Quantum Chromodynamics (QCD) and Physics of the strong interaction

(Lecture 5)

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Lecture: Mon – Wed – Fri

10:00-11:40AM

Location: B326, Main Building

Review of Lecture Four

- e^+e^- to hadrons total cross section – purely infrared safe!
  - – all hadronic information are integrated out via the unitarity
- The need of an infrared regulator for partonic calculations
  - – how the dimensional regularization works for both UV and IR divergence?
- Example: LO and NLO contribution to the total cross section
- Jets – the trace of parton – the discovery of gluon
  - – infrared safety of jets (Sterman-Weinberg jet, …)
- Jet finding algorithms: JADE, Durham, k_T, cone, …
- Event selection – physical constraints – infrared safety

Question

Does perturbative QCD work for cross sections with identified hadrons?

Facts:

Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

Energy exchange in hard collisions: $Q \gg \Lambda_{QCD}$

pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

Answer:

Perturbative QCD factorization

Cross Section with ONE Identified Hadron

- Cross section with identified hadrons is IR sensitive!
  – perturbative QCD does not work at hadronic scale

Example: Lepton-hadron deeply inelastic scattering
Inclusive single hadron production in e^+e^- collisions

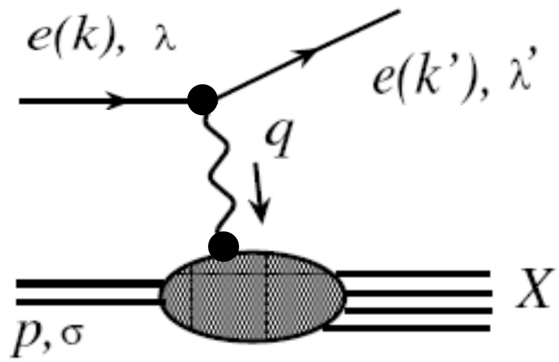
- Factorization:
  – separate hadronic from partonic physics
  – calculate the partonic physics
  – express the hadronic physics in terms of universal matrix elements and provide them with physical meaning

Example: Lepton-hadron deeply inelastic scattering
– hadron-to-parton distribution functions
Inclusive single hadron production in e^+e^- collisions
– parton-to-hadron fragmentation functions

Lepton-hadron Deep Inelastic Scattering

- Recall:

$$E'\frac{d\sigma^{\text{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$



- Hadronic tensor:

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\ \langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x_B,Q^2) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2(x_B,Q^2)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1(x_B,Q^2) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2(x_B,Q^2)\right]$$

- Structure functions – infrared sensitive:

$$F_1(x_B,Q^2),F_2(x_B,Q^2),g_1(x_B,Q^2),g_2(x_B,Q^2)$$

Perturbative QCD Factorization

- PQCD could be useful IF and Only IF the partonic dynamics can be factorized from the hadronic physics and the quantum interference between these two can be neglected

  Typical hadronic scale: $1/R \sim 1\ \mathrm{fm}^{-1} \sim \Lambda_{QCD}$

  Energy exchange in hard collisions: $Q \gg \Lambda_{QCD}$

- Factorization:

Short-distance

Power corrections

$$\sigma_{\text{phy}}(Q, 1/R) \sim \widehat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

Measured

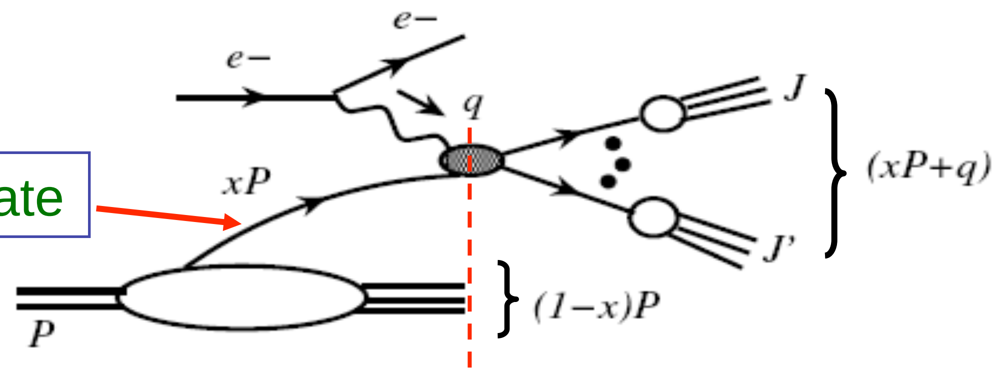
Long-distance

$\boxed{\text{Factorization} \longleftrightarrow \text{needs a “long-lived” parton state}}$

Picture of factorization for DIS

- Time evolution:

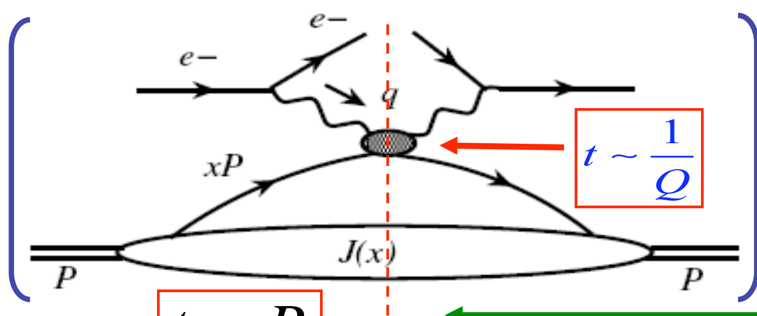
Long-lived parton state



Time: **"Past"** Now **"Future"**

- Unitarity – summing over all hard jets:

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \mathrm{Im}$$



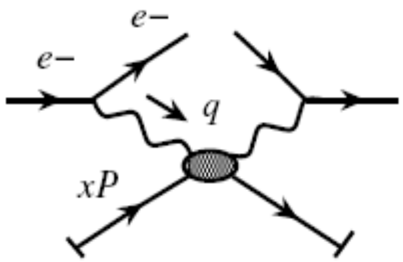
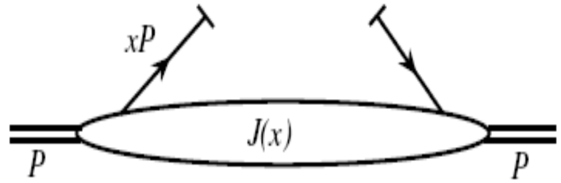
$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

Leading Power Factorization for DIS

$$\sigma_{\rm tot}^{\rm DIS} \sim$$  $$\otimes$$  $$+\, O\left(\frac{1}{QR}\right)$$

Now | Past | Connection

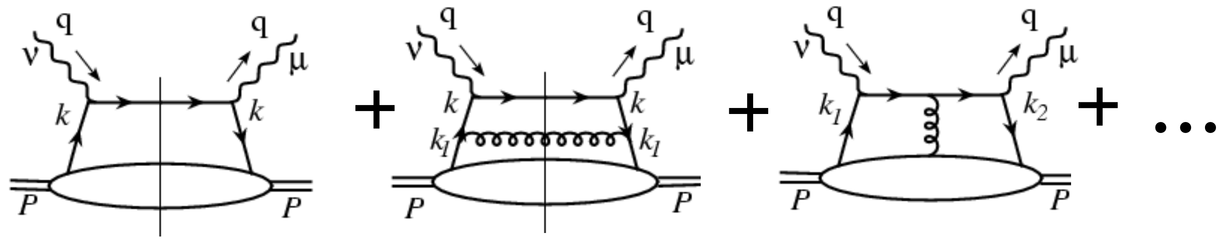
Predictive power of pQCD factorization:

- ❖ short-distance and long-distance are separately gauge invariant
- ❖ short-distance part is Infrared-Safe, and calculable
- ❖ long-distance part can be defined to be Universal
- ❖ compare observables with the same long-distance part, but, different short-distance dynamics

Long-lived Parton States

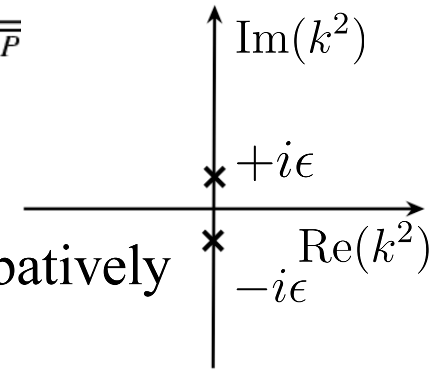
- Feynman diagram representation:

$W^{\mu\nu} \propto$



- Perturbative pinched poles:

$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$



- Perturbative factorization:

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

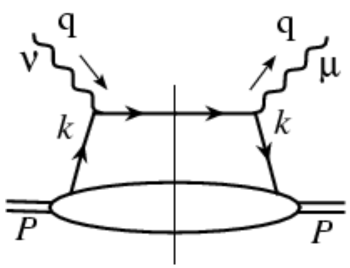
Nonperturbative matrix element

$$\int\frac{dx}{x}d^2k_T\ \mathrm{H}(Q,k^2=0)\ \boxed{\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})}$$

Short-distance

Long-lived Parton States

- Collinear approximation, if $\boxed{Q \sim xp\cdot n \gg k_T, \sqrt{k^2}}$
  - Lowest order:



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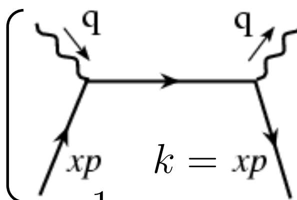
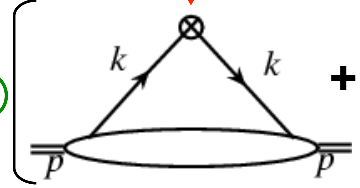


$$+O\left(\frac{k_T^2}{Q^2}\right) \otimes \int \frac{dx}{x} \quad \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$



+UVCT

$$\frac{1}{2}\gamma\cdot(xp)$$

Same as elastic x-section

Scheme dependence

Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- DIS limit: $\nu, Q^2 \to \infty$, while x_B fixed

Feynman's parton model and Bjorken scaling

$$F_2(x_B,Q^2) = x_B\sum_f e_f^2\ \varphi_f(x_B) + O(\alpha_s) + O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

Parton Distribution Funtions (PDFs)

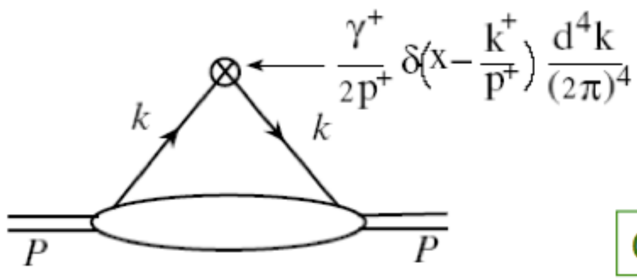
❑ PDFs as matrix elements of two parton fields:

– quark distribution as an example:

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle$$

$|h(p)\rangle$ can be a hadron, or a nucleus, or a parton state!

– corresponding diagram in momentum space:



+ UV CT

Gives the μ-dependence

But, it is not gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– need a gauge link:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle$$

❑ Predictive power of factorization relies on universality of PDFs

Necessary Conditions for Factorization

"Any uncanceled long-distance divergence of a partonic scattering cross section has to be process-independent"

On hadron state: $\sigma_{H}(Q,1/R) \sim \sum_{a} \widehat{\sigma}_{a}(Q) \otimes \varphi_{a/H}(1/R) + O(1/QR)$

On parton state: $\sigma_{p}(Q,1/R) \sim \sum_{a} \widehat{\sigma}_{a}(Q) \otimes \varphi_{a/p}(1/R) + O(1/QR)$

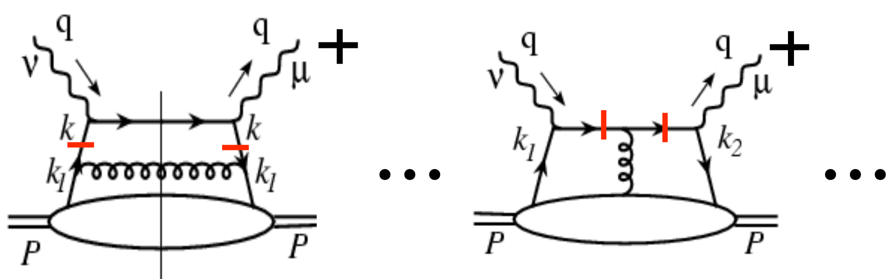
Process dependent partonic cross section (Feynman diagrams)

Process-independent Parton-level pdfs (Feynman diagrams)

LHS and RHS have the same long-distance physics!

Example:

$W_{\mu\nu}^{(1)} \propto$



All uncanceled divergences are absorbed into PDFs

An Instructive Exercise for Factorization

- Consider a cross section: $\sigma(Q^2, m^2) = \sigma_0\left[1 + \alpha_s I + O(\alpha_s^2)\right]$

- Leading quantum correction: $I = \int_0^\infty dk^2 \, \frac{1}{k^2+m^2} \, \frac{Q^2}{Q^2+k^2}$

- Analysis of the integral:

$$I = \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2+m^2} + \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2+k^2} + \mathcal{O}(m^2/Q^2)$$

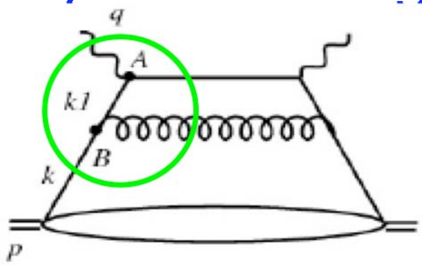
- Result for the cross section:

$$\sigma = \boxed{\left(1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2+m^2}\right)} \boxed{\left(1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2+k^2}\right)}$$
$$+\mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2)$$

$$\equiv f \times \hat{\sigma} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2).$$

Scaling Violation and Factorization

- NLO partonic diagram to structure functions:

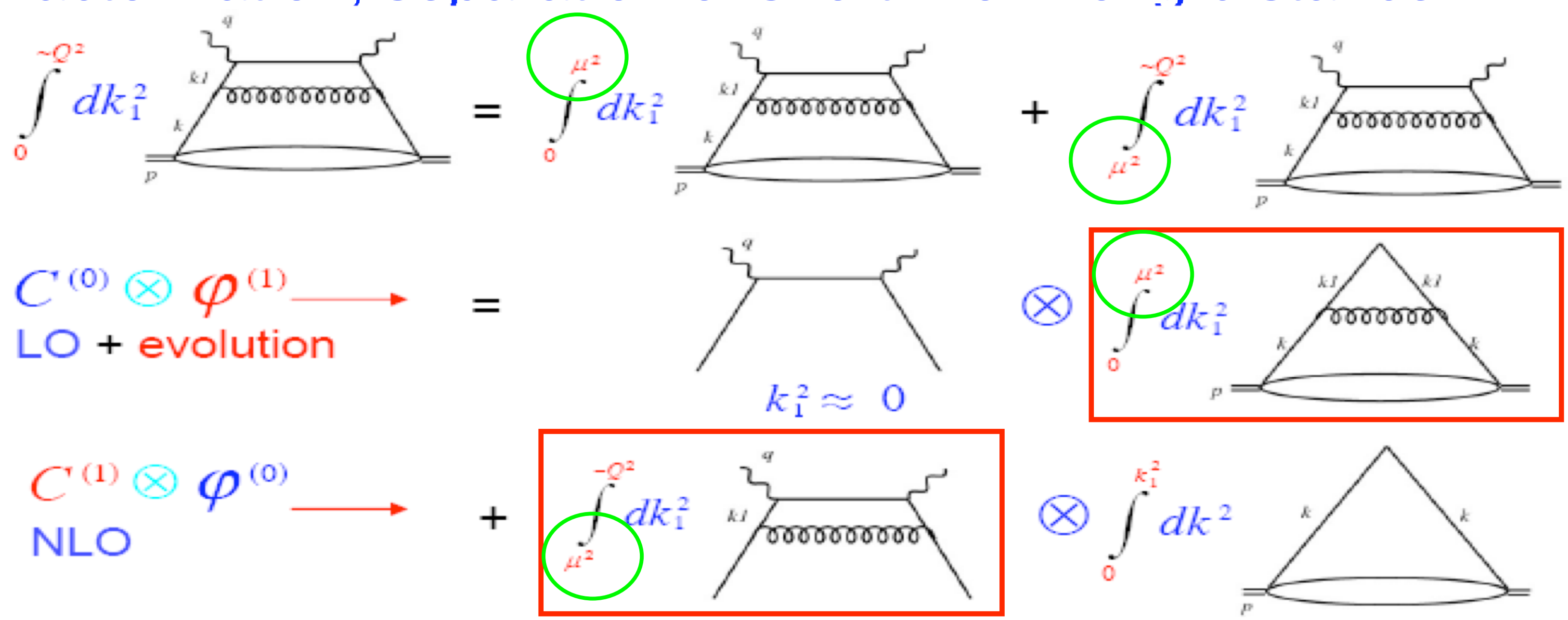


$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by $\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$

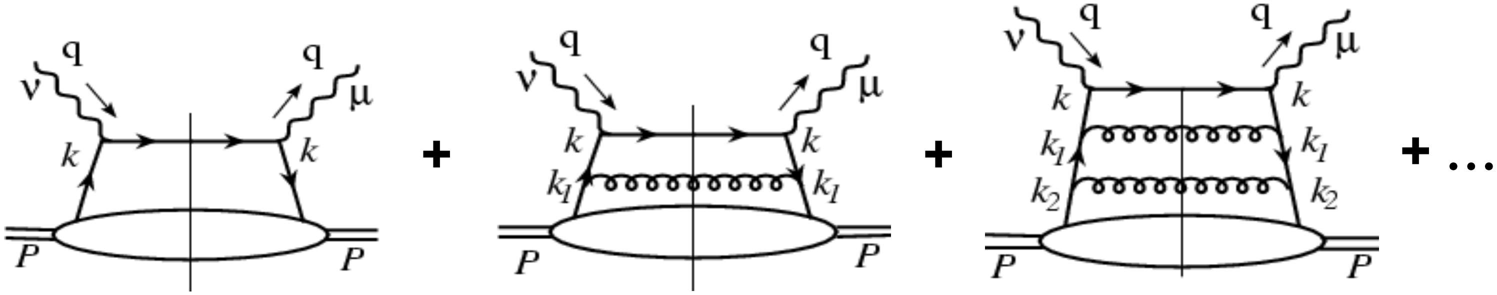
Diagram has both long- and short-distance physics

- Factorization, separation of short- from long-distance:

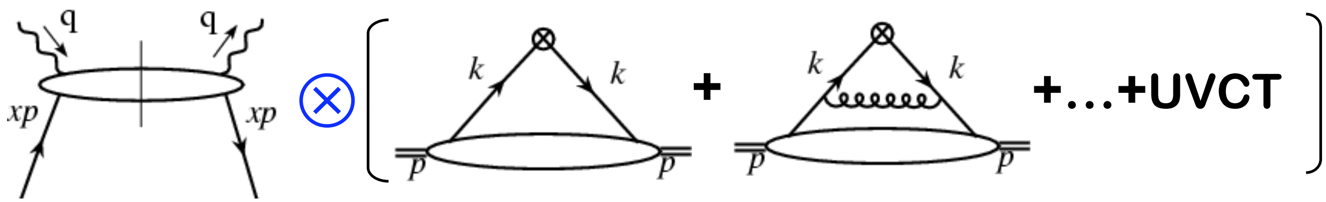


Leading Power QCD Formalism

- QCD corrections: pinch singularities in $\int d^4 k_i$



- Logarithmic contributions into parton distributions:



$$\Longrightarrow \quad F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- Factorization scale: μ_F^2

→ **To separate collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

Calculation of Perturbative Parts

- Use DIS structure function F_2 as an example:

$$F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

✧ Apply the factorized formula to parton states: $h \rightarrow q$

Feynman diagrams $\longrightarrow$ $F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu_F^2\right)$ $\longleftarrow$ Feynman diagrams

✧ Express both SFs and PDFs in terms of powers of a_s:

0th order: $F_{2q}^{(0)}(x_B,Q^2) = C_q^{(0)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu_F^2\right)$

$$\Longrightarrow \boxed{C_q^{(0)}(x) = F_{2q}^{(0)}(x)} \qquad \varphi_{q/q}^{(0)}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order: $F_{2q}^{(1)}(x_B,Q^2) = C_q^{(1)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu_F^2\right)$

$$+\,C_q^{(0)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)$$

$$\Longrightarrow \boxed{C_q^{(1)}(x,Q^2/\mu_F^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)}$$

Leading Order Coefficient Function

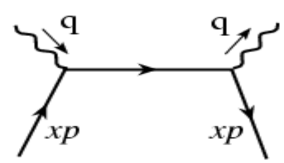
- Projection operators for SFs:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x,Q^2\right)$$

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \\ F_2(x,Q^2) &= x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \end{aligned}}$$

- 0th order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W^{(0)}_{\mu\nu,q} = x g^{\mu\nu} \left[\frac{1}{4\pi}\right.$$



$$\left.\right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p \gamma_\mu \gamma\cdot(p+q)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$\boxed{C_q^{(0)}(x) = e_q^2 x\delta(1-x)}$$

NLO Coefficient Function

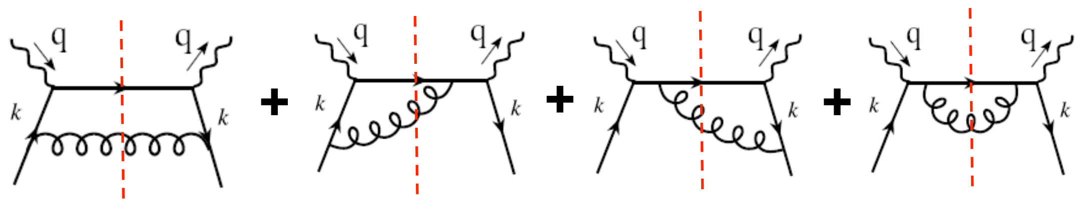
$$\boxed{C_q^{(1)}(x,Q^2/\mu_F^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)}$$

- Projection operators in n-dimension: $g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

$$\boxed{(1-\varepsilon)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}}$$

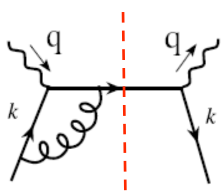
- Feynman diagrams:

$W^{(1)}_{\mu\nu,q}$

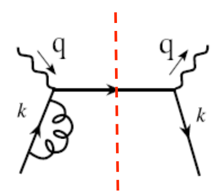


Real

Virtual $\{$ +



+ c.c. $\Big[$ +



+ c.c. **+ UV CT** $\Big]$

- Calculation: $-g^{\mu\nu}W^{(1)}_{\mu\nu,q}$ and $p^\mu p^\nu W^{(1)}_{\mu\nu,q}$

Contribution from the trace of $W_{\mu\nu}$

- Lowest order in n-dimension:

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- NLO virtual contribution:

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- NLO real contribution:

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

- The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ell n(1-z)\, f(1)$$

- One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+ C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$
$$\left.\left. + 3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

- Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

- One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

- One loop contribution to F_2 of a quark:

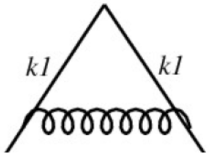
$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\Big(1+\varepsilon\,\ell n(4\pi e^{-\gamma_E})\Big) + P_{qq}(x)\,\ell n\left(\frac{Q^2}{\mu_F^2}\right)\right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \ \ \varepsilon \to 0$$

- One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu_F^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



Different choice of UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ MS scheme: $\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x) \left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

✧ $\overline{\text{MS}}$ scheme: $\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x) \left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1 + \varepsilon\,\ell n(4\pi e^{-\gamma_E})\right)$

✧ DIS scheme: choose a UV-CT, such that $C_q^{(1)}(x, Q^2/\mu_F^2)\Big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x, Q^2/\mu_F^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu_F^2\right)}$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi} \left\{ P_{qq}(x)\,\ell n\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) \right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

→ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\mathrm{QCD}}^2\right)$

Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ DGLAP evolution equation:

$$\mu_F^2 \frac{\partial}{\partial\mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

DGLAP evolution of PDFs

- DGLAP equations:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

- Splitting functions:

  - Splitting functions have to be process independent
  - Can be then derived in many different ways
    - from the logarithmic part of the C's
    - from the anomalous dimension of the nonlocal operators defining the PDFs

- Predictive power of pQCD:

Once the boundary condition is fixed by the data, the scale dependence of PDFs is a prediction of pQCD

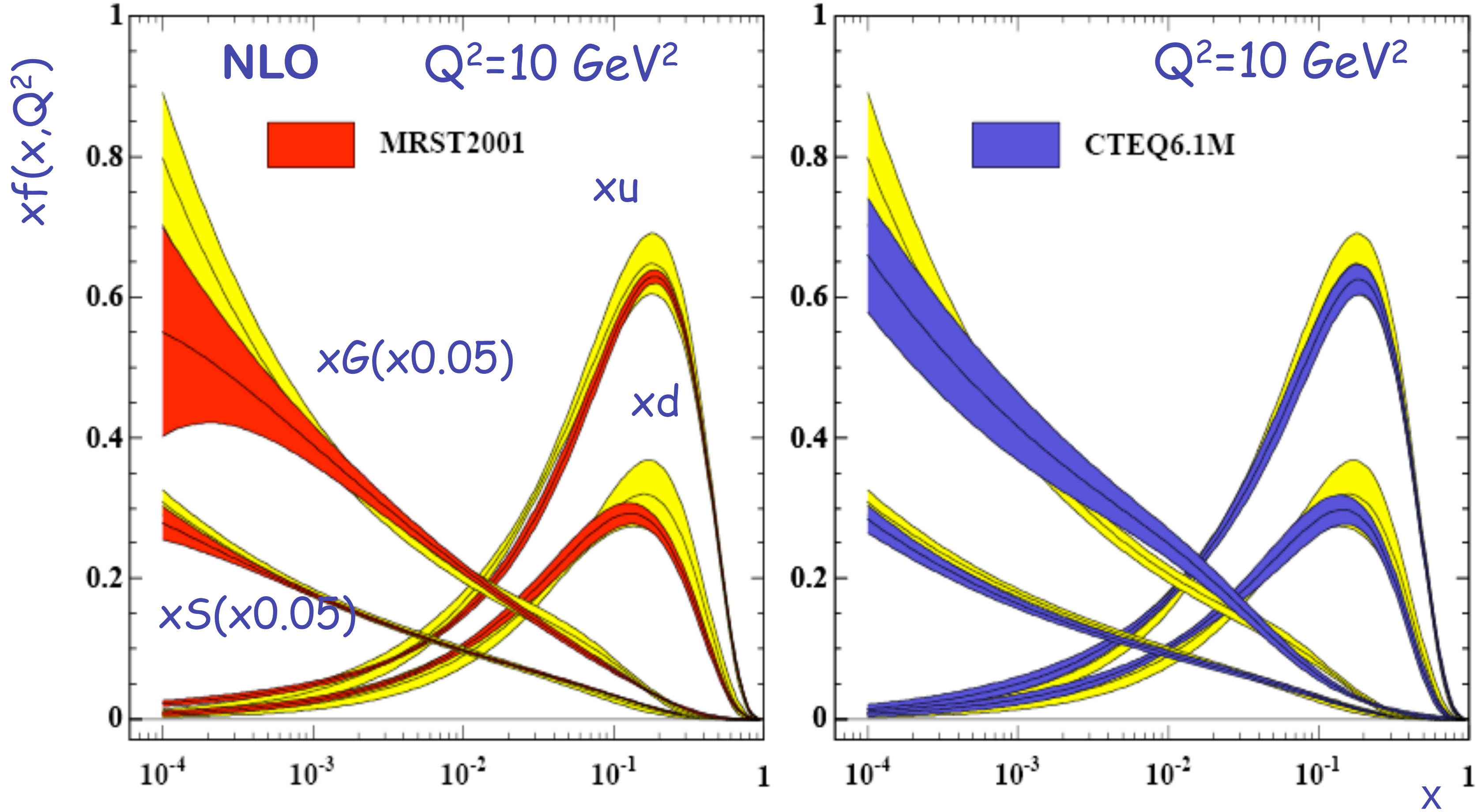
Global QCD analysis of PDFs

- PDFs are extracted by using:
  - **DGLAP** $$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$
  - **Factorized hard cross sections, e.g.** $$F_{2h}(x_B, Q^2) = \sum_q C_{q/f}\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_{f/h}\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$
  - **Data:** to fix the boundary condition of DGLAP

- The order and scheme dependence of PDFs:
  - Leading order (tree-level) C_q ⟷ **LO PDF's**
  - Next-to-Leading order C_q ⟷ **NLO PDF's**
  - Calculation of C_q at NLO and beyond depends on the UVCT → the scheme dependence of C_q → the scheme dependence of PDFs

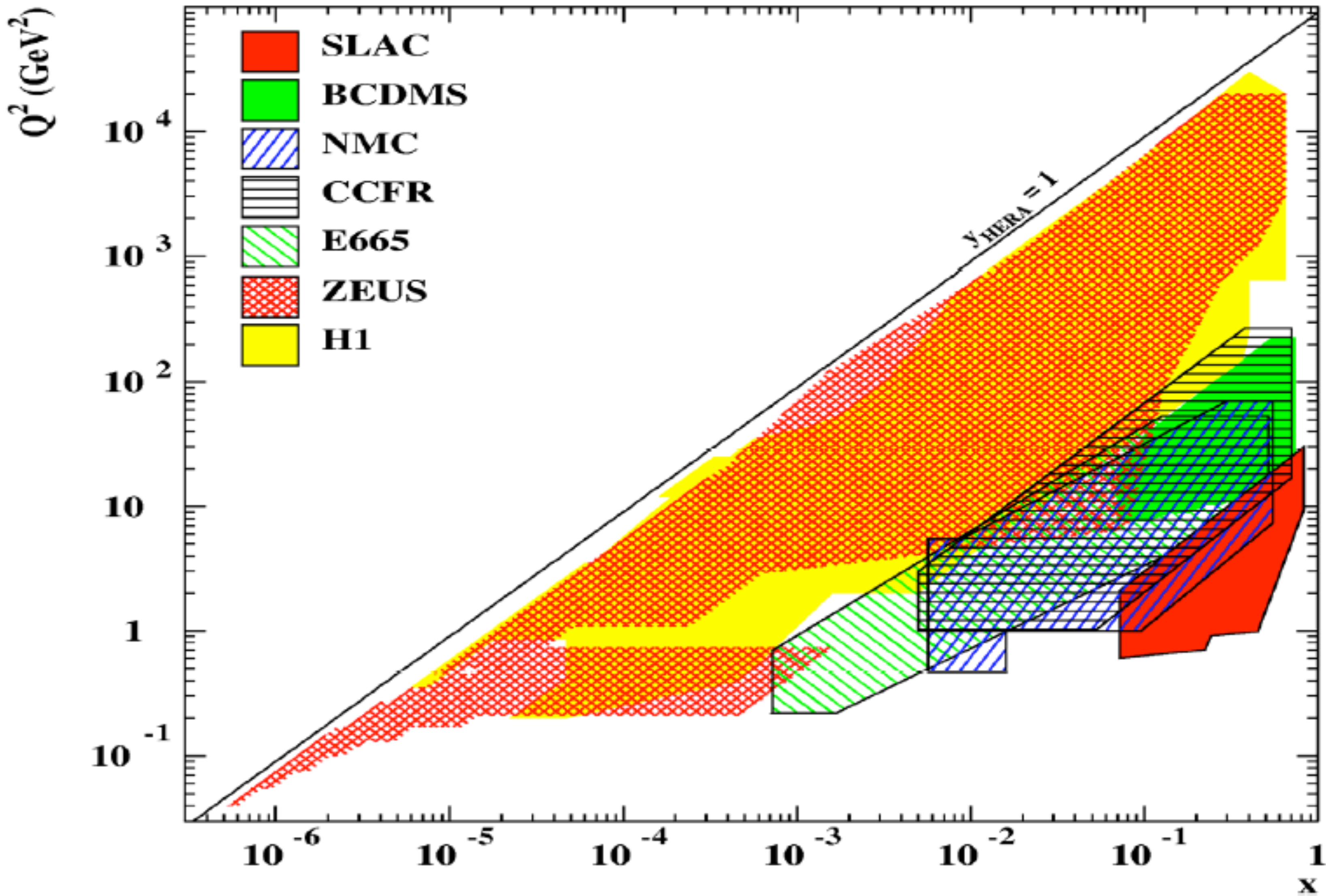
PDFs of a spin-averaged proton

- Modern sets of PDFs with uncertainties:

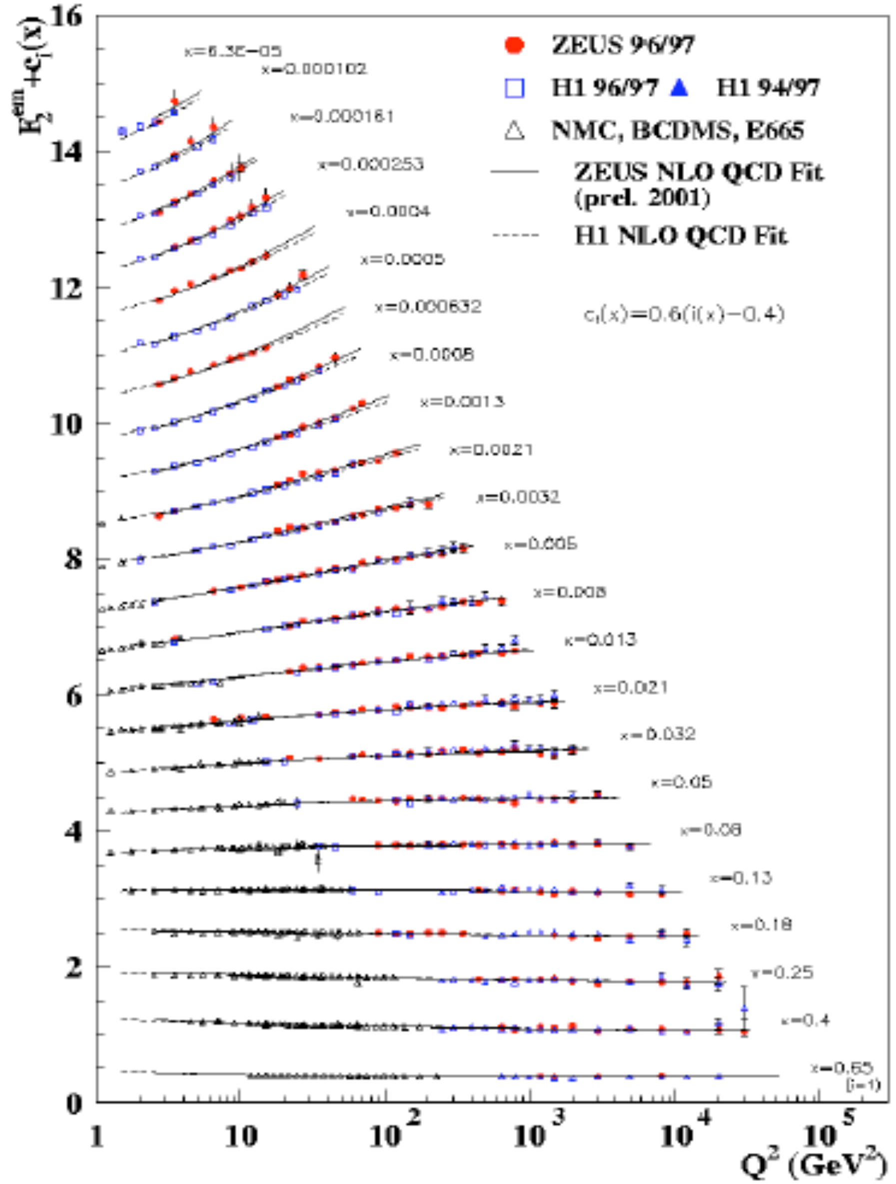


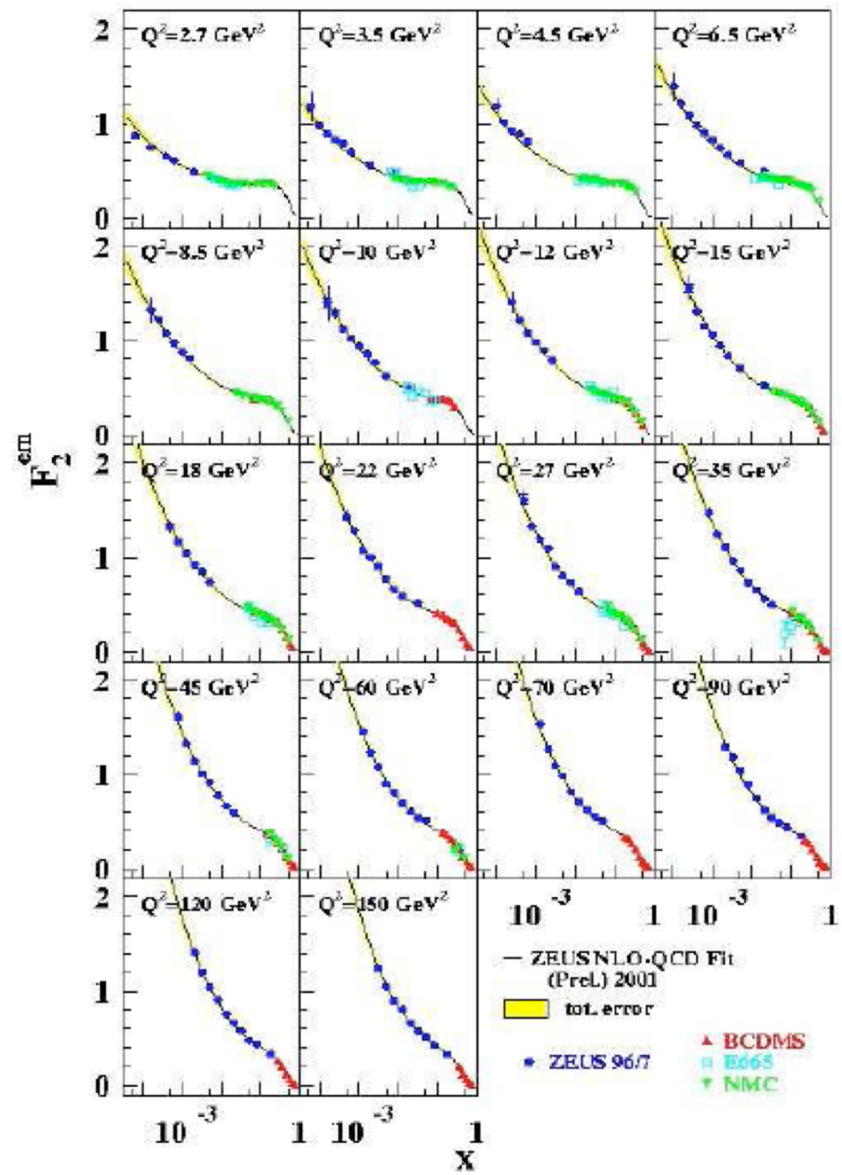
Consistently fit almost all data with Q > 2GeV

Kinematic Regions of DIS Experiments

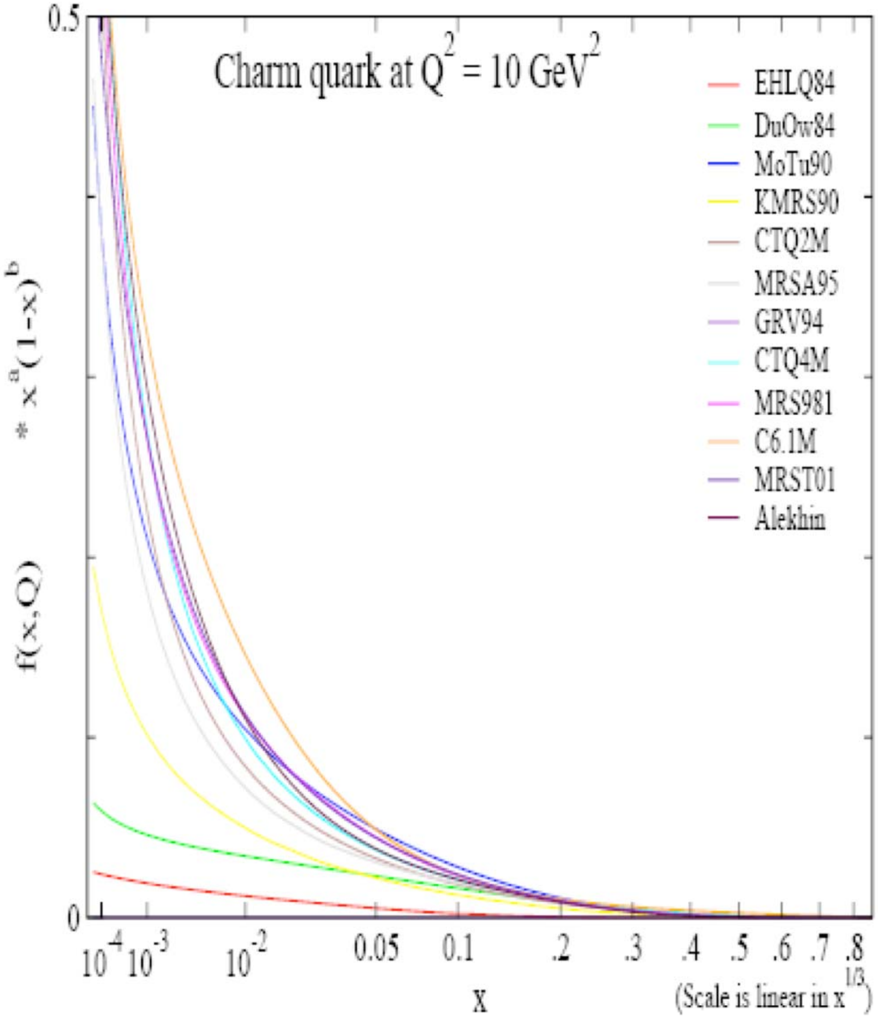


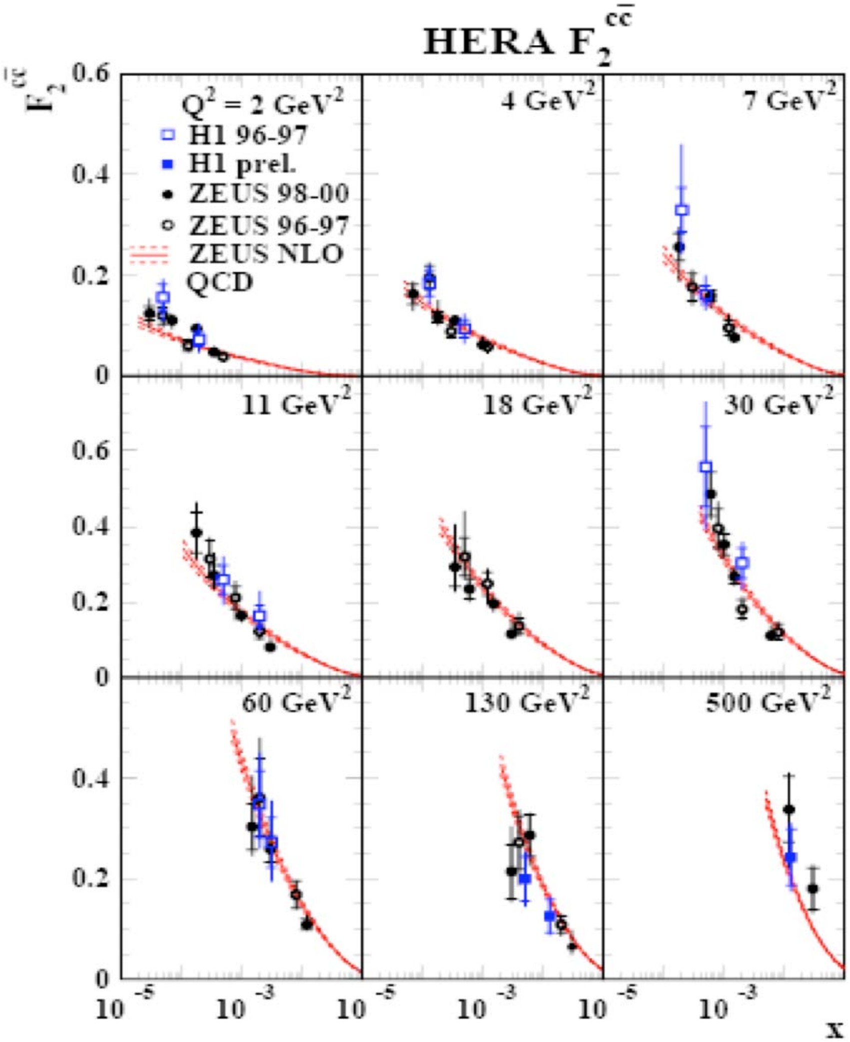
Comparison with DIS Data





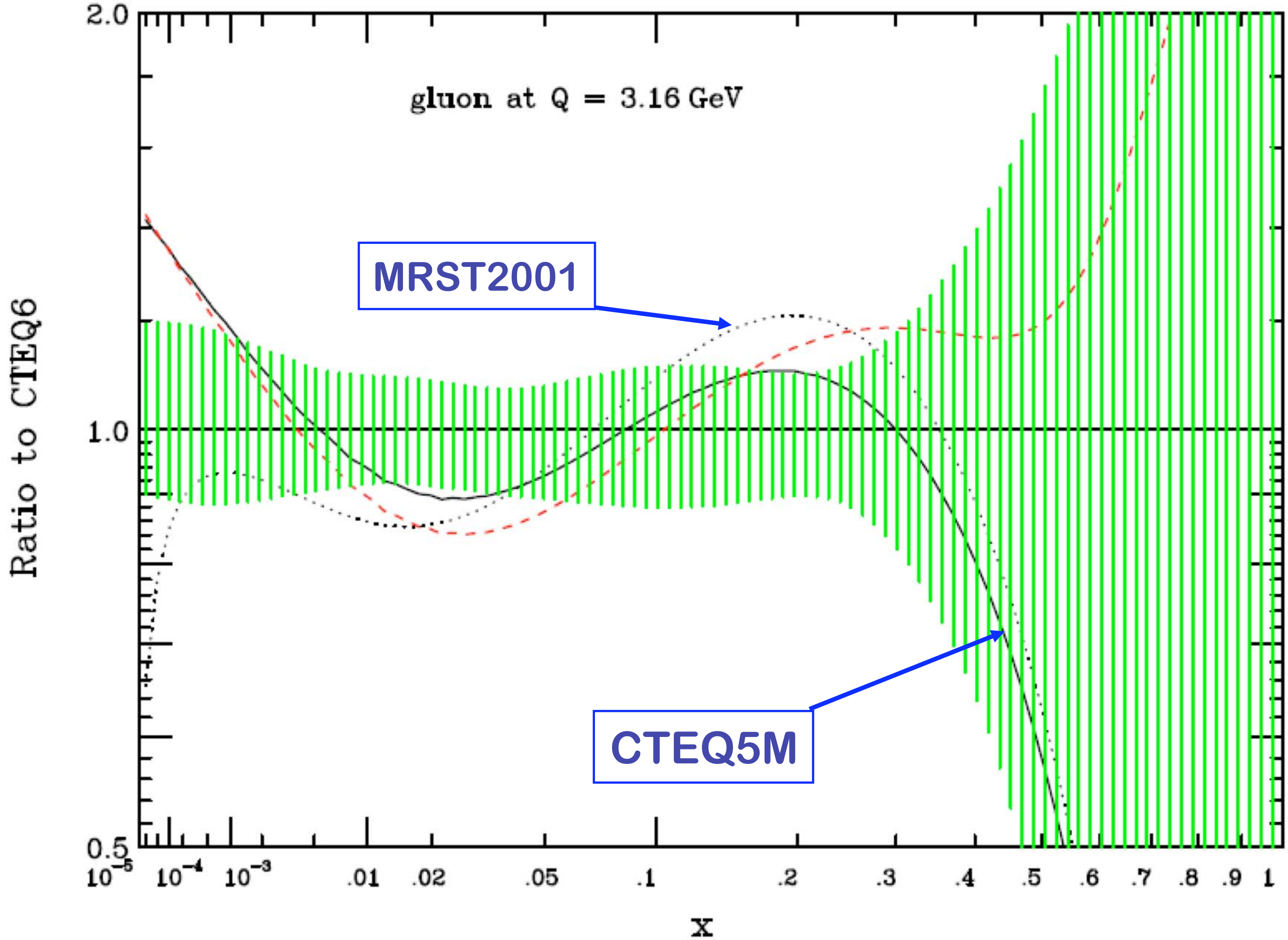
Charm Quark Distributions





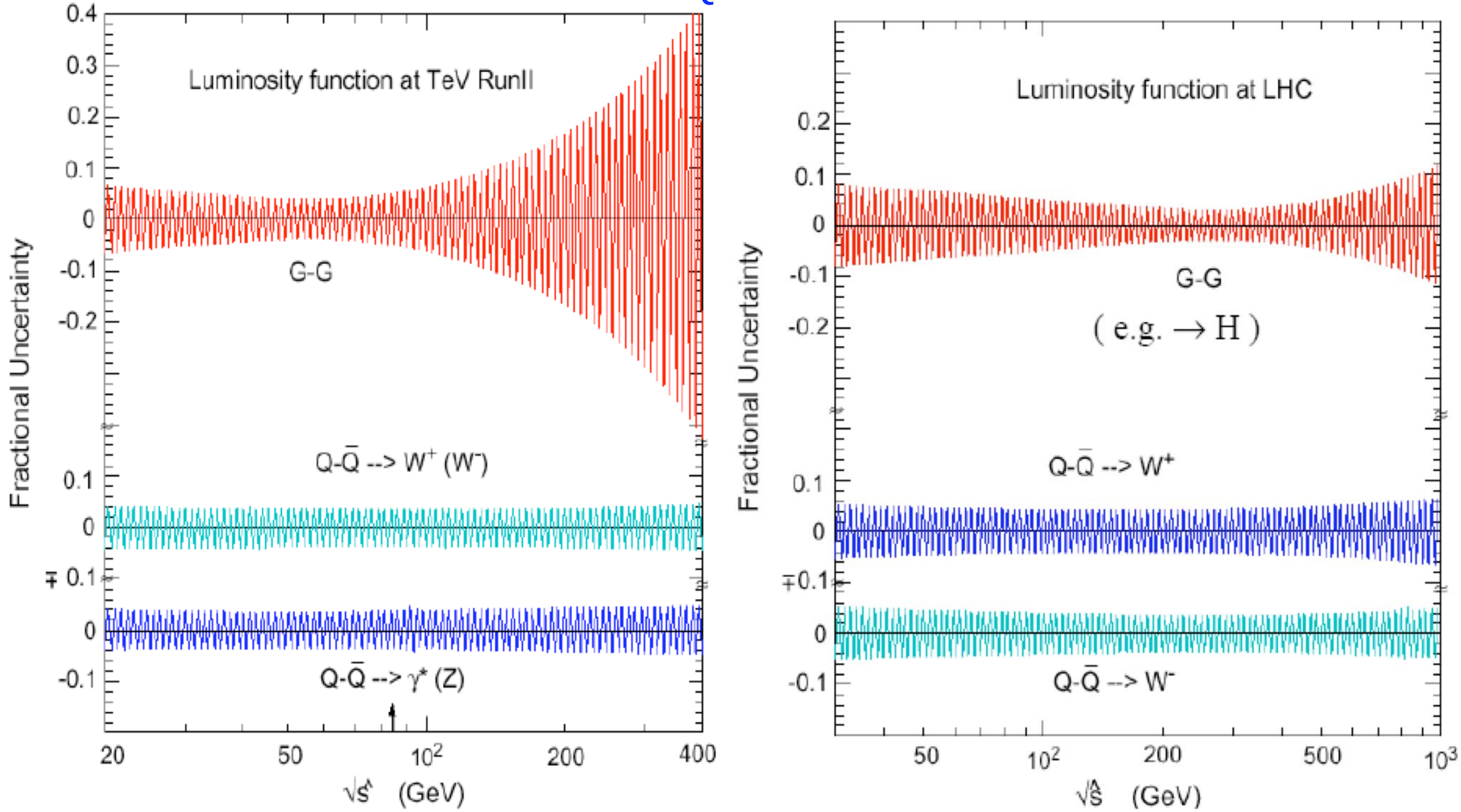
Large gluon at small-x ⟹ large charm quark distribution

Uncertainties of Gluon Distribution



PDF Uncertainty for Observables

CTEQ6 PDFs



% uncertainty for strong interaction

Recover the Effect of Non-vanishing k_T

- Sources of power corrections:
  - Parton transverse momentum: $\langle k_\perp^2 \rangle / Q^2 \sim \langle k^2 \rangle / Q^2$
  - Target and parton masses: m^2/Q^2
  - Coherent multiple scattering: $\left[\left(1/Q^2\right) / R^2 \right] \langle F^{+}_{\perp} F^{+\perp} \rangle \langle \text{Medium length} \rangle$
- Systematic of power corrections:

Leading Twist

$$\sigma^h_{phys} = \boxed{\hat{\sigma}^i_2 \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_2^{i/h}(x)}$$
$$+ \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_4^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_6^{i/h}(x)$$
$$+ \ldots$$

Perturbative

Power corrections

Factorization may not be true for power corrections!

Need to be proved for any given process

Qiu and Vitev, PRL 2004

Improvement from the Fixed Order

- Beyond the Born term (lowest order), partonic hard-parts are NOT unique – choice of the scheme – from the renormalization of parton distributions

- Once $\varphi(x,m^2)$ is fixed in one scheme, same scheme should be used for all calculations of partonic parts

- Coefficient has the logarithm: $P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)$

  Suggests to choose the scale: $\mu_F^2 \sim Q^2$

- Coefficient has potentially large logarithms:

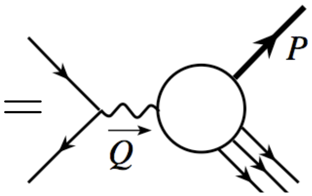
$$\ell n(x), \quad \frac{1}{(1-x)_+}, \quad \left(\frac{\ell n(1-x)}{1-x}\right)_+$$

$\Longrightarrow$ Resummation of the large logarithms

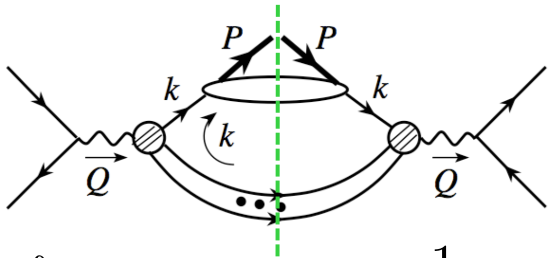
Inclusive Single Hadron Production

❑ Potential pinch singularity:

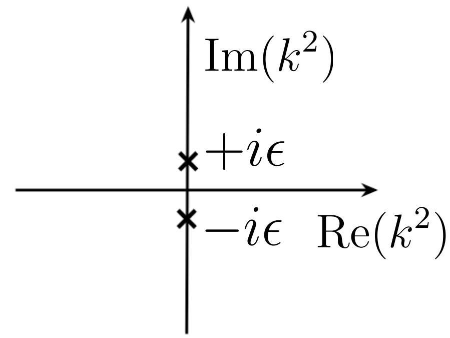
$$M_{e^+e^-\to h(P)+X} =$$



$$\sigma \propto$$



$$\propto \int d^4k\, H(Q,k)\,\frac{1}{k^2+i\epsilon}\,\frac{1}{k^2-i\epsilon}\,T(k,1/\mathrm{fm})$$

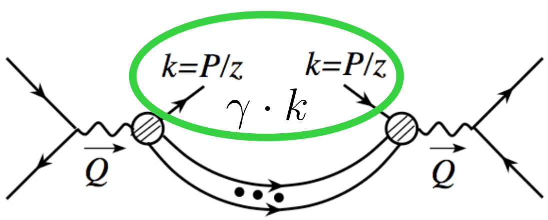


$$\to \infty \quad \text{Perturbatively}$$

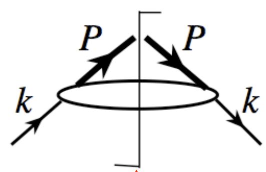
Cross section is dominated by the phase space where $k^2 \sim 0$

❑ Collinear factorization:

$$\sigma \approx$$



$$\otimes$$



$$\frac{\gamma\cdot n}{4k\cdot n}\, z^2 \delta\left(z - \frac{P\cdot n}{k\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$$= \hat{\sigma}_{e^+e^-\to q+X}(z,Q,\mu_F)\otimes D_{q\to h}(z,\mu_F) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

Summary of Lecture 5

- We can actually "see" and "count" the quarks and gluons – quark and gluon distributions

- PQCD factorization works for DIS to all orders as well as all powers due to Operator Product Expansion (OPE)

- PDFs evolves – the number of partons is sensitive to the probing scale

- PQCD global analysis for spin averaged cross sections results into the reasonably well-determined universal PDFs

What happen if there are more than one identified hadrons?

See you next time!