Quantum Chromodynamics (QCD) and Physics of the strong interaction

(Lecture 4)

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Lecture: Mon – Wed – Fri

10:00-11:40AM

Location: B326, Main Building

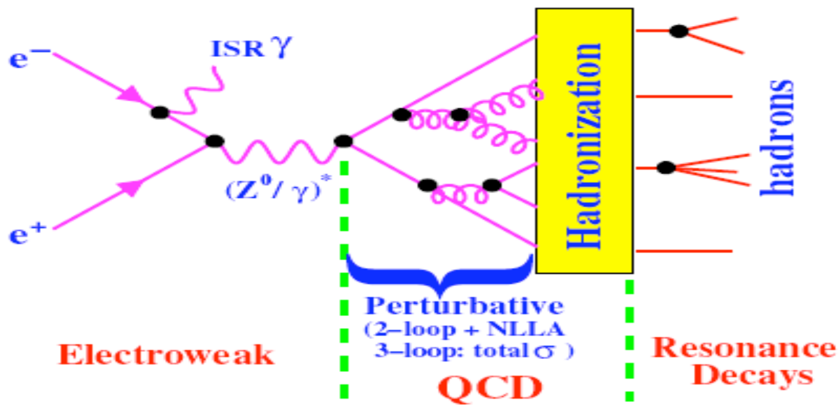
Review of Lecture Three

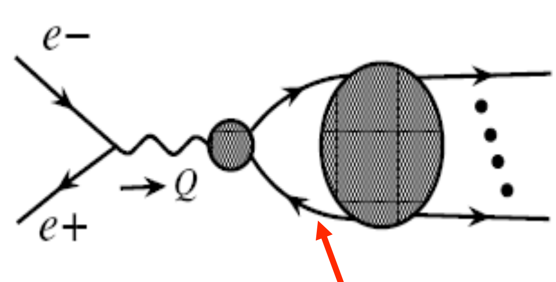
- Introduction of QCD Lagrangian
- UV divergence – Renormalization – Perturbative QCD
- Renormalization group and running coupling constant
- QCD Asymptotic Freedom – basis of pQCD
- Mass renormalization – massless theory – Infrared Safety!
- How to connect parton dynamics to hadron dynamics?
  - Hadron matrix elements of parton operators

    PQCD factorization, Effective field theory, …
  - Quark models, …
- Look for Infrared Safe Quantities! – e^+e^- total cross section

Purely Infrared Safe Cross Sections

- e+e- → hadron total cross section is infrared safe (IRS):





Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto \sum_n P_{e^+e^-\to n} = \sum_n \sum_m P_{e^+e^-\to m} P_{m\to n} = \sum_m P_{e^+e^-\to m} \sum_n P_{m\to n} = 1$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} \propto \sum_m P_{e^+e^-\to m}$$

Unitarity

$$\Rightarrow \quad \sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}$$

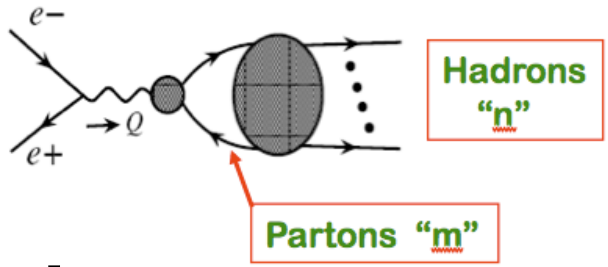
Finite in perturbation theory – KLN theorem

"Local" – of order of 1/Q

e^+e^- Total Cross Sections

- **On-shell approximation (m=2):**

$$\sigma^{\text{tot}}_{e^+e^-} = \frac{1}{2S}\left| \quad \right|^2$$



$$= \sum_{\text{hadrons}} \left[\int d^4k \, \hat{M}_{e^+e^-\to q\bar{q}}(Q,k) \, \hat{M}_{q\bar{q}\to\text{hadrons}}(k, 1/\text{fm}) \right]^2$$

On-shell approx. →

$$\approx \int \frac{d^3k}{2k_0} \hat{M}_{e^+e^-\to q\bar{q}}(Q,\hat{k}) \int \frac{d^3k'}{2k'_0} \hat{M}^*_{e^+e^-\to q\bar{q}}(Q,\hat{k}')$$

$$\times \sum_{\text{hadrons}} \int dk^2 \hat{M}_{q\bar{q}\to\text{hadrons}}(k, 1/\text{fm}) \int dk'^2 \hat{M}^*_{q\bar{q}\to\text{hadrons}}(k', 1/\text{fm}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

$$\approx \int \frac{d^3k}{2k_0} \hat{M}_{e^+e^-\to q\bar{q}}(Q,\hat{k}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

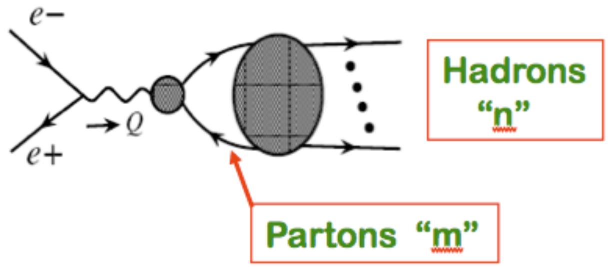
- **Total cross section (sum m):**

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \approx \sigma^{\text{tot}}_{e^+e^-\to\text{partons}} + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

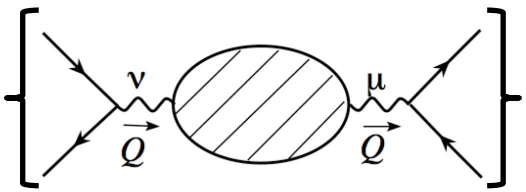
Infrared Safety of e⁺e⁻ Total Cross Sections

- ❑ Optical theorem:

$$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\left| \right.$$

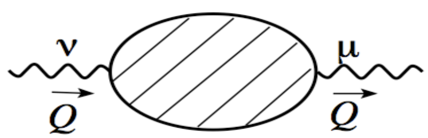


$$\left.\right|^2 \propto \text{Im}\left\{ \right.$$



$$\left.\right\}$$

- ❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

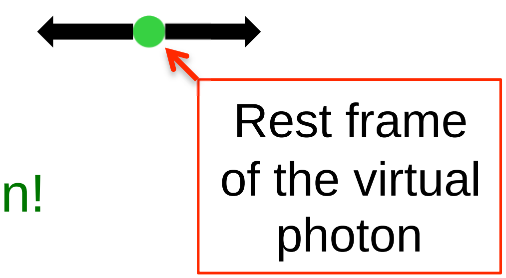
IR safety of $\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$ = IR safety of $\Pi(Q^2)$ with $Q^2 > 0$

- ❑ IR safety of $\Pi(Q^2)$:

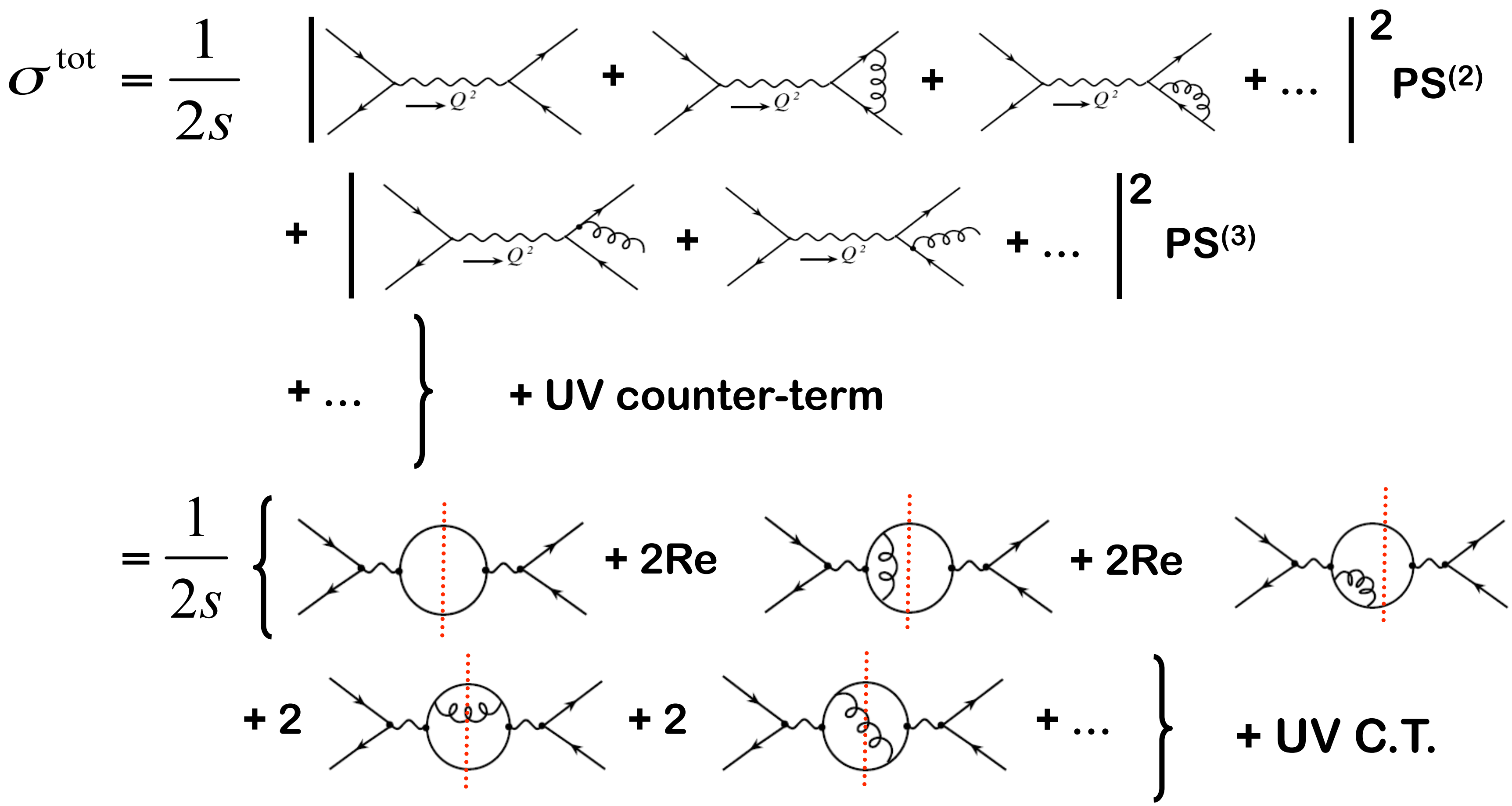
If there were pinched poles in $\Pi(Q^2)$,

✧ real partons moving away from each other

✧ cannot be back to form the virtual photon again!



One-loop Contribution to e^+e^- Cross Section



$$= \sigma_2^{(0)} + \sigma_2^{(1)} + \sigma_3^{(1)}$$

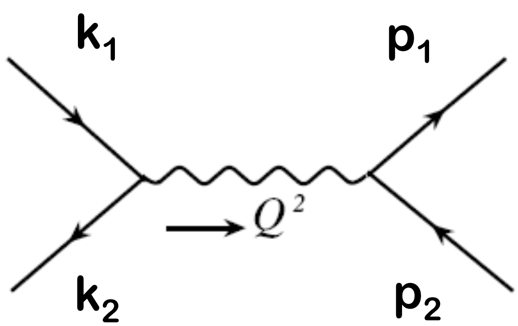
Born

O(α_s)

3-particle phase space

Lowest Order Contribution - I

❑ Lowest order Feynman diagram:



$$s = (k_1 + k_2)^2$$
$$t = (k_1 - p_1)^2$$
$$u = (k_2 - p_1)^2$$

❑ Invariant amplitude square:

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot k_2 \gamma^\mu \gamma\cdot k_1 \gamma^\nu\right]$$
$$\times \mathrm{Tr}\left[\left(\gamma\cdot p_1 + m_Q\right)\gamma_\mu \left(\gamma\cdot p_2 - m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right]$$

Keeps the final state quark mass

Lowest Order Contribution - II

- Lowest order total cross section:

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2} |\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s = Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1 + \frac{2m_Q^2}{s}\right]\sqrt{1 - \frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

- Normalized total cross section:

$$R = \frac{\sigma_{e^+e^-\to \text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \sum_Q e_Q^2 \, N_c \left[1 + \frac{2m_Q^2}{s}\right]\sqrt{1 - \frac{4m_Q^2}{s}}$$

One of the best measurements for the N_c

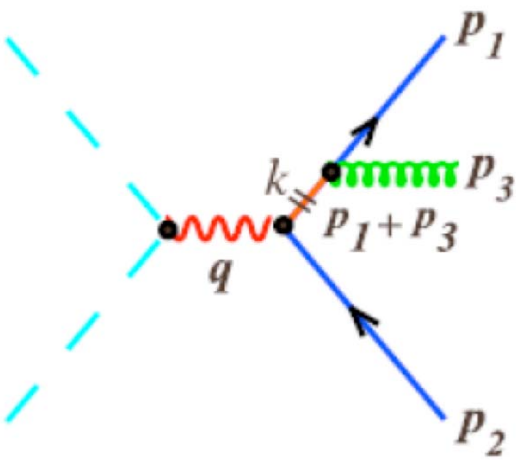
Next-to-Leading-Order Contribution - I

- Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

- Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

IR as x3→0
CO as $\theta_{13}\to 0$
$\theta_{23}\to 0$

Divergent as $x_i \to 1$
Need the virtual contribution and a regulator!

Next-to-Leading-Order Contribution - II

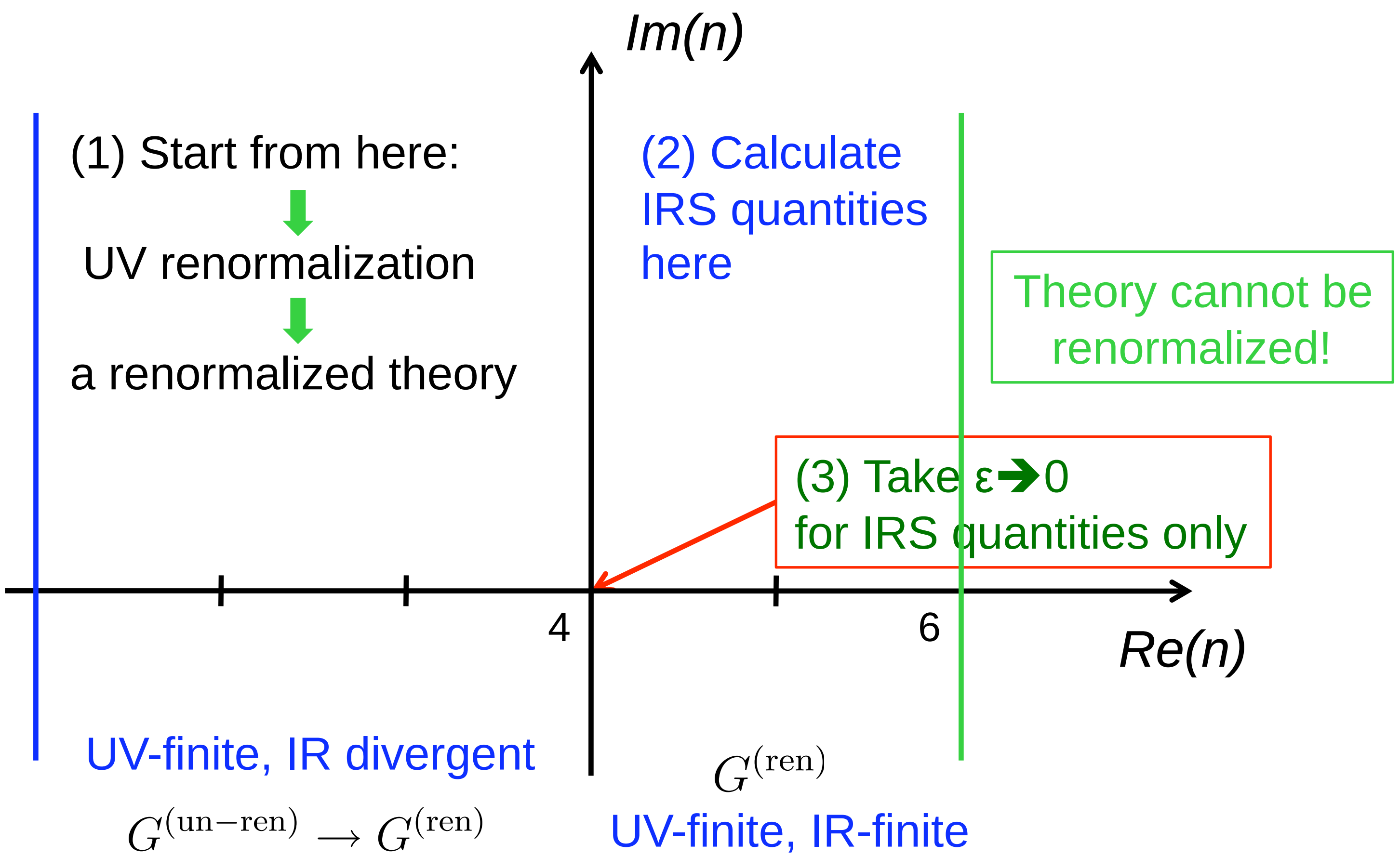
- Infrared regulator:
  - ✧ Gluon mass: $m_g \neq 0$
    – easier because all integrals at one-loop is finite: $\frac{1}{k^2} \to \frac{1}{k^2 - m_g^2}$
  - ✧ Dimensional regularization: $4 \to D = 4 - 2\varepsilon$
    – manifestly preserves gauge invariance

- Gluon mass regulator:
  - ✧ Real: $$\sigma^{(1)}_{3,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[2\ln^2\left(\frac{Q}{m_g}\right) - 3\ln\left(\frac{Q}{m_g}\right) - \frac{\pi^2}{6} - \frac{5}{2}\right]$$
  - ✧ Virtual: $$\sigma^{(1)}_{2,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[-2\ln^2\left(\frac{Q}{m_g}\right) + 3\ln\left(\frac{Q}{m_g}\right) + \frac{\pi^2}{6} - \frac{7}{4}\right]$$
  - ✧ Total: $$\sigma^{\text{tot}} = \sigma^{(0)}_2 + \sigma^{(1)}_{3,m_g} + \sigma^{(1)}_{2,m_g} + O\left(\alpha_s^2\right) = \sigma^{(0)}_2\left[1 + \frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

No m_g dependence!

How dimensional regularization works?

❑ Complex *n*-dimensional space: $\int d^n k$



Next-to-Leading-Order Contribution - III

- Dimensional regulator:

  ✧ Real:
  $$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

  ✧ Virtual:
  $$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

  ✧ NLO:
  $$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]$$

  No ε dependence!

  ✧ Total:
  $$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O\left(\alpha_s^2\right) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

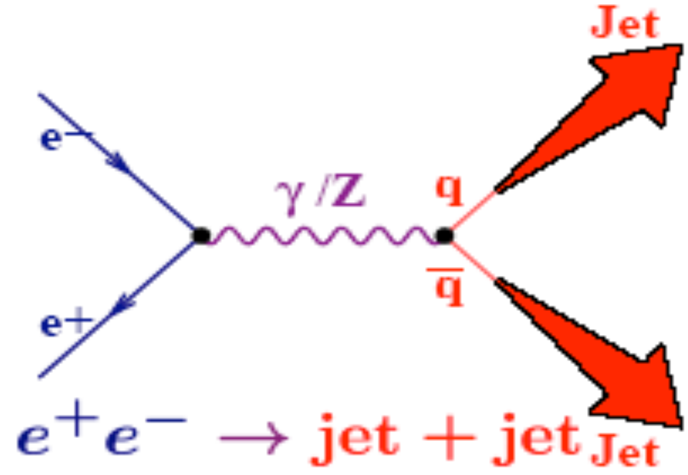
- Lesson:

  σ^{tot} is independent of the choice of IR and CO regularization

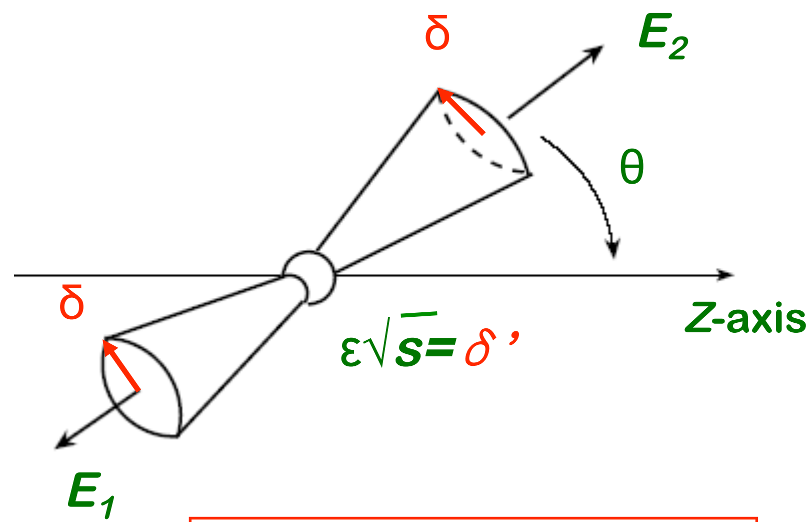
  σ^{tot} is Infrared Safe!

Jets in e^+e^- - Trace of Partons

- ❑ **Jets – Inclusive x-section with a limited phase-space**



- ❑ **Q: will IR cancellation be completed?**
  - ✧ **Leading partons are moving away from each other**
  - ✧ **Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton**



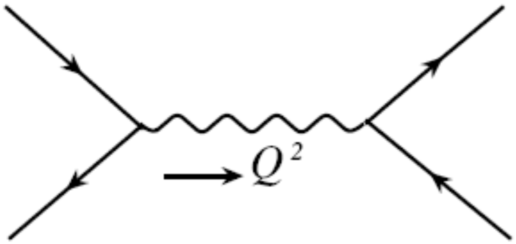
- ❑ **Jet algorithm**

Sterman-Weinberg Jet

Two-jets Cross Section in e⁺e⁻ Collisions

- Parton-Model = Born term in QCD:

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



- Two-jet in pQCD:

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n(\delta)$

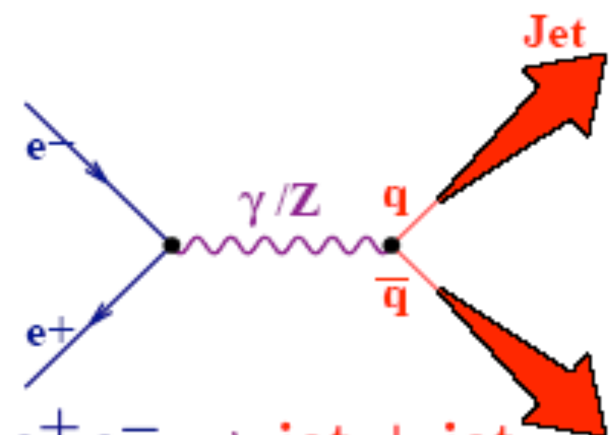
- Sterman-Weinberg jet:

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as } Q\to\infty$$

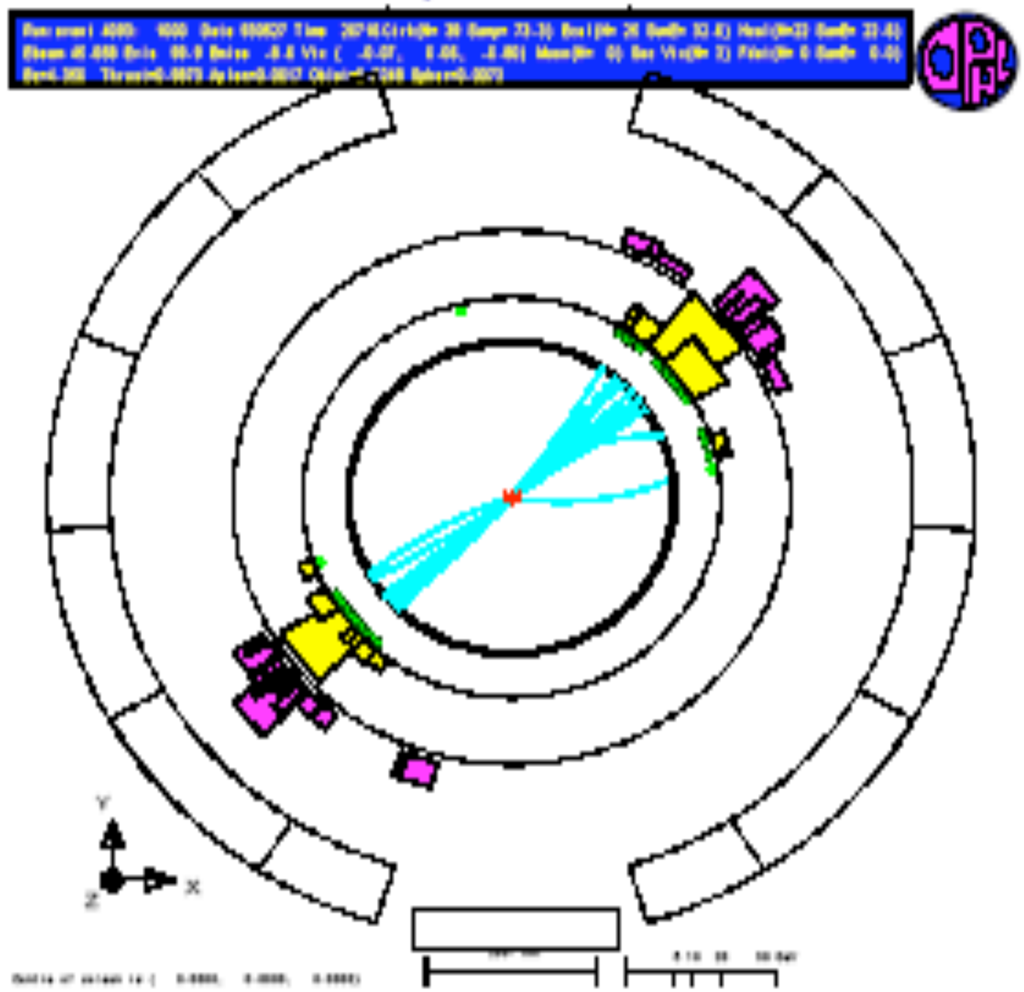
A Clean Two-jet Event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

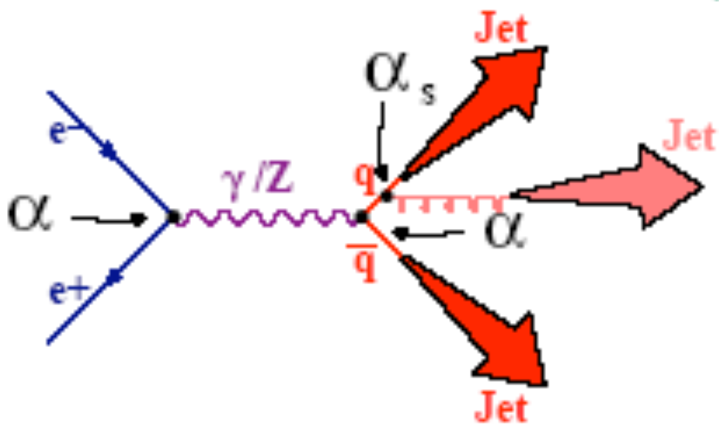
LEP ($\sqrt{s} = 90 - 205$ GeV)



A clean trace of two partons – a pair of quark and antiquark

Discovery of a Gluon Jet

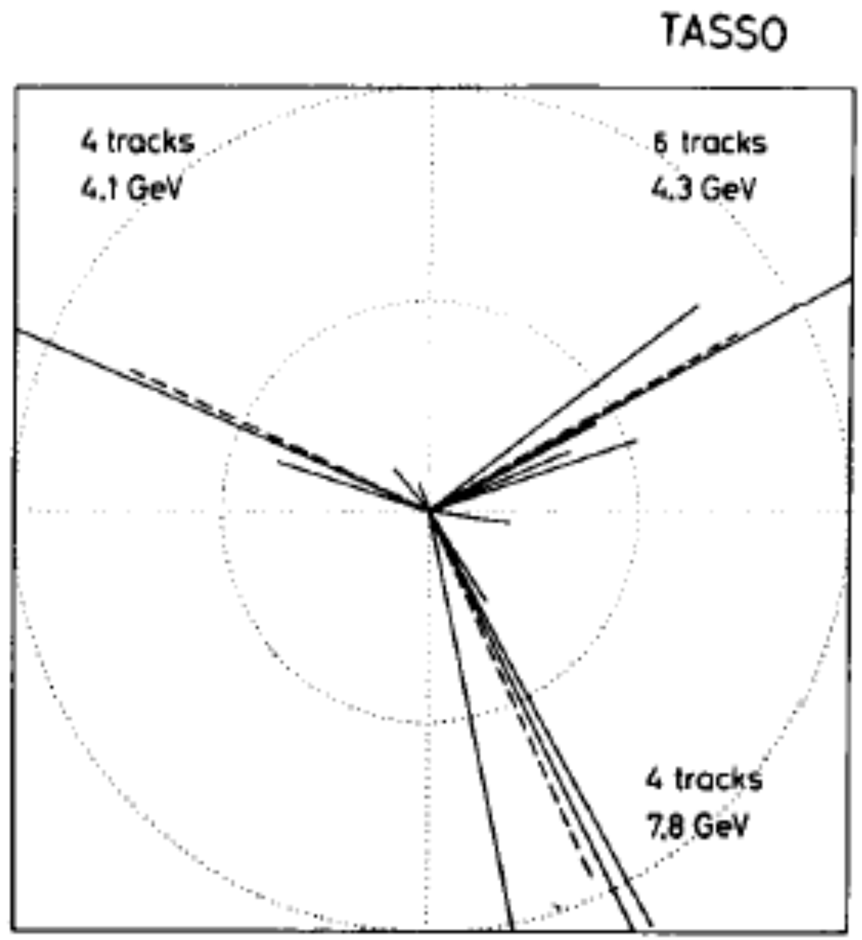
First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



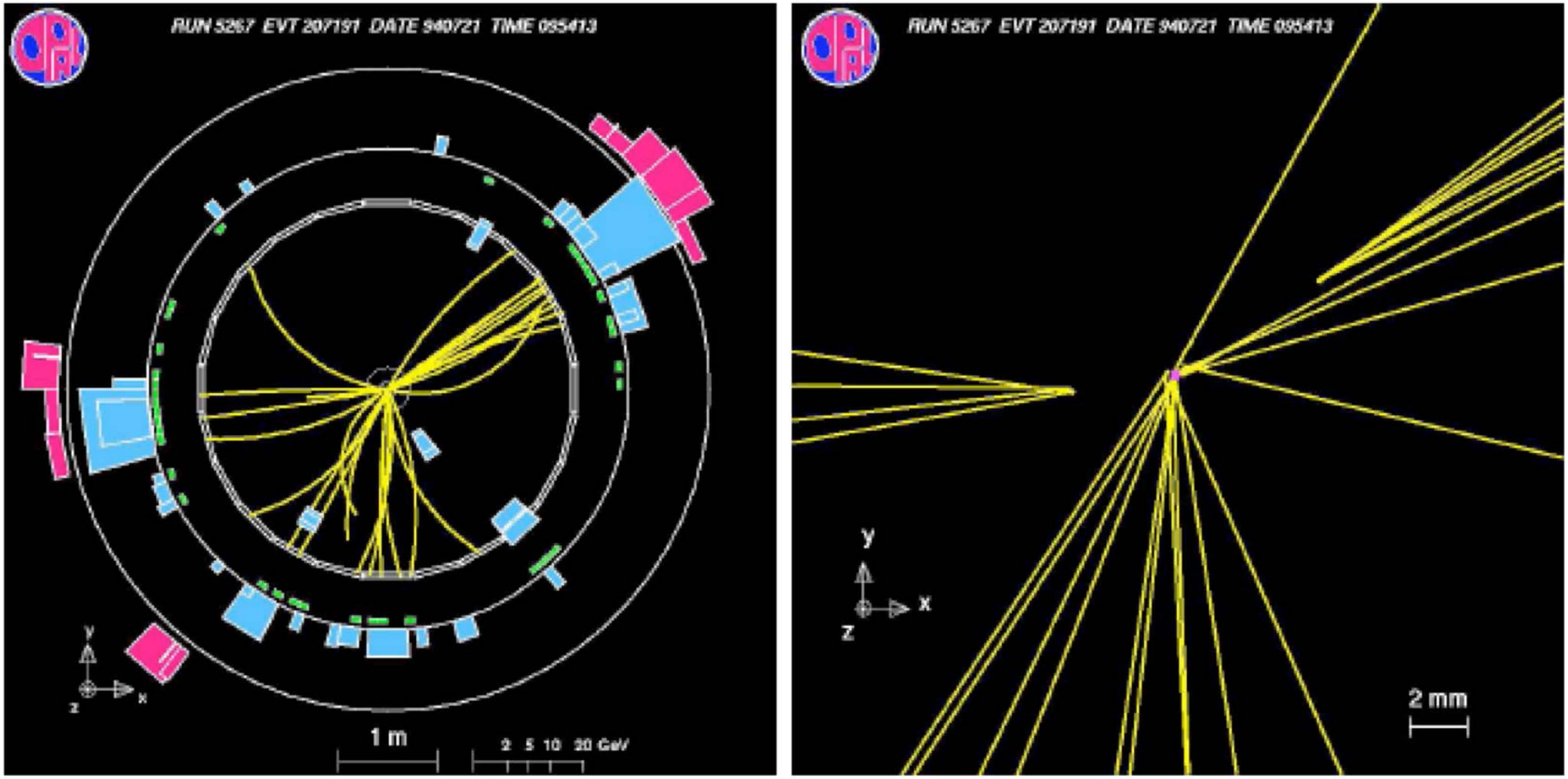
TASSO Collab., Phys. Lett. B86 (1979) 243

MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged Three-jet Event from LEP



Gluon Jet

Basics of jet finding algorithms

- Recombination jet algorithms:
  —— almost universal choice at e+e- colliders

- Recombination metric: $y_{ij} = \dfrac{M_{ij}^2}{E_{c.m.}^2}$

  $\longrightarrow$ Combine the particle pair (i,j) with the smallest y_{ij}:

$$(i,j) \to k$$

$$\underline{\text{E scheme :}} \qquad p_k = p_i + p_j \qquad \longrightarrow \quad \text{massive jets}$$

$$\underline{\text{E}_0\,\text{scheme :}} \qquad E_k = E_i + E_j$$

$$\vec{p}_k = \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|} E_k \qquad \longrightarrow \quad \text{massless jets}$$

- Iterate until all remaining pairs satisfy $y_{ij} > y_{cut}$

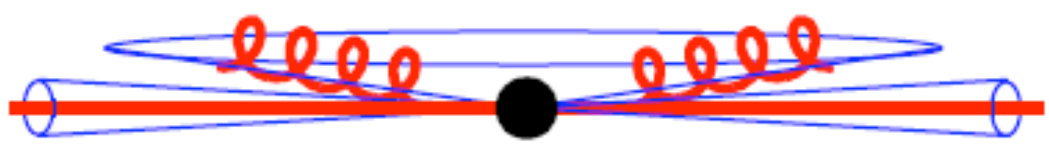
The JADE Jet Finder

[JADE Collab., Z. Phys. C33 (1986) 23]

⟶ The original recombination jet finder:

- $M_{ij}^2 = 2E_iE_j(1 - \cos\theta_{ij}) \approx$ (invariant mass)^2
- Original version based on the E_0 scheme

Sometimes leads to the formation of "junk jets"



⟶ Two-jet events with ≥ 2 soft, colinear gluons can be classified, unnaturally, as three-jet events

⟶ Prevents re-summation techniques from being applied

The Durham k_T Jet Finder

[S. Catani et al., Phys. Lett. B269 (1991) 432]

⟶ Introduced to reduce the problem of junk jets

- $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$
- E scheme combination of particles: $(i, j) \to k$

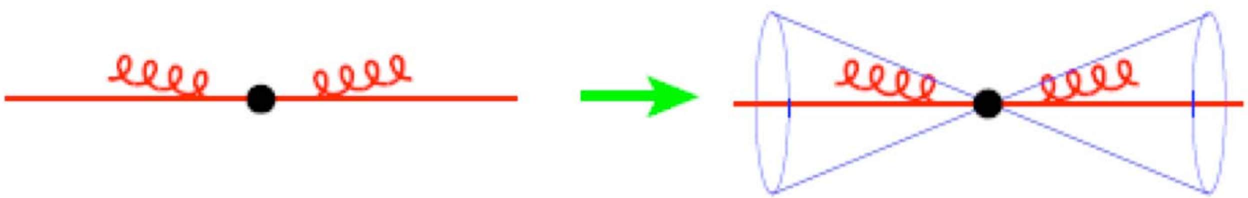
⟶ Consider small emission angles θ_{ij}:

$$M_{ij}^2 \approx 2\min(E_i^2, E_j^2)\left[1-\left(1-\theta_{ij}^2/2+\cdots\right)\right]$$

$$\approx \min(E_i^2, E_j^2)\,\theta_{ij}^2 \approx K_\perp^2$$

(min. transverse momenta of one particle w.r.t. the other)

⟶ Soft, colinear radiation is attached to the quark jet(s)

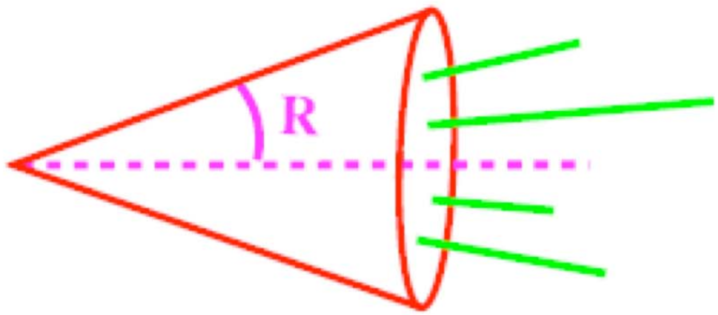


⟶ Permits re-summation

The Cone Jet Finder

CDF Collab., Phys. Rev. D45, 1448 (1992); OPAL Collab., Z. Phys. C63, 197 (1994)

- Cluster particles within a cone of half angle R into a jet



- Require a minimum visible jet energy: $E_{\text{jet}} \geq \epsilon$

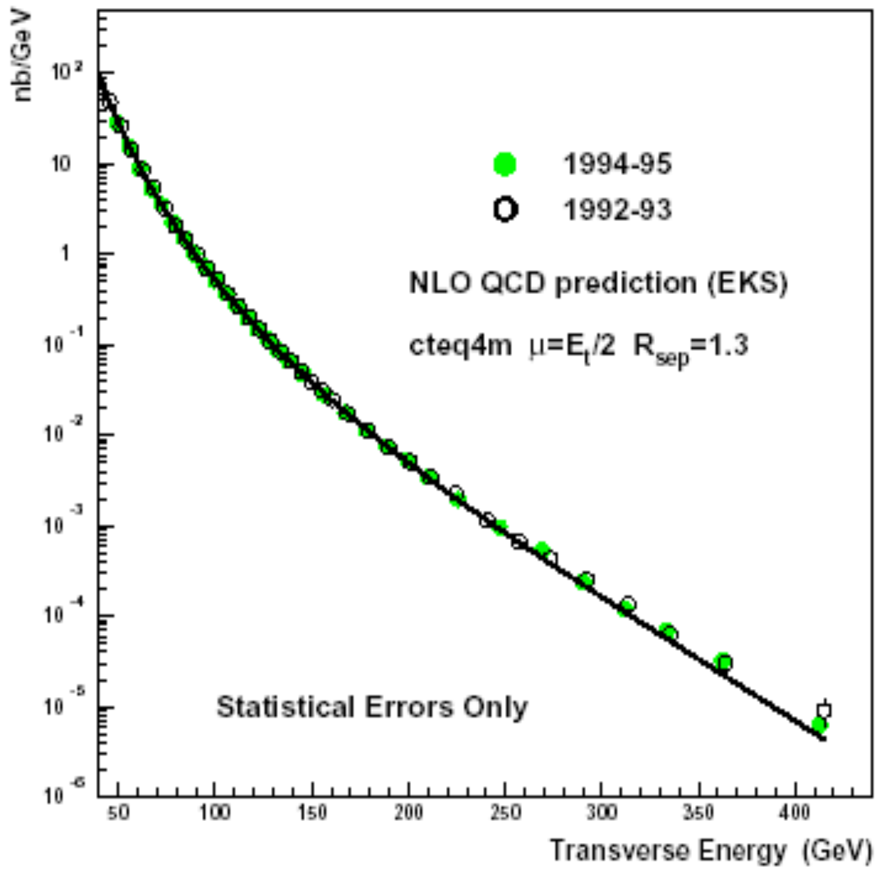
  ⟶ Two resolution parameters: R and ϵ , as opposed to re-combination algorithms which only have one (y_{cut})

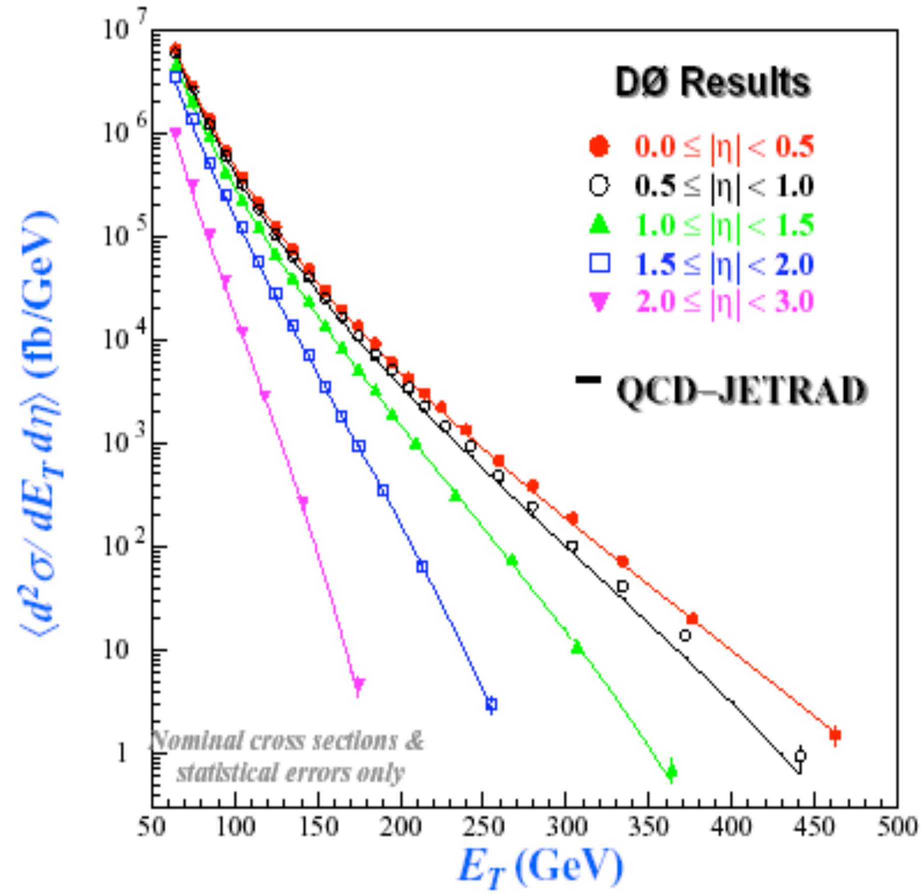
- Eliminate or merge overlapping or redundant jets

  ⟶ Unlike recombination algorithms, not all particles in an event are necessarily assigned to a jet

Inclusive Jet Cross Section at Tevatron

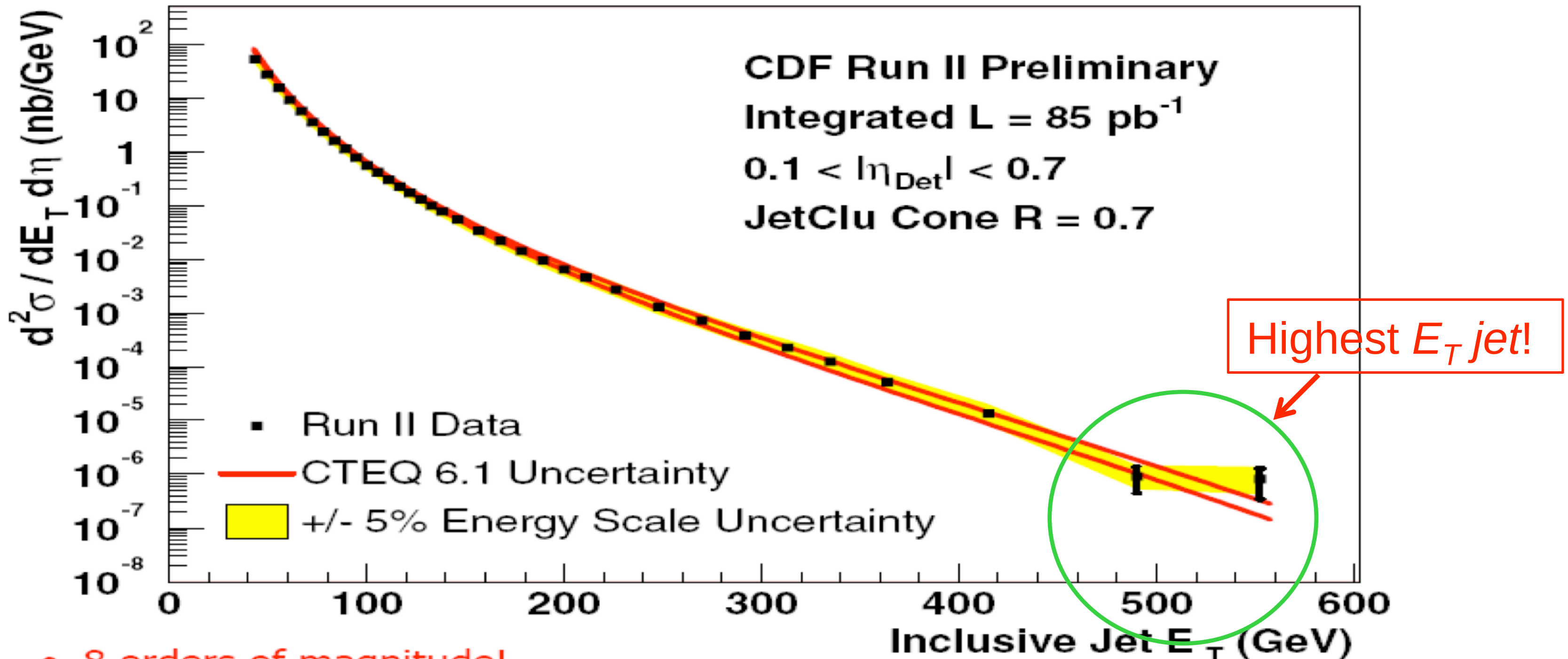
Run – 1b results



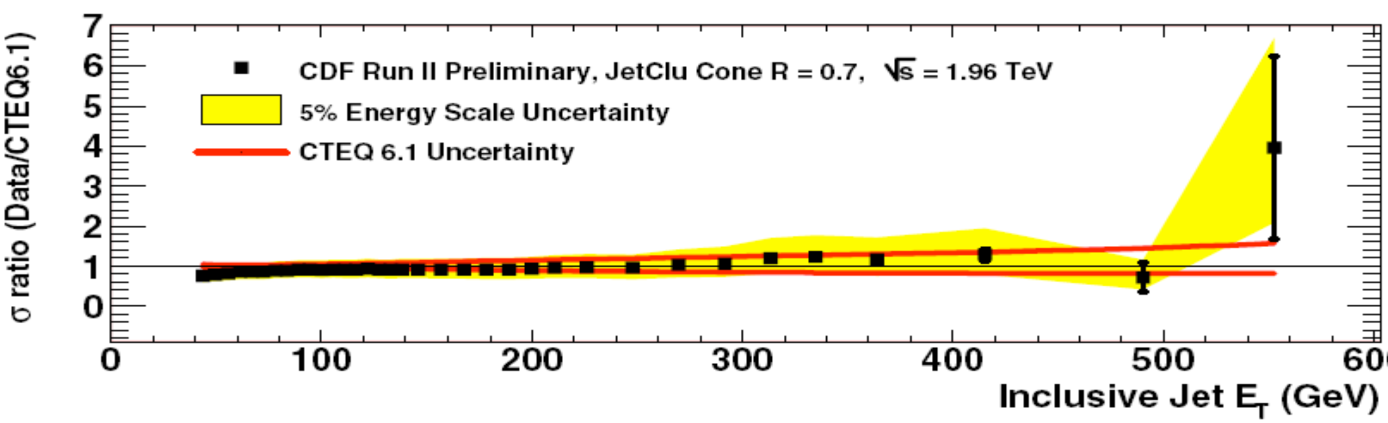


Data and Predictions span 7 orders of magnitude!

Prediction vs CDF RUN-II Data



- 8 orders of magnitude!



Infrared Safety for Jet Cross Sections

- Jet cross section = inclusive cross section with a phase-space constraint

- For any observable with a phase space constraint, Γ,

$$
\begin{aligned}
d\sigma(\Gamma) \equiv\; & \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) \\
& + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) \\
& + \ldots \\
& + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots
\end{aligned}
$$

where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of dσ(Γ):

$$
\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu, k_{n+1}^\mu = \lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right)
$$

$$
\text{with}\quad 0 \le \lambda \le 1
$$

Physics Meaning of Infrared Safety

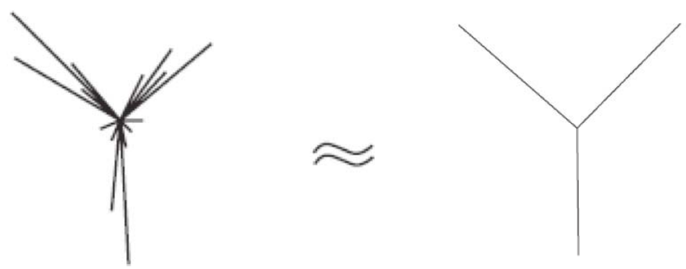
- Conditions for IRS:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,k_{n+1}^\mu=\lambda k_n^\mu\right)=\Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right)$$

$$\text{with}\quad 0\le\lambda\le 1$$

- Physical meaning:

  Measurement cannot distinguish a state
  with an additional zero/collinear momentum parton
  from a state without the parton



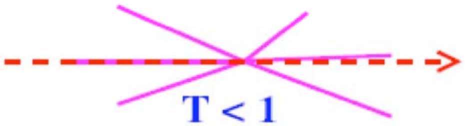
- Total cross section = a special case of jet cross section:

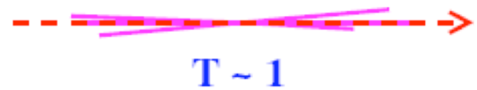
$$\Gamma_n\left(k_1,k_2,\ldots,k_n\right)=1\ \text{ for all } n \quad\Rightarrow\quad \sigma^{(\text{tot})}$$

Thrust Distribution

- Thrust axis: $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}}\left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} |\vec{p}_i|}\right)$$





- Phase space constraint:

$$\Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

✧ Contribution from p=0 particles drops out the sum

✧ Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

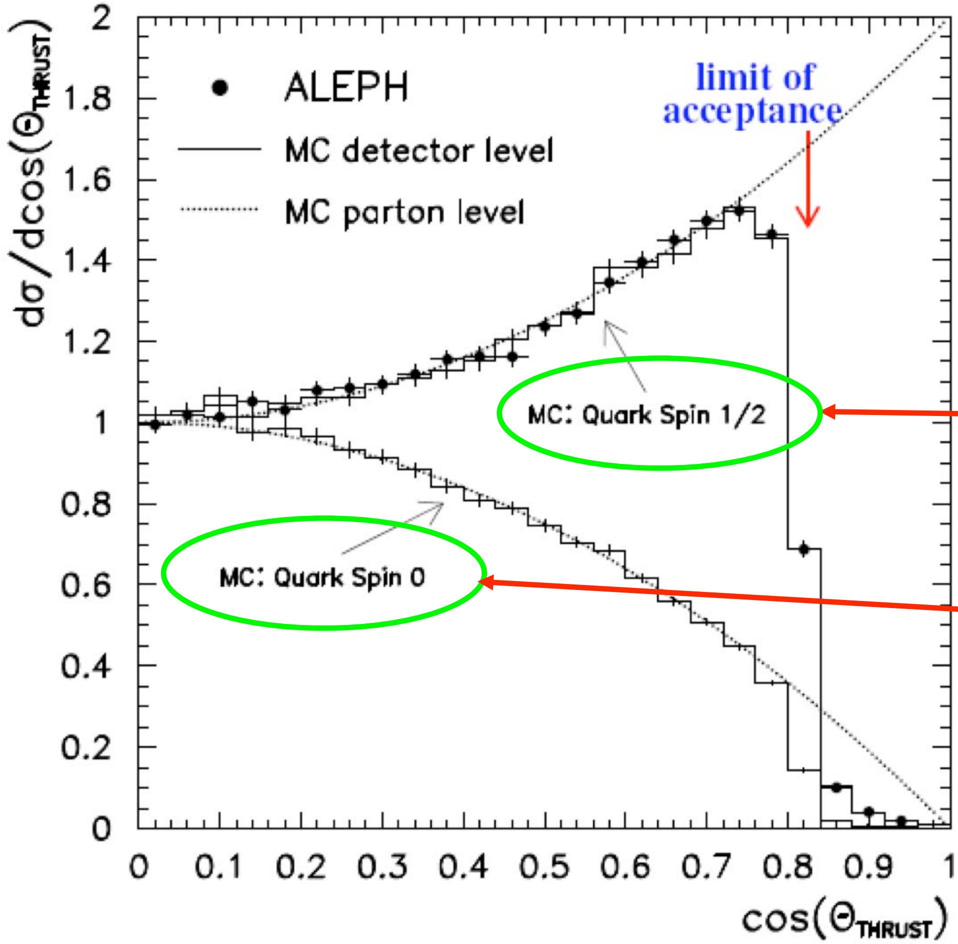
and

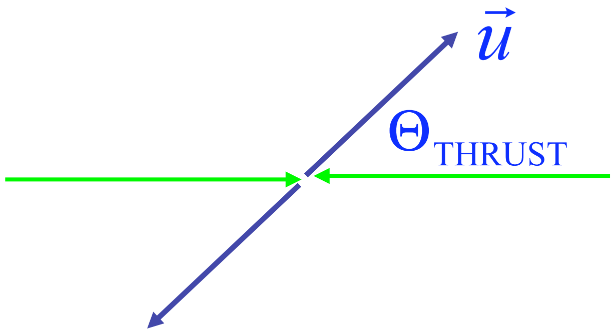
$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

Another Test of Quark Spin

- Angle between the thrust axis and the beam axis:

[ALEPH Collab., Phys. Rep. 294 (1998) 1]





At LO:

$$1+\cos^2\Theta_{THRUST}$$

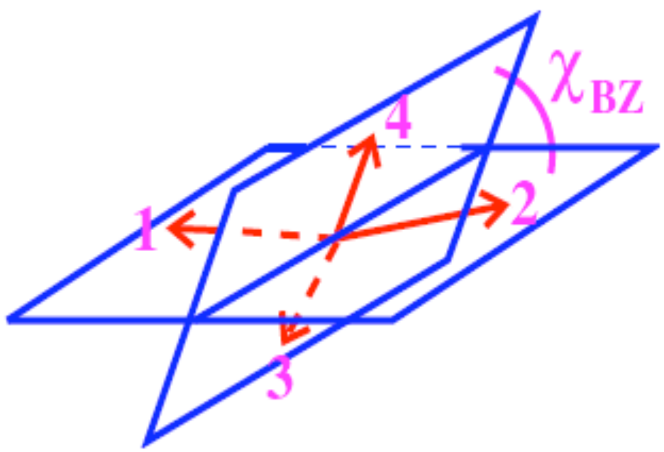
$$1-\cos^2\Theta_{THRUST}$$

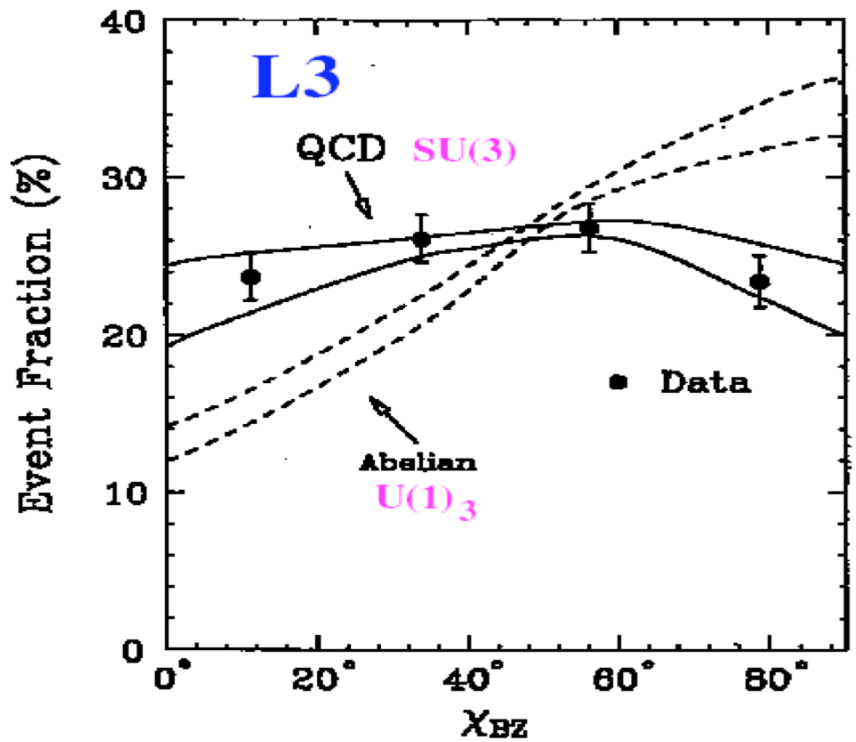
Another Test of SU(3) Color

- Select 4-jet events: $E_1 > E_2 > E_3 > E_4$

  jet-3 and jet-4 are more likely from radiation

- Bengtsson-Zerwas angle:





Question

Does perturbative QCD work for cross sections with identified hadrons?

Facts:

Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

Energy exchange in hard collisions: $Q \gg \Lambda_{QCD}$

pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

Answer:

Perturbative QCD factorization

See you next time!