Quantum Chromodynamics (QCD) and Physics of the strong interaction

(Lecture 3)

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Lecture: Mon – Wed – Fri

10:00-11:40AM

Location: B326, Main Building

Review of Lecture Two

- Introduction of Quark Model
- Constituent quarks differ from current quarks of QCD
- Constituent quarks carry current quarks' quantum numbers

  But, they have internal structure and larger mass
- Quark Model NOT equal to QCD, NOT derived from QCD

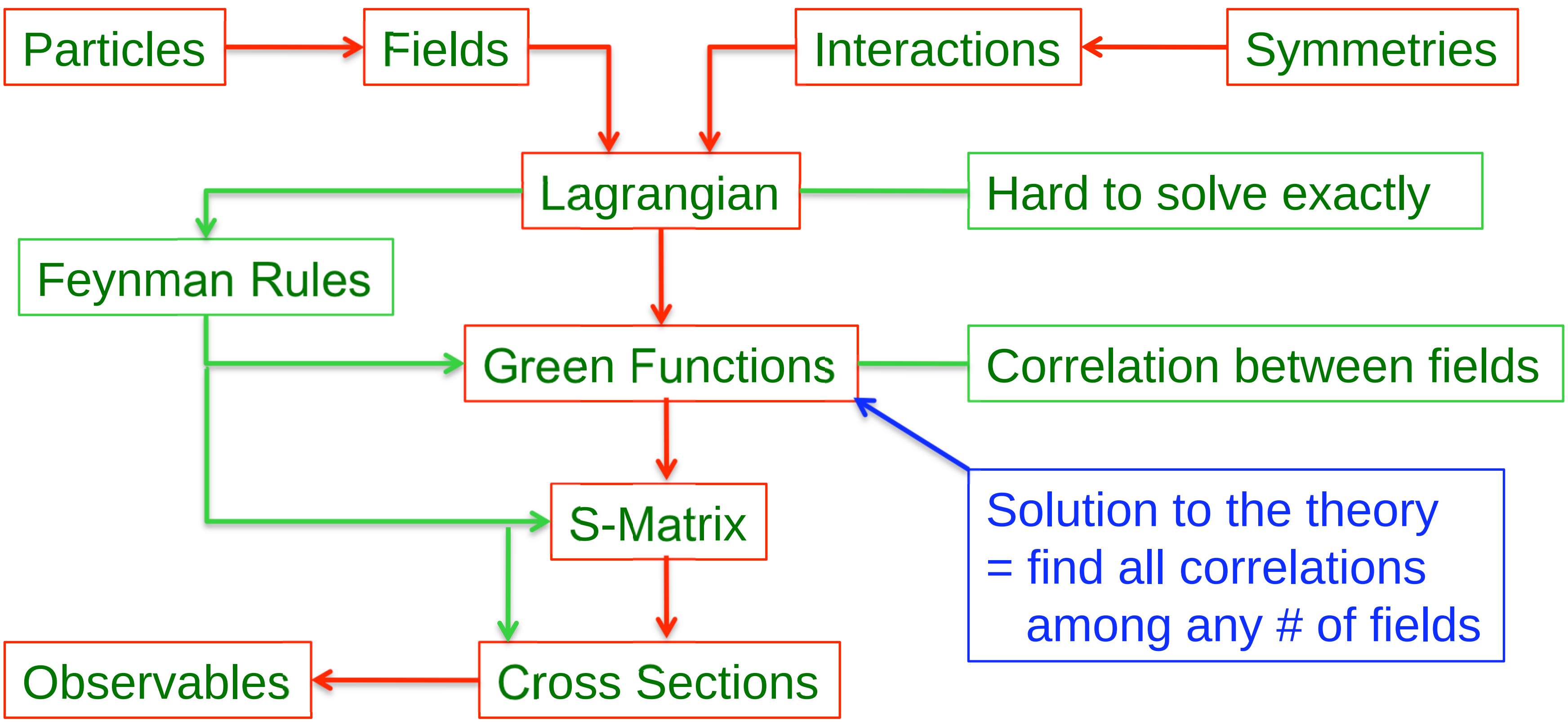
  But, it gives a clearly defined connection between

  the hadrons and the "quarks".
- Newly discovered hadronic resonances renewed our

  interests in hadron physics and its connection to QCD!

From Lagrangian to Cross Section

- Theorists: Lagrangian = "complete" theory
- Experimentalists: Cross Section ⟹ Observables
- A road map – from Lagrangian to Cross Section:



Quantum Chromodynamics (QCD)

Quantum Chromodynamics (QCD – 量子色动力学) is a quantum field theory of quarks and gluons

- Fields:

$\psi_i^f(x)$

Quark fields: spin-½ Dirac fermion (like electron)
Color triplet: $i = 1, 2, 3 = N_c$
Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$

Gluon fields: spin-1 vector field (like photon)
Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- QCD Lagrangian density:

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - g A_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2 + \text{gauge fixing} + \text{ghost terms}$$

- Color matrices:

$$[t_a, t_b] = i\, C_{abc}\, t_c$$

Generators for the fundamental representation of SU3 color

Gauge property of QCD

- Gauge Invariance:

$$\psi_i(x) \to \psi'_j(x) = U(x)_{ji}\,\psi_i(x)$$
$$A_\mu(x) \to A'_\mu(x) = U(x)\,A_\mu(x)\,U^{-1}(x) + \frac{i}{g}\,[\partial_\mu U(x)]\,U^{-1}(x)$$

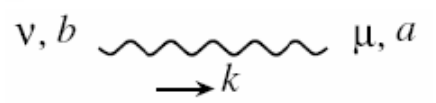
where $A_\mu(x)_{ij} \equiv A_{\mu,a}(x)(t_a)_{ij}$

$$U(x)_{ij} = \left[e^{i\,\alpha_a(x)\,t_a}\right]_{ij} \quad \text{Unitary} \quad [\,\text{det}=1,\ \text{SU}(3)]$$

- Gauge Fixing:

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}(\partial_\mu A_a^\mu)(\partial_\nu A_a^\nu)$$

Allow us to define the gauge field propagator:



$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1-\frac{1}{\lambda}\right)\right]$$

with $\lambda = 1$ the Feynman gauge

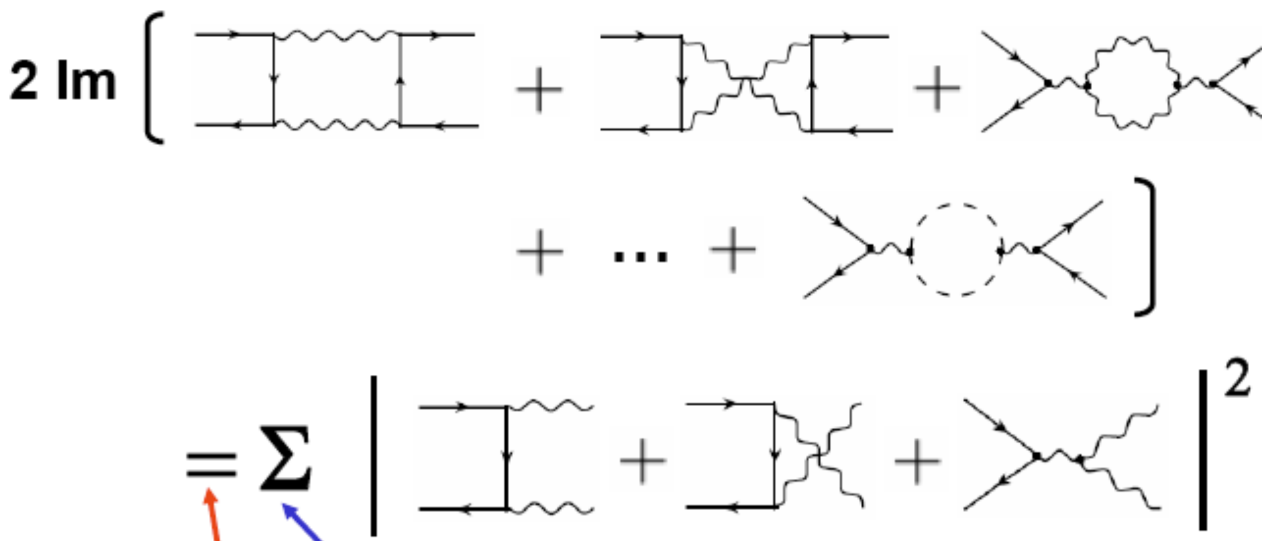
Ghost in QCD

❑ Ghost:

Ghost

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a(x))(\partial^\mu \eta_a(x) - g\, C_{abc}\, A_b^\mu(x)\, \eta_c(x)$$

so that the optical theorem (hence the unitarity) can be respected



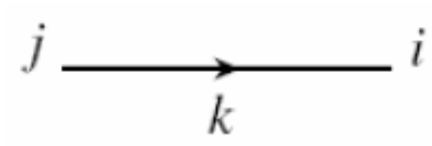
Sum over all physical polarizations

Fail without the ghost loop

Feynman rules in QCD

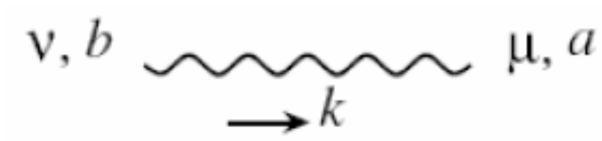
❑ Propagators:

Quark:



$$\frac{i}{\gamma \cdot k - m}\delta_{ij}$$

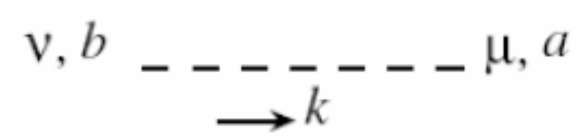
Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu \nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

for covariant gauge

Ghost:

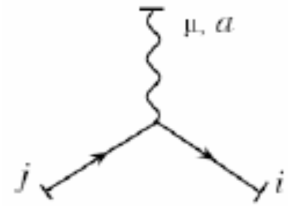


$$\frac{i\delta_{ab}}{k^2}$$

Feynman rules in QCD

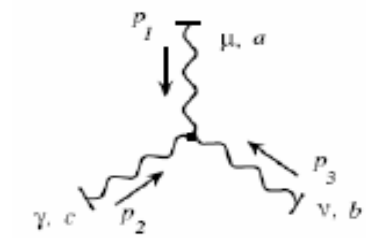
❑ Interactions:

$$-g\bar{\psi}\gamma^{\mu}A_{\mu,a}t_a\psi$$



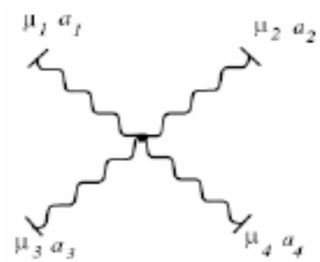
$$-ig(t_a)_{ij}\gamma_{\mu}$$

$$\frac{1}{2}gC_{abc}(\partial_{\mu}A_{\nu,a} - \partial_{\nu}A_{\mu,a})A_b^{\mu}A_c^{\nu}$$



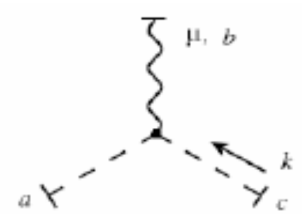
$$-gC_{abc}\left[g_{\mu\nu}(p_1-p_2)_{\gamma} + g_{\nu\gamma}(p_2-p_3)_{\mu} + g_{\gamma\mu}(p_3-p_1)_{\nu}\right]$$

$$-\frac{g^2}{4}C_{abc}C_{ab'c'} * A_b^{\mu}A_c^{\nu}A_{\mu,b'}A_{\nu,c'}$$



$$-ig^2\left[C_{ea_1a_2}C_{ea_3a_4} * (g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) + \ldots\right]$$

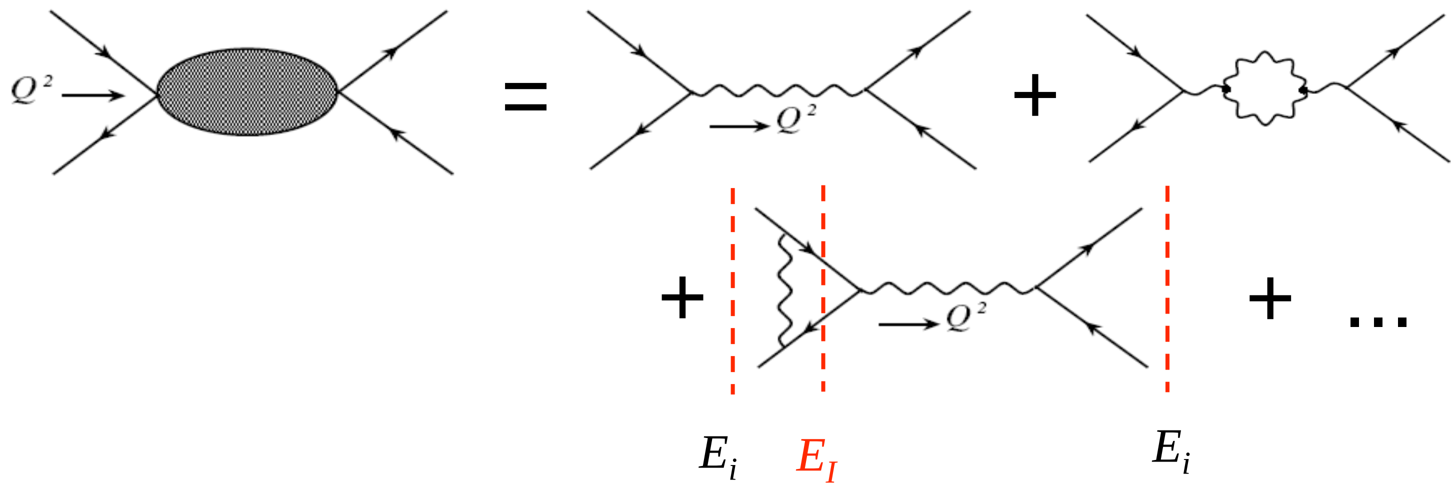
$$\partial_{\mu}\bar{\eta}_a\left(gC_{abc}A_b^{\mu}\right)\eta_c$$



$$gC_{abc}k_{\mu}$$

Why Need Renormalization

❑ Scattering amplitude:



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \dots \right) + \dots \Rightarrow \infty$$

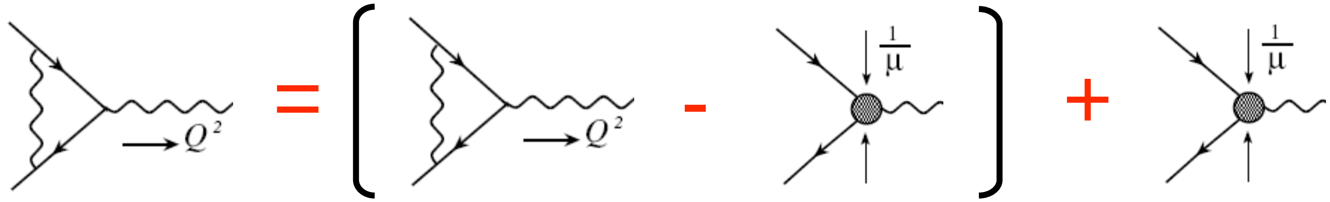
UV divergence: result of a “sum” over states of high masses

Uncertainty principle: High mass states = “Local” interactions

No experiment has an infinite resolution!

Physics of Renormalization?

- UV divergence due to "high mass" states, can not be observed

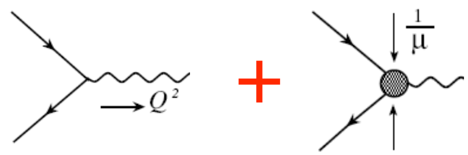


"Low mass" state

"High mass" states

- Combine the "high mass" states with LO

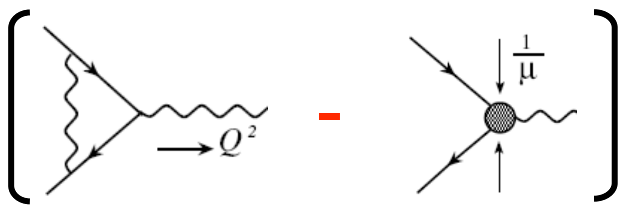
LO:



$= \; g(\mu)$

Renormalized coupling

NLO:



$+ \; \ldots$

No UV divergence!

- Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantity should not depend on the renormalization scale μ ⟹ renormalization group equation:

$$\mu^2 \frac{d}{d\mu^2}\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0 \quad \Longrightarrow \quad \sigma_{\rm Phy}(Q^2) = \sum_n \hat{\sigma}^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- Running coupling constant:

$$\mu\frac{\partial g(\mu)}{\partial \mu} = \beta(g) \qquad \alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- QCD β function:

$$\beta(g) = \mu\frac{\partial g(\mu)}{\partial\mu} = +g^3\frac{\beta_1}{16\pi^2} + \mathcal{O}(g^5) \qquad \beta_1 = -\frac{11}{3}N_c + \frac{4}{3}\frac{n_f}{2} < 0 \quad \text{for } n_f \le 6$$

- QCD running coupling constant:

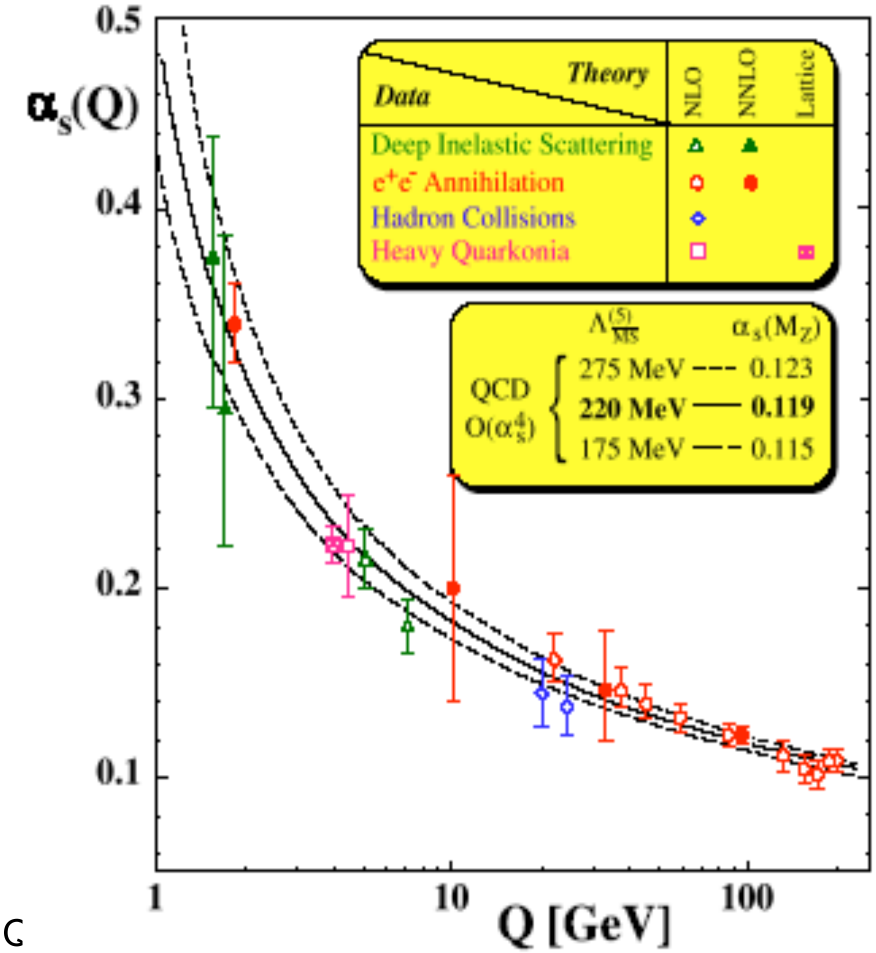
$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \quad \text{as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Asymptotic freedom!

QCD Asymptotic Freedom

❑ Λ_{QCD}: $$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{\rm QCD}^2}\right)}$$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

QCD: $$\frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

QED: $$\frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

2004 Nobel Prize in Physics

Effective Quark Mass

- Running quark mass:

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- QCD running quark mass:

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since} \quad \gamma_m(g(\lambda)) > 0$$

- Choice of renormalization scale:

$\mu \sim Q$ for small logarithms in the perturbative coefficients

- Light quark mass: $m_f(\mu) \ll \Lambda_{\rm QCD}$ for $f = u, d,$ even s

QCD perturbation theory ($Q \gg \Lambda_{\rm QCD}$) is effectively a massless theory

Infrared Safety

- Infrared safety:

$$\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right)\Rightarrow\hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+\mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = $\kappa > 0$

Asymptotic freedom is useful
only for
quantities that are infrared safe

"See" the partonic dynamics

- ❑ No ideal snap shot!

  We only see hadrons, leptons, not quarks and gluons – QCD confinement

- ❑ Need observables not sensitive to the hadronization:

  ✧ e^+e^- total cross section:

  – help of the unitarity

  ✧ Jets:

  – trace of the energetic quarks and gluons

  – infrared cancelation, the scale of $\varepsilon\sqrt{s}$

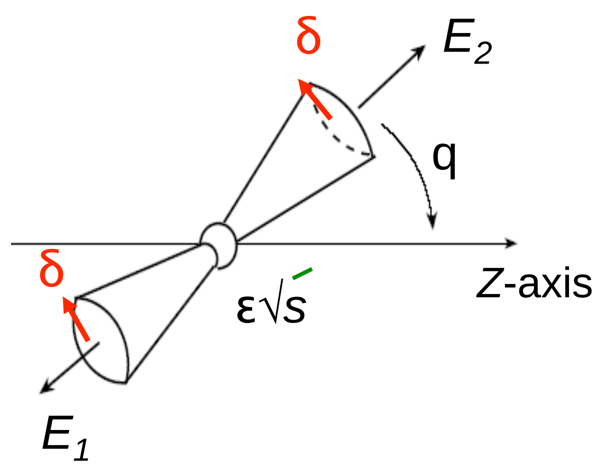
  (good jet > 50 GeV at Tevatron)

  – jet shape – resummation of shower

  – k_T jet finder – "junk" jet

  – change of the jet shape – k_T factorization

  ✧ …



Connecting the partons to the hadrons

❑ Lattice QCD can calculate partonic properties
But, cannot link partons to hadronic cross sections

❑ Effective field theories + models:

✧ Integrate out some degrees of freedom, express QCD in some effective degrees of freedom:

HQEF, SCEF, …

– approximation in field operators, still need the matrix elements to connect to the hadron states

✧ effective theory in hadron degrees of freedom, …

✧ models – Quark Models, …

❑ PQCD factorization:

✧ Connect partons to hadrons via matrix elements (PDFs, FFs, …)

$$\langle H(p,s)|\mathcal{O}(\phi,F_{\mu\nu})|H(p,s)\rangle$$

QCD, Factorization, Effective Theory

- PQCD is an effective field theory (EFT) of QCD
  - ✧ Integrate out the UV region of momentum space
  - ✧ Match the renormalized pQCD and QCD at the renormalization scale **μ** ~ Q: $\sigma(Q, g_0) = \sigma(Q/\mu, \alpha_s(\mu))$ – renormalized coupling
  - ✧ μ-independence ➡ RGE ➡ running coupling constant
- Collinear factorization – an "EFT" of pQCD
  - ✧ Integrate out the transverse momentum of active partons
  - ✧ Match the factorized form and pQCD at the factorization scale μ_F ~ Q:

$$\sigma(Q/\mu, \alpha_s(\mu)) = \hat{\sigma}(Q/\mu_F, \mu/\mu_F, \alpha_s(\mu)) \otimes \phi(\mu_F, \alpha_s(\mu)) + \mathcal{O}(1/Q)$$

  - ✧ μ_F-independence ➡ DGLAP ➡ scale dependence of PDFs
  - ✧ Power correction: 1) multi-parton correlation functions
    2) modified evolution equations in μ_F

Foundation of perturbative QCD

- **Renormalization**
  – **QCD is renormalizable**

Nobel Prize, 1999
't Hooft, Veltman

- **Asymptotic freedom**
  – **weaker interaction at a shorter distance**

Nobel Prize, 2004
Gross, Politzer, Welczek

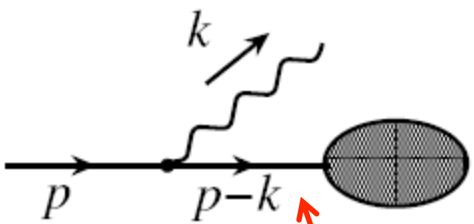
- **Infrared safety**
  – **pQCD factorization and calculable short distance dynamics**
  – **connect the partons to physical cross sections**

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe quantities!

Infrared and Collinear Divergence

- Consider a general diagram:



Singularity

$$p^2 = 0, \quad k^2 = 0$$ for a massless theory

✧ $$k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$$

$\Longrightarrow$ Infrared (IR) divergence

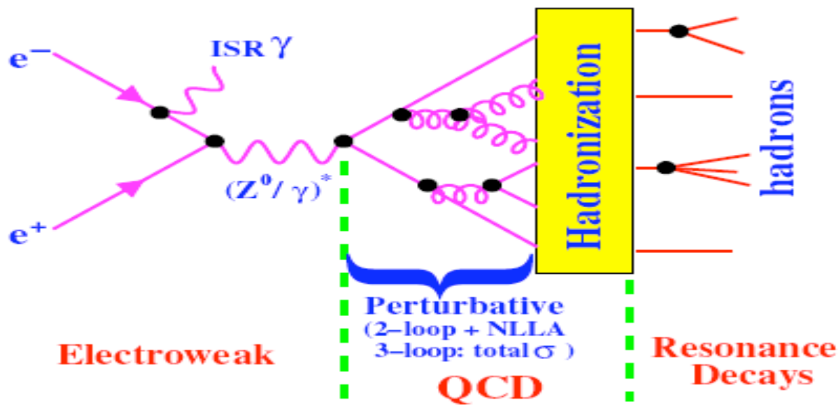
✧ $$k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \;\; 0 < \lambda < 1$$
$$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2 p^2 = 0$$

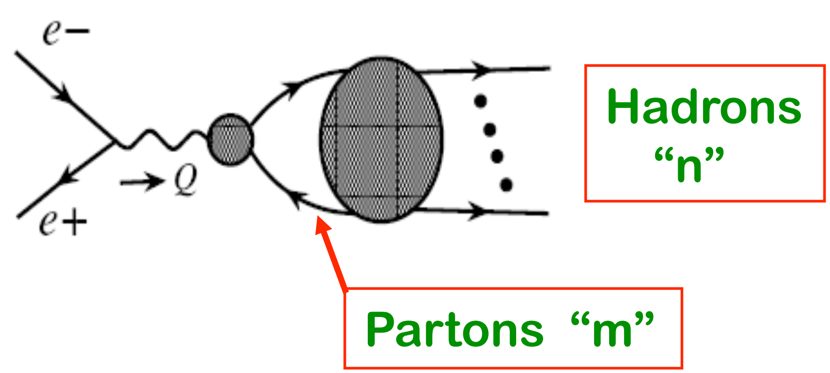
$\Longrightarrow$ Collinear (CO) divergence

IR and CO divergences are generic problems of massless perturbation theory

Purely Infrared Safe Cross Sections

- $e^+e^- \to$ hadron total cross section is infrared safe (IRS):





If there is no quantum interference between partons and hadrons,

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto \sum_n P_{e^+e^-\to n} = \sum_n \boxed{\sum_m P_{e^+e^-\to m} P_{m\to n}} = \sum_m P_{e^+e^-\to m} \boxed{\sum_n P_{m\to n}} = 1$$

Unitarity

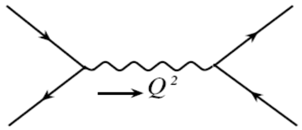
$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} \propto \sum_m P_{e^+e^-\to m}$$

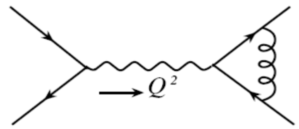
$$\Rightarrow \boxed{\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}}$$

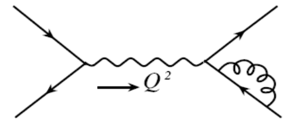
Finite in perturbation theory – KLN theorem

“Local” – of order of 1/Q

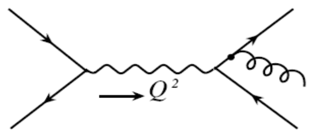
Total Cross Section for e^+e^- Collision

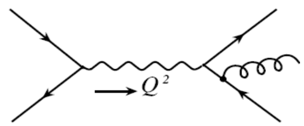
$\sigma^{\rm tot} = \dfrac{1}{2s}$ $\Big|$ 

$+$ 

$+$ 

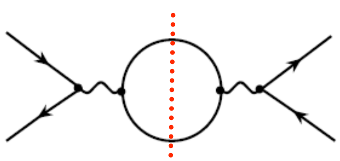
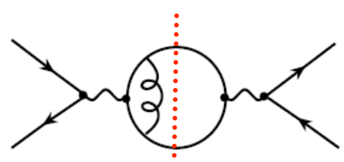
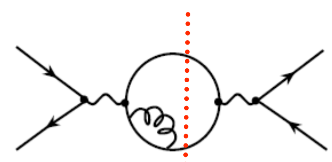
$+ \ldots \Big|^2$ $PS^{(2)}$

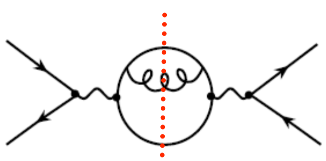
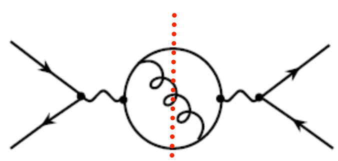
$+ \Big|$ 

$+$ 

$+ \ldots \Big|^2$ $PS^{(3)}$

$+ \ldots \Big\}$ **+ UV counter-term**

$= \dfrac{1}{2s} \Big\{$ 
+ 2Re 
+ 2Re 

+ 2 
+ 2 
$+ \ldots \Big\}$ **+ UV C.T.**

$$= \sigma_2^{(0)} + \sigma_2^{(1)} + \sigma_3^{(1)}$$

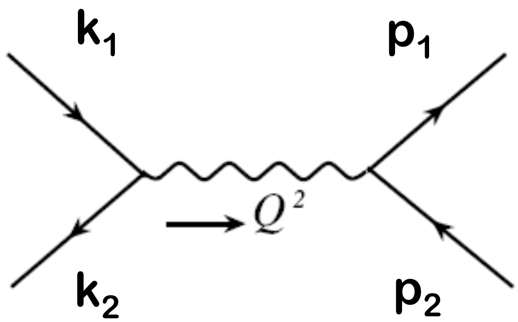
3-particle phase space

Born

O(α_s)

Lowest Order Contribution - I

- Lowest order Feynman diagram:



$$s = (k_1 + k_2)^2$$
$$t = (k_1 - p_1)^2$$
$$u = (k_2 - p_1)^2$$

- Invariant amplitude square:

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot k_2\gamma^\mu\gamma\cdot k_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot p_1 + m_Q\right)\gamma_\mu\left(\gamma\cdot p_2 - m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right]$$

Keeps the final state quark mass

Lowest Order Contribution - II

- Lowest order total cross section:

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2} |\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s = Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

- Normalized total cross section:

$$R = \frac{\sigma_{e^+e^-\to \text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \sum_Q e_Q^2\, N_c \left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best measurements for the N_c

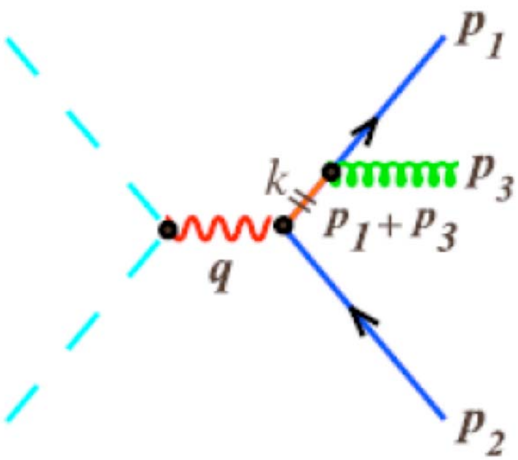
Next-to-Leading-Order Contribution - I

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \quad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

IR as x3→0
CO as $\theta_{13}\to 0$
$\theta_{23}\to 0$

Divergent as $x_i \to 1$
Need the virtual contribution and a regulator!

Next-to-Leading-Order Contribution - II

- Infrared regulator:
  - ✧ Gluon mass: $m_g \neq 0$
    – easier because all integrals at one-loop is finite
  - ✧ Dimensional regularization: $4 \to D = 4 - 2\varepsilon$
    – manifestly preserves gauge invariance

- Gluon mass regulator:
  - ✧ Real: $$\sigma^{(1)}_{3,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[2\ln^2\left(\frac{Q}{m_g}\right) - 3\ln\left(\frac{Q}{m_g}\right) - \frac{\pi^2}{6} - \frac{5}{2}\right]$$
  - ✧ Virtual: $$\sigma^{(1)}_{2,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[-2\ln^2\left(\frac{Q}{m_g}\right) + 3\ln\left(\frac{Q}{m_g}\right) + \frac{\pi^2}{6} - \frac{7}{4}\right]$$
  - ✧ Total: $$\sigma^{\text{tot}} = \sigma^{(0)}_2 + \sigma^{(1)}_{3,m_g} + \sigma^{(1)}_{2,m_g} + O\left(\alpha_s^2\right) = \sigma^{(0)}_2\left[1 + \frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

No m_g dependence!

Next-to-Leading-Order Contribution - III

- Dimensional regulator:

  - ✧ Real: $$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

  - ✧ Virtual: $$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

  - ✧ NLO: $$\boxed{\sigma_{3,\varepsilon}^{(1)}+\sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]}$$

    No ε dependence!

  - ✧ Total: $$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O\left(\alpha_s^2\right) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

- Lesson:

  σ^{tot} is independent of the choice of IR and CO regularization

  σ^{tot} is Infrared Safe!

See you next time!