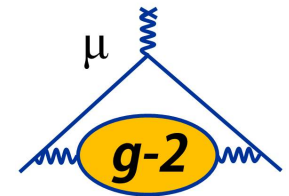


# Review of g-2: theory



UNIVERSITY OF  
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Thomas Teubner



- Introduction
- QED and weak contributions
- $a_{\mu}^{\text{had}}$  : HLbL and VP status, work in progress
- BSM?!
- Outlook

# Introduction: Lepton Dipole Moments

- Dirac equation (1928) combines non-relativistic Schroedinger Eq. with rel. Klein-Gordon Eq. and describes **spin-1/2** particles and interaction with EM field  $A_\mu(x)$ :

$$(i\partial_\mu + eA_\mu(x)) \gamma^\mu \psi(x) = m \psi(x)$$

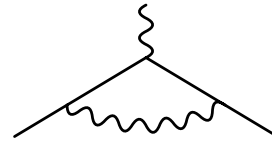
with gamma matrices  $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} I$  and 4-spinors  $\psi(x)$ .

- Great success: Prediction of **anti-particles** and **magnetic moment**  $\vec{\mu} = g \frac{Qe}{2m} \vec{s}$  with  $g = 2$  (and not 1) in agreement with experiment.
- Dirac already discussed electric dipole moment together with MDM:  $\vec{\mu} \cdot \vec{H} + i\rho_1 \vec{\mu} \cdot \vec{E}$  but discarded it because imaginary.
- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with  $g_s = 2.00229 \pm 0.00008$ .

# Introduction: Lepton Dipole Moments

- 1948: Schwinger calculates the famous radiative correction:  
that  $g = 2(1+a)$ , with

$$a = (g-2)/2 = \alpha/(2\pi) = 0.001161$$



“ If you can’t join ‘em, beat ‘em “

- The anomaly  $a$  (Anomalous Magnetic Moment) is from the Pauli term:

$$\delta\mathcal{L}_{\text{eff}}^{\text{AMM}} = -\frac{Qe}{4m} a \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

This is a dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But it occurs through radiative corrections and is calculable in perturbation theory.

- Similarly, an EDM can come from a term  $\delta\mathcal{L}_{\text{eff}}^{\text{EDM}} = -\frac{d}{2} \bar{\psi}(x) i \sigma^{\mu\nu} \gamma_5 \psi(x) F_{\mu\nu}(x)$

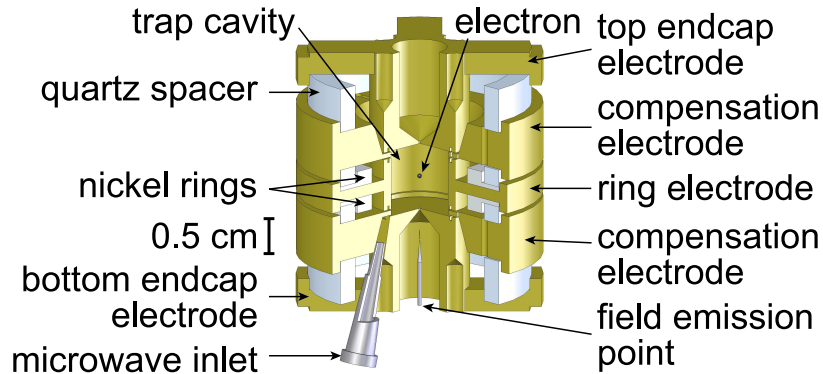
# Magnetic Moments: $a_e$ vs. $a_\mu$

$$a_e = 1\,159\,652\,180.73 (0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$

Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

$$a_\mu = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron



- $a_e^{\text{EXP}}$  more than 2000 times more precise than  $a_\mu^{\text{EXP}}$ , but for  $e^-$  loop contributions come from very small photon virtualities, whereas muon 'tests' higher scales
  - dimensional analysis: **sensitivity to NP** (at high scale  $\Lambda_{\text{NP}}$ ):  $a_\ell^{\text{NP}} \sim \mathcal{C} m_\ell^2 / \Lambda_{\text{NP}}^2$
- $\mu$  wins by  $m_\mu^2 / m_e^2 \sim 43000$  for NP, but  $a_e$  provides best determination of  $\alpha$

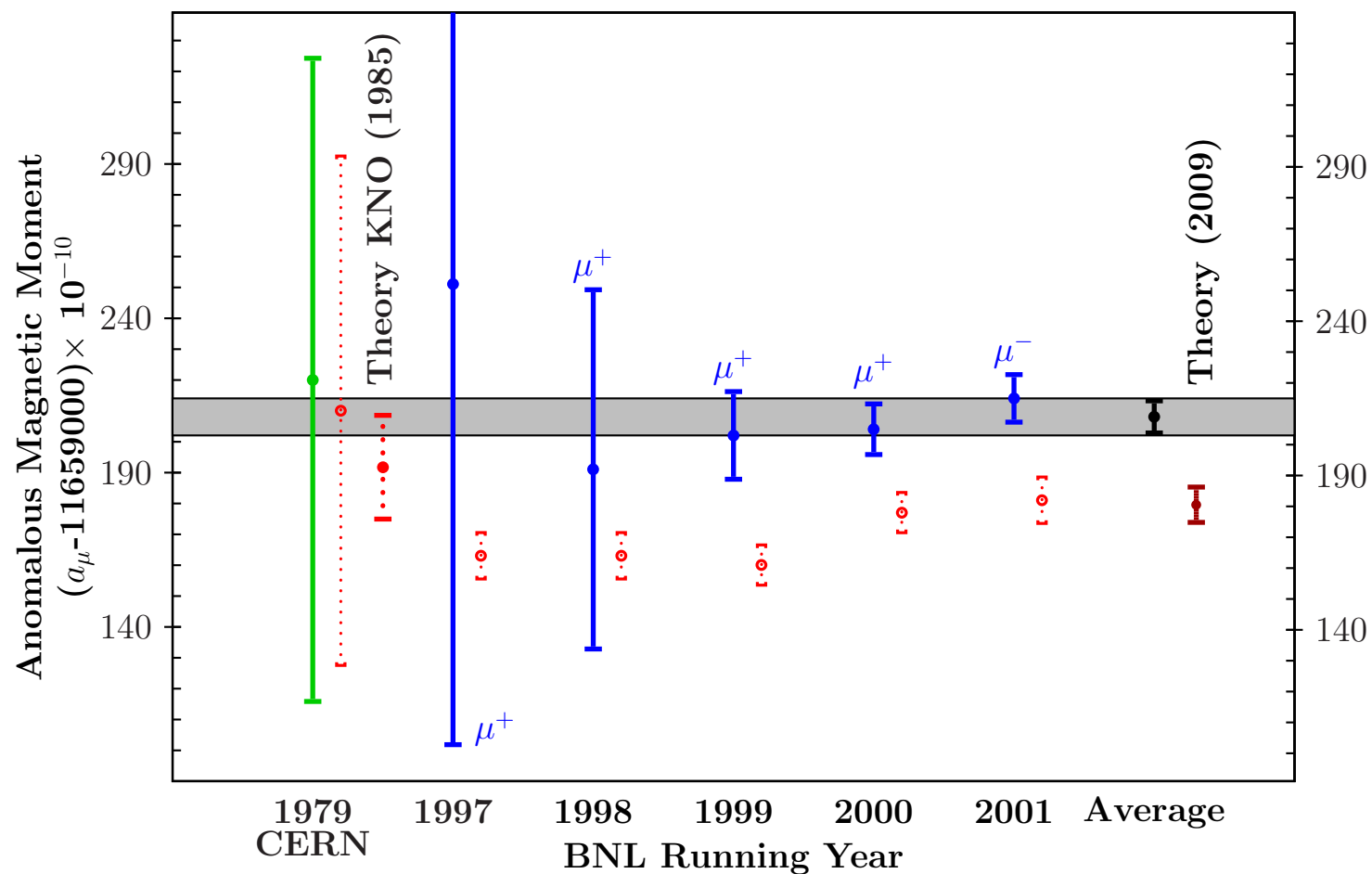


# Magnetic Moments: $a_\mu$ history

g-2 history plot and

book motto from Fred Jegerlehner:

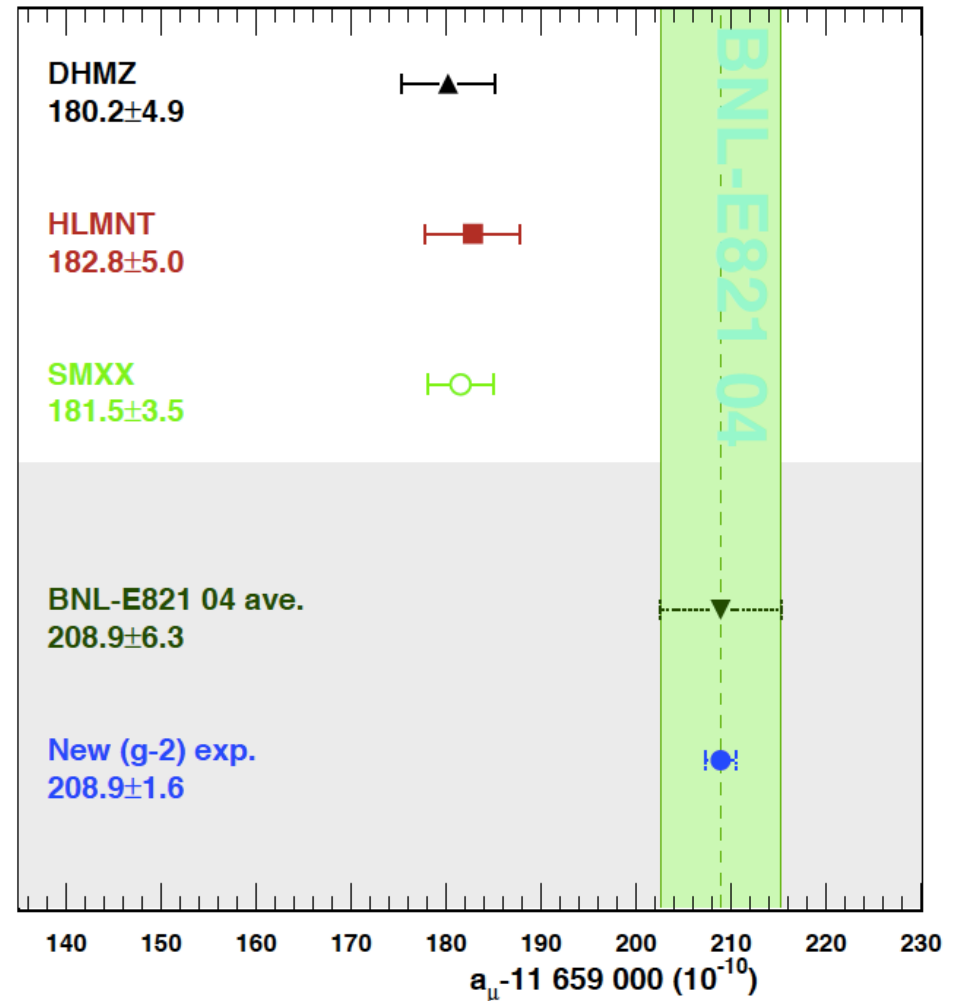
'The closer you look the more there is to see'



# $a_\mu$ : Status and future projection → charge for TH

## Future picture:

- if mean values stay and with  $a_\mu^{\text{SM}}$  improvement:  
5 $\sigma$  discrepancy
- if also EXP+TH can improve  $a_\mu^{\text{SM}}$   
'as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8 $\sigma$
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...



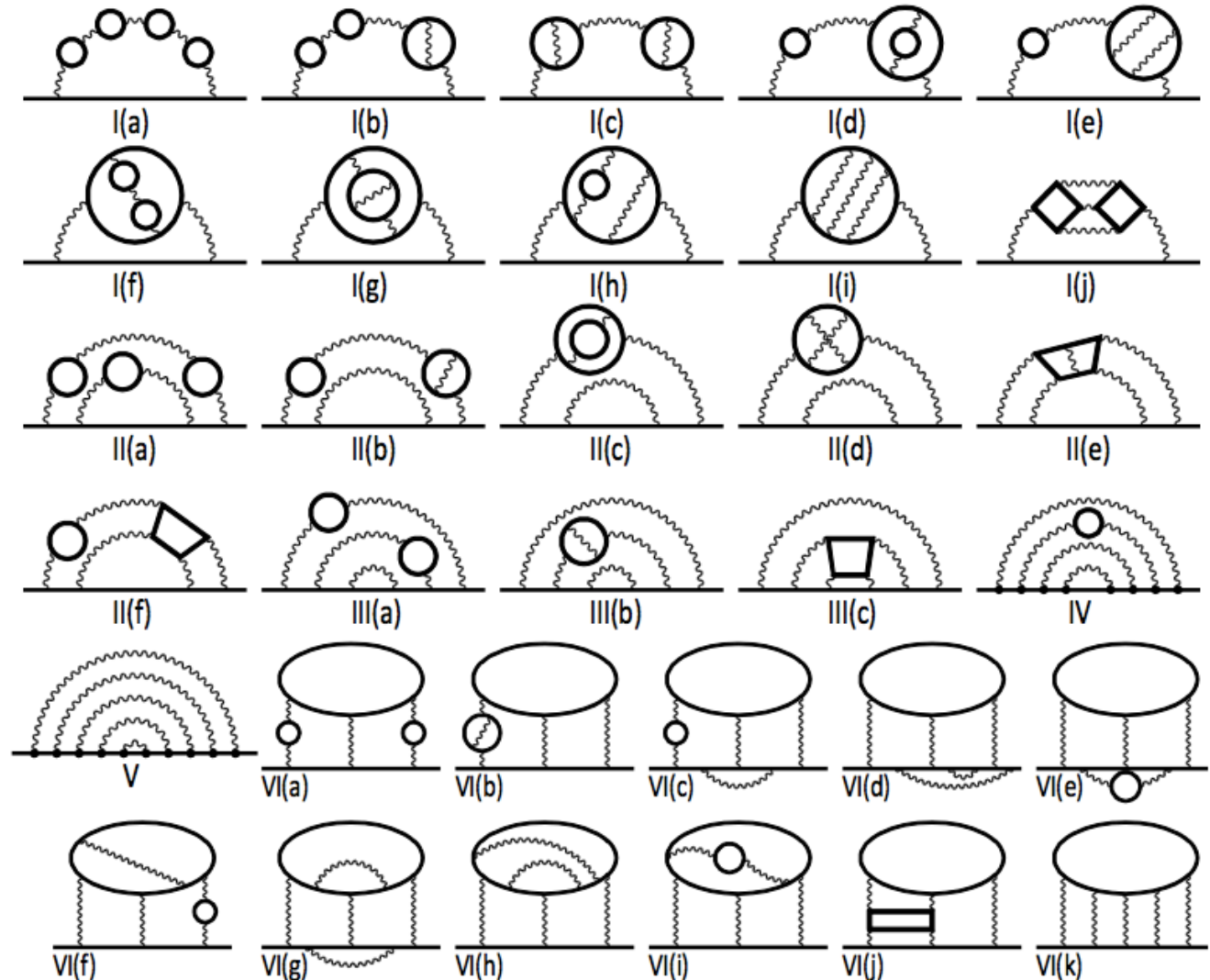
# $a_\mu^{\text{SM}}$ : status of the SM prediction

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

T. Aoyama, M. Hayakawa,  
T. Kinoshita, M. Nio (PRLs, 2012)

A triumph for perturbative QFT and computing!

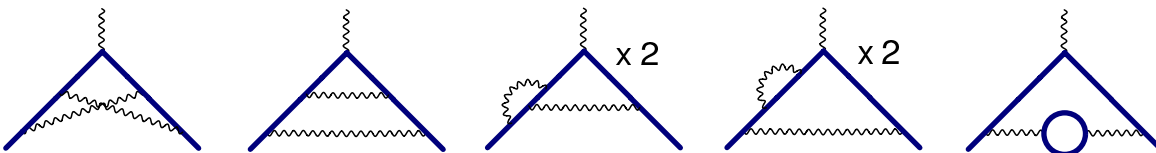
10<sup>th</sup>  
12672  
diagrams



- code-generating code, including renormalisation
- multi-dim. numerical integrations

# $a_\mu^{\text{QED}}$

- **Schwinger 1948:** 1-loop  $a = (g-2)/2 = \alpha/(2\pi) = 116\,140\,970 \times 10^{-11}$

- 2-loop graphs: 

- 72 3-loop and 891 4-loop diagrams ...

- **Kinoshita et al. 2012:** 5-loop completed numerically (12672 diagrams):

$$a_\mu^{\text{QED}} = 116\,584\,718.951\, (0.009)\, (0.019)\, (0.007)\, (0.077) \times 10^{-11}$$

errors from: lepton masses, 4-loop, 5-loop,  $\alpha$  from  $^{87}\text{Rb}$

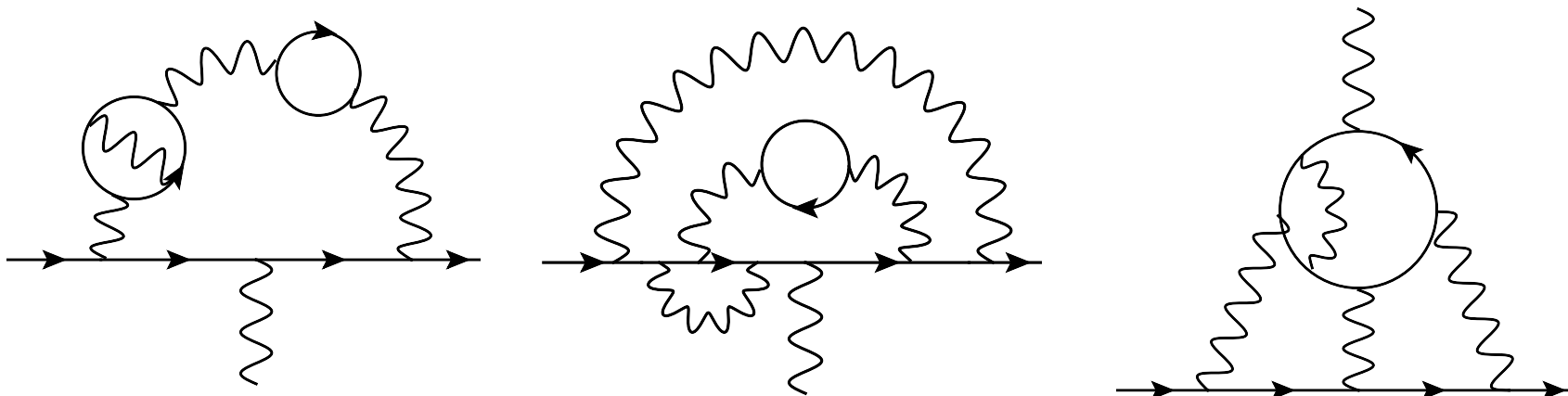
- QED extremely accurate, and the series is stable:  $a_\mu^{\text{QED}} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$

$$C_\mu^{2,4,6,8,10} = 0.5, 0.765857425(17), 24.05050996(32), 130.8796(63), 753.29(1.04)$$

- Could  $a_\mu^{\text{QED}}$  still be wrong?

Some classes of graphs known analytically ([Laporta](#); [Aguilar](#), [Greynat](#), [deRafael](#)),

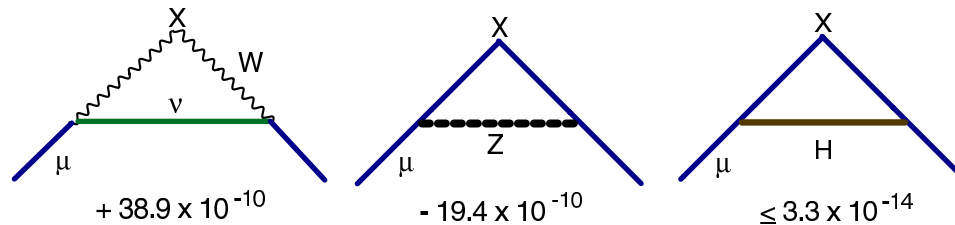
- ... but 4-loop and 5-loop rely heavily on numerical integrations
- Recently several independent checks of 4-loop and 5-loop diagrams:  
[Baikov, Maier, Marquard](#) [NPB 877 (2013) 647], [Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser](#) [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:
- all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., arXiv:160202785)

- ... and agree with Kinoshita et al.'s results
- remaining, not yet checked 4-loop universal (purely photonic) term is small, of the same order as the 5-loop contribution, and less than  $\frac{1}{4}$  of the discrepancy), so
- **QED on safe ground.**

- Electro-Weak 1-loop diagrams:



$$a_\mu^{\text{EW}(1)} = 195 \times 10^{-11}$$

- known to 2-loop (1650 diagrams, the first EW 2-loop calculation):  
Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

- agreement,  $a_\mu^{\text{EW}}$  relatively small, 2-loop relevant:  $a_\mu^{\text{EW}(1+2 \text{ loop})} = (154 \pm 2) \times 10^{-11}$

- Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,

PRD 88 (2013) 053005

$$a_\mu^{\text{EW}(1+2 \text{ loop})} = (153.6 \pm 1.0) \times 10^{-11} \quad \checkmark$$

compared with  $a_\mu^{\text{QED}} = 116\,584\,718.951(80) \times 10^{-11}$

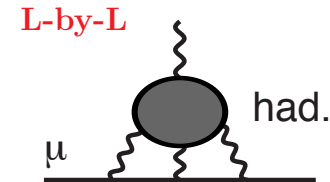
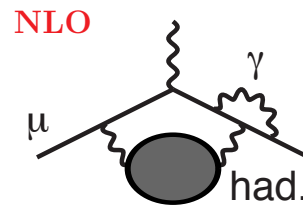
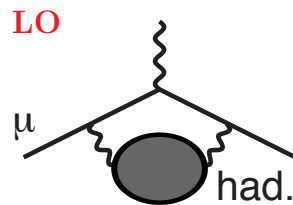


# $a_\mu^{\text{hadronic}}$

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

- QED: [Kinoshita et al. 2012](#): 5-loop completed (12672 diags) [some 4-l checks] ✓
- EW: 2-loop (and SM Higgs mass now known) ✓
- Hadronic: **non-perturbative, the limiting factor of the SM prediction** ✗

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

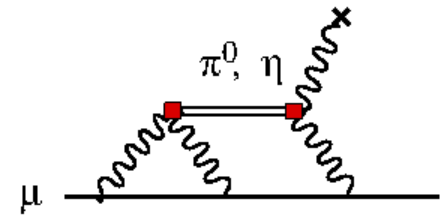


- L-by-L:**
- so far use of [model calculations](#), form-factor data will help improving,
  - also [lattice QCD](#), and
  - new [dispersive](#) approach

→ talks by Taku Izubuchi, Fu-Guang Cao, Christoph Redmer

# $a_{\mu}^{\text{had, L-by-L}}$ : Light-by-Light (I)

- **L-by-L**:  $\gamma \rightarrow \text{hadrons} \rightarrow \gamma^* \gamma^* \gamma^*$  non-perturbative, impossible to fully measure **X**
- so far use of **model calculations**, based on large  $N_c$  limit, Chiral Perturbation Theory, plus **short distance constraints** from OPE and pQCD
- **meson exchanges** and **loops** modified by form factor suppression, but with limited experimental information:
  - in principle off-shell form-factors ( $\pi^0, \eta, \eta', 2\pi \rightarrow \gamma^* \gamma^*$ ) needed
  - at most possible, directly experimentally:  $\pi^0, \eta, \eta', 2\pi \rightarrow \gamma \gamma^*$
- additional quark loop, pQCD matching; theory not fully satisfying conceptually ☹
- several independent evaluations, different in details, but **good agreement for the leading  $N_c$  ( $\pi^0$  exchange) contribution**, differences in sub-leading bits
- mostly used recently:
  - 'Glasgow consensus' by Prades+deRafael+Vainshtein:  
$$a_{\mu}^{\text{had, L-by-L}} = (105 \pm 26) \times 10^{-11}$$
  - compatible with Nyffeler's  $a_{\mu}^{\text{had, L-by-L}} = (116 \pm 39) \times 10^{-11}$



# $a_\mu^{\text{had, L-by-L}}$ : Overview from A Nyffeler @ Frascati 2016

## HLbL scattering: Summary of selected results for $a_\mu^{\text{HLbL}} \times 10^{11}$

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
$\pi^0, \eta, \eta'$	$85 \pm 13$	$82.7 \pm 6.4$	$83 \pm 12$	$114 \pm 10$	—	$114 \pm 13$	$99 \pm 16$
axial vectors	$2.5 \pm 1.0$	$1.7 \pm 1.7$	—	$22 \pm 5$	—	$15 \pm 10$	$22 \pm 5$
scalars	$-6.8 \pm 2.0$	—	—	—	—	$-7 \pm 7$	$-7 \pm 2$
$\pi, K$ loops	$-19 \pm 13$	$-4.5 \pm 8.1$	—	—	—	$-19 \pm 19$	$-19 \pm 13$
$\pi, K$ loops +subl. $N_C$	—	—	—	$0 \pm 10$	—	—	—
quark loops	$21 \pm 3$	$9.7 \pm 11.1$	—	—	—	2.3 (c-quark)	$21 \pm 3$
Total	$83 \pm 32$	$89.6 \pm 15.4$	$80 \pm 40$	$136 \pm 25$	$110 \pm 40$	$105 \pm 26$	$116 \pm 39$

BPP = Bijns, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijns, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

- **Pseudoscalar-exchanges dominate numerically.** Other contributions not negligible. **Cancellation** between  $\pi, K$ -loops and quark loops !
- Note that recent reevaluations of axial vector contribution lead to much smaller estimates than in MV:  $a_\mu^{\text{HLbL}; \text{axial}} = (8 \pm 3) \times 10^{-11}$  (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). This would shift central values of compilations downwards:  $a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$  (PdRV) and  $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$  (N, JN).
- **PdRV:** Analyzed results obtained by different groups with various models and suggested new estimates for some contributions (shifted central values, enlarged errors). **Do not consider dressed light quark loops as separate contribution. Added all errors in quadrature !**
- **N, JN:** **New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors.** Took over most values from BPP, except axial vectors from MV. **Added all errors linearly.**

# $a_\mu^{\text{had, L-by-L}}$ : Light-by-Light (III): Prospects

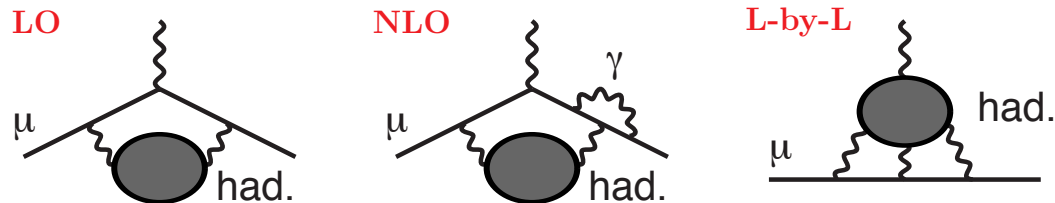
- Transition FFs can be measured by **KLOE-2** and **BESIII** using small angle taggers:  
 $e^+e^- \rightarrow e^+e^- \gamma \gamma^* \rightarrow \pi^0, \eta, \eta', 2\pi$  expected to constrain leading pole contributions from  $\pi, \eta, \eta'$  to  $\sim 15\%$  Nyffeler, arXiv:1602.03398
- or calculate on the lattice:  $\pi^0 \rightarrow \gamma^* \gamma^*$  Gerardin, Meyer, Nyffeler, arXiv:1607.08174
- New dispersive approaches promising Pauk, Vanderhaeghen, PRD 90 (2014) 113012  
Colangelo et al., see e.g. EPJ Web of Conf. 118 (2016) 01030
  - dispersion relations formulated for the general HLbL tensor or for  $a_\mu$  directly
  - allowing to constrain/calculate the HLbL contributions from data
  - e.g. Colangelo et al. have first results for the  $\pi$ -box contribution from data for  $F_V^\pi(q^2)$
- Ultimately: 'First principles' full prediction from **lattice QCD+QED**
  - several groups: **USQCD, UKQCD, ETMC, ...** much increased effort and resources
  - within 3-5 years a 10% estimate may be possible, 30% would already be useful
  - first results encouraging, proof of principle already exists, more news later...
- Conservative prediction: we will at least be able to defend/confirm the error estimate of the Glasgow consensus, and possibly bring it down significantly. ✓

# $a_\mu^{\text{had, VP}}$ : Hadronic Vacuum Polarisation

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

- QED: ✓
- EW: ✓
- Hadronic: **the limiting factor of the SM prediction** ✗

$$a_\mu^{\text{had}} = a_\mu^{\text{had, VP LO}} + a_\mu^{\text{had, VP NLO}} + a_\mu^{\text{had, Light-by-Light}}$$



**HVP**: - most precise prediction by **using  $e^+e^-$  hadronic cross section (+ tau) data** and well known **dispersion integral**

- done at LO and NLO (see graphs)

- now even at NNLO [Steinhauser et al., PLB 734 (2014) 144] ➔

- alternative: lattice QCD, but also need QED corrections; systematics <1% ?

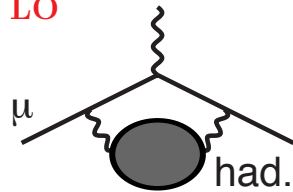
➔ next talk, by Bipasha Chakraborty

# $a_{\mu}^{\text{had, VP}}$ : Hadronic Vacuum Polarisation

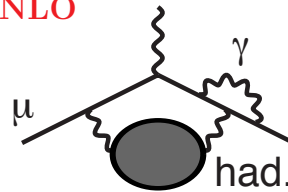
- HVP: still the largest error in the SM prediction ✗

$$a_{\mu}^{\text{had}} = a_{\mu}^{\text{had, VP LO}} + a_{\mu}^{\text{had, VP NLO}} + a_{\mu}^{\text{had, Light-by-Light}}$$

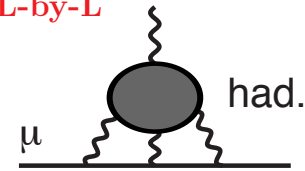
LO



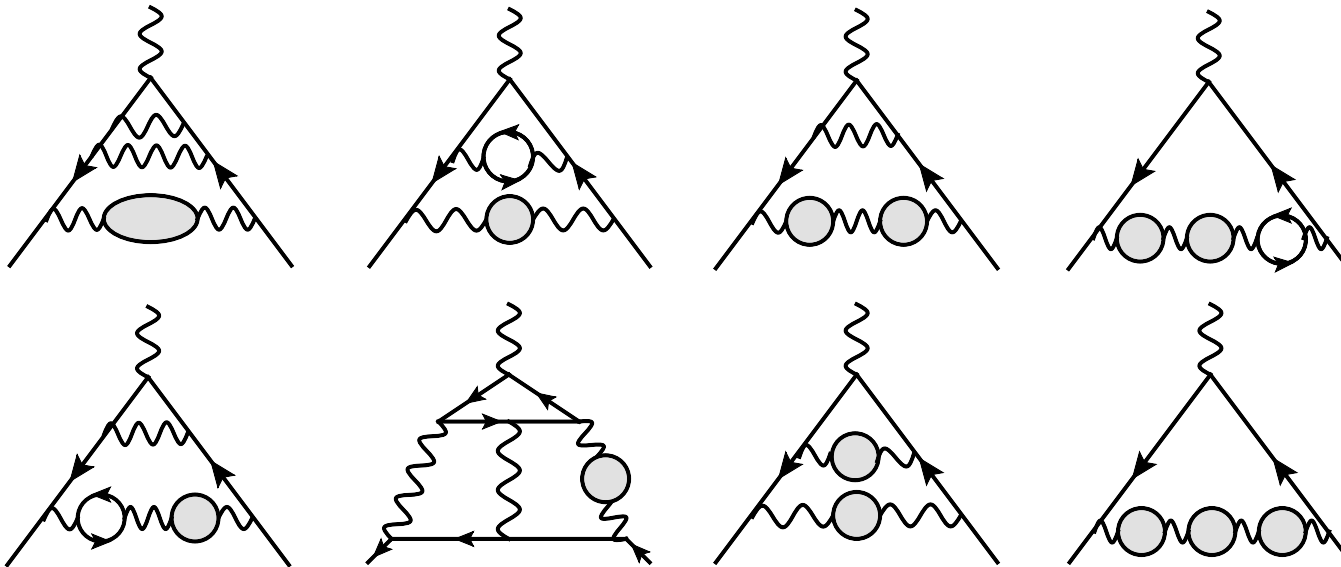
NLO



L-by-L

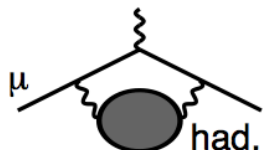


HVP at NNLO

 by Steinhauser et al.:  $a_{\mu}^{\text{HVP, NNLO}} = +1.24 \times 10^{-10}$  not so small


# Hadronic Vacuum Polarisation, essentials:

## Use of data compilation for HVP:



pQCD not useful. Use the **dispersion relation** and the **optical theorem**.

$$\text{had. bubble} = \int \frac{ds}{\pi(s-q^2)} \text{Im} \text{had. bubble}$$

$$2 \text{Im} \text{had. bubble} = \sum_{\text{had.}} \int d\Phi \left| \text{had. bubble} \right|^2$$

$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$

- Weight function  $\hat{K}(s)/s = \mathcal{O}(1)/s$   
 $\Rightarrow$  **Lower** energies **more important**  
 $\Rightarrow \pi^+\pi^-$  channel: 73% of total  $a_{\mu}^{\text{had,LO}}$

How to get the most precise  $\sigma_{\text{had}}^0$ ?  **$e^+e^-$  data:**

- Low energies: **sum ~ 25 exclusive channels**,  $2\pi, 3\pi, 4\pi, 5\pi, 6\pi, KK, KK\pi, KK\pi\pi, \eta\pi, \dots$ , use iso-spin relations for missing channels
- Above  $\sim 1.8$  GeV: can start to use **pQCD** (away from flavour thresholds), supplemented by narrow resonances ( $J/\psi, Y$ )
- Challenge of **data combination (locally in  $\sqrt{s}$ )**: many experiments, different energy bins, stat+sys errors from different sources, **correlations**; must avoid **inconsistencies/bias**
- traditional '**direct scan**' (tunable  $e^+e^-$  beams) vs. '**Radiative Return**' [+  $\tau$  spectral functions]
- $\sigma_{\text{had}}^0$  means 'bare'  $\sigma$ , but WITH FSR: **RadCorrs**  
 [ HLMNT '11:  $\delta a_{\mu}^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$  !]



# $a_\mu^{\text{SM}}$ : overview, numbers as of HLMNT '11

- Several groups have produced hadronic compilations over the years
  - At present 3–4  $\sigma$  discrepancy; HVP still dominates the SM error
  - Many more precise data in the meantime and more expected for near future
- ➔ for details/update/comparison, see M Davier's talk tomorrow

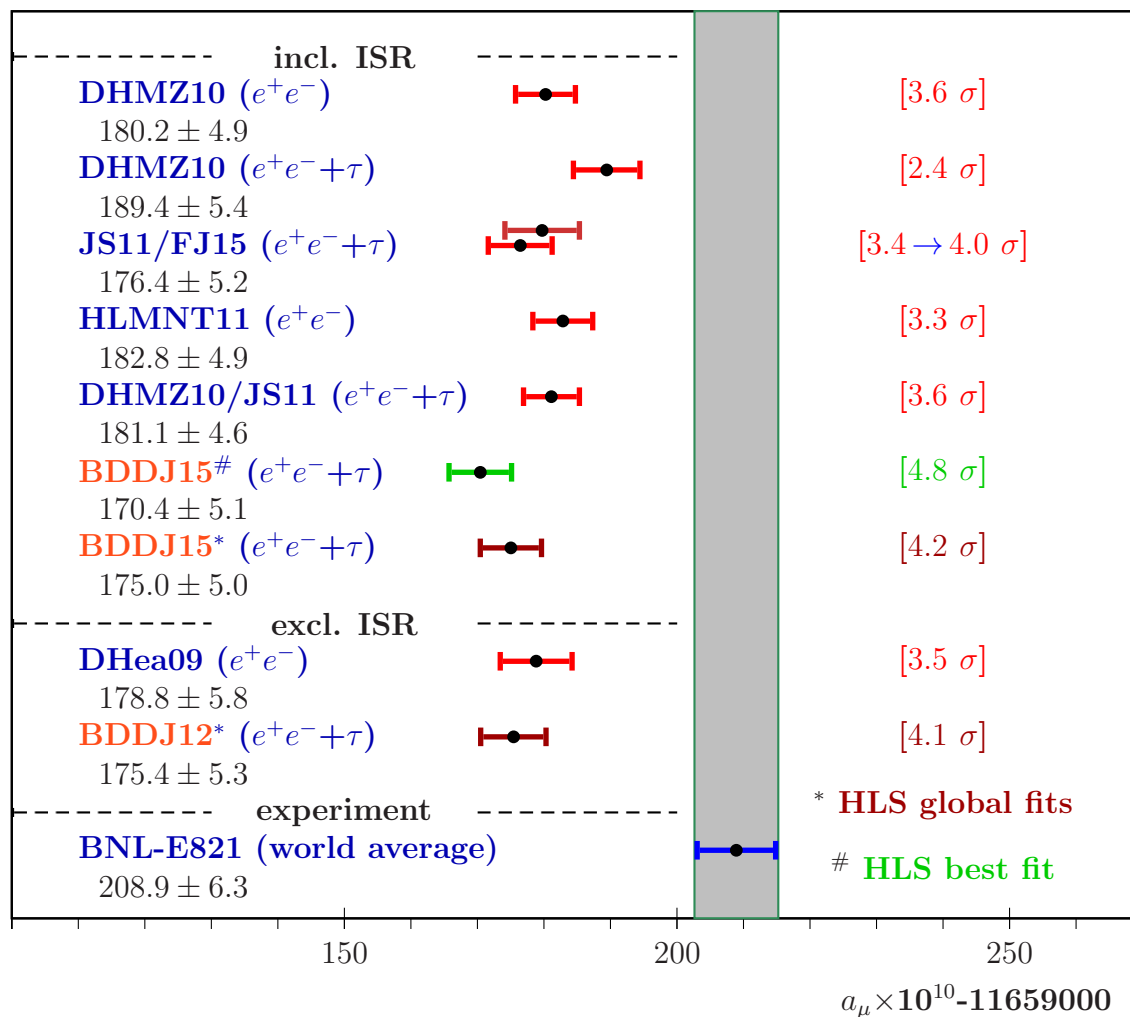
<b>QED</b> contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
<b>EW</b> contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
<b>Hadronic</b> contribution		
<b>LO</b> hadronic	694.9 (4.3) $\times 10^{-10}$	HLMNT11
<b>NLO</b> hadronic	−9.8 (0.1) $\times 10^{-10}$	HLMNT11
<b>light-by-light</b>	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
<b>Theory TOTAL</b>	11 659 182.8 (4.9) $\times 10^{-10}$	
<b>Experiment</b>	11 659 208.9 (6.3) $\times 10^{-10}$	world avg
<b>Exp – Theory</b>	26.1 (8.0) $\times 10^{-10}$	3.3 $\sigma$ discrepancy

(Numbers taken from HLMNT11, arXiv:1105.3149)

# $a_\mu^{\text{SM}}$ : overview, recent analyses

→ for more details see M Davier's talk tomorrow

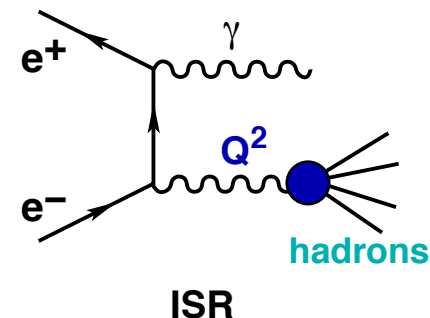
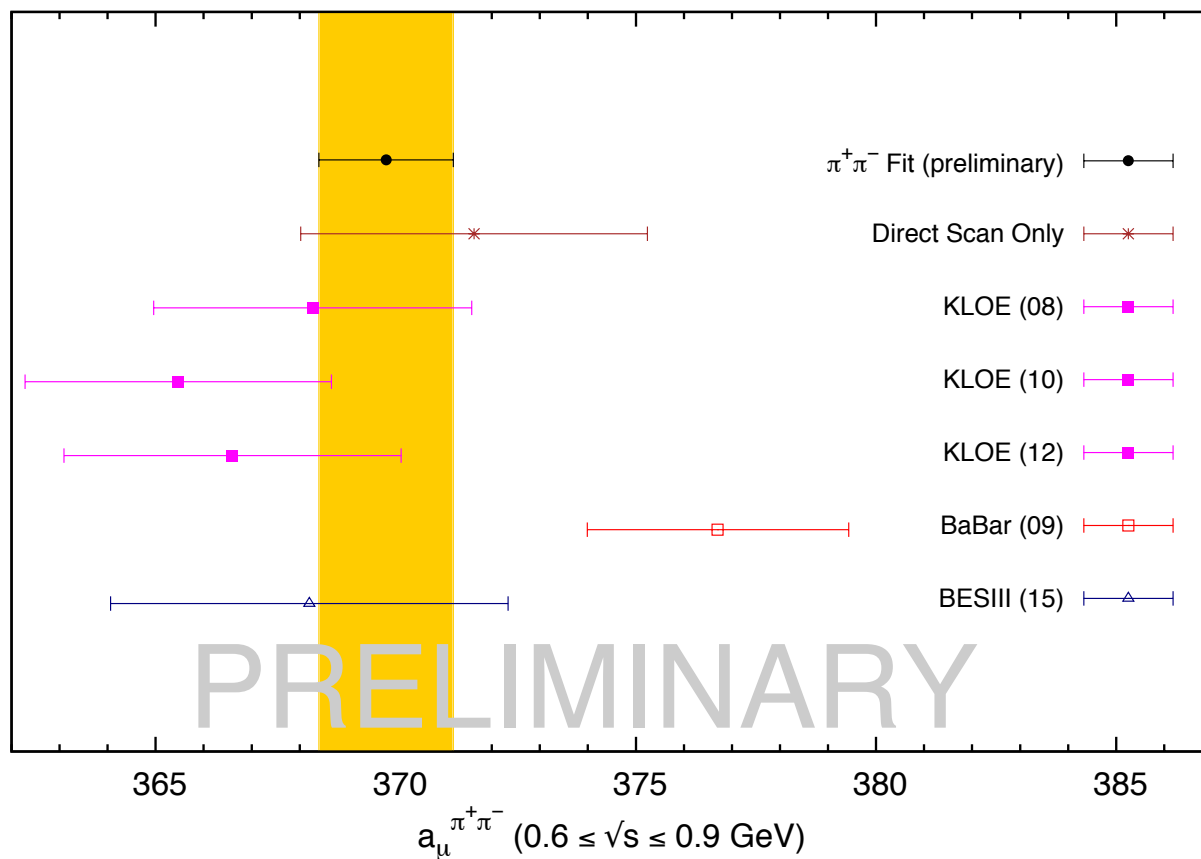
From Fred Jegerlehner's arXiv:1511.04473:



# HVP: HLMNT -> HKMNT in preparation

$\pi^+\pi^-$  channel: + KLOE12, + BES III from Rad. Ret.:

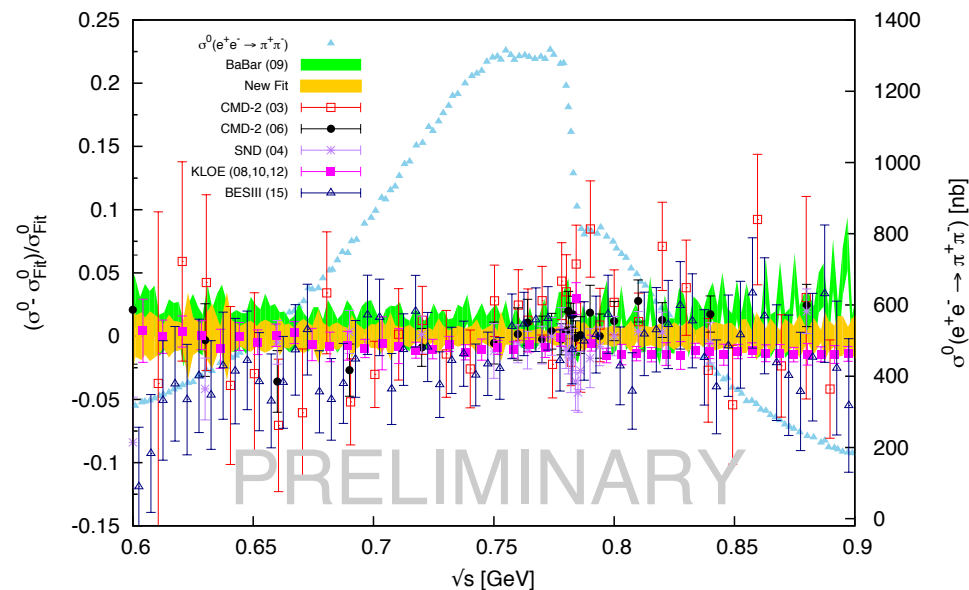
Prel. HKMNT combination w. full cov.-matrices:



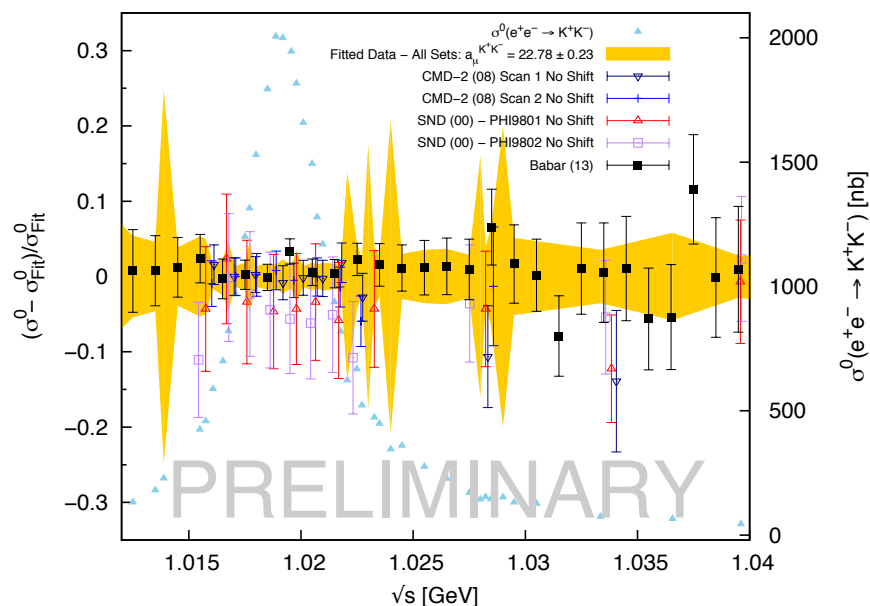
- $\chi^2_{\min}/\text{d.o.f.} = 1.4$
- further improvements expected from CMD-3, more also from BaBar?
- ➔ see Simon Eidelman's talk on CMD-3
- ➔ Yaquian Wang's talk on BES III  $\pi$  FF & ISR

# HVP: HLMNT -> HKMNT in preparation

## $\pi^+\pi^-$ channel



## $K^+K^-$ channel with recent BaBar



- Many new data sets and an improved combination algorithm, which takes fully into account all available covariance matrices, give significantly reduced errors and a slightly smaller mean value
- Previously sizeable additional (conservative) error from uncertainty in treatment of radiative corrections (VP + FSR), mainly from older data sets, gets reduced
- More exclusive data in multi-pion and K channels reduce uncertainty from estimate based on Iso-spin correlations

# Further improvements for $a_\mu^{\text{HVP}}$ :

## 1. Data input:

- Most important  $2\pi$ :
  - more from CMD-3 and BaBar
  - if discrepancy with BaBar persists, could direct scan & ISR be done in the same experiment?
- The 'subleading'  $3\pi$  (in resonance regions) and in particular  $\pi^+\pi^-\pi^0\pi^0$  need more & newer/final data
- Inclusive measurements from KEDR and BES-III at higher energies are/will be important
- Lattice simulation are becoming more and more competitive

## 2. Analysis techniques

- Refined treatment of errors and correlations make maximum use of the data
- MC studies for impact of FSR, VP refinements
- Global fits based on Hidden Local Symmetry (M. Benayoun et al.) bring in further constraints and lead to a smaller error and larger discrepancy
- Analyses based on HLS or using  $\rho$ - $\gamma$  mixing directly see no discrepancy between  $e^+e^-$  and  $\tau$  spectral function data, but gain from including  $\tau$

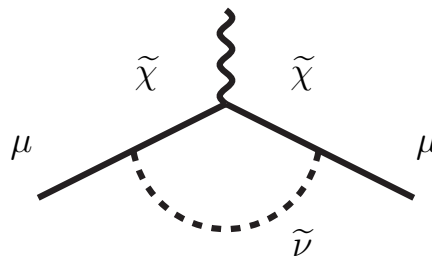
➔ I believe we can half the HVP error in time for the new g-2

# New Physics?

- Many BSM studies use  $g-2$  as constraint or even motivation

- SUSY could easily explain  $g-2$ :

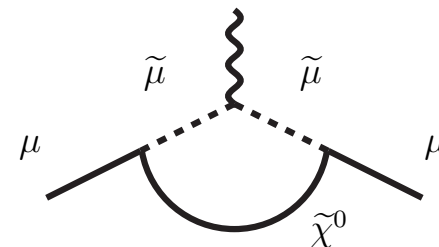
- Main 1-loop contributions:



- Simplest case:

$$a_{\mu}^{\text{SUSY}} \simeq \text{sgn}(\mu) 130 \times 10^{-11} \tan \beta \left( \frac{100 \text{ GeV}}{\Lambda_{\text{SUSY}}} \right)^2$$

- Needs  $\mu > 0$ , 'light' SUSY-scale  $\Lambda$  and/or large  $\tan \beta$  to explain  $260 \times 10^{-11}$
- This is already excluded by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large  $\chi^2$  in simultaneous SUSY-fits with LHC data and  $g-2$
- However note: SUSY does not have to be minimal (w.r.t. Higgs), could have large mass splittings (with lighter sleptons), or corrections (to  $g-2$  and Higgs mass) different from simple models, or not be there at all, but don't write it off yet...



# New Physics? just five of many other recent examples

- Don't need full MSSM (like coded in GM2Calc [by Athron, ..., Stoeckinger et al., EPJC 76 (2016) 62], which includes all latest two-loop contributions), an
  - **extended Higgs sector** could do, see, e.g. Stoeckinger et al., arXiv:160706292, 'The muon magnetic moment in the 2HDM: complete two-loop result'
- lesson: 2-loop contributions can be highly relevant in both cases; a one-loop analysis can be misleading
- **1 TeV Leptoquark** Bauer + Neubert, PRL 116 (2016) 14, 141802
    - one new scalar could explain several anomalies seen by BaBar and LHC in the flavour sector (violation of lepton universality in  $B \rightarrow K\ell\ell$ , enhanced  $B \rightarrow D\tau\nu$ ) and solve  $g-2$ , while satisfying all bounds from LEP and LHC
  - **light  $Z'$**  can evade many searches involving electrons by non-standard couplings preferring heavy leptons (but see BaBar's arXiv:1606.03501 direct search limits in a wide mass range), or invoke flavour off-diagonal  $Z'$  to evade constraints [Altmannshofer et al., arXiv:1607.06832]
  - **'dark photon'-like fifth force particle** [Feng et al., PRL 117 (2016) 7, 071803],
  - or **axion-like particle (ALP)**, contributing like  $\pi^0$  in HLbL [Marciano et al., arXiv:1607.01022]



# Conclusions/Outlook:

- All sectors of the Standard Model prediction of  $g-2$  have been scrutinised a lot in recent years
- The basic picture has not changed, but recent data, many from IRS, significantly improve the prediction for  $\Delta a_\mu^{\text{HVP}}$ , and a
- discrepancy  $> 3\sigma$  is firmly consolidated
- With further anticipated hadronic data, also on FF for HLbL, and with [efforts from lattice](#), the goal of halving  $\Delta a_\mu^{\text{SM}}$ , to stay competitive with the new  $g-2$  experiments, is in reach
- Many approaches to explain the discrepancy with NP, linking  $g-2$  with other precision observables, the flavour sector, dark matter, and many direct searches, [but where is the NP?](#)



# Aside I: Hadronic VP for running $\alpha(q^2)$

- Dyson summation of Real part of one-particle irreducible blobs  $\Pi$  into the effective, real running coupling  $\alpha_{\text{QED}}$ :

$$\Pi = \text{diagram: a wavy line with a } \gamma^* \text{ label and momentum } q \text{ enters a shaded oval blob, and another wavy line exits it.}$$

Full photon propagator  $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \dots$

$$\rightsquigarrow \alpha(q^2) = \frac{\alpha}{1 - \text{Re}\Pi(q^2)} = \alpha / (1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2))$$

- The Real part of the VP,  $\text{Re}\Pi$ , is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section,  $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow \text{hadrons})$ :

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha} \text{P} \int_{m_\pi^2}^{\infty} \frac{\sigma_{\text{had}}^0(s) ds}{s - q^2}, \quad \sigma_{\text{had}}(s) = \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi|^2}$$

[ $\rightarrow \sigma^0$  requires ‘undressing’, e.g. via  $\cdot(\alpha/\alpha(s))^2 \rightsquigarrow$  iteration needed]

- Observable cross sections  $\sigma_{\text{had}}$  contain the |full photon propagator|<sup>2</sup>, i.e. |infinite sum|<sup>2</sup>.  
 $\rightarrow$  To include the subleading Imaginary part, use dressing factor  $\frac{1}{|1 - \Pi|^2}$ .

## Aside II: Lepton EDMs and MDMS: $d_\mu$ vs. $a_\mu$

- One more reason to push for best possible muon EDM measurement:  
 $\mu$ EDM could in principle fake muon AMM ‘The g-2 anomaly isn’t’ (Feng et al. 2001)

$$\vec{\omega} = \vec{\omega}_a + \vec{\omega}_\eta$$



$$\omega = \sqrt{\vec{\omega}_a^2 + \vec{\omega}_\eta^2}$$

- Less room than there was before E821 improved the limit, **still want to measure**

