Analyses of $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta^{(\prime)} \nu_\tau$ decays

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Outline

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   - $\tau^- \to K^- \eta^{(\prime)} \nu_\tau$
   - Combined analysis of $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$
4. **Conclusions**

Decay Spectrum of the τ lepton

$\tau^- \to \nu_\tau$+strange

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^-\overline{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^-\overline{K}^0\pi^0\pi^0\nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0h^-h^-h^+\nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^-\eta\nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^-\pi^0\eta\nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^-\overline{K}^0\eta\nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^-\omega\nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^-\phi\nu_\tau(\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

- $\tau^- \to K^-\eta\nu_\tau$[1] measured by BaBar and Belle

Mode	BaBar – Belle Normalized Difference ($\#\sigma$)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	+1.4
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	−2.9
$K^-K^+\pi^-\nu_\tau$	−2.9
$K^-K^+K^-\nu_\tau$	−5.4
$\eta\ K^-\nu_\tau$	−1.0
$\phi\ K^-\nu_\tau$	−1.3
$\pi^-K_S^0K_S^0\nu_\tau$	−0.2
$\pi^-K_S^0K_S^0\pi^0\nu_\tau$	−1.0

- To predict $\tau^- \to K^-\eta'\nu_\tau$[1]
- Combined analysis[2] of $\tau^- \to K^-\eta\nu_\tau$ and $\tau^- \to K_S\pi^-\nu_\tau$[3] to constraint the $K^*(1410)$ resonance parameters (mass and width)
- To study the second-class current $\tau \to \pi^-\eta^{(\prime)}\nu_\tau$ decays[4]

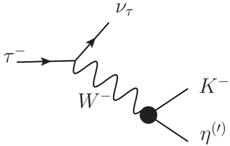
1 Escribano, Gonzàlez-Solís, Roig JHEP 1310 (2013) 039

2 Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014) 042

3 (Jamin, Pich, Portolés PLB 640 (2006), Boito, Escribano, Jamin JHEP 1009 (2010) 031, Moussallam EPJ C53 (2008))

4 Escribano, Gonzàlez-Solís, Roig hep-ph/1601.03989

$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: Amplitude and decay width



$$\overset{q^2 \ll M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{us} \bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu(1-\cancel{\gamma^5})u|0\rangle$$

$$\underbrace{\cancel{\gamma^5}}_{0^-,1^+ \nrightarrow 0^+,1^-}$$

$$\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle = \left[(p_{\eta^{(\prime)}} - p_K)^\mu + \frac{\Delta_{K^-\eta^{(\prime)}}}{s} q^\mu\right] C^V_{K\eta^{(\prime)}} F_+^{K^-\eta^{(\prime)}}(s) + \frac{\Delta_{K\pi}}{s} q^\mu C^S_{K\eta^{(\prime)}} F_0^{K^-\eta^{(\prime)}}(s)$$

$$\frac{d\Gamma\left(\tau^- \to K^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{32\pi^3 s} S_{EW} \underbrace{|V_{us}}_{\text{suppression}} F_+^{K^-\eta^{(\prime)}}(0)|^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

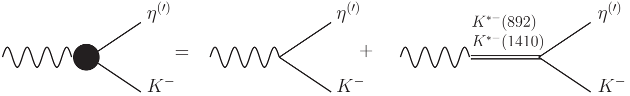
$$\left\{\left(1 + \frac{2s}{M_\tau^2}\right) q^3_{K\eta^{(\prime)}}(s)|\widetilde{F}_+^{K^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{K\eta^{(\prime)}}}{4s} q_{K\eta^{(\prime)}}(s)|\widetilde{F}_0^{K^-\eta^{(\prime)}}(s)|^2\right\}$$

$F_+^{K^-\eta}(0) = F_+^{K^-\pi}(0)\cos\theta_P\,,\ F_+^{K^-\eta'}(0) = F_+^{K^-\pi}(0)\sin\theta_P\,,$

$\theta_P = (-13.3 \pm 0.5)^\circ$

$V_{us} \cdot F_+^{K^-\pi}(0) = 0.2163(5)\,,\ K_{\ell 3}$

Vector Form Factor: Resonance Chiral Theory

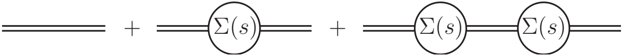


$$F_+^{K^-\eta}(s) = \cos\theta\left(1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_{K^\star}^2 - s} + \frac{F'_V G'_V}{F_\pi^2}\frac{s}{M_{K^{\star\prime}}^2 - s}\right) = \cos\theta F_+^{K^-\pi^0}(s)$$

- Requirement: $F_+^{K\pi}(s)$ vanish for $s \to \infty \Rightarrow F_V G_V + F'_V G'_V = F_\pi^2$

$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s} \quad ; \quad \gamma = -\frac{F'_V G'_V}{F_\pi^2} = \frac{F_V G_V}{F_\pi^2} - 1$$

- Breit-Wigner parameterization:



$$+ \ldots = \frac{i}{s - M_{K^\star}^2 + \Sigma(s)}$$

$$F_+^{K\pi}(s) = \frac{M_{K^\star}^2 + \gamma s}{M_{K^\star}^2 - s - iM_{K^\star}\Gamma_{K^\star}(s)} - \frac{\gamma s}{M_{K^{\star\prime}}^2 - s - iM_{K^{\star\prime}}\Gamma_{K^{\star\prime}}(s)}$$

Vector Form Factor: Dispersive representation

- Three subtractions: helps the convergence of the form factor and suppresses the the high-energy region of the integral

$$F_+^{K\pi}(s) = P(s)\exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right]$$

- $\alpha_1 = \lambda_+^{'}$ and $\alpha_1^2 + \alpha_2 = \lambda_+^{''}$ low energies parameters

$$F_+^{K\pi}(t) = 1 + \frac{\lambda_+^{'}}{M_{\pi^-}^2} t + \frac{1}{2}\frac{\lambda_+^{''}}{M_{\pi^-}^4} t^2$$

- s_{cut}: cut-off to check stability

- $\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im}F_+^{K\pi}(s)}{\mathrm{Re}F_+^{K\pi}(s)}\right]$; $\widetilde{F}_+^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})}$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n Re\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s)$$

Scalar Form Factor

- Central unitarity relation

$$\mathrm{Im}F_i(s) = \sigma_j(s)F^j(s)T^{i\to j}(s)^*$$

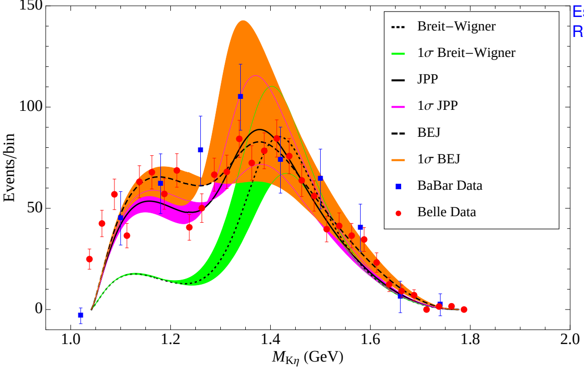
- Coupled channels dispersion relations (Jamin, Oller, Pich Nucl.Phys.B622 (2002))

$$F_0^{K\pi}(s) = \frac{1}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\sigma_{K\pi}(s')F_0^{K\pi}(s')T^*_{K\pi\to K\pi}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{K\eta}}^{\infty} ds' \frac{\sigma_{K\eta}(s')F_0^{K\eta}(s')T^*_{K\eta\to K\pi}(s')}{s'-s-i\varepsilon}$$

$$F_0^{K\eta}(s) = \frac{1}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\sigma_{K\pi}(s')F_0^{K\pi}(s')T^*_{K\pi\to K\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{K\eta}}^{\infty} ds' \frac{\sigma_{K\eta}(s')F_0^{K\eta}(s')T^*_{K\eta\to K\eta}(s')}{s'-s-i\varepsilon}$$

Fits to the $\tau^- \to K^-\eta\nu_\tau$ BaBar and Belle data

- Decay dominated by the vector Form Factor (~ 96% of the BR)



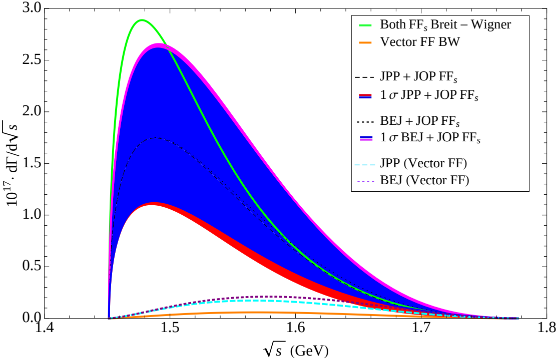
Escribano, Gonzàlez-Solís, Roig, JHEP 1310 (2013) 039

Source	Branchig ratio·10^4	χ^2/dof	$K^\star(1410)$ Mass	$K^\star(1410)$ Width	γ
Breit-Wigner	$\left(0.96^{+0.21}_{-0.15}\right)$	5.0			-0.174 ± 0.007
Exponential representation	(1.42 ± 0.04)	1.4	1332^{+16}_{-18} MeV	220^{+26}_{-24} MeV	$-0.078^{+0.012}_{-0.014}$
Dispersion relation	(1.55 ± 0.08)	0.8	1327^{+30}_{-38} MeV	213^{+72}_{-118} MeV	$-0.051^{+0.012}_{-0.036}$
Experimental value	(1.52 ± 0.08)	-			

Channel	$K^\star(1410)$ Mass	$K^\star(1410)$ Width	γ
$K\pi$ mode	1277^{+35}_{-41} MeV	218^{+95}_{-66} MeV	$-0.049^{+0.019}_{-0.016}$
$K\eta$ mode (this work)	1330^{+27}_{-41} MeV	217^{+68}_{-122} MeV	$-0.065^{+0.025}_{-0.050}$

Predictions for the $\tau^- \to K^- \eta' \nu_\tau$ decay

- Decay dominated by the scalar Form Factor ($\sim$ 90% of the BR)

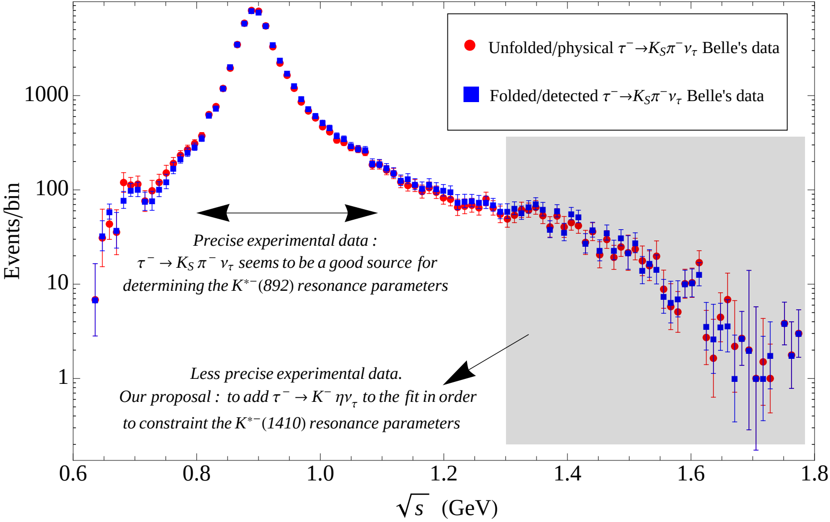


Source	Branchig ratio
Breit-Wigner	$(1.45^{+3.80}_{-0.87}) \cdot 10^{-6}$
Exponential representation	$(1.00^{+0.37}_{-0.29}) \cdot 10^{-6}$
Dispersion relation	$(1.03^{+0.37}_{-0.29}) \cdot 10^{-6}$
Experimental bound	$< 2.4 \cdot 10^{-6}$ at 90% C.L.

Combined analysis of $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$

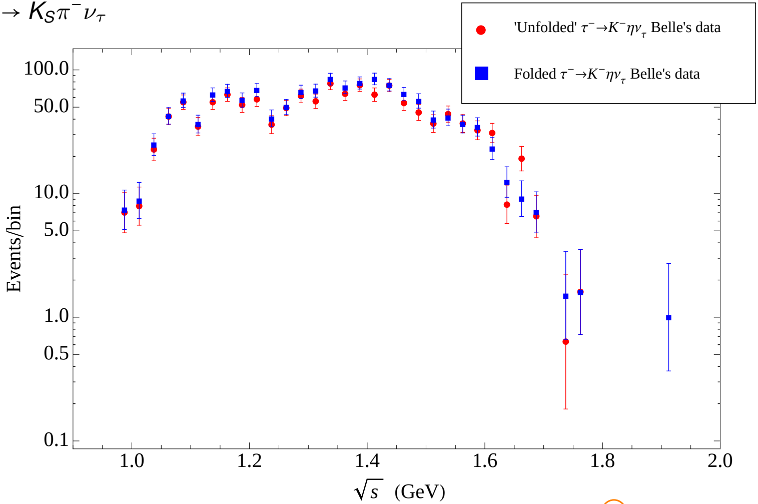
- Reason for a simultaneous fit to $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ Belle data

(Epifanov et. al. Phys. Lett. **B 654 (2007) 65**)



Combined analysis of $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$

- Unfolding $\tau^- \to K^-\eta\nu_\tau$ Belle's data through an "unfolding" function from $\tau^- \to K_S\pi^-\nu_\tau$



∘ Experimentalist: To provide unfolded data would be really useful ☺

∘ Theorists: To provide theoretical models to be fitted by experimentalists

- We relate the experimental data with the differential decay distribution from theory through

$$\frac{dN_{events}}{d\sqrt{s}} = N_{events}\Delta_{bin}\frac{1}{\Gamma_\tau BR(\tau \to P^- P^0\nu_\tau)}\frac{d\Gamma\left(\tau^- \to P^- P^0\nu_\tau\right)}{d\sqrt{s}}$$

$$\frac{d\Gamma\left(\tau^- \to P^- P^0\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{32\pi^3 s} S_{EW}|V_{us}F_+^{P^-P^0}(0)|^2\left(1-\frac{s}{M_\tau^2}\right)^2$$

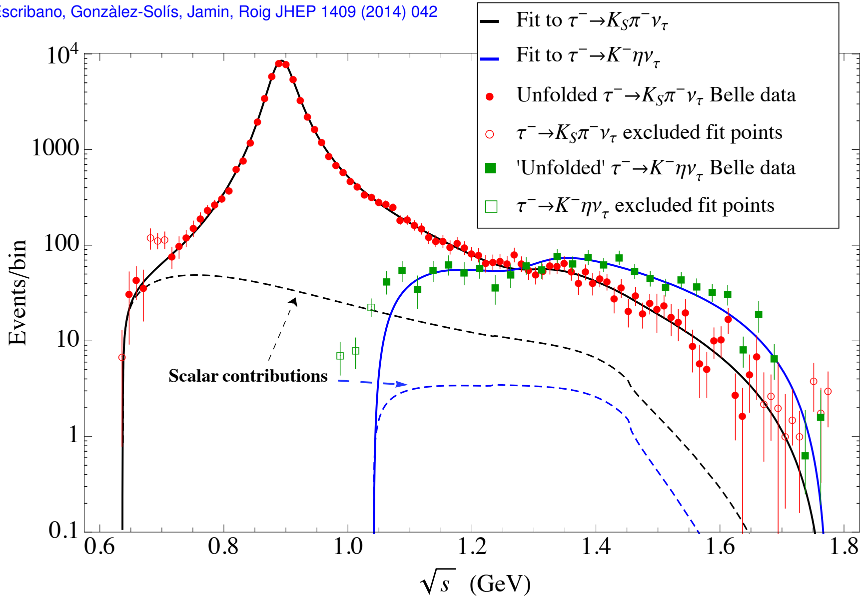
$$\times\left\{\left(1+\frac{2s}{M_\tau^2}\right)q^3_{P^-P^0}(s)|\widetilde{F}_+^{P^-P^0}(s)|^2 + \frac{3\Delta^2_{P^-P^0}}{4s}q_{P^-P^0}(s)|\widetilde{F}_0^{P^-P^0}(s)|^2\right\}$$

- $P^-P^0 = K_S\pi^- \to BR_{exp}^{Belle} = 0.404\%$ $\quad N_{events} = 53113$ $\quad \Delta_{bin} = 0.0115$ GeV/bin
- $P^-P^0 = K^-\eta \to BR_{exp}^{Belle} = 1.58\cdot 10^{-4}$ $\quad N_{events} = 1271$ $\quad \Delta_{bin} = 0.025$ GeV/bin
- $\Gamma_\tau = 2.265\cdot 10^{-12}$
- Function minimised in our fit

$$\chi^2 = \sum_{bin}\left(\frac{\mathcal{N}^{th}-\mathcal{N}^{exp}}{\sigma_{\mathcal{N}^{exp}}}\right)^2 + \sum_{K_S\pi^-,K^-\eta}\left(\frac{\bar{B}^{th}-\bar{B}^{exp}}{\sigma_{\bar{B}^{exp}}}\right)^2$$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014) 042



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4$ GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
χ^2/n.d.f.	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Reference fit results obtained for different values of s_{cut}

Parameter \ $s_{\rm cut}$(GeV^2)	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
χ^2/n.d.f.	105.8/105	108.1/105	111.0/105	111.1/105

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Central results including the largest variation of s_{cut}

$M_{K^{*-}(892)} = 892.03 \pm 0.19$ MeV
$\Gamma_{K^{*-}(892)} = 46.18 \pm 0.44$ MeV
} no gain

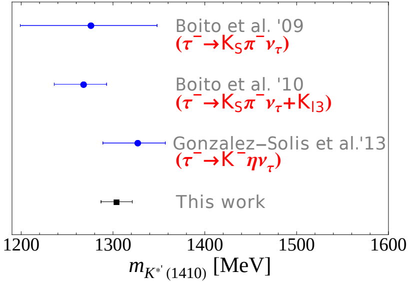
$M_{K^{*-}(1410)} = 1305^{+16}_{-18}$ MeV
$\Gamma_{K^{*-}(1410)} = 168^{+65}_{-59}$ MeV
} improvement

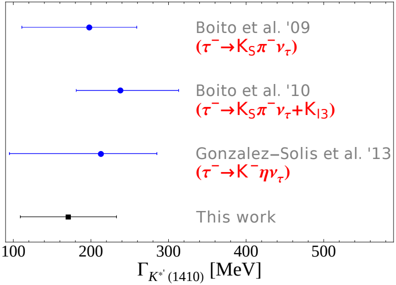
$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$

$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$

$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$

$\chi^2/d.o.f = 108.1/105 = 1.03$





Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

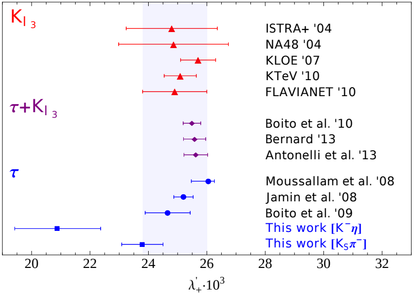
- Central results including the largest variation of s_{cut}

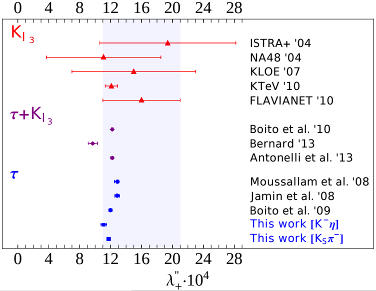
$$\left.\begin{aligned}\lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3}\\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3}\end{aligned}\right\}\text{isospin violation?}$$

$$\left.\begin{aligned}\lambda''_{K\pi} &= (11.8 \pm 0.2)\cdot 10^{-4}\\ \lambda''_{K\eta} &= (11.1 \pm 0.5)\cdot 10^{-4}\end{aligned}\right\}\text{isospin violation?}$$

$$\Uparrow$$

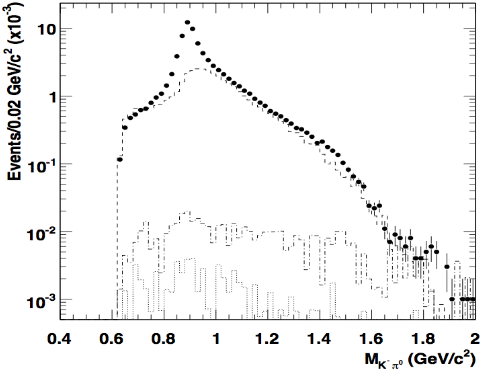
$$\tau^- \to K^-\pi^0\nu_\tau \& K_{\ell 3}$$





Prospects of improvement

- Call 1: to release $\tau^- \to K^-\eta\nu_\tau$ acceptance corrected
- Call 2: to provide $\tau^- \to K^-\pi^0\nu_\tau$ data (acceptance corrected)



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Determination of resonance parameters

- To look for a zero of the propagator in the complex plane

$$M_{K^\star}^2 - s_{\text{pole}} - \frac{3}{2} M_{K^\star}^2 \,\text{Re}\widetilde{H}_{K\pi}(s_{\text{pole}}) - iM_{K^\star}\Gamma_{K^\star}(s_{\text{pole}}) = 0\,;\; s_{\text{pole}} = \left(M_{\text{phys}} - \frac{i}{2}\Gamma_{\text{phys}}\right)^2$$

$$M_{K^*_{1410}} = 1378(24)\,\text{MeV}\,,\; \Gamma_{K^*_{1410}} = 197(70)\,\text{MeV} \Rightarrow \text{Model}$$

$$M_{K^*_{1410}} = 1305^{+16}_{-18}\,\text{MeV}\,,\; \Gamma_{K^*_{1410}} = 168^{+65}_{-59}\,\text{MeV} \Rightarrow \text{Pole}$$

$K^*(1410)$ $\qquad I(J^P) = \frac{1}{2}(1^-)$

$K^*(1410)$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	CHG	COMMENT
1414±15 OUR AVERAGE	Error includes scale factor of 1.3.				
1380±21±19	ASTON	88	LASS	0	11 $K^- p \to K^- \pi^+$
1420± 7±10	ASTON	87	LASS	0	11 $K^- p \to \overline{K}^0 \pi^+ \pi$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
1276^{+72}_{-77}	[1,2] BOITO	09	RVUE		$\tau^- \to K^0_S \pi^- \nu_\tau$
1367±54	BIRD	89	LASS	−	11 $K^- p \to \overline{K}^0 \pi^- p$
1474±25	BAUBILLIER	82B	HBC	0	8.25 $K^- p \to \overline{K}^0 2\pi$
1500±30	ETKIN	80	MPS	0	6 $K^- p \to \overline{K}^0 \pi^+ \pi^-$

[1] From the pole position of the $K\pi$ vector form factor in the complex s-plane an
EPIFANOV 07 data.
[2] Systematic uncertainties not estimated.

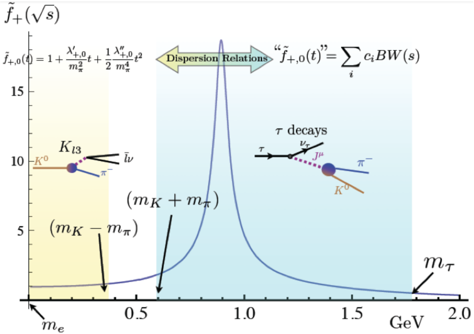
$K^*(1410)$ WIDTH

VALUE (MeV)	DOCUMENT ID		TECN	CHG	COMMENT
232± 21 OUR AVERAGE	Error includes scale factor of 1.1.				
176± 52±22	ASTON	88	LASS	0	11 $K^- p \to$
240± 18±12	ASTON	87	LASS	0	11 $K^- p \to$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
198^{+61}_{-87}	[3,4] BOITO	09	RVUE		$\tau^- \to K^0_S \pi$
114±101	BIRD	89	LASS	−	11 $K^- p \to$
275± 65	BAUBILLIER	82B	HBC	0	8.25 $K^- p \to$
500±100	ETKIN	80	MPS	0	6 $K^- p \to$

[3] From the pole position of the $K\pi$ vector form factor in the complex s-
EPIFANOV 07 data.
[4] Systematic uncertainties not estimated.

Applications of the $K\pi$ Form Factors

- Dispersive representation of the $K\pi$ form factor suited to describe both $\tau \to K\pi\nu_\tau$ and $K_{\ell 3}$ decays



- $K_{\ell 3}$ decays are the main route towards the determination of $|V_{us}|^2$

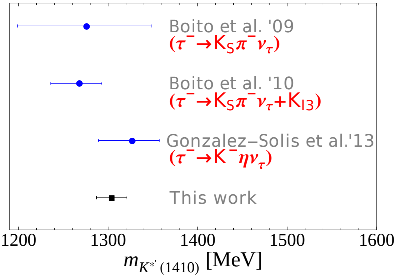
$$\Gamma_{K_{\ell 3}} \propto |V_{us}|^2 |F_+(0)|^2 I_{K_{\ell 3}}, \qquad I_{K_{\ell 3}} = \frac{1}{m_K^8}\int dt(p.s.)\left[\widetilde{F}_+(t)^2 + \eta(t, m_\ell)\widetilde{F}_0(t)^2\right]$$

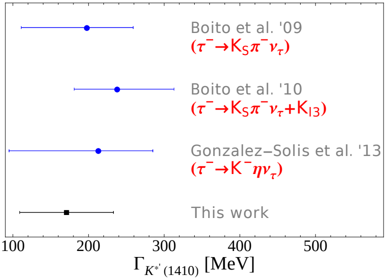
$F_+(0)$ ↓ ChPT, lattice

$\widetilde{F}_+(t)$, $\widetilde{F}_0(t)$ ↓ RChT+ dispersion relations

Conclusions

- $K\pi$ Vector Form Factor from dispersion relations with subtractions
  - First detailed analysis of $\tau^- \to K^-\eta^{(\prime)}\nu_\tau$ decays
  - Combined analysis of the decays $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$





- Prospects of improvement:
  - To provide $\tau^- \to K^-\eta\nu_\tau$ acceptance corrected
  - To release $\tau^- \to K^-\pi^0\nu_\tau$ (acceptance corrected) to access isospin violations

Back-up

Hadronic Matrix Element

- Taking the divergence we obtain on the L.H.S

$$\langle 0|\partial_\mu(\bar{s}\gamma^\mu u)|K^+\eta^{(\prime)}\rangle = i(m_s - m_u)\langle 0|\bar{s}u|K^+\eta^{(\prime)}\rangle = i\Delta_{K\pi} C^S_{K^-\eta^{(\prime)}} F_0^{K^-\eta^{(\prime)}}(s) \quad (1)$$

where $\Delta_{PQ} = M_P^2 - M_Q^2$, $C^S_{K^-\eta} = 1/\sqrt{6}$, $C^S_{K^-\eta'} = 2/\sqrt{3}$

- on the R.H.S (vector current not conserved)

$$iq_\mu\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle = iC^V_{K\eta^{(\prime)}}\left[(m^2_{\eta^{(\prime)}} - m^2_{K^-})F_+^{K^-\eta^{(\prime)}}(s) - sF_-^{K^-\eta^{(\prime)}}(s)\right] \quad (2)$$

- Equating eqs. (1,2) allows us to relate $F_-^{K^-\eta^{(\prime)}}(s)$ with $F_0^{K^-\eta^{(\prime)}}(s)$ as

$$F_-^{K^-\eta^{(\prime)}}(s) = -\frac{\Delta_{K^-\eta^{(\prime)}}}{s}\left[\frac{C^S_{K\eta^{(\prime)}}}{C^V_{K\eta^{(\prime)}}}\frac{\Delta_{K\pi}}{\Delta_{K^-\eta^{(\prime)}}}F_0^{K^-\eta^{(\prime)}}(s) + F_+^{K^-\eta^{(\prime)}}(s)\right] \quad (3)$$

- The hadronic matrix element finally reads ($q^\mu = (p_{\eta^{(\prime)}} + p_{K^-})^\mu$+ and $q^2 = s$)

$$\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle =$$

$$\left[(p_{\eta^{(\prime)}} - p_K)^\mu + \frac{\Delta_{K^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{K\eta^{(\prime)}}F_+^{K^-\eta^{(\prime)}}(s) + \frac{\Delta_{K\pi}}{s}q^\mu C^S_{K\eta^{(\prime)}}F_0^{K^-\eta^{(\prime)}}(s) \quad (4)$$

Predictions for the $\tau^- \to K^-\eta^{(\prime)}\nu_\tau$ decays

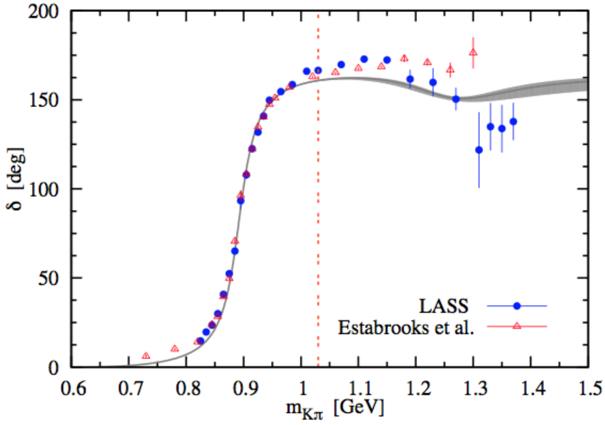
- $\tau^- \to K_S\pi^-\nu_\tau$ Fit results (Boito-Escribano-Jamin Eur.Phys.J. C59 (2009))

	$s_{\rm cut} = 3.24$ GeV^2	$s_{\rm cut} = 4$ GeV^2	$s_{\rm cut} = 9$ GeV^2	$s_{\rm cut} \to \infty$
m_{K^*} [MeV]	943.32 ± 0.59	943.41 ± 0.58	943.48 ± 0.57	943.49 ± 0.57
γ_{K^*} [MeV]	66.61 ± 0.88	66.72 ± 0.86	66.82 ± 0.85	66.82 ± 0.85
$m_{K'^*}$ [MeV]	1407 ± 44	1374 ± 30	1362 ± 26	1362 ± 26
$\gamma_{K'^*}$ [MeV]	325 ± 149	240 ± 100	216 ± 86	215 ± 86
$\gamma \times 10^2$	-5.2 ± 2.0	-3.9 ± 1.5	-3.5 ± 1.3	-3.5 ± 1.3
$\lambda'_+ \times 10^3$	24.31 ± 0.74	24.66 ± 0.69	24.94 ± 0.68	24.96 ± 0.67
$\lambda''_+ \times 10^4$	12.04 ± 0.20	11.99 ± 0.19	11.96 ± 0.19	11.96 ± 0.19
χ^2/n.d.f.	74.2/79	75.7/79	77.2/79	77.3/79

- Our $K\pi$ system is $K^-\pi^0$ instead of $K_S\pi^-$
- Mass difference ($\sim$ 10 MeV) strongly correlated with λ'_+ and λ''_+
- No $\tau^- \to K^-\pi^0\nu_\tau$ data available. We fit $\tau^- \to K_S\pi^-\nu_\tau$ data using $K^-\pi^0$ masses

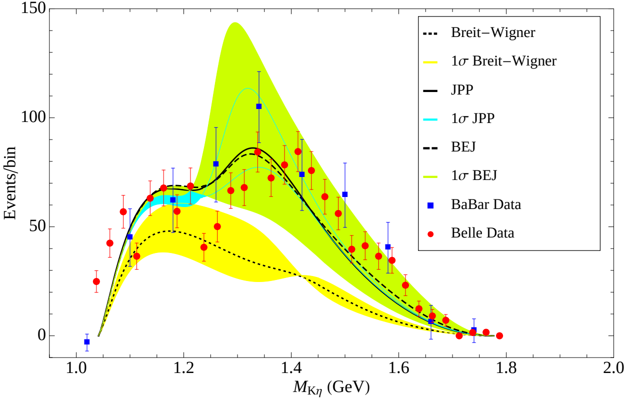
Parameter	Best fit with $K^-\pi^0$ masses	Best fit
$\lambda'_+ \times 10^3$	22.2 ± 0.9	24.7 ± 0.8
$\lambda''_+ \times 10^4$	10.3 ± 0.2	12.0 ± 0.2
$M_{K^\star}$ (MeV)	892.1 ± 0.6	892.0 ± 0.9
$\Gamma_{K^\star}$ (MeV)	46.2 ± 0.5	46.2 ± 0.4
$M_{K^{\star\prime}}$ (GeV)	1.28 ± 0.07	1.28 ± 0.07
$\Gamma_{K^{\star\prime}}$ (GeV)	$0.16^{+0.10}_{-0.07}$	$0.20^{+0.06}_{-0.09}$
γ	-0.03 ± 0.02	-0.04 ± 0.02

$K\pi$ scattering phase



Predictions for the $\tau^- \to K^- \eta \nu_\tau$ decay

- Decay dominated by the vector Form Factor (~ 96% of the BR)



Source	Branchig ratio	χ^2/dof
Breit-Wigner	$\left(0.78^{+0.17}_{-0.10}\right) \cdot 10^{-4}$	8.3
Exponential representation	$\left(1.47^{+0.14}_{-0.08}\right) \cdot 10^{-4}$	1.9
Dispersion relation	$(1.49 \pm 0.05) \cdot 10^{-4}$	1.5
Experimental value	$(1.52 \pm 0.08) \cdot 10^{-4}$	-