Tau lepton production and decays: perspective of multi-dimensional distributions and Monte Carlo methods

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- **(1)** The τ lepton decays: fascinating laboratory for intermediate energy QCD

- **(2)** In itself, developing models can be very tempting.

NEVER FORGET: precision of experimental data is substantially better, than theory predictions.

- **(3)** How to optimize work of inhomogeneous community.

How to profit from programing language opportunities, how to avoid traps.

- **(4)** I will use TAUOLA, its associated projects and updates as examples.

- **(5)** I want to adress the question what can/should be the role of MC in this respect.

- **My talk would not be possible without effort of many people and experiments. Many things originate from work and discussions in Karlsruhe: R. Decker, J. Kuhn, E. Mirkes**

TAUOLA: implementation of M.E., had. currents of τ dec.

1. **Nothing new with respect to Aachen so far...**

2. TAUOLA with new hadronic currents, up to 500 decay channels, which can be manipulated by user:
   http://annapurna.ifj.edu.pl/~tprzedzinski/tmp/TAUOLA-FORTRAN-2016-09-15.tgz
   Practically no changes were introduced to distribution I have presented 2 years ago. We have made public documentation for this code: https://arxiv.org/abs/1609.04617, in the following weeks I will clean the code and it will be placed on my web page.

3. Direction for work is essentially set. I have not received much feed-backs, but thre were no objections too.

4. Code can be translated piece after piece into C++, or other language. Whenever a need will arrive I can speed up an effort.

5. Initialization of hadronic decay channel is compatible with defaults as BaBar was using, but this is just for archivization purposes. I suspect users will now work on their own hadronic currents.

6. **Theoretical uncertainty of the models can be $\frac{1}{N_C}$, $\frac{1}{N_C^2}$ or $\cdots$,** but experimental precision has to be assumed to be better than 0.001. That is a factor of 100 better.

Theory model simulation and technical precision: easy.

Channel	Width [GeV]	reference	In tauola/RChL-currents directory channel's current: file $\to$ routine
$\pi^- \pi^0$	$5.2678 \cdot 10^{-13} \pm 0.01\%$	Subsection 2.4	frho_pi.f $\to$ CURR_PIPI0
$K^- \pi^0$	$5.853 \cdot 10^{-15} \pm 0.02\%$	Subsection 2.4	fkpipl.f $\to$ CURR_KPI0
$\pi^- K^0$	$1.1025 \cdot 10^{-14} \pm 0.03\%$	Subsection 2.4	fkpipl.f $\to$ CURR_PIK0
$K^- K^0$	$2.415 \cdot 10^{-15} \pm 0.02\%$	Subsection 2.4	fk0k.f $\to$ CURR_KK0
$\pi^- \pi^- \pi^+$	$2.08 \cdot 10^{-12} \pm 0.017\%$	Subsection 2.1	f3pi_rcht.f $\to$ F3PI_RCHT*
$\pi^0 \pi^0 \pi^-$	$2.126 \cdot 10^{-12} \pm 0.017\%$	Subsection 2.1	f3pi_rcht.f $\to$ F3PI_RCHT*
$K^- \pi^- K^+$	$3.8467 \cdot 10^{-15} \pm 0.04\%$	Subsection 2.2	fkkpi.f $\to$ FKKPI*
$K^0 \pi^- \bar{K}^0$	$3.5935 \cdot 10^{-15} \pm 0.03\%$	Subsection 2.2	fkkpi.f $\to$ FKKPI*
$K^- \pi^0 K^0$	$2.769 \cdot 10^{-15} \pm 0.04\%$	Subsection 2.3	fkk0pi0.f $\to$ FKK0PI0*
			* The F_i of form-factors.

Table 1: Collection of numerical results from paper: O. Shekhovtsovaa, T. Przedzinski, P. Roig and Z. Was *Resonance Chiral Lagrangian currents and τ decay Monte Carlo*, Phys.Rev. D86 **(2012)** 113008. References to subsections of that paper. Last column includes references to routines of the currents code. It looked like mission accomplished. Just fine tuning of some parameters.

Theory model simulation and technical precision: easy.

- New hadronic currents (more than 88 % of hadronic τ decay width) version installed with the 0.05 % technical tag:
  O. Shekhovtsovaa, T. Przedzinski, P. Roig and Z. Was *Resonance Chiral Lagrangian currents and τ decay Monte Carlo*, Phys.Rev. D86 (2012) 113008
- **But** physics precision was definitely **NOT** as good as 0.05 %.
- Over the first two years we worked on preparing confrontation env. with the data keeping precision in mind.
- But despite partial success for 3π modes, we are far from satisfactory solution.

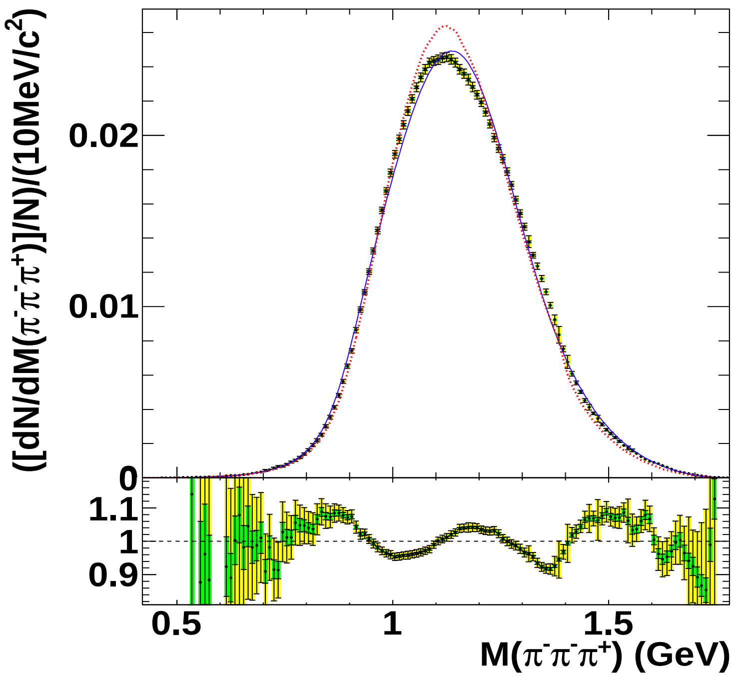
- **Useful for further work:**
- We have investigated technical aspects for fitting using weights.
  It is of interest in case when experimental cuts are present, multidimensional distributions are used and no semi-analytical results can be easily obtained.
- We have used semi-analytical 1-dim distributions for fits: I.M. Nugent, T. Przedzinski, P. Roig, O. Shekhovtsova, Z. Was, Phys.Rev. D88 (2013) 093012
- Such distributions are essential for technical tests of our code, but also for fits and evaluation how experimental errors propagate to parameters of the models. Unfortunately we were missing 3 dimensional distributions...

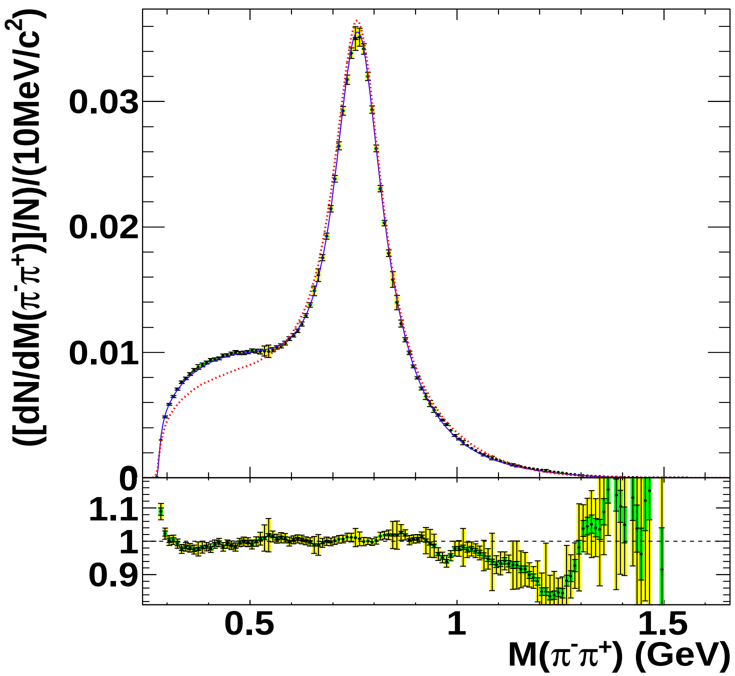
Attempt: comparison with experimental distr.

New currents for $\tau \to 3\pi$ and $\tau \to 2\pi$ decays

Currents based on Resonance Chiral Lagrangian approach and fits to BaBar data.

Experimental systematic errors considered. Software environment for fits was prototyped but used in non automated way. From: *Resonance Chiral Lagrangian Currents and Experimental Data for* $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$, I.M. Nugent, T. Przedzinski, P. Roig, O. Shekhovtsova, Z. Was, Phys. Rev. D 88, 093012 (2013).





Attempt: comparison with experimental distr.

To progress in case of $\tau \to 3\pi\nu_\tau$ we had to:

- Modify the model (contribution of σ)
- Work simultaneously with fits using weights (at this time only to cross-check results for big mistakes). We had difficulties with stability because of strong correlations of parameters. Template method I have learned at ALEPH time requires better understanding if model parameters are strongly correlated and for some of them dependencies is weak. Necessity to linearize dependencies because of CPU-time constraints in case when model was not giving perfect predictions complicated things further.
- We relied on fitting semi-analytical formulas.
  - We had to assure that derivaties of results are continuous.
  - We had to speed up calculations using different methods of pretabulation/interpolation of results for Q-dependent a_1 width (unitarity constraint).
  - We relied on 1-dimensional invariant mass distributions.

Attempt: comparison with experimental distr.

- Not anymore separating work into theoretical, experimental and computing aspects. Even for the simple case of 1-dimensional unfolded distribution.
- **NONETHELESS:**
- We got improvement for 3π modes *when only 1-dim histograms*.
- Control of experimental systematic errors.
- No control of systematic due to limitation to 1-dim histograms.
- Experience for the future steps, but no organized software solution.
- What is the best form of input from experimental side?
- Multidimensional histograms, number of bins comparable with size of measured sample? Moments, bias due to model assumptions?
- How to coordinate work?
- **Not acceptable: theorist/experimentalist have to wait for ...**

Attempt: comparison with experimental distr.

- **Achieved:**
- TAUOLA MC with up to 500 decay channels, solution similar as presented on TAU04 and used by BaBar. Neutrinoless channels available.
- **Default BaBar Tauola initialization.**
- Alternatively, for 2 and 3 π's, new currents with comparison with experimental data prepared.
- Theoretically motivated currents, 4 and 5 π's decay modes, also as alternative.
- No fits to global properties such as average charged energy. For alternatives, no experimental quality stamps.
- User can re-initialize TAUOLA with own (C++ coded) currents (or matrix elements).

- **Non complete tasks:**
- The 3-scalar modes with K's. require better control of multidim. distr. than 3π.
- Many alternative parametrizations, eg. for 2K 2π modes (BaBar) are not incorporated, even though these are missing channels, at present only flat phase space.
- Environments for fits are not well structured for model independent use.

General formula for tau production and decay.

Formalism for $\tau^+\tau^-$: nothing changes

- Because narrow τ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

- This formalism is fine, but because of over 20 τ decay channels we have over 400 distinct processes. Also picture of production and decay are mixed.

- Below only τ spin indices are explicitly written:

$$\mathcal{M} = \sum_{\lambda_1 \lambda_2 = 1}^{2} \mathcal{M}^{prod}_{\lambda_1 \lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Cross section can be re-written into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

General formalism for semileptonic decays

- Matrix element used in TAUOLA for semileptonic decay

$$\tau(P,s) \to \nu_\tau(N)X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(v+a\gamma_5)u(P)J_\mu$$

- J_μ the current depends on the momenta of all hadrons

$$|\mathcal{M}|^2 = G^2\frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2}m_\nu M(J^*\cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2}m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho P_\sigma$$

Hadronic currens dominate uncertainty

- Improvements for ρ channel are technically straightforward: single real function to be fitted: $J^\mu = (p_{\pi^\pm} - p_{\pi^0})^\mu F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^\mu F_S(Q^2)$ ($F_S \simeq 0$).
- For 3-scalar states: 4 complex function 3 variables each. Role of theoretical assumptions is essential. Agreement on 1-dim distribution is a consistency check.
- No go for model independent measurements? Not necessarily. Use of all dimensions for data distributions: invariant masses Q^2, s_1, s_2 as arguments of form-factors. Angular asymmetries help to separate currents: scalar $J_4^\mu \sim Q^\mu = (p_1 + p_2 + p_3)^\mu$, vector $J_1^\mu \sim (p_1 - p_3)^\mu|_{\perp Q}$ and $J_2^\mu \sim (p_2 - p_3)^\mu|_{\perp Q}$ and finally pseudovector $J_5^\mu \sim \epsilon(\mu, p_1, p_2, p_3)$.
- Model independent methods, template methods, neural networks, multidimensional signatures. It was easier for Cleo. There, τ's were produced nearly at rest, ν_τ four-momentum was easy to reconstruct.
- **I will not rely on any further symmetries like isospin symmetry. It should not be part of the Monte Carlo design.**
- Fitting in complex situation is ... **well complex !**

Warning message

- Biases in art, Giuseppe Arcimboldo (1572 - 1593).

- Already for 3-scalar final states theoretical predictions and experimental data: distributions over 8-dimensional space. We fit 1- (2-) dim. histos. Result depend on model assumptions. Models inspired with results ... Fitting setup $\rightarrow$ biases.
- Our algorithms are far less elaborate than human eye/brain.
- Who in charge? (TH, EXP?) My doubts expressed in: Z. Was, J. Zaremba *Study of variants for Monte Carlo generators of* $\tau \rightarrow 3\pi\nu$ *decays*, Eur.Phys.J. C75 (2015) 566
- Guess for today: experiments ...

We are not alone with the problem

Figure 1: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don't.

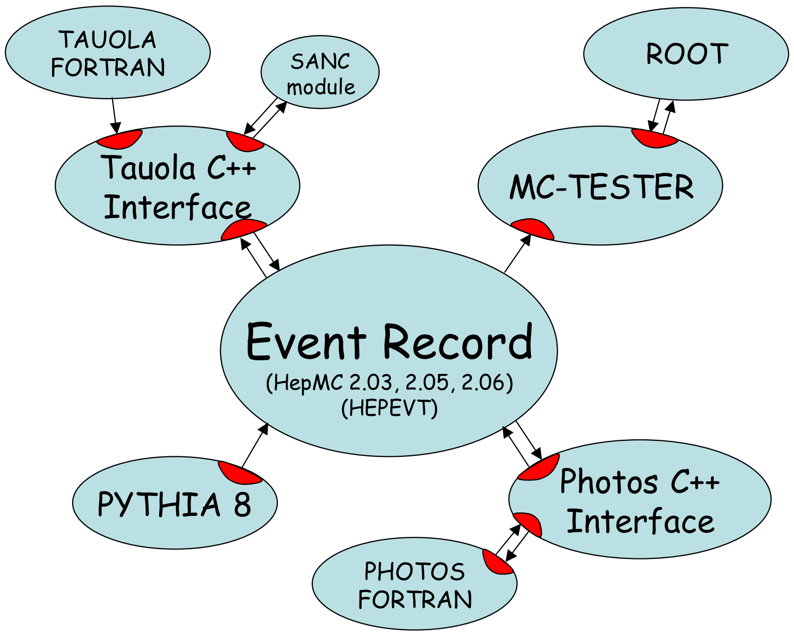
From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

Pattern recognition is an active field and deep concern and not only for us.

Toward techniques using weighted events, template fitting.

Communication though event record: (for program inerfaces or data files).

Solution for phase space $\times |M|^2$ algorithms.



Parts:

- hard process: (Born, weak, new physics),
- parton shower,
- τ decays
- QED bremsstrahlung

- High precision achieved

- Detector studies: acceptance, resolution lepton with or without photon.

Such organization requires:

- Good control of factorization (theory)
- Good understanding of tools on user side.

Techniques of weighted events

`TauSpinner`

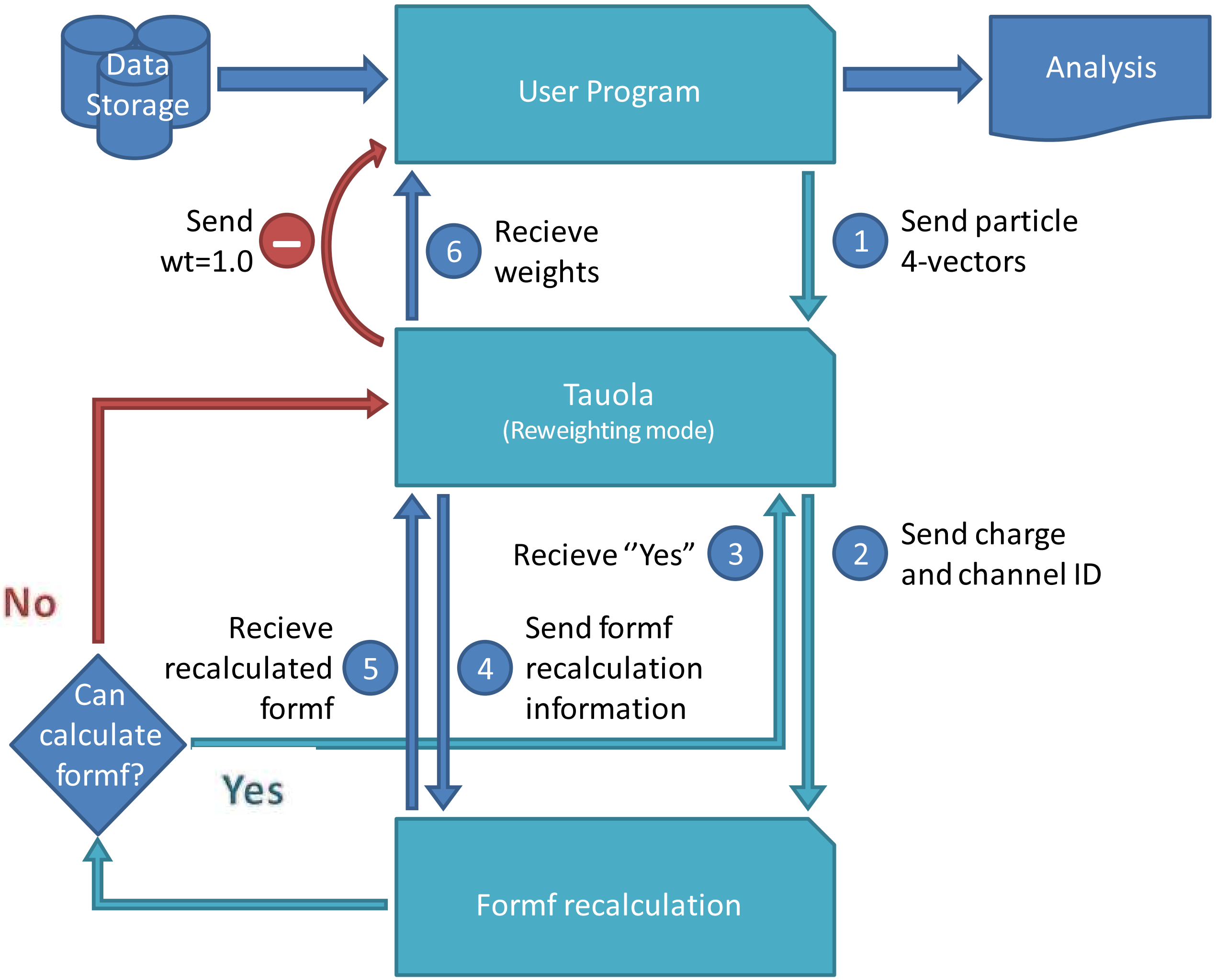


Figure 2: Flow chart for communication when already stored events are modified with the weights. Useful at LHC and at low energy applications as well.

Results relevant for fitting and for τ leptons.

1. *W production at LHC: lepton angular distributions and reference frames for probing hard QCD*, E. Richter-Was and Z. Was, arXiv:1609.02536
2. *Potential for optimizing Higgs boson CP measurement in H to tau tau decay at LHC and ML techniques* , R. Jozefowicz, E. Richter-Was and Z. Was,arXiv:1608.02609
3. *Separating electroweak and strong interactions in Drell—Yan processes at LHC: leptons angular distributions and reference frames*, E. Richter-Was and Z. Was, Eur.Phys.J. C76 (2016) 473
4. *“. Production of tau lepton pairs with high pT jets at the LHC and the TauSpinner reweighting algorithm”*, J. Kalinowski, W. Kotlarski, E. Richter-Was and Z. Was, arXiv:1604.00964
5. *“TauSpinner Program for Studies on Spin Effect in tau Production at the LHC”*, Z. Czyczula, T. Przedzinski and Z. Was, Eur. Phys. J. C **72**, 1988 (2012)

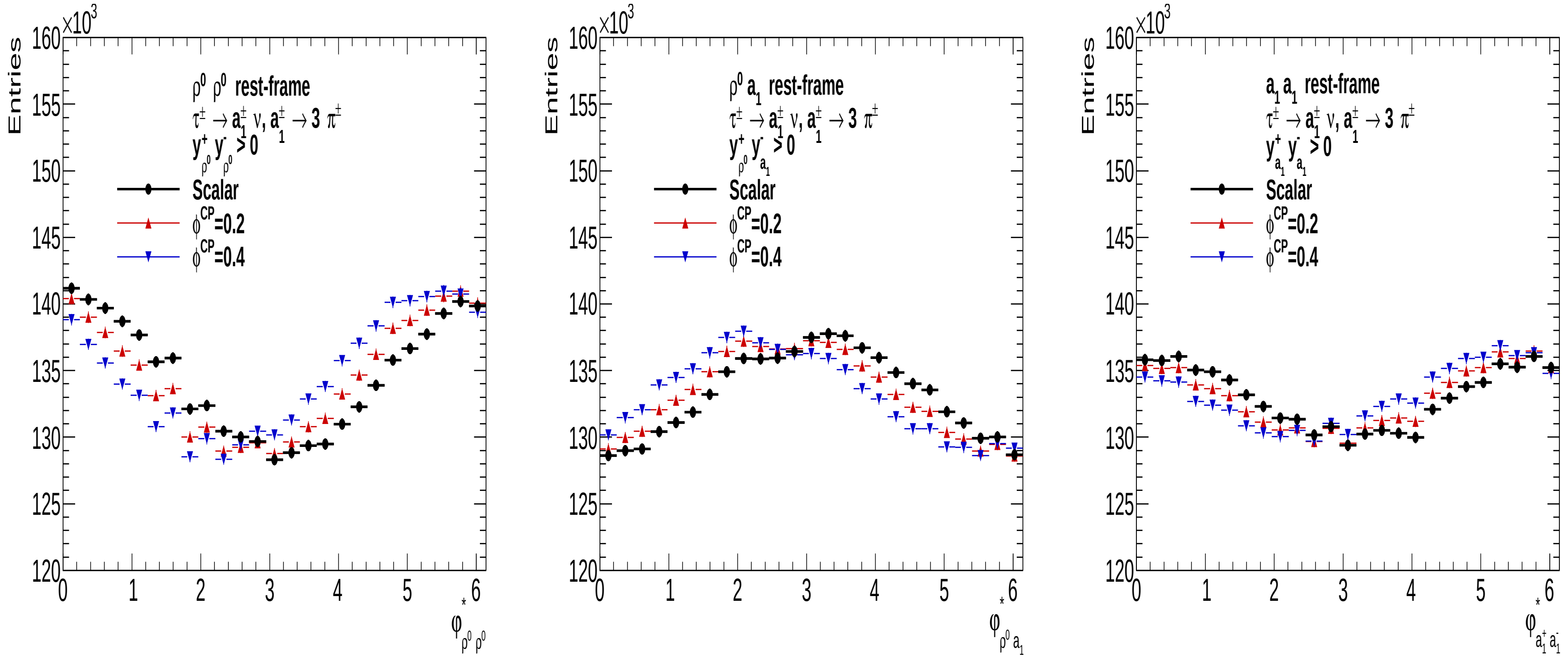
Migration to C++:

1. *PHOTOS Interface in C++: Technical and Physics Documentation ”*,N. Davidson, T. Przedzinski, Z. Was Comput.Phys.Commun. 199 (2016) 86

Neural Network for CP parity of Higgs,arXiv:1608.02609

Acoplanarity angles of oriented half decay planes: $\varphi^*_{\rho^0\rho^0}$ (left), $\varphi^*_{a_1\rho^0}$ (middle) and $\varphi^*_{a_1 a_1}$ (right), for events grouped by the sign of $y^+_{\rho^0} y^-_{\rho^0}$, $y^+_{a_1} y^-_{\rho^0}$ and $y^+_{a_1} y^-_{a_1}$ respectively. Shown are distributions for three values of mixing angle ϕ^{CP} = 0.0 (scalar), 0.2 and 0.4.

Up to 16 plots like that have to be measured, correlations understood. But physics model depends on 1 parameter only and effect of ϕ^{CP}, the higgs mixing scalar pseudoscalar angle, is always a linear shift.



If it was like at Born level, most of the coefficents would equal zero:

$$\frac{d\sigma}{dp_T^2 dY d\cos\theta d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^2 dY}[(1+\cos^2(\theta)) + 1/2 A_0(1-3\cos^2(\theta))$$
$$+A_1 \sin(2\theta)\cos(\phi) + 1/2 A_2 \sin^2(\theta)\cos(2\phi)$$
$$+A_3 \sin(\theta)\cos(\phi) + A_4\cos(\theta) + A_5\sin^2(\theta)\sin(2\phi)$$
$$+A_6\sin(2\theta)\sin(\phi) + A_7\sin(\theta)\sin(\phi)]$$

angles θ, ϕ define orientation of leptons in the lepton pair rest frame. At Born level, only A_4 depends on electroweak parameters.

Usually it is not the case, but do EW and QCD effects separate in multijet processes?

One can investigate that using analytic, first order results.

Some of result originate from properties of Lorentz group and are quite universal.

Definition of `Mustraal` frame relies on geometrical properties. Stochastich choice of frame does not depend on any couplings! We hope ro reduce complexity and simplify fitting.

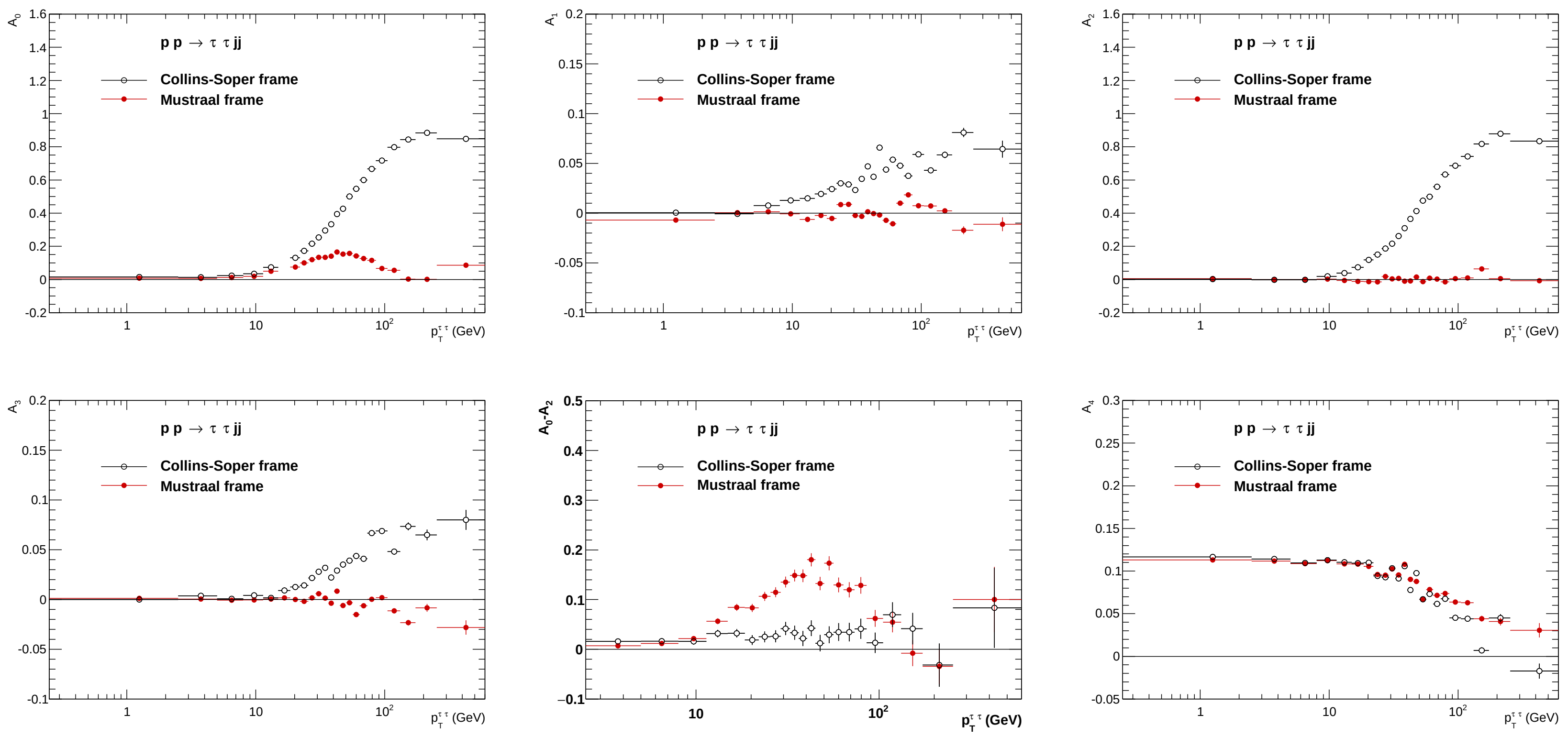


Figure 3: The A_i coefficients of Eq. (1)) calculated in Collins-Soper (black) and in `Mustraal` (red) frames for $pp \to \tau\tau jj$ process generated with `MadGraph`. Thanks to exploitation of single gluon amplitude geometrical properties, in these two-jet configurations, we got shapes more like of Born. Separation into electroweak-stron geffects is cleaner.

$W \to l\nu_l$ production at LHC, 1609.02536

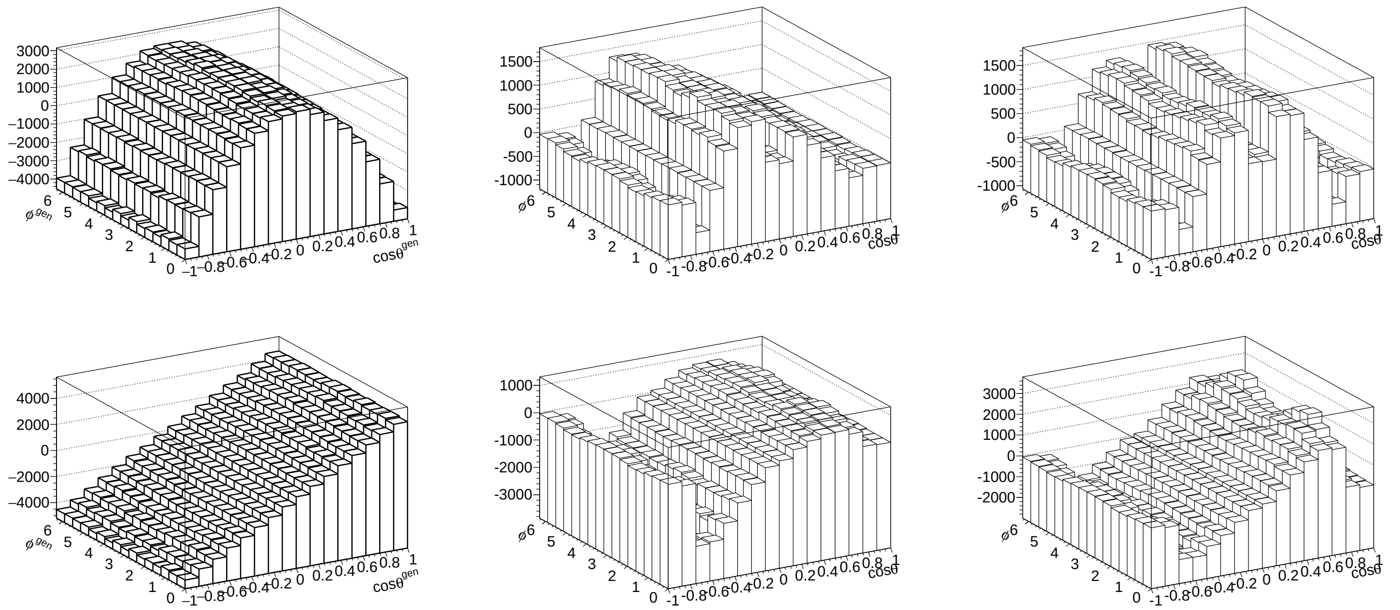


Figure 4: Analytical shape of the polynomial P_0 (top) and P_4 (bottom) in the full phase-space (left) and templates for polynomials after reconstructing p_Z^ν and fiducial selection for: W^- (middle) and W^+ (right). Original spherical harmonics of second order for $W \to l\nu$ decay angles are strongly deformed, but can be measured even for W. For the benefit of initial state hadronic interaction.

I do not have as convincing results.

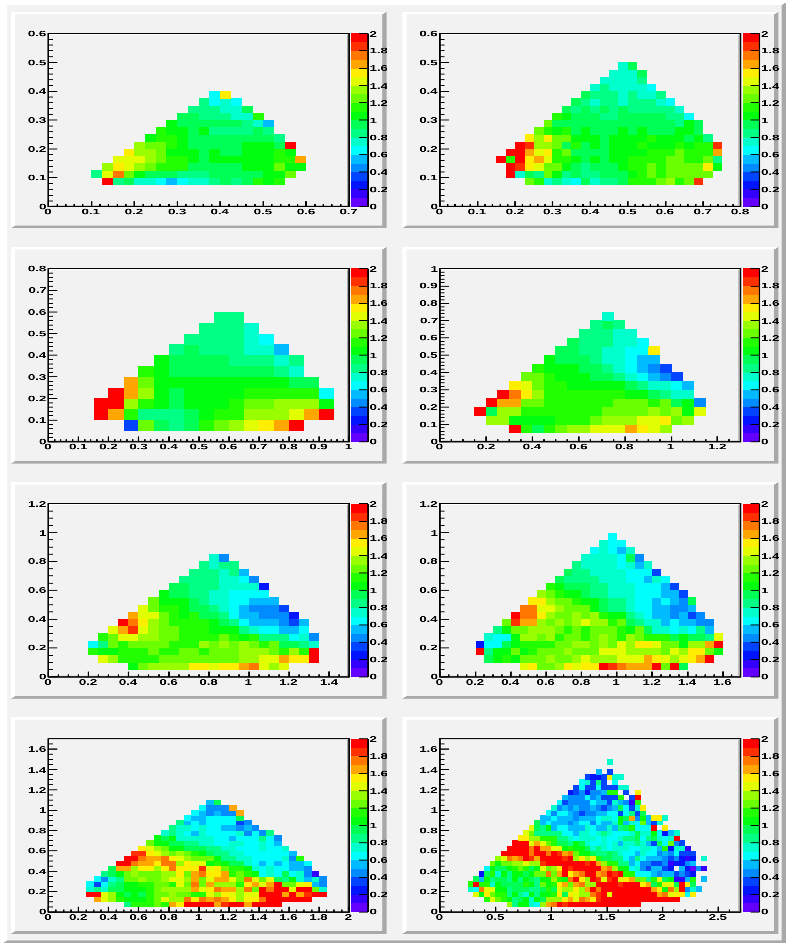
1. For fits, dynamic of τ decay is more challenging, than for Higgs,Z,W ...
2. Mdels to confront with data, may have more than 10 parameters and their impact on measured quantities may be strongly correlated.
3. Models have a lot of ambiguities in their definitions ...
4. Experimental data have uncertainties and usually represent at best one or 2 dimensional distributions available for fits.
5. The way out is to fit distributions which have systematic errors controlled, but at the same time cross check multidimensional distributions.
6. Theorists are motivated to fit, even 1-dimensional distributions. Beware of matrix elements, typically for 11 dimensional phase-space.
7. One need to understand limitations and never forget of cross checks.

Migration to C++:

1. *PHOTOS Interface in C++: Technical and Physics Documentation "*,N. Davidson, T. Przedzinski, Z. Was Comput.Phys.Commun. 199 (2016) 86

Warning already from 3-dim. distr. CLEO style.

Rations of Dalitz plots for **TAUOLA RChL** and **TAUOLA CLEO**. The Q^2 in range: 0.36-0.81, 0.81-1.0, 1.0-1.21, 1.21-1.44, 1.44-1.69, 1.69-1.96, 1.96-2.25, 2.25-3.24 GeV^2. Where red, blue; $\sim$50% off.



Summary

- How should we proceed to get most from experimental data for understanding intermediate energy hadronic interactions?
- (i) Experimental systematic errors (ii) Theoretical systematic errors
- Systematic errors due to cross biasing.
- What are the constraints on organization of Monte Carlo and fitting environments?
- Intermediate version of TAUOLA with decays still in F77 but C++ compatible.
- Flexibility for re-definition of, dynamic of tau decays and initialization based on work of BaBar/Belle collaborations, with the help of plug-ins.
- I delegate details to private discussions.
- We have collected some experience on requirements for building fitting environments, but we are not at the level of automated approach.
- Context of systematic errors, in case of fits to multi-dimensional representation of data, require systematic approach if we aim at 1% or better precision.
- Question of manpower and training as well as motivation of involved people.
- Use of τ leptons for high energy applications.