CURRENT STATUS OF THE EXTRACTION OF V_{us} FROM HADRONIC τ DECAY

IHEP, Beijing, October 22, 2007

Kim Maltman

[See also PLB639 (2006) 283 and PLB650(2007) 27]

OUTLINE

- *Basics of the hadronic τ decay approach to V_{us}*
- *Some key technical issues*
- *Results and prospects (especially impact of new 2007 data/hints of possible non-SM contributions)*

BACKGROUND/NOTATION/TERMINOLOGY

- V,A $ij = ud,\ us$, $(J) = (0+1),\ (0)$ spectral functions accessible from experimental τ decay distributions

$$\boxed{\begin{aligned} R_{V/A;ij} \;&=\; 12\pi^2\,|V_{ij}|^2 S_{EW} \int_0^1 dy_\tau \left[w_T(y_\tau)\,\rho^{(0+1)}_{V/A;ij}(s) \right. \\ &\qquad \left. + w_L(y_\tau)\,\rho^{(0)}_{V/A;ij}(s) \right] \end{aligned}}$$

with $R_{V/A;ij} \equiv \frac{\Gamma[\tau\to\nu_\tau\,\mathrm{hadrons}_{V/A;ij}\,(\gamma)]}{\Gamma[\tau^-\to\nu_\tau e^-\bar{\nu}_e(\gamma)]}$, $y_\tau = s/m_\tau^2$

$w_T(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$,

- for J^μ the flavor $ij = ud$ or us V or A current, scalar correlators $\Pi^{(J)}_{V/A;ij}$ defined,

$$\begin{aligned}\Pi^{\mu\nu}_{V/A;ij}(q^2) &= i\int d^4x\, e^{iq\cdot x}\langle 0|T\left(J^\mu(x)J^{\nu\,\dagger}(0)\right)|0\rangle \\ &= \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}(q^2) + q^\mu q^\nu\,\Pi^{(0)}(q^2)\end{aligned}$$

the "spectral functions", $\rho^{(J)}_{V/A;ij}(s)$ in the integral above are just $\frac{1}{\pi} Im\, \Pi^{(J)}_{V/A;ij}$

- TECHNICAL ASIDE: in QCD, the combinations $s\rho^{(0)}_{V/A;ij}$ and $\rho^{(0+1)}_{V+A;ij} \equiv \rho^{(0)}_{V/A;ij} + \rho^{(1)}_{V/A;ij}$, above correspond to scalar correlator combinations with no "kinematic singularities" ($\Rightarrow$ singularities in the complex-s plane only for $s = m^2 \geq 0$, with m the mass of some physical hadronic state)

- SOME COMMON TERMINOLOGY
  - "longitudinal": pure $(J) = (0)$ term in (0), $(0+1)$ decomposition
  - "(k, m) spectral weights": experimental distribution multiplied by $(1 - y_\tau)^k\, y_\tau^m$ before integration
  - "$(0, 0)$ spectral weight: kinematic weight case ($R^{(0,0)}_{V/A;ij}$ obtainable from sum over branching fractions)
  - "inclusive analysis": one with $J = 0 + 1$ and $J = 0$ spectral contributions in a combination proportional to the kinematically weighted one above (contrast to non-inclusive analyses, which require a separation of $\rho^{(0)}_{V/A;ij}$, $\rho^{(0+1)}_{V/A;ij}$ contributions)

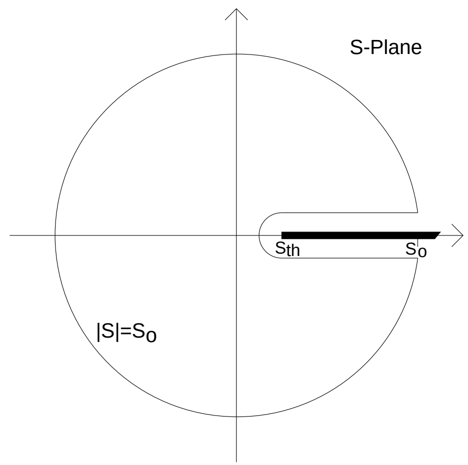
- Chiral constraints on longitudinal spectral contributions

  - with f_X for a $J=0$, flavor ij $X=S/PS$ state defined by $\langle 0|V/A^\mu|X(q)\rangle = i\, f_X\, q^\mu$, the X contribution to $\rho^{(0)}_{V/A;ij}$ is $\propto f_X^2$,
  - in QCD
    - f_π, f_K NON-zero in the chiral limit
    - Ward identities $\Rightarrow$ for ALL scalars, $f^2_{S_{ij}} \propto (m_i - m_j)^2$, and for ALL pseudoscalars (other than π, K), $f^2_{PS_{ij}} \propto (m_i + m_j)^2$
  - $\Rightarrow$ double chiral suppression $\propto (m_i \pm m_j)^2$ of ALL "continuum" (non-π/K-pole) $\rho^{(0)}_{V/A;ud,us}$ contributions

EXTRACTING V_{us} (and m_s)

- Basic FESR relation

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$



(valid for any Π without kinematic singularities, any s_0, and any $w(s)$ analytic in the region of the contour)

- data on LHS, OPE (hence SM parameters) on RHS for s_0 large enough

- In what follows, $R^w_{ij}(s_0)$ denotes a generic (V, A or V+A; $(J) = (0+1)$ or (0); $ij = ud$ or us) spectral integral weighted using the analytic function $w(s)$ over the interval $0 < s \leq s_0 \leq m_\tau^2$

- Access to V_{us}, m_s is possible by forming flavor-breaking combinations of $R^w_{ud}(s_0)$ and $R^w_{us}(s_0)$ with the same $w(s)$ and s_0 for both [next slide for why this approach can yield high accuracy V_{us}]

- V_{us} (and m_s) from flavor-breaking combinations

$$\delta R^w(s_0) = \left[R^w_{ud}(s_0)/|V^2_{ud}|\right] - \left[R^w_{us}(s_0)/|V^2_{us}|\right]$$

  - $[\delta R^w(s_0)]^{OPE}_{D=0} = 0$ *(for physical V_{us} only)* ($D = 0 \leftrightarrow$ massless perturbative contributions)
  - incorrect $V_{us} \Rightarrow$ residual (large) $D = 0$ OPE contribution, hence leverage for V_{us} [Gamiz et al., JHEP 0301: 060 (2003)]
  - High precision possible because $D = 0$ OPE contribution for R^w_{ud}, R^w_{us} separately $>>$ $D = 2$ and higher contributions
  - (Leading OPE contribution, $[\delta R^w(s_0)]^{OPE}_{D=2} \propto m_s^2 \Rightarrow$ joint fit for V_{us}, m_s also possible)

- With input m_s from other sources, V_{us} via

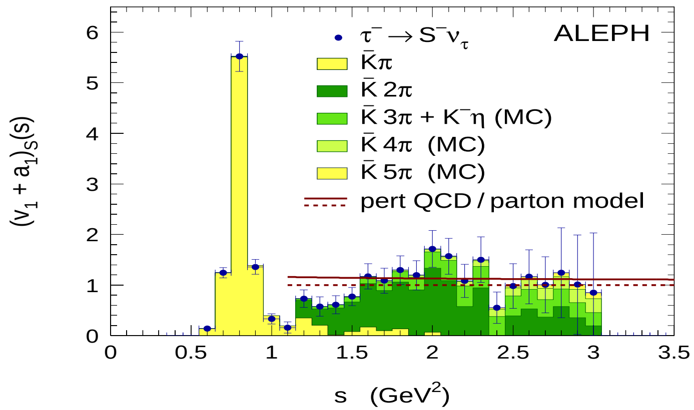
$$\boxed{|V_{us}| = \sqrt{R^w_{us}(s_0) / \left[\frac{R^w_{ud}(s_0)}{|V^2_{ud}|} - \delta R^w_{OPE}(s_0)\right]}}$$

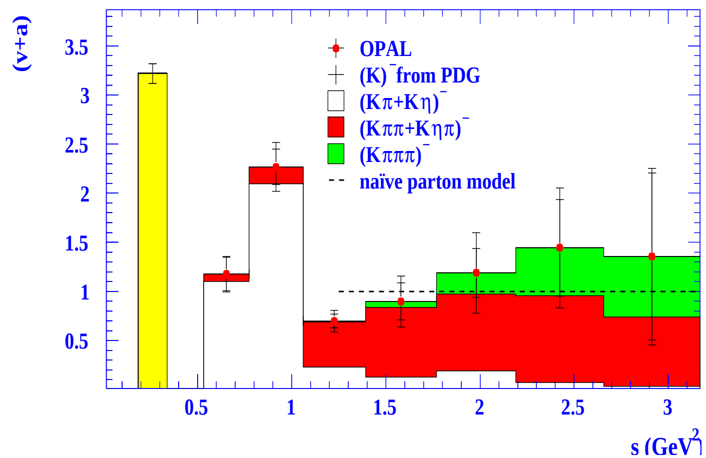
  - KEY POINT: $R^w_{ud}(s_0)$ typically $>>$ $\delta R^w_{OPE}(s_0) \Rightarrow$
    - $*$ fractional OPE-induced error on $V_{us} \sim \frac{\delta R^w_{OPE}(s_0)}{2R^w_{ud}(s_0)}$ $<<$ that on $\delta R^w_{OPE}(s_0)$ itself
    - $*$ good precision V_{us} from modest precision OPE
    - $*$ NOTE: for $s_0 = m^2_\tau$, $(0,0)$ spectral weight, V_{us} from total B_{ud}, B_{us} *IF OPE RELIABLE*

- Experimental Considerations
  - no experimental us V/A separation, ambiguity for ud $K\bar{K}\, n\pi$ $(n\geq 1)$ states $\Rightarrow$ work with ud, us V+A combinations
  - pre-2007 data errors: $\sim 0.5\%$ on R^w_{ud} ($\Rightarrow \sim 0.25\%$ on $|V_{us}|$); $3-4\%$ on R^w_{us} ($\Rightarrow 1.5-2\%$ on $|V_{us}|$)
  - significant reductions in progress from BABAR and BELLE ($\sim 10^3$ times more data than LEP experiments, MUCH improved particle ID; 2007 results restricted to branching fractions, but full us distribution in progress)

THE CURRENT us DISTRIBUTION SITUATION

[Davier, Hocker, Zhang review hep-ph/0507078]





COMPLICATIONS/CONVERGENCE ISSUES

THE MAIN ISSUES

- *severe* problems with longitudinal OPE representation and need for "longitudinal subtraction"

- problems associated with slow convergence of integrated $(0+1)$ $D=2$ OPE series

- (Not discussed here: possible $D>6$ OPE contributions [see supplementary pages])

PROBLEMS WITH THE LONGITUDINAL OPE

- integrated longitudinal $D = 2$ OPE series badly non-convergent for all kinematically-allowed scales

- (even worse) *ALL* truncation schemes employed in the literature *BADLY* violate longitudinal continuum spectral positivity [KM, J. Kambor, PRD64: 093014]

$\Rightarrow$ *MUST subtract longitudinal spectral contributions from experimental distribution and work with* $(0+1)$ *sum rules*

THE LONGITUDINAL SUBTRACTION

- K, π pole terms very accurately known and dominant for chiral+kinematic reasons

- residual ud "continuum" contributions $O[(m_d \pm m_u)^2]$ and hence numerically negligible

- residual us "continuum" subtraction

  - Jamin, Oller, Pich coupled-channel dispersive analysis, with short-distance QCD and long-distance ChPT constraints, for us scalar $K\pi$, $K\eta$, $K\eta'$ contributions

  - KM, Kambor combined FESR, Borel sum rule analysis for excited us PS decay constants

– both involve well-behaved OPE representations

– *both determinations strongly constrained by implications of the results for* m_s*, which turn out to be in good agreement with recent lattice determinations* (⇒ subtractions cannot be much larger than estimated)

– NONETHELESS, good upper bounds, or an experimental determination of the us $J = 0$ contributions would be preferrable

  ∗ job for a machine with near-threshold capabilities (like BESIII)

  ∗ HOWEVER, requires angular distributions, INCLUDING detection of the τ direction

SLOW $D = 2$ $(0+1)$ OPE SERIES CONVERGENCE

- $\delta R^{w,(0+1)}(s_0)$, and the OPE thereof involve

$$\Delta\Pi(Q^2) \equiv \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$$

  and the corresponding spectral function $\Delta\rho(s)$

- $D = 2$ OPE series, $\bar{m}_s = m_s(Q^2)$, $\bar{a} = \alpha_s(Q^2)/\pi$, $\overline{MS}$ scheme [Baikov, Chetyrkin, Kuhn PRL95:012003]

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \cdots\right]$$

- $a(m_\tau^2) \sim 0.10-0.11$ hence series *very* slowly converging at spacelike point on $|s| = s_0 = m_\tau^2$

- running of $\alpha_s(Q^2) \Rightarrow$ convergence improved away from spacelike point

- $s = s_0 e^{i\phi}$, $y = s/s_0 \Rightarrow |1-y| = 2\,sin(\phi/2) \Rightarrow$ higher $(k,0)$ spectral weights ($\propto (1-y)^{k+2}$) strongly peaked in spacelike (slowest convergence) direction

- can improve convergence with non-spectral weights [w_{20}, $\hat{w}_{10}$, w_{10} (KM, Kambor PRD62(2000)093020), w_8]

- s_0-instability of physical output wrt s_0 as sign of premature truncation of slowly converging series

- (s_0-stability tests also important re neglect of $D > 6$ OPE terms [Supplementary pages])

CONVERGENCE/STABILITY STUDIES

- ALEPH us data, covariances; mode-by-mode rescaling for new BR's [Davier et al EPJC22 (2001) 31 strategy]

- significant shifts in central values from recent B-factory results, especially $B[\bar{K}^0\pi^-]$ (BELLE, arXiv:0706.2231), $B[K^-\pi^0]$ (BABAR, arXiv:0707.2922), $B[K^-\pi^+\pi^-]$ (BABAR, arXiv:0707.2981) [all hep-ex]

- Table for new 2007 WA's (from Banerjee arXiv:0707.3058 [hep-ex] plus newer BELLE $K\eta$, $K^*\eta$ results)

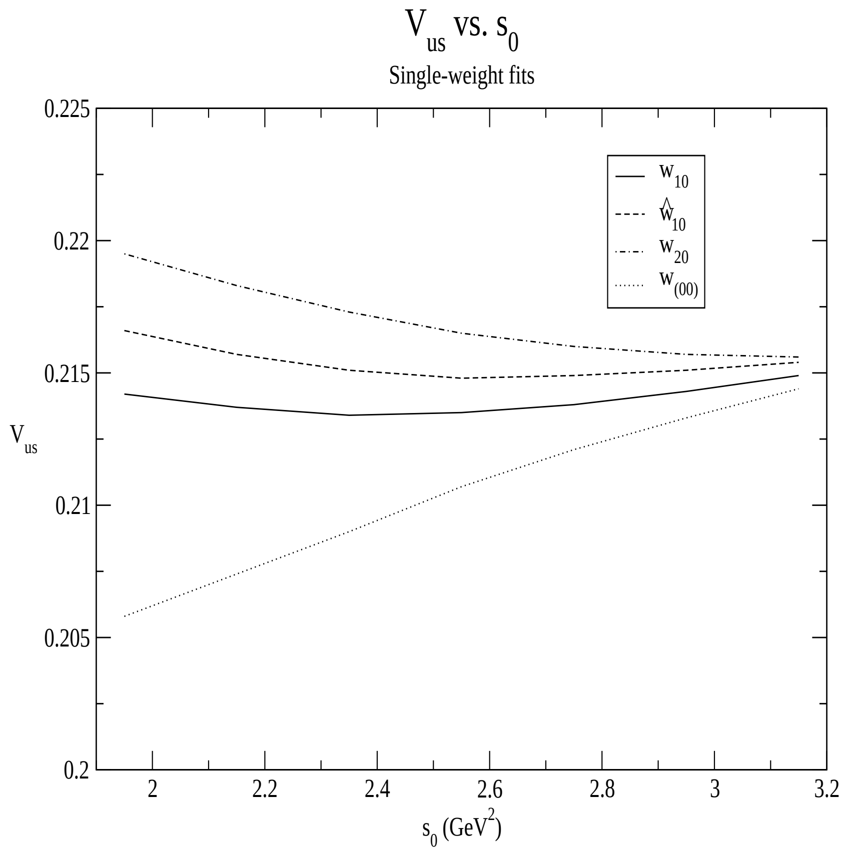
LEP+CLEO +BELLE+BABAR us $\mathcal{B}$ VALUES

Mode	$\mathcal{B}_{PDG06}$ (%)	$\mathcal{B}_{WA,2007}$ (%)
K^- [τ decay]	0.685 ± 0.023	[**]
(Alt: [$K_{\mu 2}$])	(0.715 ± 0.003)	
$K^-\pi^0$	0.454 ± 0.030	0.426 ± 0.016
$\bar{K}^0\pi^-$	0.878 ± 0.038	0.831 ± 0.028 $(S = 1.3)$
$K^-\pi^0\pi^0$	0.058 ± 0.024	[**]
$\bar{K}^0\pi^0\pi^-$	0.360 ± 0.040	[**]
$K^-\pi^-\pi^+$	0.330 ± 0.050	0.280 ± 0.016 $(S = 1.9)$
$K^-\eta$	0.027 ± 0.006	0.016 ± 0.002 $(S = 1.8)$
$(\bar{K}3\pi)^-$ (est'd)	0.074 ± 0.030	[**]
$K_1(1270) \to K^-\omega$	0.067 ± 0.021	
$(\bar{K}4\pi)^-$ (est'd)	0.011 ± 0.007	
$K^*\eta$	0.029 ± 0.009	0.012 ± 0.004 $(S = 2.0)$
TOTAL	2.969 ± 0.086	2.815 ± 0.074
	(3.003 ± 0.083)	(2.848 ± 0.071)

- for each $w(y)$, can fix m_s, $s_0 = m_\tau^2$, solve for V_{us}, then
  - look for m_s giving consistent V_{us} for different weights
  - check OPE/spectral integral match versus $s_0 < m_\tau^2$ for each weight separately with given m_s, resulting V_{us}, to see if any m_s yields s_0-stability

- results show [see Figures]
  - no consistency, no s_0-stability, no m_s with OPE/spectral integral match for $(k, 0)$ spectral weights
  - much improved situation (in all respects) for non-spectral weights

V_{us} s_0 STABILITY



$D=2$ OPE convergence/non-convergence behavior

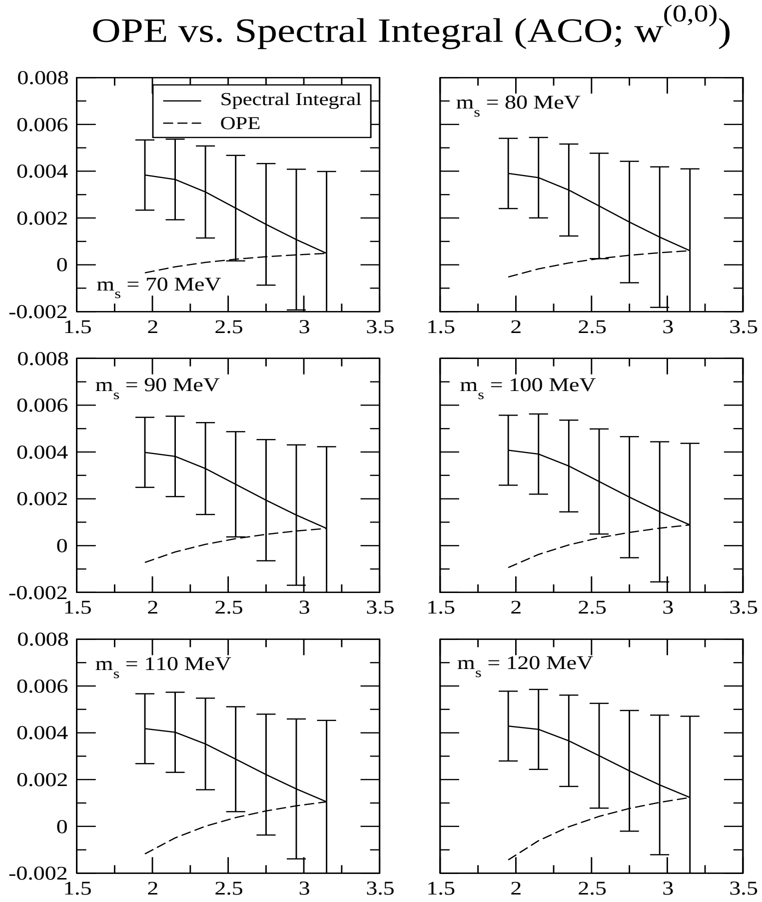
$O(a^N)$-truncated $D=2$ correlator/Adler function difference as alternate estimate of truncation uncertainty

- $r_k^w(s_0)$: $O(a^k)$ (correlator-Adler)/correlator ratio

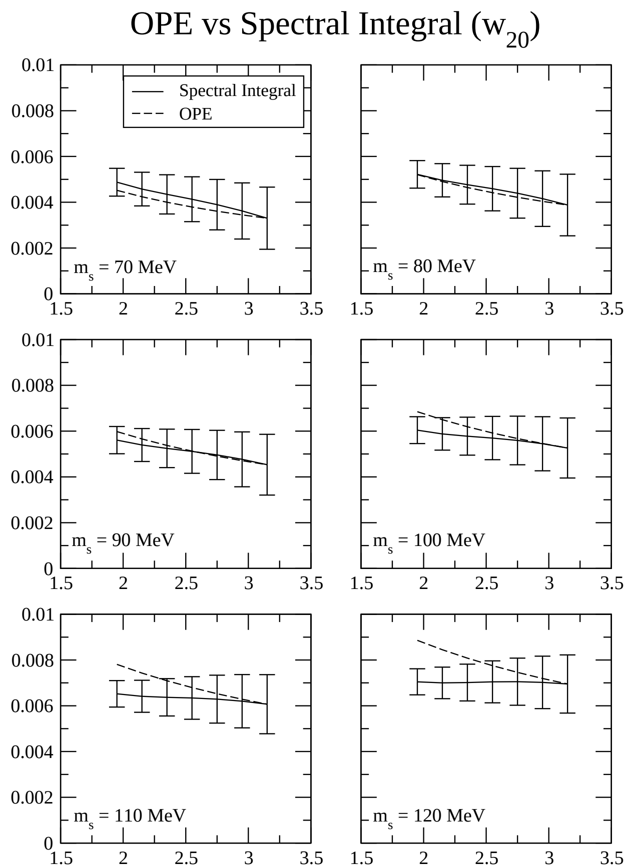
Weight	$r_1^w(m_\tau^2)$	$r_2^w(m_\tau^2)$	$r_3^w(m_\tau^2)$	$r_4^w(m_\tau^2)$
$w_{J=0+1}^{(0,0)}$	-0.01	0.06	0.20	0.67
$\hat{w}_{10}$	-0.11	-0.07	-0.05	-0.03
w_{20}	-0.11	-0.08	-0.05	-0.03
w_{10}	-0.10	-0.06	-0.03	-0.01

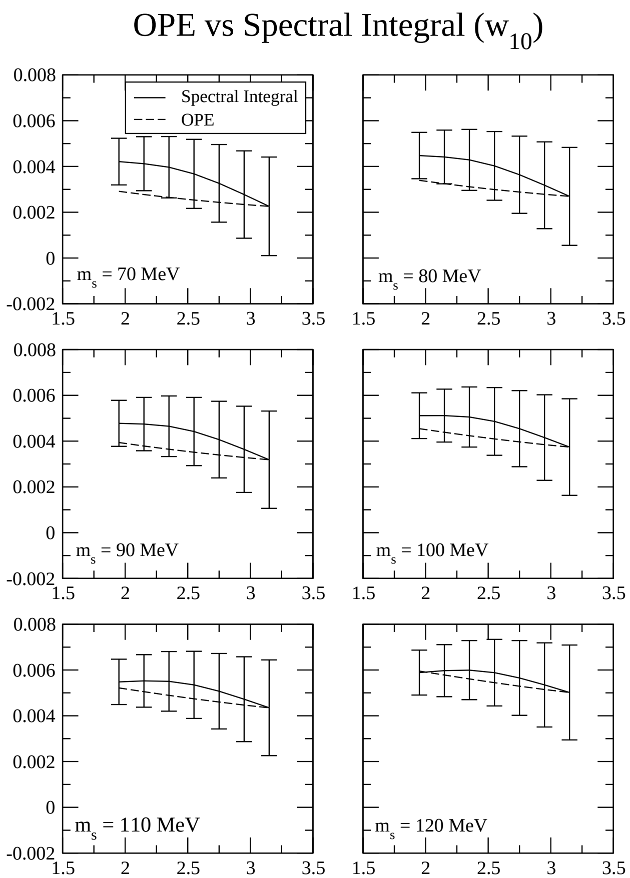
- s_0-instability $\Rightarrow$ (unfortunately) theory errors significantly underestimated for the $(0,0)$ spectral weight

$w^{(0,0)}$ OPE/SPECTRAL INTEGRAL MATCHES

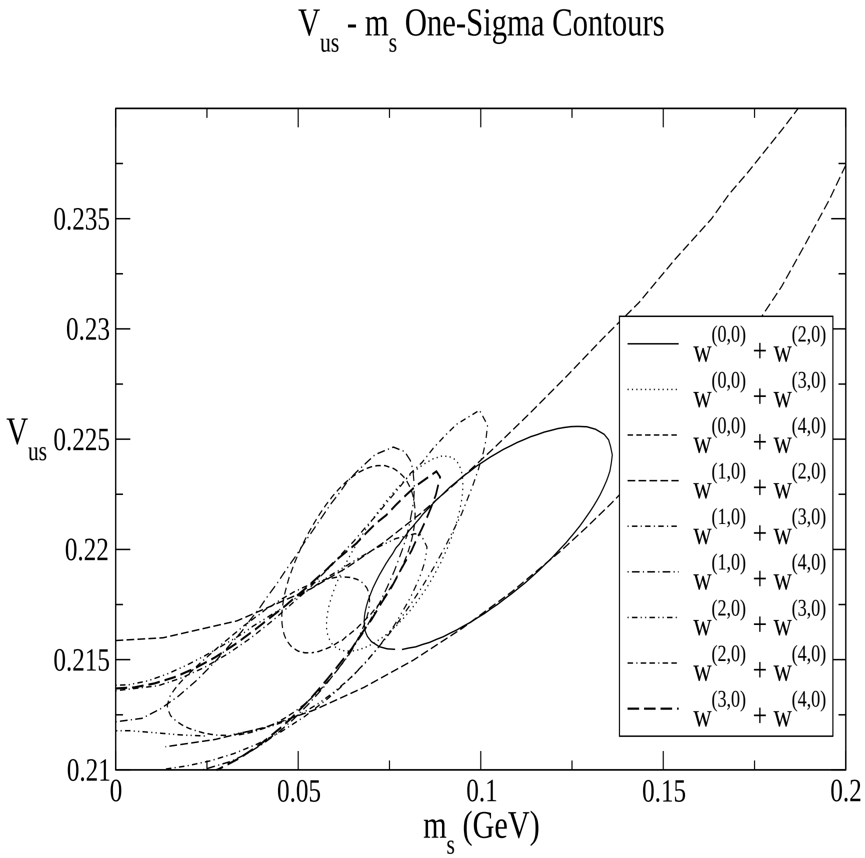


w_{20}, w_{10} OPE/SPECTRAL INTEGRAL MATCHES

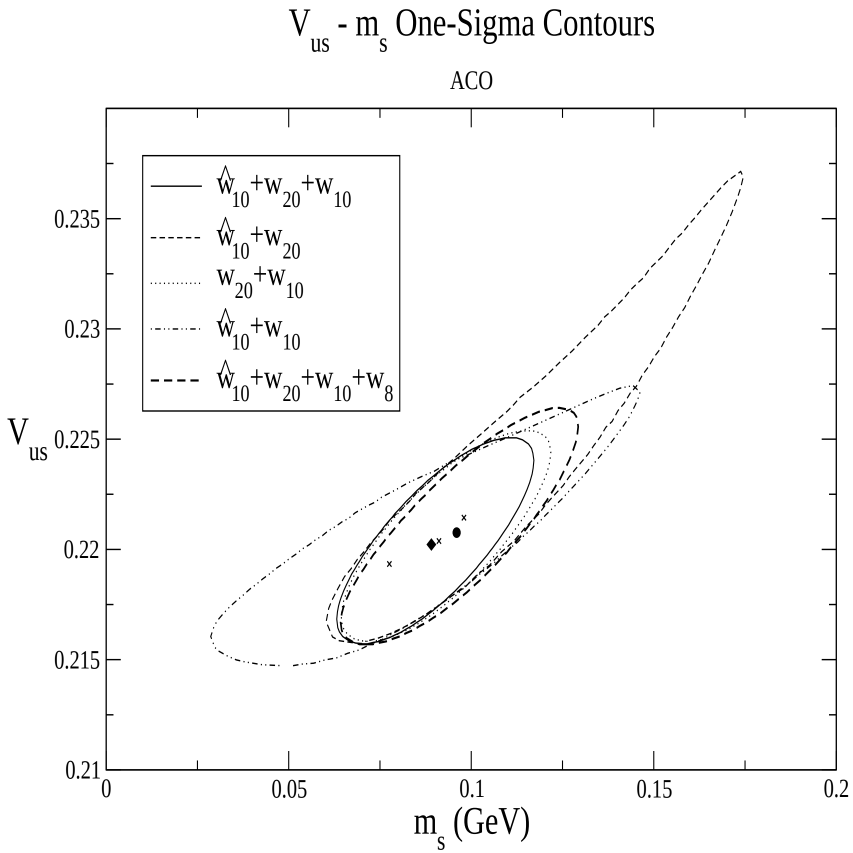




FIT CONTOURS, $(k, 0)$ SPECTRAL WEIGHTS



FIT CONTOURS, NON-SPECTRAL WEIGHTS



RESULTS WITH 2006 DATA

- conventional $s_0 = m_\tau^2$, 1-weight fits for $|V_{us}|$ with $m_s(2\ \text{GeV}) = 94 \pm 6$ MeV, PDG2006 ave BF's as input

$$0.2210 \pm 0.0030_{exp} \pm 0.0010_{th}\ (\hat{w}_{10})$$
$$0.2209 \pm 0.0029_{exp} \pm 0.0017_{th}\ (w_{20})$$
$$0.2206 \pm 0.0032_{exp} \pm 0.0007_{th}\ (w_{10})$$
$$0.2218 \pm 0.0037_{exp} \pm 0.0009_{th}\ (w_8)$$

- Combined $s_0 = m_\tau^2$, w_{20}, $\hat{w}_{10}$, w_{10} fit results

$$\boxed{\begin{array}{l} m_s(2\ \text{GeV}) = 89 \pm 26\ \text{MeV} \\ |V_{us}| = 0.2202 \pm 0.0046 \end{array}}$$

- m_s result in excellent agreement with recent average, (94 ± 6 MeV) of strange scalar, strange PS, $n_f = 2+1$ lattice results [Gamiz et al. hep-ph/0610246]

- 3-fold fit with $m_s(2\text{ GeV}) = 94 \pm 6$ MeV input:

$\vert V_{us}\vert = 0.2209 \pm 0.0031$ (WA B$[K^-\pi^+\pi^-]$
$\vert V_{us}\vert = 0.2232 \pm 0.0031$ (CO ave B$[K^-\pi^+\pi^-]$)

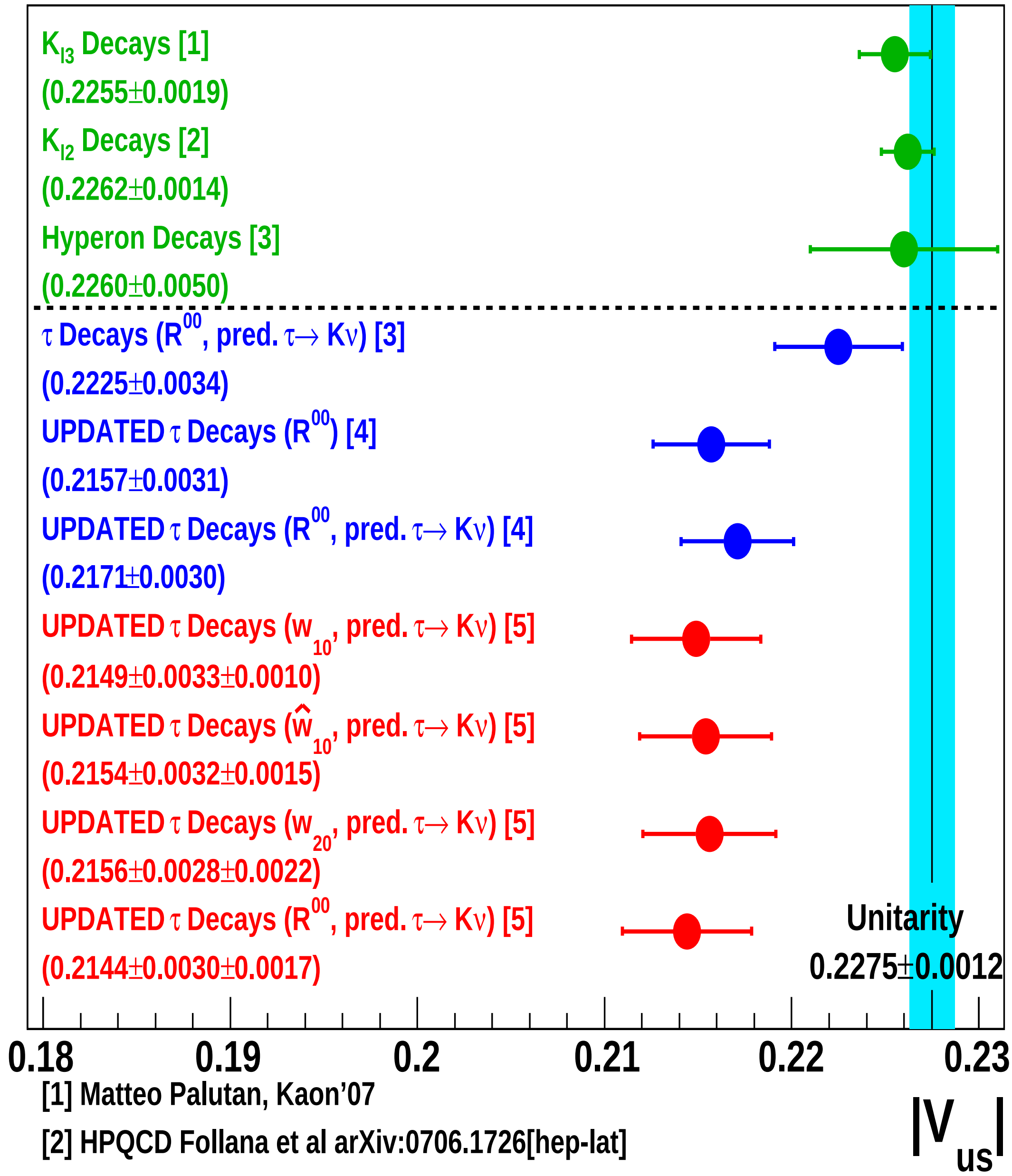
- c.f. unitarity expectation 0.2258 ± 0.0012 (NOTE: decrease from 0.2275 ± 0.0012 due to Hardy, Towner Oct. 19/07 V_{ud} update)

RESULTS WITH NEW 2007 us DATA

- conventional $s_0 = m_\tau^2$, 1-weight fits for $|V_{us}|$ with updated input $m_s(2\text{ GeV}) = 96 \pm 10$ MeV

$$0.2154 \pm 0.0032_{exp} \pm 0.0015_{th} \ (\hat{w}_{10})$$
$$0.2156 \pm 0.0028_{exp} \pm 0.0022_{th} \ (w_{20})$$
$$0.2149 \pm 0.0033_{exp} \pm 0.0010_{th} \ (w_{10})$$
$$0.2144 \pm 0.0030_{exp} \pm 0.0017_{th} \ (w_{(0,0)})$$

- *$\sim 3\sigma$ from 3-family unitarity expectations, most recent $K_{\ell 3}$, lattice-supplemented $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$*



[1] Matteo Palutan, Kaon'07

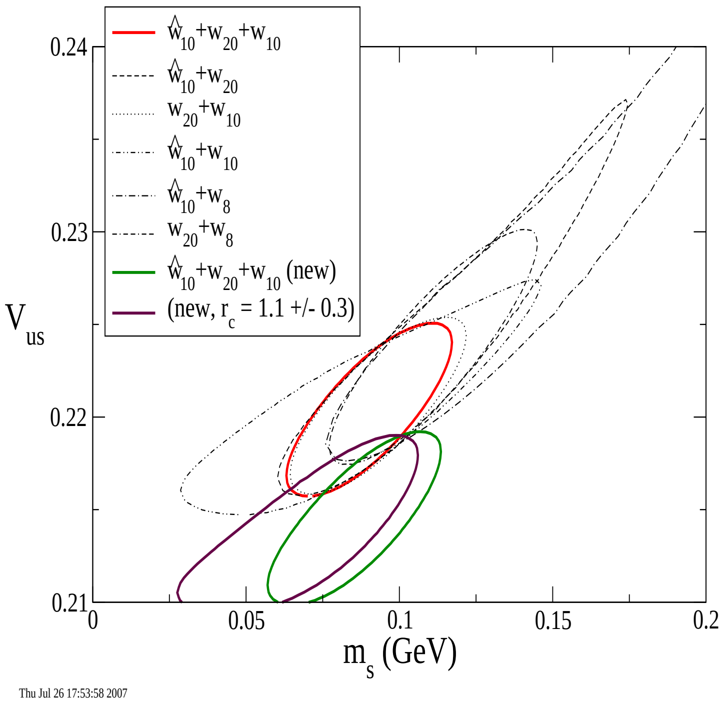
[2] HPQCD Follana et al arXiv:0706.1726[hep-lat]

[3] Jamin, Moriond EW07

[4] Swagato Banerjee, arXiv:0707.3058 [hep-ex] (R^{00} moment)

[5] Maltman and Wolfe, Private Communications (ALEPH99 us+ud rescaled with new BR)

- combined non-spectral weight fit impact mostly on V_{us}



- shifts in B_{us} appear small (sub-0.1%) but

  - shift in $\mathcal{B}[K^-\pi^0\nu_\tau]+\mathcal{B}[\bar{K}^0\pi^-\nu_\tau]$ (-0.075%) is $\sim 2.6\%$ of total us branching fraction $\leftrightarrow\ \sim 1.3\%$ (~ 0.0029) reduction in V_{us}

  - shift in $\mathcal{B}[K^-\pi^-\pi^+\nu_\tau]$ (-0.050%) is $\sim 1.8\%$ of total us branching fraction $\leftrightarrow\ \sim 0.9\%$ (~ 0.0020) reduction in V_{us}

  $\Rightarrow\ \sim 0.0050$ reduction in V_{us} c.f. 2006 analysis values

- LESSON: will need all strange modes (some no doubt new) with BF's down to $\sim$ a few 10^{-5} level

ERRORS AND THE FUTURE

- many us BF errors already much reduced, others soon to be much reduced by B factory analyses ($> 10^3$ times LEP statistics each for BABAR and BELLE)

- ingredients for full re-measurement of actual us spectral distribution in place and work in progress

- some obvious targets for near term BABAR, BELLE work, *especially* K^-, $K_S\pi^-\pi^0$ [Table]

Mode	$\mathcal{B}_{PDG06}$ (%)	New 2007 $\mathcal{B}$ (%)
K^- [τ decay]	0.685 ± 0.023	[**]
(Alt: [$K_{\mu 2}$])	(0.715 ± 0.003)	
$K^-\pi^0$	0.454 ± 0.030	0.416 ± 0.018
$\bar{K}^0\pi^-$	0.878 ± 0.038	0.808 ± 0.026
$K^-\pi^0\pi^0$	0.058 ± 0.024	[**]
$\bar{K}^0\pi^0\pi^-$	0.360 ± 0.040	[**]
$K^-\pi^-\pi^+$	0.330 ± 0.050	0.273 ± 0.009
$K^-\eta$	0.027 ± 0.006	0.0162 ± 0.0010
$(\bar{K}3\pi)^-$ (est'd)	0.074 ± 0.030	[**]
$K_1(1270) \to K^-\omega$	0.067 ± 0.021	
$(\bar{K}4\pi)^-$ (est'd)	0.011 ± 0.007	
$K^*\eta$	0.029 ± 0.009	0.0113 ± 0.0020
ϕK^-		0.00405 ± 0.00036
		0.00339 ± 0.00034

- need all strange modes with B to few-10^{-5} level
  - $B[K^-\phi]$ at few $\times 10^{-5}$ already reported
  - missing modes: higher multiplicity, higher s region
  - total $B_{us} \sim 3\%$ $\Rightarrow$ neglected 10^{-4} mode lowers $|V_{us}|$ by ~ 0.0004 for $w^{(0,0)}$, somewhat less for $w(s)$ with stronger high-s suppression

- poor convergence of $D = 2$ $(0,0)$ spectral weight OPE series implies need re-measured spectral *distribution* NOT just improved branching fractions (unfortunately)

- ud data also relevant [e.g. slightly lower BELLE central $B_{\pi\pi}$ (e^+e^- $B_{\pi\pi}$) would raise $|V_{us}|$ by ~ 0.0004 (0.0018)] (relation to $(g-2)_\mu$ question)

- additional non-spectral weight possibilities for exploration, improvement of both m_s, $|V_{us}|$ once BABAR, BELLE us distributions (with reduced errors above K^*) available

- s_0-stability tests absolutely crucial given the unavoidable slow convergence of the $D = 2$ $J = 0 + 1$ OPE series)

- Possibilities for BESIII?

  – NOTE: the 2007 BABAR and BELLE results are typically strongly systematics, NOT statistics, limited $\Rightarrow$ BESIII can contribute if systematics are better, in spite of reduced statistics [Table for examples]

Mode	$\mathcal{B}_{PDG06}$ (%)	New 2007 $\mathcal{B}$ (%)
$\pi^+\pi^-\pi^0$	8.99 ± 0.08	$8.83 \pm 0.01 \pm 0.13$
$K^-\pi^0$	0.452 ± 0.027	$0.416 \pm 0.003 \pm 0.018$
$\bar{K}^0\pi^-$	0.878 ± 0.038	$0.808 \pm 0.004 \pm 0.026$
$K^-\pi^-\pi^+$	0.330 ± 0.050	$0.273 \pm 0.002 \pm 0.009$
$K^-\eta$	0.027 ± 0.006	$0.0162 \pm 0.0005 \pm 0.0009$

  – If BESIII DOES has a systematics advantage interesting possibilities to consider are:

* non-strange and strange branching fractions and distributions (relevant to m_s, V_{us})

* experimental separation of $J = 0$, 1 components for $K\pi$, $K\pi\pi$ states for m_s, V_{us} studies ($J = 0$ subtraction)

  · HOWEVER [see Kuhn and Mirkes ZPC56 (1992) 661, erratum C67 (1995) 364] $J = 0/1$ separation requires detection of τ direction

  · HOWEVER^2, if possible, the V/A separation for the $K\pi\pi$ states can be done at the same time

* non-strange $\pi\pi$ and 4π distributions (relevant to V_{us}, the EM-τ disagreement for the $I = 1$ spectral function, implications for the LO hadronic contribution to $(g-2)_\mu$ in the SM)

* V/A separation for $K\bar{K}\pi$ states

  - dominant source of ambiguity in $I = 1$ sector

  - $J = 0$ negligible due to $O[(m_d + m_u)^2]$ suppression, hence separation possible WITHOUT detecting τ direction

  - theoretical disagreement over which of V, A should dominate

  - improvements for studies of classical Weinberg V-A sumrules, DGLY V-A sum rule for π EM self-energy in the $SU(3)$ chiral limit, experimental extraction of $K \to \pi\pi$ electroweak penguin ME's in $SU(3)$ chiral limit

CONCLUSIONS

- Current $\sim 3\sigma$ discrepancy for V_{us}, compared with 3-family unitarity expectations $\Rightarrow$
  - high priority for experimental work, especially on remaining larger modes (K, $K_S\pi^-\pi^0$)
  - desirability of results from BABAR, BELLE (and anyone else) on all us modes
  - need for detailed studies of higher multiplicity modes ($\bar{K}3\pi$, $\bar{K}4\pi, \cdots$)
  - importance of also actually re-measuring the full us distribution, not just branching fractions

- Theoretical issues:
  - *Longitudinal subtraction/modelling unavoidable*
  - non-spectral weights superior to spectral weights (including $(0,0)$); improvement via additional non-spectral weight choices almost certainly possible
  - slow convergence of basic $D = 2$ OPE $0 + 1$ correlator series $\Rightarrow$ s_0-stability checks essential
  - *Polemical stance: window of s_0 MUST exist with $|V_{us}|$ instability less than estimated theory uncertainty. If not $\Rightarrow$ theory error estimate insufficiently conservative*

- Two possible scenarios:
  - Continuing discrepancy with unitarity expectations (obviously the most interesting possibility)
  - New mode results, shifts for remaining modes restore agreement, in which case
    - $|V_{us}|$ to sub-± 0.0010 accuracy from m_s to ± 5 MeV, better than $1/5$ us data error reduction
    - combined m_s, V_{us} fit to check consistency of m_s important (slow $D = 2$ OPE convergence)
    - $|V_{us}|$ uncertainties *completely* independent of those for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice chiral extrapolation), hence further improvement by averaging

SUPPLEMENTARY PAGES

- Increase in statistics at B factory experiments
- Details on the handling of potential $D > 6$ OPE contributions

B factory vs. LEP statistics

Experiment	# $\tau^+\tau^-$ pairs
LEP	$\sim 3 \times 10^5$
BABAR	$\sim 3 \times 10^8$
BELLE	$\sim 5 \times 10^8$

(plus improved K ID at BABAR, BELLE)

HIGHER D OPE CONTRIBUTIONS

- rough estimates for $D = 6$ condensates, $D > 6$ combinations unknown, usually assumed negligible

- $w(y) = \sum_m c_m y^m$, $y = s/s_0$ $\Rightarrow$ integrated $D = 2k + 2$ OPE $\propto c_k/s_0^k$ (up to logs) $\Rightarrow$ avoid large c_k, $k \geq 2$

- neglect of non-negligible higher D terms $\Rightarrow$ s_0-instability of output $\Rightarrow$ *need to study output as function of* s_0

- NOTE: growth of coefficients in $(k, 0)$ spectral weights

$$
\begin{aligned}
w^{(0,0)}(y) &= 1-3y^2+2y^3 \\
w^{(1,0)}(y) &= 1-y-3y^2+5y^3-2y^4 \\
w^{(2,0)}(y) &= 1-2y-2y^2+8y^3-7y^4+2y^5 \\
w^{(3,0)}(y) &= 1-3y+10y^3-15y^4+9y^5-2y^6 \\
w^{(4,0)}(y) &= 1-4y+3y^2+10y^3-25y^4+24y^5 \\
&\quad -11y^6+2y^7
\end{aligned}
$$

- contrast 4 *non-spectral weights* used in literature (also normalized to 1 at $y = 0$): w_{20}, $\widehat{w}_{10}$, w_{10}, w_8, with largest $k \geq 2$ coefficients $c_4 = 2.087$ (w_{20}), $c_5 = 1.206$ ($\widehat{w}_{10}$), $c_5 = 2$ (w_{10}), $c_5 = 1.182$ (w_8)