# Quantum Chromodynamics (QCD)
# and
# Physics of the strong interaction


Name: Jianwei Qiu (邱建伟)

Office: Rm A402 – 实验物理中心

Phone: 010-88236061

E-mail: jwq@iastate.edu


Lecture: Mon – Wed – Fri
10:00-11:40AM

Location: B326, Main Building

# Lecture Plan

1. Introduction and review
2. Quark model
3. Fundamentals of QCD
4. QCD in $e^+e^-$ annihilation
5. QCD in lepton-hadron collisions
6. QCD in hadron-hadron collisions
7. …

# Fundamental Forces

## ❑ Known "fundamental" interactions:

AdS/CFT

"Strong" – QCD

Weak – Fermi's theory, …

Electromagnetic – QED

…

Gravity – GR, SuperGravity, String, …

Electro-Weak

Standard "Model"

## ❑ Status:

- ✧ QED is the best tested, but, can only be an effective theory
- ✧ Electro-weak is only tested at low energy (as an effective theory)
- ✧ GR is successful, what is the theory for quantum gravity?
- ✧ QCD is successful at short distance, but, connection to hadrons?

# Quantum Chromodynamics (QCD)

Quantum Chromodynamics (QCD – 量子色动力学) is a quantum field theory of quarks and gluons

❑ Fields: $\psi_i^f(x)$

Quark fields: spin-½ Dirac fermion (like electron)
Color triplet: $i = 1, 2, 3 = N_c$
Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$

Gluon fields: spin-1 vector field (like photon)
Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

❑ QCD Lagrangian density:

$$\begin{aligned}\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f &\left[ (i\partial_\mu \delta_{ij} - g A_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f \\ &- \frac{1}{4}\left[ \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2 \\ &+ \text{gauge fixing} + \text{ghost terms}\end{aligned}$$

❑ Color matrices:

$$[t_a, t_b] = i\, C_{abc}\, t_c$$

Generators for the fundamental representation of SU3 color

# Questions

- Why do we need a quantum field theory?

  A quick review from classical mechanics

  to quantum mechanics

  to relativistic quantum mechanics

  (classical quantum field theory)

  to quantum field theory

- Why do we need quarks, gluons, and colors?

  Hadrons are NOT fundamental particles

  A quick review of the Quark Model

- Have we seen quarks and gluons?

  Jets – the trace of quarks and gluons

- How quarks and gluons make up the hadrons?

  Not sure!  Mass, spin, …    A lot of work are waiting to be done!!!

# Classical Mechanics

Dynamics system of finite physical observables

- Variables and degrees of freedom:

$$q_i(t) \text{ with } i = 1, 2, ..., N \text{ and } N \text{ is finite!}$$

- Equation of motion: $$m_i \frac{d^2}{dt^2} q_i(t) = F_i(q_j, \dot{q}_j, t)$$

- Boundary conditions: $$q_i(t_0) \text{ and } \dot{q}_i(t_0) \text{ with } i = 1, 2, ..., N$$

(2nd order differential equation)

- Predictive power:

Time-dependence of all physical observables

$$q_i(t) \text{ with } i = 1, 2, ..., N$$

# Lagrangian Mechanics

Another way to do the classical mechanics

- Variables and degrees of freedom:

$$q_i(t) \text{ with } i = 1, 2, ..., N \text{ and } N \text{ is finite!}$$

- Lagrangian: $L = L(q_i(t), \dot{q}_i(t))$

- Equation of motion (Lagrange equation):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

One for each independent degree of freedom (or each *i*)

- Advantages:
  - Treat all d.o.f equivalently and systematically
  - More systematic way to find all forces
  - Better to deal with the symmetries and conservations
  - …

# Hamiltonian Approach

## Another way to do the classical mechanics

- Variables and degrees of freedom:

  Generalized position: $q_i(t)$ with $i = 1, 2, ..., N$ and $N$ is finite!

  Generalized momentum: $p_i(t) = \dfrac{\partial L}{\partial \dot{q}_i}$

- Hamiltonian: $H(q_i, p_i) = \sum_i p_i\, \dot{q}_i - L$

- Equation of motion (Hamilton's equations):

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}, \text{ and } \dot{q}_i = \frac{\partial H}{\partial p_i}$$

- Advantages:

  - 1st order differential equations
  - explicit connection between the symmetries and conservations
  - …

# Non-relativistic Quantum Mechanics

## 1st quantization

- Classical variables: $q_i(t)$ and $p_i(t)$
- Degrees of freedom: same as classical $i = 1, 2, ..., N$
- Hamiltonian: $$H(q_i, p_i) = \sum_i p_i\, \dot{q}_i - L$$
- Quantization – assumption:
  - classical variables to operators – quantum Hamiltonian: $\hat{H}(\hat{q}_i, \hat{p}_i)$

$$\hat{q}_i(t) \text{ and } \hat{p}_i(t) \equiv -i\frac{\partial}{\partial q_i}$$

  - quantization conditions (equal time):

(the "hat" was neglected)

$$[q_i(t), p_j(t)] = i\delta_{ij}$$

$$[q_i(t), q_j(t)] = [p_i(t), p_j(t)] = 0$$

# Equation of Motion – Time Dependence

- Classical case:

  Dynamical equation of physical variables: $m_i \dot{p}_i = F(q_j, p_j, m_j, t)$

- Quantum mechanics:

  Dynamical equation of the wave function or quantum operators:

  ✧ Schrödinger equation – wave equation: $H\psi(\vec{x},t) = -i\frac{\partial}{\partial t}\psi(\vec{x},t)$

  Wave function: $\psi(\vec{x},t)$

  ❖ a property of the whole system

  ❖ not a direct physical observable/variable

  ❖ normalization condition: $\int \psi^*(x,t)\psi(x,t)dx = 1$

  ✧ Heisenberg equation – operator equation: $i\frac{d}{dt}\hat{O} = [\hat{O},\hat{H}]$

  Quantum operator: $\hat{O} = \hat{p}_i, \hat{H}, \ldots$

# Physical Observables

- Classical case:

Functions of classical variables: $O(t) = F(q_i(t), p_i(t))$

Dynamics $\Longleftrightarrow$ the time-dependence of: $q_i(t)$ and $p_i(t)$

Examples: $K(t) = \frac{1}{2m}[p_i(t)]^2$

- Quantum mechanics:

Expectation values of Hermitian operators:

$$O(t) = \langle \hat{O}(x,t) \rangle \equiv \int \psi^*(x,t)\hat{O}(x,t)\psi(x,t)dx$$

Hermitian operator: $\hat{O}(q_i, p_i) = \hat{O}^\dagger(q_i, p_i)$

Dynamics $\Longleftrightarrow$ the time-dependence of: $\psi(\vec{x}, t)$

Examples: $q_i, p_i = -i\frac{\partial}{\partial q_i}$ $\quad q_i = q_i^\dagger,\ K = \frac{p_i^2}{2m} = K^\dagger$

# Relativistic Quantum Mechanics

- Relativity: $E^2 = p^2 + m^2 \text{ with } c = \hbar = 1$
- Quantization: $E \to \hat{H},\ p \to \hat{p},\ \Rightarrow\ \hat{H}^2 = \hat{p}^2 + m^2$
- Equation of motion:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi(x,t) = 0$$ – Klein-Gordon Equation

Compare to Schrödinger equation – linear in $\hat{H} = \pm\sqrt{\hat{p}^2 + m^2}$

$$(i\gamma^\mu \partial_\mu - m)\,\psi(x,t) = 0$$ – Dirac Equation

- Physical meaning of $\phi(x,t)$ and $\psi(x,t)$ :
  - ✧ Called as relativistic fields – classical functions of space and time – analog to the wave functions of Schrödinger Eq. of non-rel. Q.M.
  - ✧ $\psi(x,t)$ is a 4X1 matrix – each element is a function of space and time – including both positive and negative energy states, antiparticle
  - ✧ Equation of motion $\Longrightarrow$ the propagator theory, … – Bjorken & Drell

# Classical Field Theory

- Relativistic quantum mechanics = classical field theory:
- Variables – classical fields: $\phi(x),\ \psi(x)$
- Lagrangian density of fields: $\mathcal{L} = \mathcal{L}(\phi(x,t), \partial_\mu \phi(x,t))$
- Lagrangian: $L(t) = \int d^3x\, \mathcal{L}(\phi(x,t), \partial_\mu \phi(x,t))$
- Action: $S = \int dt L(t)$
- Equation of motion – Euler-Lagrange's equation:

$$\delta S = 0 \quad \Longrightarrow \qquad \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

- Physical observables – Hermitian operators:

$$O(t) = \langle \hat{O}(x,t) \rangle \equiv \int \psi^*(x,t)\, \hat{O}(x,t)\, \psi(x,t)\, dx$$

- Time-dependence:

Not independent from the spatial components – Lorentz invariance

# Hamiltonian Approach

for classical field theory

- Variables and degrees of freedom:

Generalized fields: $\phi(x)$

Generalized "momentum": $\pi(x) = \dfrac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}$

- Hamiltonian density: $\mathcal{H} = \pi(x)\dot{\phi}(x) - \mathcal{L}(\phi(x), \dot{\phi}(x))$

- Hamiltonian: $H = \int \mathcal{H}(\phi(x), \pi(x))\, dx$

- Equation of motion (Hamilton equations):

$$\dot{\pi}(x) = -\frac{\partial \mathcal{H}}{\partial \phi(x)} \text{ and } \dot{\phi}(x) = \frac{\partial \mathcal{H}}{\partial \pi(x)}$$

- Compare to classical mechanics:

$$q_i \longrightarrow \phi(x),\ p_i \longrightarrow \pi(x),\ H(q_i, p_i) \longrightarrow \mathcal{H}(\phi(x), \pi(x)),\ \sum_i \longrightarrow \int dx$$

# Quantum Field Theory

## $2^{nd}$ quantization

- **Fields:** $\phi(x,t)$ and $\psi(x,t)$ $\Longrightarrow$ operators

- **Canonical quantization – equal time:**

Scalar fields:

$$[\phi(\vec{x},t),\pi(\vec{y},t)] = i\delta^3(\vec{x}-\vec{y})$$
$$[\phi(\vec{x},t),\phi(\vec{y},t)] = [\pi(\vec{x},t),\pi(\vec{y},t)] = 0$$

Dirac fields:

$$\{\psi_a(\vec{x},t),\psi_b^\dagger(\vec{y},t)\} = i\delta^3(\vec{x}-\vec{y})\delta_{ab}$$
$$\{\psi_a(\vec{x},t),\psi_b(\vec{y},t)\} = \{\psi_a^\dagger(\vec{x},t),\psi_b^\dagger(\vec{y},t)\} = 0$$

- **Physical observables:**

Expectation value of a hermitian operator is still an operator:

$$\hat{O}(t) = \int d^3x\,\psi^\dagger(x,t)\,[\hat{O}(x,t)]\,\psi(x,t)$$

Physical observable = eigenvalue of $\hat{O}(t)$ on a specific state:

$$\hat{O}(t)|p,s\rangle = O(t)|p,s\rangle \quad\Longleftrightarrow\quad O(t) = \langle p,s|\,\hat{O}(t)\,|p,s\rangle$$

# Quantum Mechanics vs Quantum Field Theory

- Variables:

QM: $q_i(t), \quad p_i(t)$ with $i = 1, 2, ..., N$

QFT: $\phi(\vec{x}, t), \quad \pi(\vec{x}, t)$ with a continuous variable $\vec{x}$

- Quantization:

QM:
$$[q_i(t), p_j(t)] = i\delta_{ij}$$
$$[q_i(t), q_j(t)] = [p_i(t), p_j(t)] = 0$$

QFT:
$$[\phi(\vec{x}, t), \pi(\vec{y}, t)] = i\delta^3(\vec{x} - \vec{y})$$
$$[\phi(\vec{x}, t), \phi(\vec{y}, t)] = [\pi(\vec{x}, t), \pi(\vec{y}, t)] = 0$$

- Physical observables:

QM: $$O(t) = \langle \hat{O}(x,t) \rangle \equiv \int \psi^*(x,t)\hat{O}(x,t)\psi(x,t)dx$$

QFT: $$O(t) = \langle p, s | \hat{O}(t) | p, s \rangle$$

- Symmetry of physical observables in QFT:

Requires the symmetry of Lagrangian AND symmetry of the state!!!

# Hadrons

- Protons, neutrons, and pions:

$$p\begin{cases} m = 938.3 \text{ MeV} \\ S = 1/2 \\ I_3 = +1/2 \end{cases} \qquad n\begin{cases} m = 939.6 \text{ MeV} \\ S = 1/2 \\ I_3 = -1/2 \end{cases}$$

Isospin doublet $N = \begin{pmatrix} p \\ n \end{pmatrix}$

$$\pi^{\pm}\begin{cases} m = 139.6 \text{ MeV} \\ S = 0 \\ I_3 = \pm 1 \end{cases} \qquad \pi^{0}\begin{cases} m = 135.0 \text{ MeV} \\ S = 0 \\ I_3 = 0 \end{cases}$$

Isospin triplet $\pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$

- "Historic" – $\pi$ as $N\overline{N}$ bound state:

$$\pi^+ = (p\bar{n}), \quad \pi^- = (n\bar{p}), \quad \pi^0 = \frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n})$$

Fermi and Yang, 1952; Nambu and Jona-Lasinio, 1960 (dynamics)

Yang-Mills theory, SU(2) Non-Abelian Gauge theory (1953)

# Hadrons are NOT Elementary

- Nucleons are not point-like spin-½ Dirac particles:

  Proton magnetic moment: $g_p \neq 2$

  Neutron magnetic moment: $g_n \neq 0$

- "Modern" – common constituents for $\pi, N$: Quarks

  ➡ **Quark Model** – Gell Mann, Zweig, 1964

- Quarks at point-like spin-½ Dirac particles:

$$u \begin{cases} Q = 2/3\,e \\ S = 1/2 \\ I_3 = +1/2 \end{cases} \qquad d \begin{cases} Q = -1/3\,e \\ S = 1/2 \\ I_3 = -1/2 \end{cases} \qquad s \begin{cases} Q = -1/3\,e \\ S = 1/2 \\ I_3 = 0 \end{cases}$$

$$\pi^+ = \left(u\bar{d}\right), \quad \pi^- = \left(d\bar{u}\right), \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

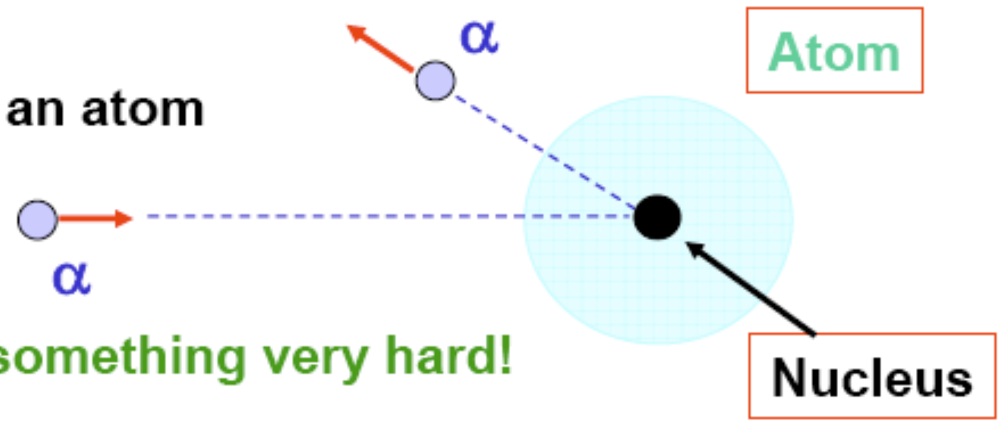
$$p = (uud), \quad n = (udd), \quad K^+ = (u\bar{s}), \ldots, \Delta^{++} = (uuu), \ldots$$

- How to "see" the quark and its dynamics, as well as the color?

# How to "See" the Substructure of a Nucleon?

- **Rutherford experiment:**

  – to see the substructure of an atom

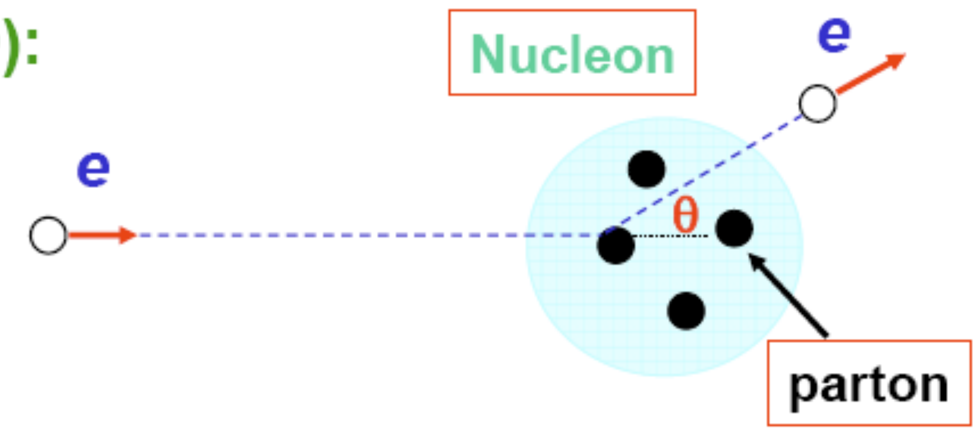



High energy α bounce off something very hard!

Discovery of nucleus inside an atom

- **SLAC experiment (1969):**

  Lepton-nucleon deeply inelastic scattering (DIS)



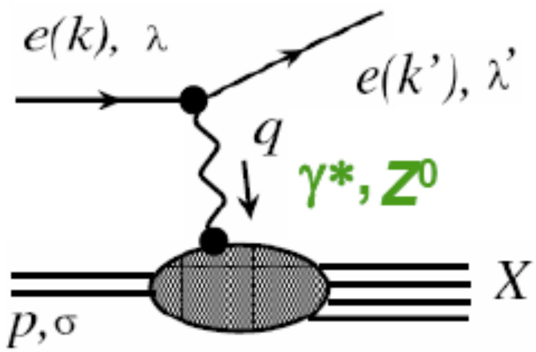

Scattering information on the θ-distribution

Discovery of the point-like spin-1/2 "partons"

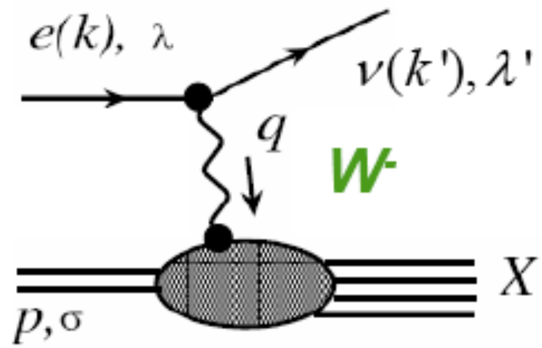
# Lepton-Hadron Deep Inelastic Scattering

- Process: $e(k,\lambda)+P(p,\sigma)\rightarrow e(k',\lambda')+X$

Neutral current (NC)



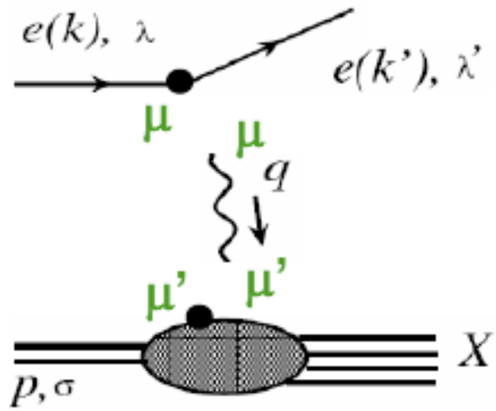

Charged current (CC)




- Kinematics:
  - **4-momentum transfer:** $Q^2=-q^2$
  - **Bjorken variable:** $x_B=\dfrac{Q^2}{2p\cdot q}$
  - **Squared CMS energy:** $s=(p+k)^2=\dfrac{Q^2}{x_B y}$
  - **Inelasticity:** $y=\dfrac{p\cdot q}{p\cdot k}$
  - **Final-state hadronic mass:** $W^2=(p+q)^2\approx\dfrac{Q^2}{x_B}(1-x_B)$

# General Analysis without QCD

- **Scattering amplitude:**




$$M(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_\mu\right]u_\lambda(k)$$

$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$

$$* \left\langle X\left|eJ_{\mu'}^{em}(0)\right|p,\sigma\right\rangle$$

- **Cross section:**

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|M(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3l_i}{(2\pi)^3 2E_i}\right]\frac{d^3k'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\Longrightarrow \quad \boxed{E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)}$$

Only unknown!

- **Leptonic tensor:**

– **known from QED**

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}\right)$$

# Hadronic Tensor and Structure Functions

- Hadronic tensor:

$$W_{\mu\nu}(q,p) = \frac{1}{4\pi}\left\{\frac{1}{2}\sum_{\sigma}\int d^4z\ e^{iq\cdot z}\ \left\langle p,\sigma\left|J^{\dagger}_{\mu}(z)J_{\nu}(0)\right|p,\sigma\right\rangle\right\}$$

EM current

- Structure functions:

  - **Parity invariance (EM current)** → $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
  - **Time-reversal invariance** → $W_{\mu\nu} = W^*_{\mu\nu}$ real
  - **Current conservation** → $q^{\mu}W_{\mu\nu} = q^{\nu}W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

**Reduced to two dimensionless scalar structure functions for spin-avgeraged DIS**

**Two more structure functions for spin-dependent DIS**

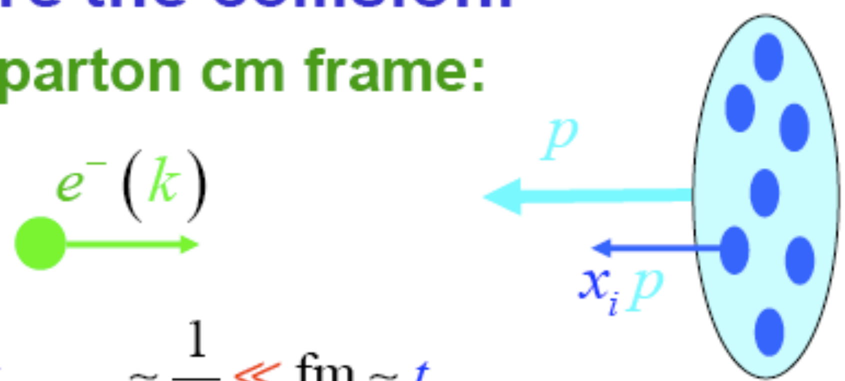
**Measure cross sections ⇔ extraction of structure functions**

# The Parton Model

Feynman, 1969, 1972

- **Before the collision:**

  in e⁻–parton cm frame:



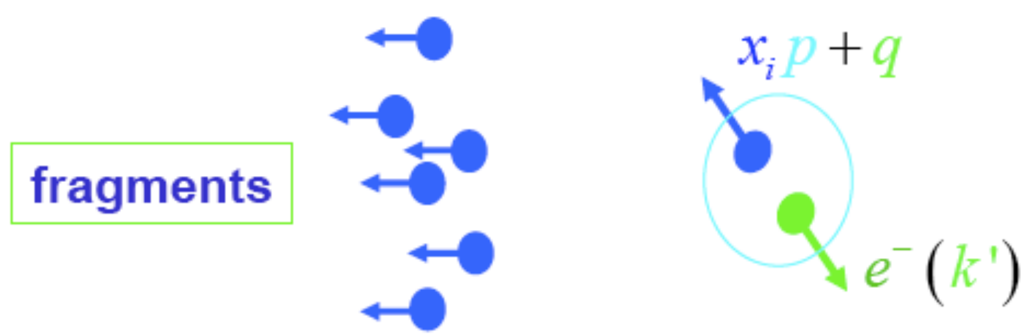

$$t_{collision} \sim \frac{1}{Q} \ll \text{fm} \sim t_{hadron}$$

$$0 \le x_i \le 1$$

$$\sum_i x_i = 1$$

Lorentz contracted
Time dilated
Effectively frozen

- **After the collision:**




$$(x_i p + q)^2 \approx 0$$

$$\left[ i.e., \ll |q^2| \right]$$

elastic collision

**"Deeply inelastic scattering"**

# Structure Functions in Parton Model

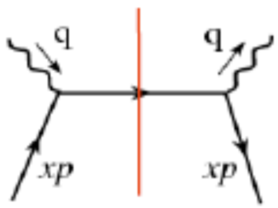
**Recall:**

$$\boxed{E'\frac{d\sigma_{eh}^{\text{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)}$$

**PM formula:**

$$W_{\mu\nu}(q,p) = \sum_{partons-f}\int_0^1 dx \left[\frac{1}{x}\hat{W}_{\mu\nu}^{\text{el}}(q,xp)\right]\phi_f(x)$$

$$\hat{W}_{\mu\nu}^{\text{el}}(q,xp) = \sum_f e_f^2 \frac{1}{4\pi}\frac{1}{2}\text{Tr}\left[\gamma_\mu\gamma\cdot(xp+q)\gamma_\nu\gamma\cdot(xp)\right](2\pi)\delta\left((xp+q)^2\right)$$




$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[e_f^2\,\frac{1}{2}\,\delta\left(1-\left(\frac{x_B}{x}\right)\right)\right]$$

$$+\frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)\left[e_f^2\; x\;\delta\left(1-\left(\frac{x_B}{x}\right)\right)\right]$$

$$\Longrightarrow\quad F_2(x_B,Q^2) = \sum_f e_f^2\; x_B\;\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

- ❑ **Callan-Gross Relation ⟶ spin ½ parton**
- ❑ **Bjorken scaling ⟶ $Q^2$-independent universal PDFs**

# Need a better Dynamical Theory!

- ❑ **Total momentum carried by the partons:**

$$F_q \equiv \sum_f \int_0^1 dx\, x\, \phi_f(x) \sim 0.5$$

**Missing momentum**

→ **Need particles not directly interact with photon (or EM charge)**

→ **the gluon?**

- ❑ **Scaling violation:**

→ $Q^2$**– dependence of structure functions?**

- ❑ **Are partons the same as the quarks?**

Feynman say: No! Gell-Mann say: Yes!

- ❑ **The birth of QCD:**

A combination of Quark Model and Yang-Mills non-Abelian gauge theory