Quantum Chromodynamics (QCD) and Physics of the strong interaction

(Lecture 6)

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Lecture: Mon – Wed – Fri 10:00-11:40AM

Location: B326, Main Building

Review of Lecture Five

- ❑ We can actually "see" and "count" the quarks and gluons – quark and gluon distributions

- ❑ PQCD factorization works for DIS to all orders as well as all powers due to Operator Product Expansion (OPE)

- ❑ PDFs evolves – the number of partons is sensitive to the probing scale

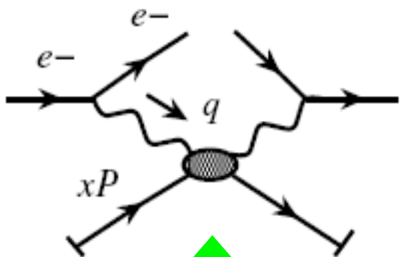
- ❑ PQCD global analysis for spin averaged cross sections results into the reasonably well-determined universal PDFs

- ❑ Hadronization – the probability – fragmentation functions

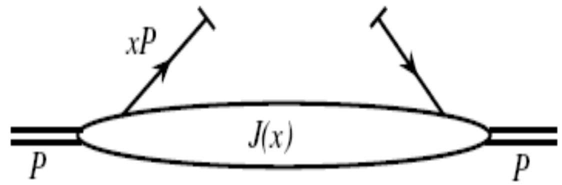
Cross Section with Identified Hadrons

❑ One hadron:

$$\sigma_{\rm tot}^{\rm DIS} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

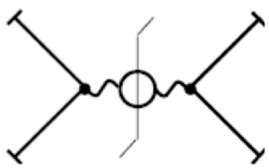
Now
Hard-part

Past
Parton-distribution

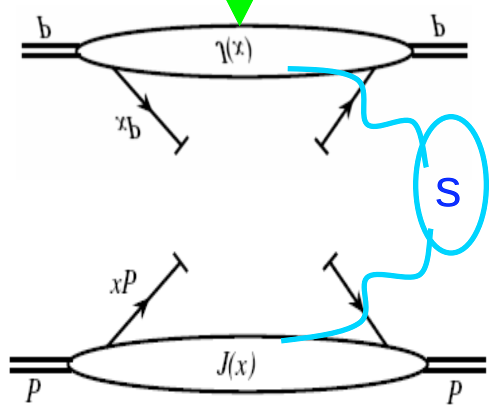
Connection
Power corrections

❑ Two hadrons:

$$\sigma_{\rm tot}^{\rm DY} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Soft interactions between incoming hadrons break the universality of PDFs

"Drell-Yan" Cross Section

- Drell-Yan process:

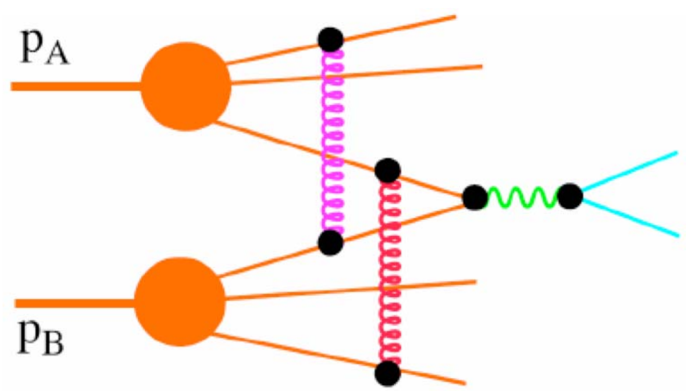
via a heavy colorless particle

$$h(p_A) + h'(p_B) \to \ell^+\ell^-(q) + X \qquad \text{with } Q^2 = q^2$$

- Parton model formula:

$$\frac{d\sigma^{\rm DY}_{hh'}(p_A, p_B, q)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx' \, \phi_f(x) \, \frac{d\hat{\sigma}^{\rm el}_{ff}(xp_A, x'p_B, q)}{dQ^2} \, \phi_{f'}(x')$$

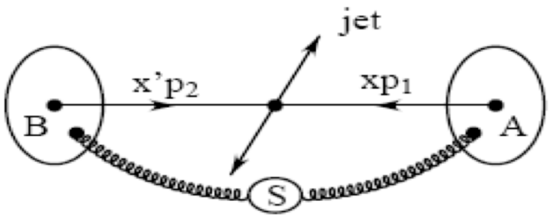
- Long-range soft interactions before the hard collision could break the PDF's universality – loss of predictive power



$$K_{\rm factor} = \frac{\sigma^{\rm DY}_{\rm Exp}}{\sigma^{\rm DY}_{\rm PM}} \sim 2$$

Long-range Soft Gluon Interactions

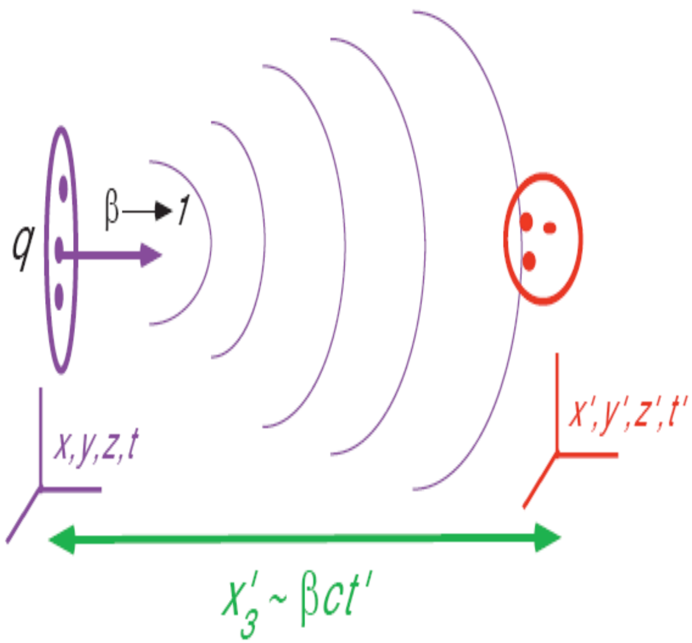
- Soft-gluon interaction takes place all the time:



Question:

What is its effect on a physical observable?

- Factorization = soft-gluon interactions are suppressed:



Field	x-Frame	x'-Frame
Scalar	$V(x)=\frac{e}{\lvert\vec{x}\rvert}$	$V'(x')=\frac{e}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Rightarrow \frac{1}{\gamma}$ "contracted like a ruler"
Gauge	$A^-(x)=\frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x')=\frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Rightarrow 1$ "not contracted!"

Field Strength is Strongly Contracted

Field	x-Frame	x'-Frame
Field Strength	$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \dfrac{-e\gamma\Delta}{(x_T^2 + \gamma^2\Delta^2)^{3/2}}$
		$\Longrightarrow \dfrac{1}{\gamma^2}$ "strongly contracted!"

- *Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!*

  the $1/\gamma^2$ translates into a suppression factor of $1/Q^4$

- *Initial-state interaction disappear at high enough energies!*

$$\sigma(Q) = \sigma_0(Q) + \sigma_2(Q)\frac{1}{Q^2} + \sigma_4(Q)\frac{1}{Q^4} + \ldots$$

- the factorization should be valid at the order of $1/Q^2$

Leading power (twist): Collins, Soper, and Sterman; Bodwin

Next leading power: Qiu and Sterman

Factorization is violated at $1/Q^4$ via explicit calculation: Taylor et al.

QCD Formalism for Drell-Yan Cross Section

❑ Factorization at leading power:

$$\frac{d\sigma^{\text{DY}}_{hh'}(p_A, p_B, q)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx' \, \phi_f(x, \mu_F^2) \, \frac{d\hat{\sigma}^{\text{DY}}_{f f}(xp_A, x' p_B, q, \mu_F^2)}{dQ^2} \, \phi_{f'}(x', \mu_F^2) + \ldots$$

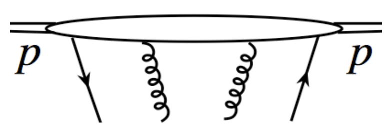
✧ This is not "leading log approximation", corrections to this factorized formula are power suppressed $1/Q^2$

✧ Parton distributions are non-perturbative, but, defined in terms of the same matrix elements as those defined in DIS

✧ $d\hat{\sigma}$ has an expansion in powers of α_s

❑ Factorization at next-to-leading power:

$$\frac{d\sigma^{(4)}_{hh'}(p_A, p_B, q)}{dQ^2} = \frac{1}{Q^2} \sum_{f_1, f_2, f'} \int dx dx_1 dx_2 \int dx' \, \mathcal{T}^{(4)}_{f_1 f_2}(x, x_1, x_2, \mu_F^2) \left[\frac{d\hat{\sigma}^{(4)}_{f_1 f_2, f'}}{dQ^2} \right] \phi_{f'}(x', \mu_F^2) + \mathcal{T} \leftrightarrow \phi$$

$\mathcal{T}^{(4)}_{qF}(x, x_1, x_2, \mu_F^2) =$ 

$=$ Twist-4 quark-gluon correlation function

Why Drell-Yan Process Makes Sense?

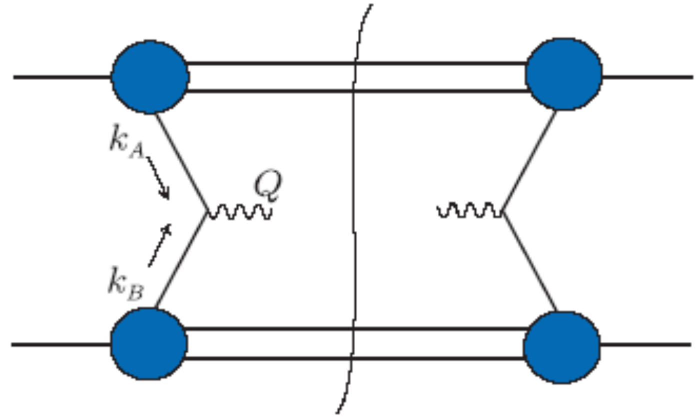
- Drell-Yan = Lowest order in QCD perturbation theory

✧ Perturbative pinch singularities

✧ Collision kinematics

✧ Large Q^2

⟹ determine the process



$$\frac{d\sigma}{dQ^2dy} = \int dk_{A,T}\, dk_{B,T} dk_A^- dk_B^+\; H_{\mu,\nu}(Q^+, Q^-, k_{A,T} + k_{B,T})$$
$$\times \mathrm{Tr}\{\gamma^\mu \Phi_A(Q^+ - \cancel{k_B^+}, k_{A,T}, k_A^-)\gamma^\nu \Phi_B(k_B^+, k_{A,T}, Q^- - \cancel{k_A^-})\}$$

Approximation:

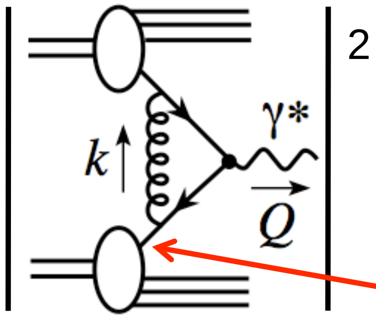
$$k_{A,T}^2\ ,\ k_{B,T}^2 \ll Q^2$$
$$k_A^- \ll Q^-$$
$$k_B^+ \ll Q^+$$

⟹ Drell-Yan formula

Trouble of Gluonic Interactions

- Virtual gluonic interaction is divergent:

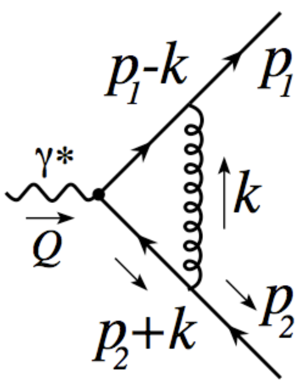
$\sigma \propto$



Virtual loop momentum *k*-integration can be divergent!

Dominated by on-shell parton momentum

- One-loop example (EM form factor):



$$I_\Delta = \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 + i\epsilon)((p_1 - k)^2 + i\epsilon)((p_2 + k)^2 + i\epsilon)}$$

$$= 2 \int \frac{d^n k}{(2\pi)^n} \int_0^1 \frac{d\alpha_1\, d\alpha_2\, d\alpha_3\, \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)}{[D(\alpha_1, \alpha_2, \alpha_3, k)]^3}$$

$$D(\alpha_1, \alpha_2, \alpha_3, k) = \alpha_1 k^2 + \alpha_2 (p_1 - k)^2 + \alpha_3 (p_2 + k)^2 + i\epsilon$$

$$I_\Delta = (-i) \left(\frac{1}{4\pi}\right)^2 \frac{1}{Q^2} \left(\frac{4\pi\mu^2}{-Q^2 - i\epsilon}\right)^\varepsilon \Gamma(1+\varepsilon) \frac{B(-\varepsilon, 1-\varepsilon)}{-\varepsilon} \longrightarrow \frac{1}{\varepsilon^2}$$

Divergent!

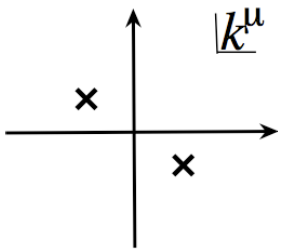
Singularities and Divergences

❑ Singularities:

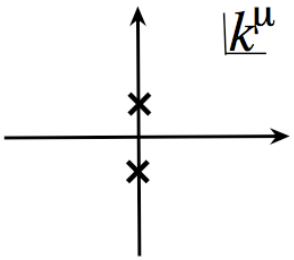
✧ Divergences from: $D(\alpha_1, \alpha_2, \alpha_3, k) = \alpha_1 k^2 + \alpha_2 (p_1 - k)^2 + \alpha_3 (p_2 + k)^2 + i\epsilon = 0$

✧ $D(\alpha_1, \alpha_2, \alpha_3, k)$ is quadratic in each component of k^μ

$\Longrightarrow$ Two poles for each component of k^μ - contour (1-up, 1-down)



No pinched poles
No real divergence



Pinched poles
Trouble!

❑ Conditions for pinched poles:

$$\begin{cases} D(\alpha_1, \alpha_2, \alpha_3, k) = 0 \\ \dfrac{\partial}{\partial k^\mu} D(\alpha_1, \alpha_2, \alpha_3, k) = 0 \qquad \text{for } \mu = 0, 1, 2, 3 \end{cases}$$

Also known as (or equivalent to) the Landau Equations

$$\Longrightarrow \begin{cases} \alpha_1 k^2 + \alpha_2 (p_1 - k)^2 + \alpha_3 (p_2 + k)^2 = 0 \\ \alpha_1 k^\mu - \alpha_2 (p_1 - k)^\mu + \alpha_3 (p_2 + k)^\mu = 0 \end{cases}$$

No pinched pole for α_i

Solutions of Landau Equations

- Landau equations:

$$\begin{cases} \alpha_1 k^2 + \alpha_2 (p_1 - k)^2 + \alpha_3 (p_2 + k)^2 = 0 \\ \alpha_1 k^\mu - \alpha_2 (p_1 - k)^\mu + \alpha_3 (p_2 + k)^\mu = 0 \end{cases}$$

Solution (1):

$$k^\mu = 0, \ \frac{\alpha_2}{\alpha_1} = \frac{\alpha_2}{\alpha_1} = 0$$

Pinched Infrared divergence!

Solution (2):

$$k^\mu = x p_1^\mu, \ \alpha_3 = 0, \ \alpha_1 \, x = \alpha_2 (1 - x), \ 0 < x < 1$$

Pinched collinear divergence!

Solution (3):

$$k^\mu = -x' p_2^\mu, \ \alpha_2 = 0, \ \alpha_1 \, x' = \alpha_3 (1 - x'), \ 0 < x' < 1$$

Pinched collinear divergence!

- Note:

✧ Having pinched singularity is a necessary condition for divergence

✧ Possible extra convergence from the numerator

✧ Divergent, but, power suppressed – high twist contribution

Infrared Power Counting

❑ Pinch surface:

A surface in the full phase-space of d^4k, on which the k is pinched

✧ Intrinsic variable – the component of the k on the pinch surface

✧ Normal variable – the component of the k out of the pinched surface

❑ Rescale all normal variables:

$$k_j \equiv \lambda^{a_j} K \qquad a_j = 1, 2, \ldots \text{ (or } \tfrac{1}{2}, 1, \ldots) \qquad K \text{ is a hard scale}$$

The momentum k^μ moves to the pinch surface when $\lambda \to 0$

Ex: In C.M. frame: $p_1^\mu = (p_1^+, 0^-, 0_\perp),\ p_2^\mu = (0^+, p_2^-, 0_\perp),$ and $p_1^+ = p_2^- = \sqrt{Q^2/2},$

If $k^\mu \parallel p_1^\mu$, rescale k^μ as $k^+ \sim \sqrt{Q^2},\ k^- \sim \lambda\sqrt{Q^2},\ k_\perp \sim \lambda\sqrt{Q^2}$

❑ Keep the lowest power in λ for each denominator:

$$\ell(k_j, \lambda)^2 \equiv \lambda^{A_j} f(k_j) + \ldots$$

Ex: $(p_2 + k)^2 = 2p_2 \cdot k + k^2 \to \lambda^0 (2p_2 \cdot k)$

❑ Degree of divergence – power of λ:

$S_I = \text{power from numerator}$

$$n_s = \sum_j a_j - \sum_i A_i + S_I \qquad n_s > 0 \text{(IR finite)},\ n_s \le 0 \text{(IR divergent)}$$

Physics of the Pinched Singularities

❑ Pinched singularity = long-lived partonic states:

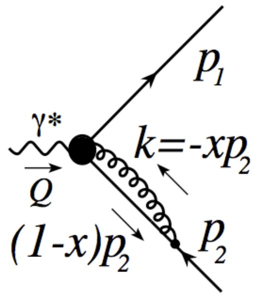
✧ Collinear divergence:

If $k^\mu \parallel p_1^\mu$, rescale k^μ as $k^+ \sim \sqrt{Q^2},\ k^- \sim \lambda^2\sqrt{Q^2},\ k_\perp \sim \lambda\sqrt{Q^2}$

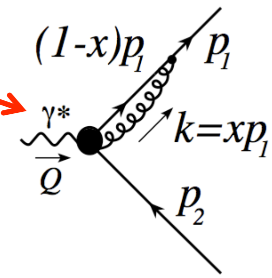
$$\Rightarrow \begin{cases} D = k^2\,(p_1-k)^2\,(p_2+k)^2 \sim k^2(-2p_1\cdot k)(2p_2\cdot k) \longrightarrow \lambda^2\cdot\lambda^2\cdot 1 = \lambda^4 \\ d^4k \longrightarrow \lambda^4 \end{cases}$$

0 0 1

Similarly for $k^\mu \parallel p_2^\mu$,



Collinear gluon

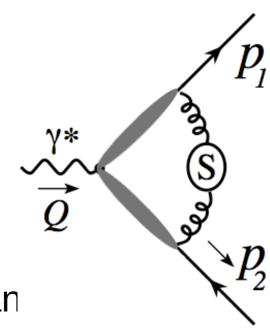


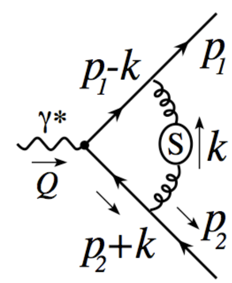
✧ Infrared divergence:

If $k^\mu \to 0$, rescale k^μ as $k^+ \sim k^- \sim k_\perp \sim \lambda\sqrt{Q^2}$

$$\Rightarrow \begin{cases} D = k^2\,(p_1-k)^2\,(p_2+k)^2 \sim k^2(-2p_1\cdot k)(2p_2\cdot k) \longrightarrow \lambda^2\cdot\lambda\cdot\lambda = \lambda^4 \\ d^4k \longrightarrow \lambda^4 \end{cases}$$

❑ General pinch surface:

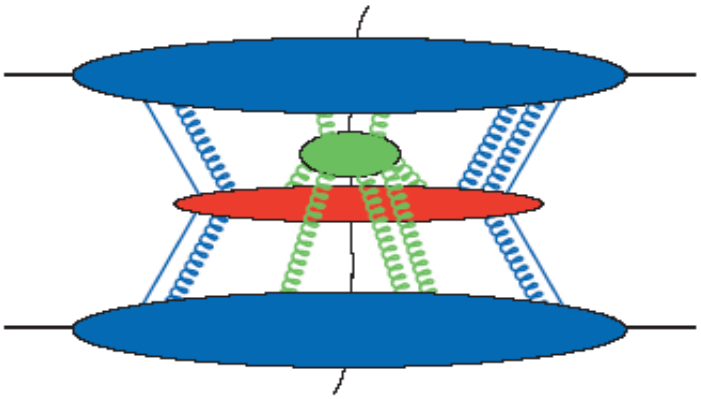




QCD Factorization for "Drell-Yan"

❑ Analysis of leading (pinch or singular) integration regions:

Power counting in partons' momentum scales gives the following separate regions:



Hard (Large P_T or way off shell) – infrared safe

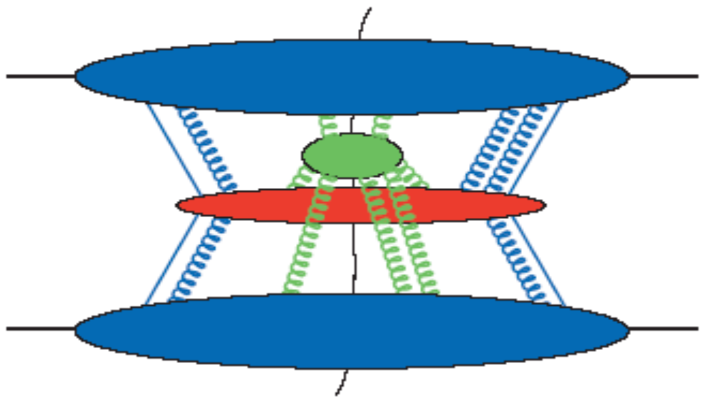
Collinear (to A or to B, small P_T) – could be a trouble

Soft (All components small, includes "Glauber.")

– a big trouble (took 20 years to solve the problem) $|k^+k^-| \ll k_\perp^2$

Eikonalization of Collinear Gluons

❑ Collinear gluons:



The extra collinear gluons could be a big problem because the factorization formula contemplates one parton from one hadron

❑ Solution:

✧ transversely polarized gluons are power suppressed

✧ longitudinally polarized gluons have $\varepsilon^{\mu}(k) \propto k^{\mu}$

Their effect can be approximated as shown with eikonal lines, with its direction, u, in the direction opposite to the hadron momentum:

u = "-" for hadron A, and u ="+" for hadron B

✧ Feynman rule for the interaction with eikonal line:

$$\text{Propagator} = \frac{i}{k \cdot u + i\epsilon} \qquad \text{Vertex} = -ig\, t_a\, u^{\mu}$$

Eikonalization of Collinear Gluons

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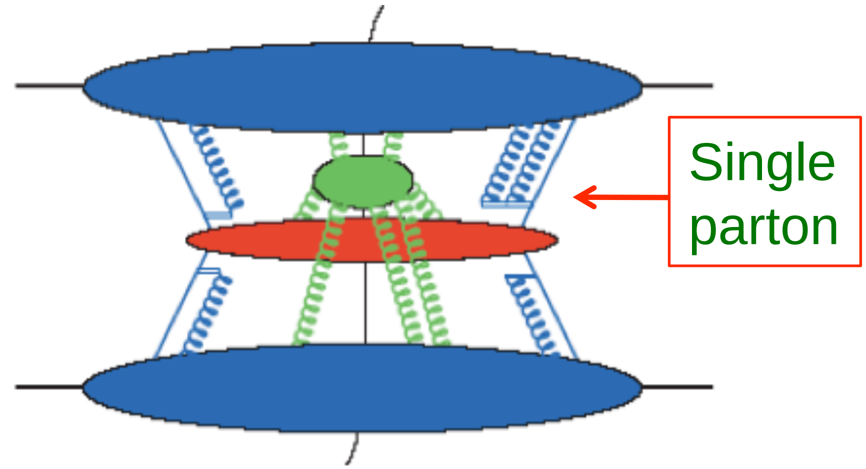
The extra collinear gluons could be a big problem because the factorization formula contemplates one parton from one hadron



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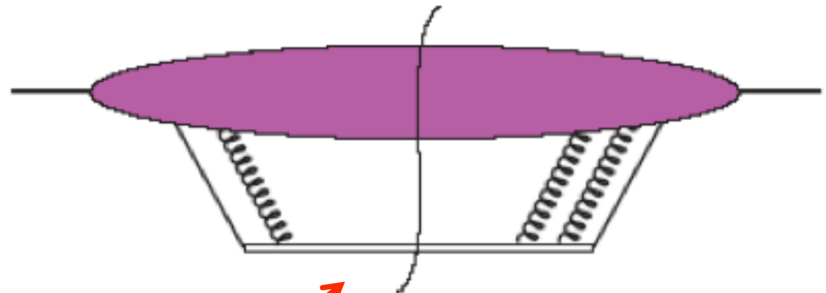
Factorization of Parton Distribution Functions

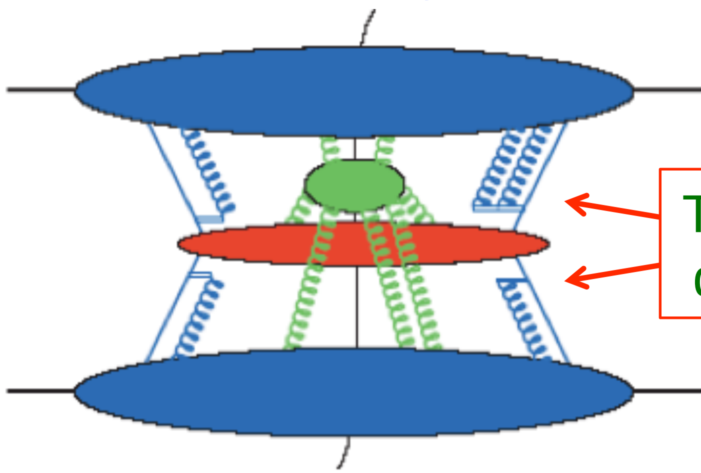
- Parton distribution function:

Takes care of collinear gluon interaction by the gauge link of PDF



$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle$$

- Factorization of collinear gluons:

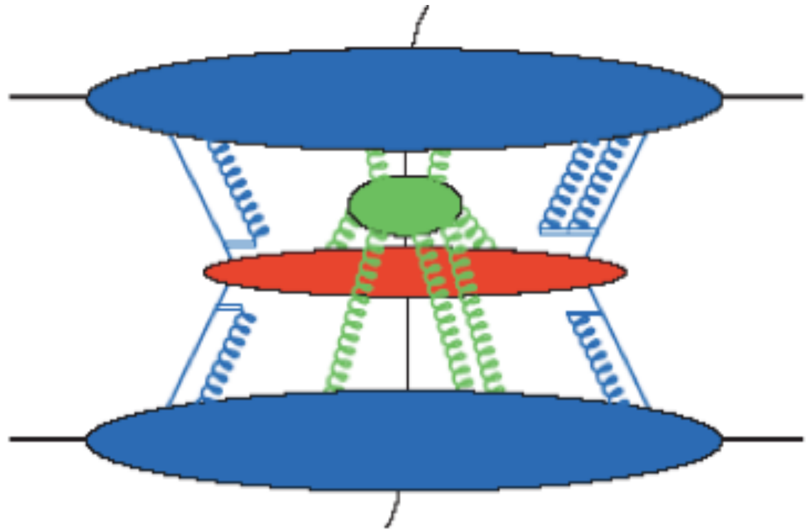


Two factorized parton distribution functions

Trouble from Soft Gluons

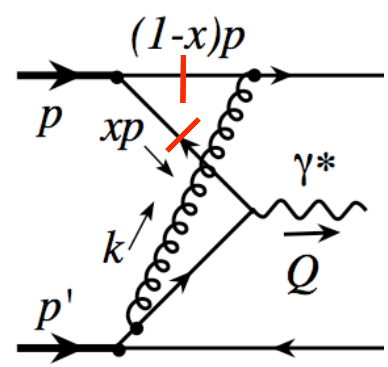
- ❑ Interaction between active quark and spectator quark:

  A soft gluon exchanged from a spectator quark in hadron A to the active quark in hadron B can rotate the quark's color, and thus, keep it from annihilating



- ❑ Additional pinch singularity:

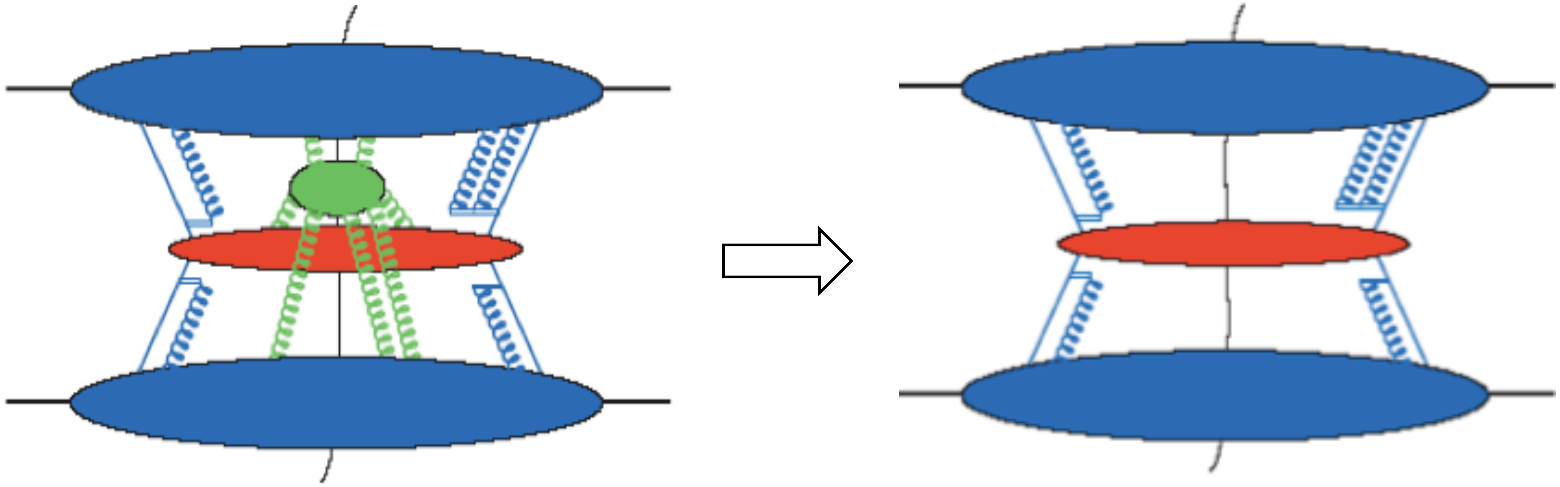
  Soft gluon approximation (with eikonal lines) requires the active parton to have large "+" (or "-") momentum. But, the contours of these momenta can be trapped in "too small" region



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$

$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

Soft gluons take care of themselves



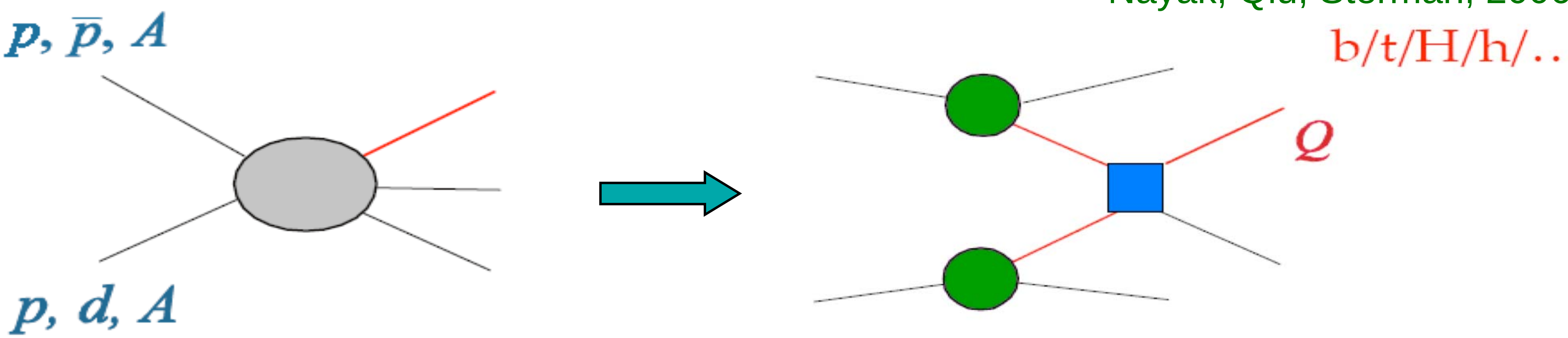
- Sum over all final-state and use of unitarity to remove all poles from upper half plane for k^--integration (or lower half plane for k^+-integration) – no-pinched poles

- Soft gluon approximation and gauge invariance to decouple soft gluon interaction from the jet-functions into the eikonal lines

- Unitarity to remove all decoupled soft-gluon interactions

Three Hadrons in a Single Hard Collision

Nayak, Qiu, Sterman, 2006



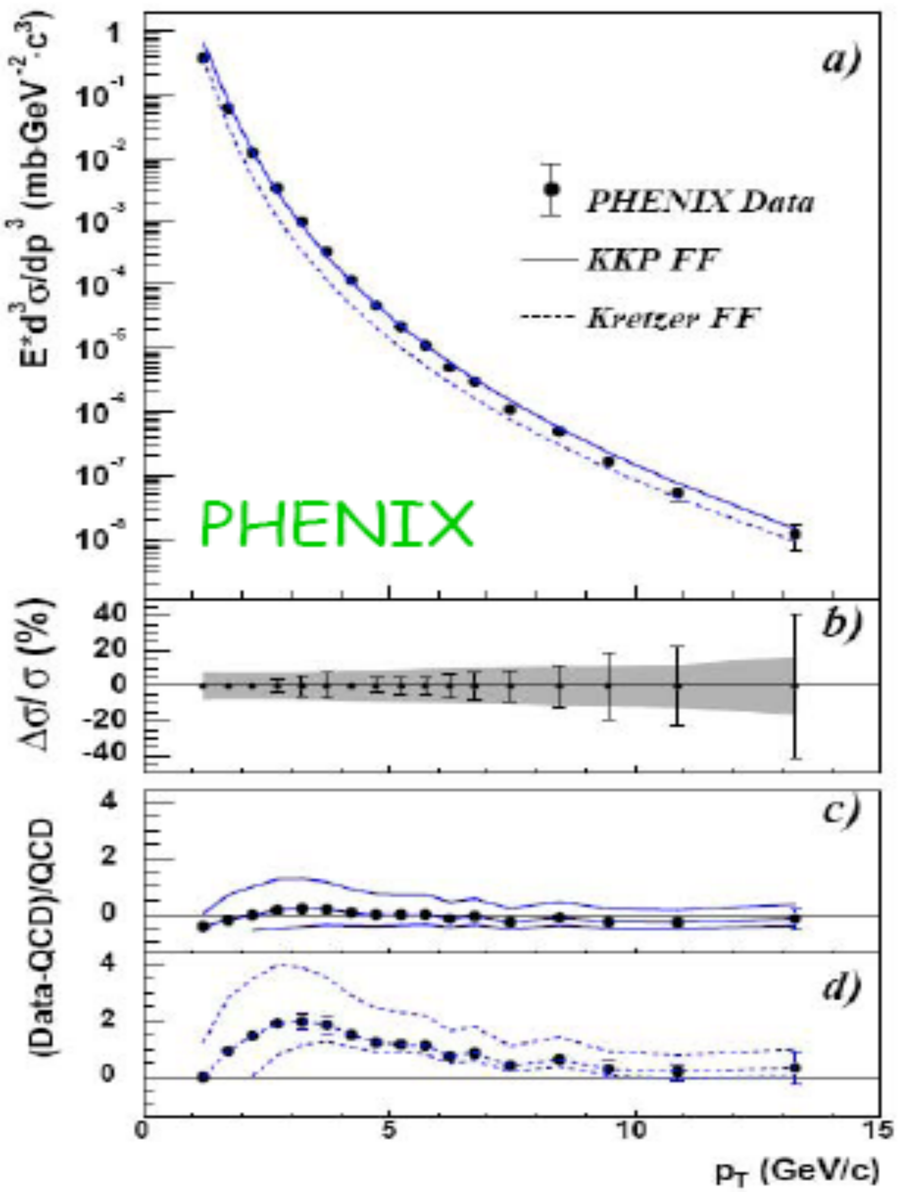
$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2)$$

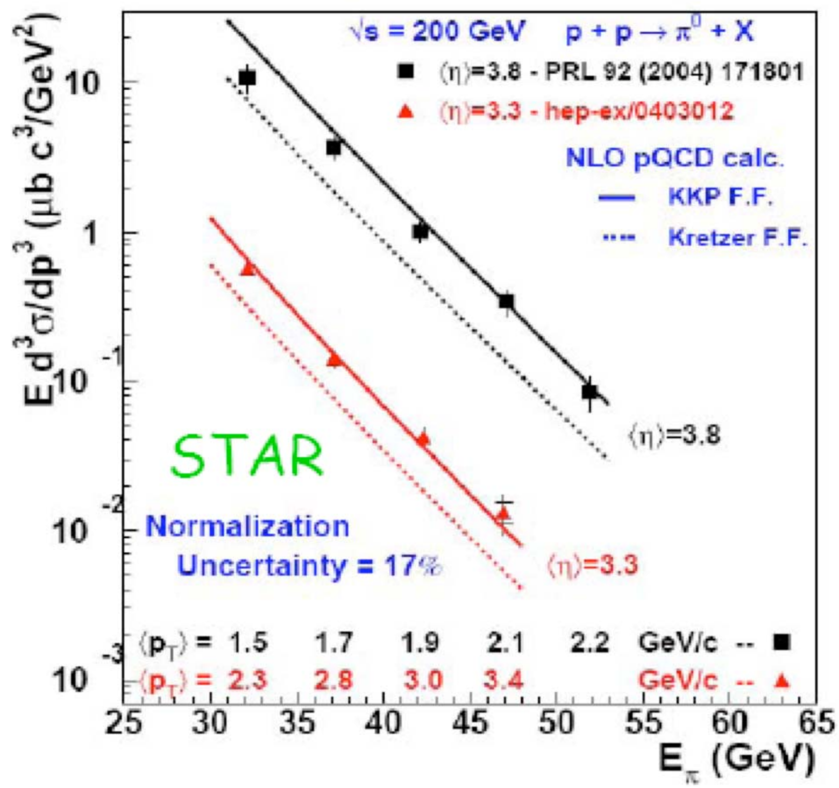
$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

- Fragmentation function: $D_{c\to C}(z, \mu_F^2)$
- Choice of the scales: $\mu_{\text{Fac}}^2 \approx \mu_{\text{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

High p_T Hadron Production at RHIC





Collinear factorization
at NLO works well
at RHIC energies

Processes with two Large Scales

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\rm QCD}^2$$

- We could choose: $\mu = Q_1$ or Q_2, or somewhere between

  ➡ $\alpha_s\left(Q_1^2\right)$ is small, $\alpha_s\left(Q_1^2\right)\ell n(Q_1^2/Q_2^2)$ is not necessary small

  Cannot remove the logarithms by choosing a proper *m*

  ➡ Resummation of the logarithms is needed

- For a massless theory, we can get <u>two</u> powers of the logarithms at each order in perturbation theory:

$$\alpha_s\left(Q_1^2\right)\ell n^2(Q_1^2/Q_2^2)$$

due to an overlap region of IR and CO divergences

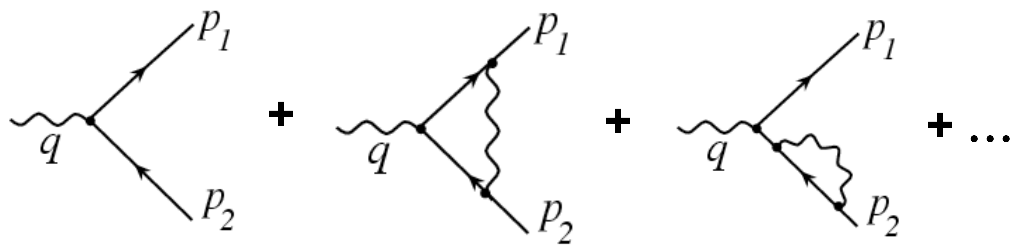
- Examples:

  ✧ pT distribution of heavy boson (Higgs, W±, Z, γ*,…) production

  ✧ jet-momentum imbalance in e^+e^-, $e^\pm h$, and hh collisions

Double Logarithms

- Consider electromagnetic form factor:



$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\ \bar{u}\ (p_1)\gamma_\mu v(p_2)\ \rho(q^2,\epsilon)$$

- For massless quark at one loop:

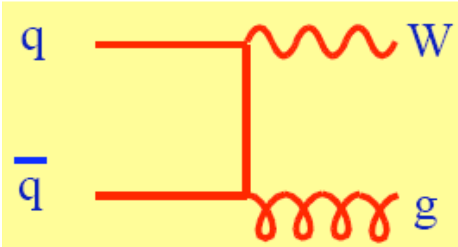
$$\rho(q^2,\epsilon) = -\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{1}{(-\epsilon)^2}-\frac{3}{2(-\epsilon)}+4\right\}$$

$$= 1-\frac{\alpha_s}{4\pi}C_F\ \ln^2(q^2/\mu^2)+\ldots$$

Overlap of IR and CO singularities $\Longrightarrow$ Double logarithms

- ❖ known as Sudakov double logarithms
- ❖ common in a massless theory

Leading Double Log Contribution to Drell-Yan



LO Differential Q_T-distribution as $Q_T \to 0$:

$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

$$\int_0^{Q^2}\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\rm real+virutal} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_W^2$$

Integrated Q_T-distribution:

$$\int_0^{Q_T^2}\left.\frac{d\sigma}{dy\,dp_T^2}\right|_{\rm real+virutal} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right]\left.\frac{d\sigma}{dy\,dp_T^2}\right|_{\rm real+virutal} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F\frac{\alpha_s}{\pi}\frac{\ell n\left(Q^2/p_T^2\right)}{p_T^2}dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \left[1 - C_F\frac{\alpha_s}{\pi}\ell n^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times \exp\left[-C_F\frac{\alpha_s}{\pi}\ell n^2\left(Q^2/Q_T^2\right)\right]$$

Effect of gluon emission

Assume this exponentiates

Resummed Q_T-distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \times \exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ell n^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- compare to the explicit LO calculation:

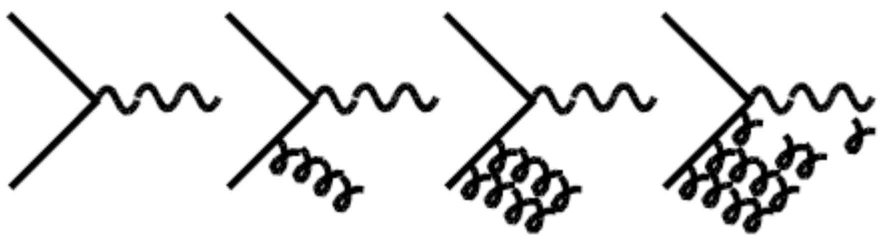
$$\frac{d\sigma}{dy\,dQ_T^2}_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2\,!} - \frac{(\alpha_s L^2)^3}{3\,!} + \ \ldots$$

$$L \propto \ell n\left(Q^2/Q_T^2\right)$$



Soft gluon emission treated as uncorrelated

Still a Wrong Q_T-distribution

- Experimental fact: $\dfrac{d\sigma}{dy\,dQ_T^2} \Rightarrow$ finite [neither ∞ nor 0!] as $Q_T \to 0$

  - Double Leading Logarithms Approximation (DLLA)

    radiated gluons are both soft and collinear with strong ordering in their transverse momenta

  - Strong ordering in transverse momenta in DLLA

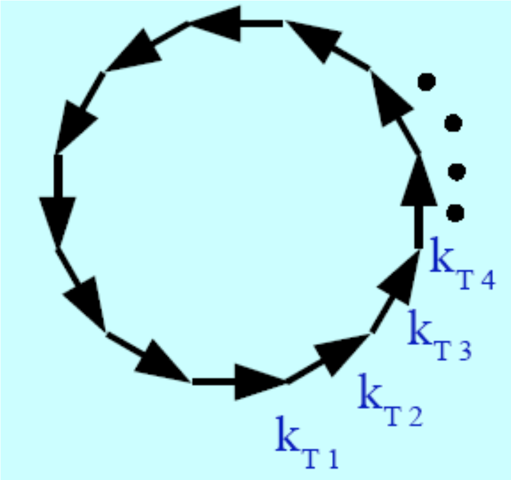
    – overly constrains the phase space ofthe emitted gluons

    – ignores the overall transverse momentum conservation

    $\Rightarrow$ DLLA over suppresses small Q_T region

Resummation of uncorrelated soft gluon emission leads to too strong suppression at Q_T=0

- Why?
  Particle can receive many finite k_T kicks via soft gluon radiation yet still have Q_T=0 – Vector sum!

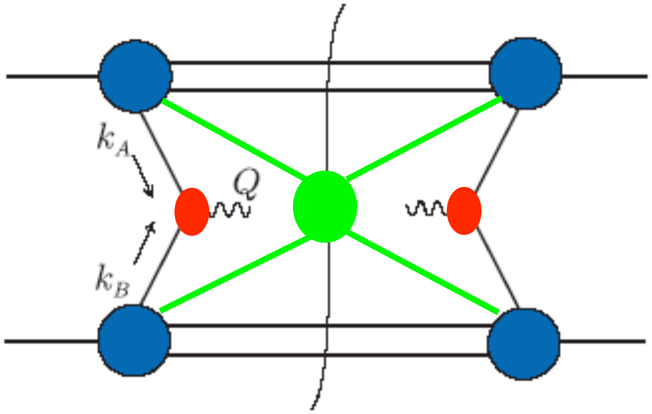


- Subleading logarithms are equally important at Q_T=0

- Solution:
  impose 4-momentum conservation at each step of soft gluon resummation

kT-factorization and Resummation

- Leading order K_T-factorized cross section:



$$\frac{d\sigma_{AB}}{dQ^2\,dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T}\, d^2k_{B_T}\, d^2k_{s,T}}{(2\pi)^6}$$
$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar f/B}(\xi_b, k_{B_T}) H_{f\bar f}(Q^2) S(k_{s,T})$$
$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\boxed{\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2}\int d^2b\, \mathrm{e}^{i\vec{b}\cdot\vec{Q}_T} \prod_i \mathrm{e}^{-i\vec{b}\cdot\vec{k}_{i,T}}}$$

- Factorized cross section in “impact parameter space”:

$$\frac{d\sigma_{AB}(Q,b)}{dQ^2} = \sum_f \int d\xi_a d\xi_b \overline{P}_{f/A}(\xi_a, b, n) \overline{P}_{\bar f/B}(\xi_b, b, n) H_{\bar f f}(Q^2) U(b,n)$$

- Resummation:

Two equations for two types log’s to resum

$$\mu_{\rm ren}\frac{d\sigma}{d\mu_{\rm ren}} = 0 \qquad\qquad n^\nu \frac{d\sigma}{dn^\nu} = 0$$

CSS b-space Resummation Formalism

- Solve those two equations and transform back to Q_T:

$$\frac{d\sigma_{AB}}{dQ^2\,dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\; \mathrm{e}^{i\vec{b}\cdot\vec{Q}_T}\,\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$

resummed

No large log’s

$$= \frac{1}{(2\pi)}\int_0^\infty db\; J_0(bQ_T)\; b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\mathrm{Pert})}}{dQ^2\,dQ_T^2} - \frac{d\sigma_{AB}^{(\mathrm{Asym})}}{dQ^2\,dQ_T^2}\right]$$

- b-space distribution: $\tilde{W}_{AB}(b,Q) \equiv \sum_{i,j}\tilde{W}_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$

The Q_T-distribution is determined by the b-space function:

$$\frac{\partial}{\partial \ln Q^2}\tilde{W}_{ij}(b,Q) = \left[K(b\mu,\alpha_s) + G(Q/\mu,\alpha_s)\right]\tilde{W}_{ij}(b,Q) \qquad (1)$$

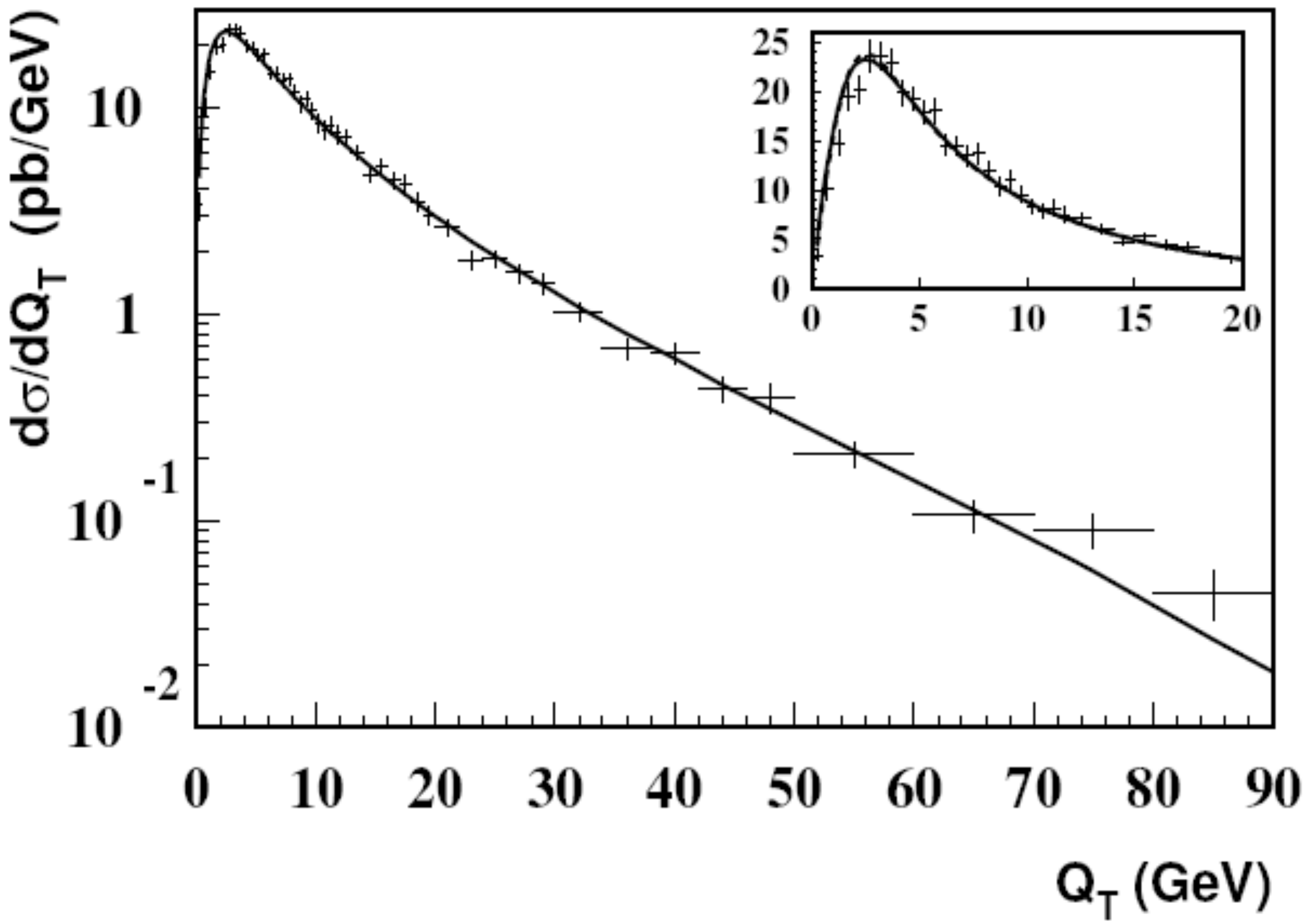
$$\frac{\partial}{\partial \ln \mu^2}K(b\mu,\alpha_s) = -\frac{1}{2}\gamma_K(\alpha_s(\mu)) \qquad (2)$$

$$\frac{\partial}{\partial \ln \mu^2}G(Q/\mu,\alpha_s) = \frac{1}{2}\gamma_K(\alpha_s(\mu)) \qquad (3)$$

Success of the Resummation Formalism

- Fermilab CDF data on Z at $\sqrt{S} = 1.8$ TeV

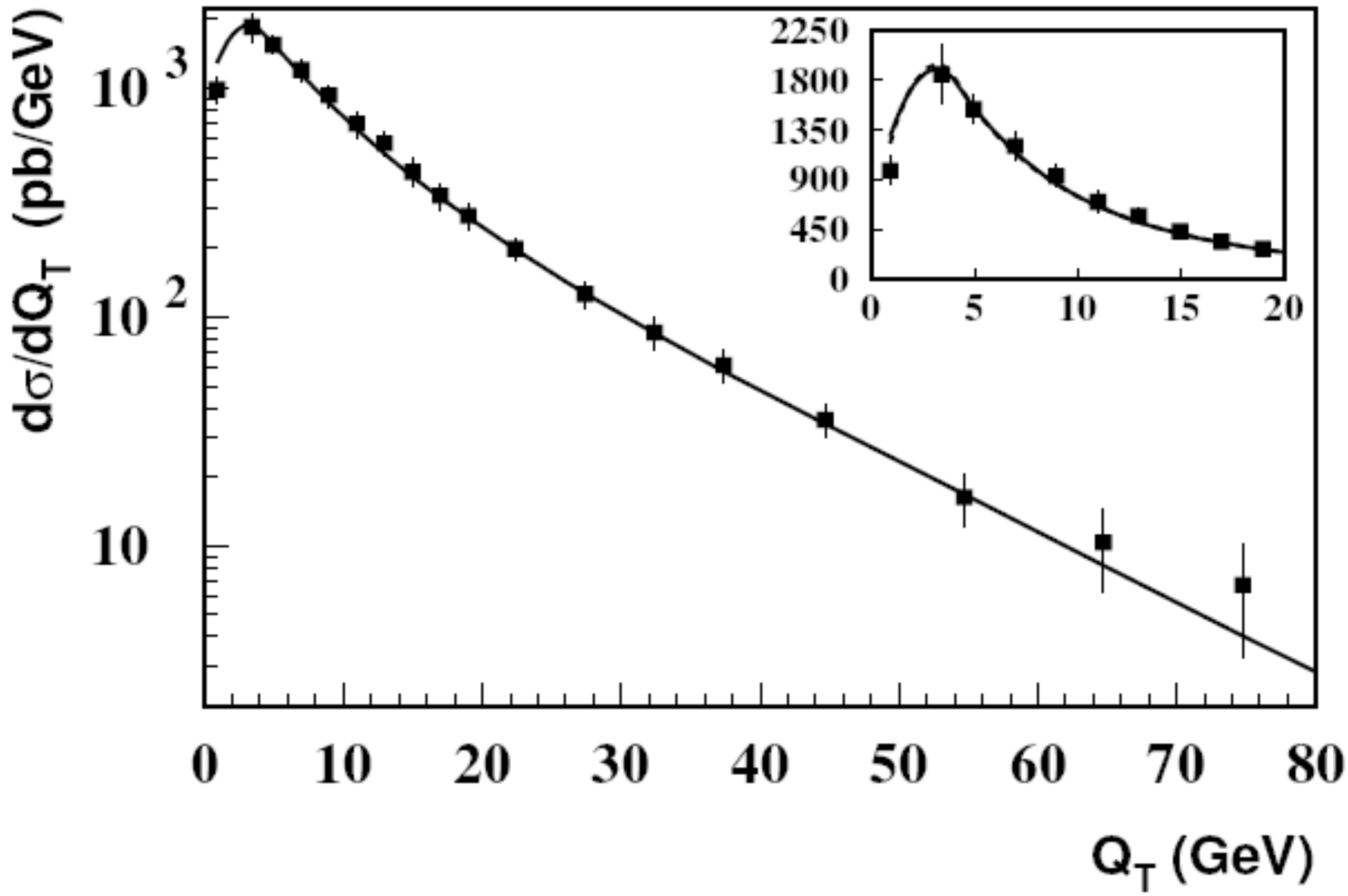
Qiu, Zhang, PRL 2001



Non-perturbative power correction is very small, excellent prediction!

- Fermilab D0 data on W at $\sqrt{S} = 1.8$ TeV

Qiu, Zhang, PRL 2001



No free fitting parameter!

Hadronic Upsilon Production

- Process: $$A(p_A) + B(p_B) \to b\bar{b}(Q)[\to \Upsilon(p) + \bar{X}] + X'$$

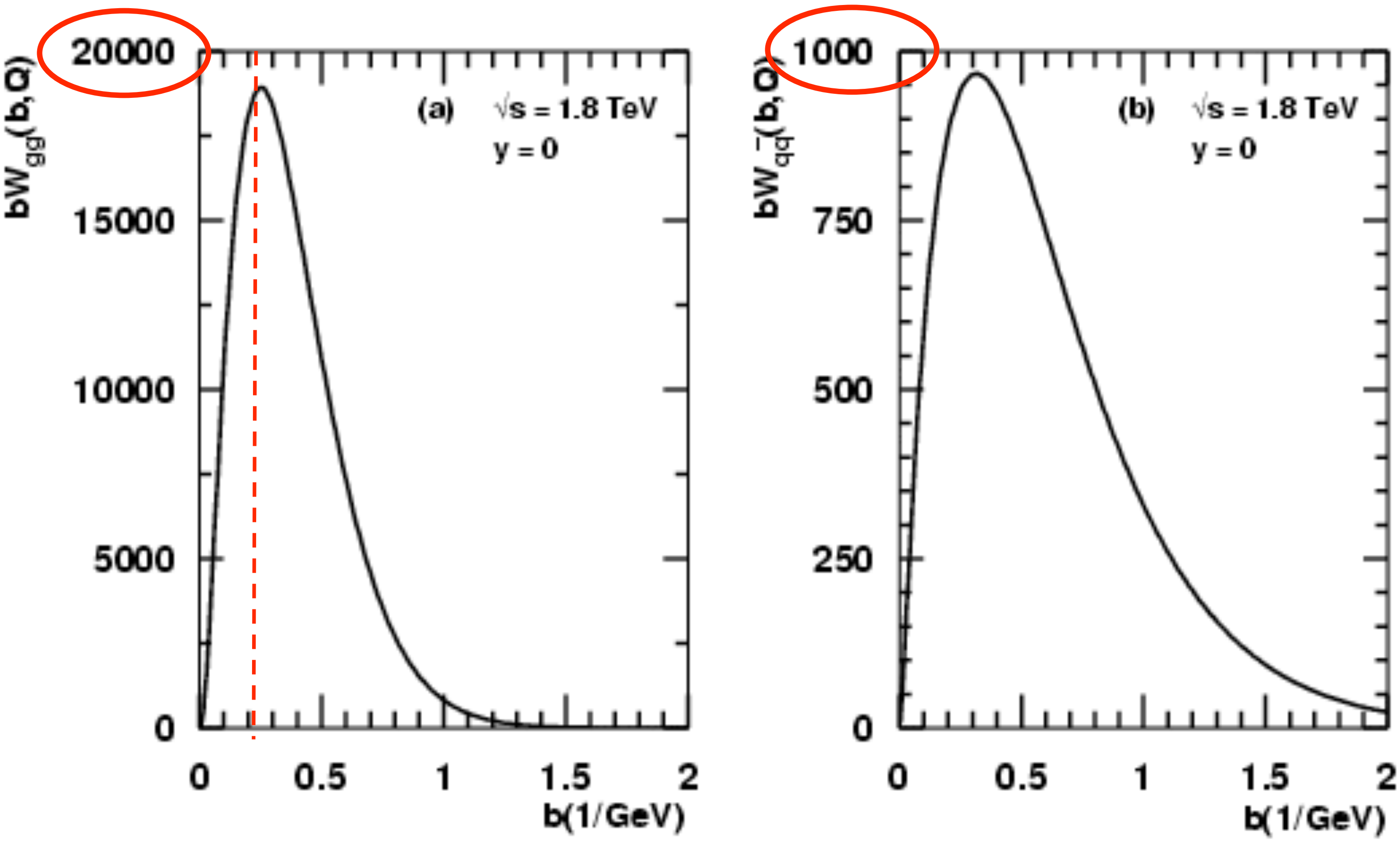
- Similarities and differences from W/Z, or Higgs

  - Events are dominated by low Q_T region
  - Gluon shower should play an important role in determine the Q_T – distribution
  - $M_Y << M_W$, or Q is now small
  - Heavy b-quark pair is not necessary color singlet
  - Additional nonperturbative physics from b-quark to Upsilon

- Key approximation:

  Gluon radiation from heavy quarks has a less effect on the Q_T-distribution than that from radiation of initial-state light partons

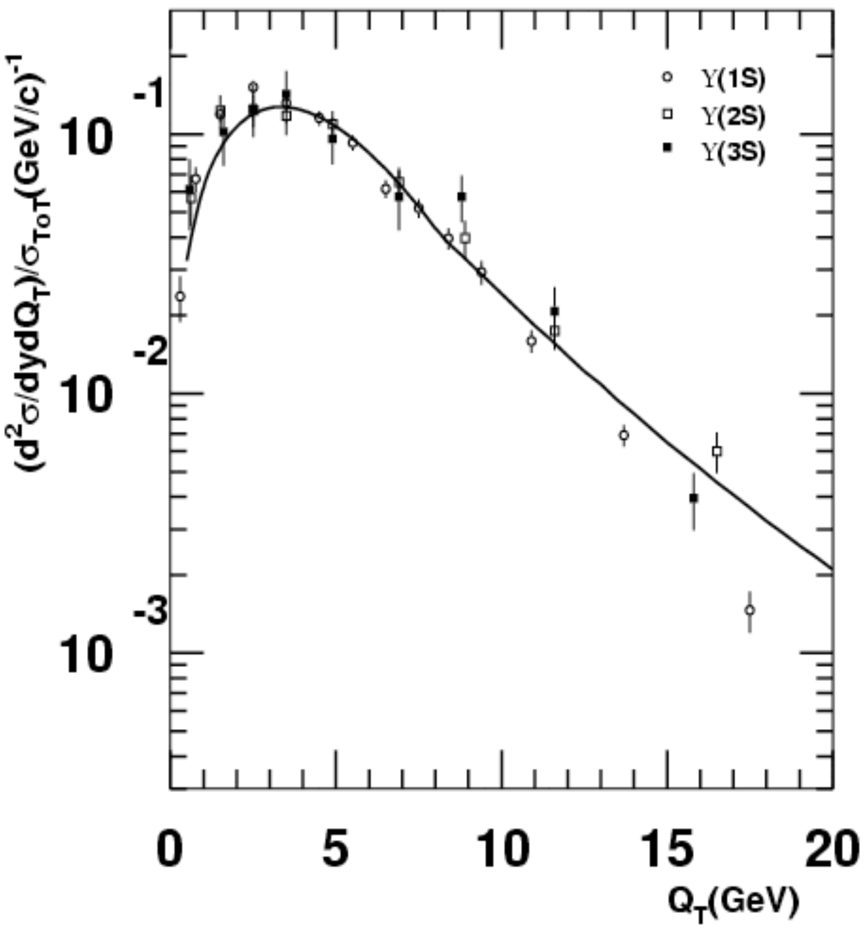
The b-space Distribution

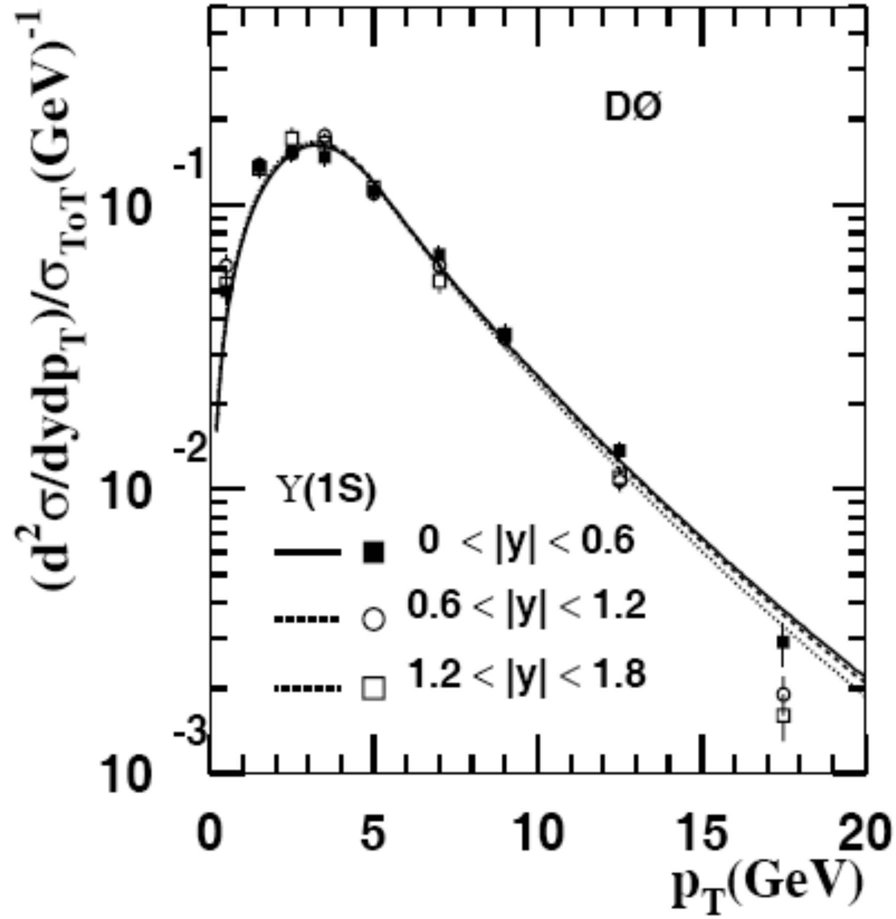


- ✧ Gluon-gluon dominate the production
- ✧ Dominated by perturbative contribution – small b region!

Upsilon Production at Tevatron

Berger, Qiu, Wang





CDF Run-I data

DO Run-II data

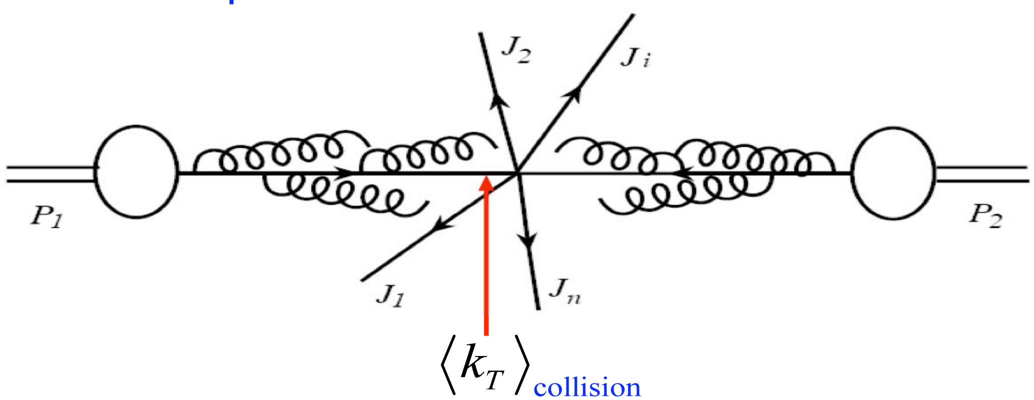
When is k_T-factorization needed?

- Recall: Necessary condition for QCD factorization:

  Scattering is dominated by the region of the phase space where the scattering partons are almost on-shell

$$k^2 = 0 \quad \Rightarrow \quad k^\mu = xp^\mu + \frac{k_T^2}{2xp\cdot n} n^\mu + k_T^\mu$$

- Need k_T-factorization if $\langle k_T \rangle_{\text{collision}} \sim xp^+$
- Need k_T-factorization if $\langle k_T \rangle_{\text{Collision}} \sim Q_{\text{observed}} \ll Q_{\text{Hard}}$



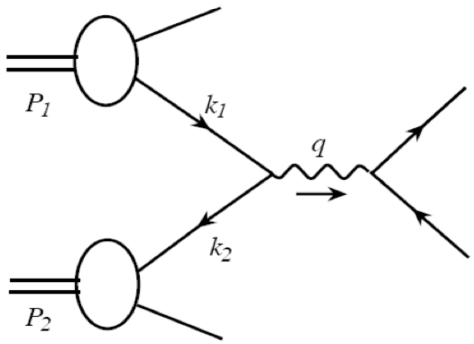
$$\langle k_T \rangle_{\text{collision}} \sim \ell n\left(Q_{\text{Hard}}\right) \propto \ell n\left(\sqrt{S}\right)$$

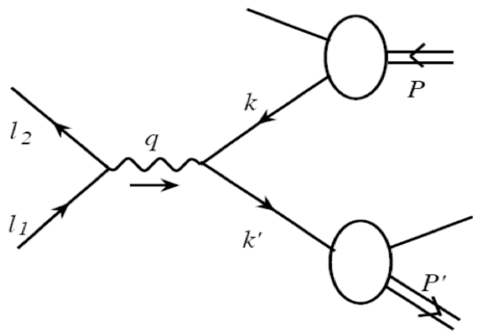
- ❖ Leading logarithm included in DGLAP to take care of the rate of partonic flux
- ❖ kinematics in transverse direction is approximated
- ❖ $\langle k_T \rangle/Q$ is also neglected

k_T-factorization can be violated

- Leading power k_T-factorization is valid for Drell-Yan and SIDIS process

Collins, Soper, …
Ji, Ma, Yuan, …

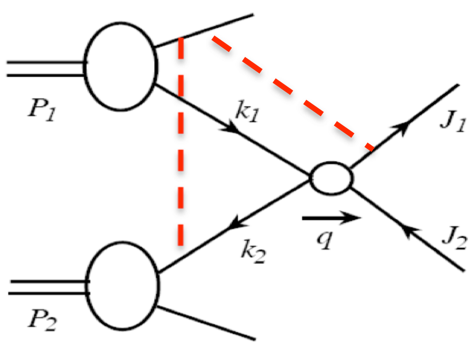




Key: color singlet boson
"universal" TMD

- Leading power k_T-factorization is violated in multiple jet production if jet momentum imbalance is of order k_T

Collins, Qiu, …



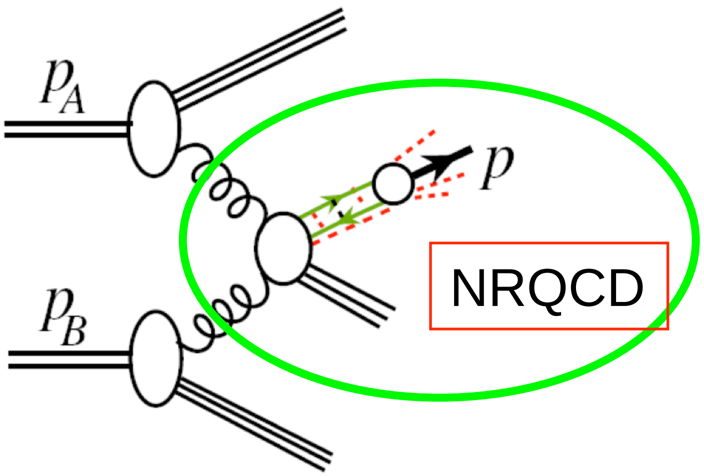
- Shown by a counter example
- Affect both spin averaged and spin dependent observables

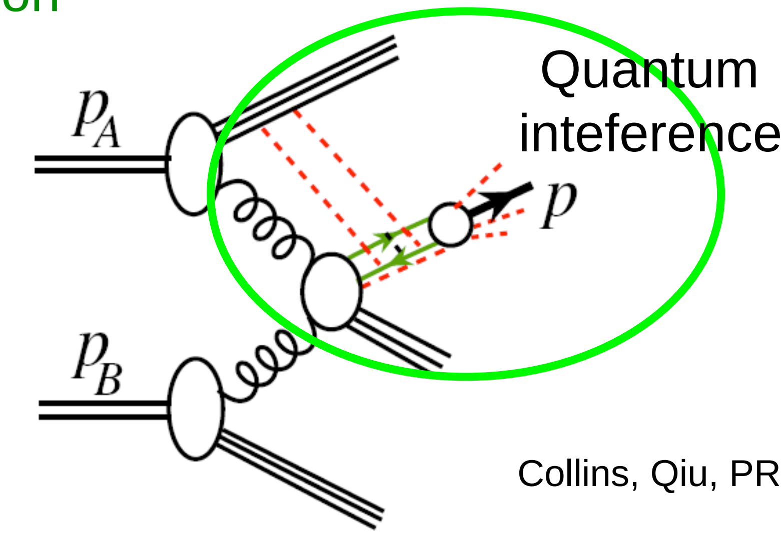
Key: color flow,
non-universal TMD

Hadronic Heavy Quarkonium Production

- NRQCD factorization has not been proved theoretically
- NRQCD Factorization fails for low p_T:

Low p_T requires pQCD k_T-factorization





Collins, Qiu, PRD 2007

- NRQCD Factorization might work for large p_T

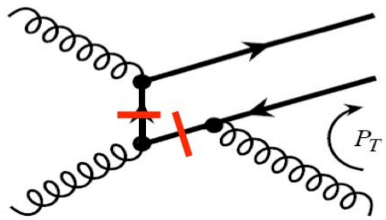
Spectator interactions are suppressed by $(1/p_T)^n$

Factorization is necessary for the predictive power

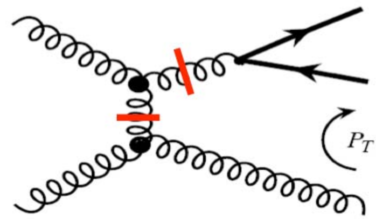
Combination of pQCD and NRQCD

- Heavy quarkonium production when $P_T \gg 2M$

Also see Lansberg
In the workshop



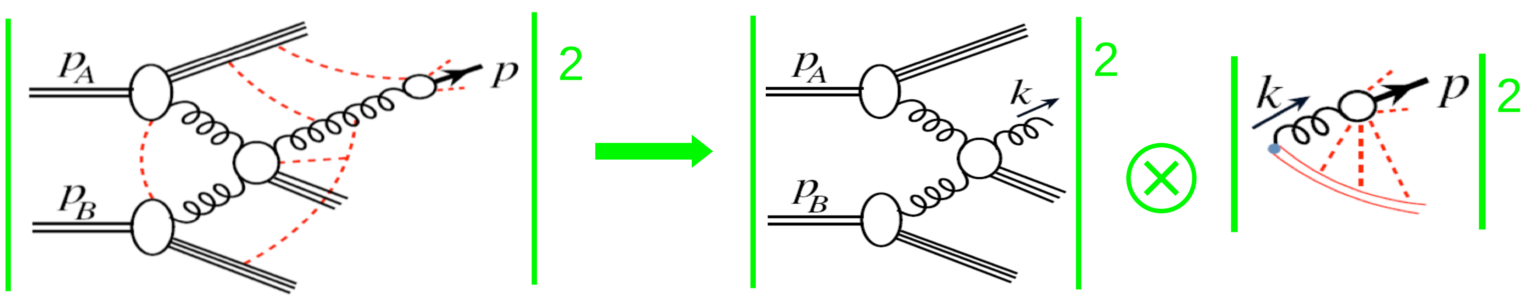
$$\propto \left(\frac{1}{P_T^2}\right)^2$$



$$\propto \frac{1}{P_T^2} \times \frac{1}{(2M)^2}$$

When $P_T^2 \gg (2M)^2$, fragmentation contribution dominates the production

- Combination of pQCD and NRQCD factorization (not proved):
  - ❖ pQCD factorization to isolate heavy quarkonium physics into the universal fragmentation function



$$+ \frac{M^2}{P_T^2}$$

  - ❖ NRQCD factorization to isolate non-perturbative physics of the fragmentation function into NRQCD matrix elements

Connection to NRQCD Factorization

- Proposed NRQCD factorization:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T)\,\langle \mathcal{O}_n^H \rangle$$

- Proved pQCD factorization for single hadron production:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

- Prove NRQCD Factorization

⟷ To prove: at $\mu_0 \sim 2m_c$

$$\boxed{D_{H/i}(z,m_c,\mu) = \sum_n d_{i\to c\bar{c}[n]}(z,\mu,m_c)\,\langle \mathcal{O}_n^H \rangle}$$

with
- ❖ $d_{g\to c\bar{c}[n]}(z,\mu,m_c)$ safe
- ❖ $\langle \mathcal{O}_n^H \rangle$ gauge invariant and universal
- ❖ independent of the direction of the Wilson lines

- An all order proof is still lacking!

Heavy Quarkonium Associated Production

- Inclusive J/ψ + charm production:

$$\sigma(e^+e^- \to J/\psi c\bar{c})$$

Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb

NRQCD-LO: ~ 0.07 pb

Kiselev, et al 1994,
Cho, Leibovich, 1996
Yuan, Qiao, Chao, 1997
…
Zhang, Chao, 2007 (NLO)

- Ratio to light flavors:

$$\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X)$$

Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

- Message:

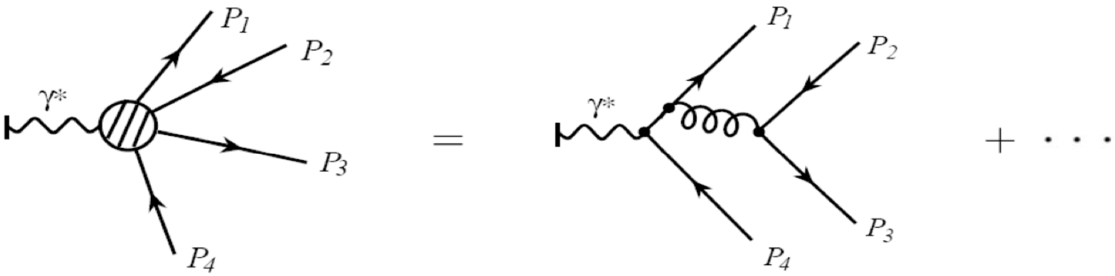
Production rate of $e^+e^- \to J/\psi c\bar{c}$ is larger than

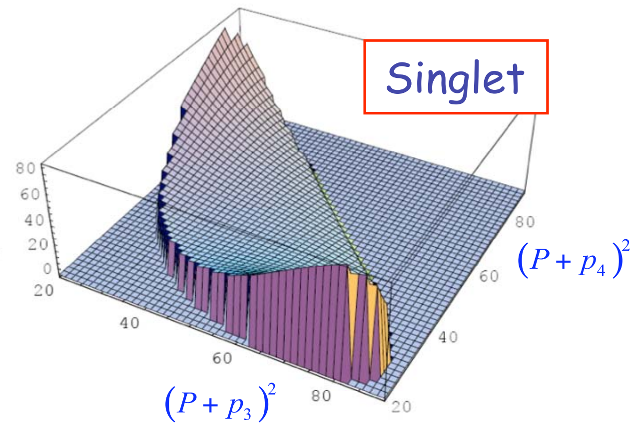
all these channels: $e^+e^- \to J/\psi gg$, $e^+e^- \to J/\psi q\bar{q}$, ...

combined ?

Associated Production at B-factory

- Kinematically preferred configuration:

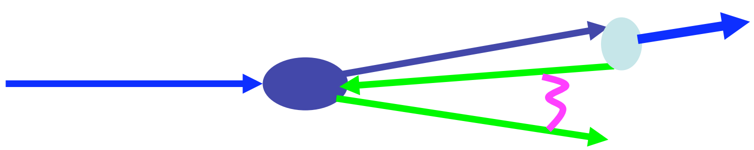




Production rate of a singlet charm quark pair is dominated by the phase space where $s_3=(P_1+P_2+P_3)^2$ or $s_4=(P_1+P_2+P_4)^2$ near its minimum

- NRQCD formalism does not apply when there are more than one heavy quark velocity involved

- Color transfer enhances associated quarkonium production

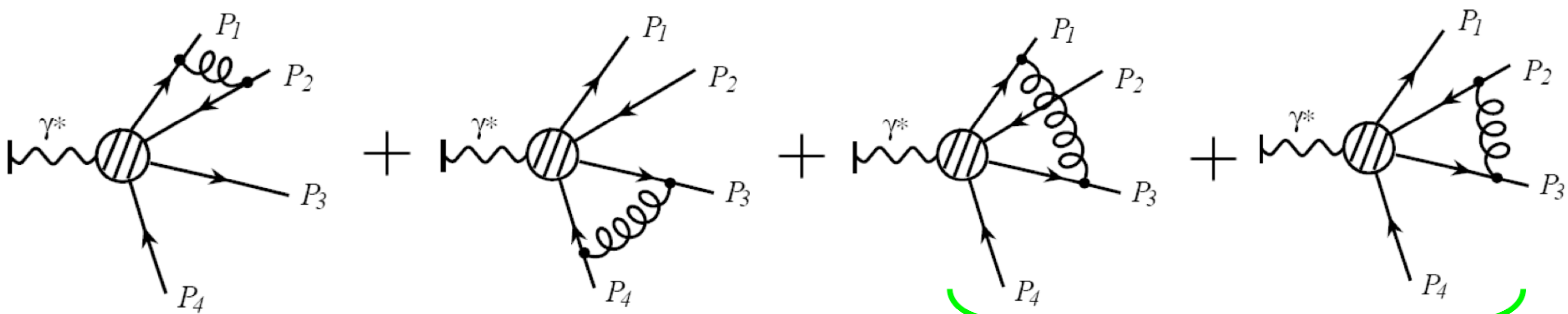


A heavy quark as a color source to enhance the transition rate for an octet pair to become a singlet pair

Nayak, Qiu, Sterman, PRL 2007

Soft-gluon Enhancement – Color Transfer

- Soft gluons between heavy quarks:



active pair: P_1, P_2; spectators: P_3, P_4

$$\beta_{ij} \equiv \sqrt{1 - 4m^2/(P_i + P_j)^2}$$

- There are three heavy quark velocities:

NRQCD approach is not well defined in this region

- Soft gluon between a heavy quark pair:

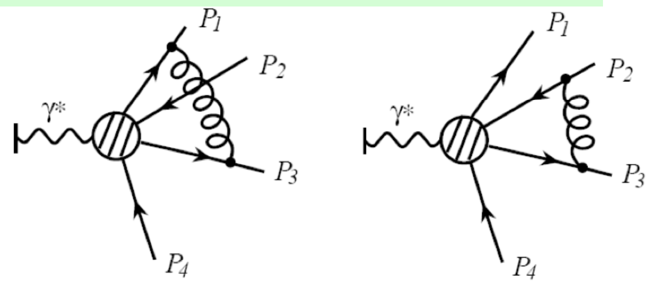
$$-i\,g^2 \int \frac{d^D k}{(2\pi)^D} \frac{4 P_i \cdot P_j}{[2P_i \cdot k + k^2 + i\epsilon][-2P_j \cdot k + k^2 + i\epsilon][k^2 + i\epsilon]}$$

$$= \frac{\alpha_s}{2\pi}\left[-\frac{1}{2\varepsilon}\left(\frac{1}{\beta_{ij}} + \beta_{ij}\right)(2\beta_{ij} - i\pi) \;+ \ldots\right] \Longrightarrow i\,\frac{1}{\varepsilon}\,\frac{\alpha_s}{\beta_{ij}}$$

Associated Production is Enhanced

- NLO correction to the amplitude:

$$\mathrm{Im}\left[\mathcal{A}_{13}+\mathcal{A}_{23}\right] = \frac{\alpha_s}{4\varepsilon}\,\mathcal{A}^{(0)}(P_i)\left[\frac{1+\beta_{13}^2}{\beta_{13}}-\frac{1+\beta_{23}^2}{\beta_{23}}\right]$$



Does not contribute to NLO production rate in NRQCD

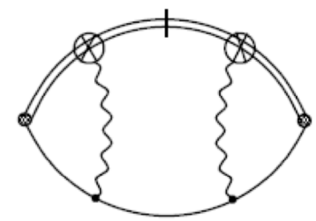
Zhang, Chao, PRL 2007

- Estimate enhancement factor from NNLO in NRQCD:

✧ Velocity expansion:

$$\frac{1}{\beta_{13}}-\frac{1}{\beta_{23}} \sim -\frac{4}{\beta_S^3}\,\frac{q_S\cdot q}{m^2} \sim \frac{4}{\beta_S^2}\,v\,\cos\phi_S$$

$$\beta_S=\sqrt{\frac{-q_S^2}{m^2-q_S^2}}$$



$$P_3^\mu=\frac{P_0^\mu}{2}\sqrt{1-\frac{q_S^2}{m^2}}+q_S^\mu$$

$$P_0^\mu=(2m,0)\ \text{and}\ q_S\cdot P_0=0$$

✧ Velocity-ordered region:

$$\beta_S<1,\quad \frac{v}{\beta_S}<1$$

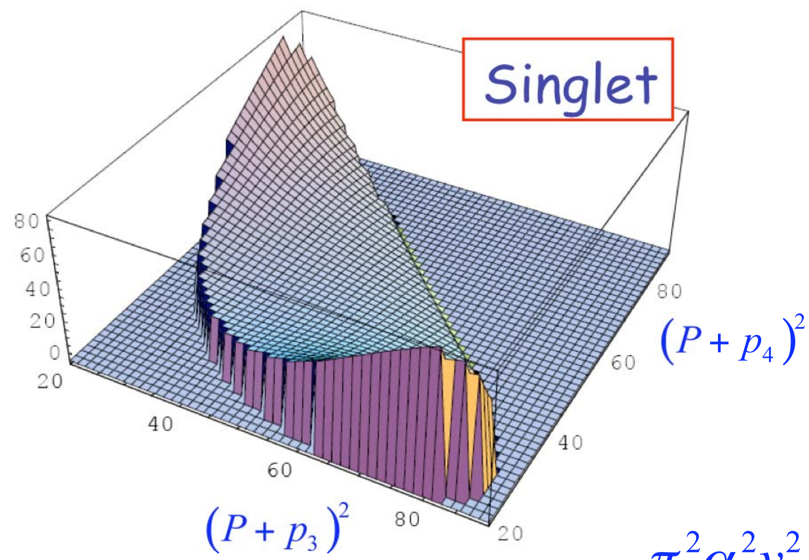
✧ the enhancement factor: $\left|A_{Singlet}^{NNLO}\right|^2 \sim \left(C_{8\to 1}\,\frac{\alpha_s^2 v^2}{\varepsilon^2}\right)\left(\frac{\pi^2}{\beta_s^4}\right)\left|A_{Octet}^{LO}\right|^2$

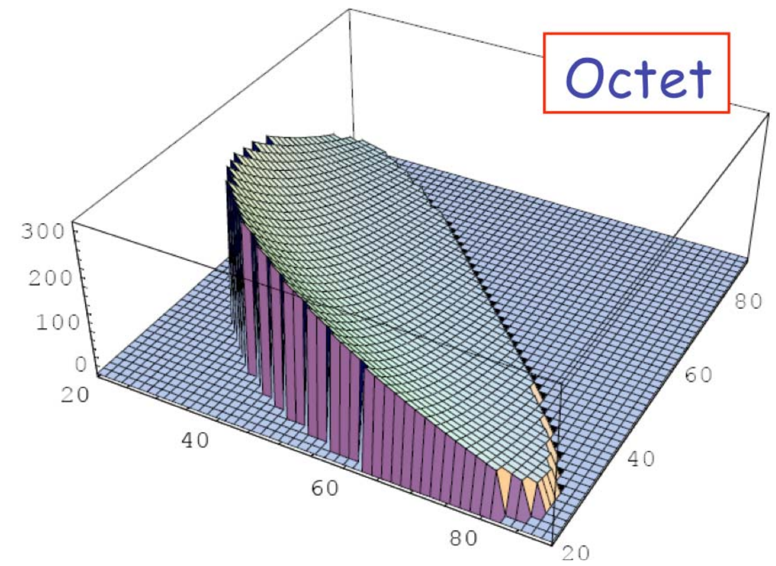
All other two-loop diagrams give a single pole !

Nayak, Qiu, Sterman, 2007

Numerical Enhancement from NNLO

- LO hard parts with color factor:





- Matrix elements: $\frac{\pi^2 \alpha_s^2 v^2}{\varepsilon^2} \Rightarrow \langle O_8 \rangle$

$$d\sigma^{\text{tot}}_{e^+e^-\to H+X}(p_H) \sim d\hat{\sigma}_{e^+e^-\to Q\bar{Q}[S_1]+Q'(\beta_S)}(p_H)\,\langle {}^3\mathcal{S}_1^H\rangle + d\hat{\sigma}_{e^+e^-\to Q\bar{Q}[S_8]+Q'(\beta_S)}(p_H)\,\frac{\langle {}^3\mathcal{S}_8^H\rangle}{\beta_S^4}$$

Two terms are equally important if $\beta_s \sim 0.3$

Same feature for heavy quark fragmentation

Nayak, Qiu, Sterman, 2007

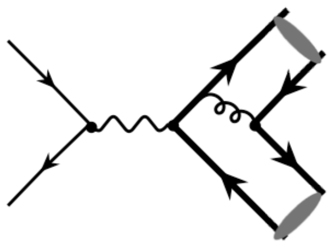
Kang, et al. 2007

Exclusive Processes

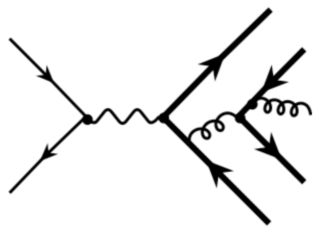
- All particles are identified:

$$e^+e^- \to J/\psi + \eta_c, \quad \pi+\pi \to \pi+\pi, \quad \pi+p \to \pi+p, \quad ...$$

- no free-space for extra radiation:



+…   But,



Not allowed unless the final-state gluon is absorbed into the matrix elements (wave functions) of quarkonia

- Conservation of quantum numbers:

It is easy and an advantage to use the conservation of fundamental Symmetry due to the small number of particles involved

Ex: charge conjugation forbids the S-wave J/ψ+J/ψ final-state

- Factorization and the color:

For factorization and the partonic calculation to work, soft interaction between produced hadrons has to be strongly suppressed!

Double Charmonia Production

❑ Exclusive production:

[4] Li, He, and Chao, [6] Braaten and Lee

$J/\psi\ c\bar{c}$	$\eta_c(1S)$	χ_{c0}	$\eta_c(2S)$
BABAR	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$16.4 \pm 3.7^{+2.4}_{-3.0}$
Belle [14]	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
NRQCD [6]	2.31 ± 1.09	2.28 ± 1.03	0.96 ± 0.45
NRQCD [4]	5.5	6.9	3.7

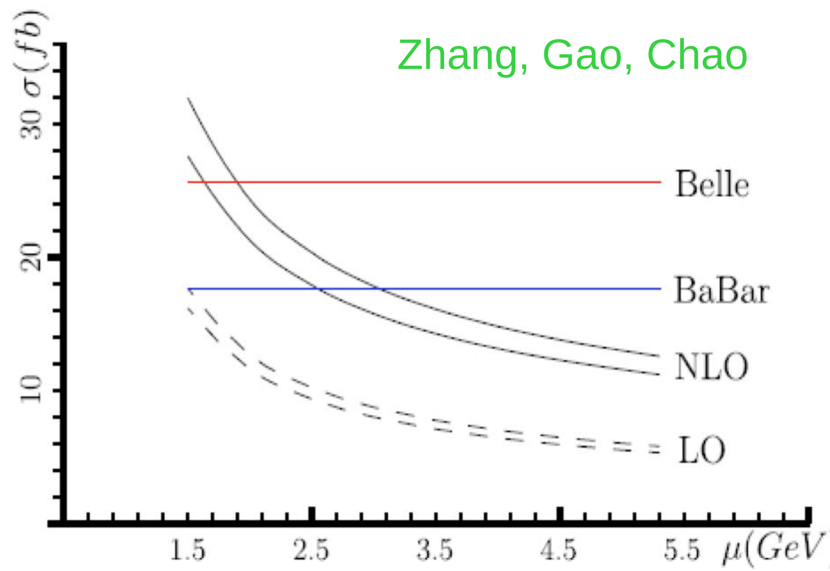
LO

❑ Possible resolution for $J/\psi+\eta_c$:

- ❖ NLO correction: $K_{factor} = 1.96$
- ❖ Relativistic Correction:
  - X-section: $K_{factor} = 1.34$
  - Wave func: $K_{factor} = 1.32$

Combined: $K_{factor} = 4.15$

$$\sigma[e^+e^- \to J/\psi + \eta_c] = 17.5 \pm 5.7\ \text{fb}$$



Bodwin et al. hep-ph/0611002

Summary

- QCD is very rich in dynamics, much more than QED, while QED is the underline theory of all excitements of CMP, …
- After 35 years, we have learned only a very small part of QCD dynamics: less than 0.1 fm, although we have been successful
- There are many research directions for exploring QCD dynamics: QCD at high density, QCD at finite temperature, Condensed QCD matter, …
- Most important, how hadrons were formed off quarks and gluons? Discovery of new hadronic resonances and their properties provides critical information on hadron formation!

Thank you for your attention!