The CKM matrix: status and perspectives

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http://ckmfitter.in2p3.fr

CP : a deep symmetry

C charge conjugation, P spatial parity,

T time reversal

C, P, T are fundamental discrete symmetries

C and P are maximally violated (chiral fermions)

CP and T are violated at the 10^{-5} level

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History of the discoveries

1964 indirect CP violation in the neutral kaon system

1998 T violation in the neutral kaon system

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CP violation is very well described by the Standard Model, but we still don't have any dynamical explanation of it

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Quark mixing

Standard Model: the quark flavors are mixed by the weak interaction

$\longrightarrow$ bi-diagonalization on the eigenstate basis *via* Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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this unitary matrix is complex ($V_{ub} \propto |V_{ub}|\, e^{-i\gamma}$) as soon as there are at least three generations of non-degenerate fermions

CKM generates CP violation

CP violation in meson decays

CP violation in mixing: $M \longrightarrow \ell^{+}X \neq \overline{M} \longrightarrow \ell^{-}X$

ε_K, $A_{sl}(B)$

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CP violation in interference between mixing and decay: $M \to \overline{M} \to f \neq \overline{M} \to M \to \bar{f}$

$B \to J/\psi K_S$, $B \to \pi\pi$, $B \to \rho\rho$

Hierarchy and Unitarity Triangle(s)

strong hierarchy of the CKM matrix:

diagonal couplings $\propto 1$

1st $\leftrightarrow$ (resp. 2nd $\leftrightarrow$ 3rd) generation
$\propto \lambda \sim 0.22$ (resp. $\propto \lambda^2$)

1st $\leftrightarrow$ 3rd generation $\propto \lambda^3$

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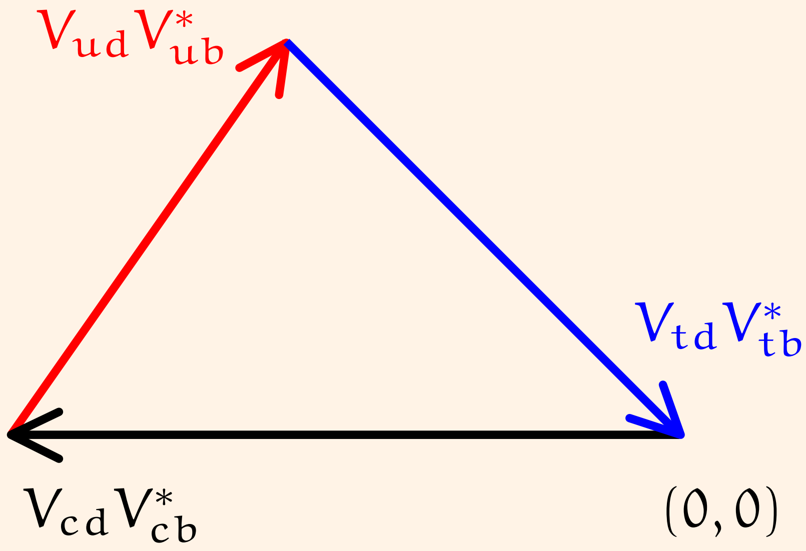
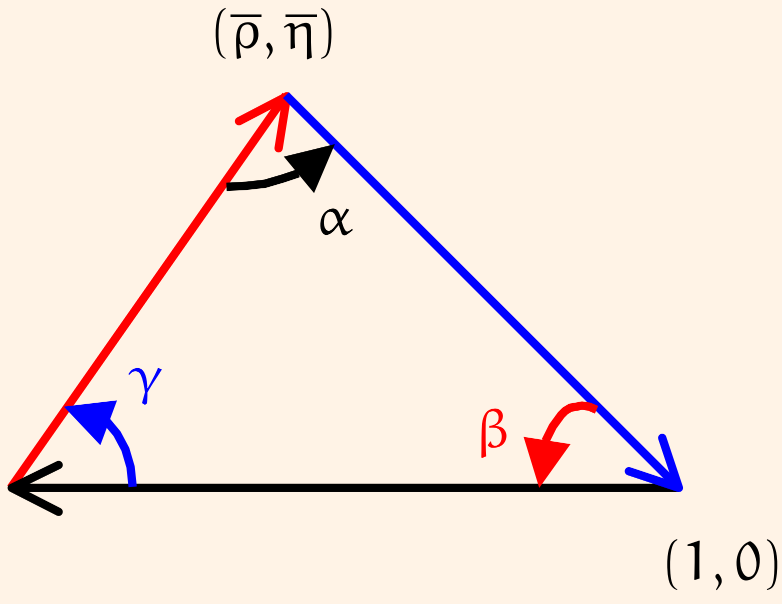
CKM unitarity $\Rightarrow$ six triangles in the complex plane, of which four are quasi flat, two are non flat and quasi degenerate





unitary-exact and convention-independent version of the Wolfenstein parametrization

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

note the East/West conversion: $\alpha = \phi_2, \ \beta = \phi_1, \gamma = \phi_3$

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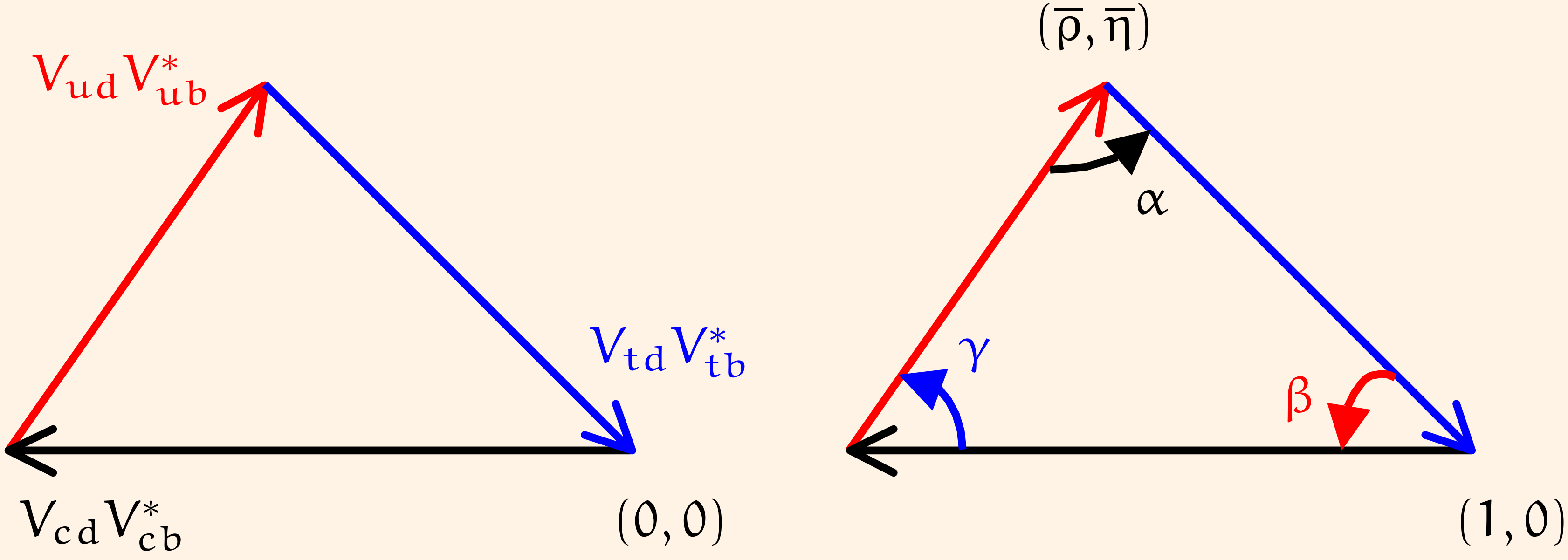
there is no need to stop at $\mathcal{O}(\lambda^4)$!

$$\bar{\rho} + i\,\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



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Extracting Standard Model parameters

tree-level charged transitions are well constrained

FCNC in mixing ($\Delta S = \Delta D = 2$, $\Delta B = \Delta D = 2$ and now $\Delta B = \Delta S = 2$) are well constrained

QCD and the extraction of CKM elements

CP-conserving couplings are extracted from observables that also depend on hadronic matrix elements, which have to be computed from theory (*e.g.* lattice simulations)

typically, $\Gamma \sim |V_{\mathrm{CKM}}|^2 \, |\langle f \,|\, O \,|\, i \rangle|^2$

in principle CP-violating CKM angles can be determined from experimental quantities only

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$$\mathcal{A}_{\mathrm{CP}} \sim \mathrm{Im}\left(V^*_{\mathrm{CKM}}/V_{\mathrm{CKM}}\right)\left(\overline{M}_{\mathrm{QCD}}/M_{\mathrm{QCD}}\right)$$

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FCNC in decay ($\Delta S = \Delta D = 1$, $\Delta B = \Delta D = 1$ and $\Delta B = \Delta S = 1$) are not so well constrained because of hadronic and/or experimental uncertainties

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SM $\longleftarrow$ QCD

$B(b) \longrightarrow D(c)\ell\nu \qquad |V_{cb}| \longleftarrow f_+^{BD}$ (OPE)

$B(b) \longrightarrow \pi(u)\ell\nu \qquad |V_{ub}| \longleftarrow f_+^{B\pi}$ (OPE)

$B \longrightarrow \ell\nu \qquad |V_{ub}| \longleftarrow f_B$

Examples

(SM: presumably SM-dominated; NP: potentially large NP contributions)

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$B \to J/\psi K_s$	β
$B \to \pi\pi$	α
$B \to DK$	γ

NP

$B \to \phi K_s$	β
$B_s \to \phi\phi$	β_s
$K \to \pi\nu\bar{\nu}$	$\overline{\rho}, \overline{\eta}$

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NP $\leftarrow$ QCD

ε_K	$\overline{\rho}, \overline{\eta} \leftarrow B_K$
$\Delta M_{d,s}$	$\lvert V_{tb} V_{td,s}\rvert \leftarrow B_B$
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$B \to DK$	γ	$B \to \ell\nu$	$\lvert V_{ub}\rvert \leftarrow f_B$

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$B \to \phi K_s$	β	ε_K	$\bar\rho, \bar\eta \leftarrow B_K$
$B_s \to \phi\phi$	β_s	$\Delta M_{d,s}$	$\lvert V_{tb}V_{td,s}\rvert \leftarrow B_B$
$K \to \pi\nu\bar\nu$	$\bar\rho, \bar\eta$	$B \to \ell^+\ell^-$	$\lvert V_{td,s}\rvert \leftarrow f_B$

theoretical errors have to be controlled quantitatively in order to test the Standard Model; there is however no systematic method to do that

The statistical framework

we use a standard frequentist approach: likelihood maximization (χ^2 minimization)

where necessary, we treat non gaussian behavior by Monte-Carlo simulation of virtual experiments

theoretical errors
no model-independent treatment available, due to lack of precise definition; we use the Rfit model: a theoretical parameter that has been computed (e.g. B_K) is assumed to lie within a definite range, without any preference inside this range
the best fit will thus be searched by moving uniformly in the theoretical parameter space

references: A. Höcker *et al.*, EPJC 21 (2001); JC *et al.*, EPJC 41 (2005); http://ckmfitter.in2p3.fr

Part I

the global CKM fit

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uses all constraints on which we think we have a good theoretical control

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$\lvert V_{ud}\rvert, \lvert V_{us}\rvert, \lvert V_{cb}\rvert$	PDG, HFAG and Flavianet WG
ε_K	exp: KTeV/KLOE, theo: CKM06
$\lvert V_{ub}\rvert$	our average
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α	exp: last $\pi\pi$, $\rho\pi$, $\rho\rho$ WA, theo: SU(2)
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$B \to \tau\nu$	exp: last WA, theo: CKM06

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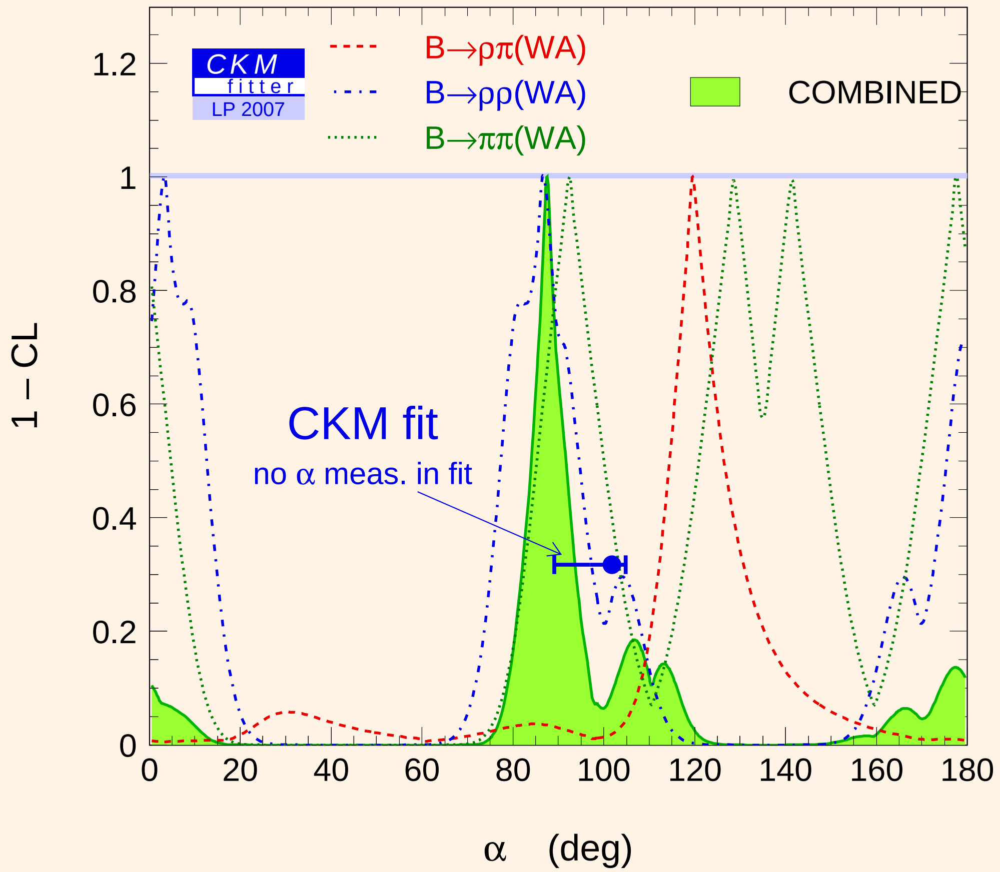
More on selected inputs. . .

the angle α

the best constraint comes from the $\rho\pi$ and $\rho\rho$ modes, which show a tendency to different central values



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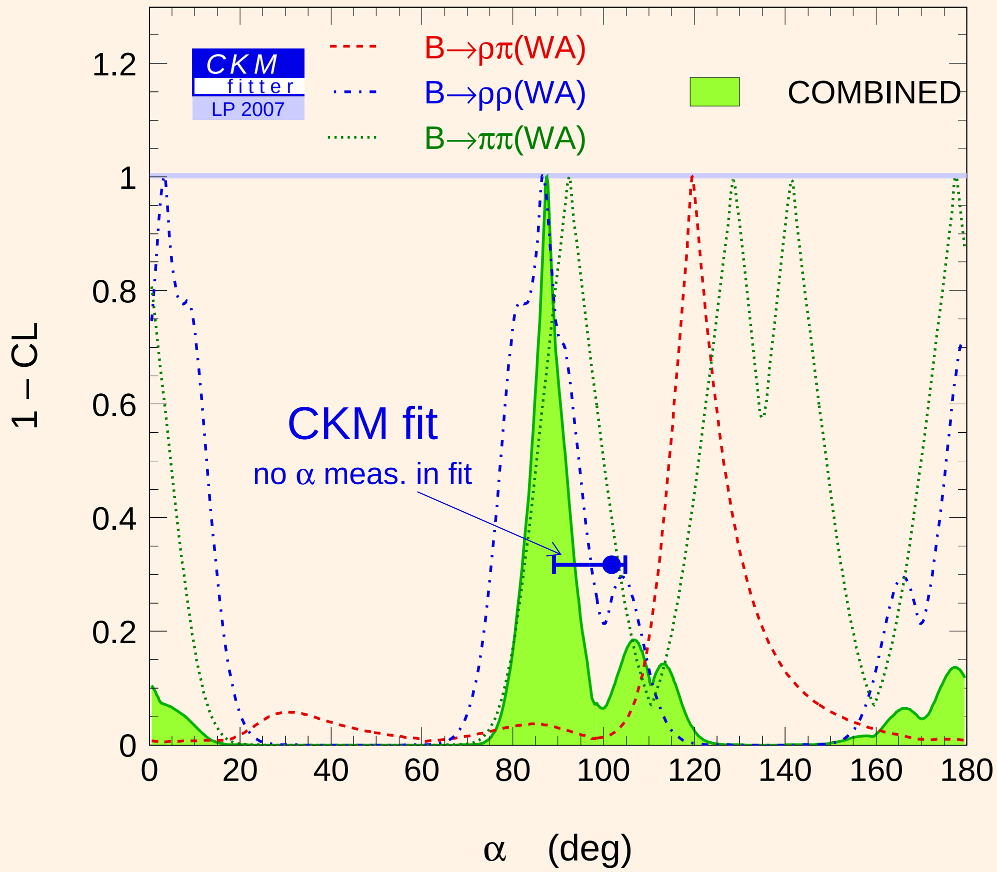

new average $\alpha = (87.7^{+6.4}_{-5.3})^\circ$

…more on selected inputs…

the angle γ (preliminary)

the analysis is non trivial: naive interpretation of χ^2 in terms of the error function underestimates the error on γ because of the bias on r_B due to r_B compatible with 0; both Babar and Belle use their own frequentist approach, while we use a different one

meanwhile the central value of r_B has decreased

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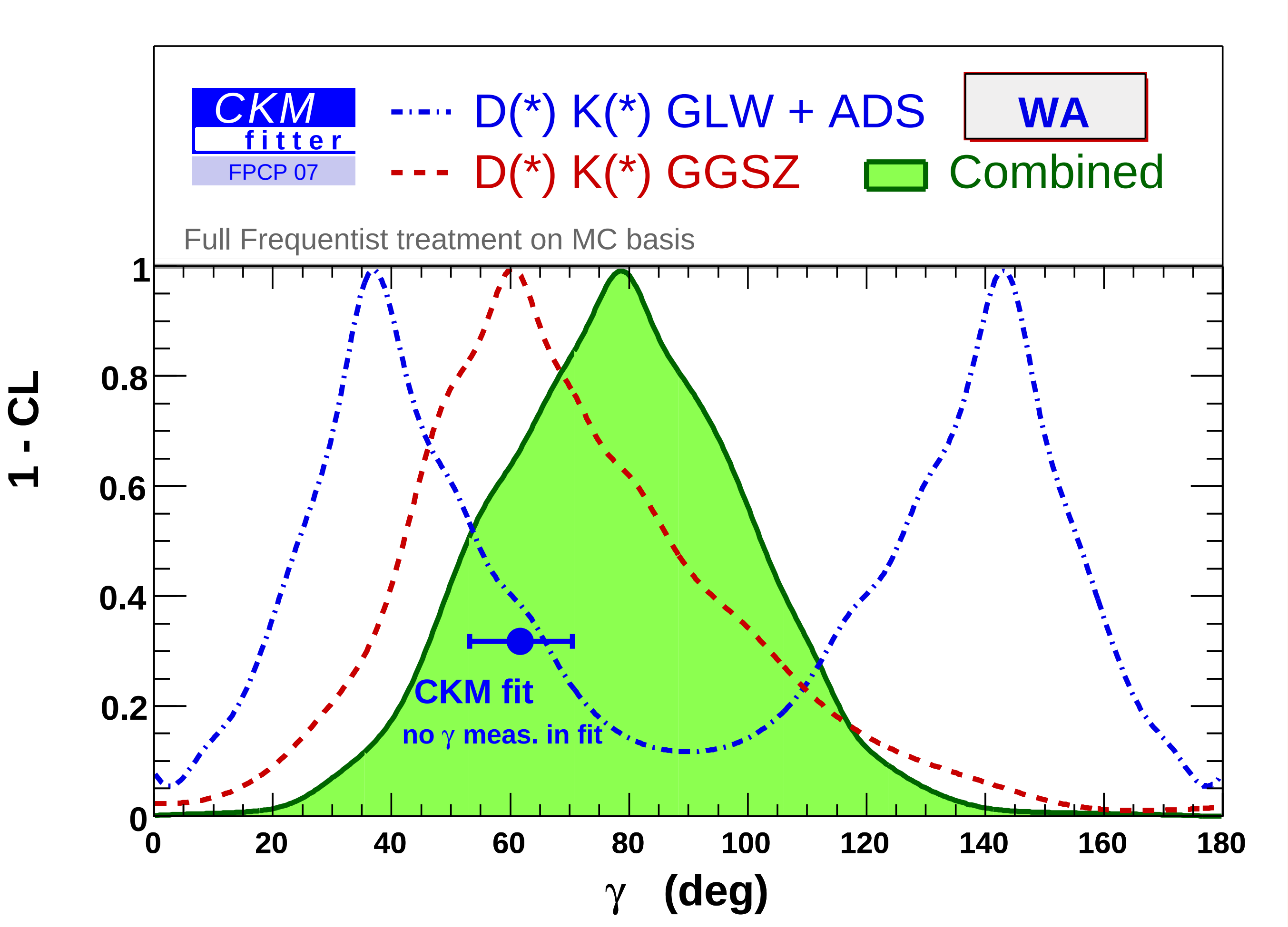
we find a somewhat loose constraint, with $\gamma = (77^{+30}_{-32})^\circ$



…more on selected inputs

the oscillation frequency Δm_s

Fermilab, winter 2006:
first two-sided bound by D0
first evidence for oscillation by CDF;

…more on selected inputs

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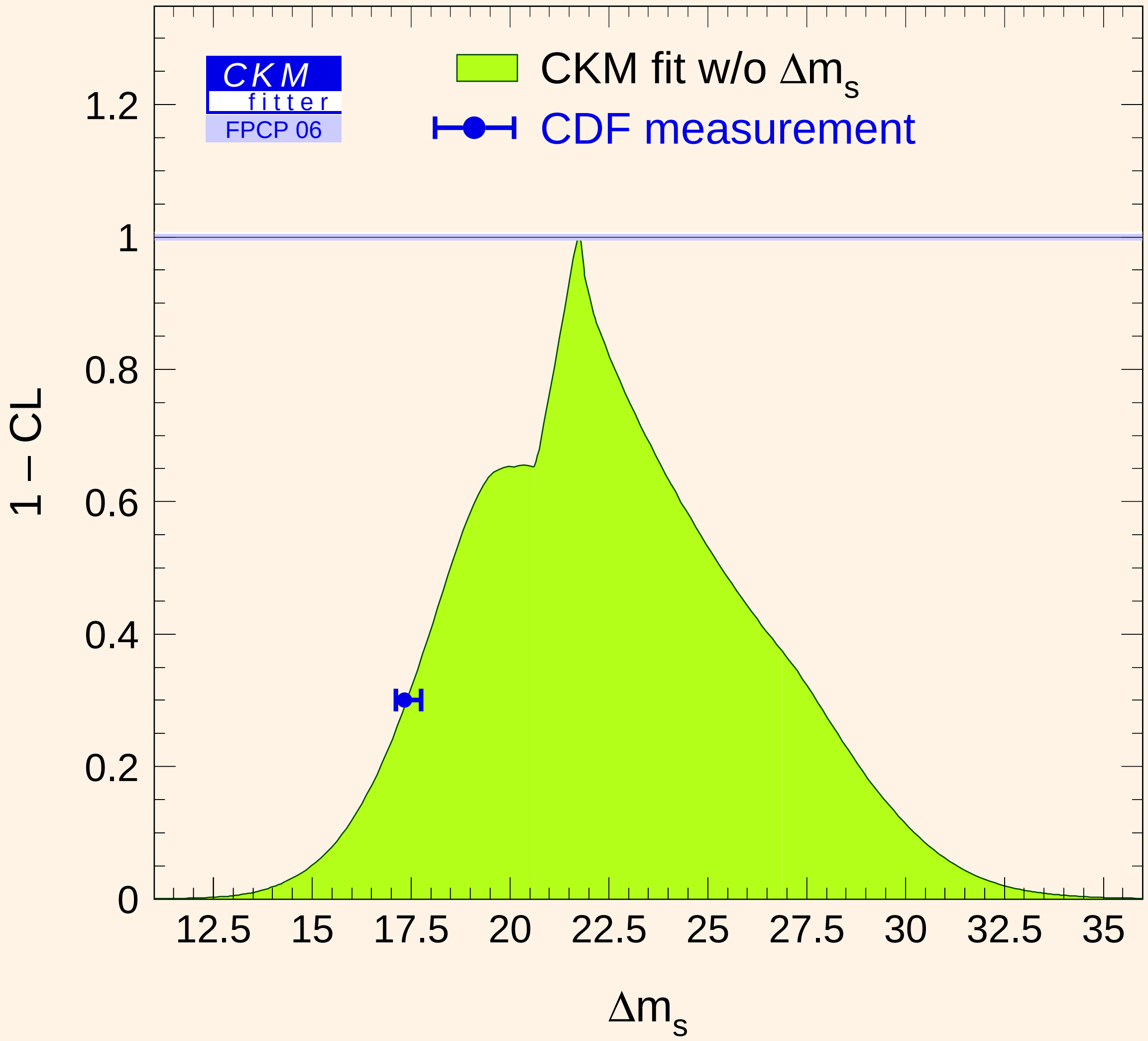
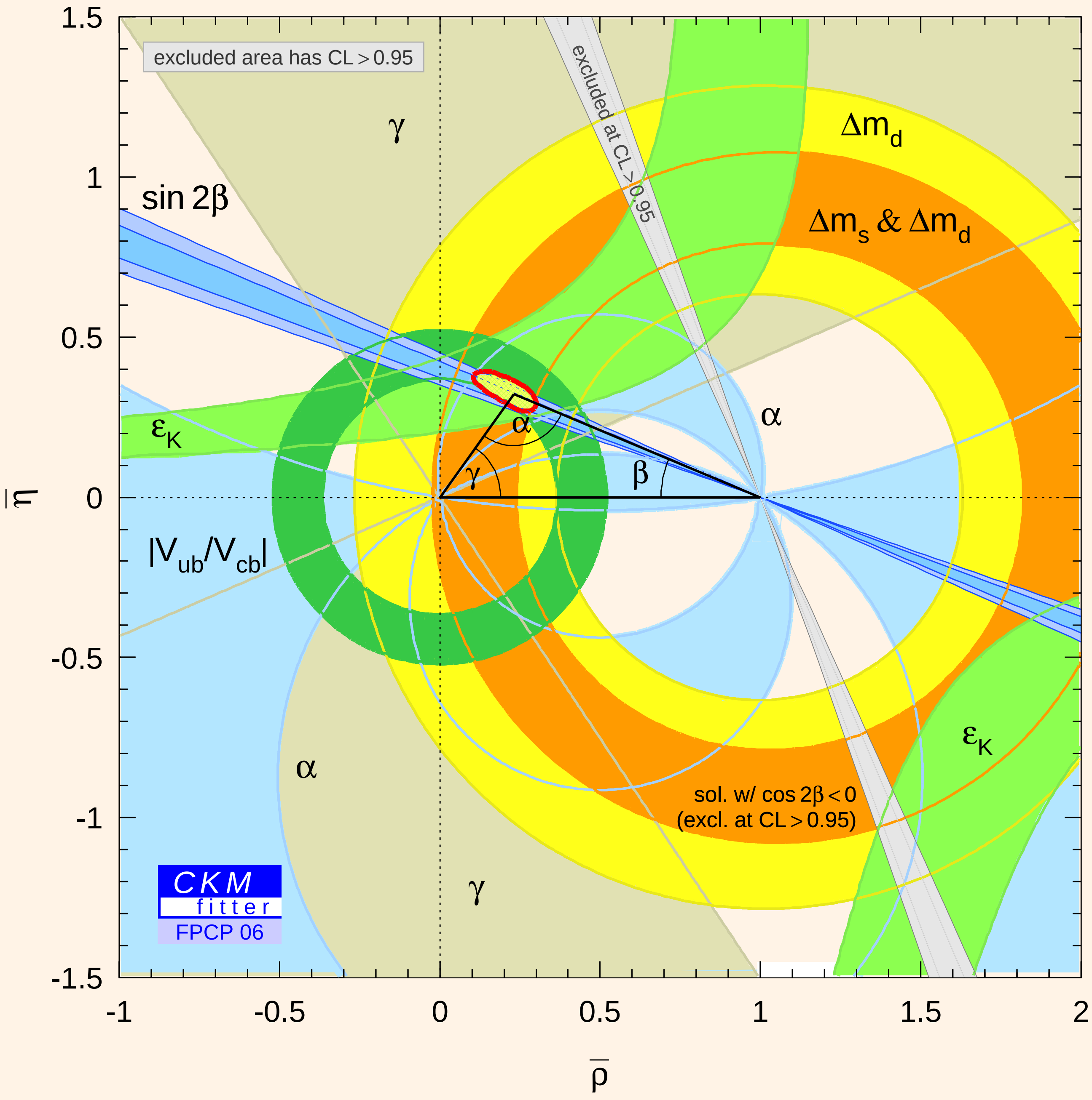
just look at this plot !



The global CKM fit: results...



FPCP06

without Δm_s(CDF)

all constraints together

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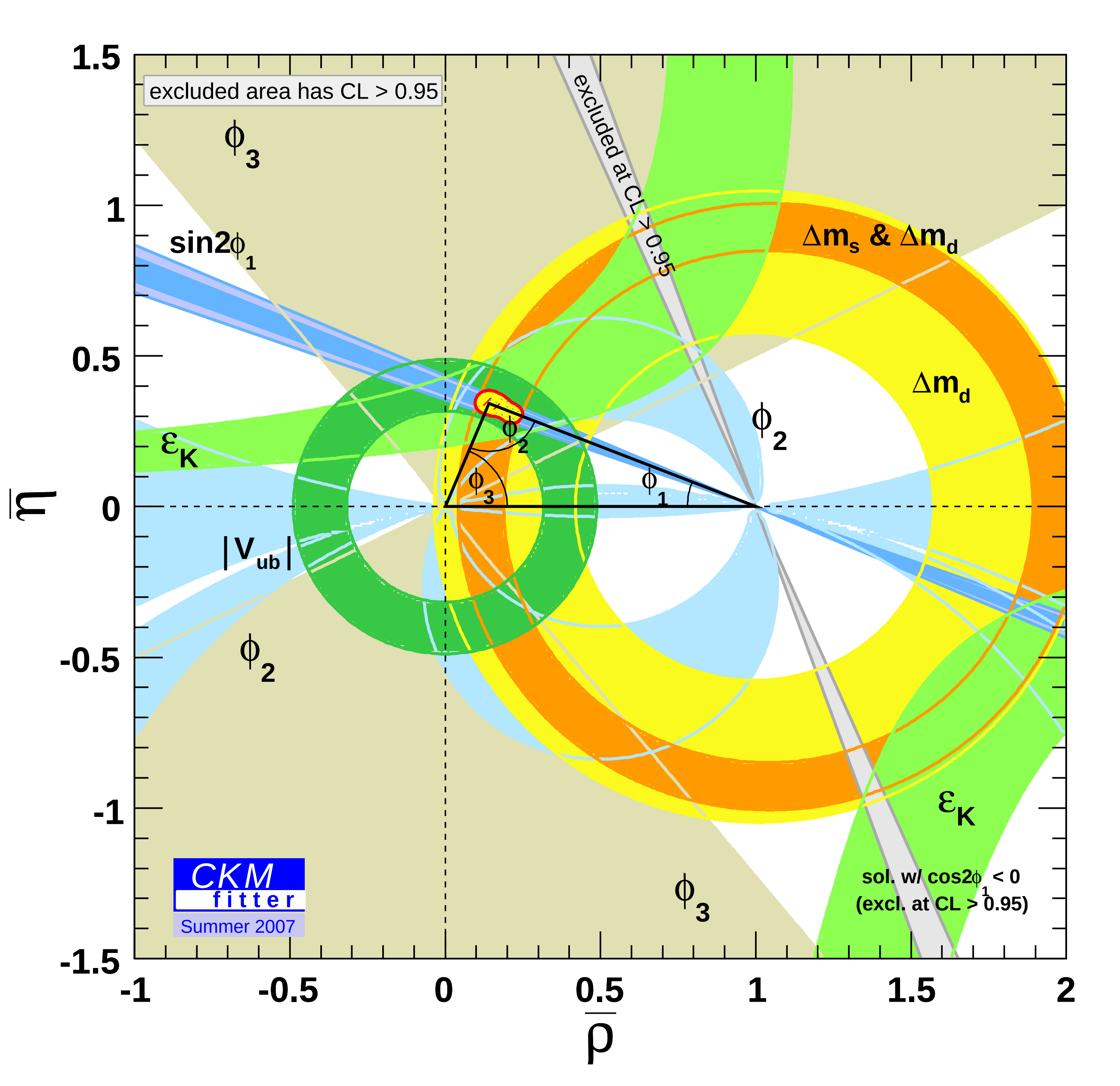
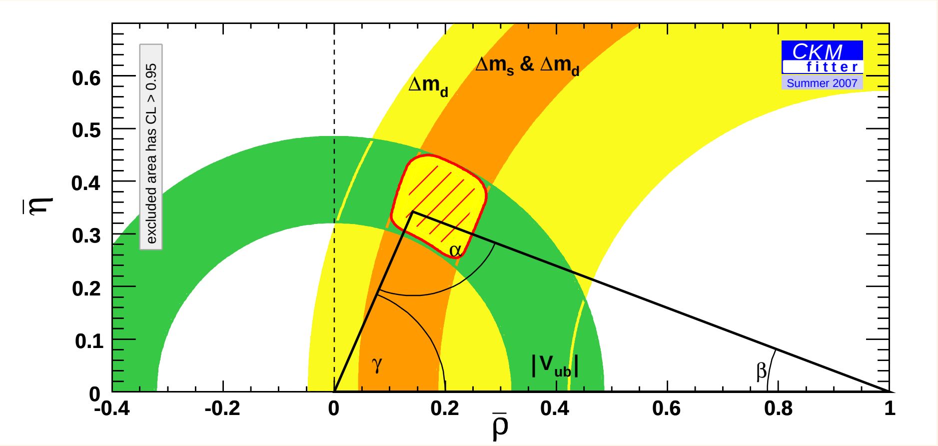


Summer 07

with Δm_s(CDF)

all constraints together

Testing the CKM paradigm



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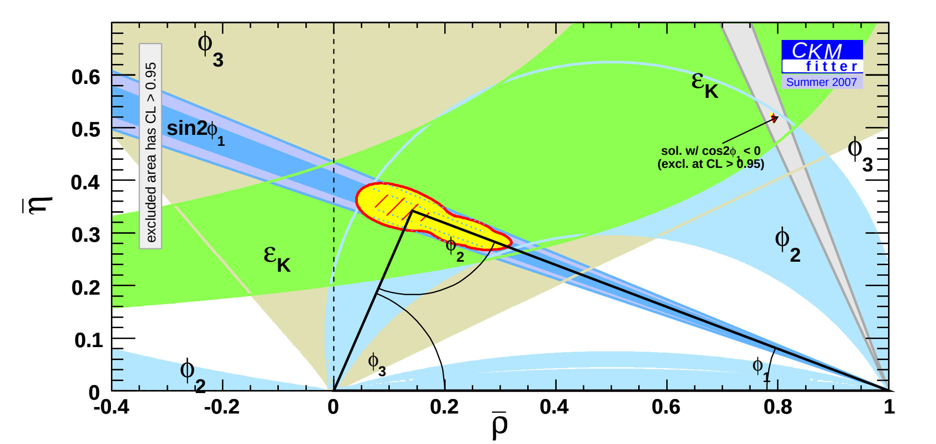

…vs. CP-violating

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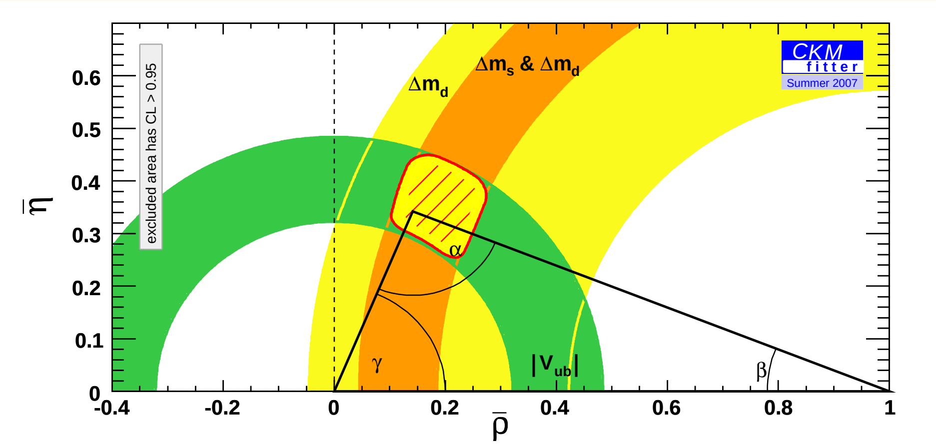

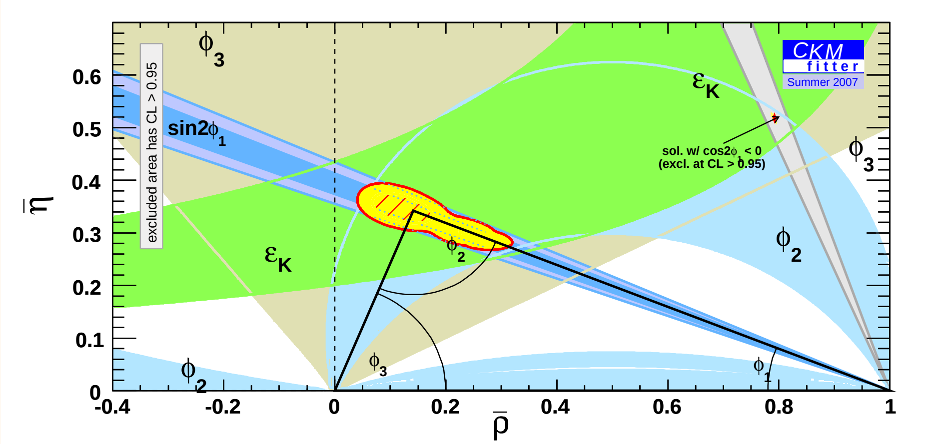

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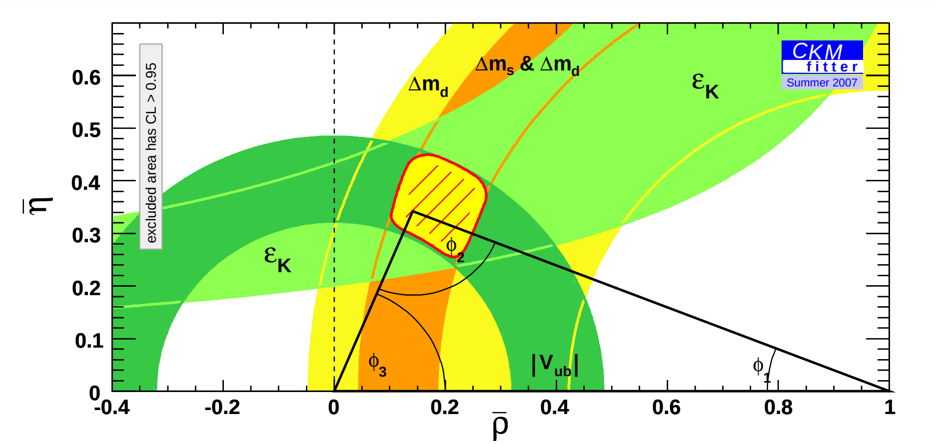

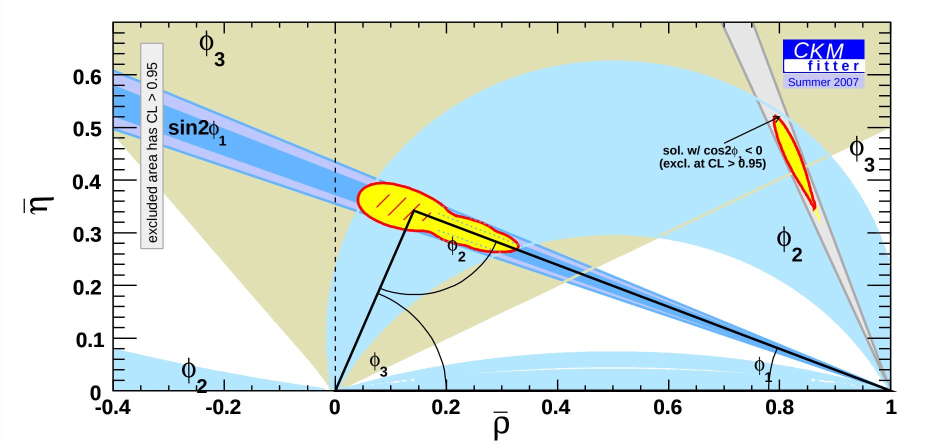

no angles (with theory)...

...vs. angles (without theory)

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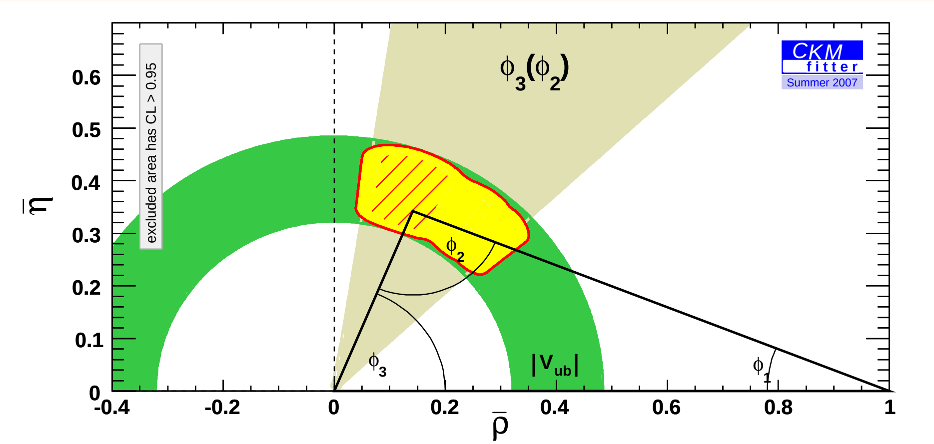

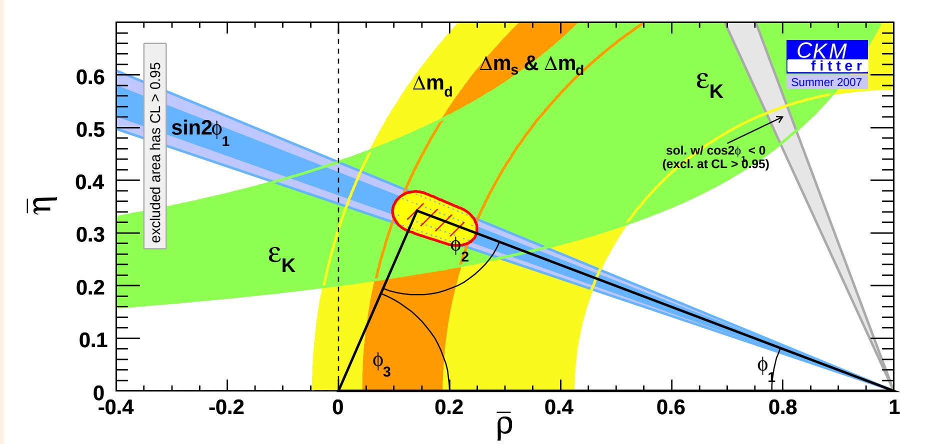

tree… …vs. loop

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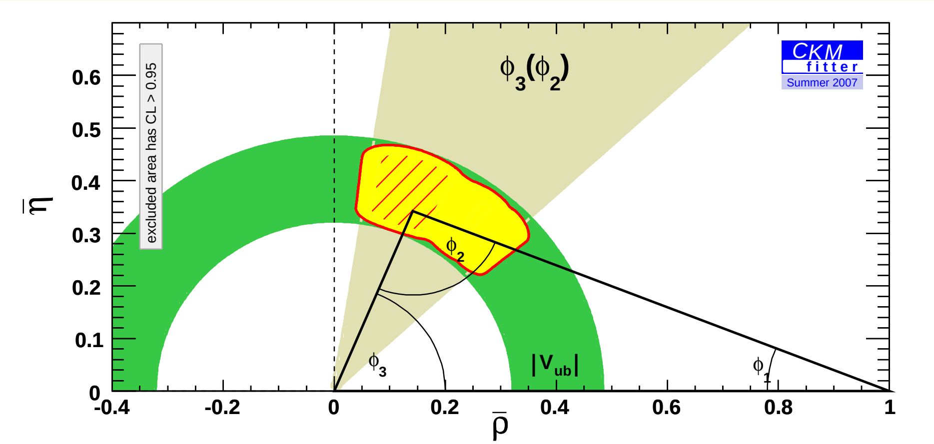

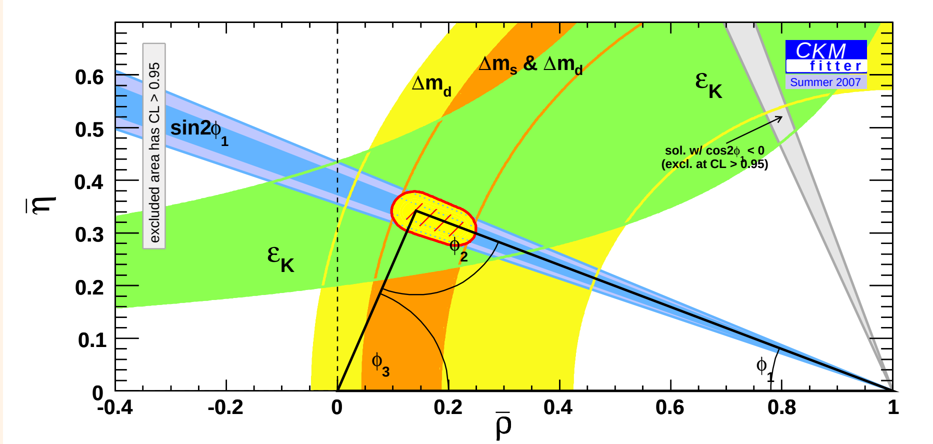

tree… …vs. loop

the $(\bar{\rho}, \bar{\eta})$ plane is not the whole story, still the overall agreement is impressive !

Selected fit predictions

the Wolfenstein parameters

$$\lambda = 0.2265^{+0.0008}_{-0.0008} \qquad A = 0.807^{+0.018}_{-0.018}$$

$$\bar{\rho} = 0.141^{+0.029}_{-0.017} \qquad \bar{\eta} = 0.343^{+0.016}_{-0.016}$$

the Jarlskog invariant

$$J = (3.01^{+0.19}_{-0.18}) \times 10^{-5}$$

the UT angles

$$\alpha = (90.7^{+4.5}_{-2.9})^\circ \qquad \beta = (21.70^{+0.97}_{-0.97})^\circ \qquad \gamma = (67.6^{+2.8}_{-4.5})^\circ$$

$B_s - \bar{B}_s$ mixing

$$\Delta m_s = 17.7^{+6.4}_{-2.1}\ \mathrm{ps}^{-1} \text{(indirect)} \qquad 17.77 \pm 0.10 \pm 0.07\ \mathrm{ps}^{-1} \text{(CDF, direct)}$$

B leptonic decay

$$\mathcal{B}(B \to \tau\nu) = (9.3^{+1.1}_{-1.2}) \times 10^{-5} \text{(indirect)} \qquad (14.1^{+4.3}_{-4.2}) \times 10^{-5} \text{(WA, direct)}$$

Part II

Depuzzling $B \to K\pi$

β and γ can be determined from tree-level decays without any theoretical input

this is not the case for α, that needs (at least) assuming isospin symmetry

also many B-decays are sensitive to the CKM matrix through loop processes

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the global "reference" fit is not the whole story !

Dynamical approaches to non-leptonic B-decays

based on the heavy mass expansion:

pQCD (diagrammatic) vs. QCDF (diagrammatic) and SCET (operator-based)

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the accuracy of the expansion is not settled yet, because the errors are large and the input parameters poorly known

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despite recent progress, it is generally unsafe to use these calculations to obtain constraints on the CKM matrix and/or New Physics

Phenomenological methods

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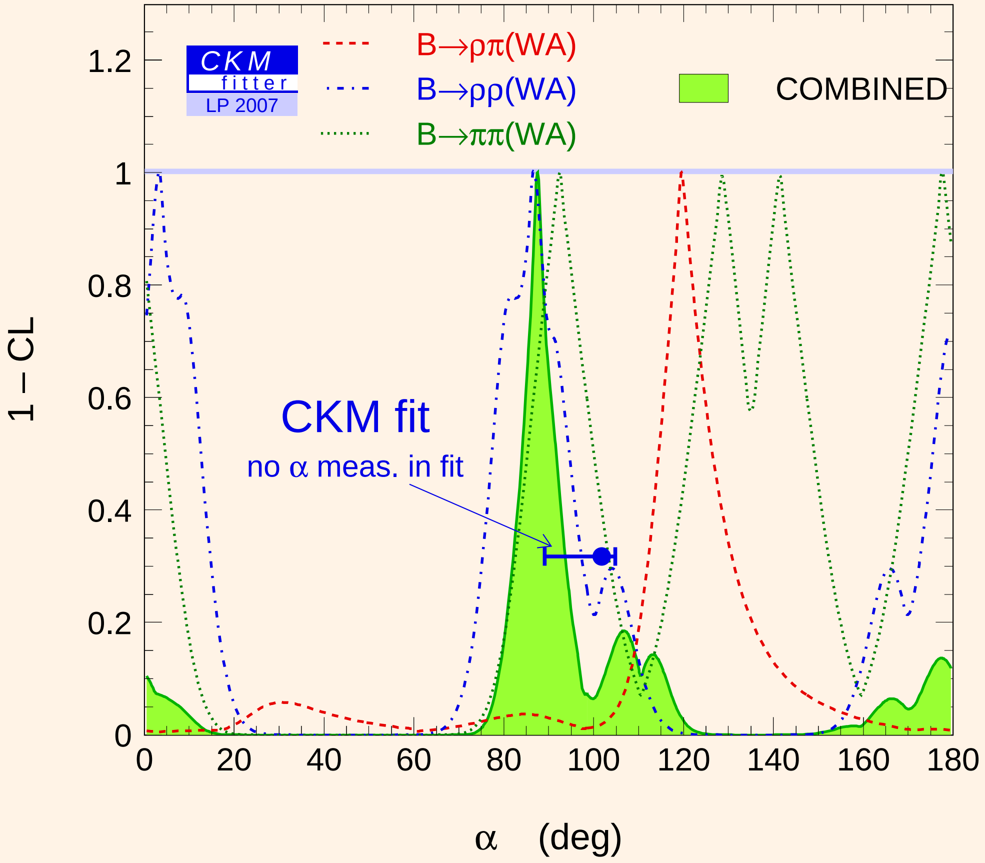
e.g. determination of α



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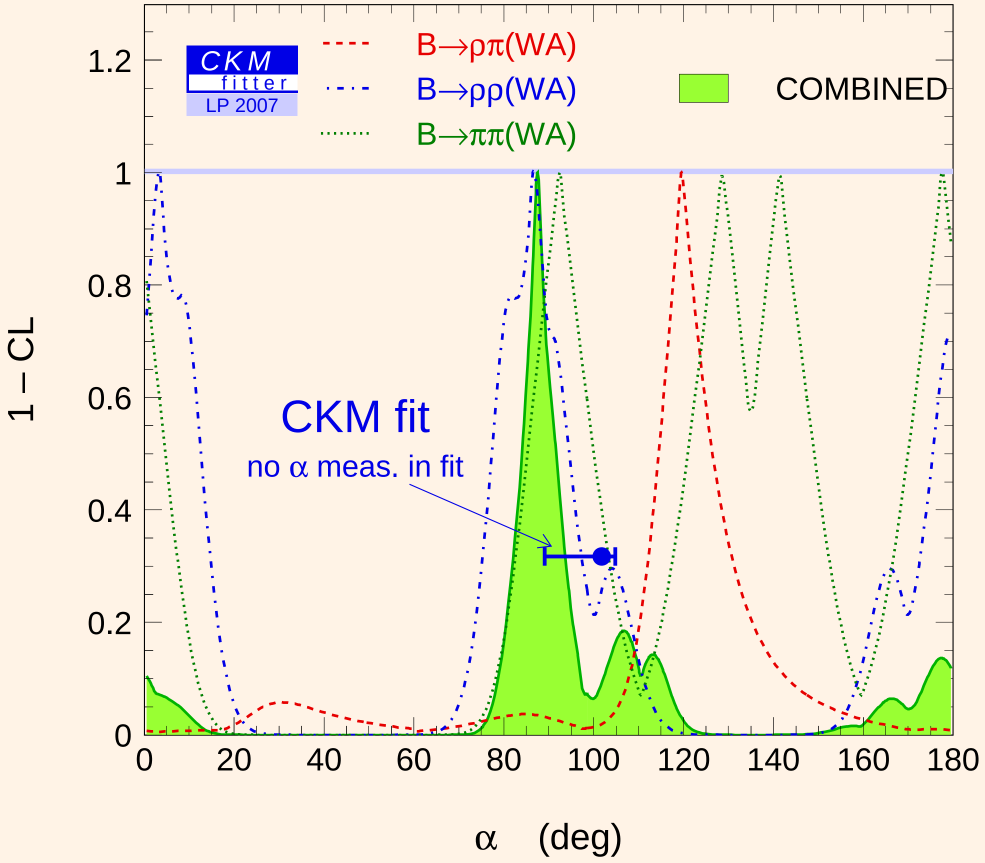

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what about SU(3) ?

Flavor SU(3) in $B \to \pi\pi, K\pi, K\overline{K}$

a long story:

Silva and Wolfenstein, 1993

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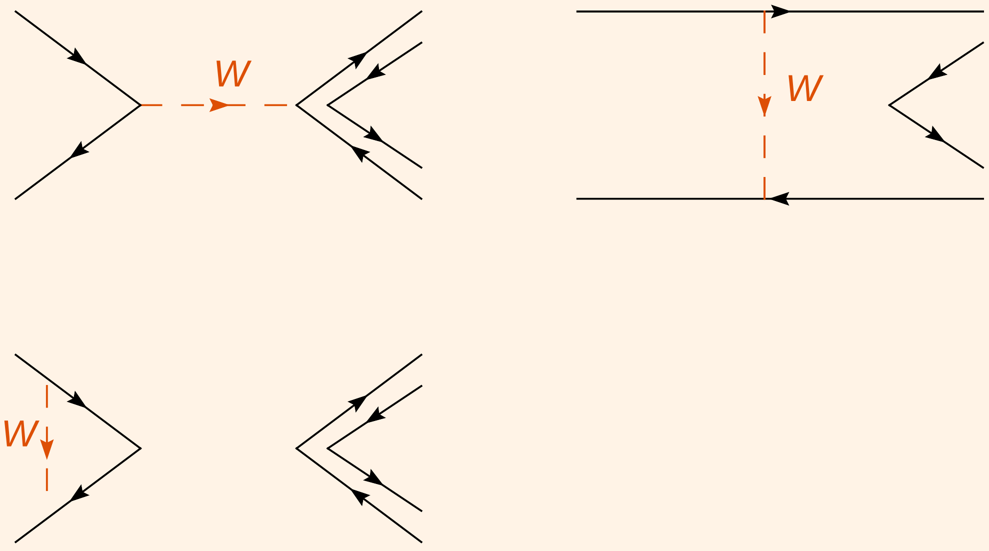
Buras *et al.* (BFRS), 2003-2005

many other works !

however due to lack of information on B_s decays, most of these works assume in addition that some or all of the following annihilation/exchange topologies are negligible (exception: Wu and Zhou, 2005)



Annihilation/exchange diagrams in heavy meson decays

these topologies are power-suppressed

the amplitude ratios

$$\left|\frac{A(D^0 \to K^0\overline{K^0})}{A(D^0 \to K^+K^-)}\right| \quad \text{and} \quad \left|\frac{A(B^0 \to D_s^- K^+)}{A(B^0 \to D^-\pi^+)}\right|$$

are both formally of order $(1/N_c)(\Lambda/m_Q)$;

but the first one is $\sim 43\%$ while the second one is $\sim 12\%$!

in charmless B-decays the only direct constraint is

$$\left|\frac{A(B^0 \to K^+K^-)}{A(B^0 \to \pi^+\pi^-)}\right| < 0.24$$

The origin of the $B \to K\pi$ puzzle

measurement of branching ratios by CLEO, BaBar and Belle were found to be not completely consistent with naïve expectations based on a specific quark diagram hierarchy (Buras and Fleischer, Gronau and Rosner, BFRS, ...)

these arguments are confirmed by a more detailed analysis (BFRS) taking input from $B \to \pi\pi$ and SU(3), but neglecting annihilation/exchange contributions

the effect is quite insensitive to what happens in the $\Delta S = 1$, $\Delta I = 0$ channel and would point towards non standard electroweak penguins

however the discrepancy was not statistically convincing and the rôle of the neglected contributions was not clear

General parametrization in the strict SU(3) limit

$$\begin{aligned}
A(K^+\pi^-) &= V_{us}V_{ub}^* T^{+-} + V_{ts}V_{tb}^* P \\
A(K^0\pi^+) &= V_{us}V_{ub}^* N^{0+} + V_{ts}V_{tb}^* (-P + P_C^{EW}) \\
\sqrt{2}A(K^+\pi^0) &= V_{us}V_{ub}^* (T^{+-} + T^{00} - N^{0+}) + V_{ts}V_{tb}^* (P + P^{EW} - P_C^{EW}) \\
\sqrt{2}A(K^0\pi^0) &= V_{us}V_{ub}^* T^{00} + V_{ts}V_{tb}^* (-P + P^{EW}) \\
A(\pi^+\pi^-) &= V_{ud}V_{ub}^* (T^{+-} + \Delta T) + V_{td}V_{tb}^* (P + PA) \\
\sqrt{2}A(\pi^0\pi^0) &= V_{ud}V_{ub}^* (T^{00} - \Delta T) + V_{td}V_{tb}^* (-P - PA + P^{EW}) \\
\sqrt{2}A(\pi^+\pi^0) &= V_{ud}V_{ub}^* (T^{+-} + T^{00}) + V_{td}V_{tb}^* P^{EW} \\
A(K^+K^-) &= V_{ud}V_{ub}^* \Delta T + V_{td}V_{tb}^* PA \\
A(K^0\overline{K}^0) &= V_{ud}V_{ub}^* \Delta P + V_{td}V_{tb}^* (-P - PA + P_C^{EW} - \frac{1}{3}P_{K\overline{K}}^{EW}) \\
A(K^+\overline{K}^0) &= V_{ud}V_{ub}^* N^{0+} + V_{td}V_{tb}^* (-P + P_C^{EW})
\end{aligned}$$

Electroweak penguins

the $Q_{7,8}$ operators are suppressed by their Wilson coefficients with respect to $Q_{9,10}$ (Neubert and Rosner; Buras and Fleischer; Gronau, Pirjol and Yan) so that their $\Delta I = 3/2,\ 1$ hadronic matrix elements are not independent parameters in the SU(3) limit

$$\begin{aligned}
P^{EW} &= R^{+}(T^{+-} + T^{00}) \\
P_{C}^{EW} &= \frac{R^{+}}{2}(T^{+-} + T^{00} + N^{0+} - \Delta T - \Delta P) \\
&- \frac{R^{-}}{2}(T^{+-} - T^{00} + N^{0+} + \Delta T + \Delta P) \\
P_{K\overline{K}}^{EW} &= R^{+}(N^{0+} - \Delta T - \Delta P)
\end{aligned}$$

with

$$R^{\pm} = -\frac{3}{2}\frac{c_9 \pm c_{10}}{c_1 \pm c_2} = (1.35 \pm 0.13)10^{-2}$$

Parameter counting

neglecting annihilation/exchange diagrams would imply $\Delta T = PA = N^{0+} - \Delta P = 0$, in which case there are 7 hadronic parameters ($+(\bar{\rho}, \bar{\eta})$) and 15 independent measured observables

the exact SU(3) limit need 6 additional parameters but introduces only 4 new measured observables (among which one upper limit)

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this seems hopeless ! however it is not...

in addition there are useful constraints coming from ratios of BR's by CDF (among which two new independent observables, $B_s \to K^+K^-$ and $B_s \to K^+\pi^-$, and one upper limit, $B_s \to \pi^+\pi^-$)

in total we have 13+2 parameters for 24 independent observables

SU(3) breaking

dominant factorizable SU(3) breaking is easy to identify, it is related to ratios of decay constants; we normalise $B \to K\pi$, $B_s \to K^+K^-$ and $B_s \to K^+\pi^-$ with respect to $B \to \pi\pi$ through the factors $N_{K\pi} \sim f_K/f_\pi$, $N_{K\bar{K}} \sim (f_{B_s}/f_B)(f_K/f_\pi)^2$ and $N^s_{K\pi} = (f_{B_s}/f_B)(f_K/f_\pi)$; take conservative theoretical errors

$$N_{K\pi} = 1.22 \pm 0.22$$
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remaining factorizable SU(3) breaking, such as $(f_\pi F^{B\to K})/(f_K F^{B\to\pi})$ is much smaller (a few %) and is neglected

non factorizable Λ/m_b-suppressed SU(3) breaking effects are neglected

A side remark: direct tests of SU(3) in heavy meson observables

decay constants: f_{D_s}/f_D (exp, latt) and f_{B_s}/f_B (latt) are of the same order as f_K/f_π

once dominant sources are identified (phase space, pole contribution to the form factor), $D \to \pi$ and $D \to K$ semileptonic decays do not indicate large corrections (Fajfer)

still, information is incomplete and one cannot exclude new mechanisms of SU(3) breaking

Notation

in the tree dominance approximation, $B^0(t) \to \pi^+\pi^-$ measures α, so write the time-dependent CP-asymmetry

$$\begin{aligned} a_{CP}(t) &= C\cos\Delta m t + S\sin\Delta m t \\ &= C\cos\Delta m t + \sqrt{1-C^2}\sin 2\alpha_{\text{eff}}\sin\Delta m t \end{aligned}$$

in the penguin dominance approximation, $B^0(t) \to K_S\pi^0$ measures β, so write the time-dependent CP-asymmetry

$$a_{CP}(t) = C\cos\Delta m t + \sqrt{1-C^2}\sin 2\beta_{\text{eff}}\sin\Delta m t$$

Understanding the constraint shape in the $(\bar\rho, \bar\eta)$ plane: the "α" subsystem

the subsystem $B \to \pi^+\pi^-$, $B \to K^\pm\pi^\mp$, $B \to K^+K^-$ approximately measures α

neglecting annihilation and exchange, there is a simple analytical solution

$$\sqrt{1 - C^2_{\pi\pi}}|\mathcal{D}|\cos(2\alpha - 2\alpha_{\text{eff}} - \epsilon) = (1+\lambda^2)^2 - 2\lambda^2 \sin^2\gamma \left[1 + \frac{BR(K^+\pi^-)}{BR(\pi^-\pi^+)}\right]$$

and $BR(K^+\pi^-)C(K^+\pi^-) + BR(\pi^+\pi^-)C(\pi^+\pi^-) = 0$

where $\mathcal{D} \equiv |\mathcal{D}|e^{i\epsilon} = (1+\lambda^2)(1+\lambda^2 e^{i\gamma})$

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where $\mathcal{D} \equiv |\mathcal{D}|e^{i\epsilon} = (1+\lambda^2)(1+\lambda^2 e^{i\gamma})$

taking power-suppressed contributions into account, the system of equations remain closed and solvable, and can be approximately viewed as a bound on $|\alpha - \alpha_{\text{eff}}|$.

the bound would become an equality if the time-dependent CP-asymmetry in $B \to K^+K^-$ is measured

the "β" subsystem

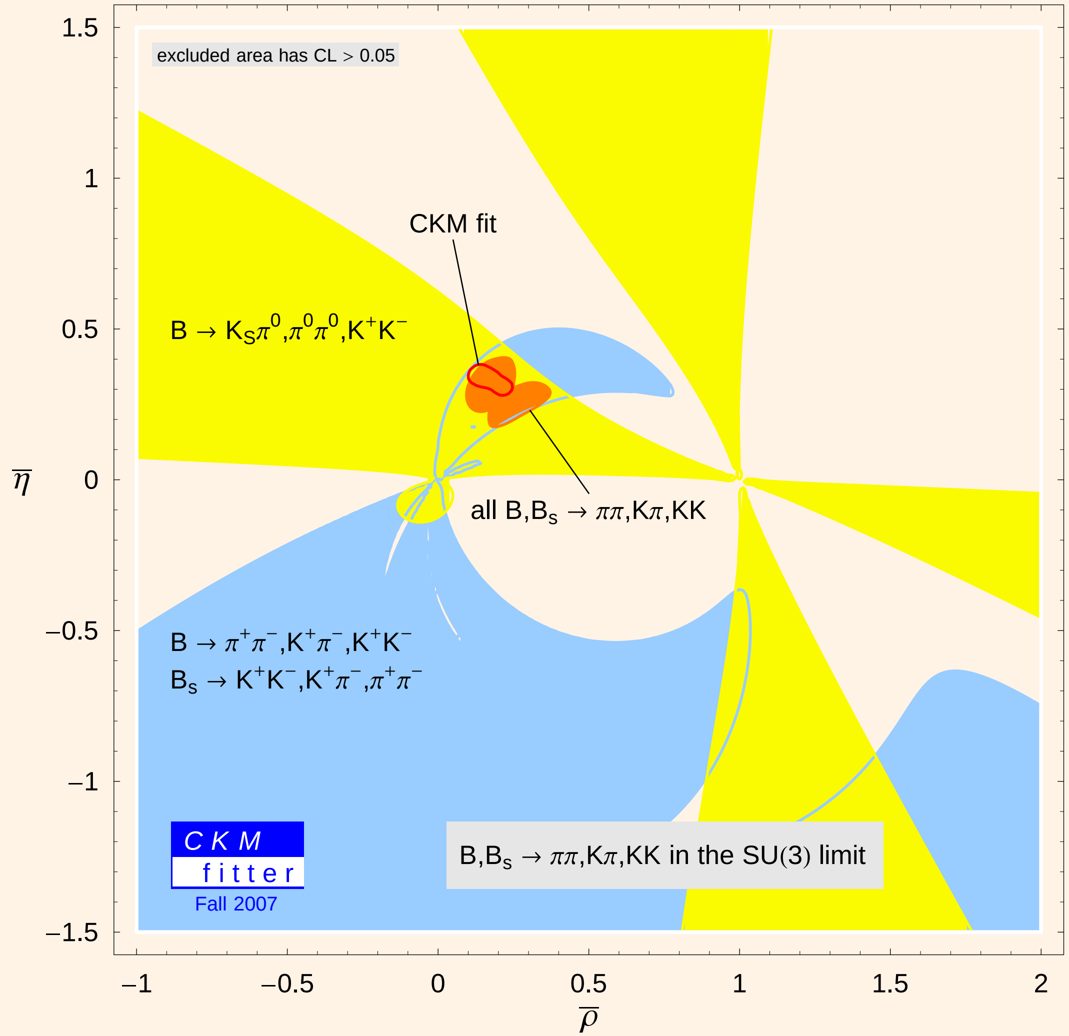
replace $B \to \pi^+\pi^-$ by $B \to K_S\pi^0$, $B \to K^\pm\pi^\mp$ by $B \to \pi^0\pi^0$, and α by β

$$\sqrt{1 - C^2_{K_S\pi^0}}|\mathcal{D}|\cos(2\beta - 2\beta_{\mathrm{eff}} + \epsilon) = (1+\lambda^2)^2 - 2\lambda^2 \sin^2\gamma \left[1 + \frac{BR(\pi^0\pi^0)}{BR(K_s\pi^0)}\right]$$

and $BR(K_S\pi^0)C(K_S\pi^0) + BR(\pi^0\pi^0)C(\pi^0\pi^0) = 0$

taking annihilation/exchange into account, this translates into a bound that improves the result of Gronau, Grossman and Rosner that is not optimal

Constraint in the $(\bar{\rho}, \bar{\eta})$ plane from the partial and full input sets



combination of constraints stronger than the naïve product $\alpha \otimes \beta$: the correlation comes mainly from the electroweak penguin coefficients $R^{\pm}$

the α and β subsystems dominate the constraint; other inputs help in disfavouring mirror solutions

The pValue of the analysis within the Standard Model

in frequentist statistics, the pValue is a well-defined interpretation of $\chi^2_{\rm min}/N_{\rm dof}$; assuming a given theory (here, $(\bar\rho,\bar\eta)_{\rm SM}$+SU(3)), the pValue is the probability that one obtains a less good fit if one performs many similar experiments

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more information can be obtained by comparing the indirect fit prediction for a given observable with the direct experimental measurement (on the way...)

Outlook

in the near future, thanks to CDF and LHCb, there may be up to 38 measured observables depending on the very same 13+2 parameters; this will allow to fit part of SU(3) breaking and to study different New Physics scenarios

Part III

New Physics in $B\overline{B}$ mixing

Model-independent parametrization

$$\left\langle B_q \,\middle|\, \mathcal{H}^{\mathrm{SM+NP}}_{\Delta B=2} \,\middle|\, \bar{B}_q \right\rangle \equiv \left\langle B_q \,\middle|\, \mathcal{H}^{\mathrm{SM}}_{\Delta B=2} \,\middle|\, \bar{B}_q \right\rangle \times \left(1 + x_q^{\mathrm{NP}} + i y_q^{\mathrm{NP}}\right)$$

(SM is thus located at $(x_d^{\mathrm{NP}}, y_d^{\mathrm{NP}}) = (0, 0)$)

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(SM is thus located at $(x_d^{\mathrm{NP}}, y_d^{\mathrm{NP}}) = (0, 0)$)

Strategy and inputs

assume that tree-level transitions are 100% SM

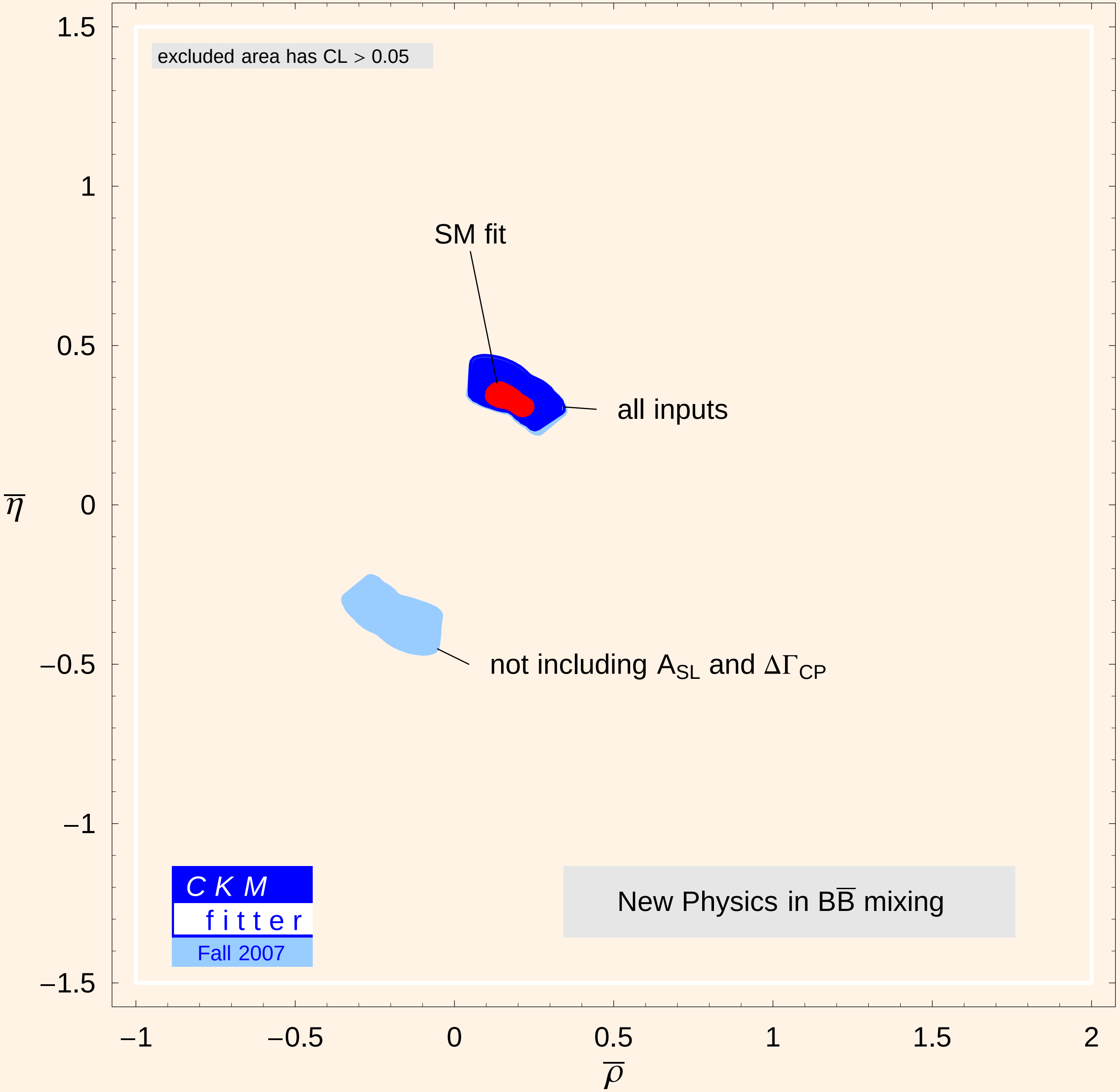
fix SM parameters with $|V_{ud}|$,$|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, γ and $\alpha = \pi - \gamma - \beta_{\mathrm{eff}}(\Psi K_S)$

$(x_d^{\mathrm{NP}}, y_d^{\mathrm{NP}})$ are then constrained by Δm_d (circle)
and by $2\beta_{\mathrm{eff}}(\Psi K_S) = 2\beta + \arg(1 + x_d^{\mathrm{NP}} + i y_d^{\mathrm{NP}})$ (straight line)

$(x_s^{\mathrm{NP}}, y_s^{\mathrm{NP}})$ are constrained by Δm_s (circle) (no phase measurement up to now)

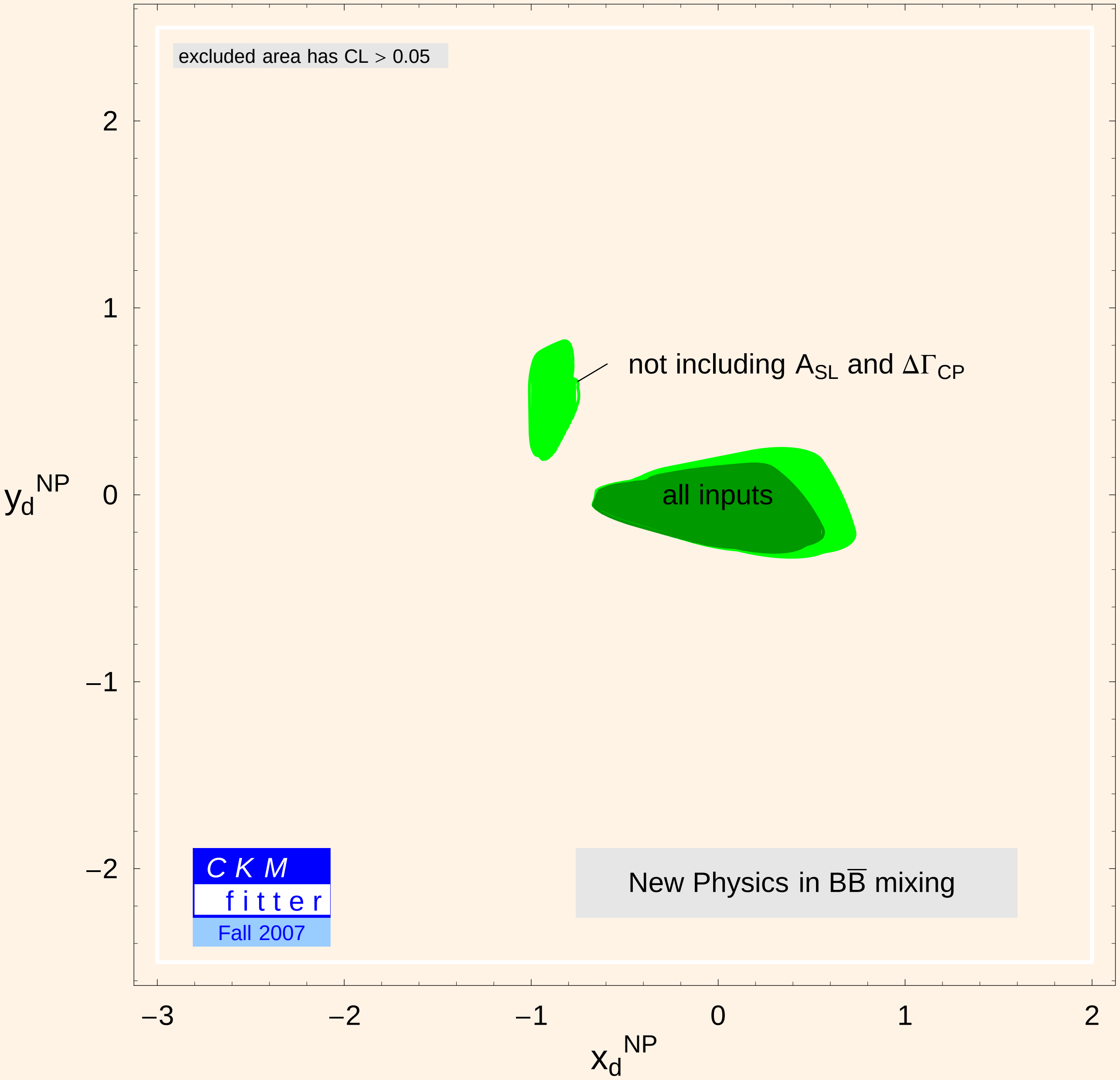
additional information is brought by the measurement of the semileptonic asymmetries A_{SL}^d, A_{SL}^s and by $\Delta\Gamma_{s,\mathrm{CP}} = \frac{(x_s^{\mathrm{NP}})^2}{(x_s^{\mathrm{NP}})^2 + (y_s^{\mathrm{NP}})^2} \Delta\Gamma_{s,\mathrm{SM}}$

Results in the $(\bar{\rho}, \bar{\eta})$ plane



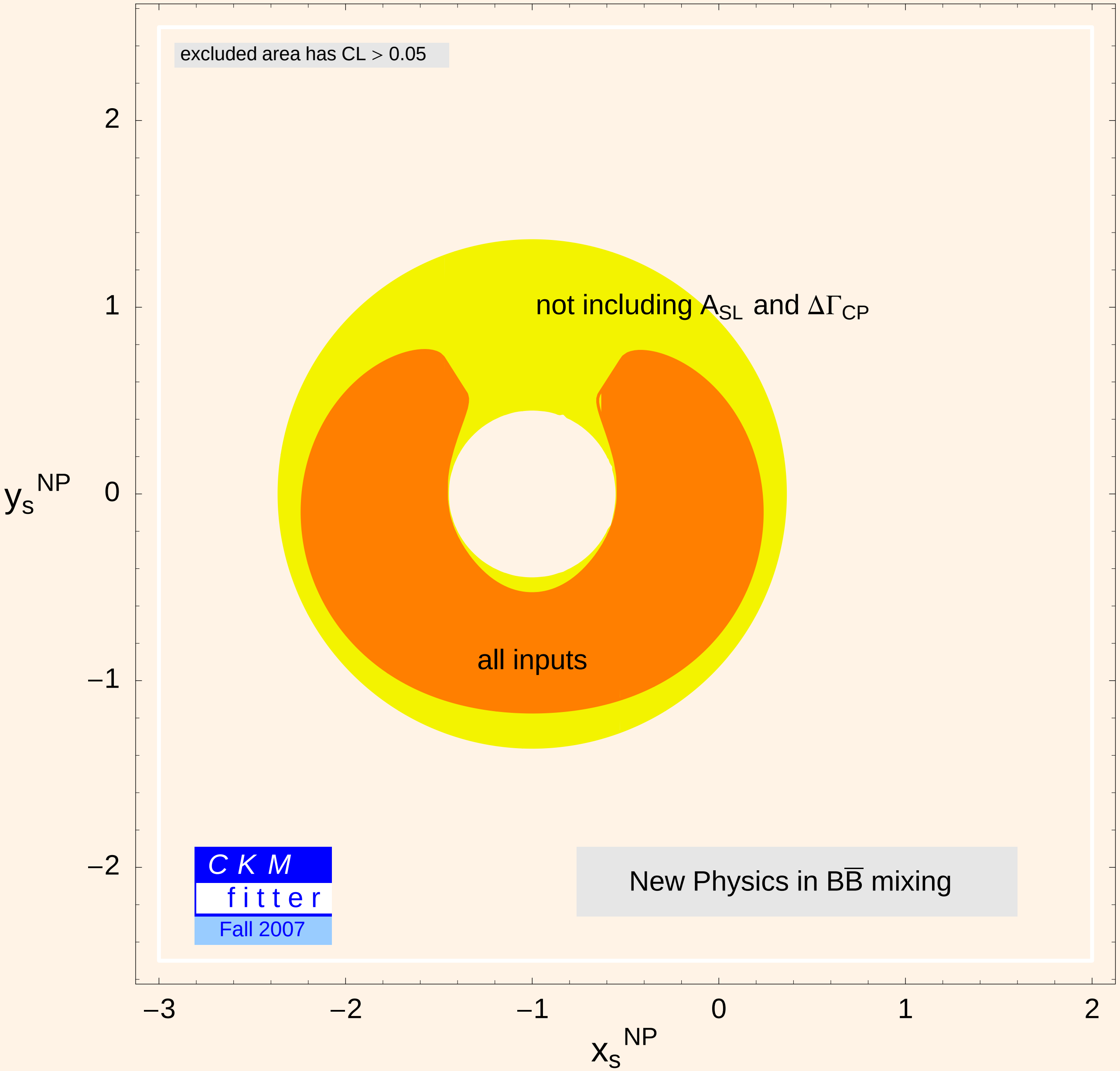
no evidence for New Physics...

Results in the x_d^{NP}, y_d^{NP} plane



no evidence for New Physics, but sizable contributions are allowed

Results in the x_s^{NP}, y_s^{NP} plane



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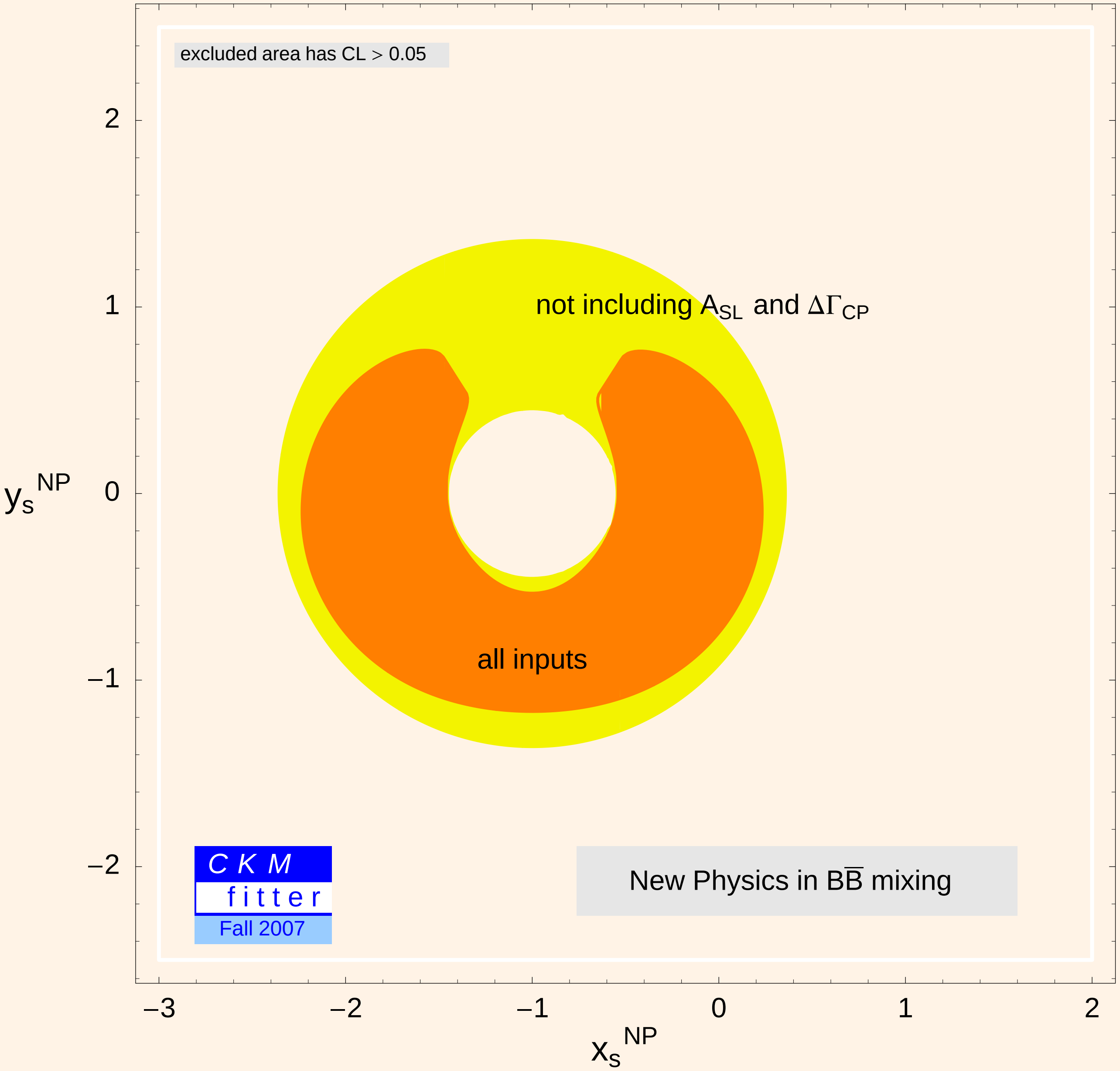

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wait for the measurement of the $B_s\overline{B}_s$ mixing phase !

A few words on statistics

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- theoretical uncertainties for the quantities that are computed within QCD
- discrete ambiguities that correspond to physical maxima of the Likelihood
- physical bounds, e.g. $|\sin 2\beta| < 1$
- nuisance parameters, that is you may want $(1 - \mathrm{CL})(\gamma)$ while the Likelihood depends on many other parameters

Bayesian vs. frequentist inference

in both cases the experimental information is encoded in the Likelihood function

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however the output is conceptually different:

Bayesian statistics answers the question whether the theory is likely, given the data. This is attractive, but meaningless because theory parameters are not random variables

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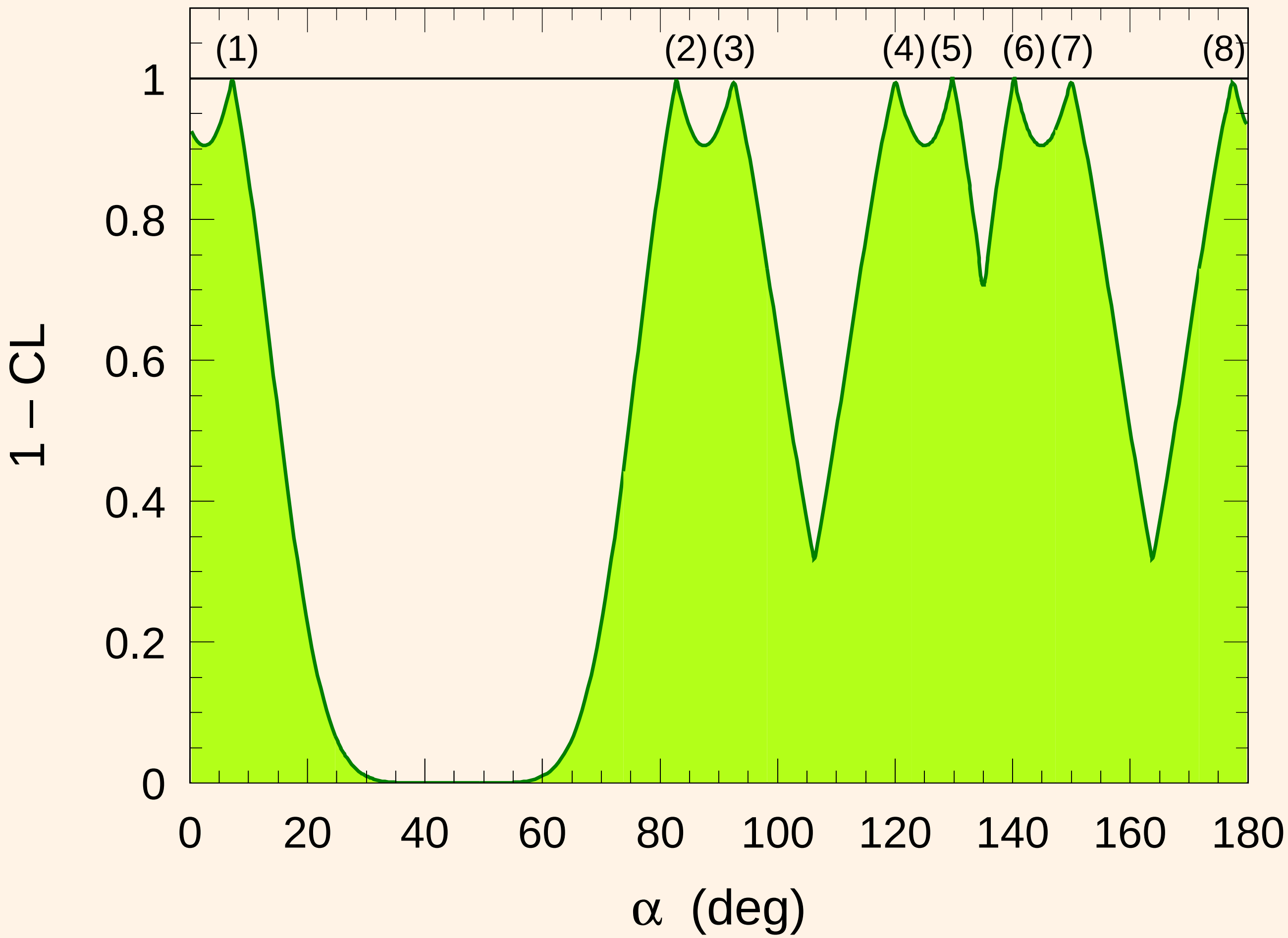
Bayesian statistics is technically simpler (no minimization), and solves in part the difficulties mentioned in the previous slide. However the drawback is the non invariance with respect to the parametrization, and the possible violation of the symmetries of the problem !

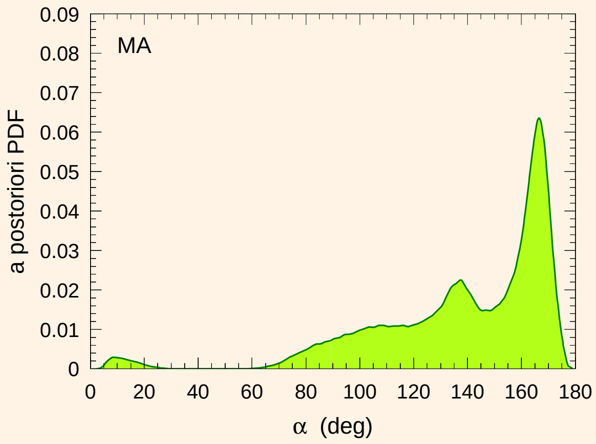
Frequentist result for the $B \to \pi\pi$ isospin analysis



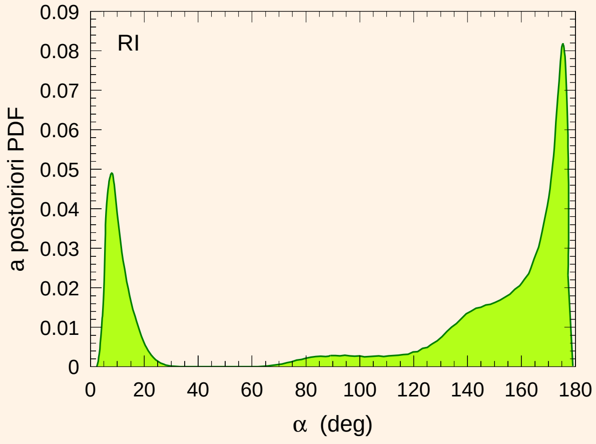
frequentist analysis is invariant with respect to the parametrization, and shows explicitly the eight-fold discrete ambiguity that can be computed analytically

Bayesian result(s)

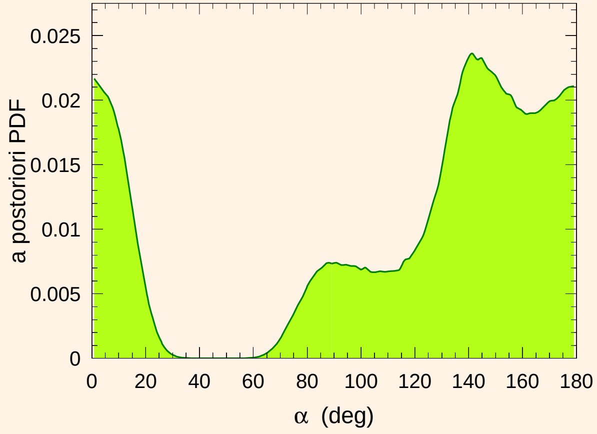
Modulus and Argument parametrization



Real and Imaginary part parametrization)



yet another parametrization



cf. discussion in hep-ph/0607246, hep-ph/0701204, hep-ph/0703073

Challenge: nuisance parameters in toy analyses

asymptotically (small Gaussian errors), when one repeats a large number of times the same experiment, the distribution of $\Delta\chi^2(\bar{\rho},\bar{\eta}) = \text{Min}_\mu\,\chi^2(\bar{\rho},\bar{\eta};\mu) - \chi^2_{\text{min}}$ follows a $N_{\text{dof}} = 2$ χ^2-distribution and does not depend on the true (unknown) value of the SM parameters μ

however in presence of physical boundaries and/or large non-linearities, the above statement is no longer true, one must compute numerically the actual distribution and study the dependence wrt to μ

this is technically very demanding, but is mandatory to get a sensible answer for specific analyses: γ from $B \to DK$, α from $B \to \rho\pi$, among others

we are implementing these techniques within a general algorithm in CKMfitter so that virtually any problem can be treated transparently

Conclusion

the global CKM fit, which uses well controlled inputs only, does confirm the CKM mechanism as the dominant contribution to flavor- and CP-violating transitions

the three main FCNC transitions ($s \to d$, $b \to d$ and $b \to s$) have now been tested and are in good to excellent agreement with SM predictions

some important observables (very rare kaon and B decays, CP violation in B_s decays ...) remain to be measured and interpreted: will be done at future experiments !

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the overall pattern of B decays to two pseudoscalars is reasonably described by simple phenomenological approaches, but its details and dynamics challenges the theory

present understanding makes unclear the disentanglement of statistical fluctuations, hadronic effects (flavor symmetry breaking) and possible New Physics effects

however due to the large number of experimentally accessible observables, new information from non leptonic B decays is expected in a close future

New Physics in $\Delta B = 2$ transitions can be parametrized model-independently and constrained with non trivial results: non standard contributions are not necessary to describe the data but ae allowed up to sizable values

frequentist approach to CKM fits is very demanding technically and asks difficult statistical questions, that we are now trying to assess in a more systematic way

Backup

The statistical method to extract γ

the observables depend on γ and μ where $\mu = (r_B, \delta)$

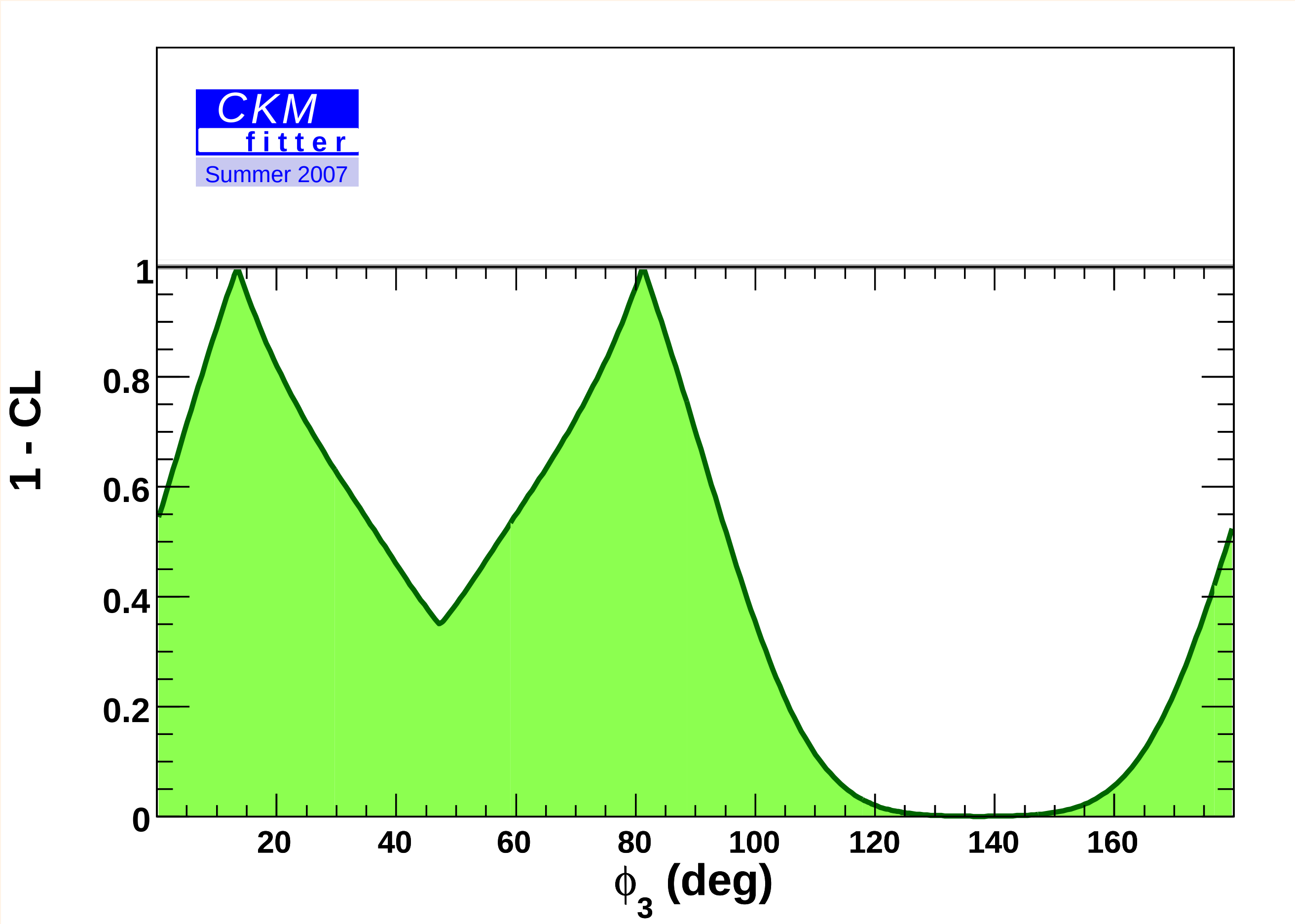
1. minimize $\chi^2(\gamma, \mu)$ with respect to μ and substract the minimum $\longrightarrow \Delta\chi^2(\gamma)$
2. assume that the true value of μ is $\mu_t \longrightarrow \mathrm{PDF}\left[\Delta\chi^2(\gamma)\,|\,\gamma, \mu_t\right]$
3. compute $(1 - \mathrm{CL})_{\mu_t}(\gamma)$ via toy Monte-Carlo
4. maximize with respect to $\mu_t \longrightarrow (1 - \mathrm{CL})(\gamma)$

this is a quite general, but very expensive, procedure; coverage must be (and is being) checked

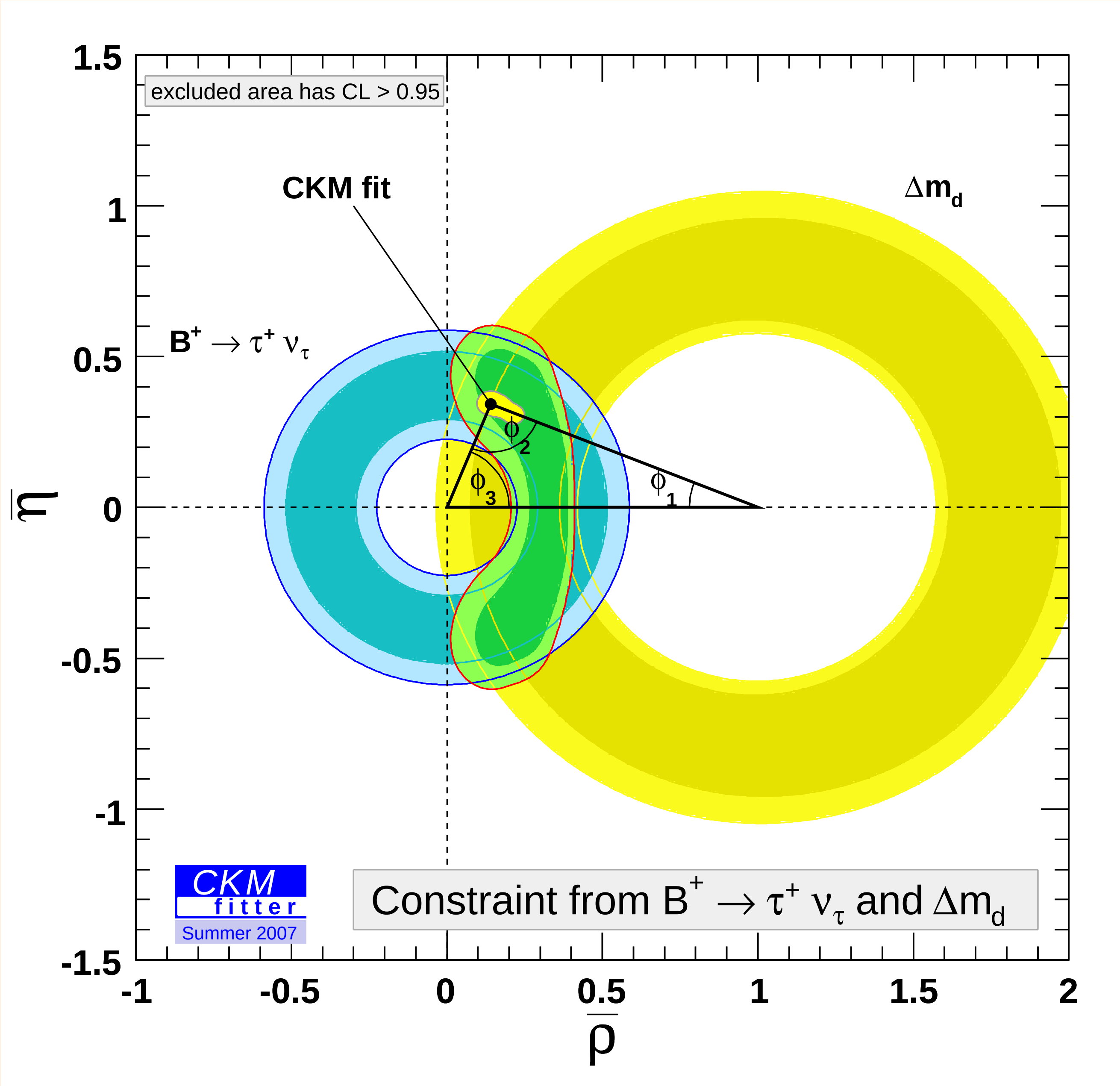
another possibility is to assume that the best value of μ corresponds to the one that minimizes $\Delta\chi^2(\gamma, \mu)$ for the fixed γ

$|\sin(2\beta+\gamma)|$

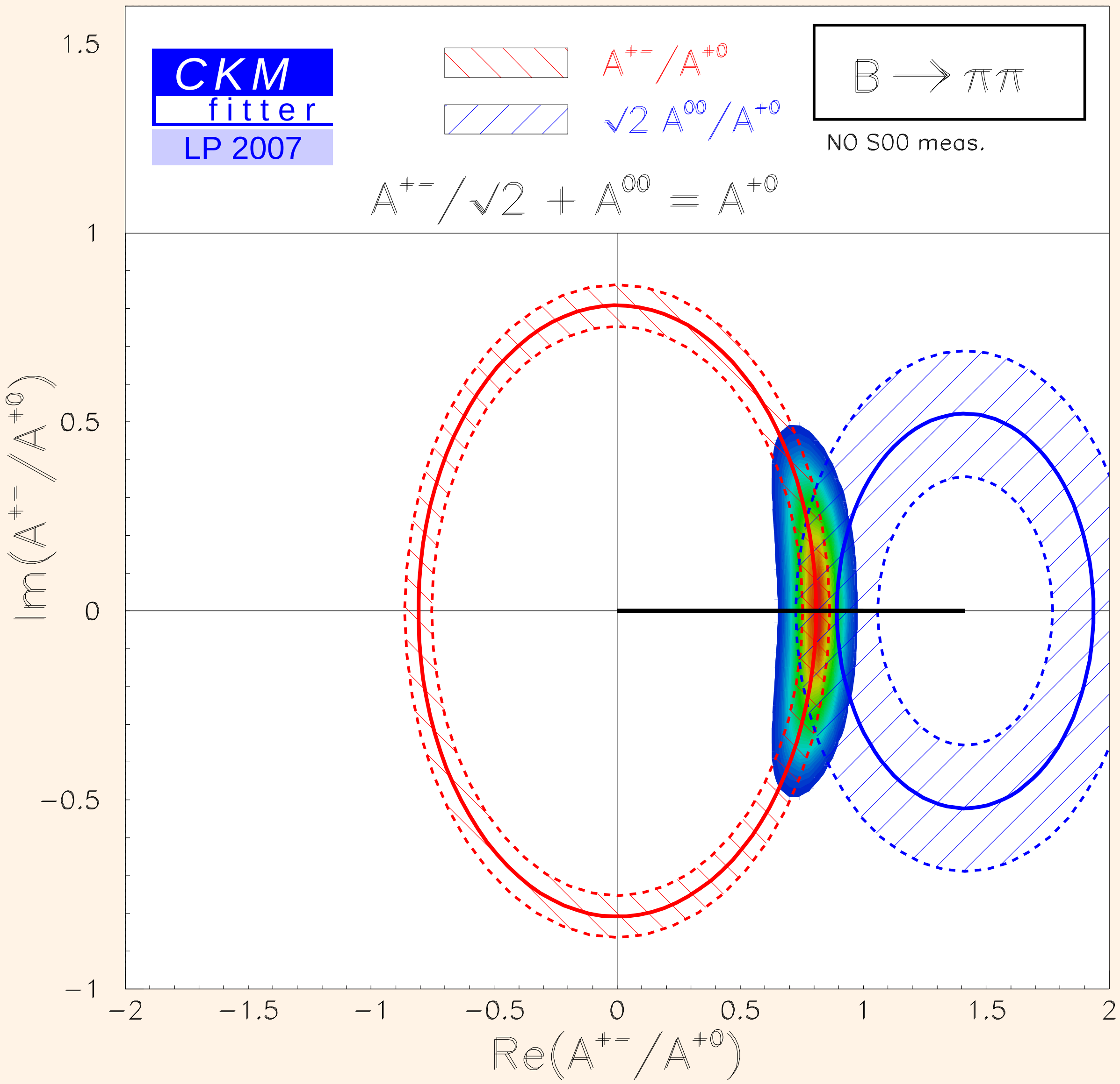
from $b \to c\bar{u}d,\ u\bar{c}d$



$B \to \tau\nu$ vs. Δm_d



Isospin triangle: $B \to \pi\pi$



Isospin triangle: $\overline{B} \to \pi\pi$