Precision QCD measurement at CEPC

Hua Xing Zhu

SLAC

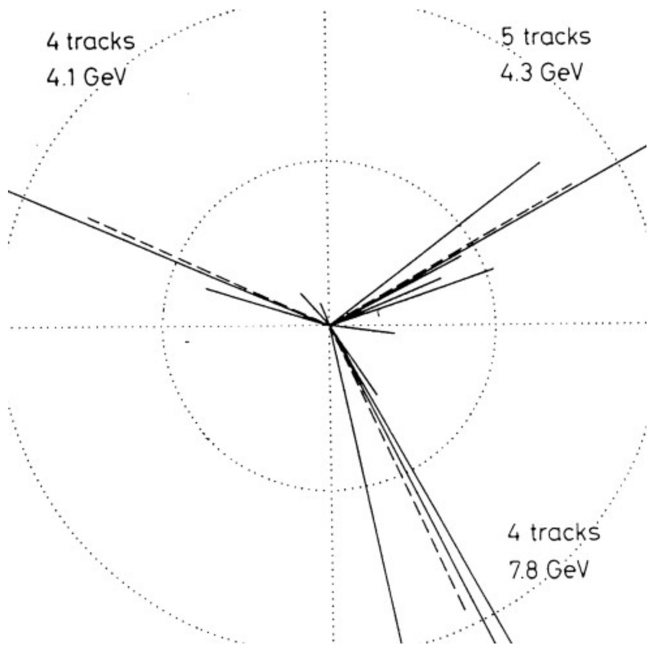
Workshop on physics at the CEPC
August 12, 2015

Outline

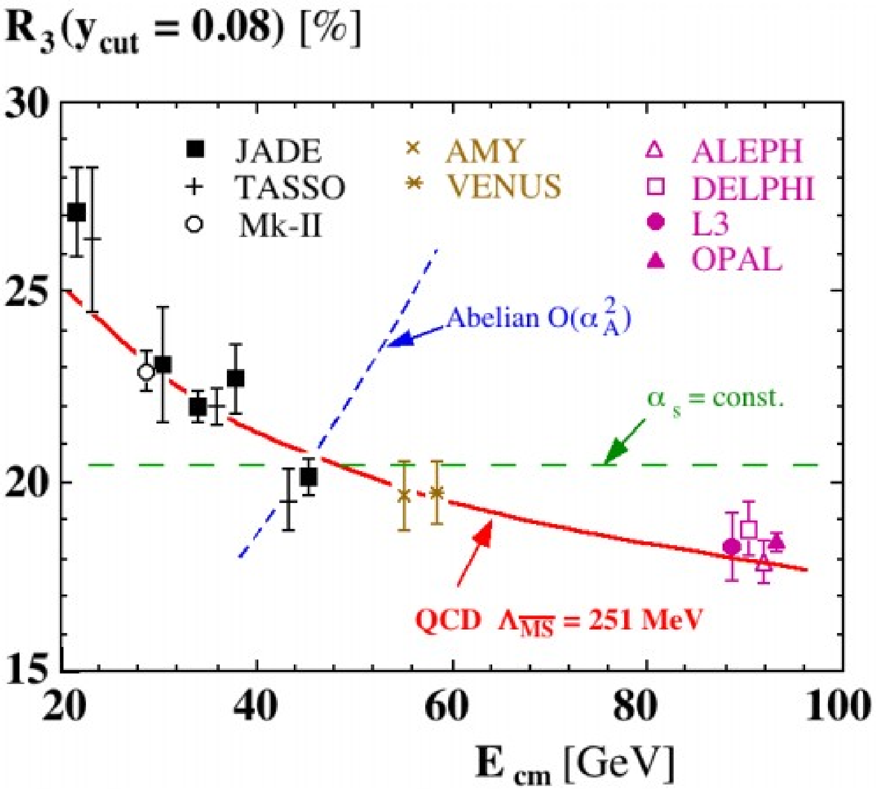
- Brief history of QCD measurement at e+e- collider
- Precision alphas measurement from e+e- global event shape and jet rates
- Non-global event shape at e+e- collider
- Summary

History of QCD measurement at e+e- Collider

- The first experiment evident of quark-parton came from deep inelastic scattering experiment around 1970s
- Around the same time study on QCD hadron production in e+e- collider started



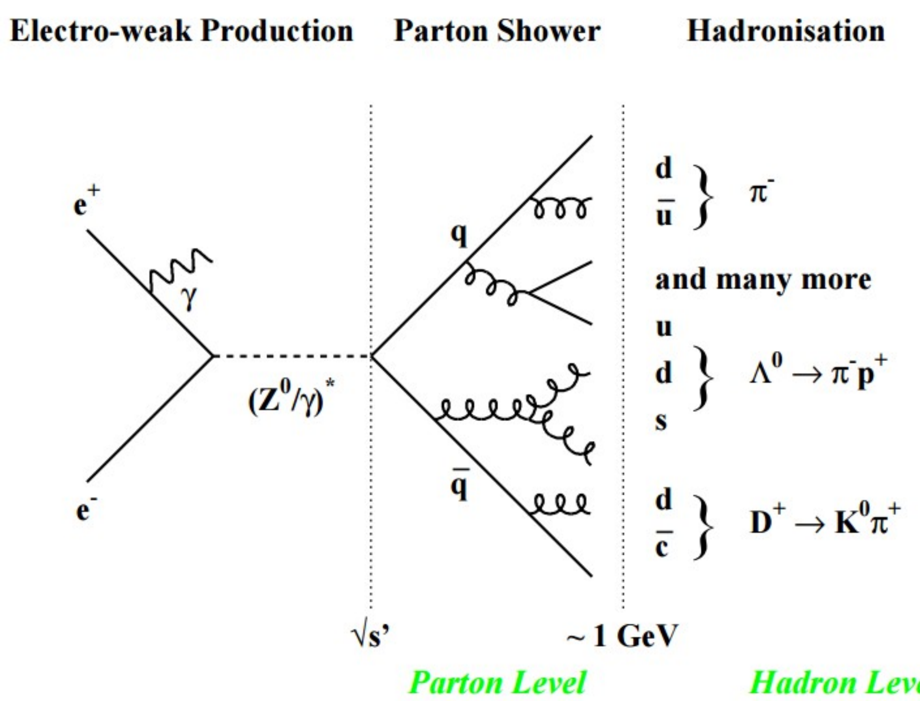
First 3 jets event observed at PETRA



Energy dependence of three jet rates

Advantage of QCD study at e+e- collider

- e+e- collider is an ideal laboratory for studying QCD
  - No interference between initial state and final state
  - In the absent of significant QED radiation, four momentum of initial state fully transferred to final state
  - Usually clean experimental conditions. No multiple interactions in a given bunch crossing.



Kluth 2006

Precision observables at e+e- collider

- Strong coupling constant is perhaps the most important parameter of QCD. Can be measured from a number of precision observables at e+e- collider
- Inclusive observables:
  - $R_{e^+e^-} = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$
  - $R_Z = \Gamma(Z^0 \to \text{hadrons})/\Gamma(Z^0 \to \text{leptons})$
  - $R_\tau = \Gamma(\tau \to \text{hadrons})/\Gamma(\tau \to \text{leptons})$

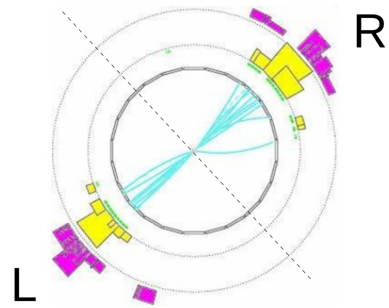
  $\Big\} \; 1 + \alpha_S + \ldots$
- Exclusive observables
  - e+e- event shape
  - Jets rate

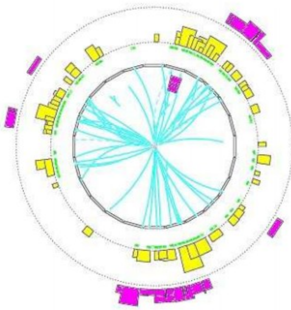
  } **LO is sensitive to alphas**

Global event shape observables

- Global e+e- event shape are usually designed as approaching zero for pencil-like events
- Thrust: $1 - T = 1 - \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_j |\vec{p}_j|}$

1 – T ~ 0



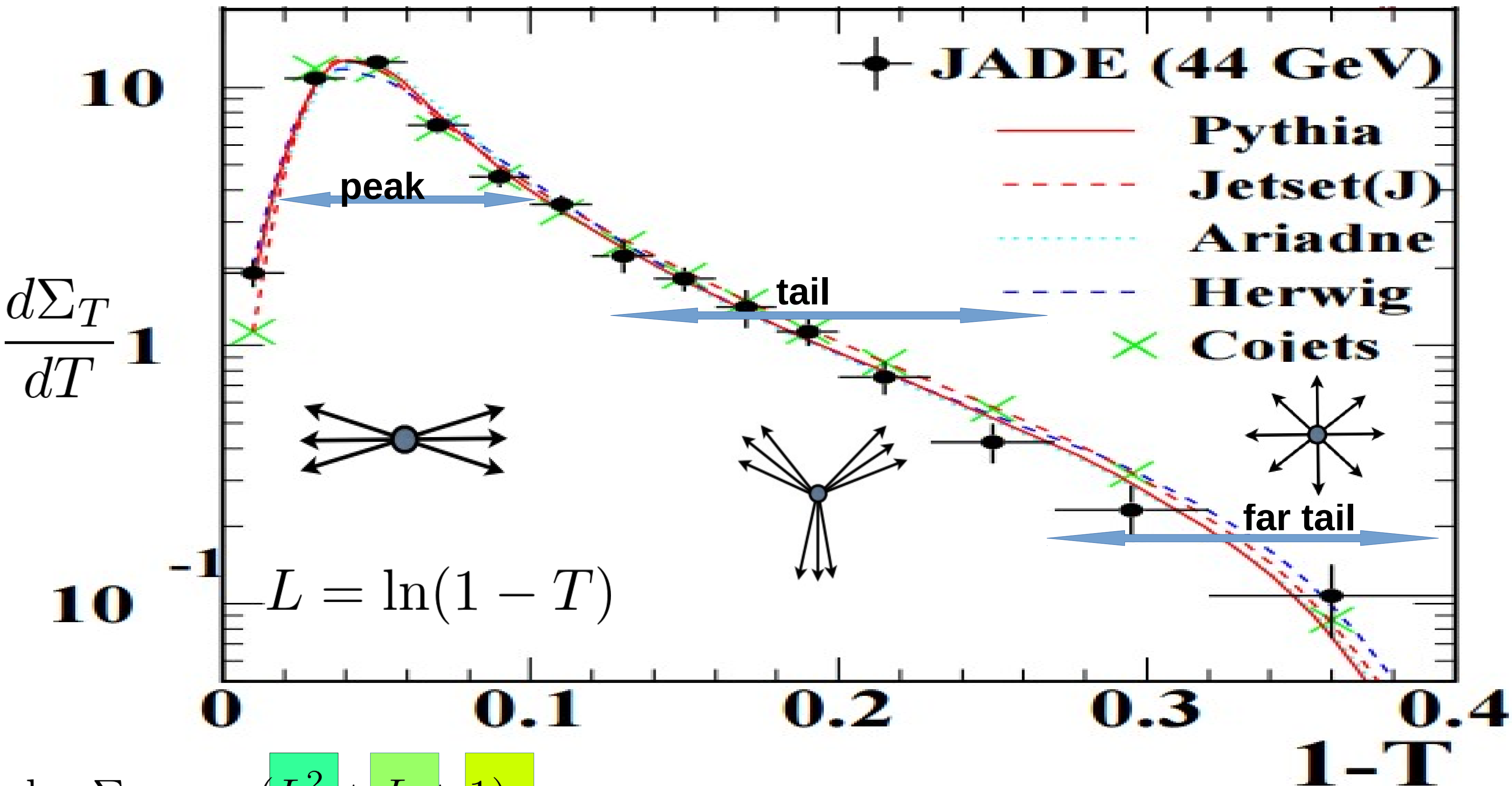


1 – T ~ 1/3

- Heavy jet mass : $\frac{\max(M_L^2, M_R^2)}{Q^2}$
- Total jet broadening : $\frac{\sum_i |\vec{p}_i \times \vec{n}_T|}{\sum_j |\vec{p}_j|}$
- C parameter: $\frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_j |\vec{p}_j|)^2}$

Large nonperturbative power corrections in event shape obs. $\mathcal{O}(\Lambda/Q)$
Large center of mass energy very welcome!

Typical feature of event shape



$$
\begin{aligned}
\log \Sigma_T = &\alpha_S(L^2 + L + 1) \\
&\alpha_S^2(L^3 + L^2 + L + 1) \\
&\alpha_S^3(L^4 + L^3 + L^2 + \dots)
\end{aligned}
$$

LL NLL NNLL

$$\alpha_S + \alpha_S^2 + \alpha_S^3 + \dots$$

LO + NLO + NNLO

Highlights of precision calculation for event shape in the past 10 years

- NNLO QCD corrections to three jet production at e+e- collider:
  - Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, 2007
  - See also Weinzierl, 2008

  Application of Soft-Collinear Effective Theory (SCET) to event shape resummation
  - Thrust: Becher, Schwartz, 2008; Abbate, Fickinger, Hoang, Mateu, Stewart, 2010
  - Heavy jet mass: Chien, Schwartz, 2010
  - C parameter: Hoang, Kolodrubets, Mateu, Stewart, 2014,15

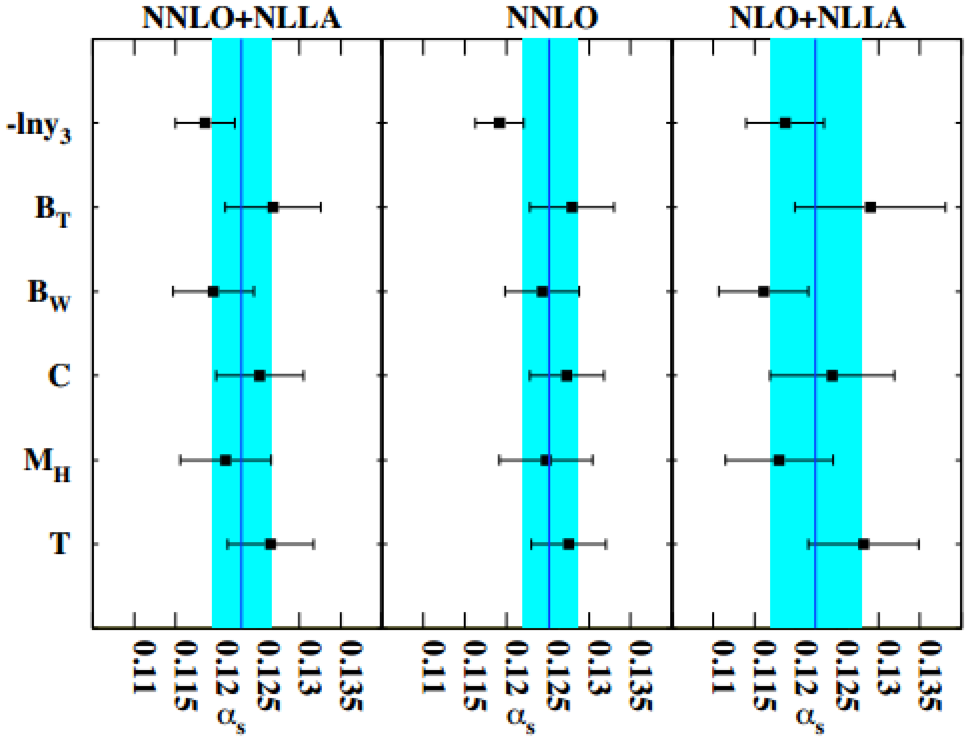
NNLO QCD corr. to 3 jet production

- Known for a long time that the limiting factor of theoretical uncertainty is the missing NNLO QCD corrections
- A heroic calculation span many years
- Techniques developed and applied in this calculation has far reaching impact
  - Two-loop for point integral with one off-shell leg from differential equation: Gehrmann, Remiddi, 2000-2001
  - Two-loop helicity amplitude for e+e- to 3 jets: Garland, Gehrmann, Glover, Koukoutsakis, Remiddi, 2001-2002
  - Antenna method for IR subtraction at NNLO: Gehrmann-De Ridder, Gehrmann, Glover, 2005-2006
  - Physical results: Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, 2007

alphas from NNLO + NLLA

Dissertori, et al, 2007. 2009

- NNLO QCD corrections improved with NLL resummation (Dissertori et al, 2007 – 2009)
- Fit to six event shape
- Including NLO mass effects
- Hadronization corrections model by Monte Carlo generator



$\alpha_s = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{had}) \pm 0.0035(\text{th})$

2% lower than world average

Hadronization correction limit the accuracy

Hoang, Kolodrubets, Mateu, Stewart, 2015

NNNLL' resummation for C parameter in SCET

- Definition of C parameter doesn't refer to thrust axis $\frac{3}{2}\frac{\sum_{i,j}|\vec{p}_i||\vec{p}_j|\sin^2\theta_{ij}}{(\sum_j|\vec{p}_j|)^2}$
- The full partonic cross section separate into singular and nonsingular contribution

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}C} = \frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}C} + \frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}C}$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}C} = \frac{Q}{6}H(Q,\mu)\int \mathrm{d}s\, J_\tau(s,\mu) S_{\widetilde{C}}\left(\frac{QC}{6} - \frac{s}{Q}, \mu\right)$$

Hard function   Jet function   Soft function

Hard scale $\mu_H \sim Q$ $\ln\frac{\mu_H^2}{\mu^2} \to \ln\frac{Q^2}{\mu^2}$

jet scale $\mu_J \sim Q\sqrt{C}$ $\ln\frac{\mu_J^2}{\mu^2} \to \ln\frac{Q^2 C}{\mu^2}$

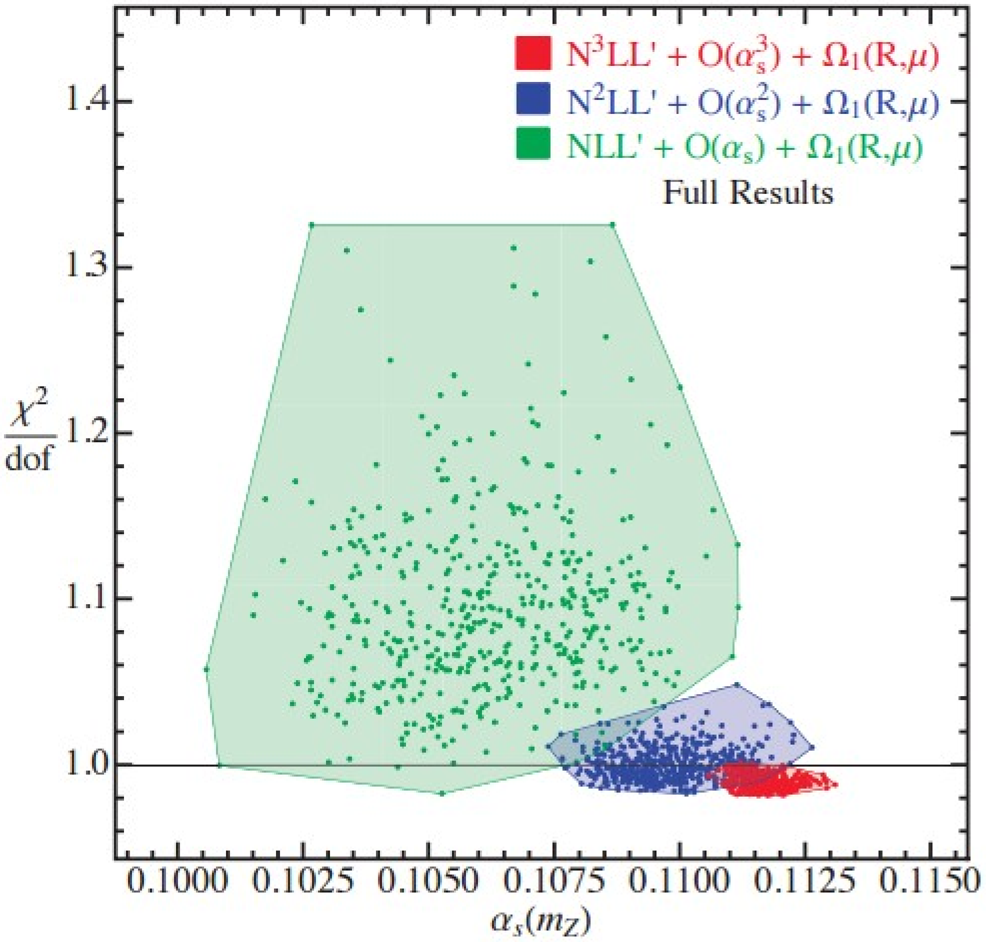
soft scale $\mu_S \sim QC$ $\ln\frac{\mu_S^2}{\mu^2} \to \ln\frac{Q^2 C^2}{\mu^2}$

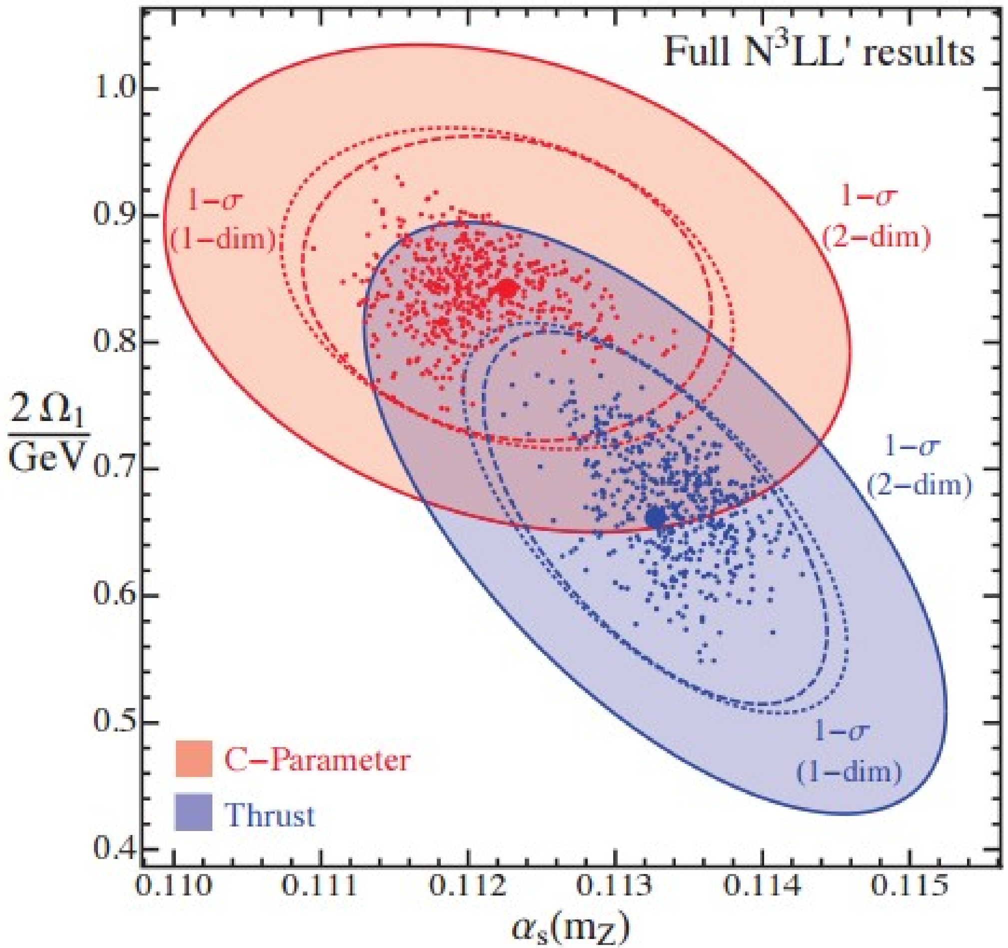
Leading power corrections amounts to a shift of the distribution

$$\frac{1}{\sigma_0}\frac{\hat{\sigma}}{dC} \xrightarrow{\text{hadr.}} \frac{1}{\sigma_0}\frac{\hat{\sigma}}{dC}\left(C - \overline{\Omega}_1^C\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

The quality of SCET alphas fit

Hoang, Kolodrubets, Mateu, Stewart, 2015

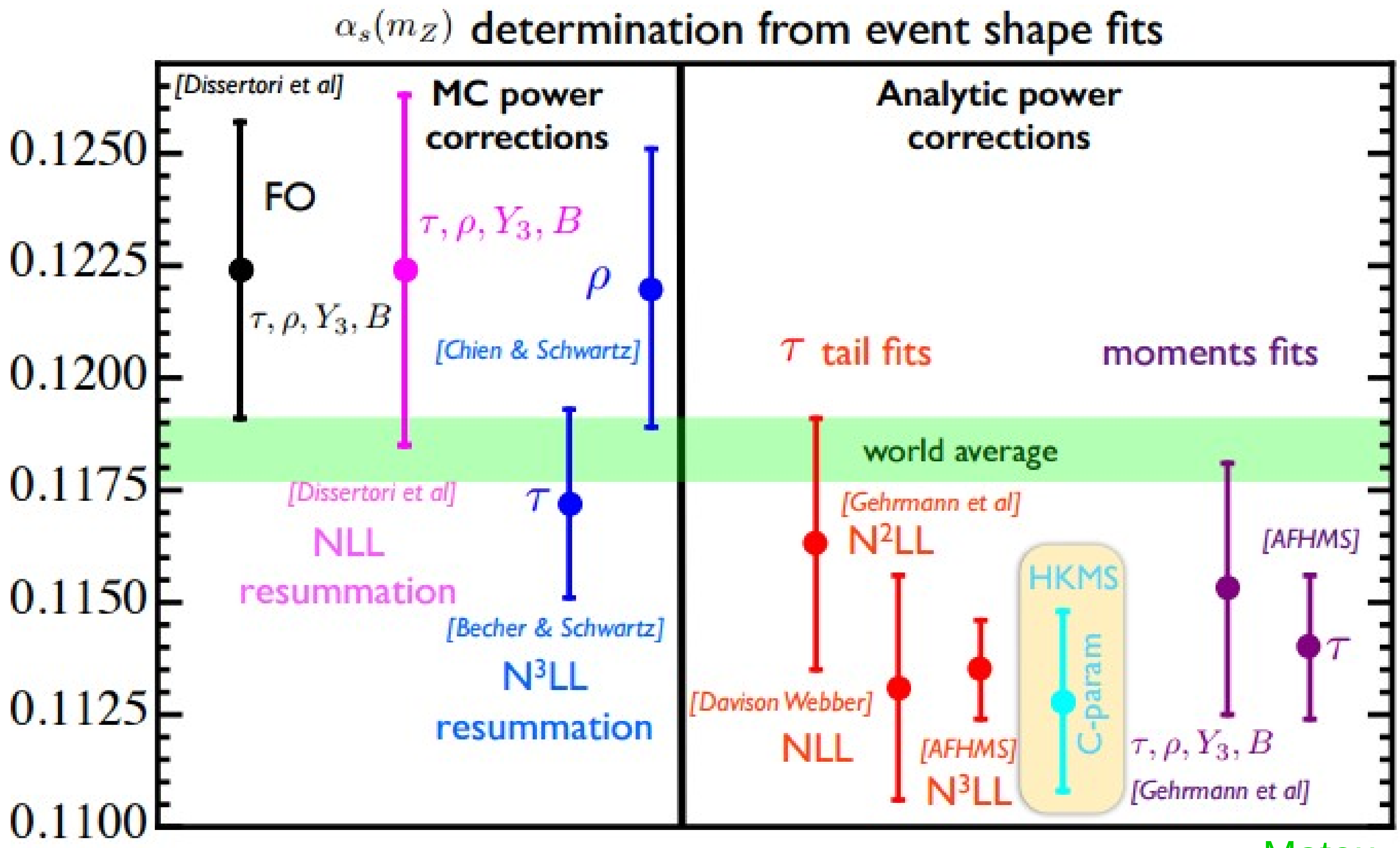




$$\alpha_S(m_Z) = 0.1134 \pm 0.0002(\mathrm{exp}) \pm 0.0005(\mathrm{hadr}) \pm 0.0011(\mathrm{pert})$$

So far the alphas fit with smallest error. But lower than world average...

Comparison of event shape fits



Mateu, ICHEP 2014

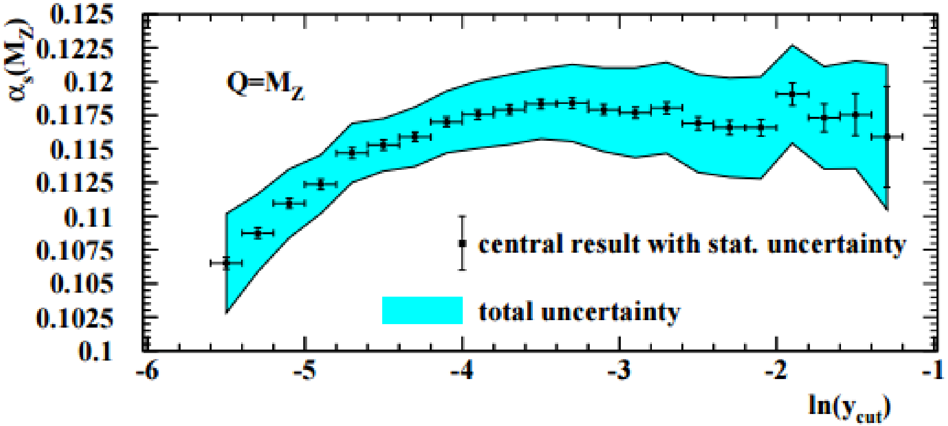
Fits with analytical power corrections seem to systematically lower than world average. The source of this disagreement is an open question.

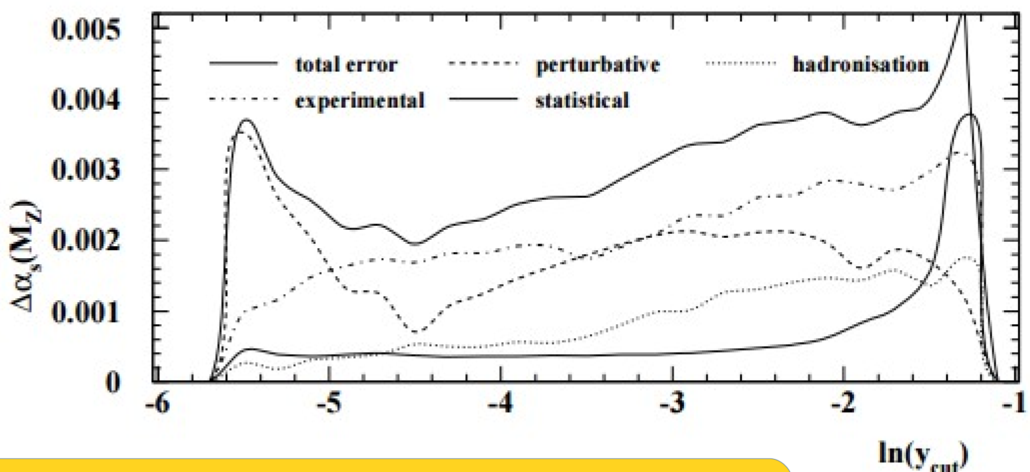
alphas from 3 jet rates

- Instead of fitting from event shape, one can extract alphas from three jet rates using NNLO results
- Hadronization uncertainties for this observable turn out to be small
- Jets cluster with Durham jet algorithm with measure

$$y_{ij,D} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{E_{\rm vis}^2}$$

Dissertori, et al, 2009

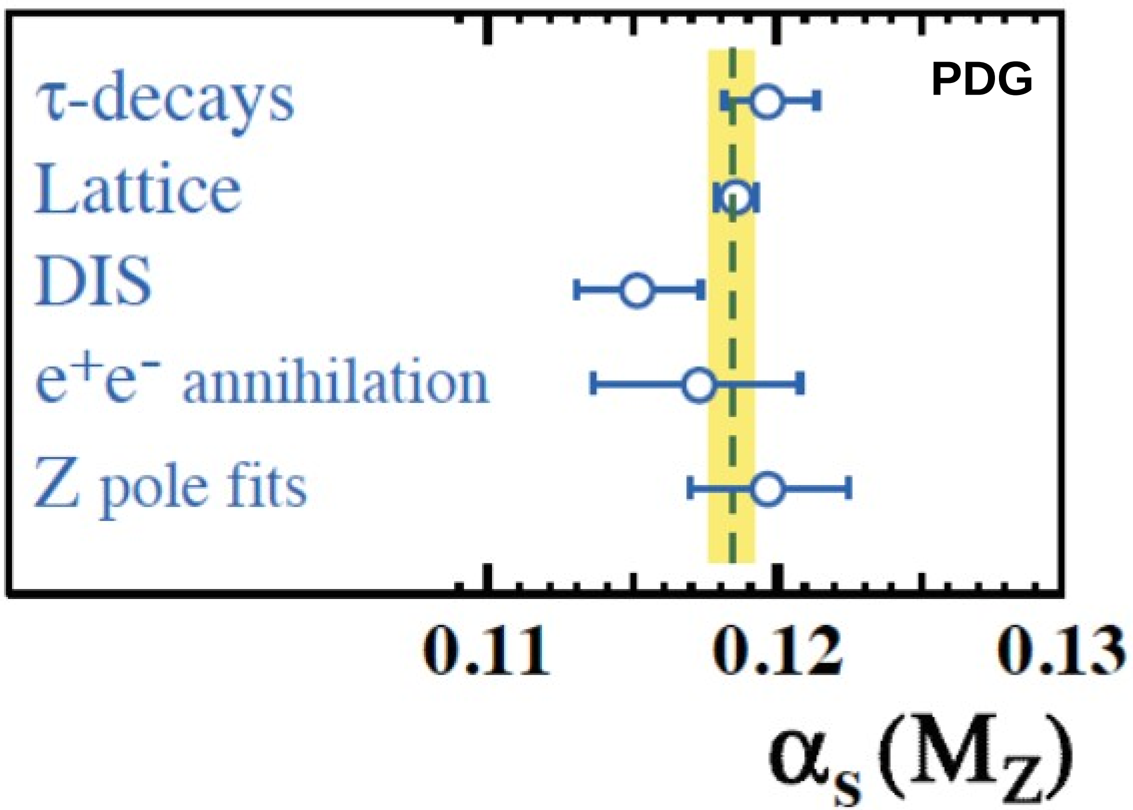




$$\alpha_S(M_Z) = 0.1175 \pm 0.0020(\text{exp}) \pm 0.0015(\text{theo})$$

Similar experimental and theory error. Room for improvement for both!

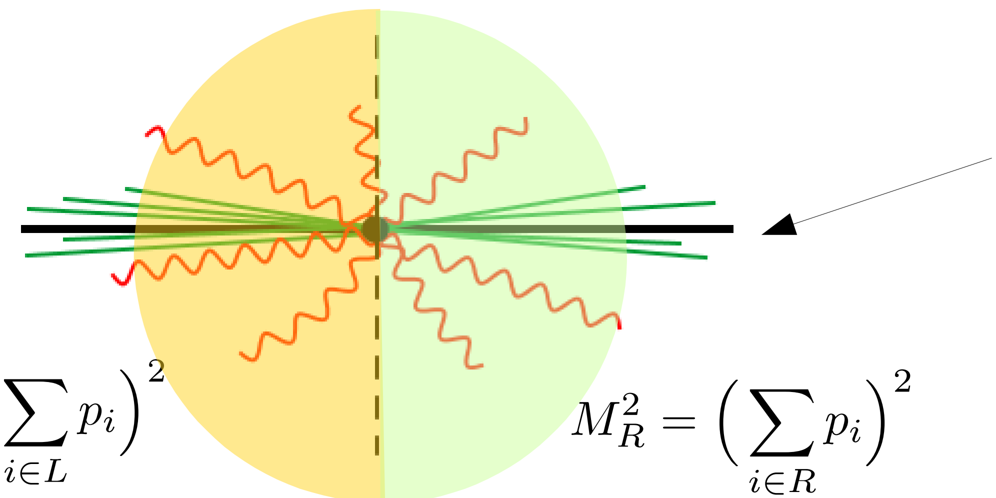
World average on alphas



- Dominated my Lattice results
- O(100-1fb) at CEPC v.s. O(100-1pb) at LEP, plus higher energy, smaller power corrections, good news for event shape analysis.
- New challenges to theorists. NNLO corrections to four jet rates? Completing the NNNLL resummation by computing the four loop cusp anomalous dimension? …

What are non-global logarithms?

- Observables which only sensitive to a restricted region of phase space. (Salam, Dasgupta, 2001)
- A best studied example is the hemisphere mass distribution in e^+e^-. The hemisphere is defined by thrust axis.



$$M_L^2 = \Big(\sum_{i\in L} p_i\Big)^2 \qquad M_R^2 = \Big(\sum_{i\in R} p_i\Big)^2$$

Thrust axis, defined by minimization

$$\min_{\vec{n}} \left(1 - \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{\sum_j |\vec{p}_j|}\right)$$

- Dijet limit: $M_L^2 \ll Q^2$ and $M_R^2 \ll Q^2$

Q: center of mass energy

In the dijet limit thrust is the sum of left and right hemisphere mass

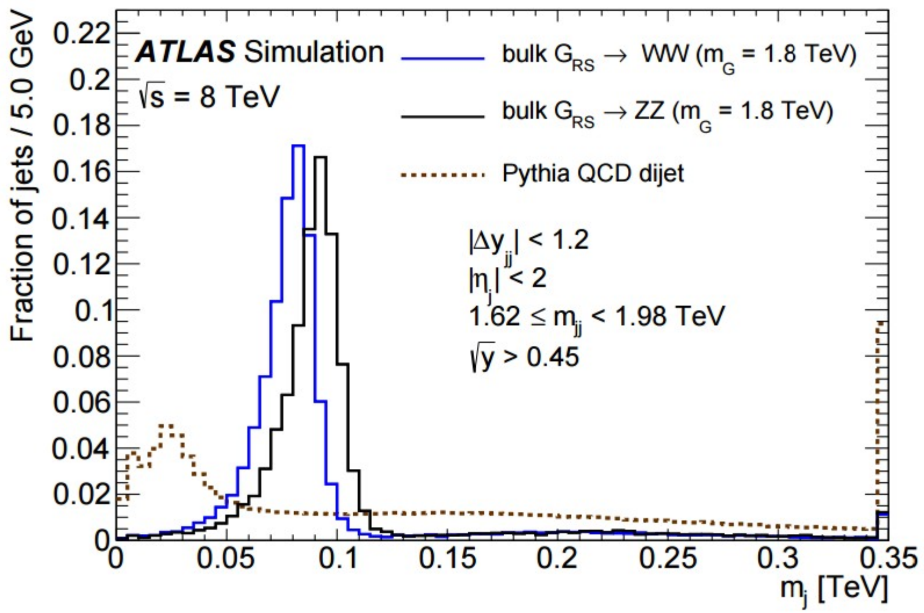
$$1 - T = \frac{M_L^2 + M_R^2}{Q^2}$$

T is a global observable

ML and MR are non-global observables

Why studying non-global logarithms

- Jet substructure has evolved into a standard tool at the LHC
- Perhaps the most important jet substructure is the jet mass
- A recent example is the 2 TeV excess observed at ATLAS



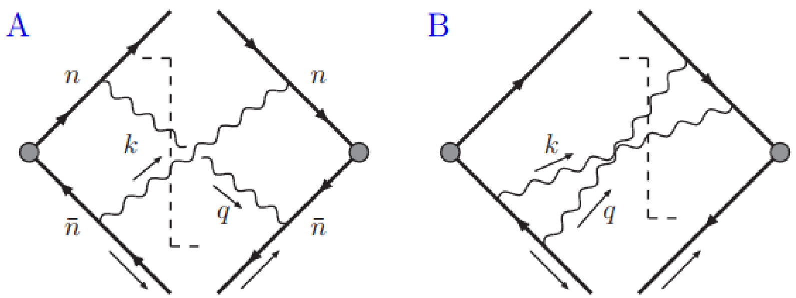
Groomed jet mass is used to distinguish boosted boson jet from QCD dijet background

ATLAS Collaboration, 1506.000962

- Most the substructure analysis are based on Monte Carlo tool
- First principle QCD computation of jet mass is interesting. Main obstacle: non-global logarithms in jet mass distribution

Origin of non-global logarithms

- The non-global logarithms originated from soft gluon corrections. First show up at $\mathcal{O}(\alpha_S^2)$



+ other diagrams

$$\left.\frac{d^2\sigma^{(2)}}{dM_L^2\,dM_R^2}\right|_{\mathrm{NGL}} = -C_F C_A \frac{4\pi^2}{3} \ln^2 \frac{M_L^2}{M_R^2}$$

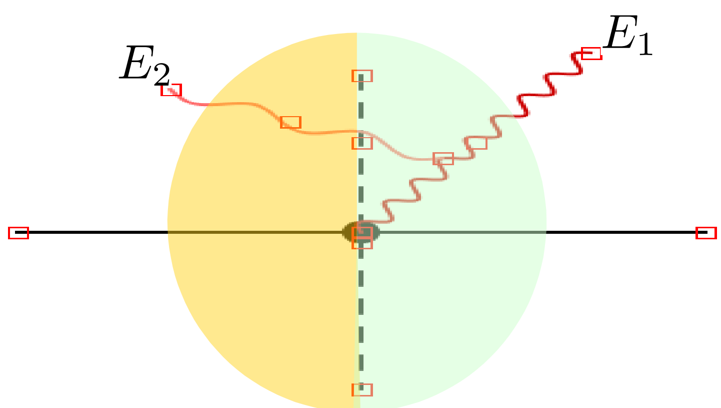
Leading non-global logarithms

$$+\left[C_F C_A\left(-8\zeta_3+\frac{44\pi^2}{9}-\frac{4}{3}\right)+C_F N_F T_F\left(-\frac{16\pi^2}{9}+\frac{8}{3}\right)\right]\left|\ln\frac{M_L}{M_R}\right|$$

Subleading NGLs

+ global logarithms

Kelly, Schwartz, Schabinger, HXZ, 2011
Hornig et al., 2011



Leading non-global logarithms arise from configurations where E1>>E2 (Salam, Dasgupta, 2001). The difference in soft gluon energy leads to large hierarchy in left and right invariant mass $M_R \gg M_L$

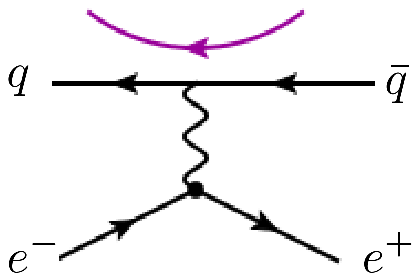
Non-global logarithms and Banfi-Marchesini-Syme (BMS) equation

- In general the evolution of non-global logs is very complicated. Simplify significant in the large Nc approximation

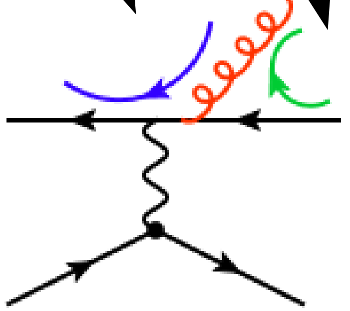
$$\bar{\rho} = \alpha_S N_c \ln \frac{M_L}{M_R}$$

BMS equation, 2002

$$\frac{d}{d\ln\bar{\rho}} G_{n\bar{n}} = \int \frac{d\Omega_j}{4\pi} \frac{(p_n \cdot p_{\bar{n}})}{(p_n \cdot p_j)\,(p_j \cdot p_{\bar{n}})} \left[\theta(j \in H_R) G_{nj} \cdot G_{j\bar{n}} - G_{n\bar{n}} \right]$$



Integral for j restricted to right hemisphere



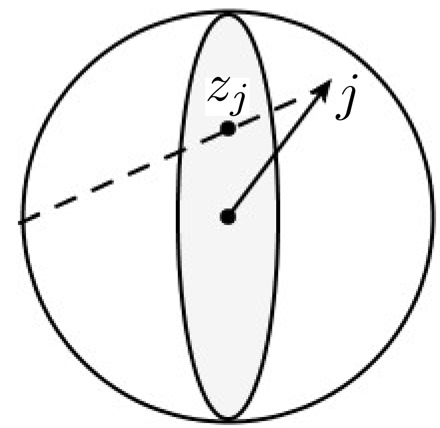
- Formally very similar to the Balitsky-Korchegov (BK) equation
- Compared with the linearized BFKL equation: $\frac{d}{d\bar{\rho}} B_{n\bar{n}} = \int \frac{d\Omega_j}{4\pi} \left[B_{nj} + B_{j\bar{n}} - B_{n\bar{n}} \right]$
- No analytical solution for BMS equation due to its non-linear property

SL(2,R) invariance of the BMS eq.

- Just as BK equation, the BMS equation has a very nice SL(2,R) symmetry, which is most obvious after stereographic projection

$$\frac{\vec{p}_j}{|\vec{p}_j|} = (\sin\theta_j \sin\phi_j, \sin\theta_j \cos\phi_j, \cos\theta_j)$$

$$z_j = \tan\frac{\theta_j}{2} e^{i\phi_j}$$



$$\frac{d}{d\bar{\rho}} G_{n\bar{n}} = \int \frac{dz_j d\bar{z}_j}{2\pi} \frac{|z_n - z_{\bar{n}}|^2}{|z_n - z_j|^2 |z_j - z_{\bar{n}}|^2} \Big[\theta(|z_j| \le 1) G_{nj} \cdot G_{j\bar{n}} - G_{n\bar{n}} \Big]$$

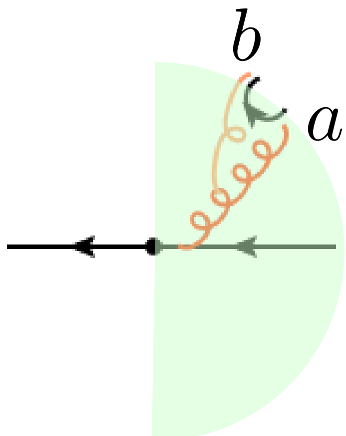
- Invariant under linear fractional transformation

Hatta, Ueda, 2009

$$z \to \frac{az+b}{cz+d}, \quad ad - bc = 1, \qquad a, b, c, d \in R$$

Perturbative solution of BMS eq.

- The symmetry of the BMS equation can be exploited to compute its perturbative solution (Schwartz, HXZ, 2014).



$$\frac{\vec{p}_a}{|\vec{p}_a|} = (\sin\theta_a \sin\phi_a, \sin\theta_a \cos\phi_a, \cos\theta_a)$$

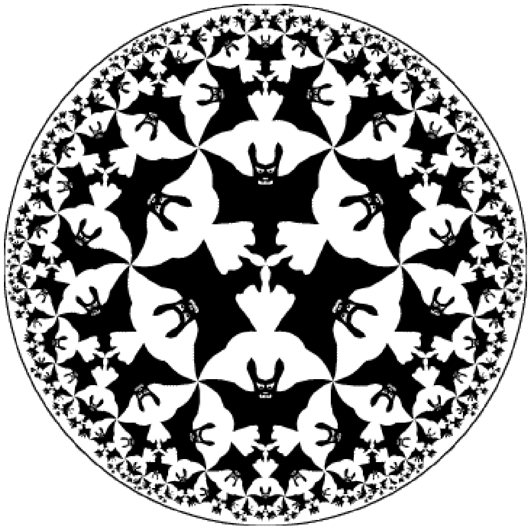
$$\frac{\vec{p}_b}{|\vec{p}_b|} = (\sin\theta_b \sin\phi_b, \sin\theta_b \cos\phi_b, \cos\theta_b)$$

G_{ab} depends on $\theta_a, \phi_a, \theta_b, \phi_b$ **?**

- But the SL(2,R) symmetry can be used to eliminate three degree of freedom.

G_{ab} is only a function of

$$\Delta = \frac{|z_a - z_b|^2}{(1-|z_a|^2)(1-|z_b|^2)}$$



Perturbative solution of BMS eq.

- Indeed the perturbative solution exhibits this symmetry

$$G_{ab}^{2-\mathrm{loop}}\Big|_{\mathrm{NGL}} = \rho^2\Big(-\frac{1}{4}H_{-1,-1}(\Delta)+\frac{1}{4}H_{-1,0}(\Delta)\Big)$$

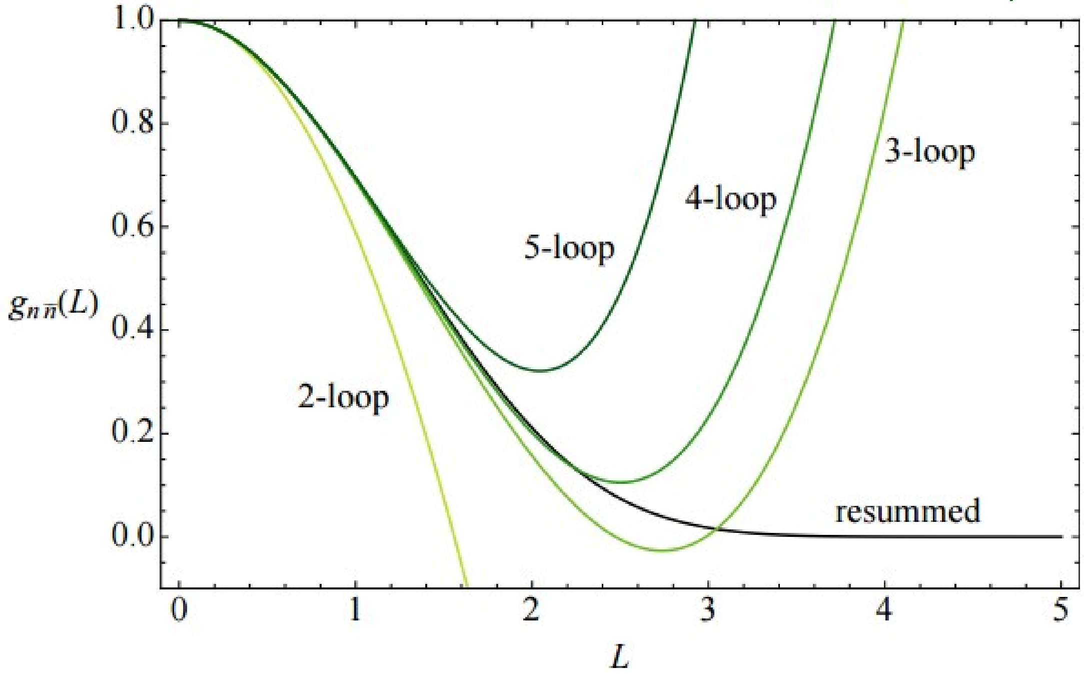
$$G_{ab}^{3-\mathrm{loop}}\Big|_{\mathrm{NGL}} = \rho^3\Big(\frac{\pi^2}{36}H_{-1}(\Delta)-\frac{1}{4}H_{-1,-1,-1}(\Delta)+\frac{1}{4}H_{-1,-1,0}(\Delta)+\frac{1}{12}H_{-1,0,-1}(\Delta)-\frac{1}{12}H_{-1,0,0}(\Delta)\Big)$$

$$\begin{aligned} G_{ab}^{4-\mathrm{loop}}\Big|_{\mathrm{NGL}} = \rho^4\Big(&\frac{\pi^2}{36}H_{-1,-1}(\Delta)-\frac{\pi^2}{144}H_{-1,0}(\Delta)-\frac{3}{16}H_{-1,-1,-1,-1}(\Delta)+\frac{3}{16}H_{-1,-1,-1,0}(\Delta)\\ &+\frac{1}{12}H_{-1,-1,0,-1}(\Delta)-\frac{1}{12}H_{-1,-1,0,0}(\Delta)+\frac{1}{48}H_{-1,0,-1,-1}(\Delta)-\frac{1}{96}H_{-1,0,-1,0}(\Delta)\\ &-\frac{1}{32}H_{-1,0,0,-1}(\Delta)+\frac{1}{48}H_{-1,0,0,0}(\Delta)-\frac{\zeta_3}{16}H_{-1}(\Delta)\Big)\end{aligned}$$

$$H_{0w}(z)=\int_0^z dt\,\frac{H_w(t)}{t},\quad H_{1w}(z)=\int_0^z dt\,\frac{H_w(t)}{1-t},\quad H_{-1w}(z)=\int_0^z dt\,\frac{H_w(t)}{1+t}$$

Schwartz, HXZ, 2014

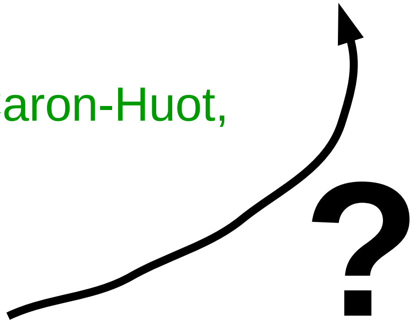
- Unfortunately the perturbative series quickly diverge from the resummed one already at low loops
- Example of asymptotically series
- Hopefully the perturbative solution can shed light on further understanding of BMS equation



Subleading non-global evolution

$$\left.\frac{d^2\sigma^{(2)}}{dM_L^2\,dM_R^2}\right|_{\rm NGL} = -\,C_F C_A \frac{4\pi^2}{3}\ln^2\frac{M_L^2}{M_R^2} + \Big[C_F C_A\Big(-8\zeta_3 + \frac{44\pi^2}{9} - \frac{4}{3}\Big) + C_F N_F T_F\Big(-\frac{16\pi^2}{9} + \frac{8}{3}\Big)\Big]\left|\ln\frac{M_L}{M_R}\right|$$

Subleading nonglobal logs



An evolution equation governing subleading nonglobal derived by (Caron-Huot, 2015)

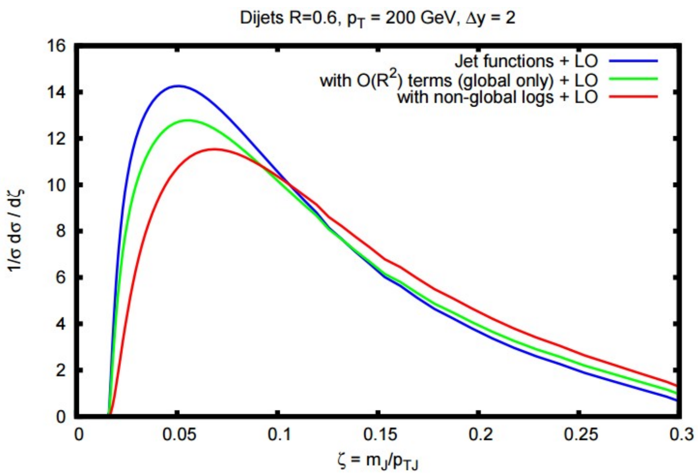
$$K^{(2)} = \int_{i,j,k}\int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K^{(2)\ell}_{ijk;00'}\, i f^{abc}\left(L^a_{i;0}L^b_{j;0'}R^c_k - R^a_{i;0}R^b_{j;0'}L^c_k\right)$$
$$+ \int_{i,j}\int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K^{(2)N=4,\ell}_{ij;00'}\left(f^{abc}f^{a'b'c'}U_0^{bb'}U_{0'}^{cc'} - \frac{C_A}{2}(U_0^{aa'} + U_{0'}^{aa'})\right)(L^a_i R^{a'}_j + R^{a'}_i L^a_j)$$
$$+ \int_{i,j}\int \frac{d^2\Omega_0}{4\pi}\frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}\gamma_K^{(2)}\left(R^a_{i;0}L^a_j + L^a_{i;0}R^a_j\right) + K^{(2)N\neq 4}. \qquad (3.32)$$

A direct verification of this equation at two loops with the explicit subleading non-global logarithms will establish the resummation of hemisphere mass distribution at NNLL.

Is non-global logarithms relevant?

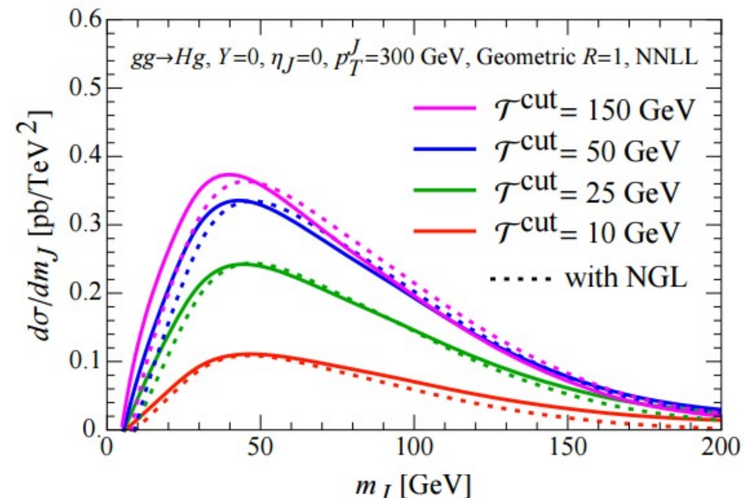
- There are different opinion on the importance of non-global in jet mass distribution

Non-global logs are important

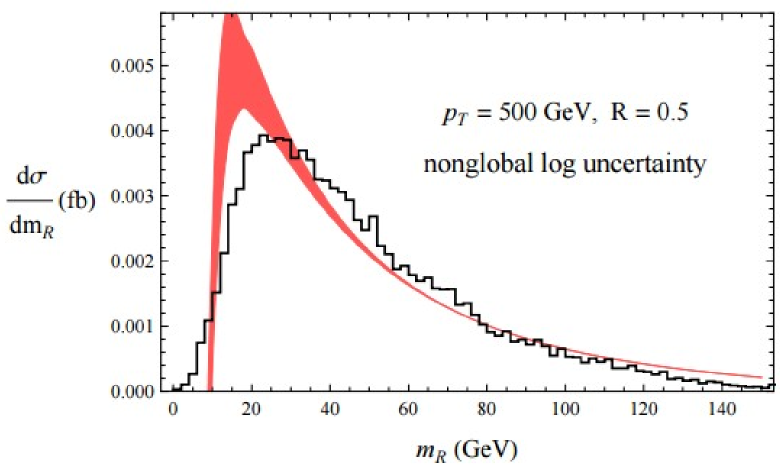


Dasgupta, Khelifa-Kerfa, Marzani, Spannowsky, 2012

Non-global logs are unimportant



Jouttenus, Stewart, Tackmann, Waalewijn, 2013



Chien, Kelley, Schwartz, HXZ, 2012

- A direct measurement and of hemisphere mass distribution and compare with theoretical prediction will be important
- Large C.O.M energy important for creating large mass hierarchy $Q \gg M_R \gg M_L \gg \Lambda$
- CEPC will be an ideal laboratory for this study

Summary

- At CEPC, precision measurement of alphas using event shape variables or jet rates will be interesting
  - Event shape
    - Sensitive to power corrections. Goes to large C.O.M energy helps a lot
    - Discrepancy between analytical power corrections method and world average. More work need.
  - Jet rates
    - Ideal observable for measuring alphas. Insensitive to nonperturbative phys.
    - Comparable exp. and theo. uncertainties at LEP. Room for improvement for both experimental and theoretical sides
- High energy e+e- collider ideal laboratory for studying non-global logarithms. Important for precision jet substructure
- Despite being developed for 40 years QCD and jet physics is still an actively evolving subject. CEPC will stimulate their study for many years to come

Thank you for your attention!