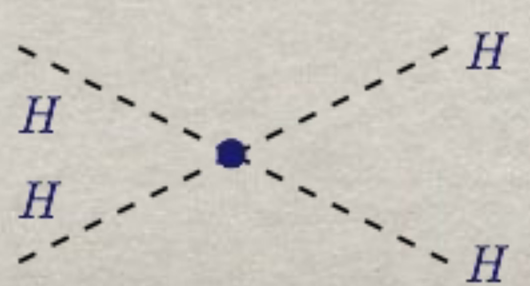
$$g_{HVV} = 2M_V^2/v = 2(\sqrt{2}G_\mu)^{1/2} M_V^2 \qquad \times (-ig_{\mu\nu})$$



$$g_{HHVV} = 2M_V^2/v^2 = 2\sqrt{2}G_\mu M_V^2 \qquad \times (-ig_{\mu\nu})$$



$$g_{HHH} = 3M_H^2/v = 3(\sqrt{2}G_\mu)^{1/2} M_H^2 \qquad \times (i)$$



$$g_{HHHH} = 3M_H^2/v^2 = 3\sqrt{2}G_\mu M_H^2 \qquad \times (i)$$

Exercise 6: Verify the above Feynman rules by invoking the low-energy theorem:

$$m_i \rightarrow m_i\left(1 + \frac{H}{v}\right) \quad \text{for} \quad p_H < v.$$

Goldstone-Boson Equivalence Theorem:

At high energies E>>M_W, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!)

Caution: Very often, we say at high energies, M_W 0. Rigorously speaking, we mean: g, M_W 0, but M_W/g v/2.

Exercise 7: Verify the Goldstone-boson Equivalence Theorem by examining the HWW vertex.
Hint: Use $\epsilon_L^\mu \rightarrow p_H^\mu / M_W$. It should give you HHH vertex.

Lecture III: Higgs Physics and Beyond

A. What Does THIS Higgs Tell us?

B. SM Higgs Sector at Higher Energies & the Need for New Physics

----

C. Higgs Boson Decays

D. Higgs Physics at the LHC Colliders

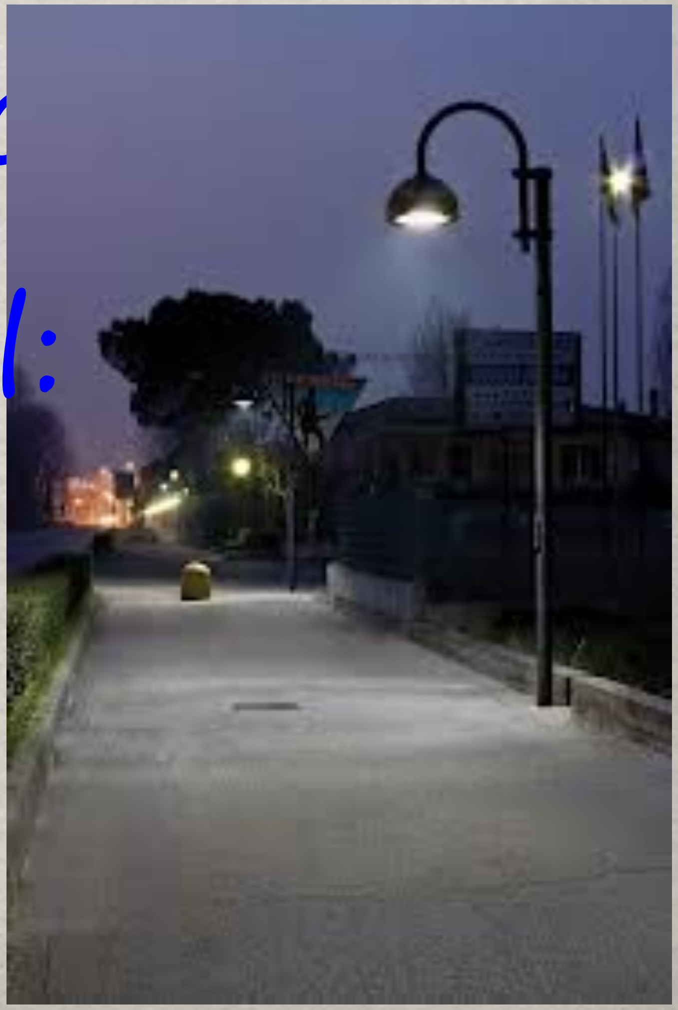
E. Higgs Physics at an e+e- Collider

This discovery opens up a new era in HEP!

In these Lectures, I wish to convey to you:

- This is truly an "LHC Revolution", ever since the "November Revolution" in 1974 for the J/ψ disc

- It strongly argues for new physics beyond the Standard Model: Under the Higgs lamp post.



A. A Weakly Coupled Light Higgs?

1. The Higgs Mechanism DOES NOT require a Higgs boson!

"If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the Goldstone mode."

The Non-Linear realization:

$$\Phi = \frac{1}{\sqrt{2}}(v+H)U\,, \qquad U = \exp[i\pi^a\tau^a/v]$$

$\mathcal{G}_{\rm SM} = SU(2)_L \otimes U(1)_Y$, as

$$U \to U' = g_L U g_Y^\dagger\,, \qquad H \to H' = H\,,$$

$$g_L = \exp[-i\theta_L^a\tau^a/2]\,, \qquad g_Y = \exp[-i\theta_Y\tau^3/2]\,.$$

Then leave out the singlet H, the SM gauge symmetry spontaneously broken:

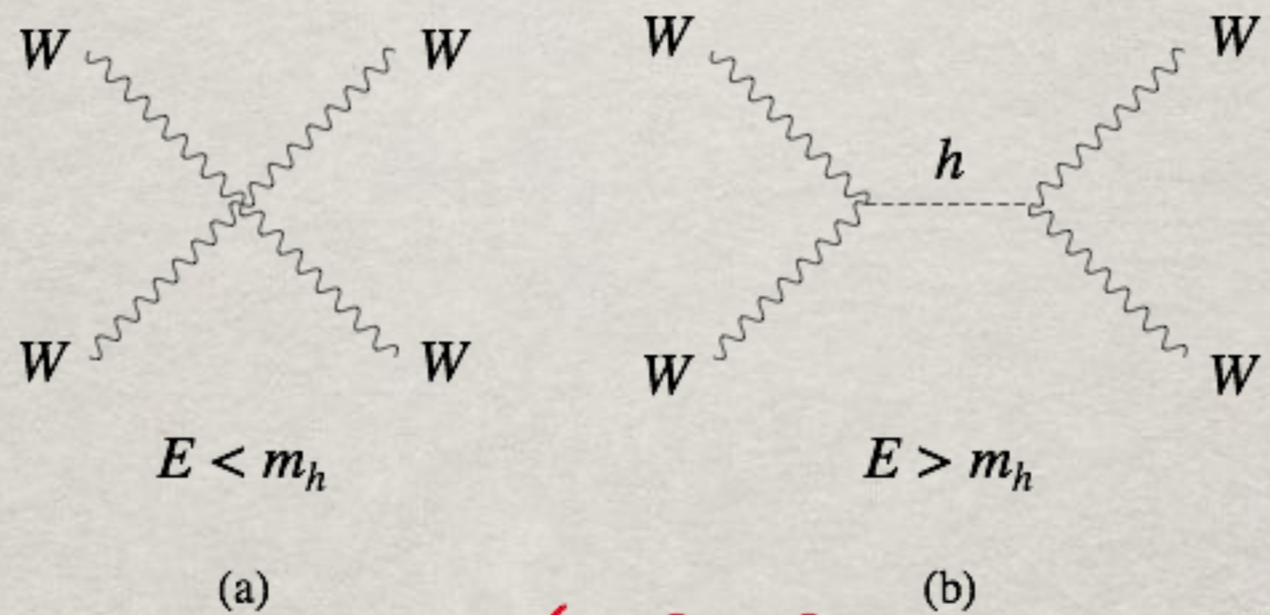
$$D_\mu U = \partial_\mu U + igW_\mu^a\frac{\tau^a}{2}U - ig'UB_\mu\frac{\tau^3}{2}$$

$$\mathcal{L} = \frac{1}{2}Tr(D_\mu\Phi^\dagger D^\mu\Phi) \Rightarrow \frac{v^2}{4}Tr(D_\mu U^\dagger D^\mu U) \longrightarrow \frac{v^2}{4}(\sum_i g^2W_i^2 + g'^2B^2)$$

(fermion masses can be accommodated similarly)

Higgs boson could be absent, but:

Consider the massive gauge boson scattering:



$$\mathcal{M}(W_L W_L \to W_L W_L) \sim \begin{cases} E_{cm}^2/v^2 & \text{no light Higgs,} \\ m_h^2/v^2 & \text{with a SM Higgs.} \end{cases}$$

Partial-wave unitarity demands

$$a_0 \;=\; \frac{1}{16\pi}\,\frac{m_h^2 \text{ or } E_{cm}^2}{v^2} \lesssim 1$$

$$\Rightarrow \; m_h \text{ or } E_{cm} \lesssim \mathcal{O}(1\ \text{TeV}).$$

Exercise 11: Verify this unitarity bound by an explicit partial wave analysis.

2. Natural dynamics prefers a heavier, broad Higgs boson!

In low-energy QCD, a generic dynamical mass is

$m \sim 4\,\pi\, f_\pi \sim 1$ GeV:

$m(f_0) \sim 0.4$ -1.2 GeV, $\Gamma \sim 0.6$ - 1.0 GeV !

$m(\rho^{\pm,0}) \sim 0.77$ GeV, $\Gamma \sim 0.15$ GeV.

Lessons from QCD and other strong dynamical models (Technicolor-like, composite, dilaton...) argue the dynamical mass to be of the order

$4\,\pi\, v \approx 2$ TeV!

And typically strong interacting: Γ(total) ≥ 20%M !

--- except the pseudo Goldstone bosons.

rather

light, weakly coupled boson:

m_h = 125-126 GeV, Γ < 1 GeV,

is truly revolutionary!

We have just discovered a "fifth (weak) force":

$\lambda \approx 1/8$! $m_H^2/2v^2$ in the SM

Hopes for uncovering a deeper theory:

- λ determined by other couplings like in SUSY? where $\lambda = (g_1^2 + g_2^2)/8$
- or dynamically generated by a new strong

Sector at Higher Energies

Recall the SM Higgs sector:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\begin{aligned} \mathcal{L}_H &= \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V \\ &= \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \end{aligned}$$

$$M_H^2 = 2\lambda v^2 = -2\mu^2$$

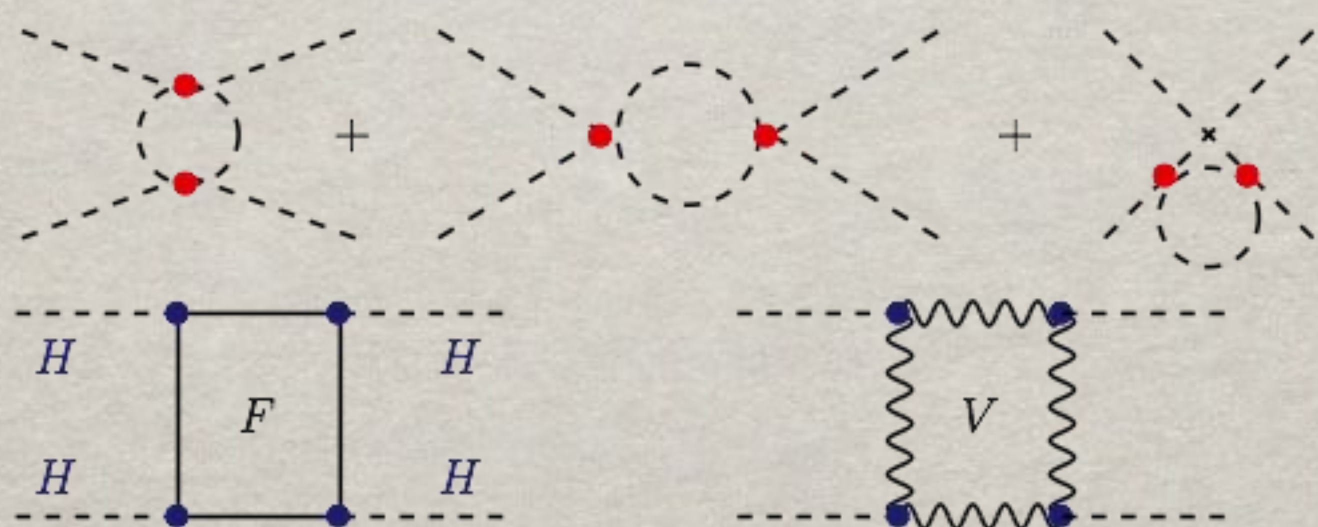
Crucial conditions:

$$\mu^2(Q^2) < 0, \quad \lambda(Q^2) > 0$$

Renormalization Group Equation Evolution at NLO:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

$$t = \ln(Q^2/Q_0^2)$$



1. Triviality bound

How large M_H (λ) can be dragged up?

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$M_H^2 = 2\lambda v^2 = -2\mu^2$$

There is a (famous) Landau Pole!

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 \quad \lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

(present in all but non-Abelian gauge theories)

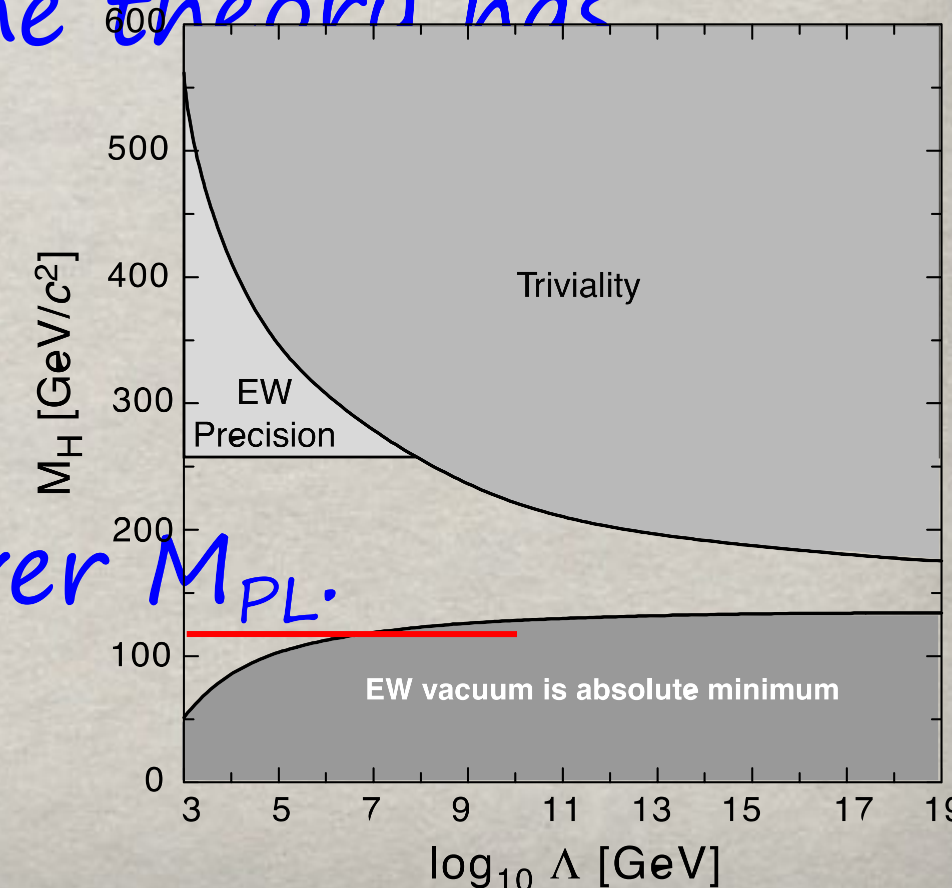
1. If SM valid to infinite energy, then $\lambda(Q_0) = 0$,

a non-interacting trivial theory!

2. Since M_H is non-zero, then the theory has a cutoff Λ,

translate to a M_H upper bound:

$$M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$$



For M_H = 125 GeV, the cutoff is over M_{PL}.

2. Vacuum stability bound

For small λ, the Top-Yukawa dominates:

$$32\pi^2 \frac{d\lambda}{dt} = -24 y_t^4 \qquad \lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right)$$

To have a stable vacuum,

$$\lambda(\Lambda) > 0 \quad \longrightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right)$$

$$\Lambda_C \sim 10^3 \text{ GeV} \quad \Rightarrow \quad M_H \gtrsim 70 \text{ GeV}$$

$$\Lambda_C \sim 10^{16} \text{ GeV} \quad \Rightarrow \quad M_H \gtrsim 130 \text{ GeV}$$

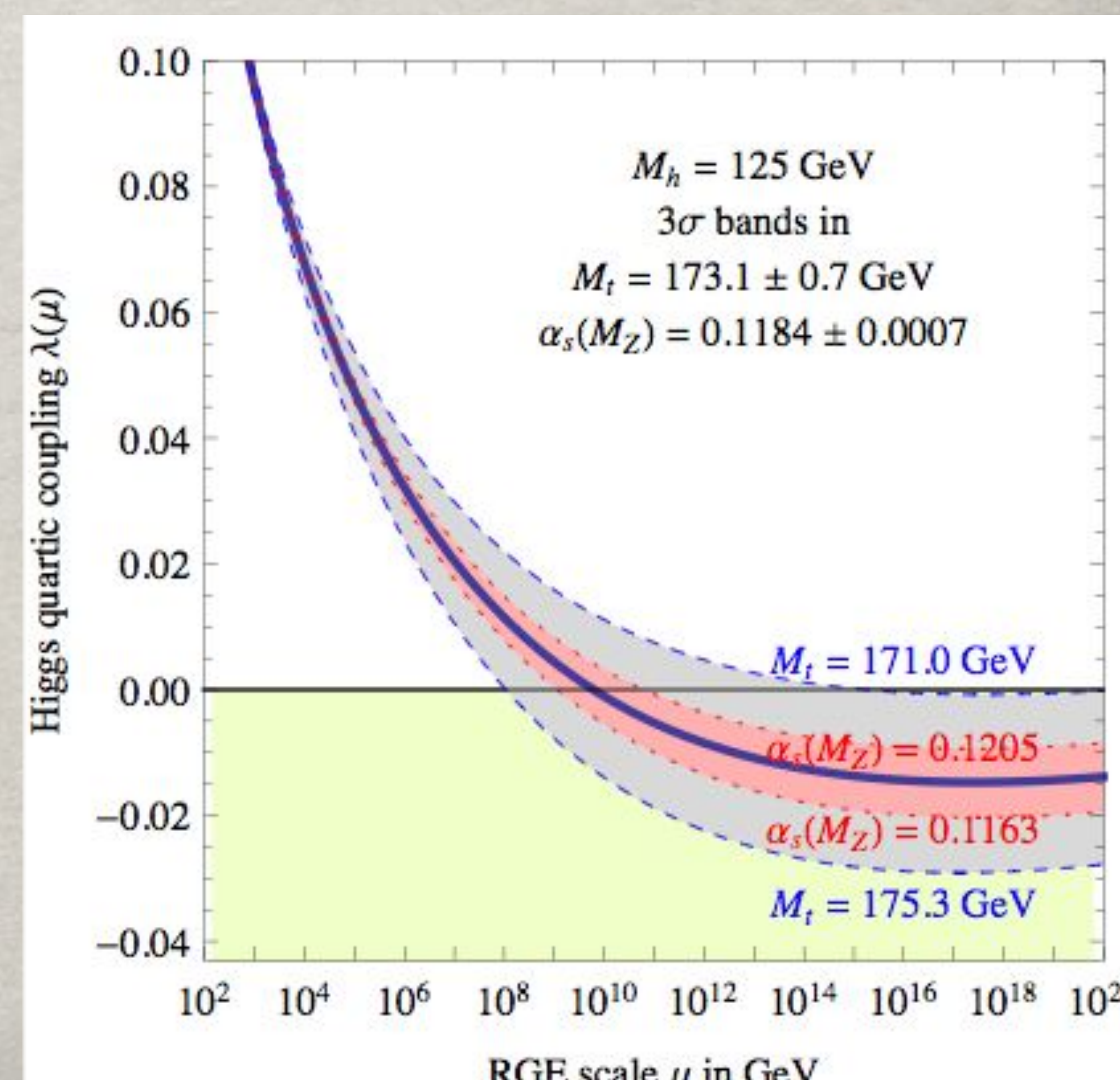
Much renewed interest, updates:[\$]

\$ G. Degrassi et al., arXiv:1205.6497

For M_H = 125 GeV,

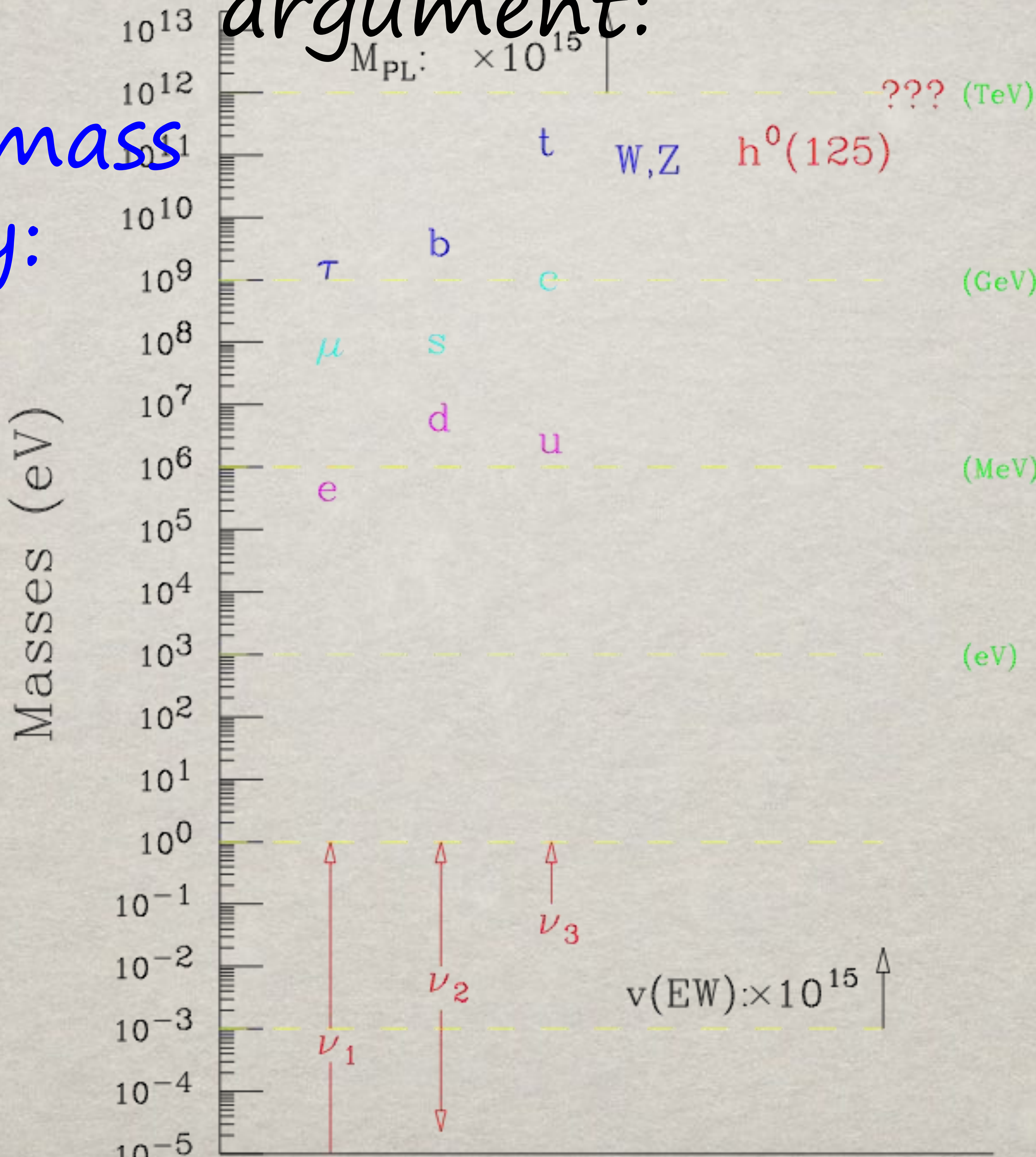
then $\Lambda(m_t=175) < 10^7$ GeV.

(but m_t=171 GeV would be fine)



3. "Naturalness" argument:

Particle mass hierarchy:



Since all the masses are generated like $\sim g\, v$.

Thus, except M_W, M_Z, M_H, $m_t \sim g\, v$,

the natural scale should be just v.

all others are unnatural: (to some extent)

$m_b \sim 5$ GeV, $m_e \sim 0.5$ MeV, $m_\nu <$ 0.1 eV ...

But, they are "technically natural":

For a given mass, if the quantum corrections are merely logarithmically dependent upon the high energy scale, then the mass parameter is said technically natural.

t'Hooft statement for "technical naturalness":

If a parameter is turned off (set to 0), the system results in an enlarged symmetry, then this parameter must be technically natural.

$$m_e \sim m^0_e [1 + 3\alpha/4\pi \ \ln(\Lambda/m_e)]$$

If m^0_e is turned off, the system possesses a chiral symmetry

Dynamical scale generation is natural!

Recall in QCD: coupling runs logarithmically between vastly separated scales:

$$\alpha_s(\Lambda^2) \approx \frac{1}{\ln \frac{\Lambda^2}{\Lambda_{QCD}^2}} \quad e.g. \quad (\frac{\Lambda}{\Lambda_{QCD}})^2 \approx (\frac{E_{LHC}}{\Lambda_{QCD}})^2 \approx 10^8.$$

Dynamical scale can be generated by "dimensional transmutation":

$$\Lambda_{TC} \approx \Lambda \exp(-\frac{1}{2\alpha_{TC}}) \approx 4\pi v.$$

However, this picture (Technicolor and variations) doesn't work (well) in EW:

* It is strong interaction, not seen in EW physics.
* Fermion masses/mixing a real killer.
* No fundamental scalar (at least not a light one).

"... scalar particles are the only kind of free particles
whose mass term does not break either an internal
or a gauge symmetry." -- Ken Wilson, 1970.

Quantum corrections to the potential or to m_H^2

Tree-level SM Higgs potential:

$$V(H) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

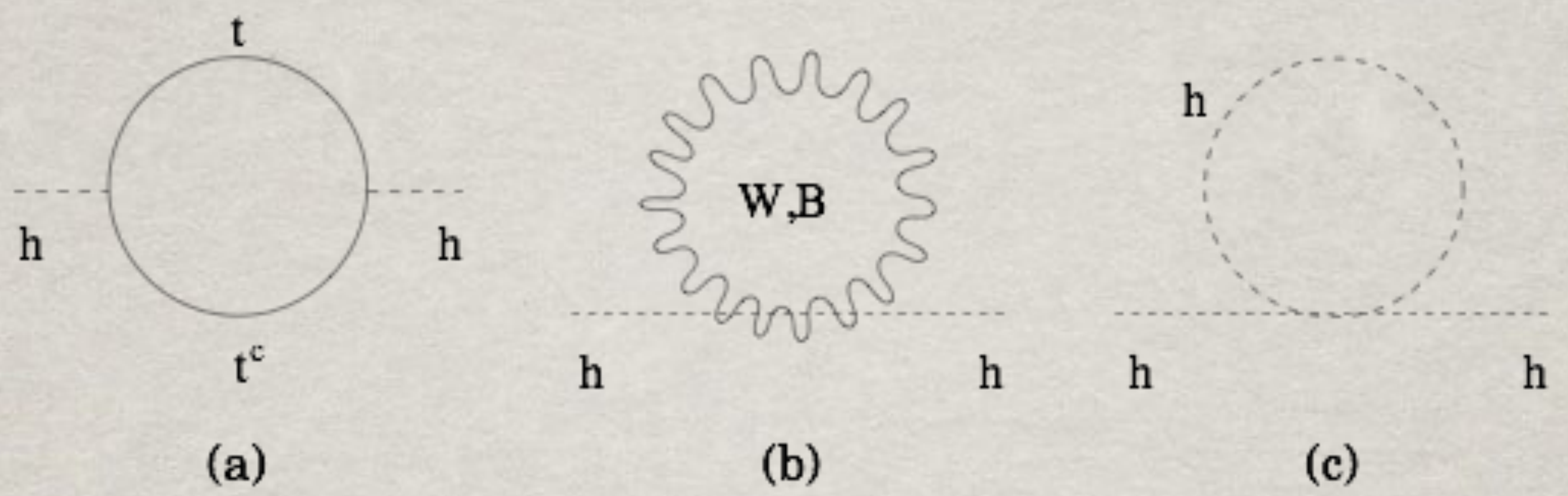
$$m_H^2 = 2\mu^2 = 2\lambda v^2 \quad \Rightarrow \quad \mu \approx 89 \text{ GeV}, \quad \lambda \approx \frac{1}{8}.$$

Quantum corrections to y_t:

$$\delta\mu^2 = -\frac{3y_t^2}{8\pi^2}\Lambda^2$$

is "un-natural": quadratic (not log) correctio

The “naturalness” problem?



$$m_H^2 = m_{H0}^2 \quad - \frac{3}{8\pi^2} y_t^2 \Lambda^2 \quad + \frac{1}{16\pi^2} g^2 \Lambda^2 \quad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

If $\Lambda^2 \gg m_H^2$, then unnaturally large cancellations must occur.

Cancelation in perspective:

$m_H{}^2$ =

36,127,890,984,789,307,394,520,932,878,928,933,023

-36,127,890,984,789,307,394,520,932,878,928,917,398

Amazing !

Unnatural: Fine-tuned to

0.05 mm/0.5 cm ~ 10^{-2}

A light Higgs is unnatural

"Naturalness" argument strongly indicates the existence of TeV scale new physics:

equiring less 90% cancellation → $\Lambda_t < 3$ Te

If you give up this belief, you are subscribing the "anthropic principle".

Cancellation Mechanisms ?

- Super-symmetry (SUSY) (symmetry between *opposite* spin & statistics)

  Natural cancellations: $\tilde{t}$ versus t
  $\tilde{W}$ versus W
  $\tilde{H}$ versus H
  H_d versus H_u,

$$\Delta m_H^2 \sim (M_{SUSY}^2 - M_{SM}^2)\,\frac{\lambda_f^2}{16\pi^2}\ln\left(\frac{\Lambda}{M_{SUSY}}\right).$$

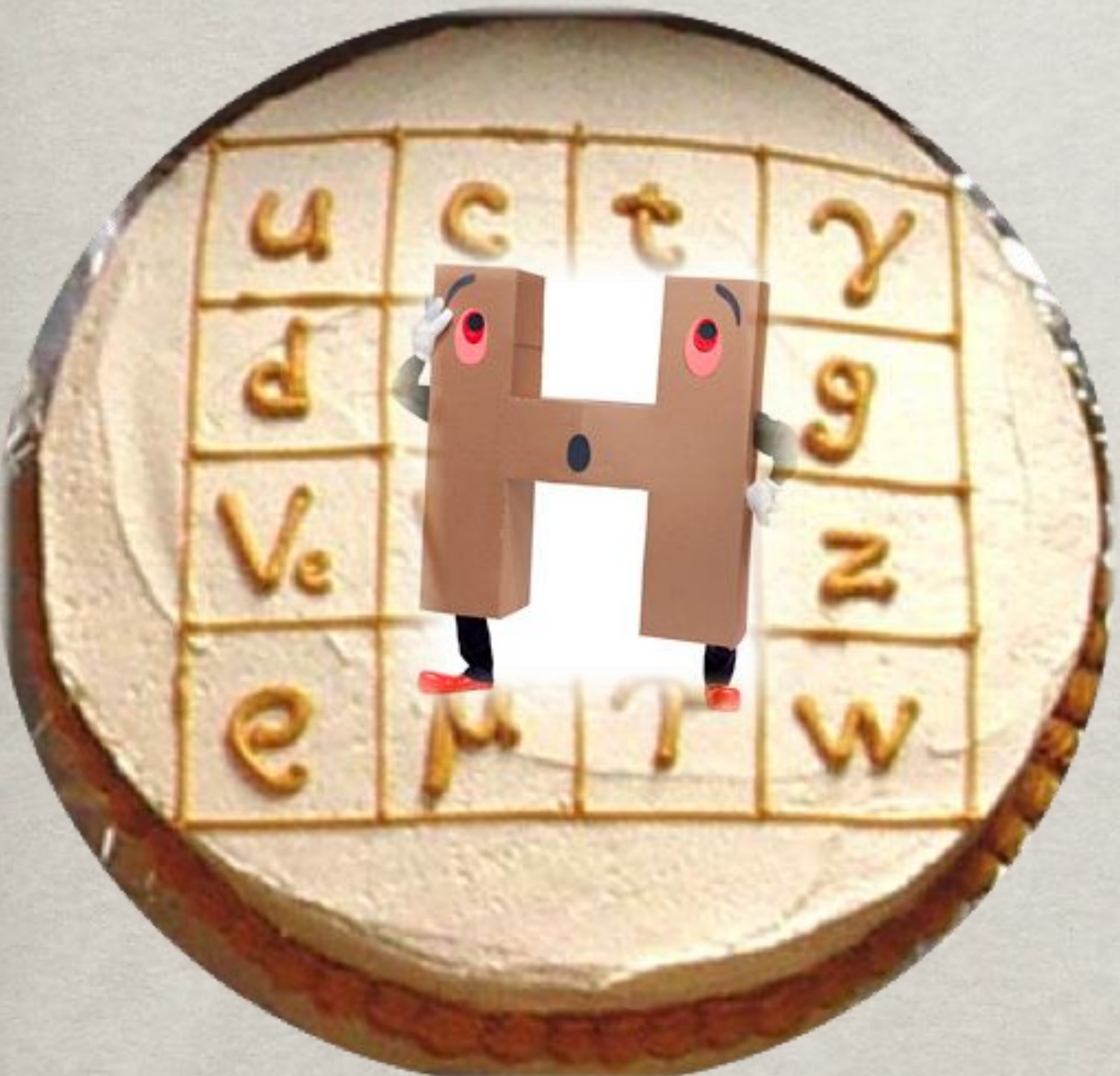
Weak scale SUSY is natural if $M_{SUSY} \sim \mathcal{O}(1\ \text{TeV})$.

- The Little Higgs idea – Strongly interacting dynamics:
  An alternative way to keep H light (naturally).
  Again, predicting new states:

$$W^{\pm}, Z, B \leftrightarrow W_H^{\pm}, Z_H, B_H; \quad t \leftrightarrow T; \quad H \leftrightarrow \Phi.$$

(cancellation among same spin states!)

A light Higgs implies new physics near 1 TeV!



Or

Summary:

- The Standard Model based on the gauge structure

  $SU(3)_c \times SU(2)_L \times U(1)_Y$

  describe our microscopic world very well: 0.1% or better up to a scale O(1 TeV)!

  And could be valid all the way to M_{PL}.

- The revolutionary discovery of the Higgs boson verified the idea of spontaneous EW symmetry breaking & the Higgs mechanism.

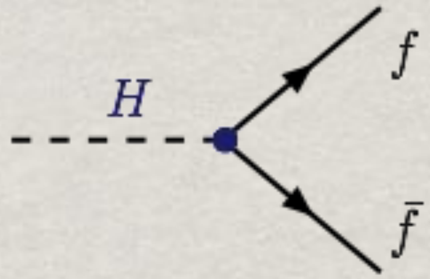
- The "Naturalness" argument indicates the need for new physics at the O(1 TeV): Go LHC & beyond!

We are a lucky generation to participate in the exciting journey!

Skip Slides

C. Higgs Boson Decay§

1. Decay to fermions:



$$\Gamma_{\rm Born}(A \to f\bar{f}) = \frac{G_\mu N_c}{4\sqrt{2}\pi} M_H\, m_f^2\, \beta_f$$

$$\Gamma = g^2\, \frac{dPS_2}{2m} \sum |M|^2 \propto \frac{g^2}{4\pi}\, m\, \beta^{2\ell+1}$$

The largest higher-order effect is the quark running mass:

$$\begin{aligned}\bar{m}_Q(\mu)_{LO} &= \bar{m}_Q(m_Q)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{\frac{2b_0}{\gamma_0}} \\ &= \bar{m}_Q(m_Q)\left(1 - \frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{\mu^2}{m_Q^2}\right) + \cdots\right)\end{aligned}$$

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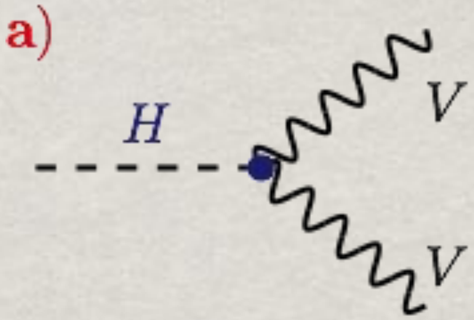
C. Higgs Boson Decay§

2. Decay to WW,ZZ:

a)



$$\Gamma(H \to VV) = \frac{G_\mu M_H^3}{16\sqrt{2}\pi}\,\delta_V\,\sqrt{1-4x}\,(1-4x+12x^2)\ ,\quad x = \frac{M_V^2}{M_H^2}$$

$$\Gamma = g^2\,\frac{dPS_2}{2m}\,\sum |M|^2 \propto \frac{g^2}{4\pi}\,m\,\beta^{2\ell+1}$$

The unusual M^3 dependence is due to the V_L: M_H/M_V.

Exercise 8:

Calculate the Higgs decay to polarized pairs
$V_T V_T$, $V_L V_T$, and $V_L V_L$.

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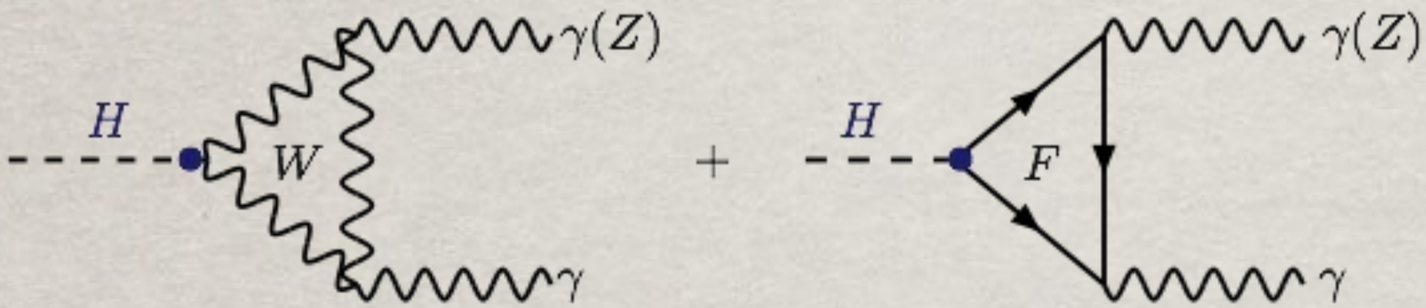
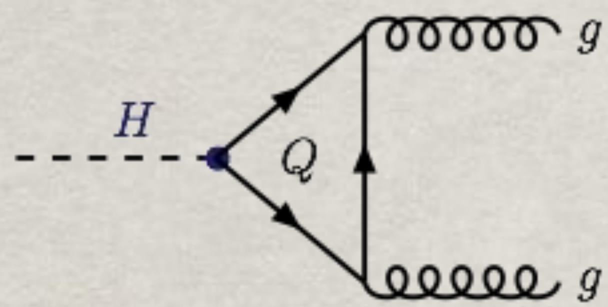
C. Higgs Boson Decay§

3. Decay through loops:

a)



b)



Sensitive to new charged (Q,L), colored (Q) heavy states in loops.

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 A_f^H(\tau_f) + A_W^H(\tau_W) \right|^2$$

$$\Gamma(H \to \gamma Z) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4} \left(1 - \frac{M_Z^2}{M_H^2}\right)^3 \left| \sum_f A_f^H(\tau_f, \lambda_f) + A_W^H(\tau_W, \lambda_W) \right|^2$$

$$\Gamma(H \to gg) = \frac{G_F \alpha_s^2 M_H^3}{36\sqrt{2}\pi^3} \left| \frac{3}{4} \sum_q A_q^H(\tau_q) \right|^2$$

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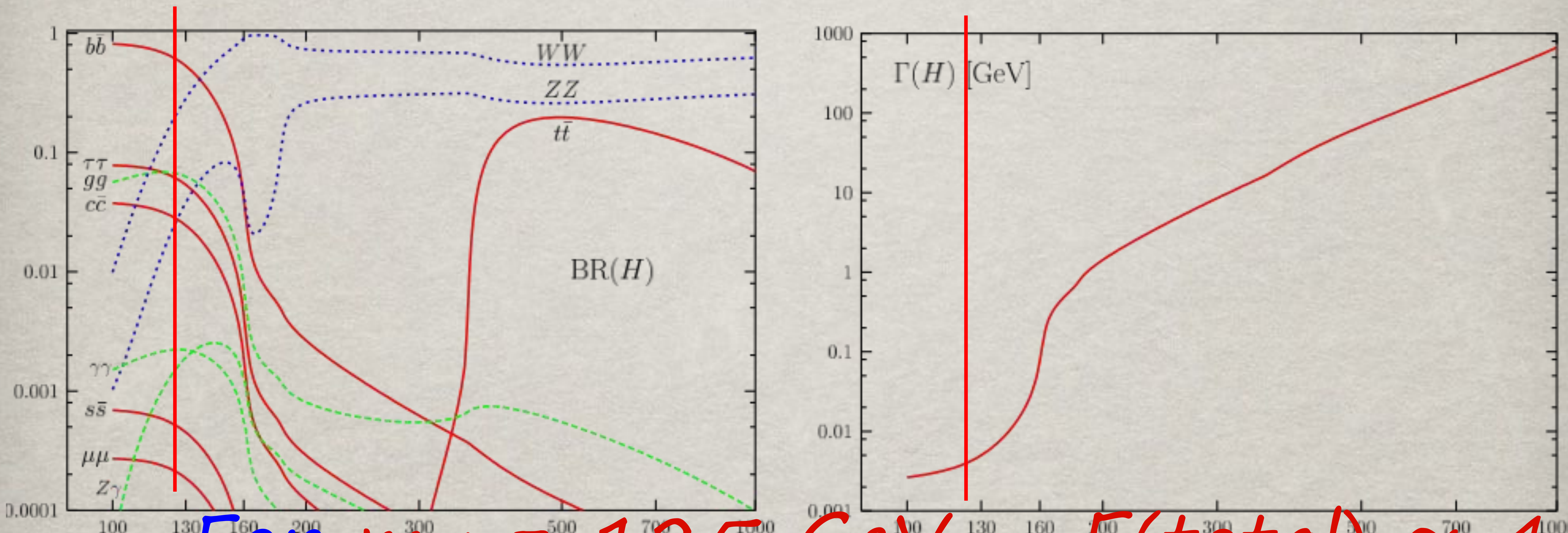
As the results for a SM Higgs: The branching fractions and total width



For m_H = 125 GeV, Γ(total) ≈ 4 MeV

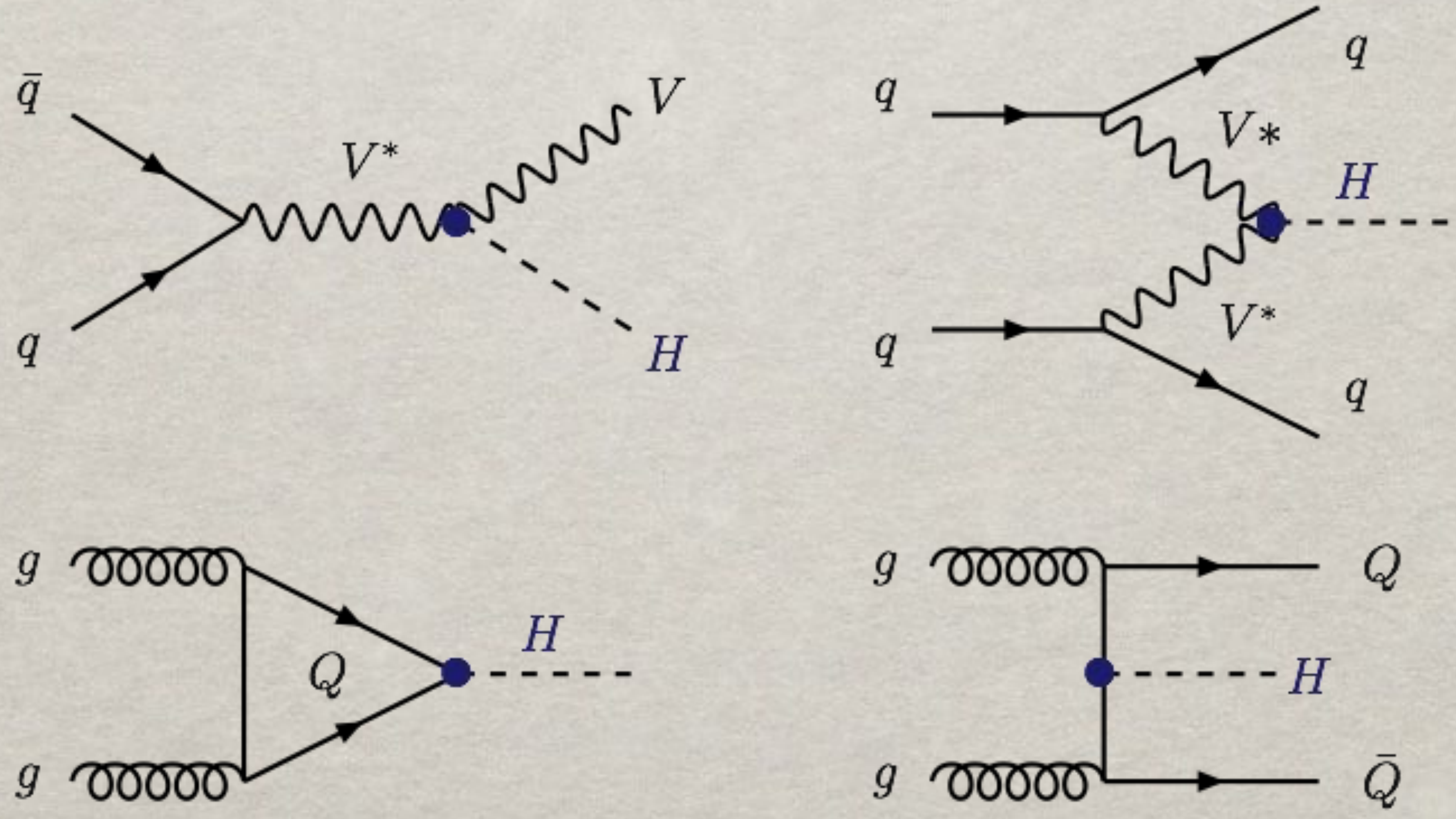
BR(bb) ≈ 60%
BR(ττ) ≈ 8%
BR(WW) ≈ 21%
BR(ZZ) ≈ 2%
BR(gg) ≈ 9%
BR(γγ) ≈ 0.22%

D. Higgs Boson Production at LHC

1. The leading channels:

Recall that the Higgs couples preferably to heavier particles.

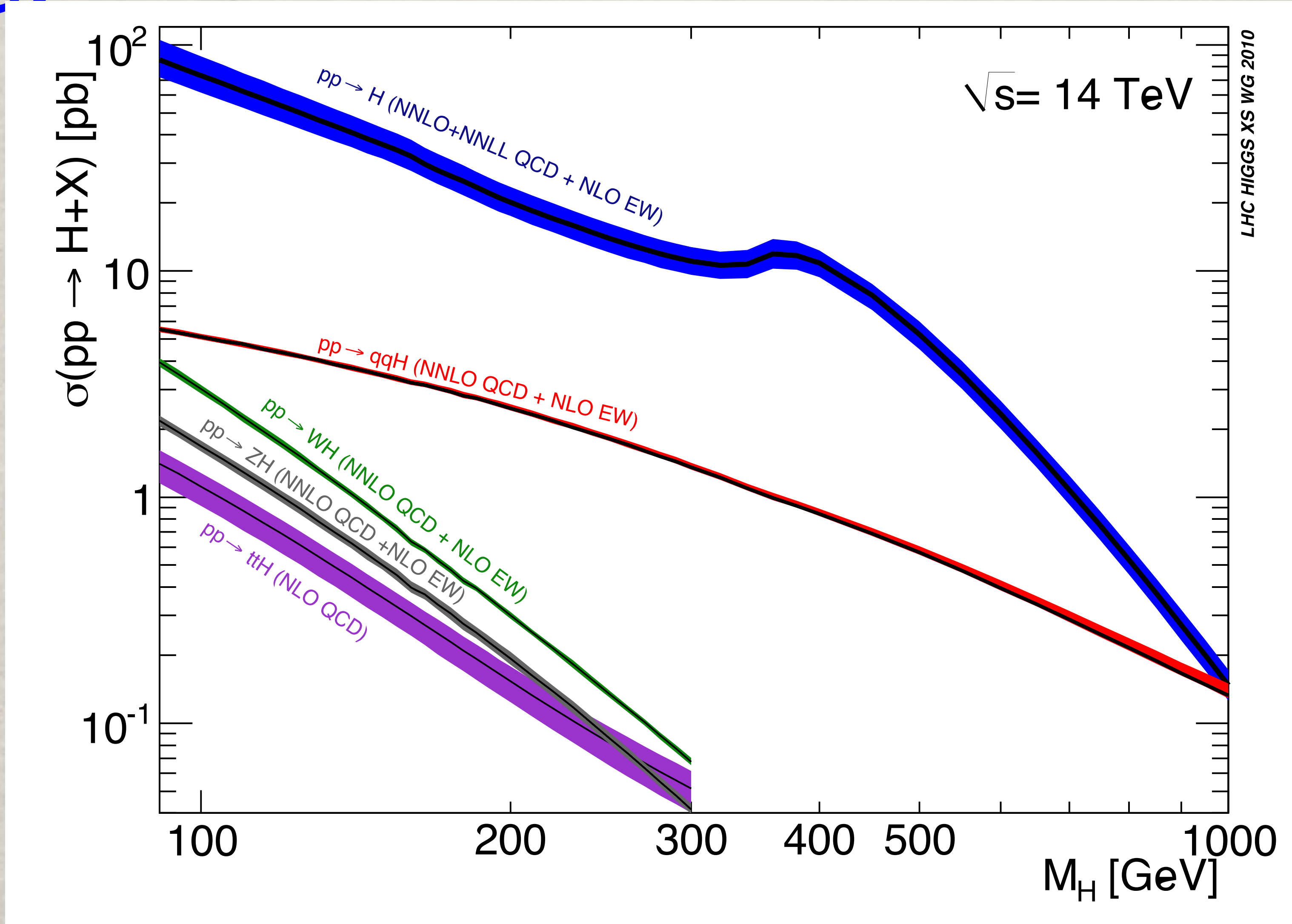
$$
\begin{aligned}
\text{associated production with } W/Z: &\quad q\bar{q} \longrightarrow V + H \\
\text{vector boson fusion}: &\quad qq \longrightarrow V^*V^* \longrightarrow qq + H \\
\text{gluon} - \text{gluon fusion}: &\quad gg \longrightarrow H \\
\text{associated production with heavy quarks}: &\quad gg, q\bar{q} \longrightarrow Q\bar{Q} + H
\end{aligned}
$$



Calculation history and references compiled by Laura Reina

process	$\sigma_{NLO,NNLO}$ by
$gg \to H$	S.Dawson, NPB 359 (1991), A.Djouadi, M.Spira, P.Zerwas, PLB 264 (1991) C.J.Glosser *et al.*, JHEP 0212 (2002); V.Ravindran *et al.*, NPB 634 (2002) D. de Florian *et al.*, PRL 82 (1999) R.Harlander, W.Kilgore, PRL 88 (2002) (NNLO) C.Anastasiou, K.Melnikov, NPB 646 (2002) (NNLO) V.Ravindran *et al.*, NPB 665 (2003) (NNLO) S.Catani *et al.* JHEP 0307 (2003) (NNLL), G.Bozzi *et al.*, PLB 564 (2003), NPB 737 (2006) (NNLL) C.Anastasiou, R.Boughezal, F.Petriello, JHEP (2008) (QCD+EW)
$q\bar{q} \to (W,Z)H$	T.Han, S.Willenbrock, PLB 273 (1991) M.L.Ciccolini, S.Dittmaier, and M.Krämer (2003) (EW) O.Brien, A.Djouadi, R.Harlander, PLB 579 (2004) (NNLO)
$q\bar{q} \to q\bar{q}H$	T.Han, G.Valencia, S.Willenbrock, PRL 69 (1992) T.Figy, C.Oleari, D.Zeppenfeld, PRD 68 (2003) M.L.Ciccolini, A.Denner,S.Dittmaier (2008) (QCD+EW) P.Bolzoni, F.Maltoni, S.O.Moch, and M.Zaro (2010) (NNLO)
$q\bar{q}, gg \to t\bar{t}H$	W.Beenakker *et al.*, PRL 87 (2001), NPB 653 (2003) S.Dawson *et al.*, PRL 87 (2001), PRD 65 (2002), PRD 67,68 (2003)
$q\bar{q}, gg \to b\bar{b}H$	S.Dittmaier, M.Krämer, M.Spira, PRD 70 (2004) S.Dawson *et al.*, PRD 69 (2004), PRL 94 (2005)
$gb(\bar{b}) \to b(\bar{b})H$	J.Campbell *et al.*, PRD 67 (2003)
$b\bar{b} \to H$	D.A.Dicus *et al.* PRD 59 (1999); C.Balasz *et al.*, PRD 60 (1999). R.Harlander, W.Kilgore, PRD 68 (2003) (NNLO)

Production cross sections at the LHC colliders:



Exercise 9: List three leading processes for SM Higgs pair production and comment on their relative sizes and SM backgrounds.

§ L. Reina, TASI lectures, 2011.
A. Djouadi, hep-ph/0503172.

2. Signal Search Strategy (in general):

Searching for the Higgs boson at the LHC

is highly non-trivial!

In theory:

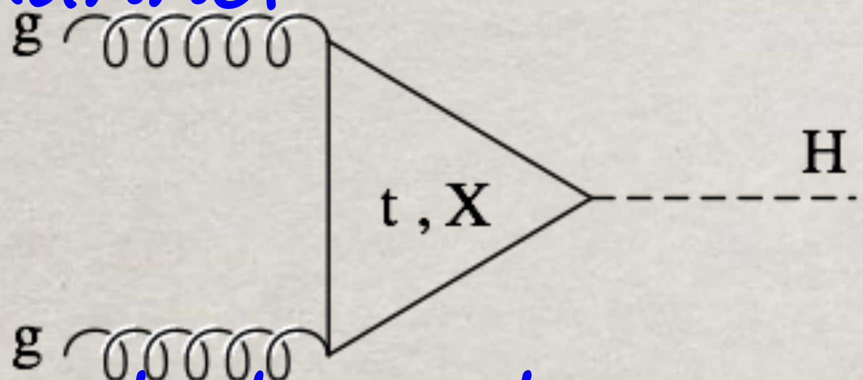
- assume a mass parameter;
- predict the production cross section;
- specify a (good) final state in H decay;
- identify the SM backgrounds;
- calculate the observability by $S/\sqrt{B}$ or alike

In experiments:

- specify a (good) final state from H decay;
- compare with the SM backgrounds;
- assume a mass parameter and compare with theory;
- estimate the sensitivity (μ signal strength, p-value)

3. Signal Characteristics:

(a). Gluon Fusion: The leading production channel



σ(125 GeV@ 8 TeV) ≈ 20 p

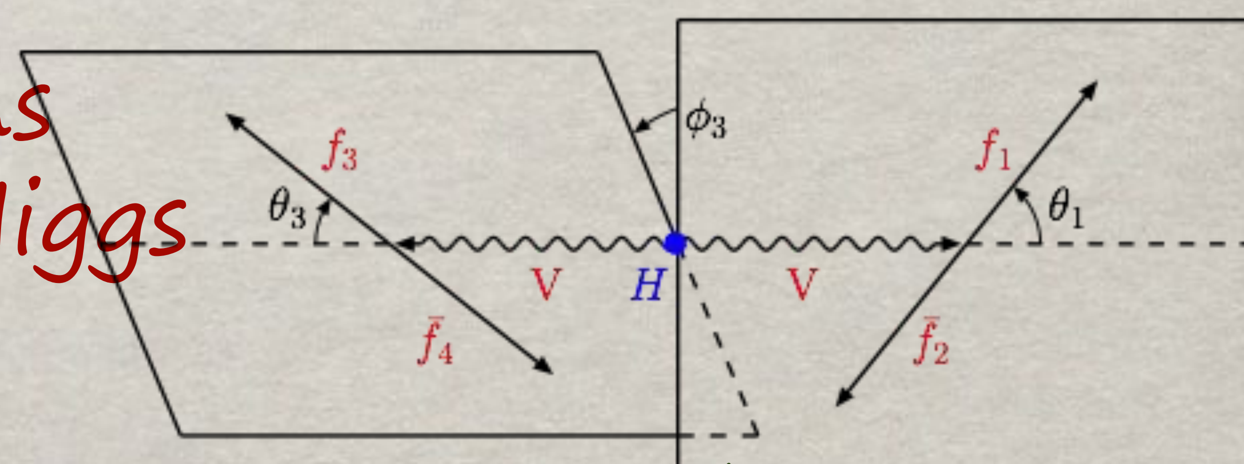
σ(125 GeV@14 TeV) ≈ 40 p

- Need clean decay modes: $\gamma\gamma$, WW, ZZ
- Effects from radiative corrections very large!§
- Sensitive to new colored particles in the loop:

gg H sensitive to new colored states: Q

H $\gamma\gamma$ sensitive to new charged states: Q, L

H ZZ 4 leptons best to study the Higgs CP properties:



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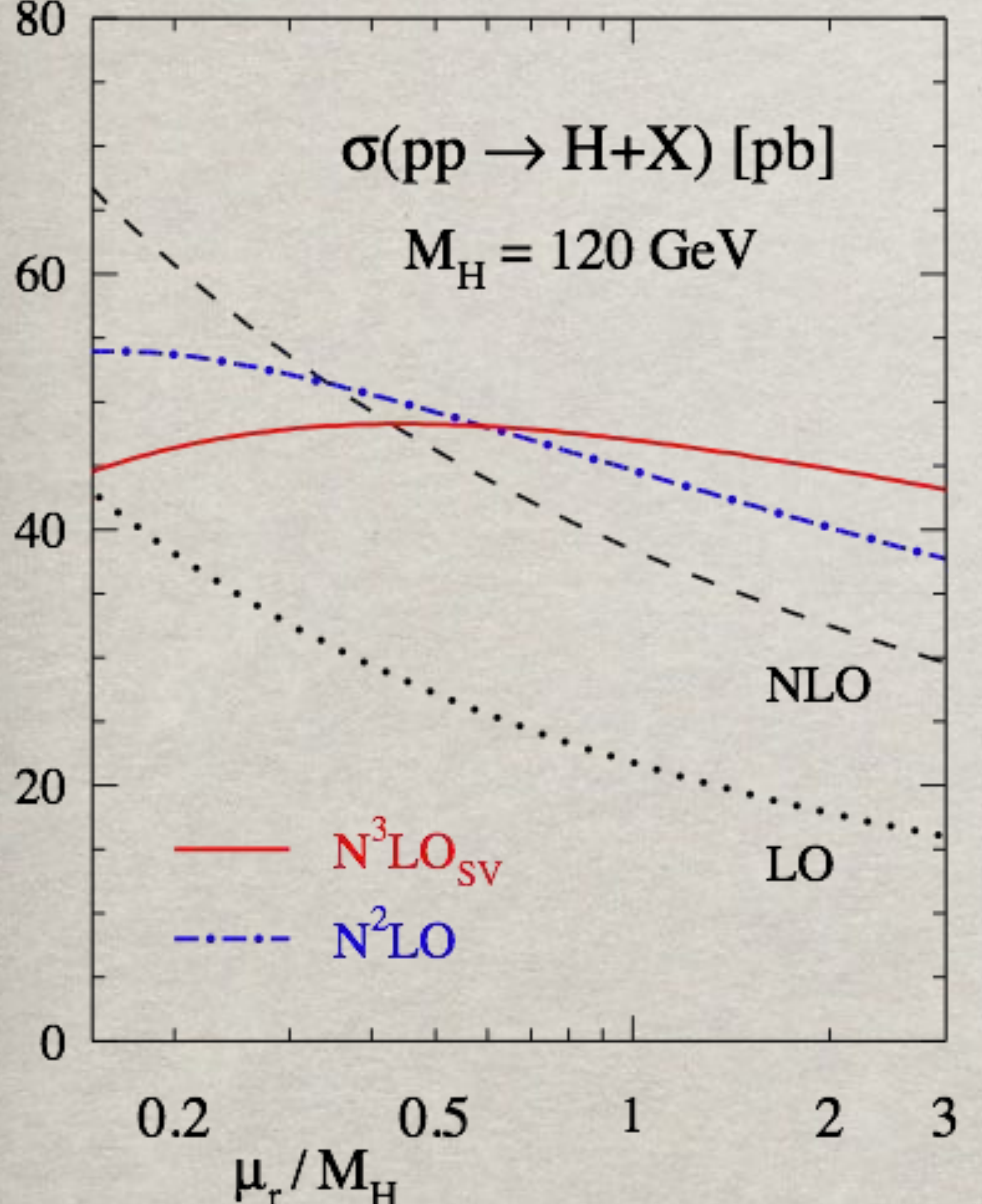
QCD corrections to $gg \to H$

Moch & Vogt, hep-ph/0508265

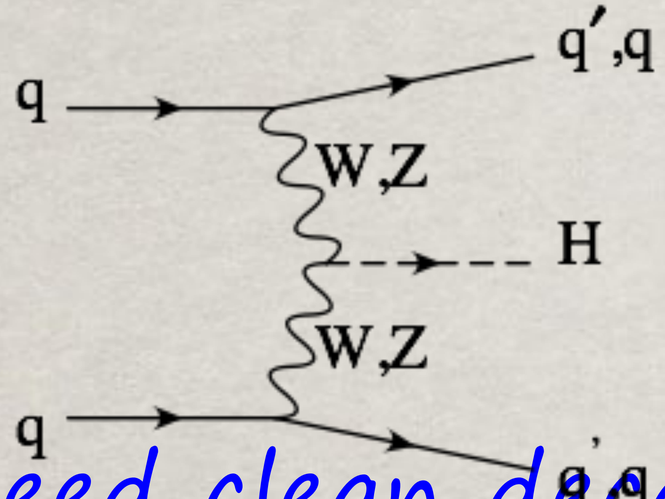


- Large QCD corrections: K-factor of about 2
- Stabilization of scale dependence needs N^3LO or at least NNLO corrections
- Cross section estimate for $m_H = 126$ GeV at 8 TeV from LHC XS WG, determined at NNLL QCD and NLO EW

$$\sigma(gg \to H) = 19.22\,\text{pb} \pm 14.7\%$$

- Error is linear combination of $\approx 7.5\%$ scale uncertainty and $\approx 7.2\%$ from gluon pdf and α_s error
- Additional uncertainty from use of effective hgg vertex (heavy top approximation) is estimated to be below 2%

(b). The Vector Boson Fusion:



$\sigma(14\ \text{TeV}) \approx 4\ pb$

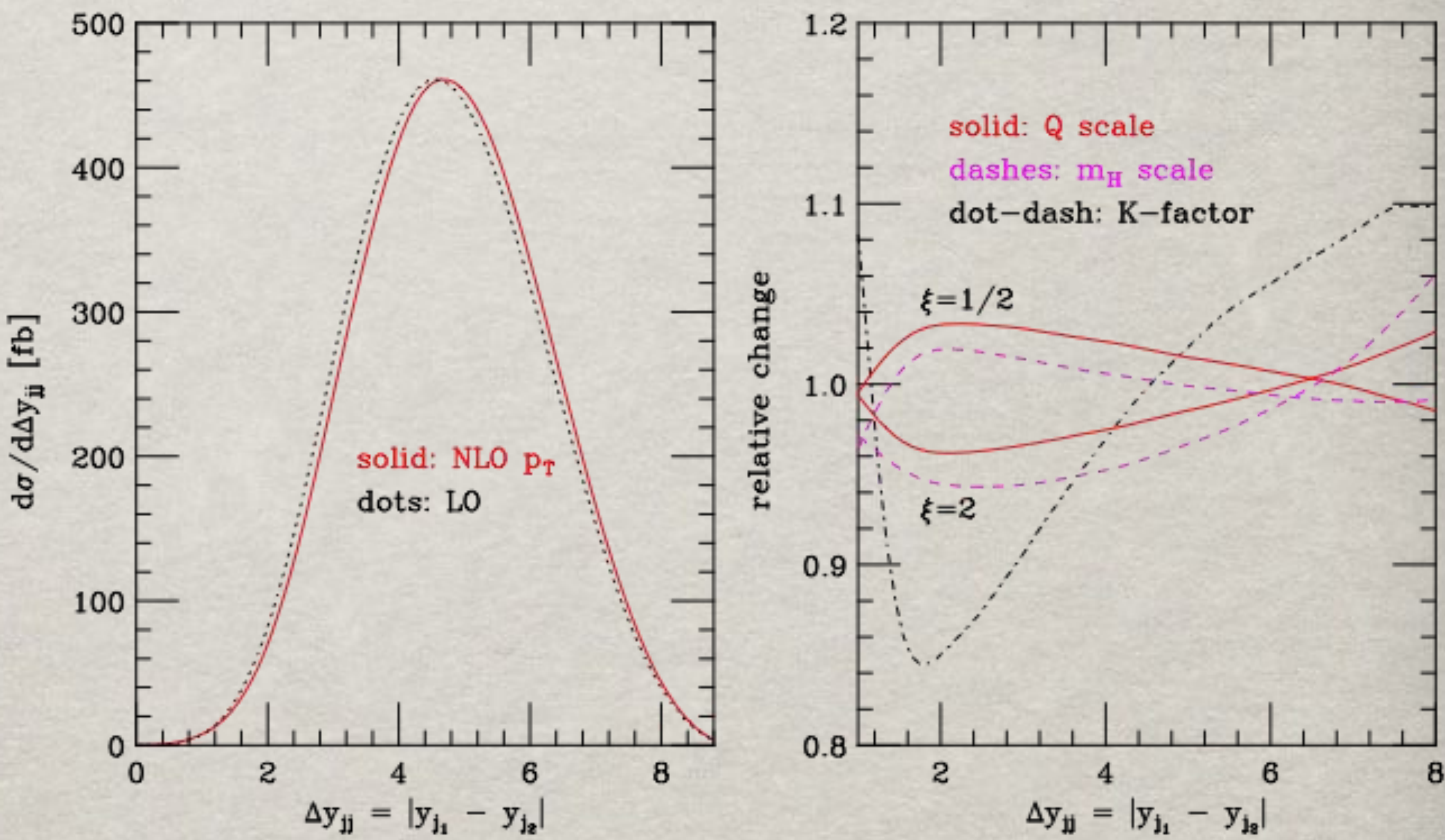
- Need clean decay modes: $\tau\tau$, WW, ZZ, $\gamma\gamma$
- Effects from radiative corrections very small!
  -> color singlet exchange, low jet activities.
- Sensitive to HWW, HZZ couplings
- Good for H $\tau\tau$, $\gamma\gamma$
- A bit lower rate, but unique kinematics

NLO corrections to VBF

- Small QCD corrections of order 10%
- Tiny scale dependence of NLO result
  - $\pm 5\%$ for distributions
  - $< 1\%$ for σ_{total}
- pdf error is below 3% since pdf's are dominated by valence quarks
- $\approx\ -5\%$ EW corrections included

  Ciccolini, Denner, Dittmaier, 0710.4749

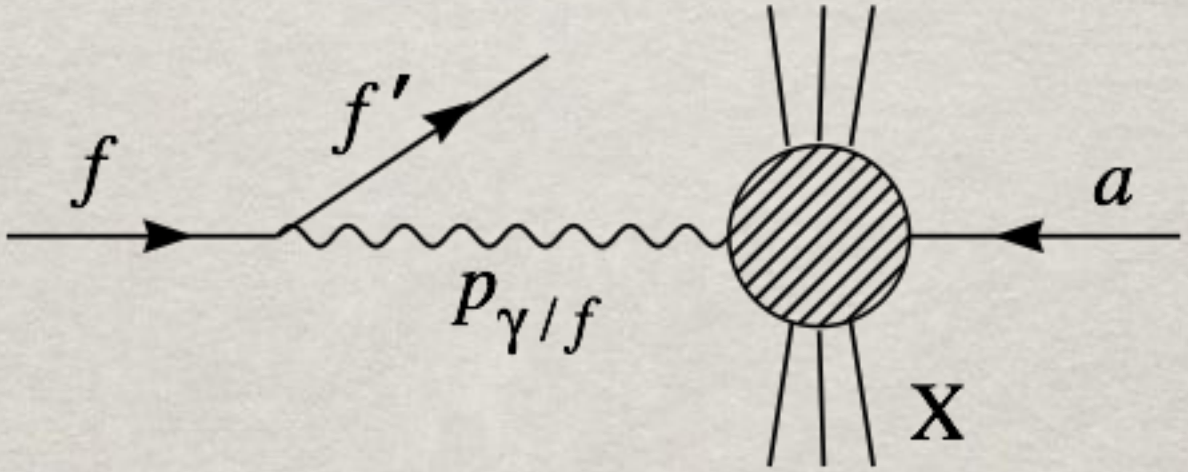
  Figy, Palmer, Weiglein arXiv:1012.4789
- Very small cross section error of about 3% for $m_H = 126$ GeV



$m_H = 120$ GeV, typical VBF cuts

Basic feature: V radiation off a quark

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \to f'X) \approx \int dx\, dp_T^2\, P_{\gamma/f}(x, p_T^2)\, \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \left(\frac{1}{p_T^2}\right)\Big|_{m_e}^{E}.$$

Exercise 10: Qualitative feature for V radiation off a quark

- Generalize to massive gauge bosons:

$$P^T_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{8\pi^2}\frac{1+(1-x)^2}{x}\frac{p_T^2}{(p_T^2+(1-x)M_V^2)^2},$$

$$P^L_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{4\pi^2}\frac{1-x}{x}\frac{(1-x)M_V^2}{(p_T^2+(1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low-p_{jT},

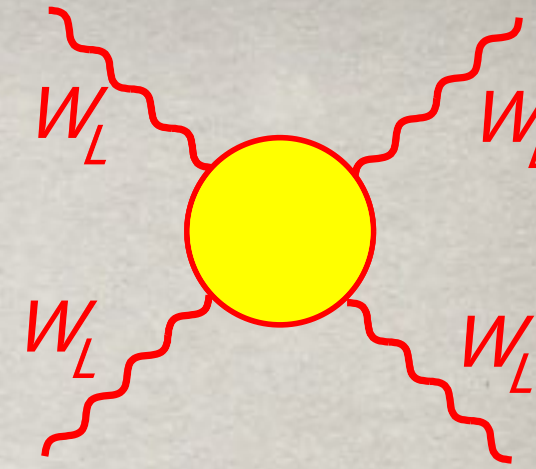
$$\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}$$

At high-p_{jT},

$$\left.\begin{array}{l} \frac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

VBF as a Probe for $W_L W_L$ Scatteting



$W_T^{\pm,3}$ Yang-Mills gauge self-interactions:

$$
\begin{aligned}
\mathcal{L}_{W3} = \; & -ig(\partial_\rho W^3_\nu) W^+_\mu W^-_\sigma [g^{\rho\mu} g^{\nu\sigma} - g^{\rho\sigma} g^{\nu\mu}] \\
& -ig(\partial_\rho W^+_\mu) W^3_\nu W^-_\sigma [g^{\rho\sigma} g^{\mu\nu} - g^{\rho\nu} g^{\mu\sigma}] \\
& -ig(\partial_\rho W^-_\sigma) W^3_\nu W^+_\mu [g^{\rho\nu} g^{\mu\sigma} - g^{\rho\mu} g^{\nu\sigma}],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{W4} = \; & \frac{g^2}{4}\left[W^+_\mu W^+_\nu W^-_\sigma W^-_\rho Q^{\mu\nu\rho\sigma} - 2 W^+_\mu W^3_\nu W^3_\sigma W^-_\rho Q^{\mu\rho\nu\sigma}\right], \\
& Q_{\mu\nu\rho\sigma} \equiv 2 g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}.
\end{aligned}
$$

Transversely polarized gauge bosons

$$\epsilon^\mu_T \sim (0, \cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta) \quad \Rightarrow \quad \mathcal{A}(W_T W_T \to W_T W_T) \sim \mathcal{O}(g^2).$$

Scattering amplitudes well behaved at high energies.

Longitudinally polarized gauge bosons

$\epsilon_L^\mu \approx p^\mu / M_W$ at high energies.

Longitudinally polarized gauge bosons scattering

$$A(W_L W_L \to W_L W_L) \sim \epsilon_{1L} \cdot \epsilon_{2L}\, \epsilon_{3L} \cdot \epsilon_{4L}$$

Naively, $A(W_L W_L \to W_L W_L) \sim g^2 (p^2)^2 / M_W^4 \sim g^2 s^2 / M_W^4$,
but miraculously canceled (due to gauge symmetry).

Next, $A(W_L W_L \to W_L W_L) \sim g^2 s / M_W^2 \sim s / v^2$,
just like the Nambu-Goldstone boson scattering, no g^2!

Goldstone-boson Equivalence Theorem: ¶

Goldstone-boson Equivalence theorem indicates that $W_L W_L$ scattering at HE, $E_W >> M_W$, is the most direct probe for EWSB.

W W Scattering

After all, at the heart of the EW SB:

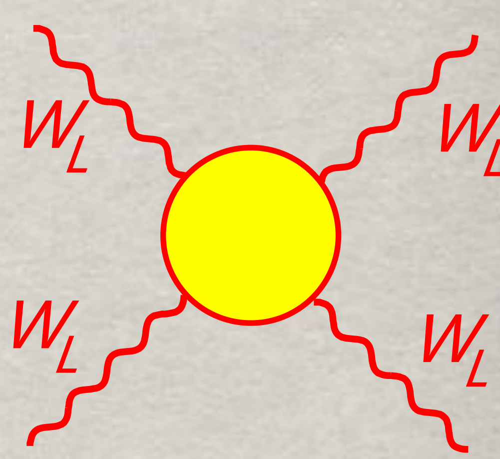
$$\mathcal{M}(w^+w^- \to w^+w^-) = \frac{1}{3}\mathcal{M}^{I=0} + \frac{1}{2}\mathcal{M}^{I=1} + \frac{1}{6}\mathcal{M}^{I=2} \qquad (H,\ \rho)$$

$$\mathcal{M}(w^+w^- \to zz) = \frac{1}{3}\mathcal{M}^{I=0} - \frac{1}{3}\mathcal{M}^{I=2} \qquad (H)$$

$$\mathcal{M}(zz \to zz) = \frac{1}{3}\mathcal{M}^{I=0} + \frac{2}{3}\mathcal{M}^{I=2} \qquad (H)$$

$$\mathcal{M}(w^\pm z \to w^\pm z) = \frac{1}{2}\mathcal{M}^{I=1} + \frac{1}{2}\mathcal{M}^{I=2} \qquad (\rho)$$

$$\mathcal{M}(w^\pm w^\pm \to w^\pm w^\pm) = \mathcal{M}^{I=2} \qquad (\text{no resonance},\ \sim s/v^2)$$

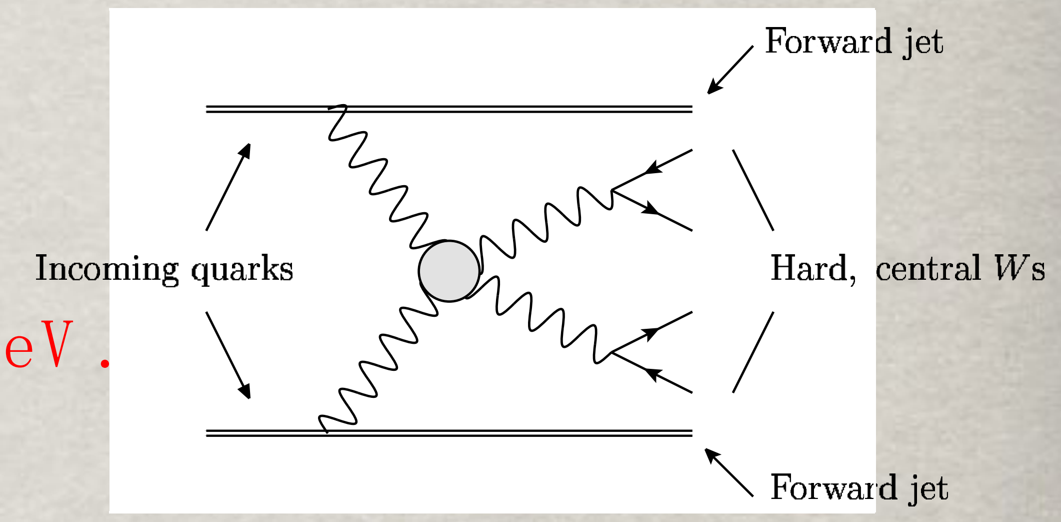


e.g., for model discrimination:

$$\frac{\sigma(w^+w^- \to w^+w^-)}{\sigma(w^+w^- \to zz)} \begin{cases} \sim 2 & \text{scalar } H^0, \\ \gg 1 & \text{vector } \rho^0_{TC}, \\ \sim 2/3 & \text{LET } \sqrt{s} \ll M. \end{cases}$$

Signals at the LHC

(A). How Do We See the Signal?



Signal features in final state:

$$pp \to W_1 W_2\, j_1 j_2\, X.$$

- high-energy gauge boson pairs $E_W \sim 0.5$ TeV.
- forward jets $E_j \sim O(1)$ TeV, $p_{Tj} \sim M_W/2$.

Challenges:

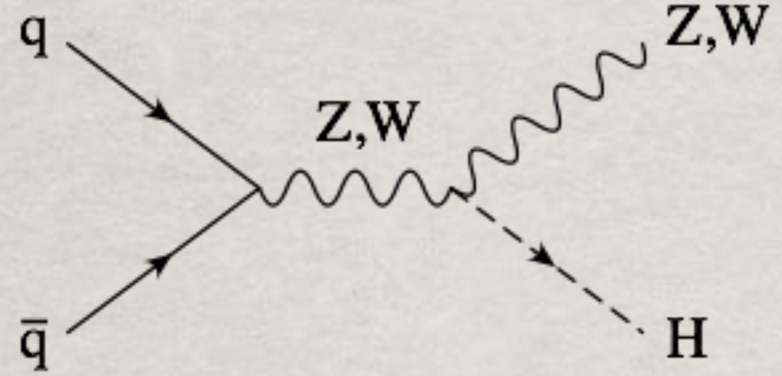
- need high-energy $s_{WW} \sim 0.5^2 s_{qq} \sim \frac{0.3^2}{4} s_{pp}$.
  for 14 TeV LHC $E_{WW} \sim 1.5$ TeV.
- identification of $W \to \ell\nu$, $Z \to \ell^+\ell^-$, $M_W(jj')$, $M_Z(jj)$.
  branching fractions, detection efficiency ...
- background, background, background!

$pp \to W_1 W_2$ QCD jets X (large, but distinctive)

$pp \to W_{T1} W_{T2}$ EW jets X (mimick signal, most difficult)

$pp \to t\bar{t}\,X \to W^+W^-\, b\bar{b}\,X$ (very large, more jetty)

(c). VH Associate production:

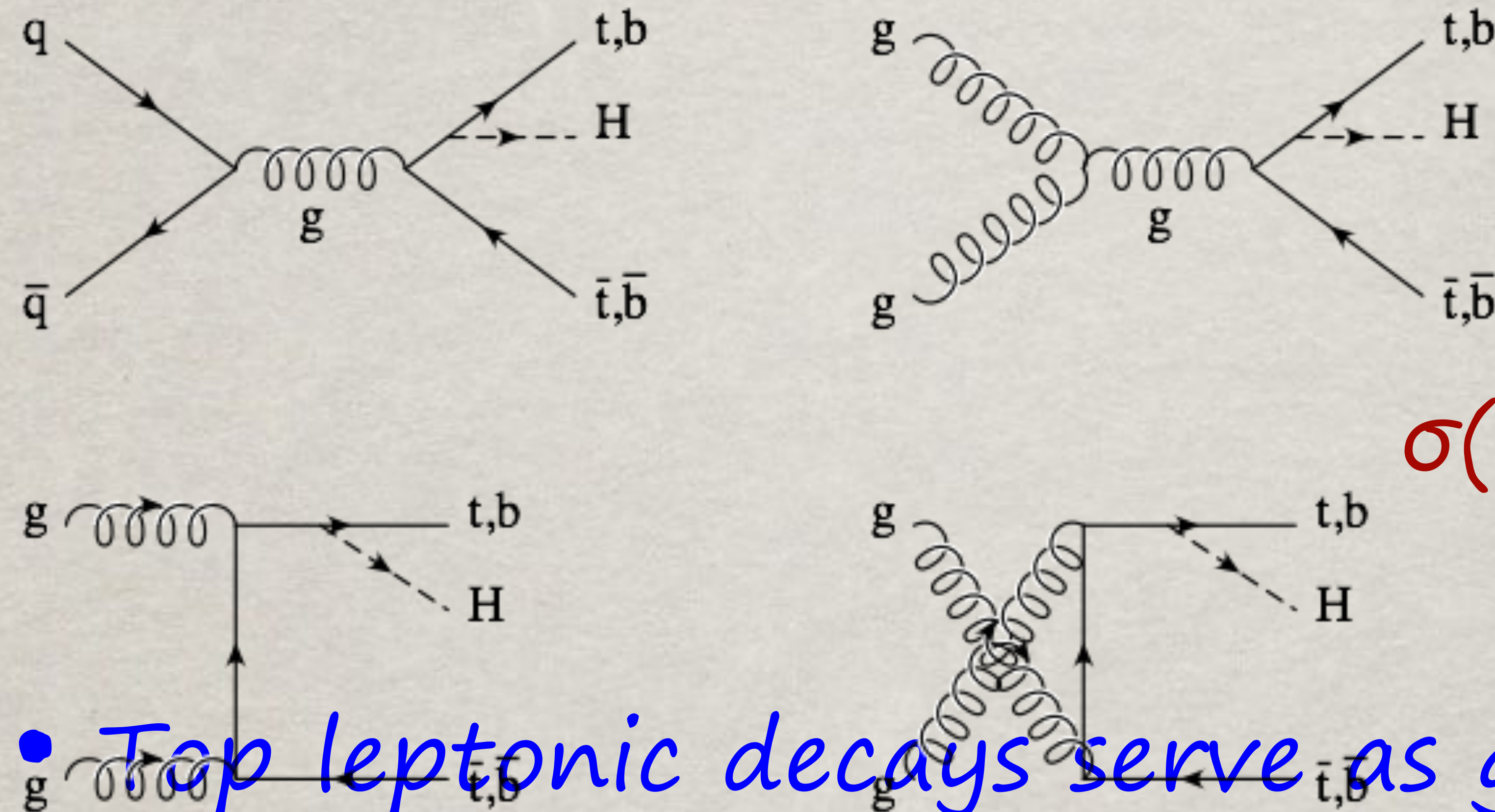


$\sigma(14\ \text{TeV}) \approx 2.2$ pb

- W/Z leptonic decays serve as good trigger.
- Effects from radiative corrections very modest.
- Sensitive to HWW, HZZ couplings
- Do not need clean decay modes: chance for b bbar !

Boosted Higgs helps for the signal ID!

(d). Top quark pair associate production:



$\sigma(14\ \text{TeV}) \approx 0.6$ pb

- Top leptonic decays serve as good trigger.
- Effects from radiative corrections can be large.
- Directly sensitive to Htt coupling
- Do not need clean decay modes: chance for b bbar !
- Combinatorics of the 4 b's are difficult to handle...

Precision Higgs Physics

In a pessimistic scenario, the LHC does not see a new particle associated with the Higgs sector, then the effects of a heavy state on Higgs coupling g_i at the scale M:

$$\Delta_i \equiv \frac{g_i}{g_{SM}} - 1 \sim \mathcal{O}(v^2/M^2)$$

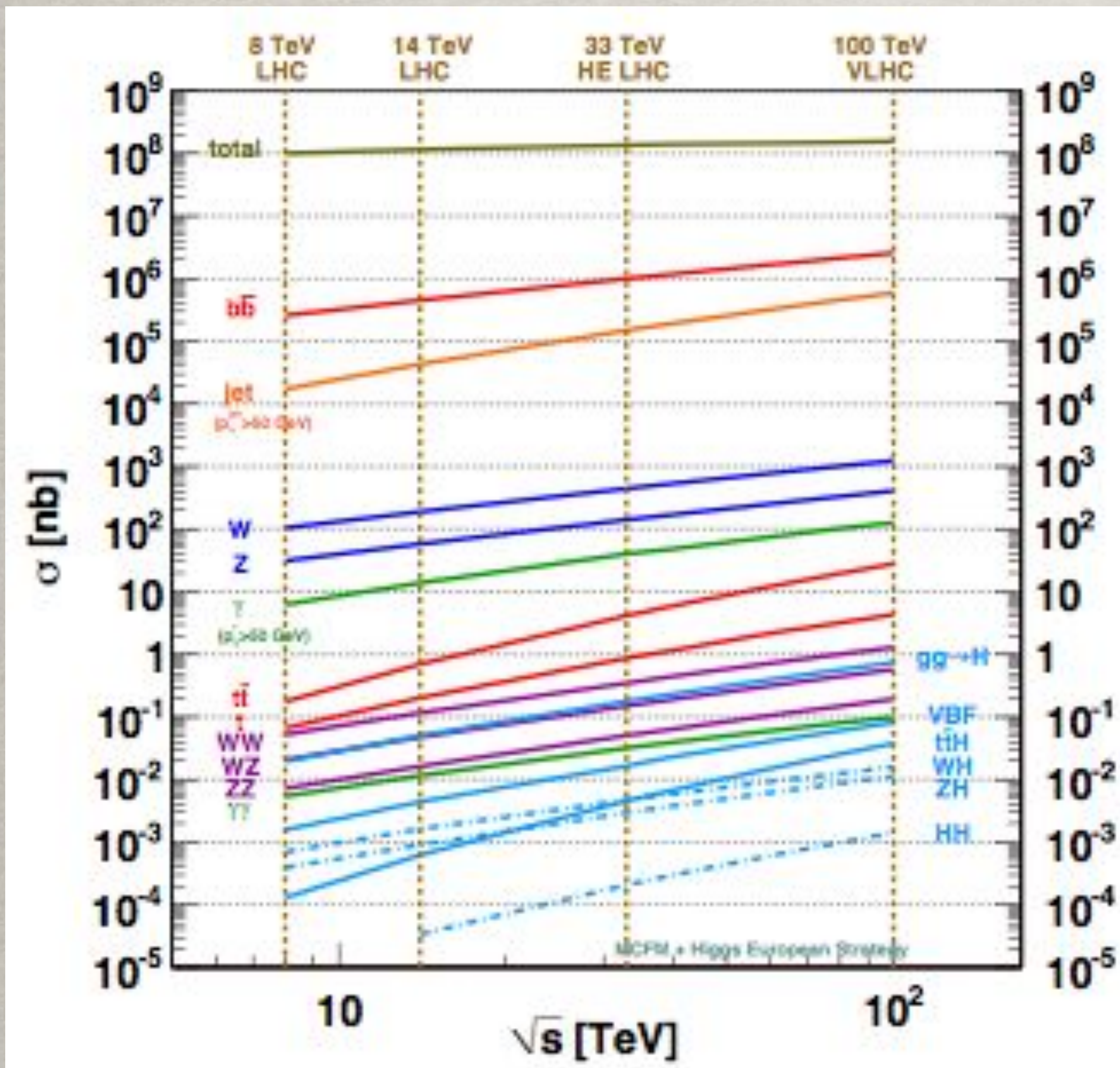
≈ a few % for M ≈ 1 TeV

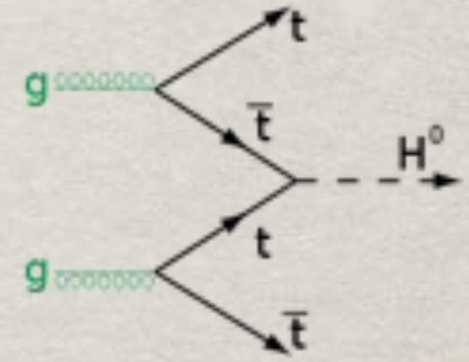
Higgs coupling deviations:

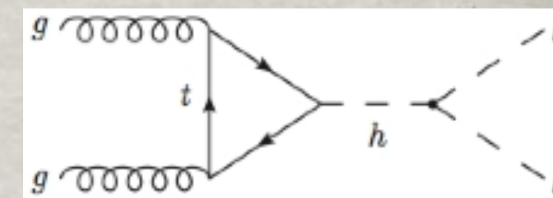
If not observed, it DEFINE it THE SM Higgs!

Δ:	VVH	bbH, ττH	ggH, γγH	HHH
Composite	(3-9)%	$(1\ \mathrm{TeV}/f)^2$ (tree-level)		100%
H^0, A^0		$6\%\ (500\ \mathrm{GeV}/M_A)^2$ (loop)		
T'			-10%	
4 TeV, 3ab⁻¹: 8%	$(1\ \mathrm{TeV}/M_T)^2$	15%	few%	

Higgs Production @ SPPC







Process	σ (100 TeV)/σ (14 TeV)
Total pp	1.25
W	~7
Z	~7
WW	~10
ZZ	~10
tt	~30
H	~15 (ttH ~60)
HH	~40
stop (m=1 TeV)	~10^3

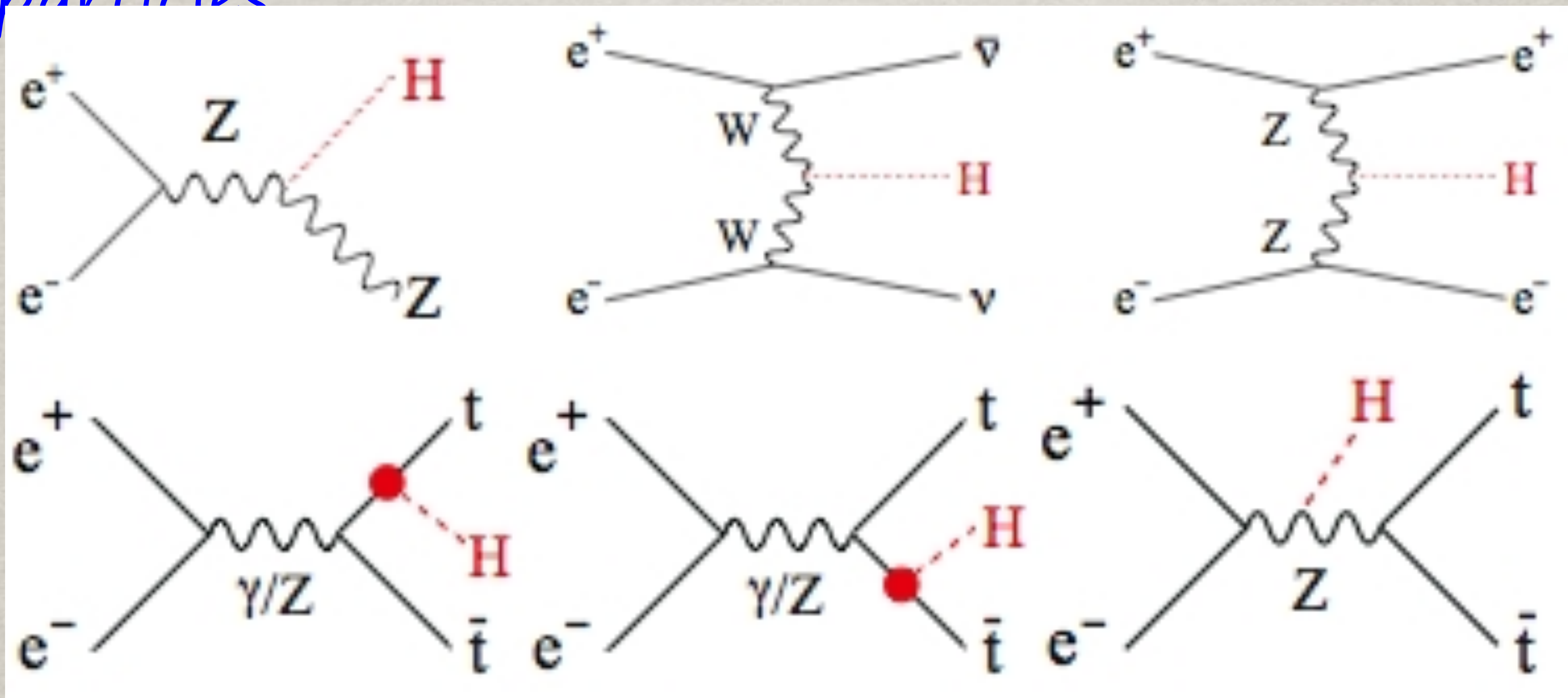
λ_t : 1%
λ : 8%

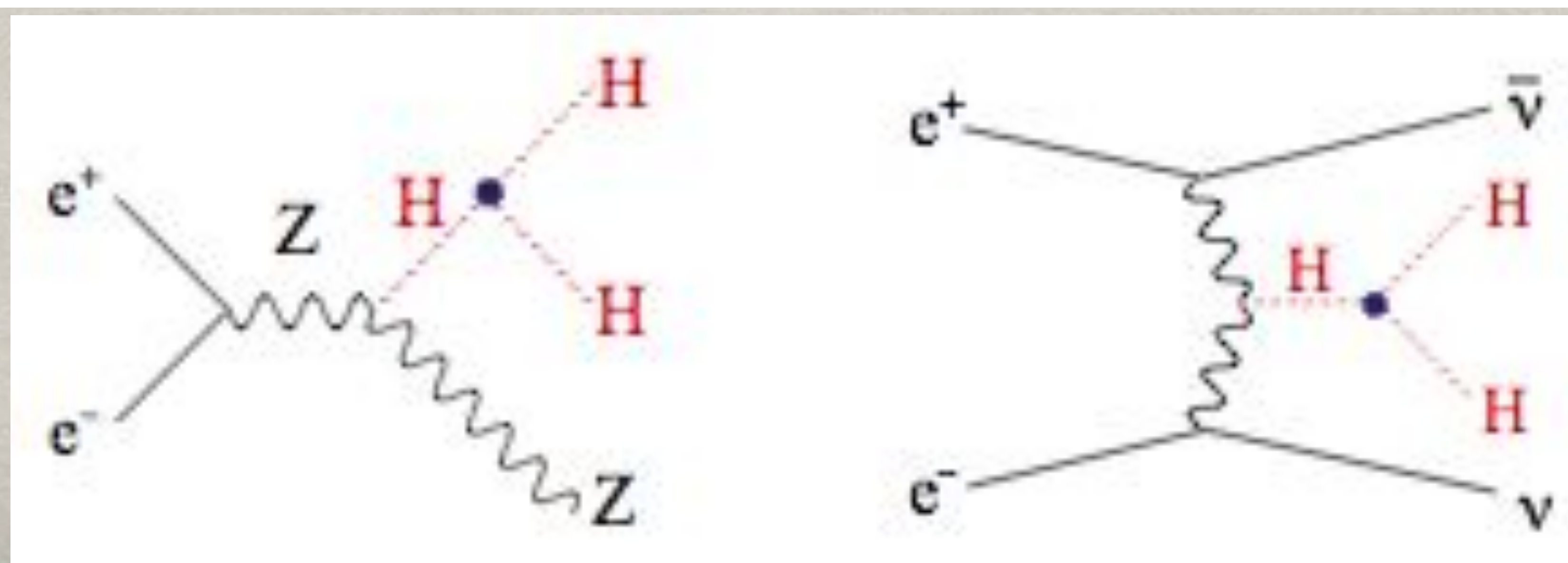
Snowmass QCD Working Group: 1310.5189

E. Higgs Boson Production at e^+e^- Colliders

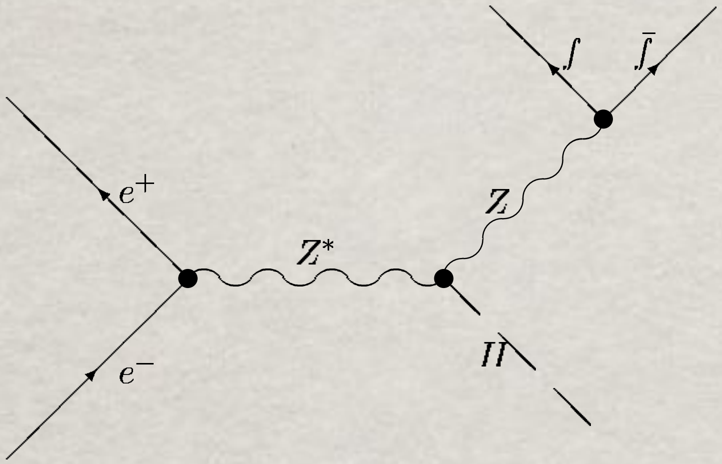
1. The leading channels:

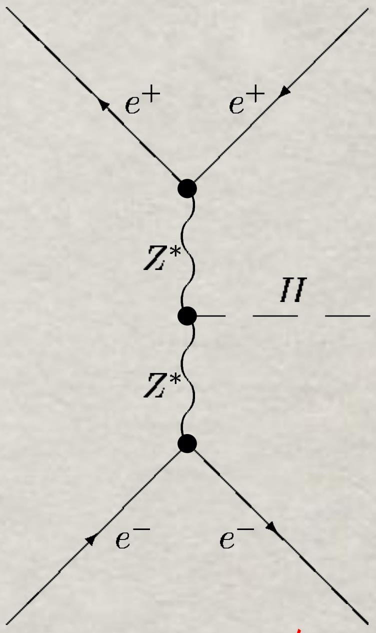
Recall that the Higgs couples preferably to heavier particles



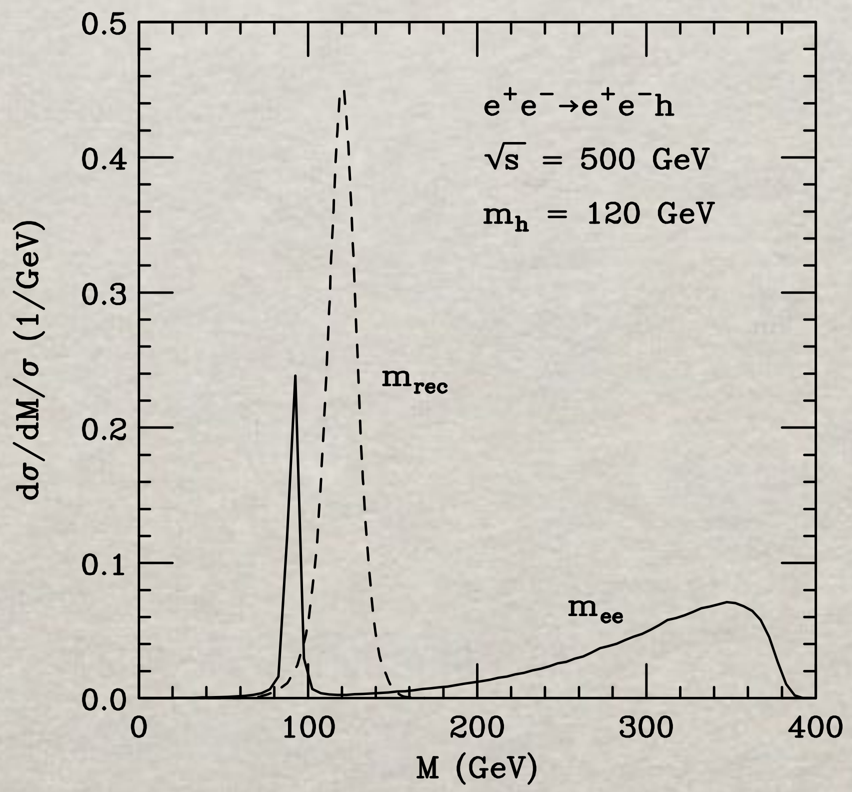


The key point for a Higgs factory: $e^+ + e^- \to f\bar{f} + h$.



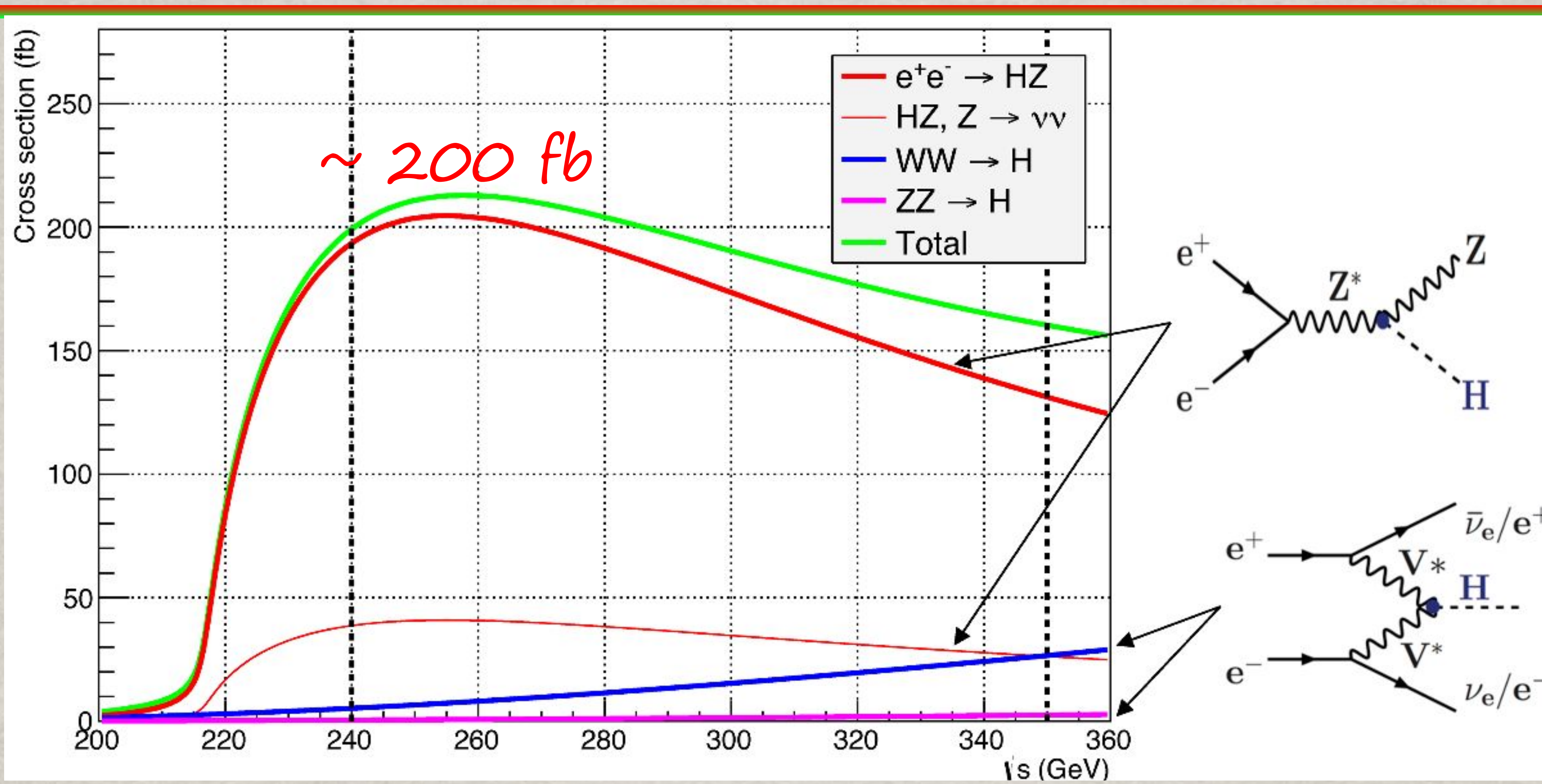


Then: $M_h^2 = (p_{e^+} + p_{e^-} - p_f - p_{\bar{f}})^2 = s + M_V^2 - 2\sqrt{s}E_{f\bar{f}}$.



Model-independent, kinematical selection of signal events!

Higgs-Factory: Mega (10^6) Higgs Physics



ILC: E_{cm} = 250 (500) GeV, 250 (500) fb^{-1}

ILC Report: 1308.6176

- Model-independent measurement:

  $\Gamma_H \sim 6\%$, $\Delta m_H \sim 30$ MeV

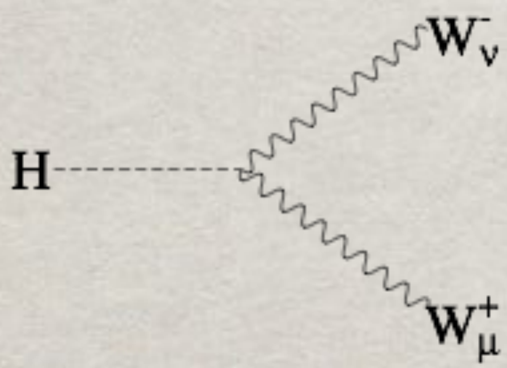
  (HL-LHC: assume SM, $\Gamma_H \sim 5\text{-}8\%$, $\Delta m_H \sim 50$ MeV)

TLEP Report: 1308.6176

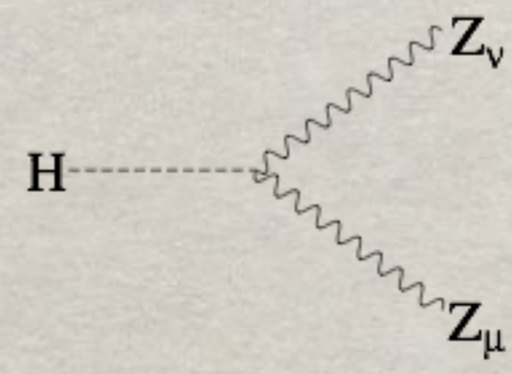
4. Higgs Coupling Deviations:

No matter there is new physics BSM seen or not, Higgs couplings need to be measured as accurate as possible.

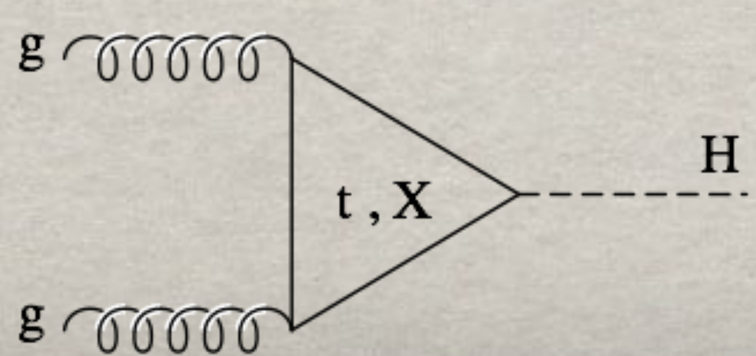
$$-i\frac{m_f}{v}\left(1+\Delta_f\right)$$

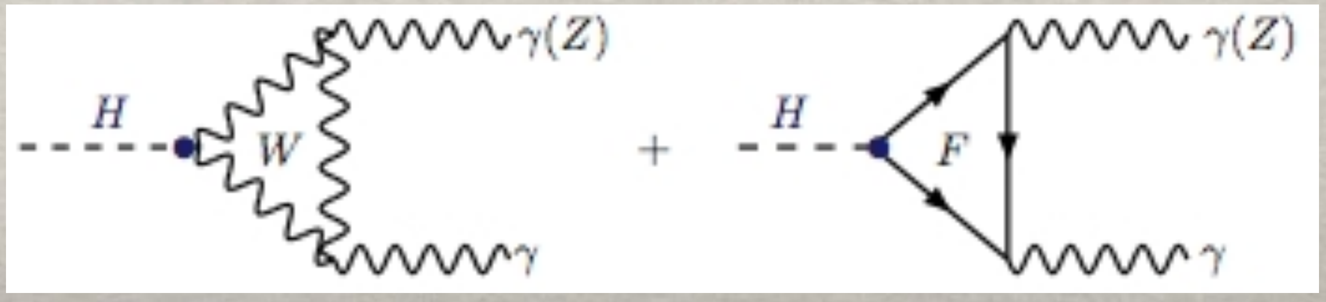


$$ig\, m_W\,(1+\Delta_W)g_{\mu\nu}$$



$$i\,g\,\frac{1}{\cos\theta_W}\,m_Z\,(1+\Delta_Z)g_{\mu\nu}$$



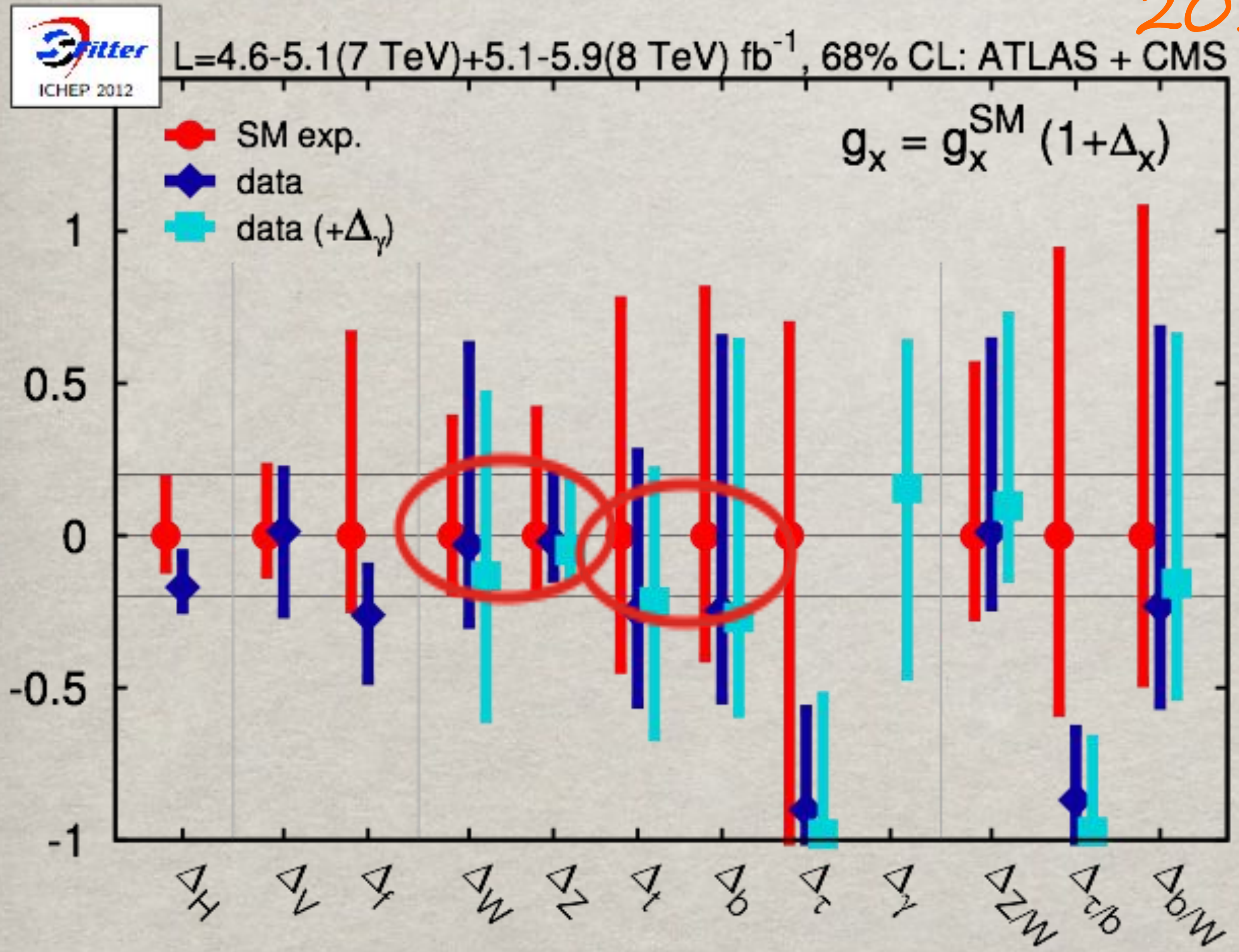


Current accuracies:

Central values and errors on couplings

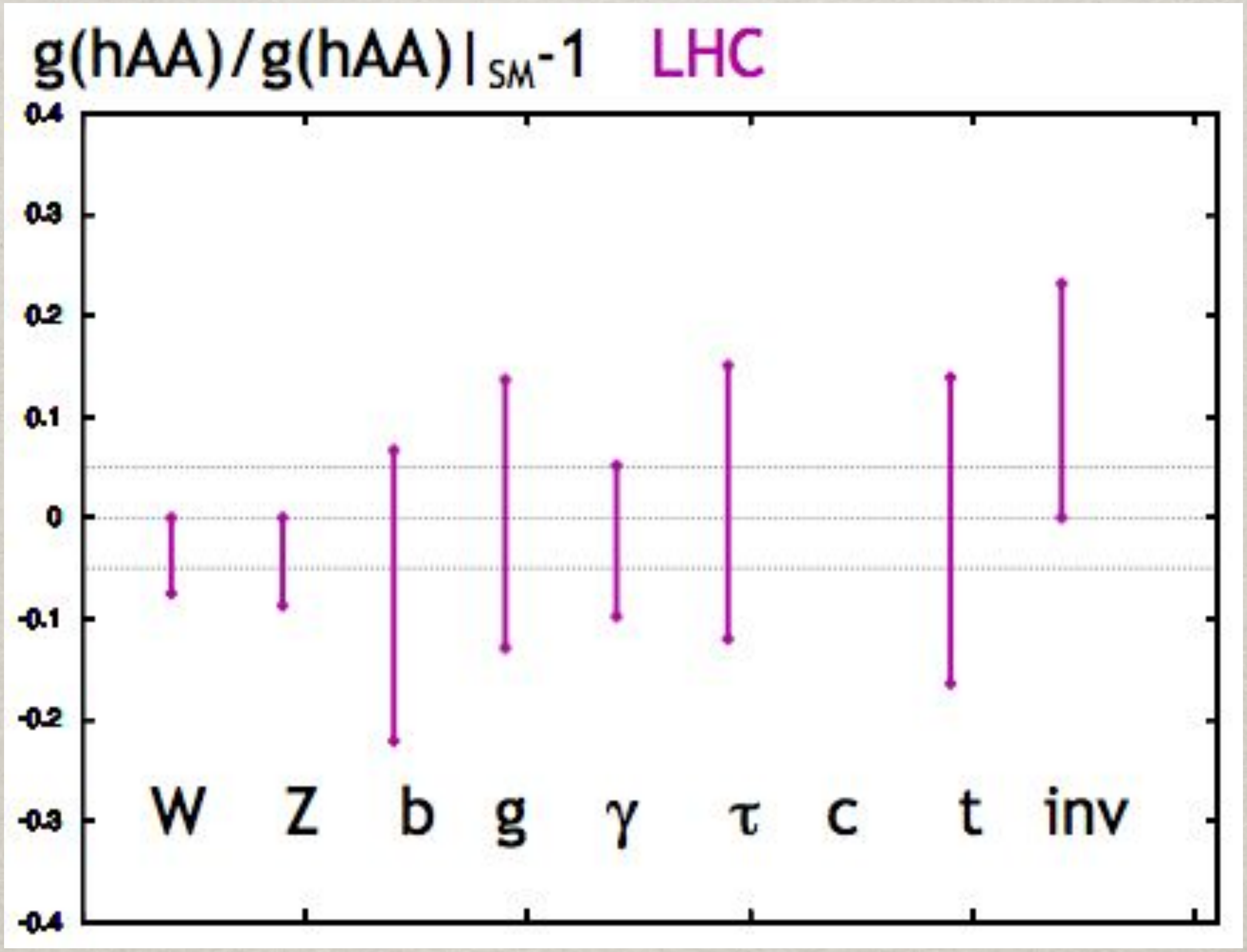
Assuming SM:

SFitter: T. Plehn et al., 2012.



- SM provides good overall description
- Two parameter fit with $\Delta_V \equiv \Delta_W = \Delta_Z$ and $\Delta_f \equiv \Delta_b = \Delta_\tau = \Delta_t$ gives improvement to $\chi^2/\text{d.o.f.} = 29.0/52$
- Five parameter fit does not give further improvement: $\chi^2/\text{d.o.f.} = 27.7/49$

Future LHC sensitivities:



14 TeV LHC with 300 fb^{-1}.

Peskin, arXiv:1207.2516; arXiv:1208.5152.

Not-So" Standard" Higgs Sector

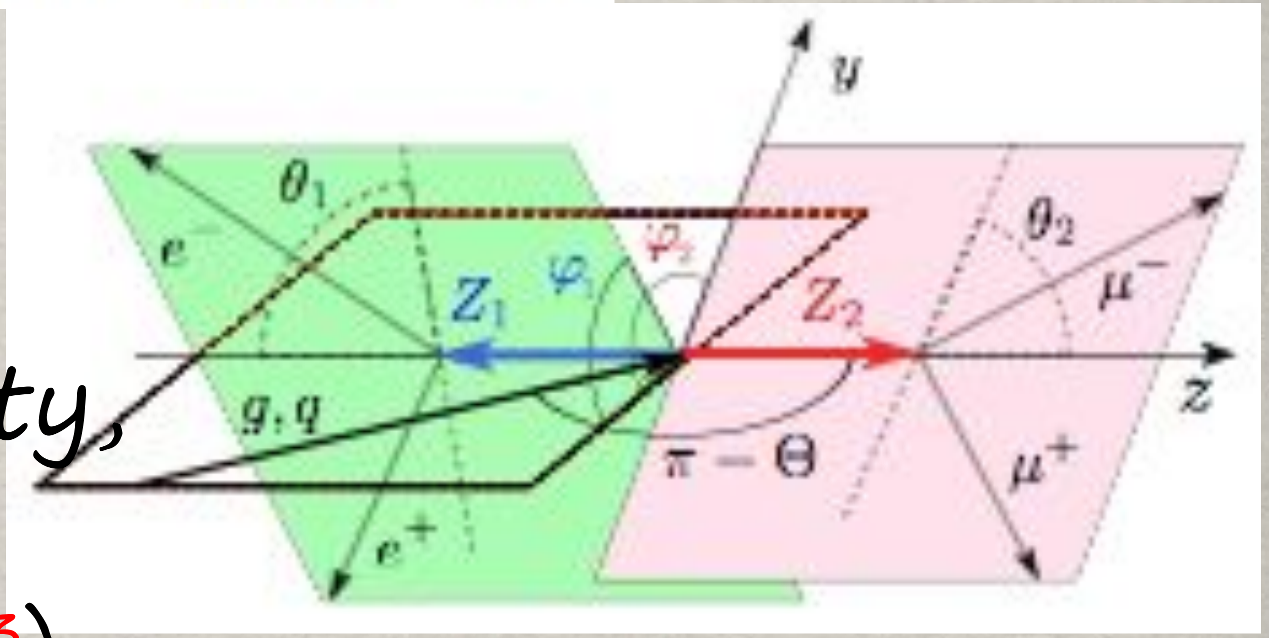
Precision measurements may be (surprisingly) rewarding !

Most general $V^\mu V^\nu H$ coupling:

$$
\begin{aligned}
T^{\mu\nu} \;=\;& a_1\, g^{\mu\nu} \;+ \\
& a_2\, \left(q_1 \cdot q_2\, g^{\mu\nu} - q_1^{\nu} q_2^{\mu}\right) + \\
& a_3\, \varepsilon^{\mu\nu\rho\sigma}\, q_{1\rho} q_{2\sigma}
\end{aligned}
$$

The $a_i = a_i(q_1, q_2)$ are scalar form factors

$$H \to Z Z^* \to \mu^+\mu^-\; e^+e^-$$



st Higgs spin-parity property, search for CP violation
may not be larger than 10^{-3}).

De Rujula, Lykken, Spiropulu et al., 2010.

Not-So "Standard" Higgs Sector

Most general $Hf\bar{f}$ coupling:

$$H\bar{\boldsymbol{t}}(a + ib\gamma_5)\boldsymbol{t}$$

$$gg,\ q\bar{q} \to \boldsymbol{t}\bar{\boldsymbol{t}}H,\ \text{with}\ H \to b\bar{b},\ \tau\bar{\tau},\ \gamma\gamma$$

Gunion and He, 1996.

It will be very challenging to study the $H\bar{\boldsymbol{t}}\boldsymbol{t}$ coupling at the LHC: **20%?**

What we need to achieve...

To go beyond the LHC direct search,

1. Precision Higgs physics at a few %:
Δ_{VVH} for composite dynamics;
$\Delta_{bbH,\ \tau\tau H}$ for decoupling H^o, A^o;
$\Delta_{ggH,\ \gamma\gamma H}$ for color/charge loops.

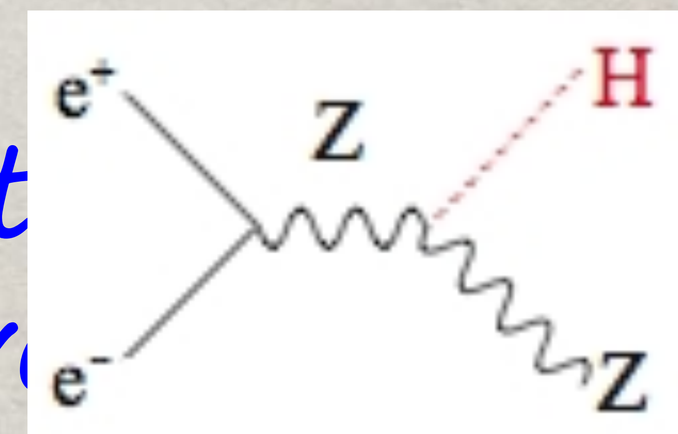
2. Reach 10% for H invisible.

3. Determine Γ_{tot} to 10%.

Expectations

1. LHC:
- $\sigma_{obs} \propto g_{in}^2 \frac{\Gamma_{final}}{\Gamma_{tot}}$ measured at 10% level.
- σ_{obs}/σ_{SM} sensitive to 20% level. $Br(h \to \bar{N}N, \chi\chi, ...)$
- No model-independent measure for Γ_i, Γ_{tot}

2. e^+e^- Higgs factory:
- model-independent for g_{ZZh} at 1.5% leve



- Extraction for $\Gamma_{tot} = \Gamma_{ZZ}/BR_{ZZ}$

3. $\mu^+\mu^-$ Higgs factory:
- Direct measurement of Γ_{tot} by scanning.

Summary:

- We are a lucky generation to have experienced the revolutionary discovery!

- We have learned a lot about Nature!
  Spontaneous symmetry breaking;
  The Higgs mechanism ...

- We are still puzzled!

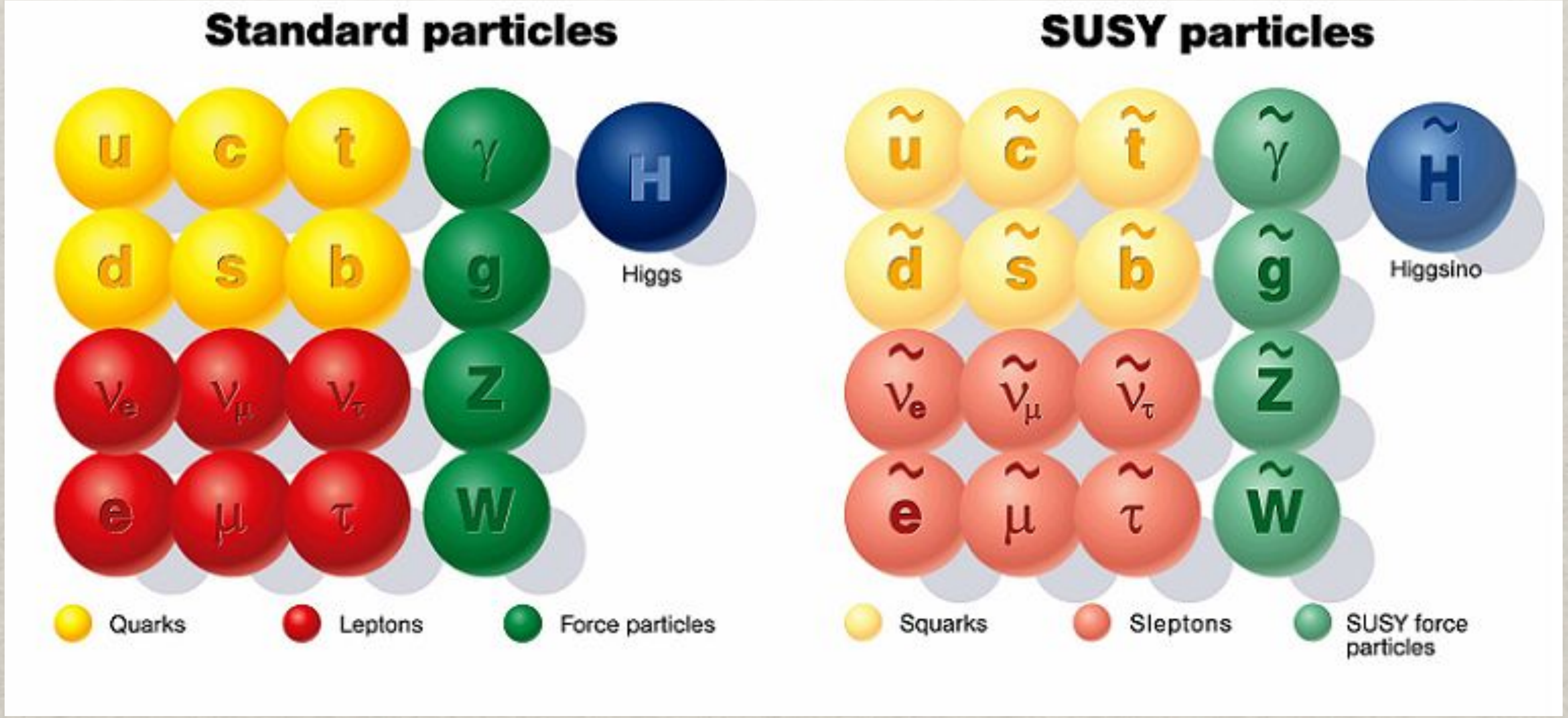
Backup Slides

Scenarios associated with the Higgs Sector

1. Supersymmetry:

The SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa

$$\mathcal{Q}|\text{Fermion}\rangle >= |\text{Boson}\rangle \quad , \quad \mathcal{Q}|\text{Boson}\rangle = |\text{Fermion}\rangle$$



See Sudhir Vempati

2. Two Higgs Doublets in the MSSM

In the MSSM, we need two doublets of complex scalar fields of opposite hypercharge

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \text{ with } Y_{H_1} = -1 \quad , \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \text{ with } Y_{H_2} = +1$$

$$\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}} \quad , \quad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}} \qquad (v_1^2 + v_2)^2 = v^2 = \frac{4M_Z^2}{g_2^2 + g_1^2} = (246 \text{ GeV})^2$$

$$\tan\beta = \frac{v_2}{v_1} = \frac{(v \sin\beta)}{(v\cos\beta)} \qquad \lambda = \frac{g_2^2 + g_1^2}{8}$$

d to 3 Goldstone bosons, and five "Higgses

$$h^0,\ H^0,\ A^0,\ H^\pm$$

Tree-level masses given by

$$M_A,\ \tan\beta$$

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

$$M_{h,H}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right]$$

$$M_h \leq \min(M_A, M_Z) \cdot |\cos 2\beta| \leq M_Z$$

3. Composite Higgs:

--- The Little Higgs Model

A very interesting idea§ is to make the Higgs a "pseudo-Nambu-Goldstone" boson.

§ H. Georgi and David B Kaplan, 1984.

A less ambitious approach: Little Higgs Models

Accept the existence of a light Higgs;

keep the Higgs boson "naturally" light (at 1-loop level).

Higgs is a pseudo-Goldstone boson from global symmetry breaking (at scale $4\pi f$)‡

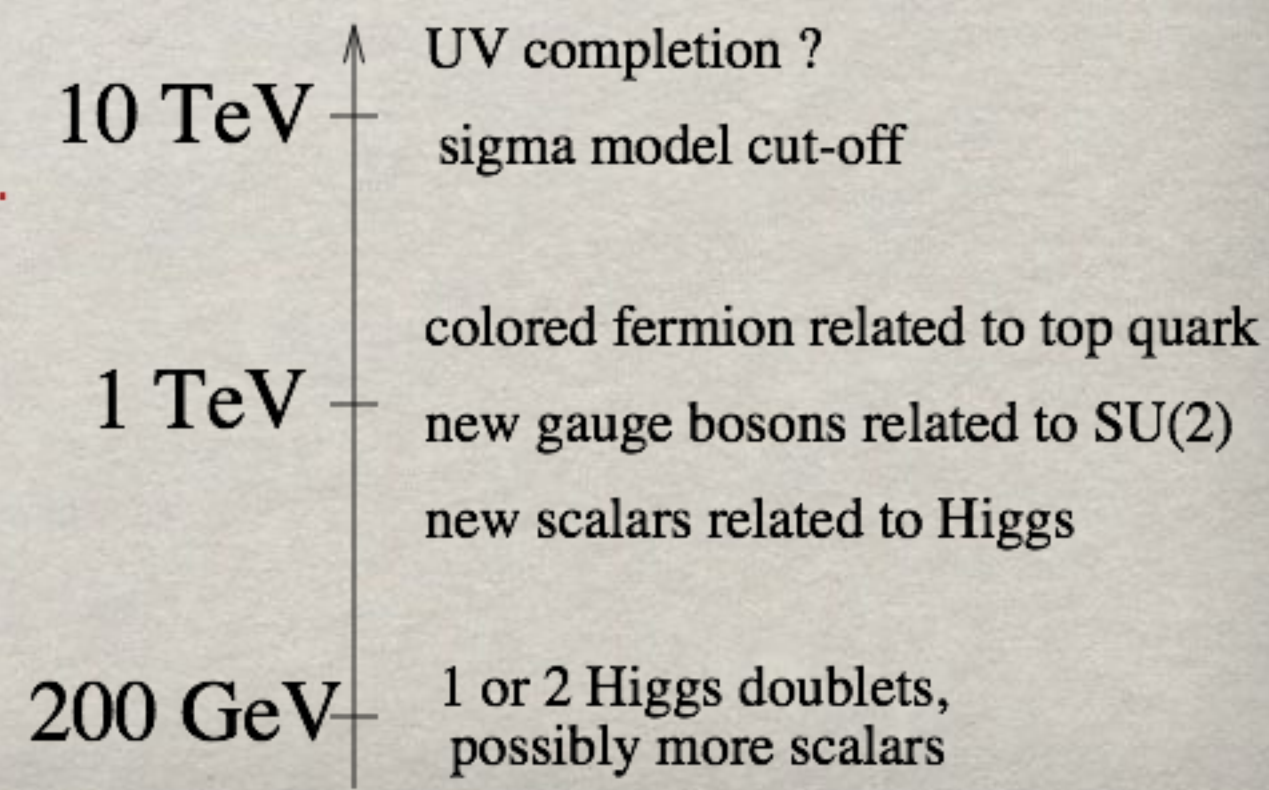
Higgs acquires a mass radiatively at the EW scale v, by collective explicit breaking

Consequently, quadratic divergences absent at one-loop level*

$W, Z, B \leftrightarrow W_H, Z_H, B_H; \quad t \leftrightarrow T; \quad H \leftrightarrow \Phi.$

(cancellation among same spin states!)

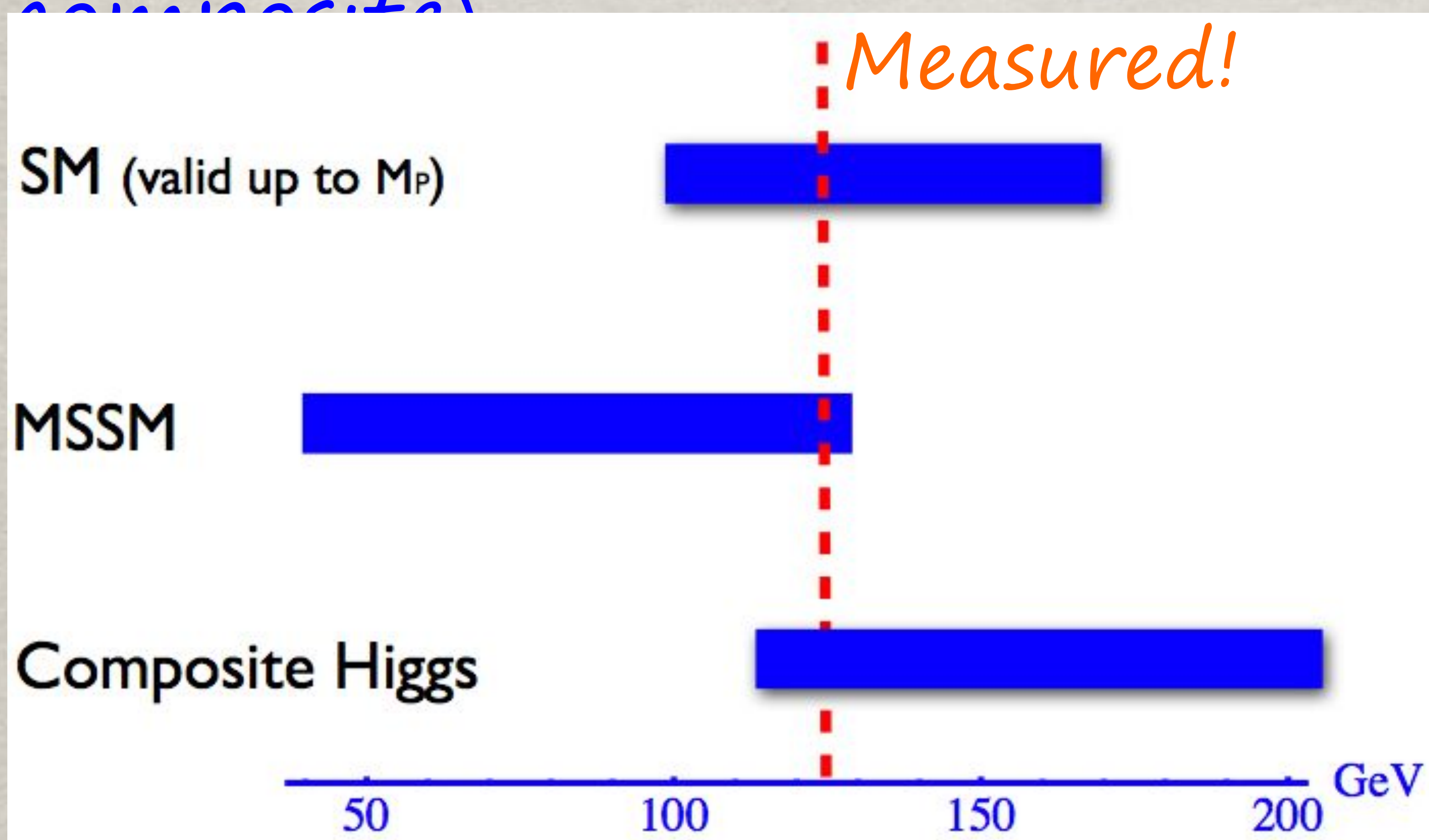
$$\lambda_{h^4} = \frac{a}{8}\left[\frac{g^2}{s^2c^2} + \frac{g'^2}{s'^2c'^2}\right] + 2a'\lambda_1^2 = \frac{1}{4}\lambda_{\phi^2}.$$



The fact that $M_H \approx 126$ GeV has already provides non-trivial test to some models

In a given theory with additional symmetries, one may be able

- to calculate (in a weakly coupled theory – SUSY)
- to (g)estimate (in a strongly coupled theory – composite)



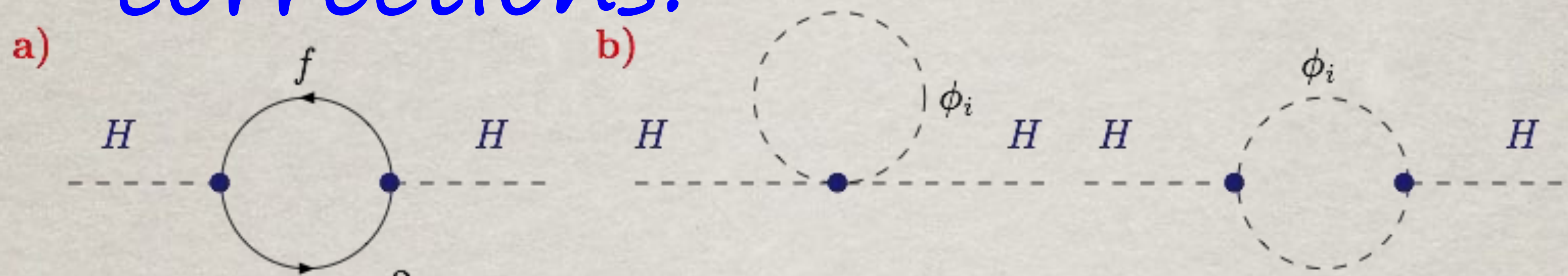
$$M_H^2 = M_Z^2 \cos^2 2\beta + \Delta_{SUSY}^2$$

$$M_H^2 \approx \frac{3}{\pi}\frac{m_t^2 M_T^2}{f^2}$$

Pomarol, ICHEP'12

Both suffer from some degree of fine-tune (already).

Thus the Higgs mass corrections:



$$\Delta M_H^2 = \frac{\lambda_f^2 N_f}{4\pi^2}\left[(m_f^2 - m_S^2)\log\left(\frac{\Lambda}{m_S}\right) + 3m_f^2\log\left(\frac{m_S}{m_f}\right)\right].$$

* In SUSY limit, the correction vanishes.
* In soft SUSY breaking case, $m_S \sim O(1$ TeV$)$.

- predict TeV scale new physics:
  light Higgs bosons, SUSY partners...
- imply a (possible) grand desert in $M_{SUSY} - M_{GUT}$, and unification
- radiative EWSB:

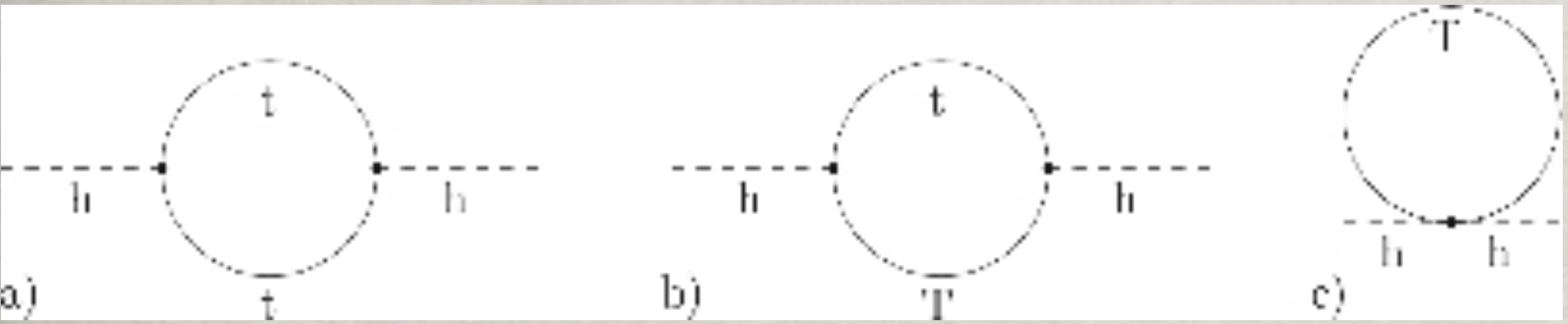
$$M_Z^2/2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2.$$

- SUSY dark matter with R-parity conserva

In Little Higgs Models,

Most interesting of all, the top fermionic partner T:

$$\mathcal{L} = -\lambda_T T_R^\dagger \tilde{H} Q_3 + \frac{\lambda_t^2 + \lambda_T^2}{2m_T}(H^\dagger H) T_L^\dagger T_R + \text{ h.c.}$$



$$m_t = \frac{i\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} v \left\{ 1 + \frac{v^2}{f^2}\left[-\frac{1}{3} + \frac{fv'}{v^2} \right. \right.$$

$$M_T = -f\sqrt{\lambda_1^2+\lambda_2^2}\left[1 + \mathcal{O}(v^2/f^2)\right].$$

=> The quadratic divergence is then cancelled at one loop level

Then the logarithmically contribution to the Higgs mass square

$$m_h^2 \sim 6\frac{\lambda_t^2 m_T^2}{8\pi^2}\log\frac{\Lambda^2}{m_T^2}$$

m_h = 125 GeV → mT < 1TeV

(J Berger, J. Hubisz and M. Perelstein, 2012)

"Naturally speaking":

- It should not be a lonely particle; has an "interactive friend circle":

and $t,\ W^{\pm},\ Z$

partners $\tilde{t},\ \tilde{W}^{\pm},\ \tilde{Z},\ \tilde{H}^{\pm,0}$ ...

- If we do not see them at the LHC, they may reveal their existence from Higgs coupling deviations from the SM values at a few percentage level.

An exciting journey ahead of us!

"... scalar particles are the only kind of free particles whose mass term does not break either an internal or a gauge symmetry." -- Ken Wilson, 1970

Quantum corrections to the potential or to m_H^2

Tree-level SM Higgs potential:

$$V(H) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

$$m_H^2 = 2\mu^2 = 2\lambda v^2 \quad \Rightarrow \quad \mu \approx 89 \text{ GeV}, \quad \lambda \approx \frac{1}{8}.$$

Coleman-Weinberg (Erick) potential:

$$V_{\rm CW}(h) = \frac{1}{2}\sum_k g_k(-1)^{F_k}\int \frac{d^4\ell}{(2\pi)^4}\log\left(\ell^2 + m_k^2(h)\right) \qquad \delta\mu^2 \equiv \frac{\delta^2 V_{\rm CW}}{\delta h^2}\Big|_{h=0}.$$

Leading contribution from y_t: $$\delta\mu^2 = -\frac{3y_t^2}{8\pi^2}\Lambda^2$$

Recollection:

The Symmetry

QED (1950's)

Yang-Mills Theory [Yang, Mills,1954]

EW unification theory [Glashow,1960]
QCD: IR & UV [Gross et al, 1973]

The Breaking

Spontaneous symmetry breaking in particle physics [Nambu,1960;Goldstone,1962]

Prediction of the existence of massive Higgs boson [Higgs,1964]

Higgs mechanism + EW + QCD unification theory
[Weinberg, 1967; Salam,1968, Gross et al. 1973]