Quantum Chromodynamics (QCD) and Physics of the strong interaction

(Lecture 2)

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Lecture:	Mon – Wed – Fri
	10:00-11:40AM
Location:	B326, Main Building

Lecture Plan

1. Introduction and review
2. Quark model
3. Fundamentals of QCD
4. QCD in e^+e^- annihilation
5. QCD in lepton-hadron collisions
6. QCD in hadron-hadron collisions
7. …

Review of Lecture One

- Introduction of QCD Lagrangian
- Evolvement of physics from classical mechanics
  to quantum field theory
- Proton and neutron are not point-like Dirac particle
  Low energy: Magnetic moment
  Theory: Quark Model – spectroscopy
  High energy: Deep inelastic scattering
  – point-like constituents
- Introduction of Feynman's Parton Model
- Are partons the same as the quarks? Yes or No?

Need a better Dynamical Theory!

- **Total momentum carried by the partons:**

$$F_q \equiv \sum_f \int_0^1 dx\, x\, \phi_f(x) \sim 0.5$$

Missing momentum

$\longrightarrow$ **Need particles not directly interact with photon (or EM charge)**

$\longrightarrow$ **the gluon?**

- **Scaling violation:**

$\longrightarrow$ Q^2**– dependence of structure functions?**

- **Are partons the same as the quarks?**

Feynman say: No! Gell-Mann say: Yes!

- **The birth of QCD:**

A combination of Quark Model and Yang-Mills non-Abelian gauge theory

Both men were "right"!

- **Gell-Mann is right:**

  Feynman's parton is now interpreted as the quark in QCD

  (We will derive Feynman's Parton Model from QCD late)

- **Feynman is also right:**

  Feynman's parton is not the same as the quark in Quark Model

Deep Inelastic Scattering	Quark Model – mass spectroscopy
Partons: point-like, "massless" more than 3 in proton	Constituent Quarks: "massive" 3 for baryon and 2 for meson
Perturbative QCD regime	Non-perturbative QCD regime
Current quarks and gluons Fundamental degrees of freedom	Constituent quarks are quasi-particles dressed with gluons and $q\bar{q}$ pairs
Point-like Constituents	Constituents with structure

Quark Model

Gell-Mann, Zweig, …

- Eightfold way:

Hadrons are bound states of two and three "constituent quarks" with approximate SU(3) flavor symmetry: u, d, s

- Constituent quarks:

Have the same spin, flavor, and color of the QCD current quarks,
But, their masses are phenomenological parameters,
are fitted by hadron mass spectroscopy

$$m_u \approx m_d = 0.2 - 0.35 \text{ GeV}, \quad m_s \approx 0.4 - 0.5 \text{ GeV}$$

$$m_c \approx 1.5 \text{ GeV}, \quad m_b \approx 5.0 \text{ GeV}$$

- Post QCD:

✧ **Gluon and** $q\bar{q}$ **degrees of freedom are frozen**

✧ **Their effects are hidden in the mass and the interaction potential**

Eightfold Way

- Flavor SU(3) – assumption:

Physical states for u, d, s, neglecting any mass difference,

are represented by 3-eigenstates of the fund'l rep'n of flavor SU(3)

- Generators for the fund'l rep'n of SU(3) – 3x3 matrices:

$$J_i = \frac{\lambda_i}{2} \qquad \textbf{with} \quad \lambda_i, i = 1, 2, ..., 8 \quad \textbf{Gell-Mann matrices}$$

- Good quantum numbers to label the states:

$$J_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad J_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

simultaneously diagonalized

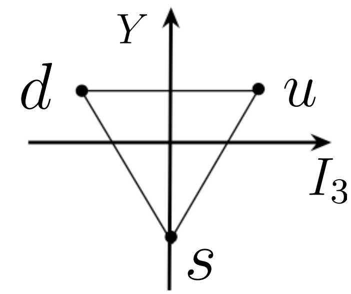
Isospin: $\hat{I}_3 \equiv J_3$, **Hypercharge:** $\hat{Y} \equiv \frac{2}{\sqrt{3}} J_8$

- Basis vectors and Eigenstates: $|I_3, Y\rangle$

$$v^1 \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^2 \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Longrightarrow d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^3 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Longrightarrow s = |0,\text{-}\tfrac{2}{3}\rangle$$

Constituent Quarks

Quark states:



$$u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad s = |0,\text{-}\tfrac{2}{3}\rangle$$

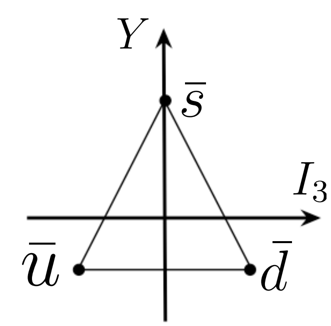
Spin: ½

Baryon #: $B = 1/3$

Strangeness: $S = Y - B$ Electric charge: $Q \equiv I_3 + \dfrac{Y}{2}$

$$u \begin{cases} Q = 2/3\, e \\ s = 1/2 \\ I_3 = 1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad d \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = -1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad s \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = 0 \\ Y = -2/3 \\ B = 1/3 \\ S = -1 \end{cases}$$

Antiquark states: $v_i \equiv \epsilon_{ijk}\, v^j v^k$



$$\hat{I}_3 v_1 = \epsilon_{123}[(\hat{I}_3 v^2)v^3 + v^2(\hat{I}_3 v^3)] + \epsilon_{132}[(\hat{I}_3 v^3)v^2 + v^3(\hat{I}_3 v^2)] = -\frac{1}{2}v_1$$

$$\hat{Y} v_1 = \epsilon_{123}[(\hat{Y} v^2)v^3 + v^2(\hat{Y} v^3)] + \epsilon_{132}[(\hat{Y} v^3)v^2 + v^3(\hat{Y} v^2)] = -\frac{1}{3}v_1$$

$$u \longrightarrow \bar{u} = |\text{-}\tfrac{1}{2},\text{-}\tfrac{1}{3}\rangle$$

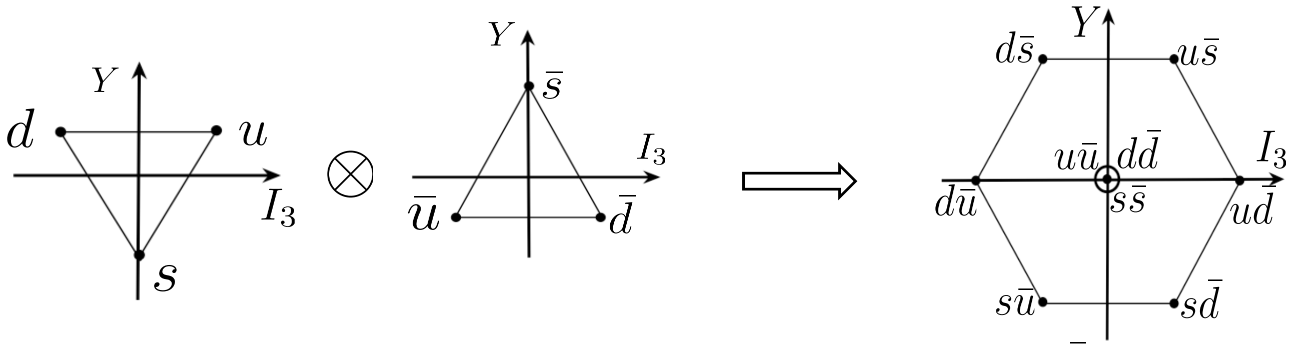
Mesons

quark-antiquark $q\bar{q}$ flavor states: $B = 0$

- Group theory says:

$$q(u,d,s) = \mathbf{3}, \quad \bar{q}(\bar{u},\bar{d},\bar{s}) = \bar{\mathbf{3}}, \quad \text{of flavor SU(3)}$$

$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ $\Longrightarrow$ 1 flavor singlet + 8 flavor octet states



There are three states with $I_3 = 0, Y = 0$: $u\bar{u}, d\bar{d}, s\bar{s}$

- Physical meson states:

(L=0, S=0)

✧ Octet states:

$$A = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \Longrightarrow \pi^0$$

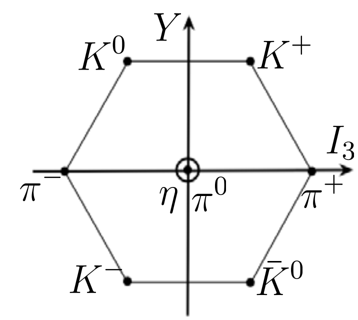
$$B = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \Longrightarrow \eta_8$$

✧ Singlet states:

$$C = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \Longrightarrow \eta_1$$

$\eta_8, \eta_1 \;\}\; \eta, \eta'$

$\Longrightarrow$

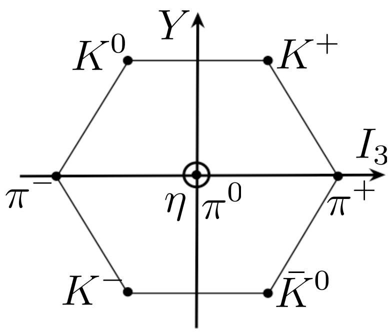


$\oplus\ \eta'$

Quantum Numbers

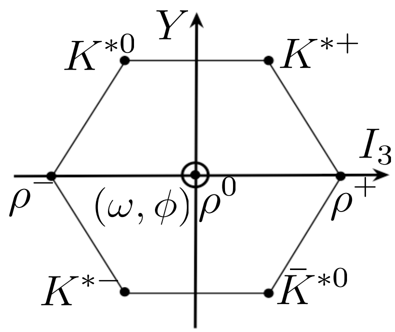
- **Meson states:** J^{PC}
  - ✧ Spin of $q\bar{q}$ pair: $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \;\longrightarrow\; S = 0, 1$
  - ✧ Spin of mesons: $J = S + L$
  - ✧ Parity: $P = -(-1)^L$
  - ✧ Charge conjugation: $C = (-1)^{L+S}$
- **L=0 states:**

$J^{PC} = 0^{-+}$: (*Y*=*S*)



$\oplus\ \eta'$

$J^{PC} = 1^{--}$: (*Y*=*S*)



$\oplus\ \mathrm{mix}(\omega, \phi)$

Flavor singlet, spin octet

Flavor octet, spin octet

- **Color:**

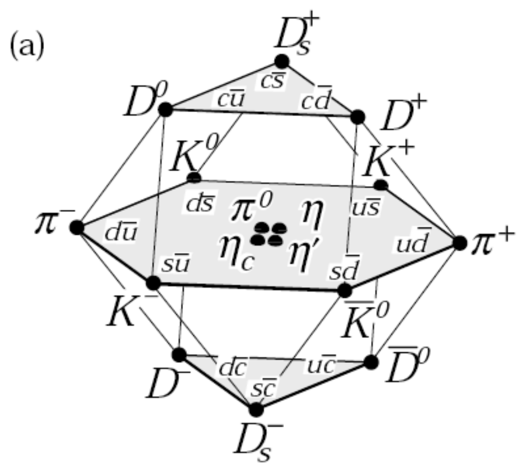
No color was introduced!

Heavy Quark Mesons

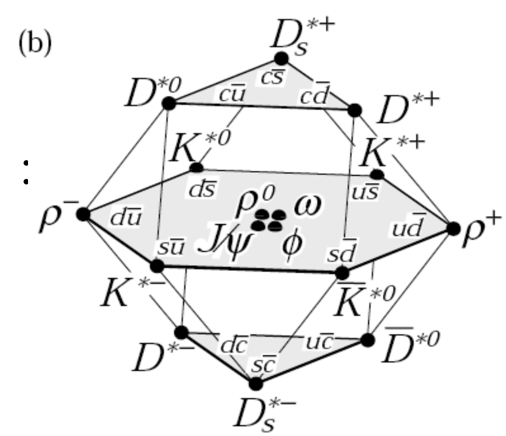
- Flavor SU(4) – Assumption:
  - ✧ All four flavor quarks: u, d, s, c are represented by the eigenstates of the fundamental representation of SU(4)
  - ✧ 3 good quantum numbers to the states – 3d representation of states
  - ✧ The symmetry is badly broken due to large mass difference

- L=0 states:

$J^{PC} = 0^{-+}:$



$J^{PC} = 1^{--}:$



- Bottom quark is too heavy to have a reasonable SU(5) flavor symmetry

Baryons

3 quark: qqq, states with $B = 1$

- Flavor SU(3): $q(u, d, s)$

27 states from qqq can be decomposed into following flavor states:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$$

$$\text{S: symmetric in all 3 q,} \quad \text{M}_S: \ \text{symmetric in 1 and 2,}$$
$$\text{M}_A: \ \text{antisymmetric in 1 and 2,} \quad A: \ \text{antisymmetric in all 3}$$

- Spin of 3 quarks:

$$\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4}_S \oplus \mathbf{2}_{M_s} \oplus \mathbf{2}_{M_A} \quad \Rightarrow \quad S = \frac{3}{2}, \frac{1}{2}, \frac{1}{2}$$

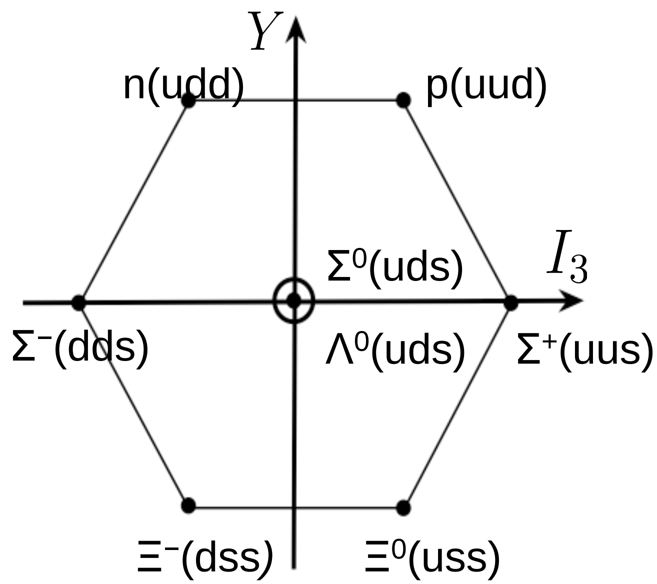
- Flavor-Spin baryon states:

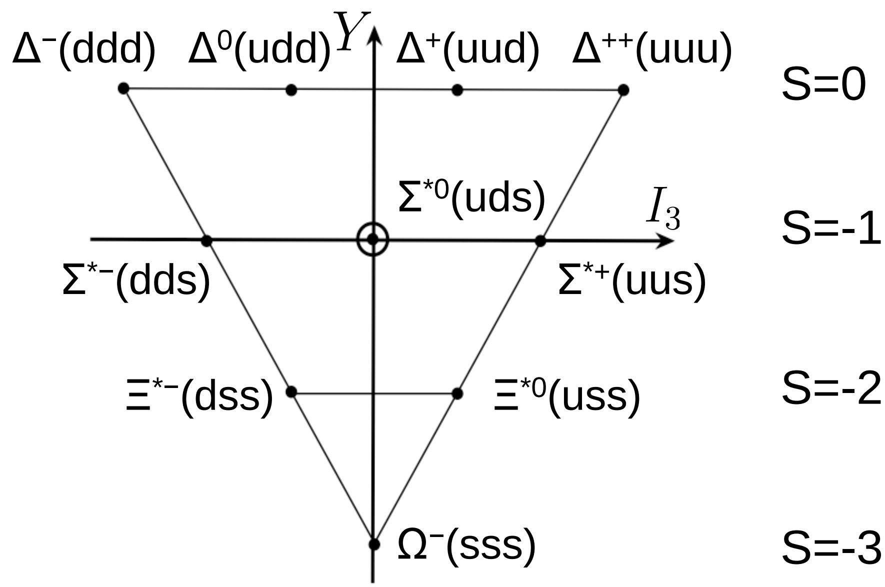
3 quarks give $8 \times 27 = 216$ baryonic states: $(\text{flavor}, \text{spin})$

216
- 56 $\quad$ S: $\quad (10_S, 4_S),\ \frac{1}{\sqrt{2}}[(8_{M_S}, 2_{M_S}) + (8_{M_A}, 2_{M_A})]$
- 70 $\quad$ M_S: $\quad (10_S, 2_{M_S}),\ (8_{M_S}, 4_S),\ \frac{1}{\sqrt{2}}[(8_{M_S}, 2_{M_S}) - (8_{M_A}, 2_{M_A})],\ (1_A, 2_{M_A})$
- 70 $\quad$ M_A: $\quad (10_S, 2_{M_A}),\ (8_{M_A}, 4_S),\ \frac{1}{\sqrt{2}}[(8_{M_S}, 2_{M_A}) + (8_{M_A}, 2_{M_S})],\ (1_A, 2_{M_S})$
- 20 $\quad$ A: $\quad (1_A, 4_S),\ \frac{1}{\sqrt{2}}[(8_{M_A}, 2_{M_S}) - (8_{M_S}, 2_{M_A})]$

Baryon Ground States

❑ Flavor – 8 and spin-1/2 and flavor-10 and spin-3/2:





❑ Difficulties of the Model:

✧ L=0: Space wave function is symmetric

✧ $(10_S, 4_S)$: Flavor-spin wave function is symmetric

Total wave function is symmetric!

✧ Δ^{++}(uuu), …: violation of the Pauli exclusive principle

Need a new quantum number!

Color

❑ Minimum requirements:

✧ Quark needs to carry at least 3 different colors

✧ Color part of the 3-quarks' wave function needs to antisymmetric

❑ SU(3) color:

Recall: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

Antisymmetric color singlet state:

$$\Longrightarrow c(\text{Red, Green, Blue})$$

$$\psi_{\text{Color}}(c_1, c_2, c_3) = \frac{1}{\sqrt{6}}[\text{RGB-GRB+RBG-BRG+GBR-BGR}]$$

❑ Baryon wave function:

$$\Psi(q_1, q_2, q_3) = \psi_{\text{Space}}(x_1, x_2, x_3) \otimes \psi_{\text{Flavor}}(f_1, f_2, f_3) \otimes \psi_{\text{Spin}}(s_1, s_2, s_3) \otimes \psi_{\text{Color}}(c_1, c_2, c_3)$$

Antisymmetric Symmetric Symmetric Symmetric Antisymmetric

A complete example: Proton

- Flavor-spin part:

$$|p\uparrow\rangle \equiv \psi_{\text{Flavor}} \times \psi_{\text{Spin}} = \frac{1}{\sqrt{2}}\left[(8_{M_S}, 2_{M_S}) + (8_{M_A}, 2_{M_A})\right]$$

$$= \frac{1}{\sqrt{18}}\left[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow)\right]$$

- Normalization:

$$\langle p\uparrow|p\uparrow\rangle = \frac{1}{18}\left[(1+1+(-2)^2) + (1+1+(-2)^2) + (1+1+(-2)^2)\right] = 1$$

- Charge: $\hat{Q} = \sum_{i=1}^{3} \hat{Q}_i$

$$\langle p\uparrow|\hat{Q}|p\uparrow\rangle = \frac{1}{18}\Big[(\tfrac{2}{3}+\tfrac{2}{3}-\tfrac{1}{3})(1+1+(-2)^2) + (\tfrac{2}{3}-\tfrac{1}{3}+\tfrac{2}{3})(1+1+(-2)^2) + (-\tfrac{1}{3}+\tfrac{2}{3}+\tfrac{2}{3})(1+1+(-2)^2)\Big] = 1$$

- Spin: $\hat{S} = \sum_{i=1}^{3} \hat{s}_i$

$$\langle p\uparrow|\hat{S}|p\uparrow\rangle = \frac{1}{18}\Big\{\big[(\tfrac{1}{2}-\tfrac{1}{2}+\tfrac{1}{2}) + (-\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}) + 4(\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2})\big] + \big[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}\big] + \big[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}\big]\Big\} = \frac{1}{2}$$

Magnetic Moments

- Quark's magnetic moment:

Assumption: Constituent quark's magnetic moment is the same as that of a point-like, structure-less, spin-½ Dirac particle

$$\hat{\mu}_i = \hat{Q}_i \left(\frac{e}{2m_i}\right)$$ for flavor "i"

- Proton's magnetic moment:

$$\mu_p = \langle p \uparrow | \sum_{i=1}^{3} \hat{\mu}_i (\hat{\sigma}_3)_i \, | p \uparrow \rangle \qquad (\hat{\sigma}_3)_i \ \text{ for quark spin direction}$$

$$= \tfrac{1}{3}[4\mu_u - \mu_d]$$

- Neutron's magnetic moment:

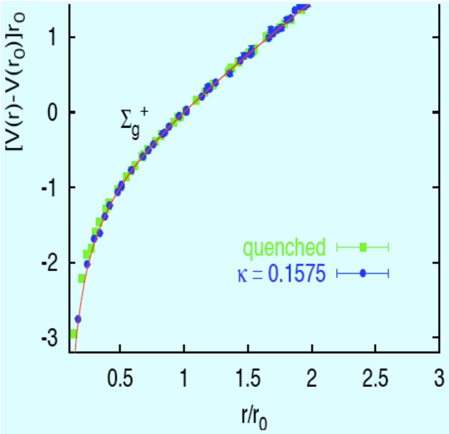
$$\mu_n = \langle n \uparrow | \sum_{i=1}^{3} \hat{\mu}_i (\hat{\sigma}_3)_i \, | n \uparrow \rangle = \tfrac{1}{3}[4\mu_d - \mu_n]$$

If $m_u = m_d,$ $\Longrightarrow \dfrac{\mu_u}{\mu_d} = \dfrac{2/3}{-1/3} = -2 \Longrightarrow \left(\dfrac{\mu_n}{\mu_p}\right)_{\text{QM}} = -\dfrac{2}{3}$

$$\left(\frac{\mu_n}{\mu_p}\right)_{\text{Exp}} = -0.68497945 \pm 0.00000058$$

Dynamics in Quark Model

- There are many, but similar, dynamical models for interactions between constituent quarks
- The first success of Constituent Quark Model is to reproduce the mass spectrum of heavy quarkonia
- Common features of the interaction potential:



Spin-dependent one gluon exchange at short-distance
+
"linear" confinement at large separation

- Sample potential for heavy quarkonia – Non-relativistic

(Cornell-type potential, spin part not shown)

$$V(r) = V_0 - \sum_{i<j}\left[\frac{3}{4} b\, r_{ij} - \frac{\alpha_s(r_{ij})}{r_{ij}}\right]\left(\frac{\lambda_j}{2}\right) \otimes \left(-\frac{\lambda_i^*}{2}\right)$$

With $r_{ij} = |\vec{r}_i - \vec{r}_j|$

Gell-Mann matrices λ_i

One Gluon Exchange Model

- Example: Spin dependent interaction from an exchange of a vector massless boson:

$$\hat{V}^{(spin)} = -\sum_{i\leq j}\left(\frac{\lambda_j}{2}\right)\otimes\left(-\frac{\lambda_i^*}{2}\right)\frac{\alpha_s(r_{ij})}{r_{ij}}\left\{\frac{8\pi}{3}\delta(\vec{r}_{ij})\,\vec{S}_i\cdot\vec{S}_j\right.$$

Spin-spin
Contact term

$$+\frac{1}{r_{ij}^3}\left[3\,\vec{S}_i\times\vec{r}_{ij}\cdot\vec{S}_j\times\vec{r}_{ij}-\vec{S}_i\cdot\vec{S}_j r_{ij}^2\right]$$

Tensor term

$$\left.+\frac{m_i m_j}{r_{ij}^3}\left[\frac{\vec{r}_{ij}\times\vec{p}_i\cdot\vec{S}_i}{2m_i^2}-\frac{\vec{r}_{ij}\times\vec{p}_j\cdot\vec{S}_j}{2m_j^2}+\frac{\vec{r}_{ij}\times\vec{p}_i\cdot\vec{S}_j}{m_i m_j}-\frac{\vec{r}_{ij}\times\vec{p}_j\cdot\vec{S}_i}{m_i m_j}\right]\right\}$$

Spin-orbit terms

- Other possible terms:
  - Spin-orbit term from Thomas-Fermi procession of the confining term
  - Color octet vs color singlet terms
  - Relativistic corrections
  - …

Understand Quark Model from QCD?

- Quark Model was proposed before QCD

  It has been reasonably successful in understanding the hadron spectroscopy

- Post QCD arguments:

  ✧ **Gluon and $q\bar{q}$ degrees of freedom are "frozen"**

  ✧ **Their effects are hidden in the mass and the interaction potential**

- Role of gluons and the color:

  To have d.o.f. "frozen" to have the quasi-stable particles:

  – a large difference in momentum scales
  (heavy quark mass, …)

  – "charge" neutral
  (constituent quarks are color charged, …)

States outside Quark Model

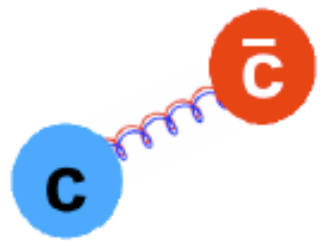
- Charmonium quantum numbers: $P = -(-1)^L \quad C = (-1)^{L+S}$

$$L = 0: \quad J^{PC} = 0^{-+}, 1^{--}$$
$$L = 1: \quad J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$$
$$L = 2: \quad J^{PC} = 2^{-+}, 1^{--}, 2^{--}, 3^{--}, ...$$

The complete list of allowed $q\bar{q}$ quantum numbers, J^{PC} , has gaps!

- Exotic J^{PC} : $0^{--}, 0^{+-}, 1^{-+}, 2^{+1}, ..., \text{etc}$

- Charmonium hybrids:



  – States with an excited gluonic degree of freedom

- If it exists,

  ✧ Link QCD dynamics of quarks and gluons to hadrons beyond the Quark Model – new insight to the formation of hadrons

  ✧ Why one "quasi-stable" gluon d.o.f.? What is the penalty to have more?

Multi-quark States

Quark Model allows bound multi-quark states

- Loosely bound meson-antimeson molecular states:

  Bound states of two or more “charge” neutral composite particles

  ✧ QED bound states – long-range multipole expansion

  ✧ QCD bound states – short-range “pion (meson)” exchange

  Key difference: localized vs non-localized “charge” sources

  Quark Model: constituent quarks represent localized color sources

  Example: $D^0(c\bar{u}) - \overline{D}^{*0}(u\bar{c})$

- Tightly bound multi-quark states:

  Tetraquark, Pentaquark, …

  Example: Diquark-diantiquark structure –$(cu) - (\bar{c}\bar{u})$

  Bound by very short-range color force – different from the molecular case

Summary

- Quark Model provides a prescription to link hadron spectroscopy to the dynamics of quarks and gluons
- With a limited number of parameters, it has been reasonably successful
- Many theoretical questions are left open:

  Why should the constituent quark exist?

  Why is the scale or dynamics to separate the interactions

  between and those within the constituent quarks?

  Are there bound states beyond those of Quark Model?

  …
- The property of XYZ and new data from BESIII and other collider experiments should bring us to a new era of strong interaction physics