

Quantum Chromodynamics (QCD) and Physics of the strong interaction (Lecture 4)

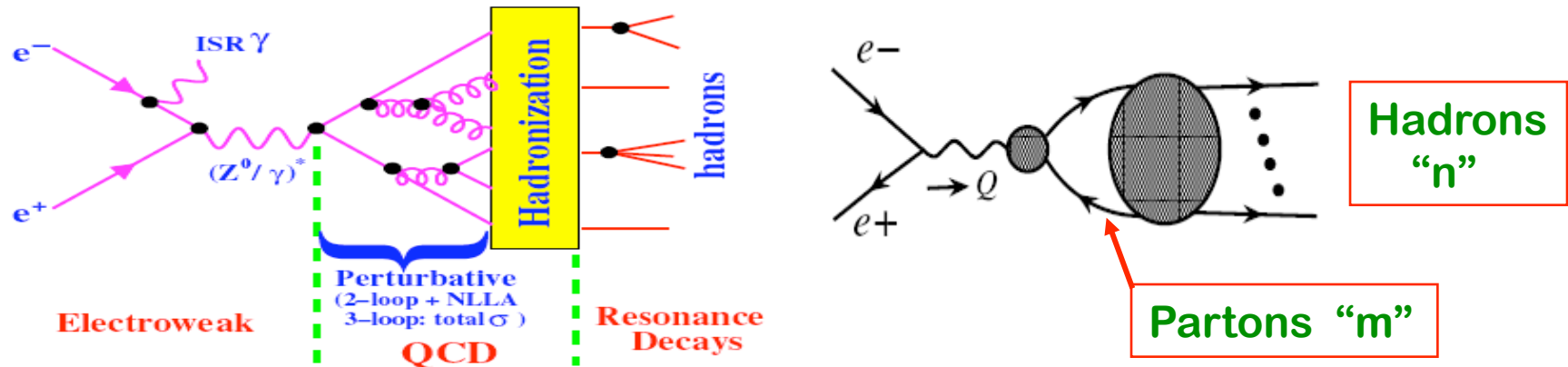
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Lecture: Mon – Wed – Fri
10:00-11:40AM
Location: B326, Main Building

Review of Lecture Three

- ❑ Introduction of QCD Lagrangian
- ❑ UV divergence – Renormalization – Perturbative QCD
- ❑ Renormalization group and running coupling constant
- ❑ QCD Asymptotic Freedom – basis of pQCD
- ❑ Mass renormalization – massless theory – Infrared Safety!
- ❑ How to connect parton dynamics to hadron dynamics?
 - Hadron matrix elements of parton operators
 - PQCD factorization, Effective field theory, ...
 - Quark models, ...
- ❑ Look for Infrared Safe Quantities! – e^+e^- total cross section

Purely Infrared Safe Cross Sections

□ $e^+e^- \rightarrow \text{hadron}$ total cross section is infrared safe (IRS):



If there is no quantum interference between partons and hadrons,

$$\sigma_{e^+e^- \rightarrow \text{hadrons}}^{\text{tot}} \propto \sum_n P_{e^+e^- \rightarrow n} = \sum_n \left[\sum_m P_{e^+e^- \rightarrow m} P_{m \rightarrow n} \right] = \sum_m P_{e^+e^- \rightarrow m} \left[\sum_n P_{m \rightarrow n} \right] = 1$$

Unitarity

$$\sigma_{e^+e^- \rightarrow \text{partons}}^{\text{tot}} \propto \sum_m P_{e^+e^- \rightarrow m}$$

$$\Rightarrow \sigma_{e^+e^- \rightarrow \text{hadrons}}^{\text{tot}} = \sigma_{e^+e^- \rightarrow \text{partons}}^{\text{tot}}$$

Finite in perturbation theory – KLN theorem

“Local” – of order of $1/Q$

e⁺e⁻ Total Cross Sections

- On-shell approximation (m=2):

[illegible]

$$= \sum_{\text{hadrons}} \left[\int d^4k \hat{M}_{e^+e^- \rightarrow q\bar{q}}(Q, k) \hat{M}_{q\bar{q} \rightarrow \text{hadrons}}(k, 1/\text{fm}) \right]^2$$

On-shell approx.

$$\begin{aligned} & \rightarrow \approx \int \frac{d^3 k}{2k_0} \hat{M}_{e^+ e^- \rightarrow q \bar{q}}(Q, \hat{k}) \int \frac{d^3 k'}{2k'_0} \hat{M}_{e^+ e^- \rightarrow q \bar{q}}^*(Q, \hat{k}') \\ & \times \sum_{\text{hadrons}} \int dk^2 \hat{M}_{q \bar{q} \rightarrow \text{hadrons}}(k, 1/\text{fm}) \int dk'^2 \hat{M}_{q \bar{q} \rightarrow \text{hadrons}}^*(k', 1/\text{fm}) + \mathcal{O}\left(\frac{\langle k^2 \rangle}{Q^2}\right) \\ & \approx \int \frac{d^3 k}{2k_0} \left| \hat{M}_{e^+ e^- \rightarrow q \bar{q}}(Q, \hat{k}) \right|^2 + \mathcal{O}\left(\frac{\langle k^2 \rangle}{Q^2}\right) \end{aligned}$$

❑ Total cross section (sum m):

$$\sigma_{e^+e^- \rightarrow \text{hadrons}}^{\text{tot}} \approx \sigma_{e^+e^- \rightarrow \text{partons}}^{\text{tot}} + \mathcal{O}\left(\frac{\langle k^2 \rangle}{Q^2}\right)$$

Infrared Safety of e^+e^- Total Cross Sections

□ Optical theorem:

$$\sigma_{e^+e^-}^{\text{tot}} = \frac{1}{2S} \left| \begin{array}{c} e^- \\ \searrow \\ e^+ \nearrow \end{array} \begin{array}{c} \rightarrow q \\ \rightarrow \bar{q} \end{array} \begin{array}{c} \text{Hadrons "n"} \\ \text{Partons "m"} \end{array} \right|^2 \propto \text{Im} \left[\begin{array}{c} \nu \\ \searrow \\ \bar{Q} \nearrow \end{array} \begin{array}{c} \text{Hadrons} \\ \text{Partons} \end{array} \begin{array}{c} \mu \\ \searrow \\ \bar{Q} \nearrow \end{array} \right]$$

□ Time-like vacuum polarization:

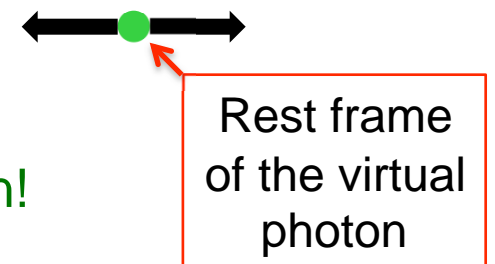
$$\begin{array}{c} \nu \\ \searrow \\ \bar{Q} \nearrow \end{array} \begin{array}{c} \text{Hadrons} \\ \text{Partons} \end{array} \begin{array}{c} \mu \\ \searrow \\ \bar{Q} \nearrow \end{array} = (Q^\mu Q^\nu - Q^2 g^{\mu\nu}) \Pi(Q^2)$$

IR safety of $\sigma_{e^+e^- \rightarrow \text{partons}}^{\text{tot}} = \text{IR safety of } \Pi(Q^2) \text{ with } Q^2 > 0$

□ IR safety of $\Pi(Q^2)$:

If there were pinched poles in $\Pi(Q^2)$,

- ✧ real partons moving away from each other
- ✧ cannot be back to form the virtual photon again!



One-loop Contribution to e^+e^- Cross Section

$$\sigma^{\text{tot}} = \frac{1}{2s} \left\{ \left| \begin{array}{c} \text{tree-level diagrams} \\ \rightarrow Q^2 \end{array} \right|^2 \text{PS}^{(2)} + \left| \begin{array}{c} \text{one-loop diagrams} \\ \rightarrow Q^2 \end{array} \right|^2 \text{PS}^{(3)} + \dots \right\} + \text{UV counter-term}$$

$$= \frac{1}{2s} \left\{ \begin{array}{l} \text{tree-level diagrams} + 2\text{Re} \text{ (one-loop diagrams)} + 2\text{Re} \text{ (two-loop diagrams)} \\ + 2 \text{ (three-loop diagrams)} + 2 \text{ (four-loop diagrams)} + \dots \end{array} \right\} + \text{UV C.T.}$$

$$= \sigma_2^{(0)} + \sigma_2^{(1)} + \sigma_3^{(1)}$$

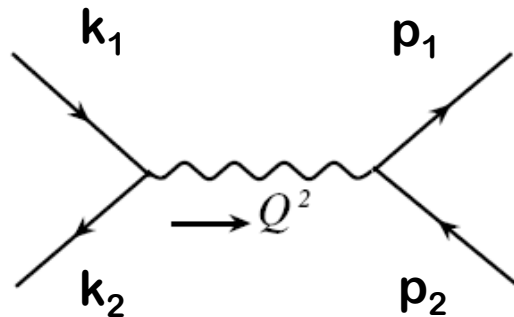
Born

$O(\alpha_s)$

3-particle phase space

Lowest Order Contribution - I

□ Lowest order Feynman diagram:



$$\begin{aligned} s &= (k_1 + k_2)^2 \\ t &= (k_1 - p_1)^2 \\ u &= (k_2 - p_1)^2 \end{aligned}$$

□ Invariant amplitude square:

$$\begin{aligned} |\bar{M}_{e^+e^- \rightarrow Q\bar{Q}}|^2 &= e^4 e_Q^2 N_c \frac{1}{s^2} \frac{1}{2^2} \text{Tr} [\gamma \cdot k_2 \gamma^\mu \gamma \cdot k_1 \gamma^\nu] \\ &\quad \times \text{Tr} [(\gamma \cdot p_1 + m_Q) \gamma_\mu (\gamma \cdot p_2 - m_Q) \gamma_\nu] \\ &= e^4 e_Q^2 N_c \frac{2}{s^2} [(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s] \end{aligned}$$

Keeps the final state quark mass

Lowest Order Contribution - II

□ Lowest order total cross section:

$$\frac{d\sigma_{e^+e^- \rightarrow Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2} |\bar{M}_{e^+e^- \rightarrow Q\bar{Q}}|^2 \quad \text{where } s = Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^- \rightarrow Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s} \left[1 + \frac{2m_Q^2}{s} \right] \sqrt{1 - \frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

□ Normalized total cross section:

$$R = \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = \frac{\sum_Q \sigma_{e^+e^- \rightarrow Q\bar{Q}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = \sum_Q e_Q^2 N_c \left[1 + \frac{2m_Q^2}{s} \right] \sqrt{1 - \frac{4m_Q^2}{s}}$$

One of the best measurements for the N_c

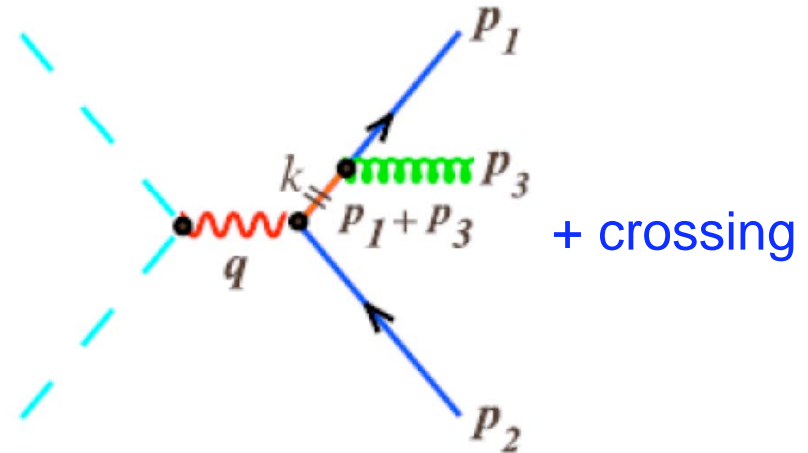
Next-to-Leading-Order Contribution - I

□ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i \cdot q}{s} \quad \text{with } i = 1, 2, 3$$

$$\sum_i x_i = \frac{2 \left(\sum_i p_i \right) \cdot q}{s} = 2$$

$$2(1 - x_1) = x_2 x_3 (1 - \cos \theta_{23}), \quad \text{cycl.}$$



□ Contribution to the cross section:

$$\frac{1}{\sigma_0} \frac{d\sigma_{e^+e^- \rightarrow Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

IR as $x_3 \rightarrow 0$
CO as $\theta_{13} \rightarrow 0$
 $\theta_{23} \rightarrow 0$

Divergent as $x_i \rightarrow 1$
Need the virtual contribution and a regulator!

Next-to-Leading-Order Contribution - II

□ Infrared regulator:

- ✧ Gluon mass: $m_g \neq 0$
 - easier because all integrals at one-loop is finite: $\frac{1}{k^2} \rightarrow \frac{1}{k^2 - m_g^2}$
- ✧ Dimensional regularization: $4 \rightarrow D = 4 - 2\epsilon$
 - manifestly preserves gauge invariance

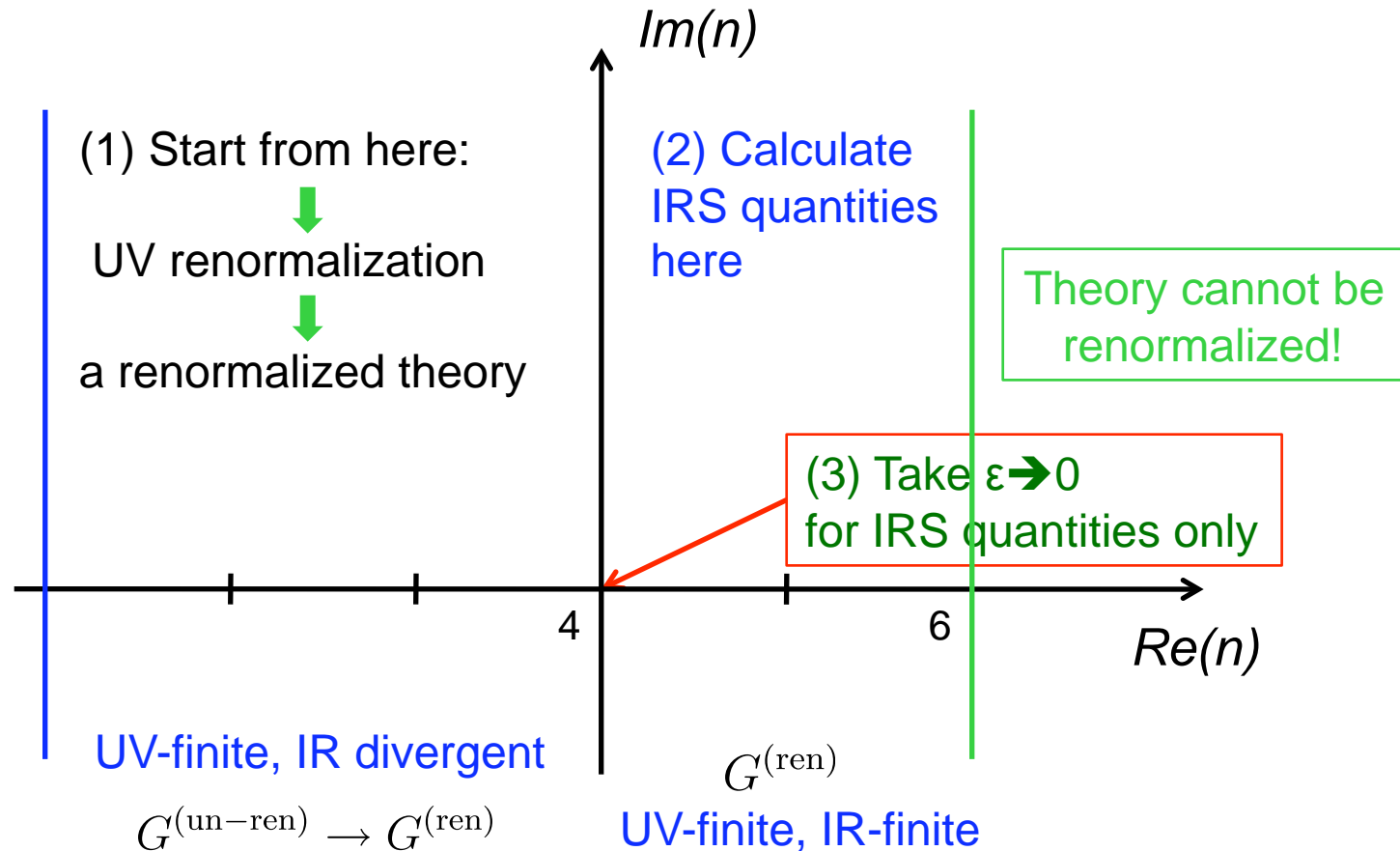
□ Gluon mass regulator:

- ✧ Real:
$$\sigma_{3,m_g}^{(1)} = \sigma_2^{(0)} \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) \left[2 \ln^2 \left(\frac{Q}{m_g} \right) - 3 \ln \left(\frac{Q}{m_g} \right) - \frac{\pi^2}{6} - \frac{5}{2} \right]$$
- ✧ Virtual:
$$\sigma_{2,m_g}^{(1)} = \sigma_2^{(0)} \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) \left[-2 \ln^2 \left(\frac{Q}{m_g} \right) + 3 \ln \left(\frac{Q}{m_g} \right) + \frac{\pi^2}{6} - \frac{7}{4} \right]$$
- ✧ Total:
$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,m_g}^{(1)} + \sigma_{2,m_g}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)} \left[1 + \frac{\alpha_s}{\pi} \right] + O(\alpha_s^2)$$

No m_g dependence!

How dimensional regularization works?

□ Complex n -dimensional space: $\int d^n k F(k, Q)$



Next-to-Leading-Order Contribution - III

□ Dimensional regulator:

✧ Real: $\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) \left(\frac{4\pi\mu^2}{Q^2} \right)^\varepsilon \left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)} \right] \left[\frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon} + \frac{19}{4} \right]$

✧ Virtual:

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) \left(\frac{4\pi\mu^2}{Q^2} \right)^\varepsilon \left[\frac{\Gamma(1-\varepsilon)^2 \Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)} \right] \left[-\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} + \frac{\pi^2}{2} - 4 \right]$$

✧ NLO: $\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)} \left[\frac{\alpha_s}{\pi} + O(\varepsilon) \right]$ No ε dependence!

✧ Total: $\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)} \left[1 + \frac{\alpha_s}{\pi} \right] + O(\alpha_s^2)$

□ Lesson:

σ^{tot} is independent of the choice of IR and CO regularization

σ^{tot} is Infrared Safe!

Jets in e^+e^- - Trace of Partons

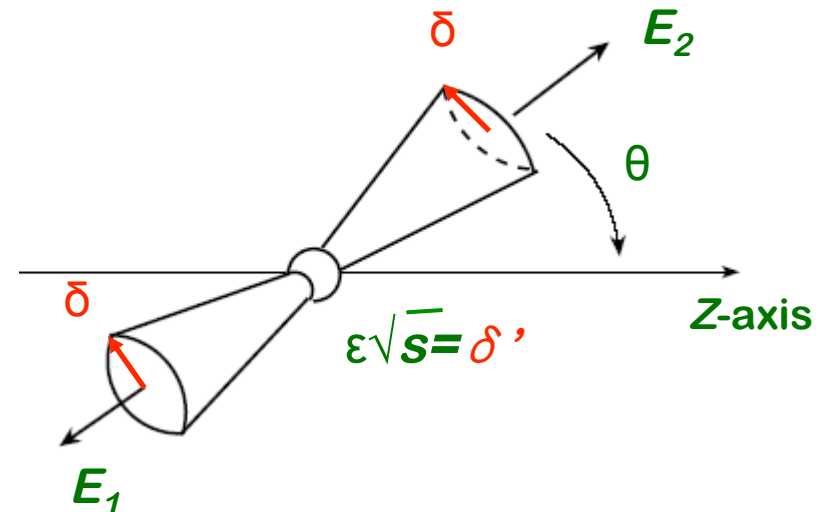
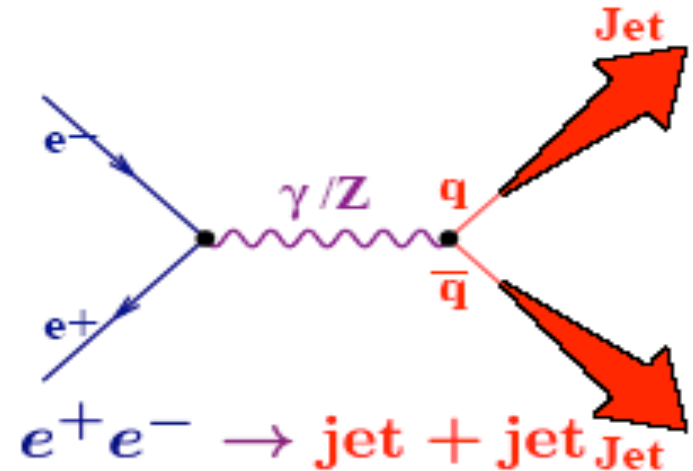
□ Jets – Inclusive x-section with a limited phase-space

□ Q: will IR cancellation be completed?

✧ Leading partons are moving away from each other

✧ Soft gluon interactions should not change the direction of an energetic parton → a “jet”
– “trace” of a parton

□ Jet algorithm

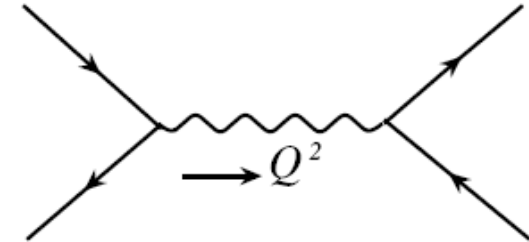


Sterman-Weinberg Jet

Two-jets Cross Section in e^+e^- Collisions

□ Parton-Model = Born term in QCD:

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8} \sigma_0 (1 + \cos^2 \theta)$$



□ Two-jet in pQCD:

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8} \sigma_0 (1 + \cos^2 \theta) \left(1 + \sum_{n=1} C_n \left(\frac{\alpha_s}{\pi} \right)^n \right)$$

with $C_n = C_n(\delta)$

□ Stermen-Weinberg jet:

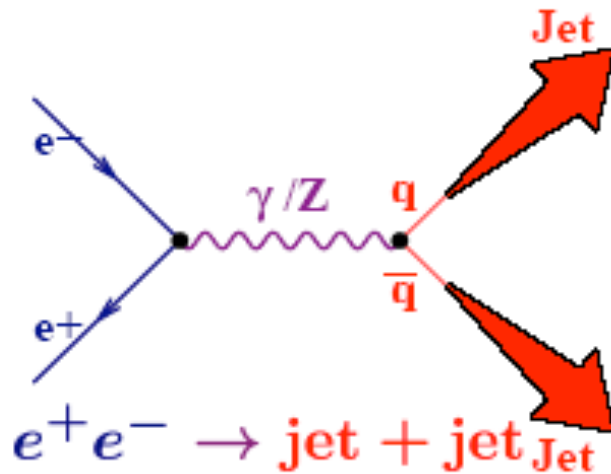
$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8} \sigma_0 (1 + \cos^2 \theta) \left[1 - \frac{4}{3} \frac{\alpha_s}{\pi} \left(4 \ln(\delta) \ln(\delta') + 3 \ln(\delta) + \frac{\pi^2}{3} + \frac{5}{2} \right) \right]$$

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \quad \text{as } Q \rightarrow \infty$$

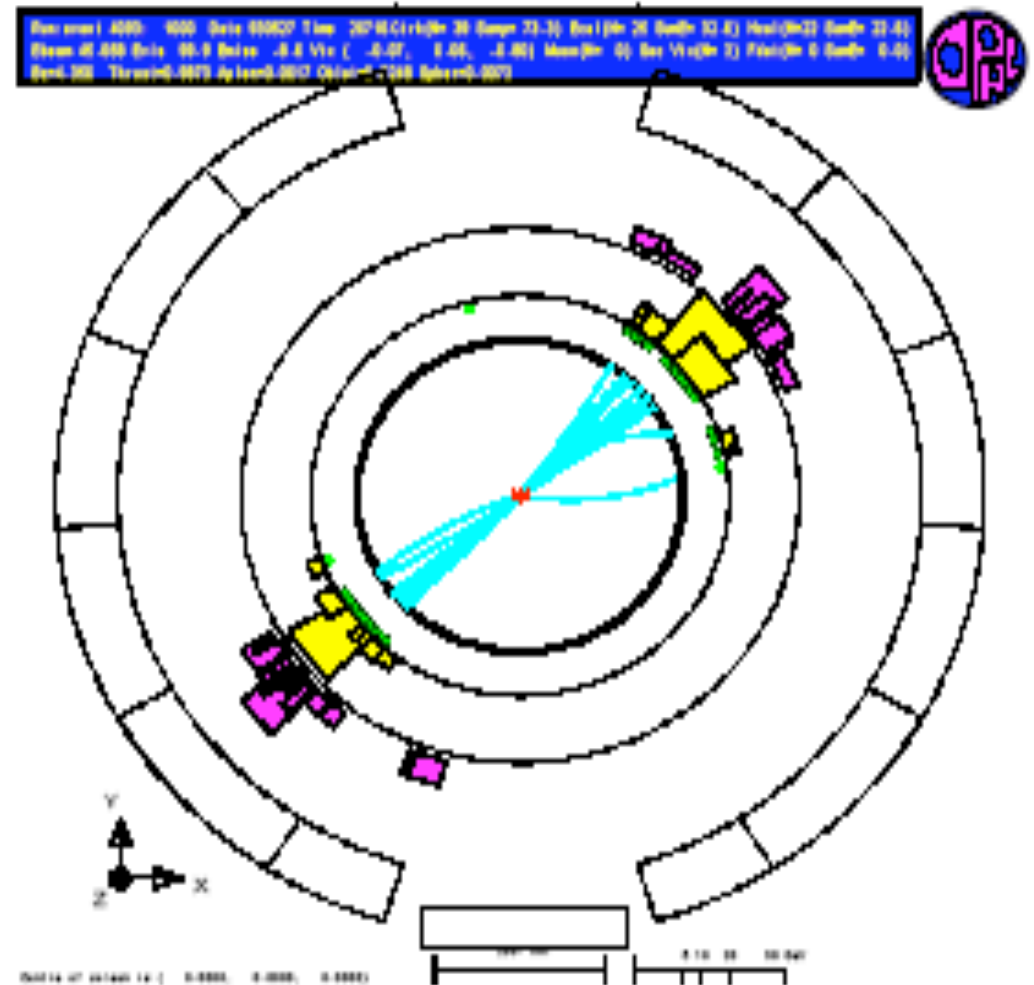
A Clean Two-jet Event

Lowest order ($\mathcal{O}(\alpha^2 \alpha_s^0)$):

LEP ($\sqrt{s} = 90 - 205 \text{ GeV}$)

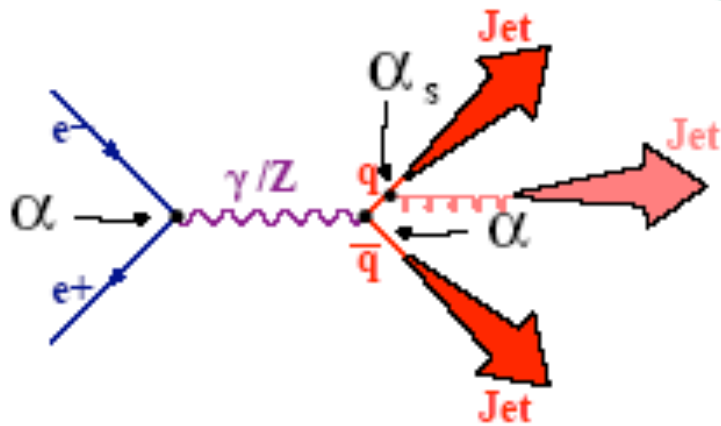


A clean trace of two
partons – a pair of
quark and antiquark



Discovery of a Gluon Jet

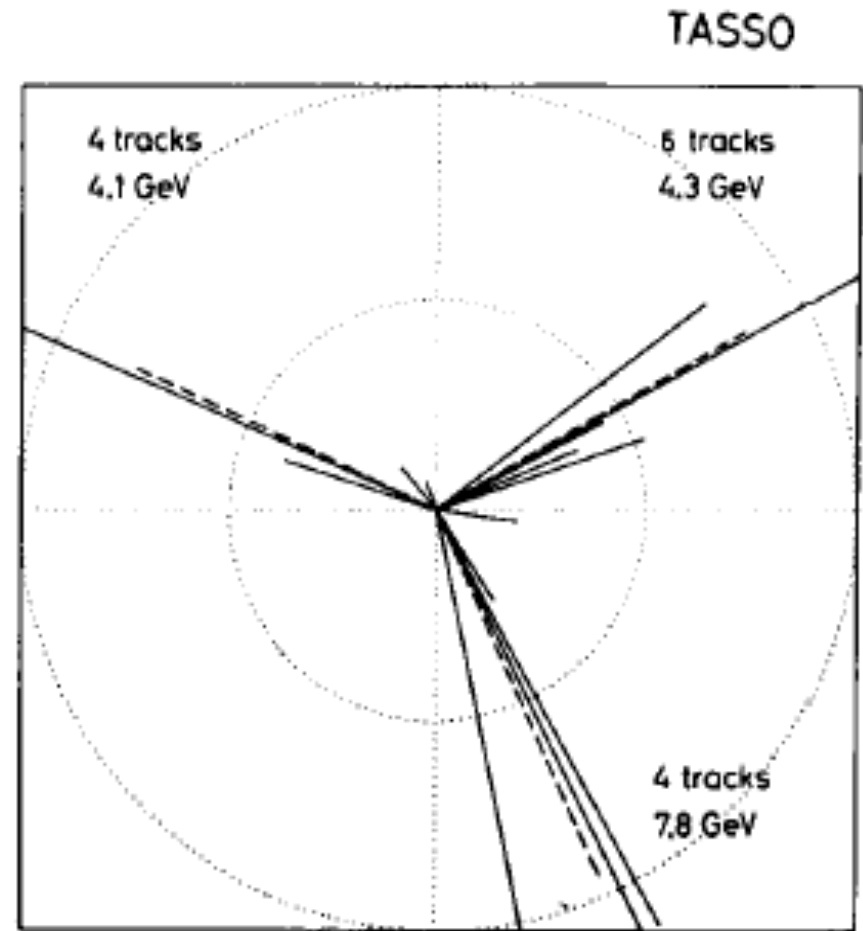
First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

PETRA e^+e^- storage ring at DESY:

$E_{\text{c.m.}} \gtrsim 15 \text{ GeV}$



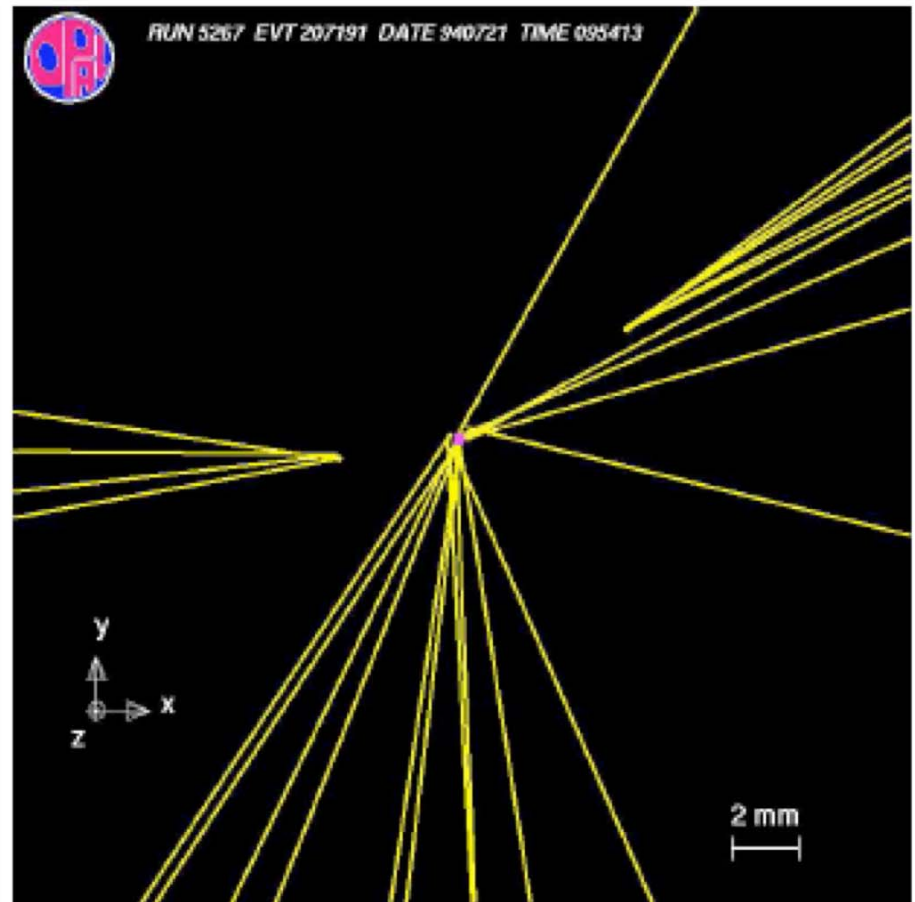
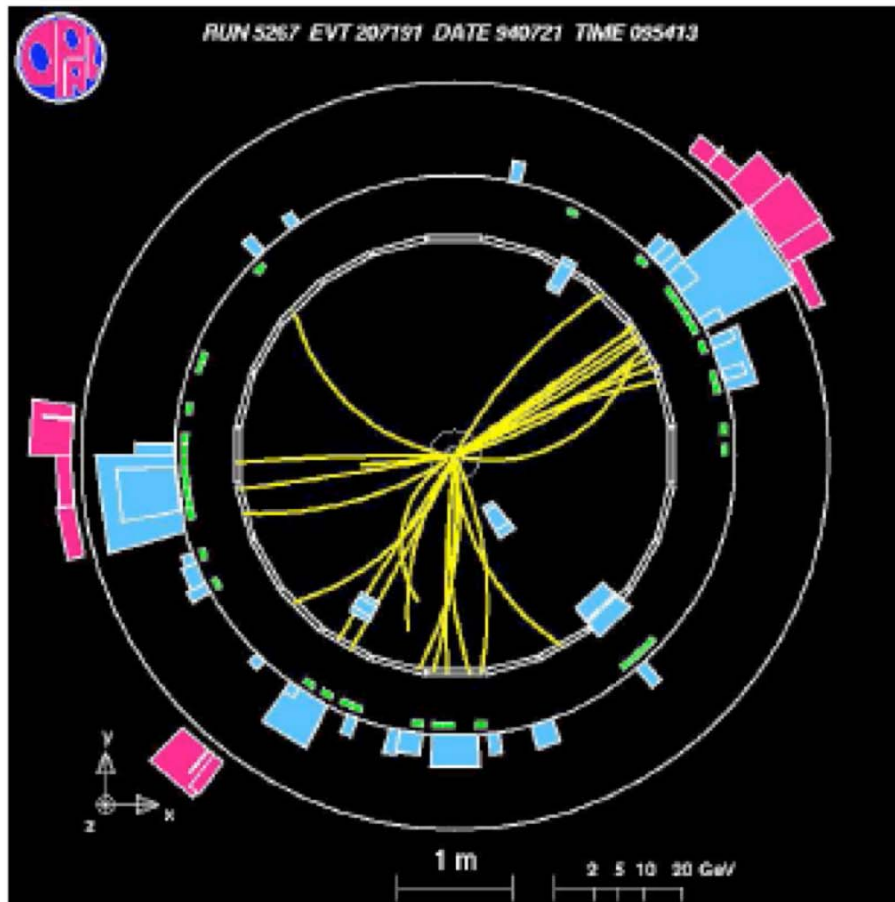
TASSO Collab., Phys. Lett. B86 (1979) 243

MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged Three-jet Event from LEP



↑
Gluon Jet

Basics of jet finding algorithms

□ Recombination jet algorithms:

— almost universal choice at e+e- colliders

- Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$

→ Combine the particle pair (i, j) with the smallest y_{ij} :

$$(i, j) \rightarrow k$$

E scheme : $p_k = p_i + p_j \rightarrow$ massive jets

E₀ scheme : $E_k = E_i + E_j$
 $\vec{p}_k = \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|} E_k \rightarrow$ massless jets

- Iterate until all remaining pairs satisfy $y_{ij} > y_{cut}$

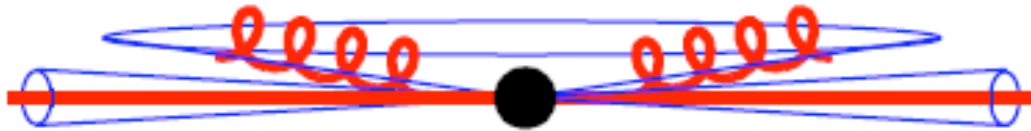
The JADE Jet Finder

[JADE Collab., Z. Phys. C33 (1986) 23]

→ The original recombination jet finder:

- $M_{ij}^2 = 2E_i E_j (1 - \cos \theta_{ij}) \approx (\text{invariant mass})^2$
- Original version based on the E_0 scheme

Sometimes leads to the formation of “junk jets”



- Two-jet events with ≥ 2 soft, collinear gluons can be classified, unnaturally, as three-jet events
- Prevents re-summation techniques from being applied

The Durham k_T Jet Finder

[S. Catani et al., Phys. Lett. B269 (1991) 432]

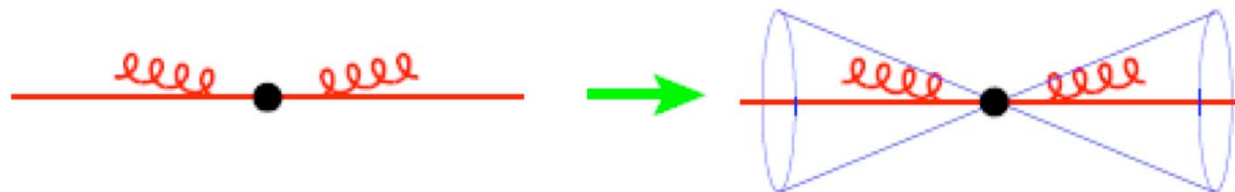
→ Introduced to reduce the problem of junk jets

- $M_{ij}^2 = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})$
- E scheme combination of particles: $(i, j) \rightarrow k$

→ Consider small emission angles θ_{ij} :

$$\begin{aligned} M_{ij}^2 &\approx 2 \min(E_i^2, E_j^2) [1 - (1 - \theta_{ij}^2/2 + \dots)] \\ &\approx \min(E_i^2, E_j^2) \theta_{ij}^2 \approx K_{\perp}^2 \\ &\quad (\text{min. transverse momenta of one particle w.r.t. the other}) \end{aligned}$$

→ Soft, colinear radiation is attached to the quark jet(s)

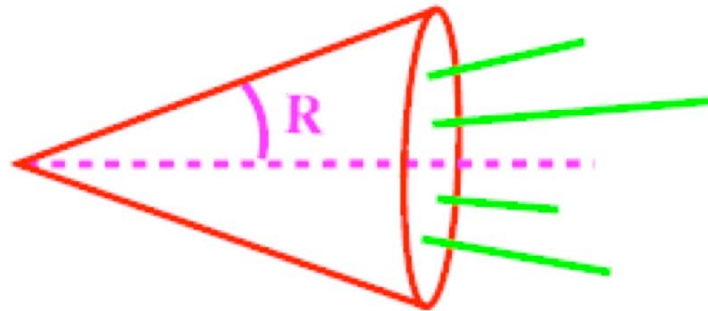


→ Permits re-summation

The Cone Jet Finder

CDF Collab., Phys. Rev. D45, 1448 (1992); OPAL Collab., Z. Phys. C63, 197 (1994)

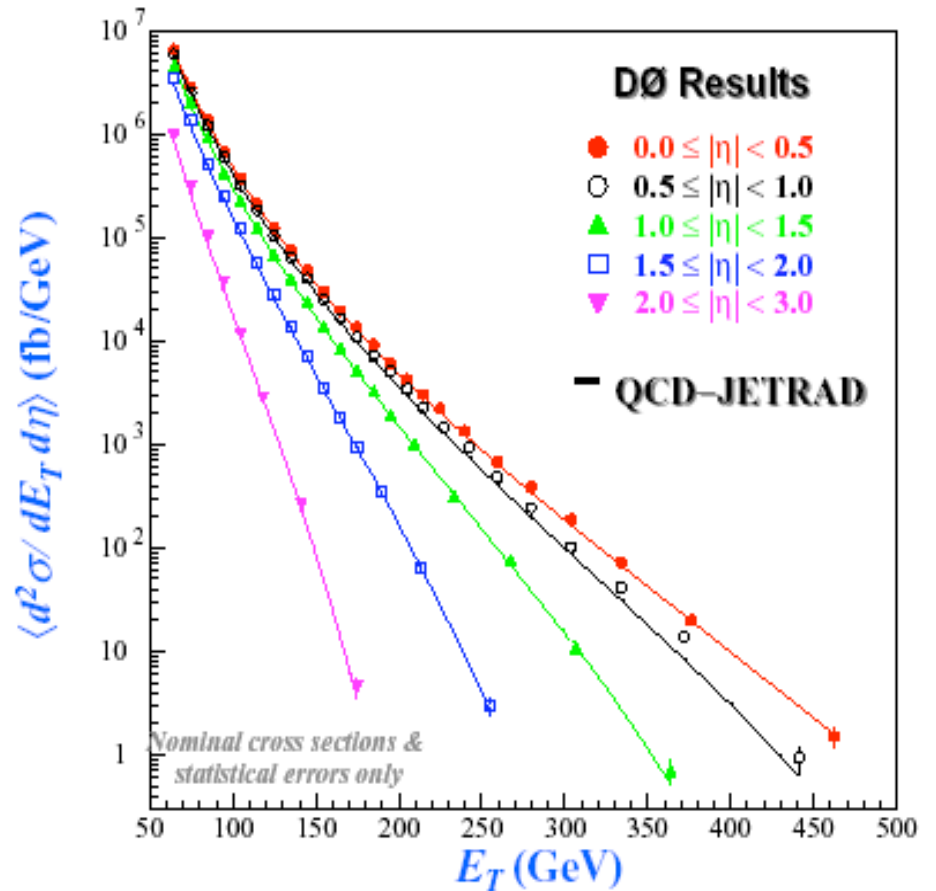
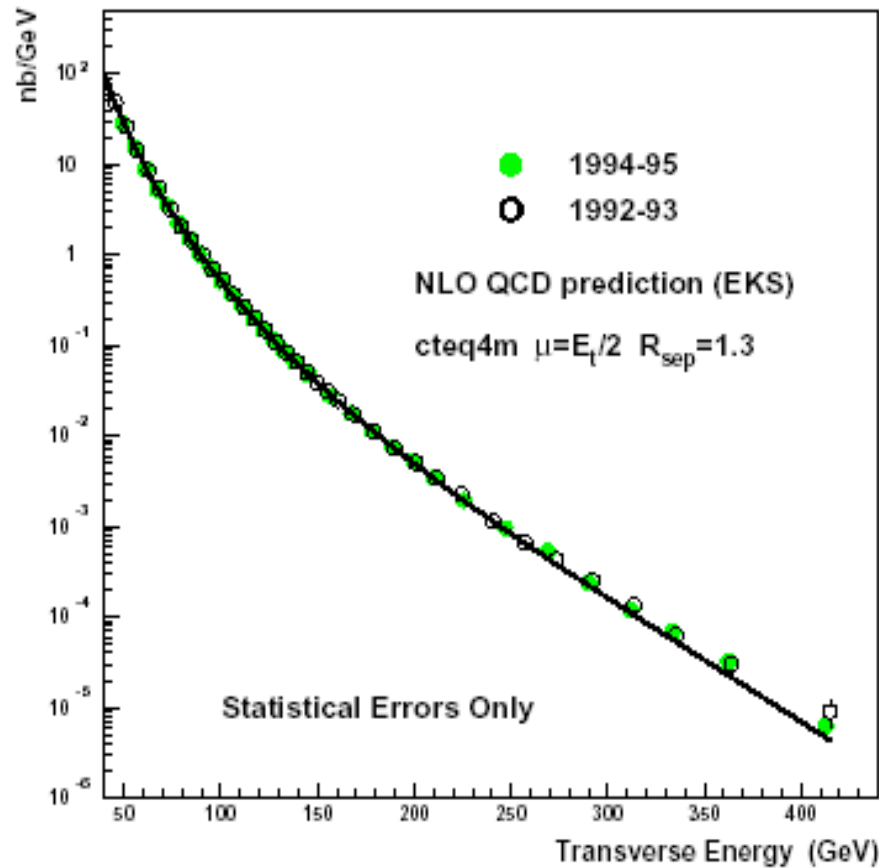
- Cluster particles within a cone of half angle R into a jet



- Require a minimum visible jet energy: $E_{\text{jet}} \geq \epsilon$
 - Two resolution parameters: R and ϵ , as opposed to re-combination algorithms which only have one (y_{cut})
- Eliminate or merge overlapping or redundant jets
 - Unlike recombination algorithms, not all particles in an event are necessarily assigned to a jet

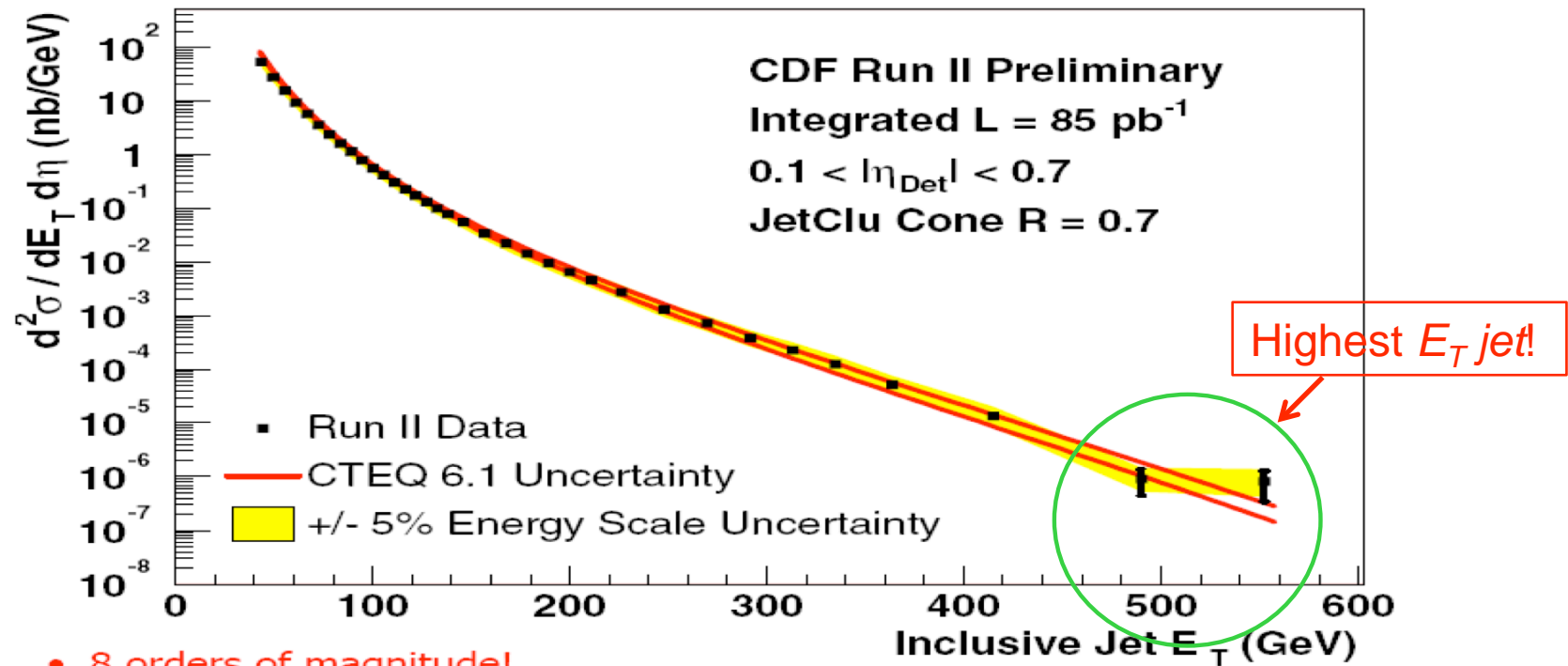
Inclusive Jet Cross Section at Tevatron

Run – 1b results

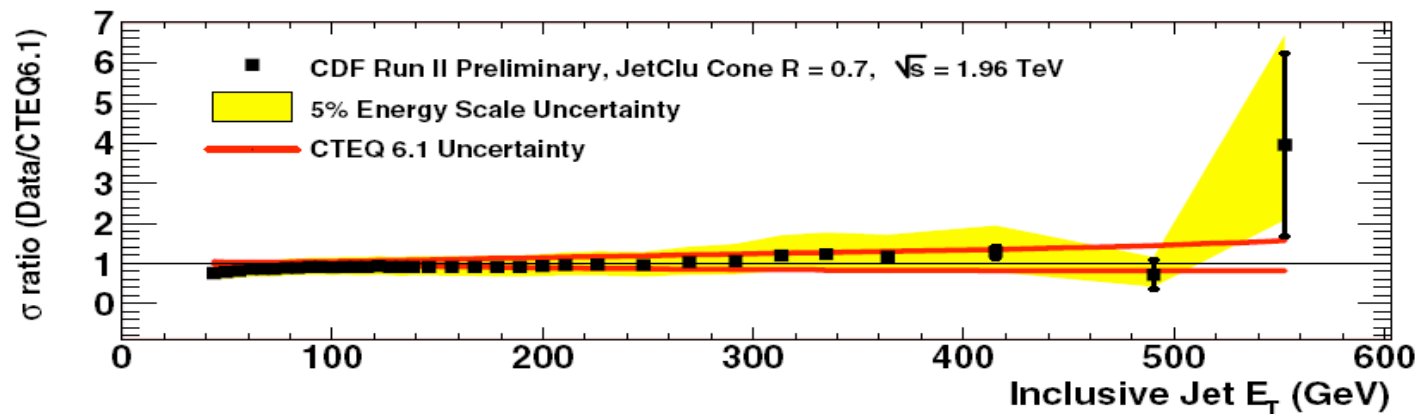


Data and Predictions span 7 orders of magnitude!

Prediction vs CDF RUN-II Data



- 8 orders of magnitude!



Infrared Safety for Jet Cross Sections

□ Jet cross section = inclusive cross section with a phase-space constraint

□ For any observable with a phase space constraint, Γ ,

$$\begin{aligned}
 d\sigma(\Gamma) &\equiv \frac{1}{2!} \int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2} \Gamma_2(k_1, k_2) \\
 &+ \frac{1}{3!} \int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3} \Gamma_3(k_1, k_2, k_3) \\
 &+ \dots \\
 &+ \frac{1}{n!} \int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n} \Gamma_n(k_1, k_2, \dots, k_n) + \dots
 \end{aligned}$$

where $\Gamma_n(k_1, k_2, \dots, k_n)$
are constraint functions
and invariant under
Interchange of n-particles

□ Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}(k_1, k_2, \dots, (1-\lambda)k_n^\mu, k_{n+1}^\mu = \lambda k_n^\mu) = \Gamma_n(k_1, k_2, \dots, k_n^\mu)$$

with $0 \leq \lambda \leq 1$

Physics Meaning of Infrared Safety

□ Conditions for IRS:

$$\Gamma_{n+1}(k_1, k_2, \dots, (1-\lambda)k_n^\mu, k_{n+1}^\mu = \lambda k_n^\mu) = \Gamma_n(k_1, k_2, \dots, k_n^\mu)$$

with $0 \leq \lambda \leq 1$

□ Physical meaning:

Measurement cannot distinguish a state
with an additional zero/collinear momentum parton
from a state without the parton



□ Total cross section = a special case of jet cross section:

$$\Gamma_n(k_1, k_2, \dots, k_n) = 1 \text{ for all } n \Rightarrow \sigma^{(\text{tot})}$$

Thrust Distribution

□ Thrust axis: \vec{u}

$$T_n(p_1^\mu, p_2^\mu, \dots, p_n^\mu) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^n \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^n |\vec{p}_i|} \right)$$



□ Phase space constraint:

$$\Gamma_n(p_1^\mu, p_2^\mu, \dots, p_n^\mu) = \delta(T - T_n(p_1^\mu, p_2^\mu, \dots, p_n^\mu))$$

✧ Contribution from $p=0$ particles drops out the sum

✧ Replace two collinear particles by one particle does not change the thrust

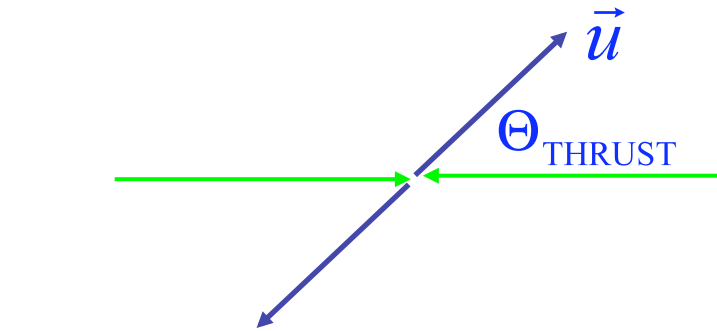
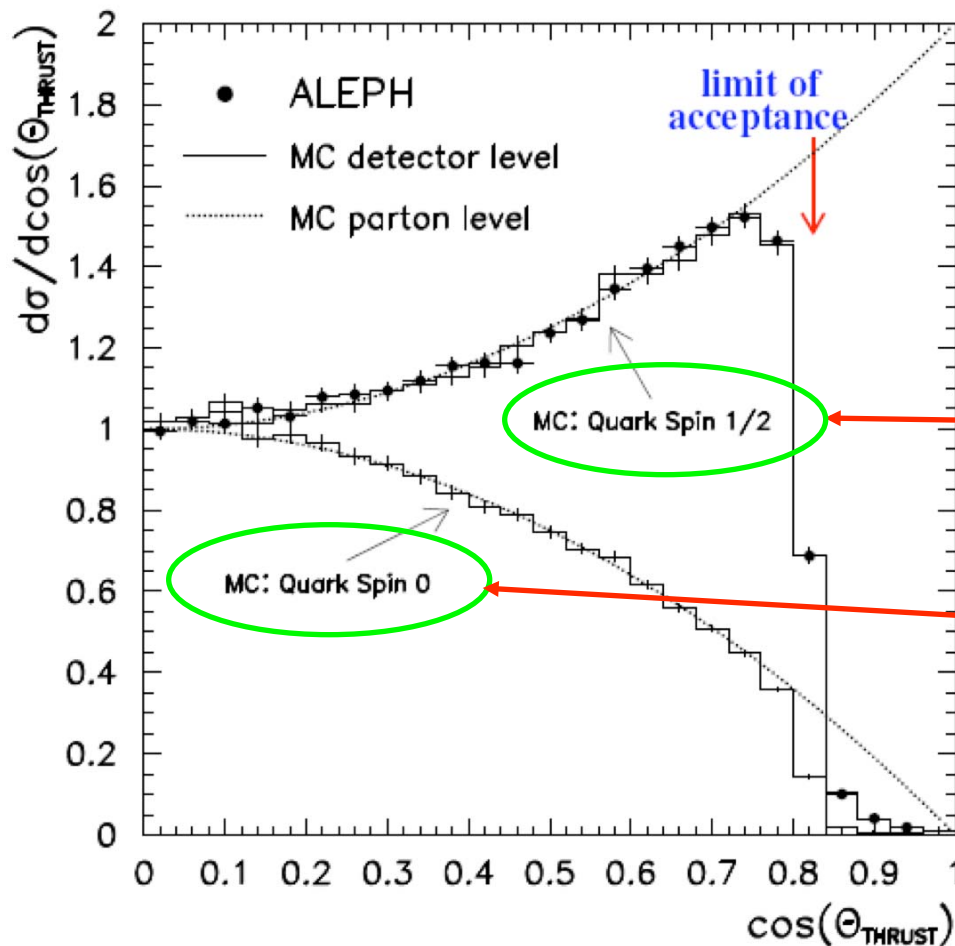
$$|(1 - \lambda) \vec{p}_n \cdot \vec{u}| + |\lambda \vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and
$$|(1 - \lambda) \vec{p}_n| + |\lambda \vec{p}_n| = |\vec{p}_n|$$

Another Test of Quark Spin

□ Angle between the thrust axis and the beam axis:

[ALEPH Collab., Phys. Rep. 294 (1998) 1]



At LO:

$$1 + \cos^2 \Theta_{\text{THRUST}}$$

$$1 - \cos^2 \Theta_{\text{THRUST}}$$

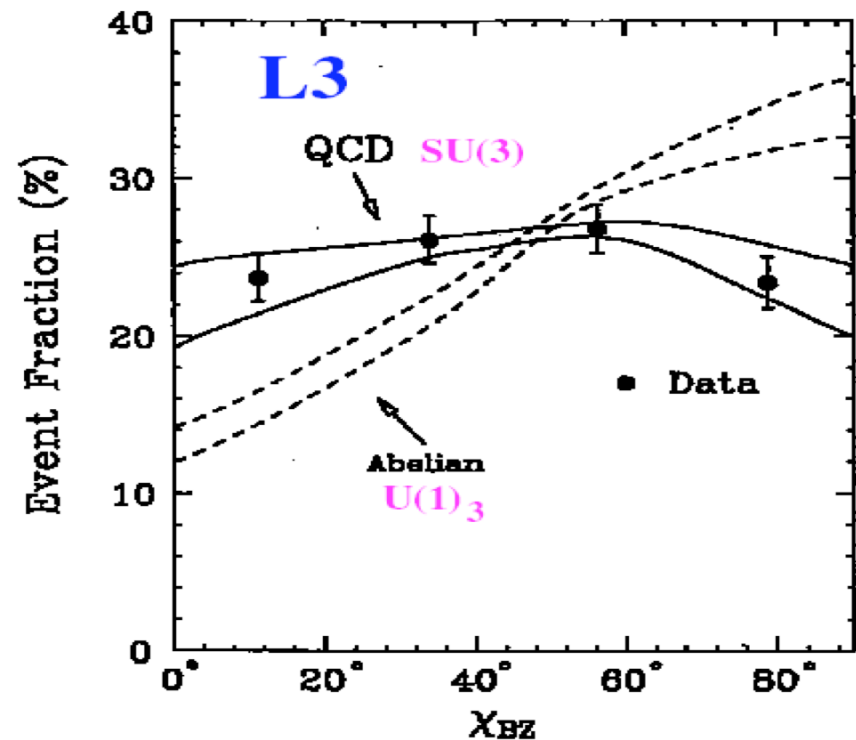
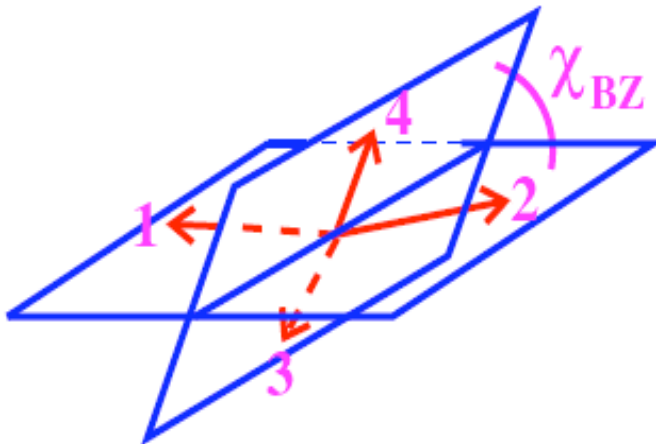
Another Test of SU(3) Color

□ Select 4-jet events: $E_1 > E_2 > E_3 > E_4$

jet-3 and jet-4 are more likely from radiation



□ Bengtsson-Zerwas angle:



Question

Does perturbative QCD work for cross sections with identified hadrons?

Facts:

Typical hadronic scale: $1/R \sim 1 \text{ fm}^{-1} \sim \Lambda_{\text{QCD}}$

Energy exchange in hard collisions: $Q \gg \Lambda_{\text{QCD}}$

pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

Answer:

Perturbative QCD factorization

See you next time!