# An overview of $\tau \to K_S \pi \nu_\tau$ and $K \eta \nu_\tau$ decays


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Purpose 1: to present a model for the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from $K_{l3}$ decays suited to describe both $\tau \to K\pi\nu_\tau$ and $K_{l3}$ decays simultaneously

Why? because a good knowledge of the $K\pi$ f.f.'s is of fundamental importance for the determination of $V_{us}$ from $K_{l3}$ decays

Purpose 2: to present a combined analysis of the $\tau \to K_S\pi\nu_\tau$ and $K\eta\nu_\tau$ decays

Why? to further constrain the properties of the $K^*(1410)$ vector resonance

## Outline:

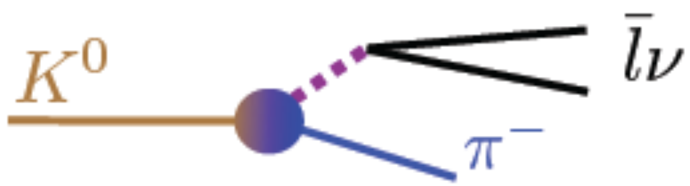
- *Introduction*
- $K\pi$ *form factors*
- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* $K_{l3}$
- *Combined analysis of* $\tau \to K_S\pi\nu_\tau$ *and* $K\eta\nu_\tau$ *decays*
- *Summary and Conclusions*

in collab. with D. R. Boito, S. González Solís, M. Jamin and P. Roig,
EPJC 59 (2009) 821, JHEP 09 (2010) 031, JHEP 10 (2013) 039 and JHEP 09 (2014) 042

## • *Introduction*

- $K_{l3}$ decays are the main route towards the determination of $|V_{us}|^2$

  H. Leutwyler and M. Roos, ZPC 25 (1984) 91




$$\Gamma_{K_{l3}} \propto |V_{us}|^2 |F_+(0)|^2 I_{K_{l3}}$$

with

$$I_{K_{l3}} = \frac{1}{m_K^8} \int dt\,(\text{p.s.}) \left[\tilde{F}_+(t)^2 + \eta(t, m_l)\tilde{F}_0(t)^2\right]$$

and $\quad \tilde{F}_{+,0}(q^2) \equiv \dfrac{F_{+,0}(q^2)}{F_+(0)}$

- $F_{+,0}(0)$ the normalization from ChPT, Lattice
- $\tilde{F}_{+,0}(q^2)$ the energy dependence from (R)ChPT, dispersion relations

## • Kπ *form factors*

vector f.f.

scalar f.f.

### Definition

$$\langle \pi^-(p)|\bar{s}\,\gamma^\mu\,u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2-m_\pi^2}{q^2}(k-p)^\mu\right] F_+(q^2) + \frac{m_K^2-m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

with $F_+(0) = F_0(0)$

### Kπ f.f. representation for $K_{l3}$ decays

$m_l^2 < q^2 < (m_K - m_\pi)^2$

slope

curvature

$$F_{+,0}(q^2) = F_{+,0}(0)\left[1 + \lambda'_{+,0}\frac{q^2}{m_{\pi^-}^2} + \frac{1}{2}\lambda''_{+,0}\left(\frac{q^2}{m_{\pi^-}^2}\right)^2 + \cdots\right]$$

In this kinematical region the f.f. are real

### Kπ f.f. representation for $\tau \to K\pi\nu_\tau$ decays

$(m_K + m_\pi)^2 < q^2 < m_\tau^2$

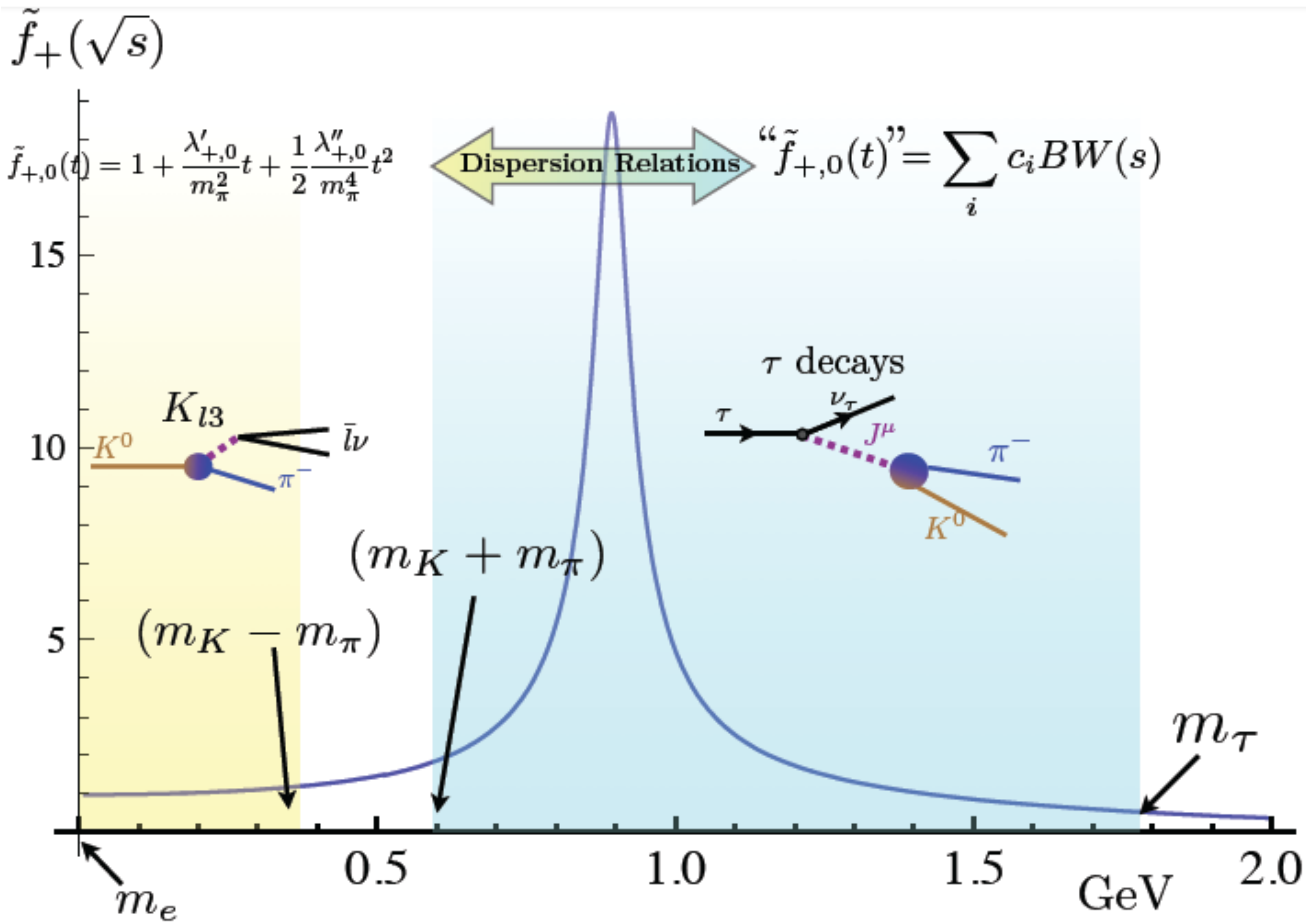
In this kinematical region the f.f. are complex

→ Taylor expansion inadmissible → more sophisticated treatments

- *Kπ form factors*

Kπ f.f. dispersive representations

Suited to described both $\tau \to K\pi\nu_\tau$ and $K_{l3}$ decays



- *Fit to* τ→Kπν$_\tau$

## Our model for the vector f.f.

After a detailed analysis in D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

Three-times-subtracted dispersion relation

cut-off to check stability

$$\tilde{F}_+(s) = \exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

with $\lambda'_+ = \alpha_1$ and $\lambda''_+ = \alpha_2 + \alpha_1^2$

## Our model for the phase

$$\delta(s) = \tan^{-1}\left[\frac{\mathrm{Im}\ \tilde{f}_+(s)}{\mathrm{Re}\ \tilde{f}_+(s)}\right]$$ where $$\tilde{f}_+(s) = \frac{\tilde{m}_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \gamma\, s}{D(\tilde{m}_{K^*}, \gamma_{K^*})} - \frac{\gamma\, s}{D(\tilde{m}_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

2 vector resonances form inspired by RChPT

M. Jamin, A. Pich and J. Portolés, PLB 640 (2006) 176 & 664 (2008) 78

and

$$D(\tilde{m}_n, \gamma_n) \equiv \tilde{m}_n^2 - s - \kappa_n \,\mathrm{Re}\, \tilde{H}_{K\pi}(s) - i\,\tilde{m}_n\gamma_n(s)$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma(\tilde{m}_n^2)^3}\frac{\gamma_n}{\tilde{m}_n} \qquad \gamma_n(s) = \gamma_n \frac{s}{\tilde{m}_n^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(\tilde{m}_n^2)}$$

$\tilde{H}_{K\pi}(s)$ is the one-loop $K\pi$ bubble integral

Physical masses and widths are obtained from

$$D(\tilde{m}_n, \gamma_n) = 0$$

for s→s$_R$ with $\sqrt{s_R} = m_R - \frac{i}{2}\Gamma_R$

R. Escribano *et. al.*, EPJC 28 (2003) 107

## • *Fit to* $\tau \to K\pi\nu_\tau$

**Differential decay distribution**

$$|V_{us}|F_+(0) = 0.2163(5)$$

M. Antonelli *et. al.*, Eur. Phys. J. C69 (2010) 399

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us} F_+(0)|^2 m_\tau^3}{32\pi^3 s} S_{\rm EW} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times$$
$$\times \left[\left(1 + 2\,\frac{s}{m_\tau^2}\right) q_{K\pi}^3\, |\tilde{F}_+(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s}\, q_{K\pi} |\tilde{F}_0(s)|^2\right]$$

with $\tilde{F}_{+,0}(q^2) \equiv \dfrac{F_{+,0}(q^2)}{F_+(0)}$

normalized vector f.f. normalized scalar f.f.

**Ansatz to analyse the data:**

$$N_i^{\rm th} = \mathcal{N}_T\, \frac{1}{2}\, \frac{2}{3}\, \Delta_{\rm b}^i\, \frac{1}{\Gamma_\tau\, \bar{B}_{K\pi}}\, \frac{d\Gamma_{K\pi}}{d\sqrt{s}}(s_{\rm b}^i)$$

with $\mathcal{N}_T = 53110$ and $\Delta_{\rm b} = 11.5$ MeV

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65

**Model for the scalar f.f.** Coupled-channel analysis (analytic and unitary)

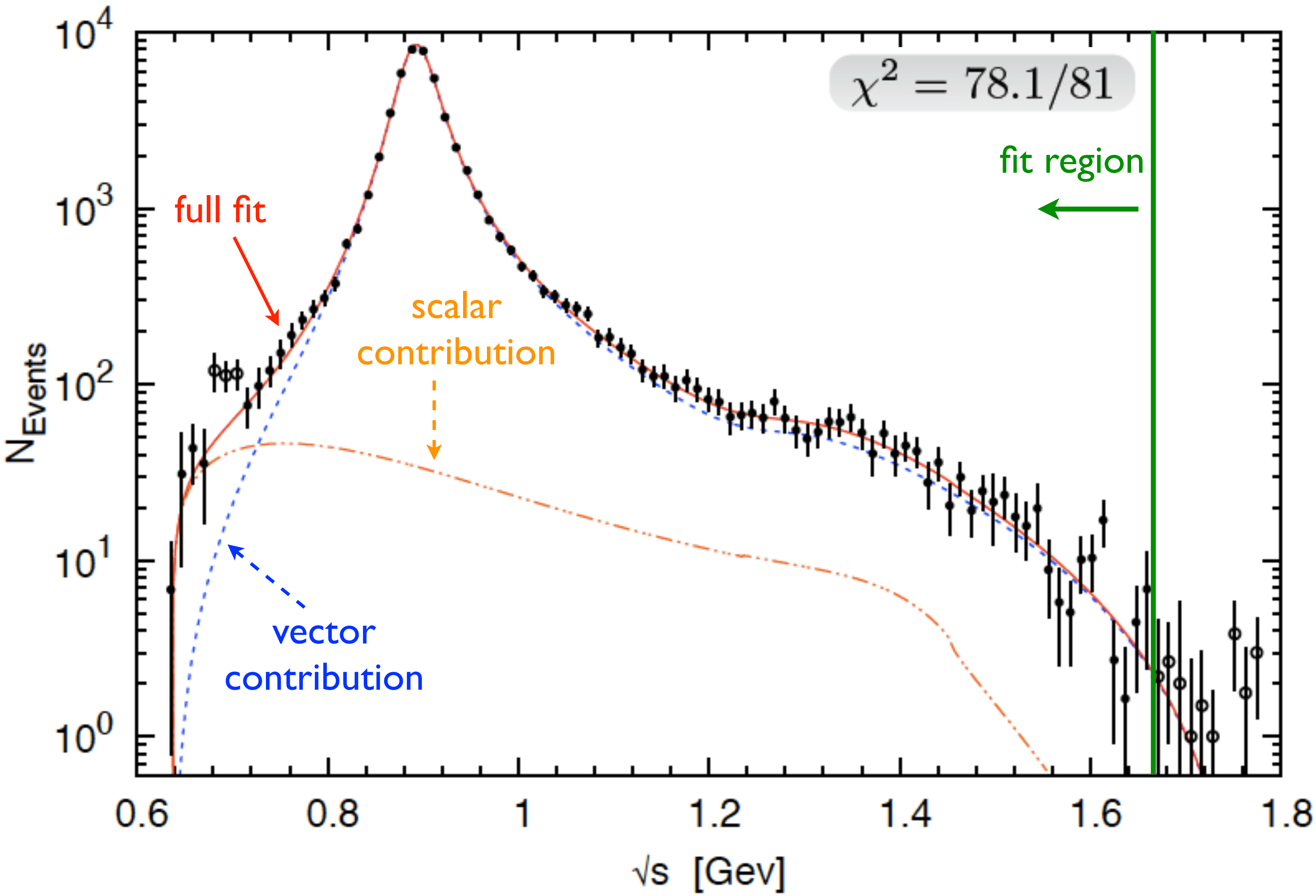
M. Jamin, J.A. Oller and A. Pich, NPB 622 (2002) 279

• *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

Fit to Belle spectrum

D. R. Boito, R. Escribano and M. Jamin, JHEP 09 (2010) 031

$s_{\text{cut}} = 4\ \text{GeV}^2$



- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

M. Antonelli *et. al.*,
Eur. Phys. J. C69 (2010) 399

Results

$$\lambda_+^{\prime\,\mathrm{exp}} = (24.9 \pm 1.1) \times 10^{-3}$$
$$\lambda_+^{\prime\prime\,\mathrm{exp}} = (16 \pm 5) \times 10^{-4}$$
$$\rho_{\lambda'_+,\lambda''_+} = -0.95$$

$$B_{K\pi}^{\mathrm{exp}} = 0.418(11)\%$$

$$\chi^2 = \sum_{i=1}^{90}{}' \left( \frac{N_i^{\mathrm{th}} - N_i^{\mathrm{exp}}}{\sigma_{N_i^{\mathrm{exp}}}} \right)^2 + \left( \frac{\bar{B}_{K\pi} - B_{K\pi}^{\mathrm{exp}}}{\sigma_{B_{K\pi}^{\mathrm{exp}}}} \right)^2 + (\boldsymbol{\lambda}_+^{\mathrm{th}} - \boldsymbol{\lambda}_+^{\mathrm{exp}})^{\mathrm{T}} V^{-1} (\boldsymbol{\lambda}_+^{\mathrm{th}} - \boldsymbol{\lambda}_+^{\mathrm{exp}})$$

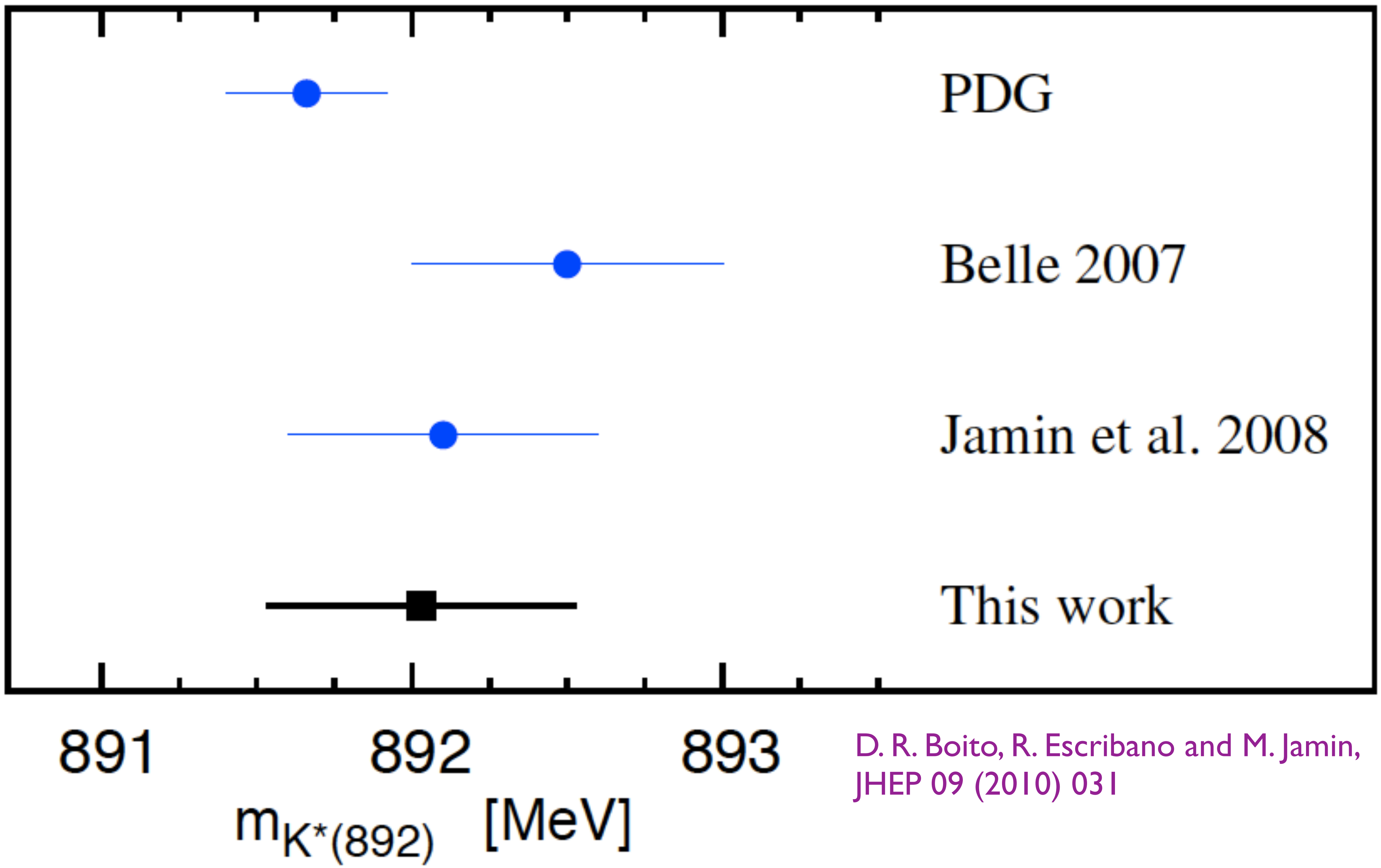
$1.8\,\mathrm{GeV} < \sqrt{s_{\mathrm{cut}}} < \infty$

| | $s_{\mathrm{cut}} = 3.24\ \mathrm{GeV}^2$ | $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$ | $s_{\mathrm{cut}} = 9\ \mathrm{GeV}^2$ | $s_{\mathrm{cut}} \to \infty$ |
|---|---|---|---|---|
| $B_{K\pi}$ | $0.429 \pm 0.009$ | $0.427 \pm 0.008\%$ | $0.426 \pm 0.008\%$ | $0.426 \pm 0.008\%$ |
| $(B_{K\pi}^{\mathrm{th}})$ | $(0.426\%)$ | $(0.425\%)$ | $(0.423\%)$ | $(0.423\%)$ |
| $m_{K^*}$ [MeV] | $892.04 \pm 0.20$ | $892.02 \pm 0.20$ | $892.03 \pm 0.19$ | $892.03 \pm 0.19$ |
| $\Gamma_{K^*}$ [MeV] | $46.58 \pm 0.38$ | $46.52 \pm 0.38$ | $46.48 \pm 0.38$ | $46.48 \pm 0.38$ |
| $m_{K^{*\prime}}$ [MeV] | $1257^{+30}_{-45}$ | $1268^{+25}_{-32}$ | $1270^{+24}_{-29}$ | $1271^{+24}_{-29}$ |
| $\Gamma_{K^{*\prime}}$ [MeV] | $321^{+95}_{-76}$ | $238^{+75}_{-57}$ | $206^{+67}_{-50}$ | $205^{+67}_{-50}$ |
| $\gamma \times 10^2$ | $-8.2^{+2.2}_{-3.5}$ | $-5.4^{+1.4}_{-2.0}$ | $-4.4^{+1.2}_{-1.6}$ | $-4.4^{+1.2}_{-1.6}$ |
| $\lambda'_+ \times 10^3$ | $25.43 \pm 0.30$ | $25.49 \pm 0.30$ | $25.55 \pm 0.30$ | $25.55 \pm 0.30$ |
| $\lambda''_+ \times 10^4$ | $12.31 \pm 0.10$ | $12.20 \pm 0.10$ | $12.12 \pm 0.10$ | $12.12 \pm 0.10$ |
| $\chi^2$/n.d.f. | $77.9/81$ | $78.1\ /81$ | $79.0\ /81$ | $79.1/81$ |

- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

K*(892)± pole mass

$$m_{K^*(892)^\pm} = 892.03 \pm (0.19)_{\rm stat} \pm (0.44)_{\rm sys}\ \mathrm{MeV}$$

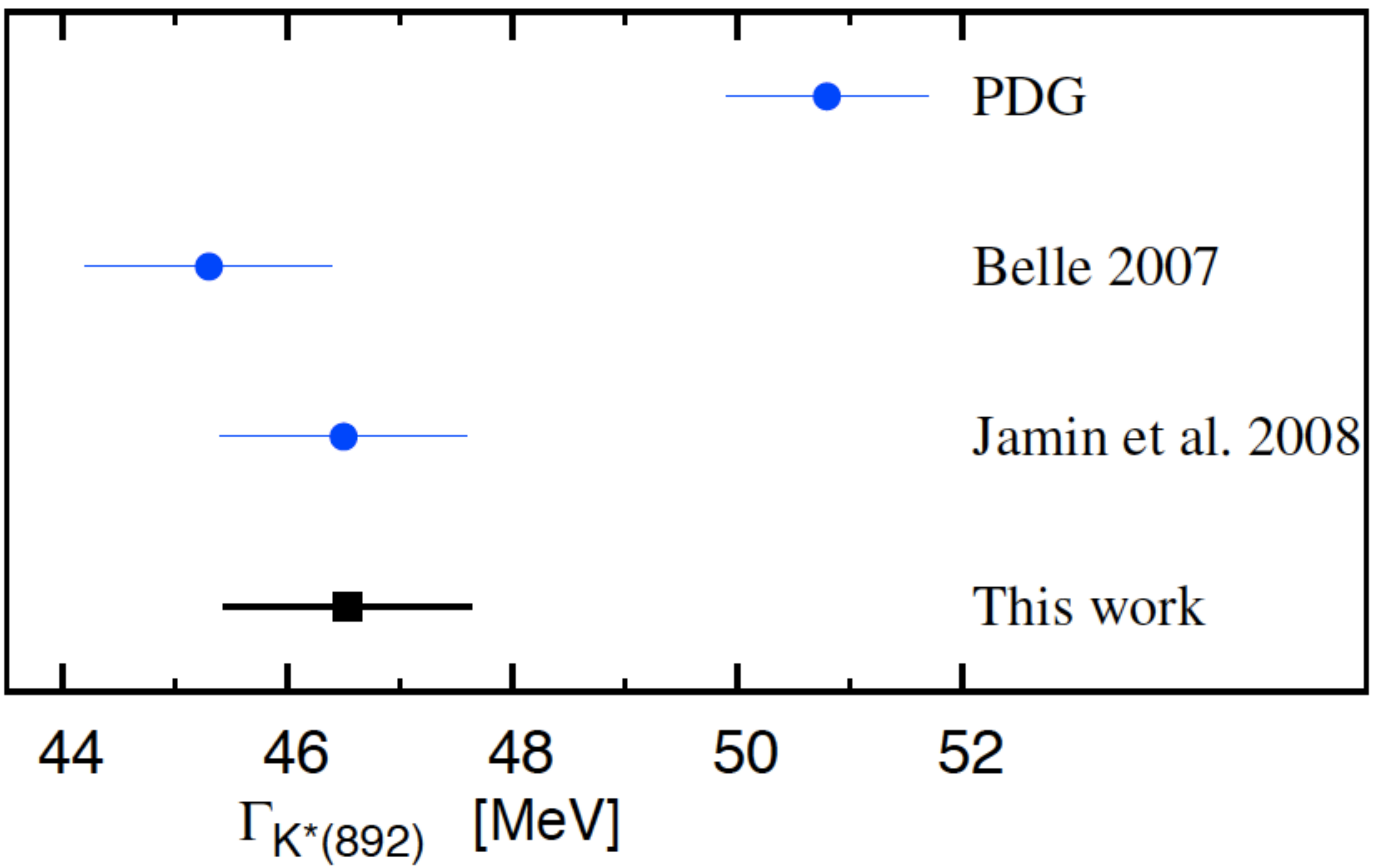



D. R. Boito, R. Escribano and M. Jamin, JHEP 09 (2010) 031

- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

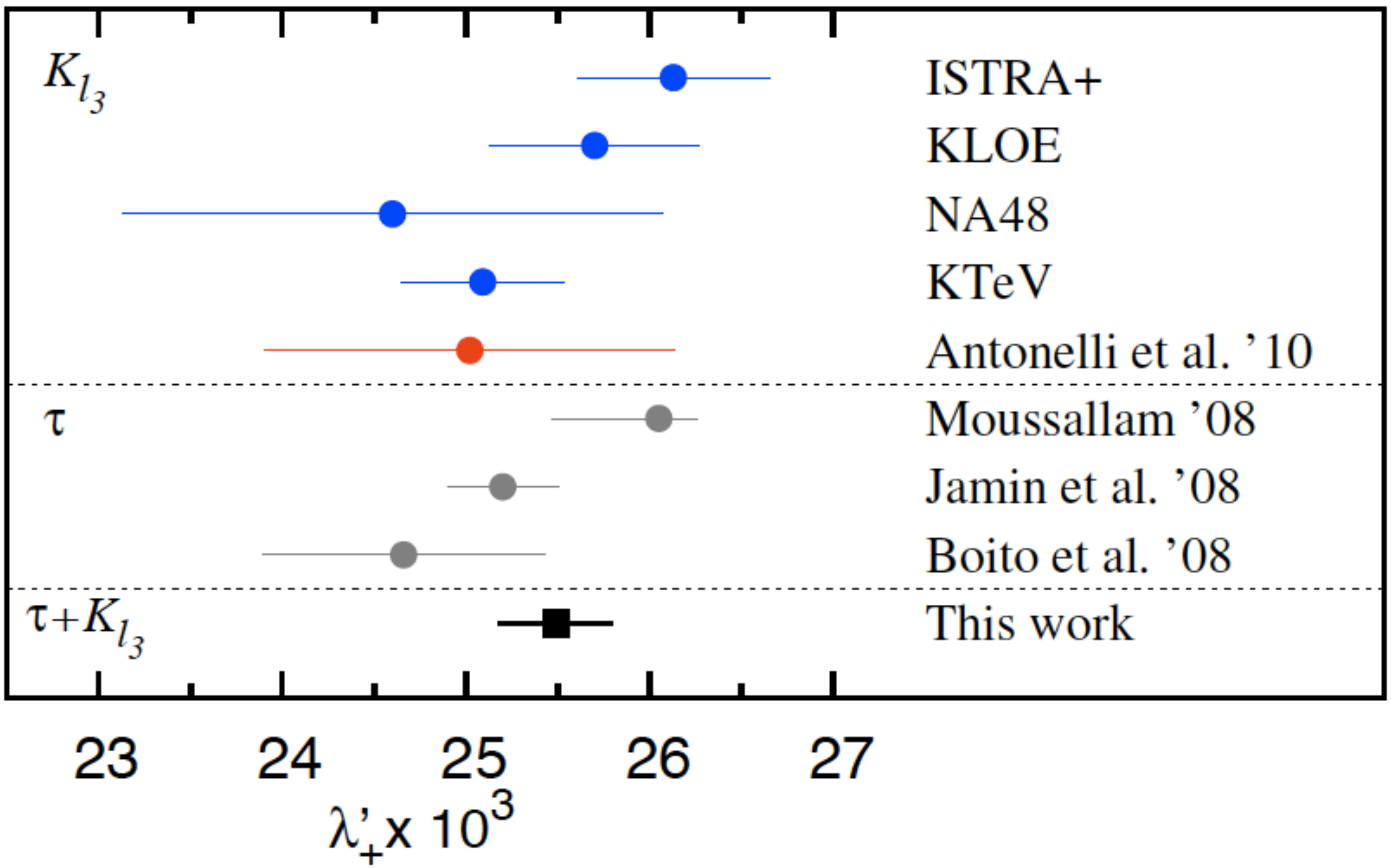
$K^*(892)^\pm$ pole width

$$\Gamma_{K^*(892)^\pm} = 46.53 \pm (0.38)_{\rm stat} \pm (1.0)_{\rm sys}\ \text{MeV}$$



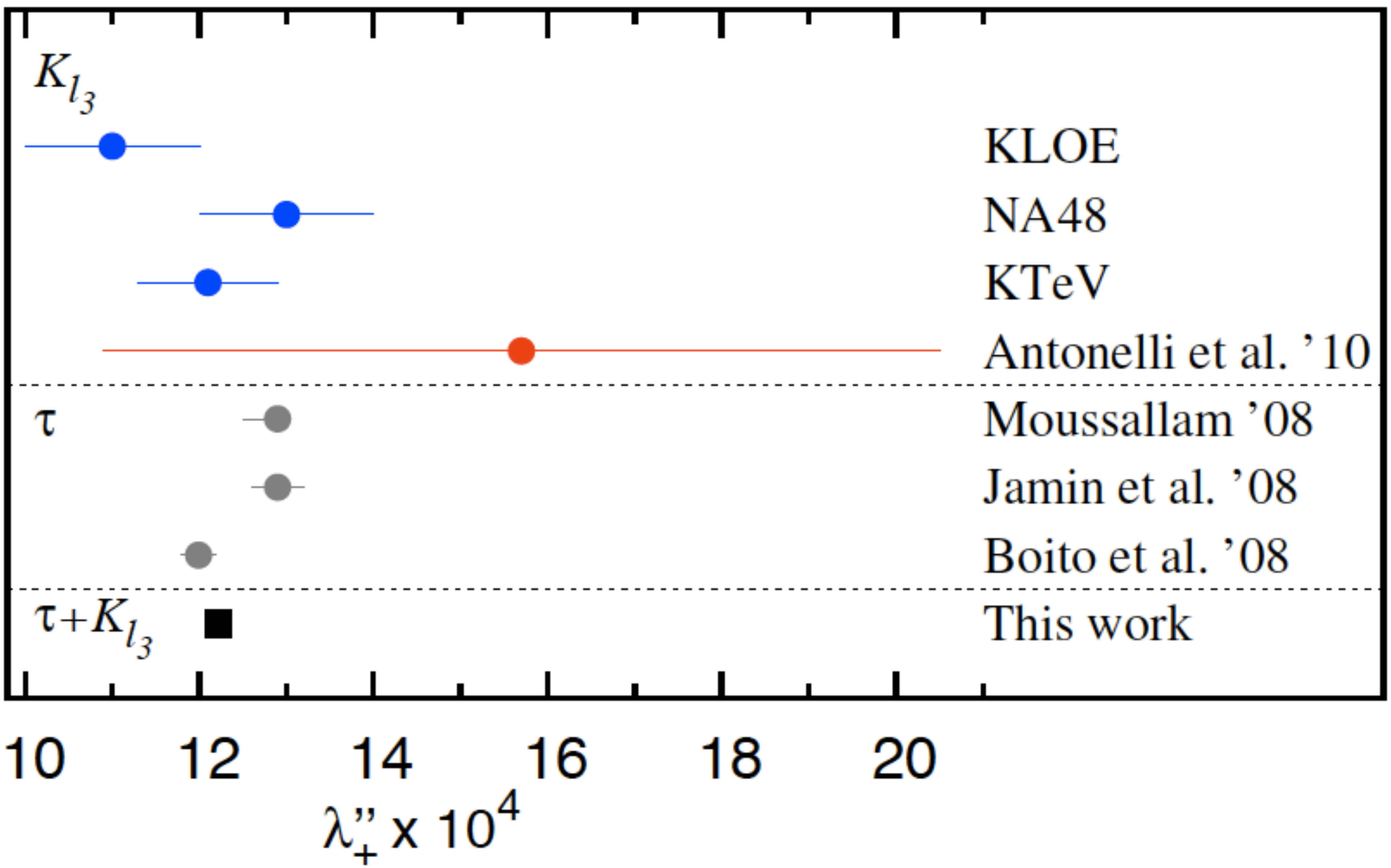
- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

$$\lambda'_+ \times 10^3 = 25.49 \pm (0.30)_{\rm stat} \pm (0.06)_{s_{\rm cut}}$$



- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

$$\lambda''_+ \times 10^4 = 12.22 \pm (0.10)_{\rm stat} \pm (0.10)_{s_{\rm cut}}$$



## • Conclusions: Intermezzo

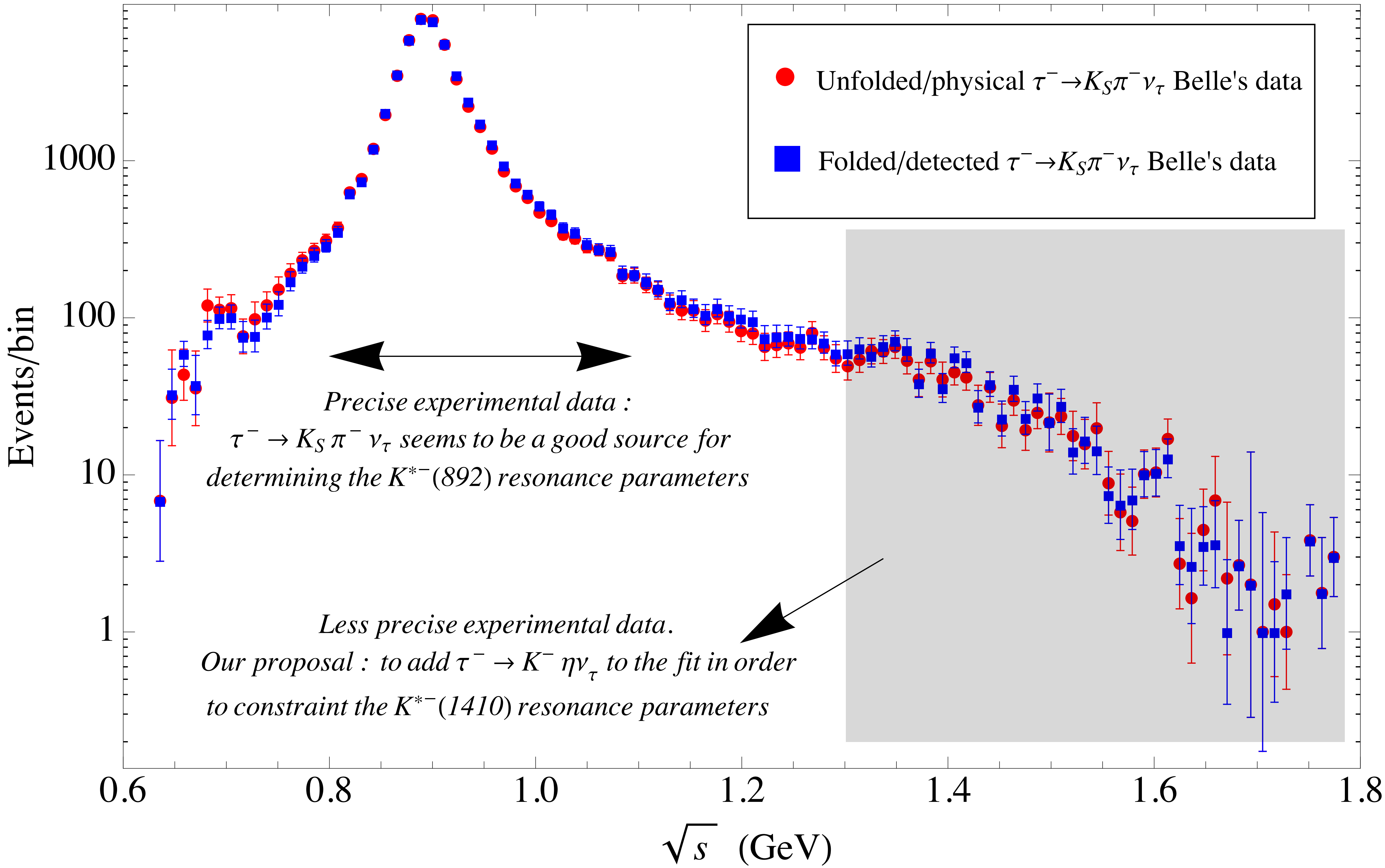
We have presented a model aimed at describing the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from $K_{l3}$ decays suited to describe both $\tau\to K\pi\nu_\tau$ and $K_{l3}$ decays simultaneously

A good detemination of the $K\pi$ vector f.f. and resonance parameters is obtained from a fit of the $\tau\to K\pi\nu_\tau$ spectrum

Competitive results for the $K^*(892)^\pm$ pole mass and width, slope and curvature parameters, $K_{l3}$ phase-space integrals, and $K\pi$ I=1/2 P-wave scattering phase and threshold parameters are obtained

A combined fit of the $\tau\to K\pi\nu_\tau$ and $K_{l3}$ spectra should be done in the future

# • Reason for a $\tau \to K\eta\nu_\tau$ analysis

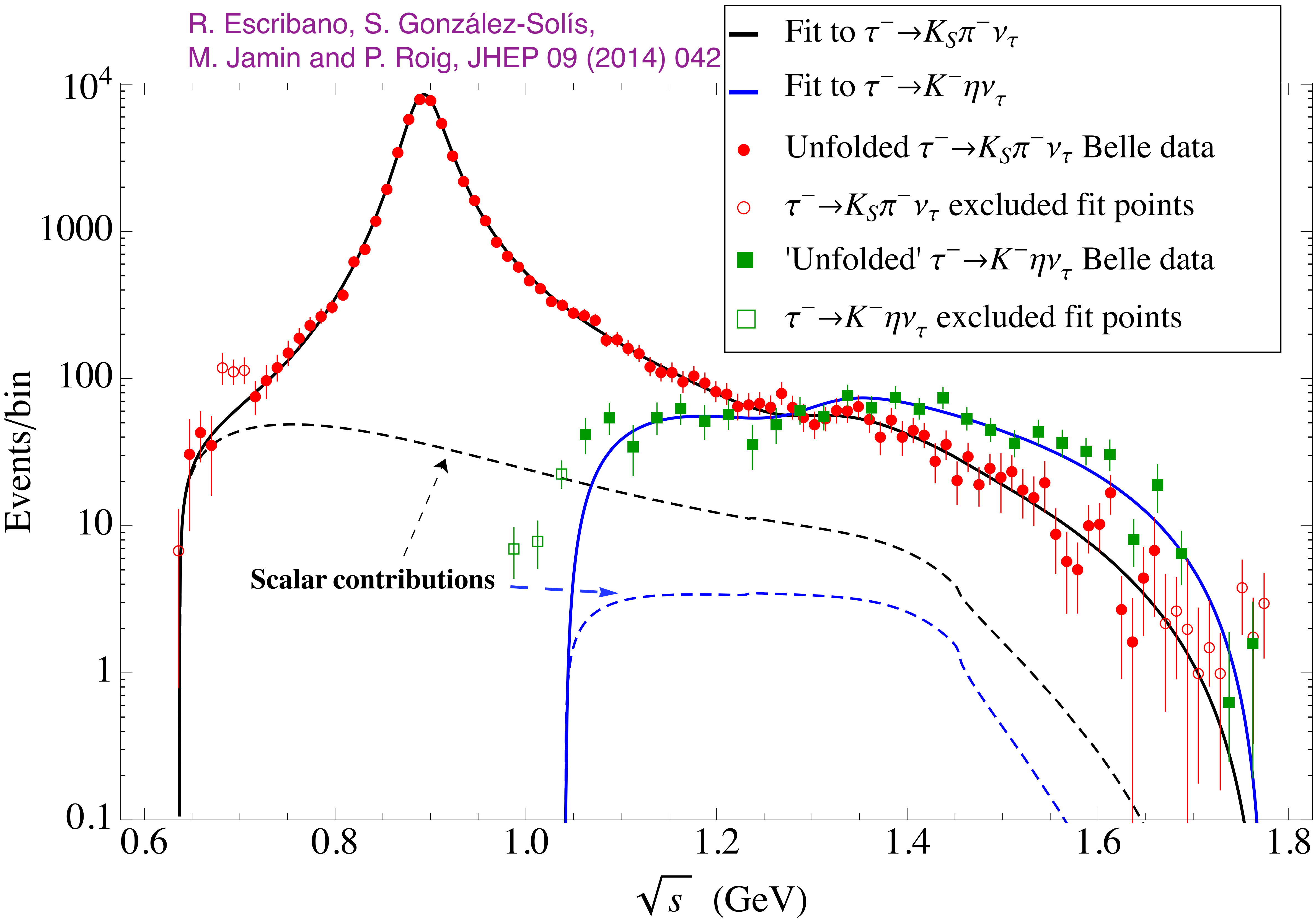


# • Results of the combined analysis

$N_{events} = 53113$ $\Delta_{bin} = 0.0115$ GeV/bin

$N_{events} = 1271$ $\Delta_{bin} = 0.025$ GeV/bin

R. Escribano, S. González-Solís,
M. Jamin and P. Roig, JHEP 09 (2014) 042



# • Results of the combined analysis

$$\left.\begin{aligned} M_{K^{*-}(892)} &= 892.03 \pm 0.19 \text{ MeV} \\ \Gamma_{K^{*-}(892)} &= 46.18 \pm 0.44 \text{ MeV} \end{aligned}\right\} \text{no gain}$$

$$\left.\begin{aligned} M_{K^{*-}(1410)} &= 1304 \pm 17 \text{ MeV} \\ \Gamma_{K^{*-}(1410)} &= 171 \pm 62 \text{ MeV} \end{aligned}\right\} \text{improvement}$$

$$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$$

$$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$$

$$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$$

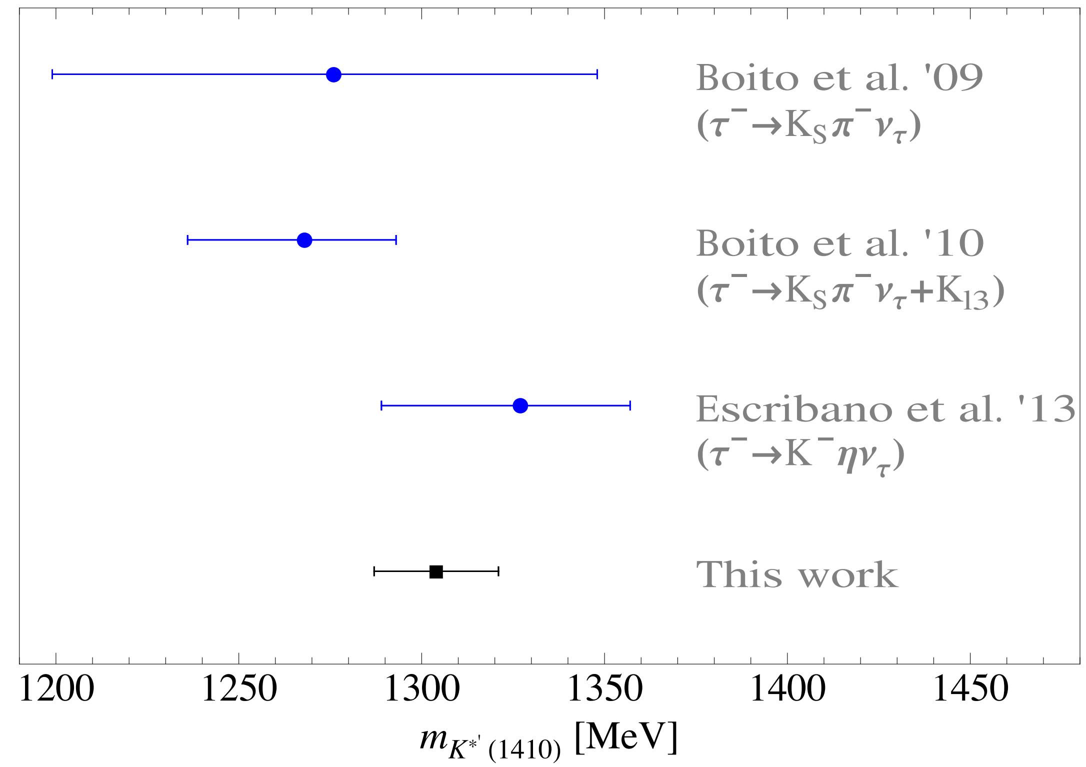
$$\left.\begin{aligned} \lambda'_{K\pi} &= (23.9 \pm 0.9) \cdot 10^{-3} \\ \lambda'_{K\eta} &= (20.9 \pm 2.7) \cdot 10^{-3} \end{aligned}\right\} \text{isospin violation?}$$

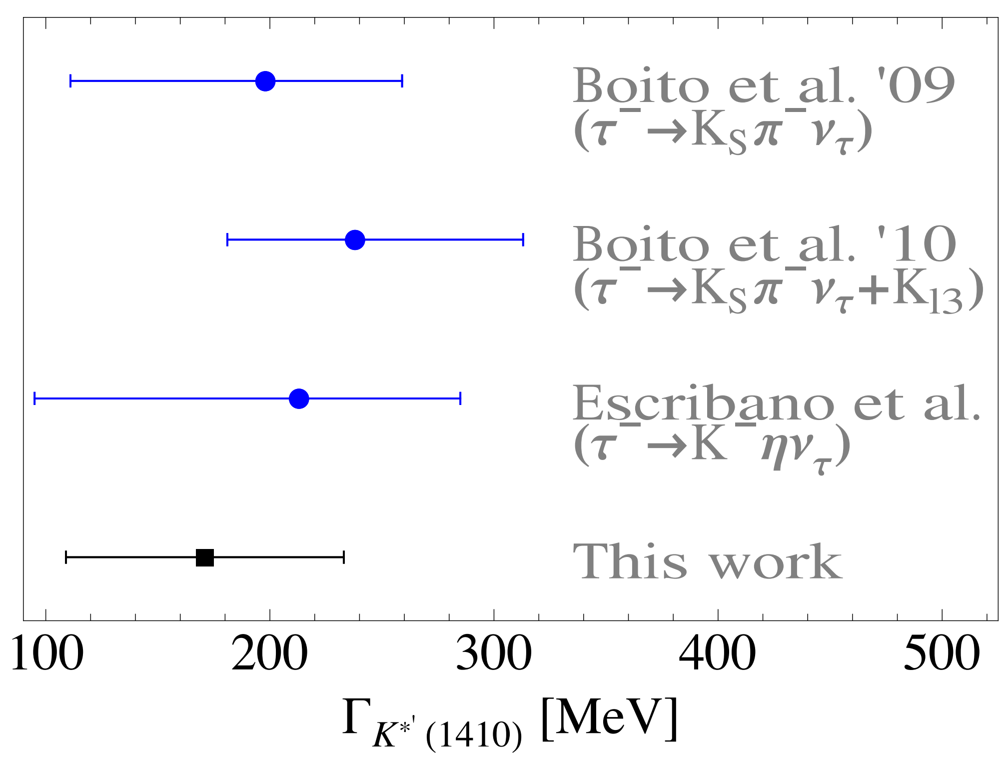
$$\left.\begin{aligned} \lambda''_{K\pi} &= (11.8 \pm 0.2) \cdot 10^{-4} \\ \lambda''_{K\eta} &= (11.1 \pm 0.5) \cdot 10^{-4} \end{aligned}\right\} \text{isospin violation?}$$

$$\Uparrow$$

$$\tau^- \to K^- \pi^0 \nu_\tau \& K_{\ell 3}$$

$$\chi^2/d.o.f = 108.1/105 = 1.03$$



## • Future prospects for Belle-I and Belle-II

| Data / Error | Current | Belle-I | Belle-I $K\pi$ | Belle-I $K\eta$ | Belle-II | Belle-II $K\pi$ | Belle-II $K\eta$ |
|---|---|---|---|---|---|---|---|
| $\bar{B}_{K\pi}(\%)$ | $0.404 \pm 0.012$ | $\pm 0.005$ | $\pm 0.005$ | $\pm 0.012$ | $^{\dagger}(0.001)$ | $^{\dagger}(0.001)$ | $\pm 0.012$ |
| $M_{K^*}$ | $892.03 \pm 0.19$ | $\pm 0.09$ | $\pm 0.09$ | $\pm 0.19$ | $^{\dagger}(0.02)$ | $^{\dagger}(0.02)$ | $\pm 0.19$ |
| $\Gamma_{K^*}$ | $46.18 \pm 0.44$ | $\pm 0.20$ | $\pm 0.20$ | $\pm 0.44$ | $^{\dagger}(0.02)$ | $^{\dagger}(0.03)$ | $\pm 0.42$ |
| $M_{K^{*\prime}}$ | $1304 \pm 17$ | $^{\dagger}(7)$ | $^{\dagger}(9)$ | $^{\dagger}(8)$ | $^{\dagger}(1)$ | $^{\dagger}(1)$ | $^{\dagger}(1)$ |
| $\Gamma_{K^{*\prime}}$ | $168 \pm 62$ | $^{\dagger}(19)$ | $^{\dagger}(24)$ | $^{\dagger}(25)$ | $^{\dagger}(3)$ | $^{\dagger}(4)$ | $^{\dagger}(11)$ |
| $\lambda'_{K\pi} \times 10^3$ | $23.9 \pm 0.9$ | $^{\dagger}(0.3)$ | $^{\dagger}(0.3)$ | $\pm 0.8$ | $^{\dagger}(0.04)$ | $^{\dagger}(0.04)$ | $\pm 0.8$ |
| $\lambda''_{K\pi} \times 10^4$ | $11.8 \pm 0.2$ | $\pm 0.07$ | $\pm 0.07$ | $\pm 0.2$ | $^{\dagger}(0.01)$ | $^{\dagger}(0.01)$ | $\pm 0.2$ |
| $\bar{B}_{K\eta} \times 10^4$ | $1.58 \pm 0.10$ | $\pm 0.05$ | $\pm 0.10$ | $\pm 0.05$ | $^{\dagger}(0.01)$ | $\pm 0.10$ | $^{\dagger}(0.01)$ |
| $\gamma_{K\eta}(= \gamma_{K\pi}) \times 10^2$ | $-3.3 \pm 1.3$ | $^{\dagger}(0.3)$ | $^{\dagger}(0.3)$ | $^{\dagger}(0.4)$ | $^{\dagger}(0.04)$ | $^{\dagger}(0.04)$ | $^{\circ}(0.3)$ |
| $\lambda'_{K\eta} \times 10^3$ | $20.9 \pm 2.7$ | $^{\dagger}(0.7)$ | $\pm 2.7$ | $^{\dagger}(0.8)$ | $^{\dagger}(0.10)$ | $\pm 2.7$ | $^{\circ}(0.4)$ |
| $\lambda''_{K\eta} \times 10^4$ | $11.1 \pm 0.5$ | $^{\dagger}(0.2)$ | $\pm 0.5$ | $^{\dagger}(0.2)$ | $^{\dagger}(0.02)$ | $\pm 0.5$ | $^{\dagger}(0.06)$ |

**Table 4.** The errors of our final results (3.3) are compared, in turn, to those achievable by analysing the complete Belle-I data sample, and updating only the $K_S\pi^-$ or $K^-\eta$ analyses. The last three columns show the potential of fitting all data collected by Belle-II and the same only for $K_S\pi^-$ or for $K^-\eta$ (assuming the other mode has not been updated to include the complete Belle-I data sample). Current Belle $K_S\pi^-$ ($K^-\eta$) data correspond to 351 (490) $\text{fb}^{-1}$ for a complete data set of $\sim 1000\,\text{fb}^{-1} = 1\ \text{ab}^{-1}$. Expectations for Belle-II correspond to 50 $\text{ab}^{-1}$. All errors include both statistical and systematic uncertainties. $^{\dagger}$ means that statistical errors (in brackets) will become negligible, while $^{\circ}$ signals a tension with the current reference best fit values. We thank Denis Epifanov for conversations on these figures and on expected performance of Belle-II at the detector and analysis levels. All errors have been symmetrised for simplicity.

## • Conclusions: Finale

A good description of the vector form factor (by analyticity+unitarity arguments) is crucial to unveil the parameters of the intermediate resonances which drive the decays

Limitations: only $\tau \to K_S \pi \nu_\tau$ is published, no access to isospin violations

$\tau \to K \eta \nu_\tau$ not very precise, convoluted with detector effects

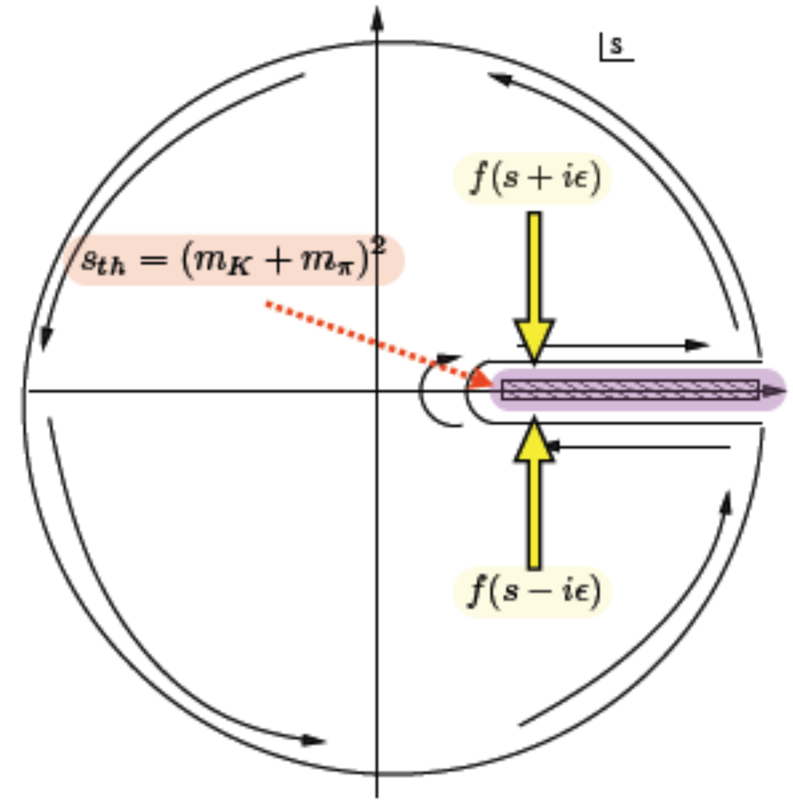
Fitting both decay spectra together we have considerable improved the determination of the $K^{*-}(1410)$ mass while we slightly reduced the uncertainty of the width

$$\boxed{M_{K^{*\prime}} = (1304 \pm 17)\ \mathrm{MeV}\,, \quad \Gamma_{K^{*\prime}} = (171 \pm 62)\ \mathrm{MeV}}$$

Call for (an unfolded) analysis of $\tau^- \to K^- \pi^0 \nu_\tau$ for unveiling possible isospin violations on the low-energy parameters $\lambda'^{('')}$

- Kπ *form factors*

Kπ f.f. dispersive representations




Analyticity →

$$f(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, f(s')}{s' - s - i\epsilon}$$

Analyticity + Unitarity

→ Muskelishivili-Omnès equation

$$f(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\tan\delta(s')\mathrm{Re}\, f(s')}{s' - s - i\epsilon}$$

solution

$$f(s) = f(0)\exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s' - s - i\epsilon}\right]$$

generalized solution (n subtractions at s=0)

$$f(s) = \exp\left[\alpha_1 + \alpha_2 s \cdots + \alpha_{n-1}s^{n-1} + \frac{s^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s')^n}\frac{\delta(s')}{s' - s - i\epsilon}\right]$$

Recent dispersive representations:

B. Moussallam, EPJC 53 (2008) 401

V. Bernard *et. al.*, PRD 80 (2009) 034034

D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

V. Bernard *et. al.*, PLB 638 (2006) 480

M. Jamin, J.A. Oller and A. Pich, NPB 587 (2000) 331 & 622 (2002) 279, PRD 74 (2006) 074009

## • *Fit to $\tau \to K\pi\nu_\tau$*

**Results**

Update of D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

$$B^{\rm exp}_{K\pi} = 0.418(11)\%$$

$$\chi^2 = \sum_{i=1}^{90}{}' \left( \frac{N_i^{\rm th} - N_i^{\rm exp}}{\sigma_{N_i^{\rm exp}}} \right)^2 + \left( \frac{\bar{B}_{K\pi} - B^{\rm exp}_{K\pi}}{\sigma_{B^{\rm exp}_{K\pi}}} \right)^2$$

$1.8\,\mathrm{GeV} < \sqrt{s_{\rm cut}} < \infty$

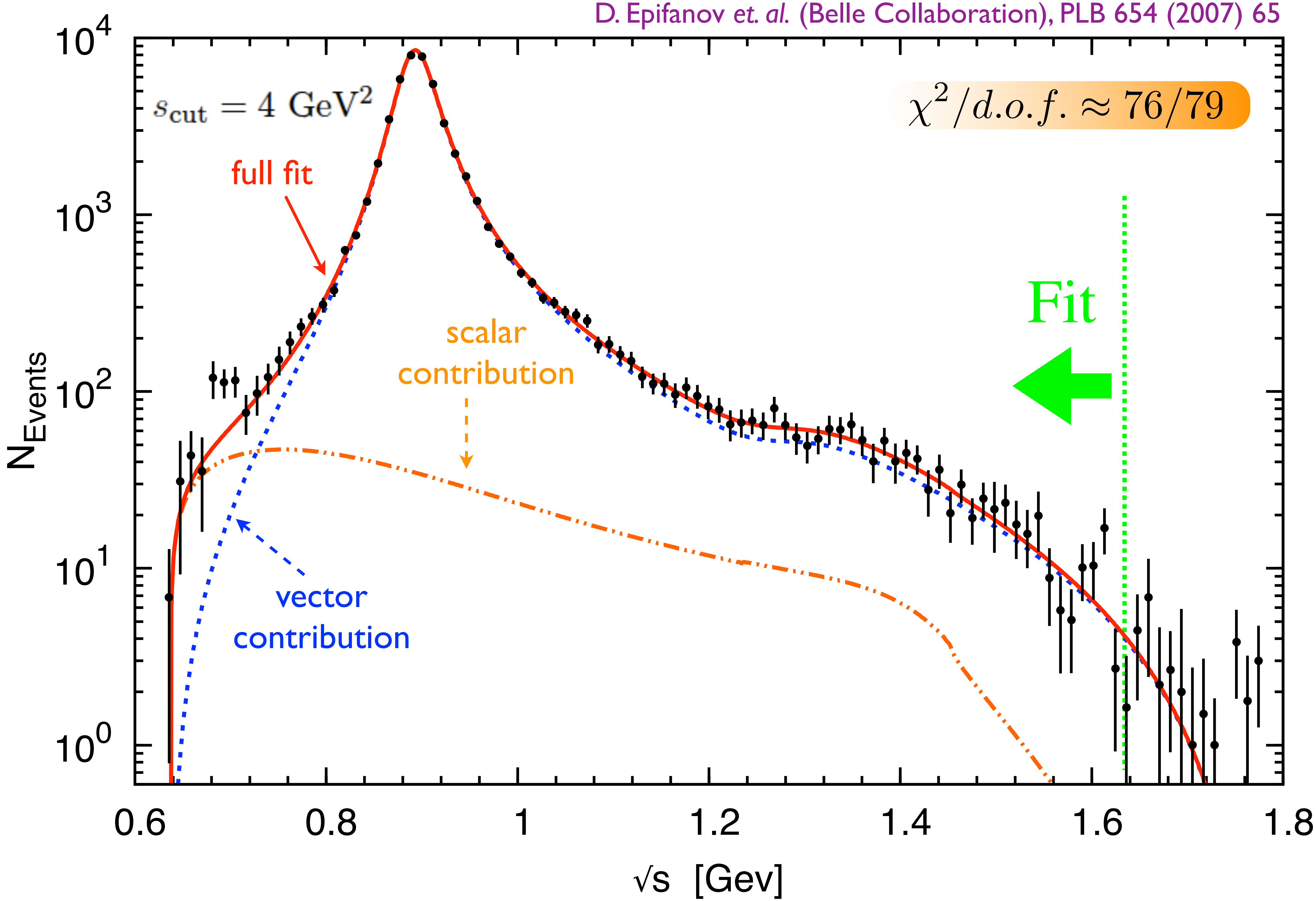
| | $s_{\rm cut} = 3.24\ \mathrm{GeV}^2$ | $s_{\rm cut} = 4\ \mathrm{GeV}^2$ | $s_{\rm cut} = 9\ \mathrm{GeV}^2$ | $s_{\rm cut} \to \infty$ |
|---|---|---|---|---|
| $\bar{B}_{K\pi}$ | $0.416 \pm 0.011\%$ | $0.417 \pm 0.011\%$ | $0.418 \pm 0.011\%$ | $0.418 \pm 0.011\%$ |
| $(B^{\rm th}_{K\pi})$ | $(0.414\%)$ | $(0.414\%)$ | $(0.415\%)$ | $(0.415\%)$ |
| $m_{K^*}$ [MeV] | $892.00 \pm 0.19$ | $892.02 \pm 0.19$ | $892.03 \pm 0.19$ | $892.03 \pm 0.19$ |
| $\Gamma_{K^*}$ [MeV] | $46.14 \pm 0.44$ | $46.20 \pm 0.43$ | $46.25 \pm 0.42$ | $46.25 \pm 0.42$ |
| $m_{K^{*\prime}}$ [MeV] | $1281^{+25}_{-33}$ | $1280^{+25}_{-28}$ | $1278^{+26}_{-27}$ | $1278^{+26}_{-27}$ |
| $\Gamma_{K^{*\prime}}$ [MeV] | $243^{+92}_{-70}$ | $193^{+72}_{-56}$ | $177^{+66}_{-52}$ | $177^{+66}_{-52}$ |
| $\gamma \times 10^2$ | $-5.1^{+1.7}_{-2.6}$ | $-3.9^{+1.3}_{-1.8}$ | $-3.4^{+1.1}_{-1.6}$ | $-3.4^{+1.1}_{-1.6}$ |
| $\lambda'_+ \times 10^3$ | $24.15 \pm 0.72$ | $24.55 \pm 0.68$ | $24.86 \pm 0.66$ | $24.88 \pm 0.66$ |
| $\lambda''_+ \times 10^4$ | $11.99 \pm 0.19$ | $11.95 \pm 0.19$ | $11.93 \pm 0.19$ | $11.93 \pm 0.19$ |
| $\chi^2$/n.d.f. | $74.1/79$ | $75.7/79$ | $77.2/79$ | $77.3/79$ |

• *Fit to $\tau \to K\pi\nu_\tau$*

Update of D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

Fit to Belle spectrum

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65



- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from $K_{l3}$*

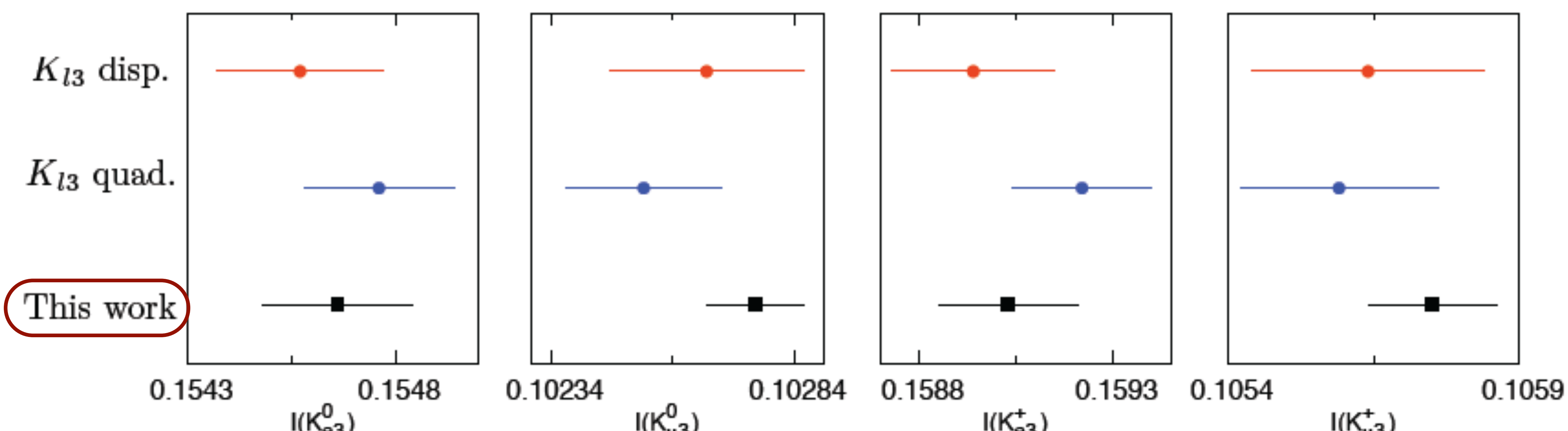
$K_{l3}$ phase-space integrals

$$I_{K_{l_3}} = \frac{1}{m_K^2} \int_{m_l^2}^{(m_K - m_\pi)^2} \mathrm{d}t\, \lambda(t)^{3/2} \left(1 + \frac{m_l^2}{2t}\right) \left(1 - \frac{m_l^2}{t}\right)^2 \left( |\tilde{f}_+(t)|^2 + \frac{3\, m_l^2 (m_K^2 - m_\pi^2)^2}{(2t + m_l^2)\, m_K^4\, \lambda(t)} |\tilde{f}_0(t)|^2 \right)$$

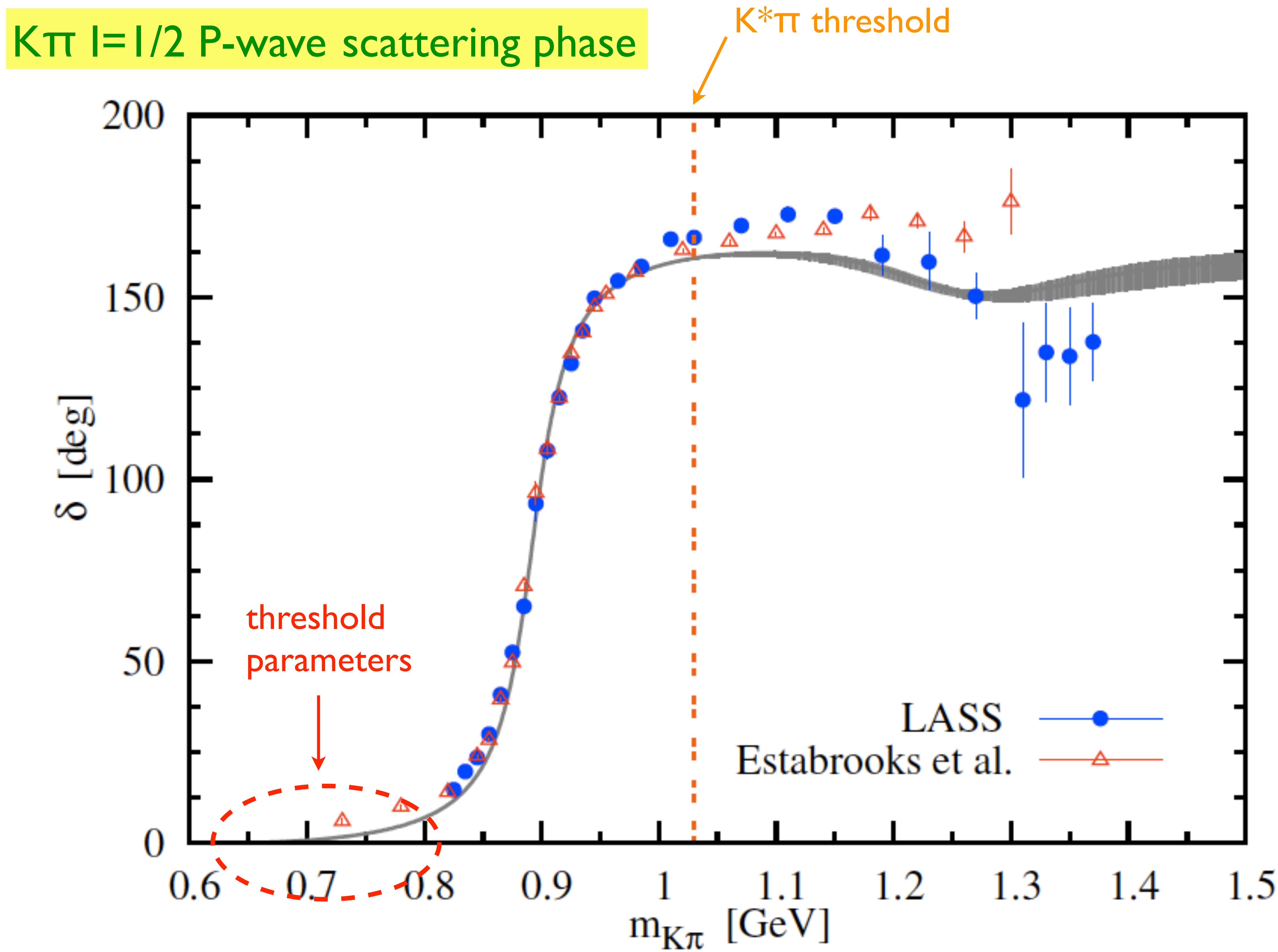
$$\lambda(t) = 1 + t^2/m_K^4 + r_\pi^4 - 2\, r_\pi^2 - 2\, r_\pi^2\, t/m_K^2 - 2\, t/m_K^2$$

| | This Work | $K_{l_3}$ disp. [9] | $K_{l_3}$ quad. [9] |
|---|---|---|---|
| $I_{K^0_{e3}}$ | 0.15466(17) | 0.15476(18) | 0.15457(20) |
| $I_{K^0_{\mu 3}}$ | 0.10276(10) | 0.10253(16) | 0.10266(20) |
| $I_{K^+_{e3}}$ | 0.15903(17) | 0.15922(18) | 0.15894(21) |
| $I_{K^+_{\mu 3}}$ | 0.10575(11) | 0.10559(17) | 0.10564(20) |

[9] M. Antonelli *et. al.*, Eur. Phys. J. C69 (2010) 399



• *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* $K_{l3}$



- *Fit to* τ→Kπν$_\tau$ *with restrictions from* K$_{l3}$

Kπ I=1/2 P-wave threshold parameters

$$\frac{2}{\sqrt{s}}\mathrm{Re}\, t_l^I(s) = \frac{1}{2q}\sin 2\delta_l^I(q) = q^{2l}\left[\underline{a_l^I} + \underline{b_l^I\, q^2} + \underline{c_l^I\, q^4} + \mathcal{O}(q^6)\right]$$

| | This work | [60] | [61] | [62] | [48] |
|---|---|---|---|---|---|
| $m_{\pi^-}^3\, a_1^{1/2} \times 10$ | 0.166(4) | 0.16(3) | 0.18 | 0.18(3) | 0.19(1) |
| $m_{\pi^-}^5\, b_1^{1/2} \times 10^2$ | 0.258(9) | - | - | - | 0.18(2) |
| $m_{\pi^-}^7\, c_1^{1/2} \times 10^3$ | 0.90(3) | - | - | - | 0.71(11) |


[48] P. Büttiker, S. Descotes-Genon and B. Moussallam, EPJC 33 (2004) 209

[60] V. Bernard, N. Kaiser and U. G. Meißner, NPB 357 (1991) 129

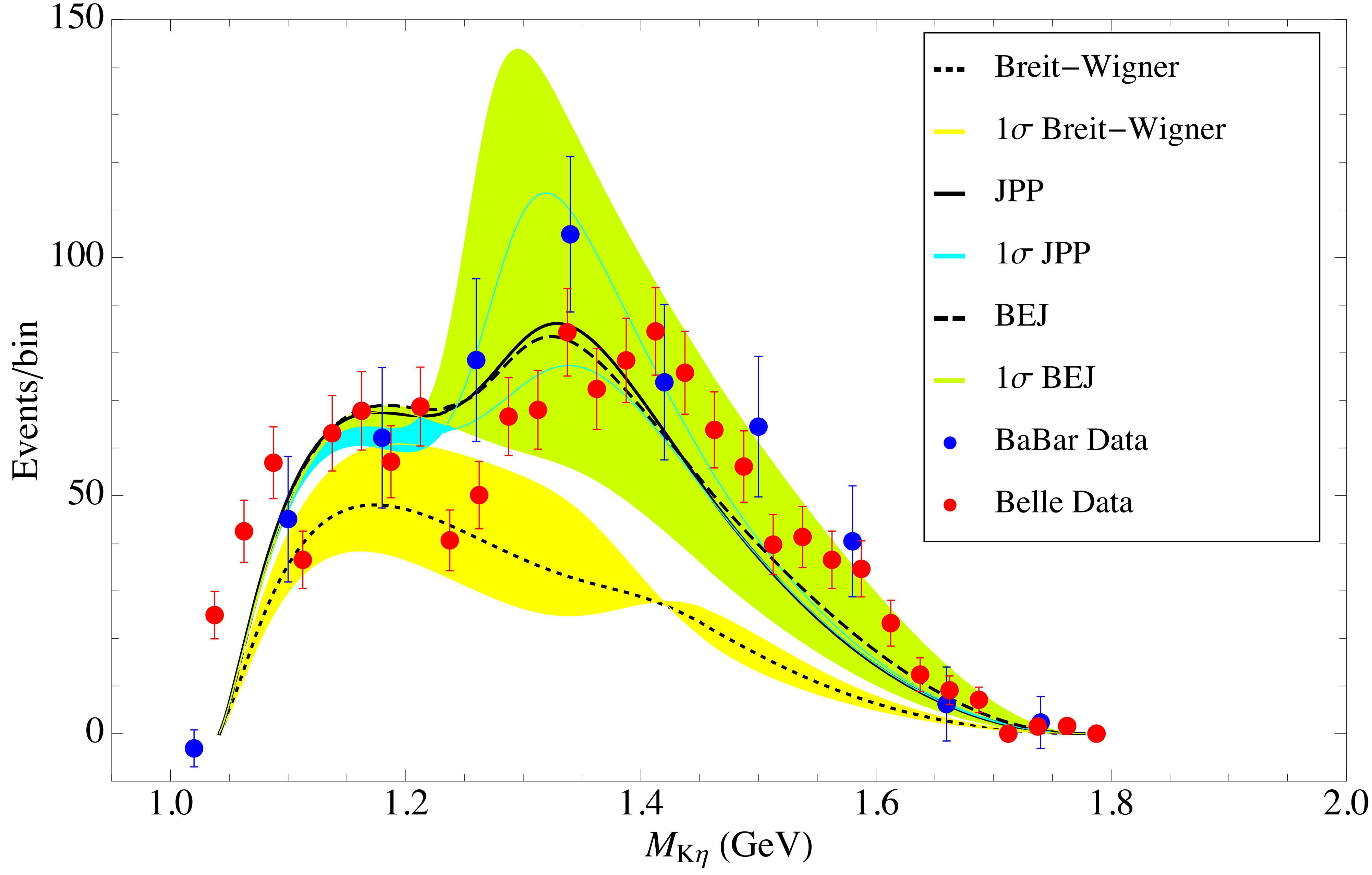
[61] J. Bijnens, P. Dhonte and P. Talavera, JHEP 05 (2004) 036

[62] V. Bernard, N. Kaiser and U. G. Meißner, NPB 364 (1991) 283

- **Results of the $\tau \to K\eta\nu_\tau$ analysis**

Predictions based on the $\tau \to K\pi\nu_\tau$ analysis

R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039



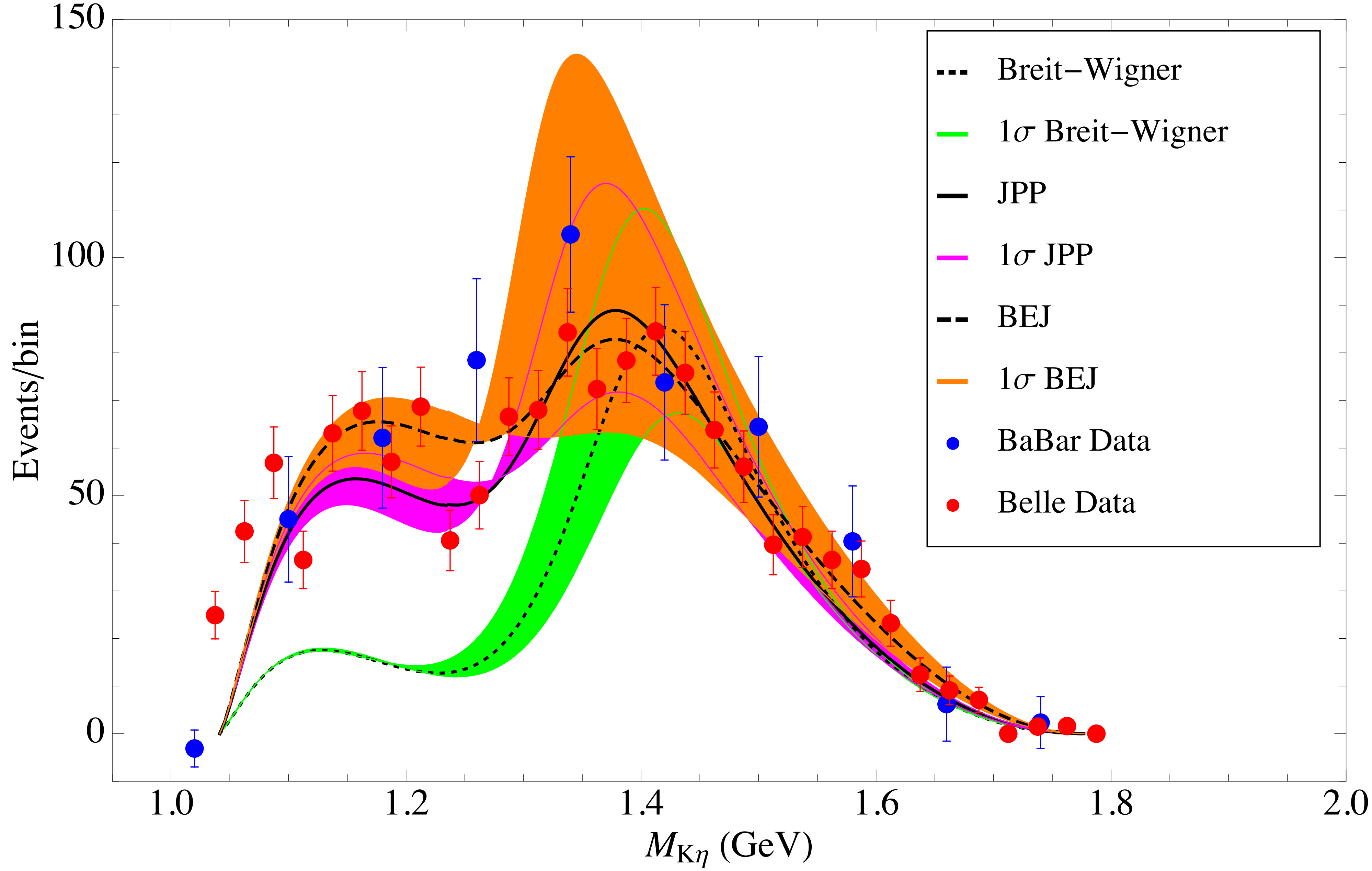

K. Inami *et. al.* (Belle Collaboration), PLB 672 (2009) 109

P. del Amo Sanchez *et. al.* (BaBar Collab.), PRD 83 (2011) 032002

# • Results of the $\tau \rightarrow K\eta\nu_\tau$ analysis

Fit to the $\tau \rightarrow K\eta\nu_\tau$ experimental data

R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039




K. Inami *et. al.* (Belle Collaboration), PLB 672 (2009) 109

P. del Amo Sanchez *et. al.* (BaBar Collab.), PRD 83 (2011) 032002

## • Results of the $\tau \to K\eta\nu_\tau$ analysis

Fit to the $\tau \to K\eta\nu_\tau$ experimental data

R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039

JPP vector form factor

$$M_{K^{\star\prime}} = 1332^{+16}_{-18}, \quad \Gamma_{K^{\star\prime}} = 220^{+26}_{-24}, \quad \gamma = -0.078^{+0.012}_{-0.014}$$

BEJ vector form factor

$$M_{K^{\star\prime}} = 1327^{+30}_{-38}, \quad \Gamma_{K^{\star\prime}} = 213^{+72}_{-118}, \quad \gamma = -0.051^{+0.012}_{-0.036}$$

JPP and BEJ averaged determinations from the $K\pi$ system

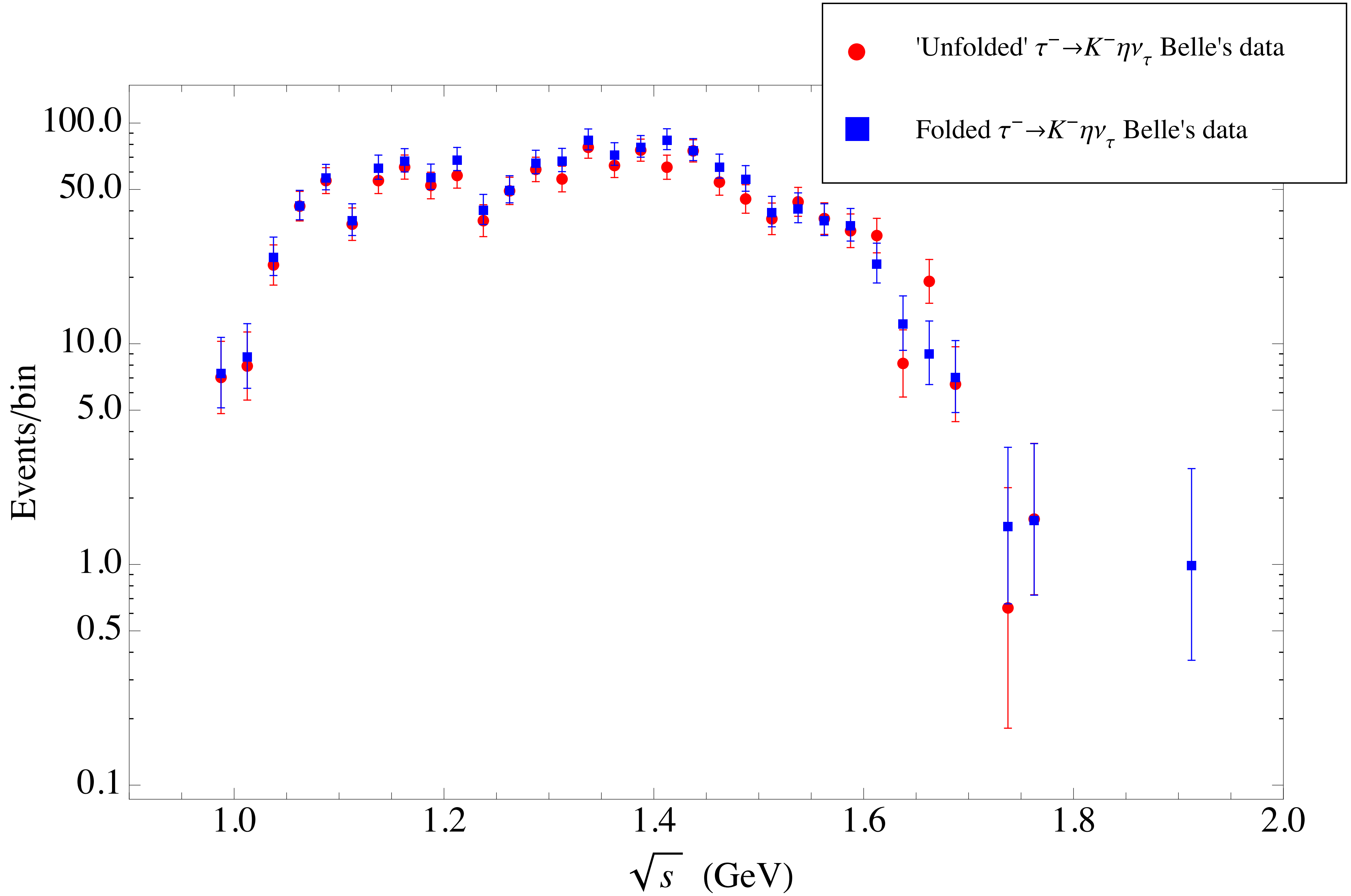
$$M_{K^{\star\prime}} = 1277^{+35}_{-41}, \quad \Gamma_{K^{\star\prime}} = 218^{+95}_{-66}, \quad \gamma = -0.049^{+0.019}_{-0.016}$$

JPP and BEJ averaged determinations from the $K\eta$ system

$$M_{K^{\star\prime}} = 1330^{+27}_{-41}, \quad \Gamma_{K^{\star\prime}} = 217^{+68}_{-122}, \quad \gamma = -0.065^{+0.025}_{-0.050}$$

# • Results of the combined analysis

Unfolding $\tau^- \to K^- \eta \nu_\tau$ Belle's data through an "unfolding" function from $\tau^- \to K_S \pi^- \nu_\tau$




- Experimentalist: To provide unfolded data would be really useful ☺
- Theorists: To provide theoretical models to be fitted by experimentalists

- **Results of the $\tau \to K\eta\nu_\tau$ analysis**

JPP vector form factor

M. Jamin, A. Pich and J. Portolés, PLB 640 (2006) 176 & 664 (2008) 78

$$f_+^{K\pi}(s) = \frac{M_{K^\star}^2}{M_{K^\star}^2 - s - iM_{K^\star}\Gamma_{K^\star}(s)} \exp\left\{\frac{3}{2} Re\left[\widetilde{H}_{K\pi}(s) + \widetilde{H}_{K\eta}(s)\right]\right\}$$

BEJ vector form factor

D. R. Boito, R. Escribano and M. Jamin,
EPJC 59 (2009) 821 & JHEP 09 (2010) 039

$$\widetilde{f}_+(s) = \exp\left[\alpha_1 \frac{s}{m_\pi^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_\pi^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

$$\delta(s) = \tan^{-1}\left[\frac{\mathrm{Im}\widetilde{f}_+(s)}{\mathrm{Re}\widetilde{f}_+(s)}\right] \qquad \widetilde{f}_+(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})}$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n Re\left[H_{K\pi}(s)\right] - i m_n \gamma_n(s)$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma^3(m_n^2)} \frac{\gamma_n}{m_n}, \quad \gamma_n(s) = \gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- **Results of the combined analysis**

## $K^*(1410)$ MASS

| Value (MeV) | | Document ID | | TECN | CHG | Comment |
|---|---|---|---|---|---|---|
| **$1414 \pm 15$** | **OUR AVERAGE** Error includes scale factor of 1.3. | | | | | |
| $1380 \pm 21 \pm 19$ | | ASTON | 1988 | LASS | 0 | 11 $K^- p \to K^- \pi^+ n$ |
| $1420 \pm 7 \pm 10$ | | ASTON | 1987 | LASS | 0 | 11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$ |
| *** We do not use the following data for averages, fits, limits, etc *** | | | | | | |
| $1276^{+72}_{-77}$ | 1, 2 | BOITO | 2009 | RVUE | | $\tau^- \to K^0_S \pi^- \nu_\tau$ |
| $1367 \pm 54$ | | BIRD | 1989 | LASS | - | 11 $K^- p \to \overline{K}^0 \pi^- p$ |
| $1474 \pm 25$ | | BAUBILLIER | 1982B | HBC | 0 | 8.25 $K^- p \to \overline{K}^0\ 2\pi n$ |
| $1500 \pm 30$ | | ETKIN | 1980 | MPS | 0 | 6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$ |

1 From the pole position of the $K\pi$ vector form factor in the complex $s$-plane and using EPIFANOV 2007 data.

2 Systematic uncertainties not estimated.

- **Results of the combined analysis**

## $K^*(1410)$ WIDTH

| Value (MeV) | | Document ID | | TECN | CHG | Comment |
|---|---|---|---|---|---|---|
| **232 ± 21** | | **OUR AVERAGE** Error includes scale factor of 1.1. | | | | |
| $176 \pm 52 \pm 22$ | | ASTON | 1988 | LASS | 0 | 11 $K^- p \to K^- \pi^+ n$ |
| $240 \pm 18 \pm 12$ | | ASTON | 1987 | LASS | 0 | 11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$ |
| *** We do not use the following data for averages, fits, limits, etc *** | | | | | | |
| $198^{+61}_{-87}$ | 1, 2 | BOITO | 2009 | RVUE | | $\tau^- \to K^0_S \pi^- \nu_\tau$ |
| $114 \pm 101$ | | BIRD | 1989 | LASS | - | 11 $K^- p \to \overline{K}^0 \pi^- p$ |
| $275 \pm 65$ | | BAUBILLIER | 1982B | HBC | 0 | 8.25 $K^- p \to \overline{K}^0 2\pi n$ |
| $500 \pm 100$ | | ETKIN | 1980 | MPS | 0 | 6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$ |

[1] From the pole position of the $K\pi$ vector form factor in the complex $s$-plane and using EPIFANOV 2007 data.

[2] Systematic uncertainties not estimated.

# • Results of the combined analysis

Reference fit results obtained for different values of $s_{cut}$

| Fitted value \ $s_{\rm cut}\,(\mathrm{GeV}^2)$ | 3.24 | 4 | 9 | $\infty$ |
|---|---|---|---|---|
| $\bar{B}_{K\pi}(\%)$ | $0.402 \pm 0.013$ | $0.404 \pm 0.012$ | $0.405 \pm 0.012$ | $0.405 \pm 0.012$ |
| $(B^{\rm th}_{K\pi})(\%)$ | $(0.399)$ | $(0.402)$ | $(0.403)$ | $(0.403)$ |
| $M_{K^*}$ | $892.01 \pm 0.19$ | $892.03 \pm 0.19$ | $892.05 \pm 0.19$ | $892.05 \pm 0.19$ |
| $\Gamma_{K^*}$ | $46.04 \pm 0.43$ | $46.18 \pm 0.42$ | $46.27 \pm 0.42$ | $46.27 \pm 0.41$ |
| $M_{K^{*\prime}}$ | $1301^{+17}_{-22}$ | $1305^{+15}_{-18}$ | $1306^{+14}_{-17}$ | $1306^{+14}_{-17}$ |
| $\Gamma_{K^{*\prime}}$ | $207^{+73}_{-58}$ | $168^{+52}_{-44}$ | $155^{+48}_{-41}$ | $155^{+47}_{-40}$ |
| $\gamma_{K\pi}$ | $= \gamma_{K\eta}$ | $= \gamma_{K\eta}$ | $= \gamma_{K\eta}$ | $= \gamma_{K\eta}$ |
| $\lambda'_{K\pi} \times 10^3$ | $23.3 \pm 0.8$ | $23.9 \pm 0.7$ | $24.3 \pm 0.7$ | $24.3 \pm 0.7$ |
| $\lambda''_{K\pi} \times 10^4$ | $11.8 \pm 0.2$ | $11.8 \pm 0.2$ | $11.7 \pm 0.2$ | $11.7 \pm 0.2$ |
| $\bar{B}_{K\eta} \times 10^4$ | $1.57 \pm 0.10$ | $1.58 \pm 0.10$ | $1.58 \pm 0.10$ | $1.58 \pm 0.10$ |
| $(B^{\rm th}_{K\eta}) \times 10^4$ | $(1.43)$ | $(1.45)$ | $(1.46)$ | $(1.46)$ |
| $\gamma_{K\eta} \times 10^2$ | $-4.0^{+1.3}_{-1.9}$ | $-3.4^{+1.0}_{-1.3}$ | $-3.2^{+0.9}_{-1.1}$ | $-3.2^{+0.9}_{-1.1}$ |
| $\lambda'_{K\eta} \times 10^3$ | $18.6 \pm 1.7$ | $20.9 \pm 1.5$ | $22.1 \pm 1.4$ | $22.1 \pm 1.4$ |
| $\lambda''_{K\eta} \times 10^4$ | $10.8 \pm 0.3$ | $11.1 \pm 0.4$ | $11.2 \pm 0.4$ | $11.2 \pm 0.4$ |
| $\chi^2/\mathrm{n.d.f.}$ | $105.8/105$ | $108.1/105$ | $111.0/105$ | $111.1/105$ |

# • Results of the combined analysis

## Correlation coefficients

| | $\bar{B}_{K\pi}$ | $M_{K^*}$ | $\Gamma_{K^*}$ | $M_{K^{*\prime}}$ | $\Gamma_{K^{*\prime}}$ | $\lambda'_{K\pi}$ | $\lambda''_{K\pi}$ | $\bar{B}_{K\eta}$ | $\gamma_{K\eta} = \gamma_{K\pi}$ | $\lambda'_{K\eta}$ | $\lambda''_{K\eta}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $M_{K^*}$ | $-0.163$ | 1 | | | | | | | | | |
| $\Gamma_{K^*}$ | 0.028 | $-0.060$ | 1 | | | | | | | | |
| $M_{K^{*\prime}}$ | $-0.063$ | $-0.104$ | $-0.142$ | 1 | | | | | | | |
| $\Gamma_{K^{*\prime}}$ | 0.126 | 0.130 | 0.292 | $-0.556$ | 1 | | | | | | |
| $\lambda'_{K\pi}$ | 0.800 | $-0.100$ | 0.457 | $-0.244$ | 0.432 | 1 | | | | | |
| $\lambda''_{K\pi}$ | 0.928 | $-0.215$ | 0.328 | $-0.166$ | 0.304 | 0.942 | 1 | | | | |
| $\bar{B}_{K\eta}$ | $-0.003$ | $-0.005$ | $-0.010$ | 0.003 | $-0.001$ | $-0.015$ | $-0.009$ | 1 | | | |
| $\gamma_{K\eta} = \gamma_{K\pi}$ | $-0.155$ | $-0.173$ | $-0.378$ | 0.498 | $-0.878$ | $-0.565$ | $-0.373$ | 0.019 | 1 | | |
| $\lambda'_{K\eta}$ | 0.058 | 0.028 | 0.117 | 0.050 | 0.337 | 0.182 | 0.128 | 0.434 | $-0.340$ | 1 | |
| $\lambda''_{K\eta}$ | 0.035 | $-0.017$ | 0.037 | 0.106 | 0.218 | 0.080 | 0.064 | 0.561 | $-0.174$ | 0.971 | 1 |

**Table 3.** Correlation coefficients corresponding to our reference fit with $s_{\rm cut} = 4\,\mathrm{GeV}^2$, second column of table 1. In the fits where $\gamma_{K\pi} = \gamma_{K\eta}$ is not enforced, their correlation coefficient turns out to be $\approx 0.67$.

## • Results of the combined analysis

Different choices regarding linear slopes and resonance mixing parameters

| Fitted value | Reference Fit | Fit A | Fit B | Fit C |
|---|---|---|---|---|
| $\bar{B}_{K\pi}(\%)$ | $0.404 \pm 0.012$ | $0.400 \pm 0.012$ | $0.404 \pm 0.012$ | $0.397 \pm 0.012$ |
| $(B^{\rm th}_{K\pi})(\%)$ | $(0.402)$ | $(0.394)$ | $(0.400)$ | $(0.394)$ |
| $M_{K^*}$ | $892.03 \pm 0.19$ | $892.04 \pm 0.19$ | $892.03 \pm 0.19$ | $892.07 \pm 0.19$ |
| $\Gamma_{K^*}$ | $46.18 \pm 0.42$ | $46.11 \pm 0.42$ | $46.15 \pm 0.42$ | $46.13 \pm 0.42$ |
| $M_{K^{*\prime}}$ | $1305^{+15}_{-18}$ | $1308^{+16}_{-19}$ | $1305^{+15}_{-18}$ | $1310^{+14}_{-17}$ |
| $\Gamma_{K^{*\prime}}$ | $168^{+52}_{-44}$ | $212^{+66}_{-54}$ | $174^{+58}_{-47}$ | $184^{+56}_{-46}$ |
| $\gamma_{K\pi} \times 10^2$ | $= \gamma_{K\eta}$ | $-3.6^{+1.1}_{-1.5}$ | $-3.3^{+1.0}_{-1.3}$ | $= \gamma_{K\eta}$ |
| $\lambda'_{K\pi} \times 10^3$ | $23.9 \pm 0.7$ | $23.6 \pm 0.7$ | $23.8 \pm 0.7$ | $23.6 \pm 0.7$ |
| $\lambda''_{K\pi} \times 10^4$ | $11.8 \pm 0.2$ | $11.7 \pm 0.2$ | $11.7 \pm 0.2$ | $11.6 \pm 0.2$ |
| $\bar{B}_{K\eta} \times 10^4$ | $1.58 \pm 0.10$ | $1.62 \pm 0.10$ | $1.57 \pm 0.10$ | $1.66 \pm 0.09$ |
| $(B^{\rm th}_{K\eta}) \times 10^4$ | $(1.45)$ | $(1.51)$ | $(1.44)$ | $(1.58)$ |
| $\gamma_{K\eta} \times 10^2$ | $-3.4^{+1.0}_{-1.3}$ | $-5.4^{+1.8}_{-2.6}$ | $-3.9^{+1.4}_{-2.1}$ | $-3.7^{+1.0}_{-1.4}$ |
| $\lambda'_{K\eta} \times 10^3$ | $20.9 \pm 1.5$ | $= \lambda'_{K\pi}$ | $21.2 \pm 1.7$ | $= \lambda'_{K\pi}$ |
| $\lambda''_{K\eta} \times 10^4$ | $11.1 \pm 0.4$ | $11.7 \pm 0.2$ | $11.1 \pm 0.4$ | $11.8 \pm 0.2$ |
| $\chi^2/\text{n.d.f.}$ | $108.1/105 \sim 1.03$ | $109.9/105 \sim 1.05$ | $107.8/104 \sim 1.04$ | $111.9/106 \sim 1.06$ |