Meson-photon Transition Form Factors and the Evaluation of Muon g-2

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1. Introduction – hadronic LxL contribution
2. Meson-photon TFFs
   - Predictions from QCD and light-front holographic QCD
   - Experimental results (BaBar and Belle)
   - Meson-virtual-photon TFFs – parameterization vs QCD
3. Evaluation of hadronic LxL contribution
4. Summary

1. Hadronic LxL contribution to muon g-2

- The discrepancy is about 3 sigma. $a_\mu = (g-2)/2$

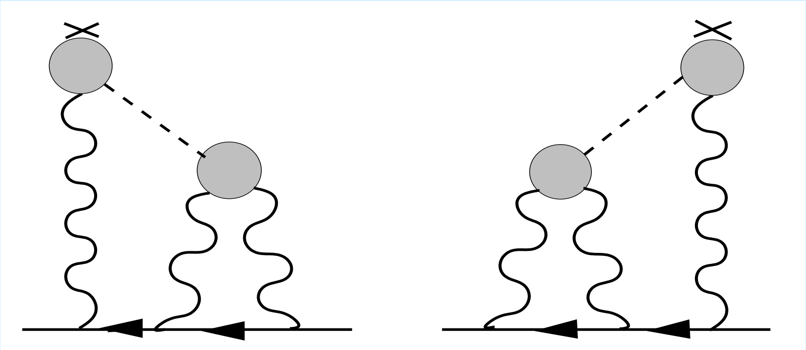
SM Contribution	Value ± Error	Ref
QED (incl. 5-loops)	116584718.951 ± 0.080	[3]
HVP LO	6949 ± 43	[4]
HVP NLO	-98.4 ± 0.7	[4, 5]
HVP NNLO	12.4 ± 0.1	[5]
HLbL	105 ± 26	[6]
Weak (incl. 2-loops)	153.6 ± 1.0	[7]
SM Total (0.51 ppm)	116591840 ± 59	[3]
Experiment (0.54 ppm)	116592089 ± 63	[2]
Difference (Exp – SM)	249 ± 87	[3]

T. Blum et al, arXiv:1510.071 [hep-lat]

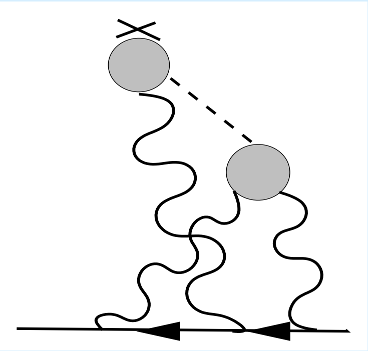
- Anomalous magnetic moment
- In the unit of 10^{-11}
- QED and week interaction contributions are well understood.
- Main source of uncertainties: HVP and HLxL

1. Hadronic LxL contribution to muon g-2

- HLxL contributions can be calculated with the help of meon-photon-photon transitions form factor (TFF).



T1



T2

$$a_\mu^{\mathrm{HLxL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3\, Q_2^3 \sum_{i=1}^{2} T_i(Q_1,Q_2,\tau)\,\Pi_i(Q_1,Q_2,\tau)$$

q_i (in Minkowski space) $\rightarrow$ Wick rotation $\rightarrow$ Q_i (in Euclidean space)

τ is the cosine of the angle between the Eucliden momenta Q_1 and Q_2.

T_i integral kernels $\qquad Q_i^2 = -q_i^2$

G. Colangelo et al, JHEP 09 (2015) 074

1. Hadronic LxL contribution to muon g-2

$$\Pi_1(Q_1,Q_2,\tau)=\frac{1}{s-M_\pi^2}\,F_{\pi\gamma^*\gamma^*}\left(-Q_1^2,-Q_2^2\right)\,F_{\pi\gamma\,\gamma^*}\left(-Q_3^2,0\right)$$

$$\Pi_2(Q_1,Q_2,\tau)=\frac{1}{t-M_\pi^2}\,F_{\pi\gamma^*\gamma^*}\left(-Q_1^2,-Q_3^2\right)\,F_{\pi\gamma\,\gamma^*}\left(-Q_2^2,0\right)$$

$$s=-Q_3^2=-\left(Q_1^2+2Q_1Q_2\tau+Q_2^2\right),\quad t=-Q_2^2$$

- Have data for pion-real-photon TFF but not for the pion-virtual-photon TFF

1. Hadronic LxL contribution to muon g-2

- Various parameterizations of TFF have been used in pervious calculations.

VMD: $$F_{\pi\gamma^*\gamma^*}\left(Q_1^2,Q_2^2\right)=\frac{1}{4\pi^2 f_\pi}\frac{M_V^2}{Q_1^2+M_V^2}\frac{M_V^2}{Q_2^2+M_V^2}$$

Does not have the correct large Q behavior!

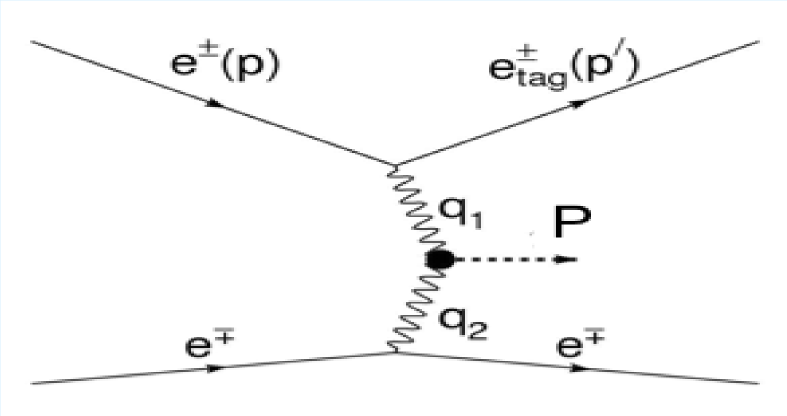
LMD: $$F_{\pi\gamma^*\gamma^*}\left(Q_1^2,Q_2^2\right)=\frac{f_\pi}{3}\frac{Q_1^2+Q_2^2+\dfrac{M_V^4}{4\pi^2 f_\pi^2}}{\left(Q_1^2+M_V^2\right)\left(Q_2^2+M_V^2\right)}$$

Does not reproduce the pion-real-photon TFF!

- Will use QCD calculations for those TFFs

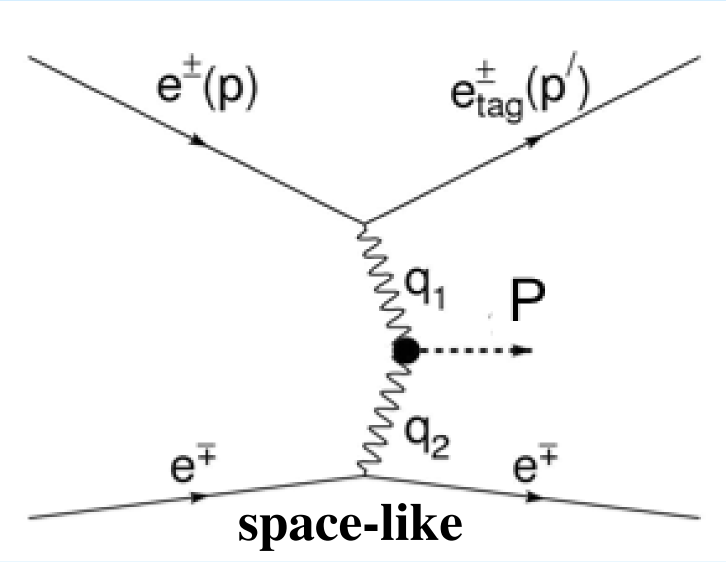
2. Meson-photon TFFs

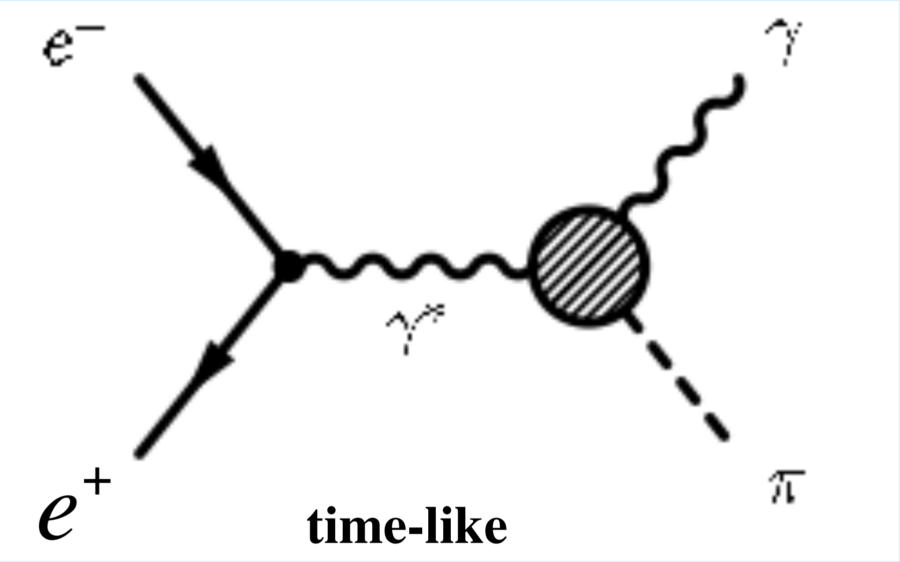
Two-photon reactions in electron-electron collisions



The simplest bound-state process in QCD

- Electrons are scattered predominantly at small angles
- For pseudoscalar meson production (P = π^0, η, η', etc) the cross section depends on only one form factor $F(q_1^2, q_2^2)$
- $q_1^2, q_2^2 \approx 0$, $\Gamma_{\gamma\gamma}$ is measured





- Single-tag mode

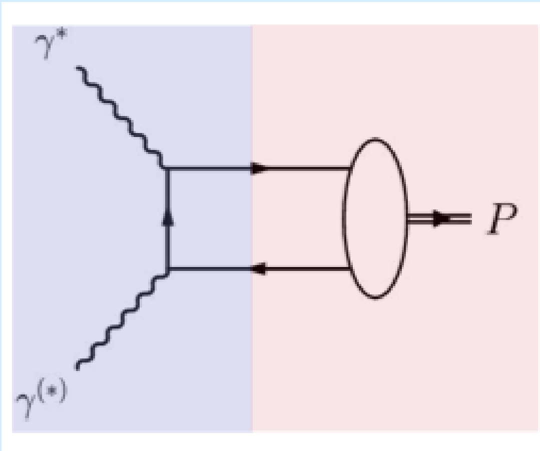
  $Q^2 = -q_1^2, q_2^2 \approx 0,$ one electron is detected and $F(Q^2)$ is measured
- Data from TPC/2γ(90), CELLO(91),

  CLEO(95,98), L3(97), BaBar(09,10), Belle (12)
- time-like=space-like (at $s = Q^2$) to LO

Pre-2009

- Theoretical foundations for exclusive processes
  - G. P. Lepage and S. J. Bordsky, Phys Rev 22 (1980) 2157
  - A. V. Efremov, and A. V. Radyushkin, Phys Lett B 94 (1980) 245
- Questions/Issues
  - Applicability of pQCD to exclusive processes
  - What form for the pion distribution amplitude (DA)
- Answers/Solutions:
  - Considering transverse momentum effects
  - Not well determined

QCD predictions



$$Q^2 F_{\pi\gamma}\left(Q^2\right) = \int_0^1 dx\, T_H(x,\mu)\,\phi\left(x,\mu\right)$$

T_H – hard scattering amplitude for $\gamma\gamma^* \to q\bar{q}$ transition

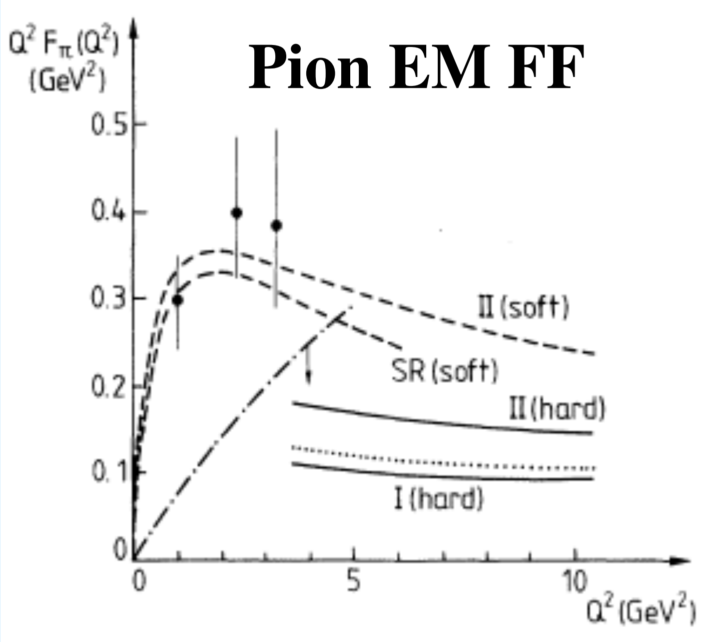
ϕ – pion distribution amplitude describing transition $\pi \to \mathrm{q\bar{q}}$

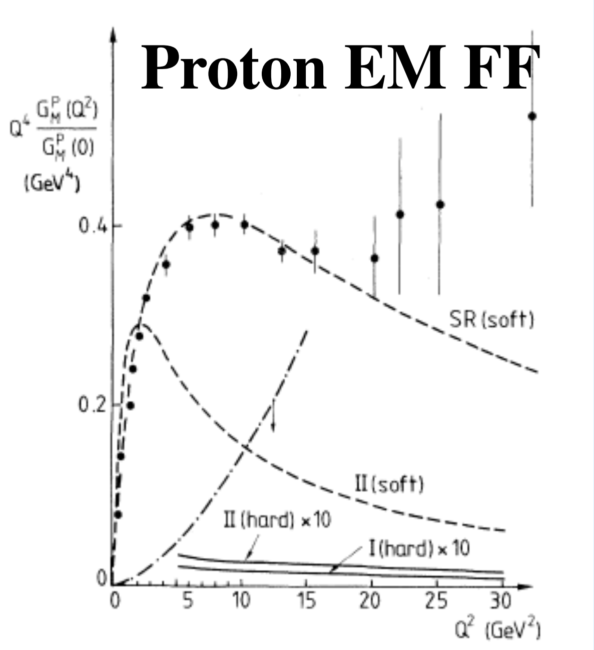
x is the fraction of the meson momentum carried by one of the quarks.

Data can be used to test phenomenological models for the meson DA.

Theoretical challenges: estimation of end-point contributions

- N. Isgur and C. H. Smith, PRL 52 (1984) 1080
- N. Isgur and C. H. Smith, NPB317 (1989) 526
- N. Isgur and C. H. Smith, PLB 52 (1989) 535





Is pQCD applicable to exclusive processes?

Solutions

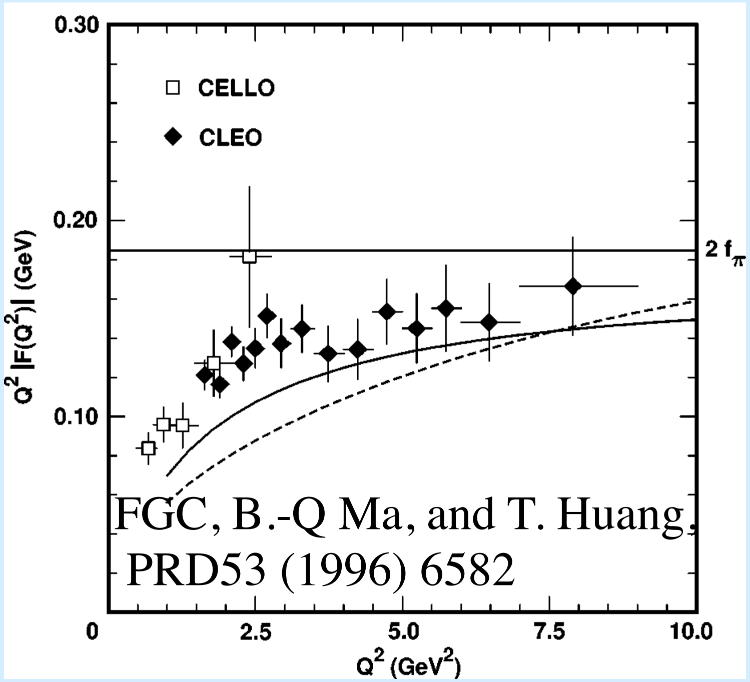
- Transverse momentum cut-off; pion EM FF

  T. Huang, Q.-X. Shen, ZPC 50 (1991) 139
- Sudakov suppression in b-space; covariant pQCD; pion EM FF
  - H.-N. Li and G. Sterman, NPB381 (1992) 129
  - FGC and T. Huang, PRD 52 (1995) 5358
- Transverse momentum effects; pion TFF
  - Sudakov suppression, R. Jakob et al, JPG 22 (1996) 22
  - LC pQCD, FGC, T. Huang and B. Q. Ma, PRD53 (1996) 6582

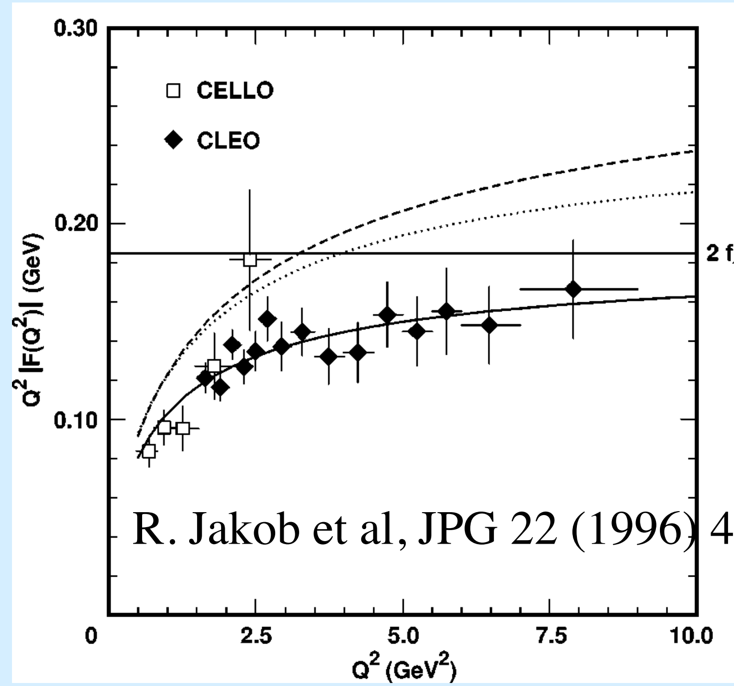
"We reanalyse the pionic form factor by using perturbative QCD theory and contributions from endpoint regions. We find that the perturbative QCD can be applied to the pionic form factor as $Q^2 > 4$ GeV^2 and they become unreliable as $Q^2 \leqq 4$ GeV^2. Therefore the applicability of perturbative QCD to the form factor is questionable only as $Q^2 \leqq 4$ GeV^2."

[ZPC 50 (1991) 139]

Pion-photon transition form factor



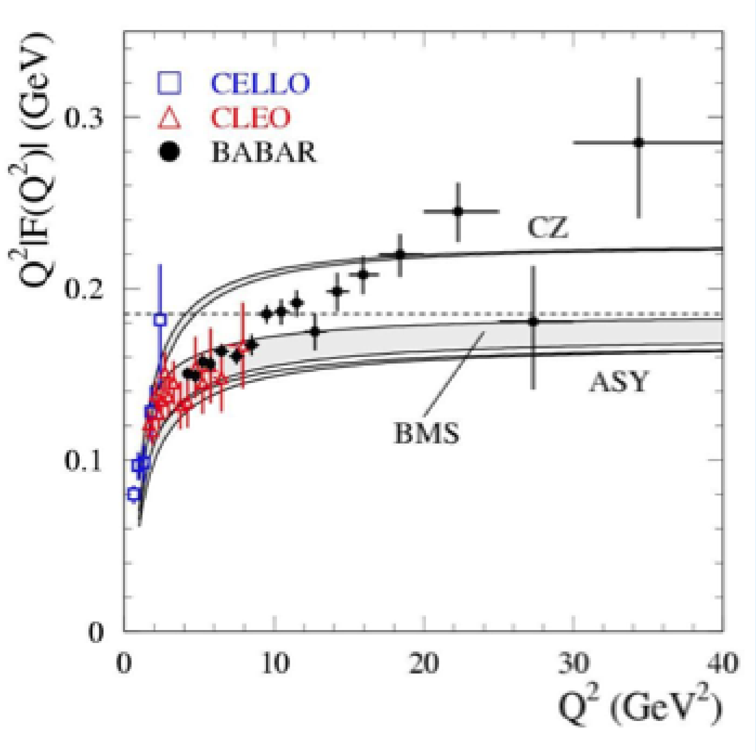
FGC, B.-Q Ma, and T. Huang, PRD53 (1996) 6582



R. Jakob et al, JPG 22 (1996) 45

- It seems ok to apply pQCD to exclusive processes.
- The pion DA is not well determined via only pion TFF.
- pQCD has been applied to many other exclusive processes.

BaBar surprise and ways out



The data show a rapid growth with Q^2, which is a surprise and hard to explain.

[Phys. Rev. D 80 (2009) 052002]

Ways out

- Extraordinary form for the pion DA: flat form
- Determining the coefficients for the pion DA
- Examining corrections to pQCD calculations
- Non-perturbative calculations: sum rules, AdS/QCD

Models suggested for the pion distribution amplitude

(a) Asymptotic form

$$\phi^{AS}(x,\mu_0) = \sqrt{3} f_\pi x(1-x);$$

(b) AdS/QCD form

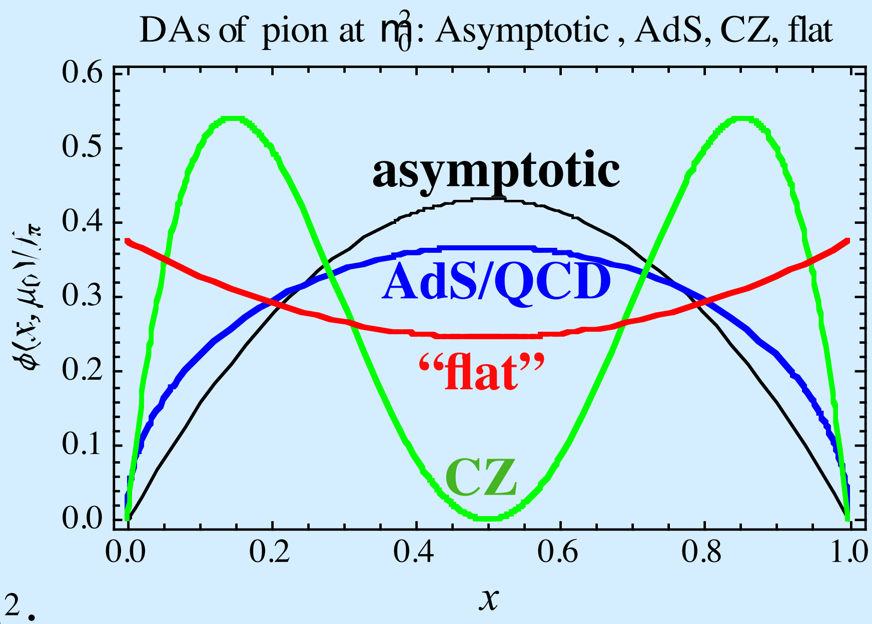
$$\phi^{AdS}(x,\mu_0) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)};$$

(c) Chernyak-Zhitnitsky form

$$\phi^{CZ}(x,\mu_0) = 5\sqrt{3} f_\pi x(1-x)(1-2x)^2;$$

(d) "Flat" form

$$\phi^{flat}(x,\mu_0) = \frac{f_\pi}{2\sqrt{3}}\left[N + 6(1-N)x(1-x)\right].$$



QCD prediction: leading-order results

Construction of the pion wave function

BHL prescription 1980

$$\psi_{q\bar{q}/\pi}^{\text{soft}}\left(x,k_\perp^2\right)=\phi(x)\frac{8\pi}{\kappa^2}\frac{1}{x(1-x)}\exp\left(-\frac{k_\perp^2}{2\kappa^2 x(1-x)}\right)$$

$$\phi\left(x,Q\right)=\int_0^{Q^2}\frac{d^2k_\perp^2}{8\pi^2}\psi_{q\bar{q}/\pi}^{\text{soft}}\left(x,k_\perp^2\right)$$

Soft evolution

$$\phi\left(x,Q\right)=\phi\left(x\right)\left[1-\exp\left(-\frac{Q^2}{2\kappa^2 x(1-x)}\right)\right]$$

Hard evolution is governed by the ERBL equation.

QCD prediction

Leading order $$Q^2F_{\pi\gamma}\left(Q^2\right)=\frac{4}{\sqrt{3}}\int_0^1 dx\frac{\phi(x,\bar{x}Q)}{\bar{x}}\left[1-\exp\left(\frac{\bar{x}Q}{2\kappa^2 x}\right)\right]$$

LB result in 80' $$Q^2F_{\pi\gamma}\left(Q^2\right)=\frac{4}{\sqrt{3}}\int_0^1 dx\frac{\phi\left(x,Q\right)}{\bar{x}}\left[1+O\left(\alpha_s,\frac{m_q^2}{Q^2}\right)\right]$$

The behavior at the asymptotic limit $Q^2\rightarrow\infty$

is well predicted, $Q^2F_{\pi\gamma}(Q^2\rightarrow\infty)=2f_\pi$.

Corrections

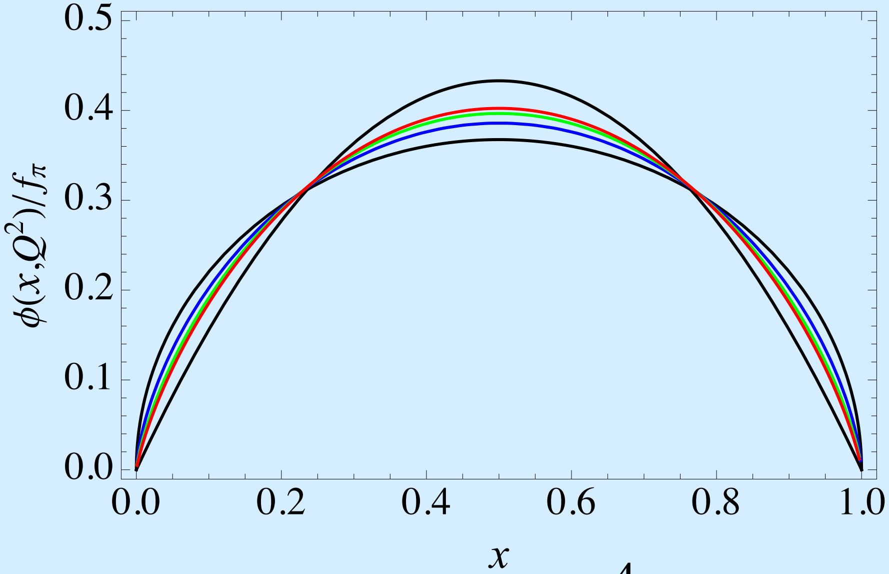
i. Replacement of $\phi(x,\bar{x}Q)$ with $\phi(x,Q)$ and evolution effect

ii. Higher order contributions

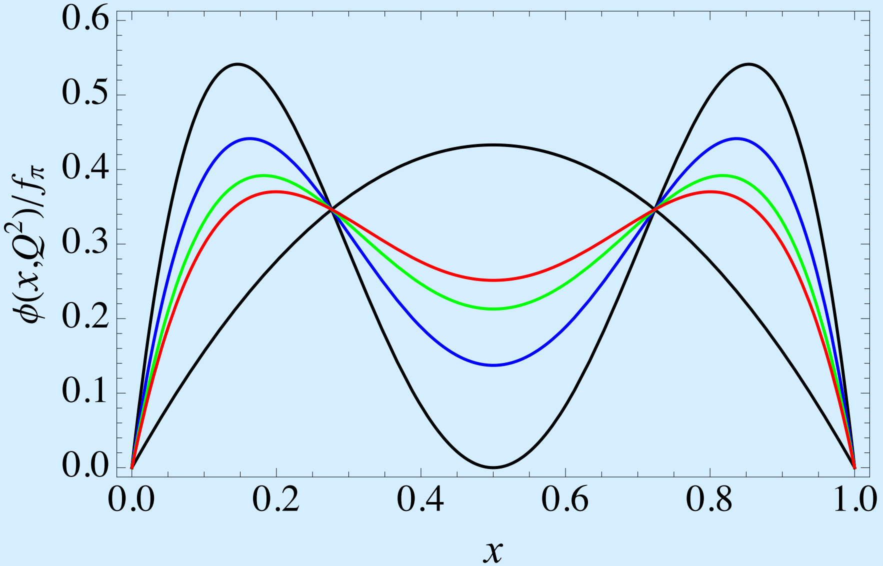
iii. Higher twist contributions

"Evolved QCD predictions for the meson-photon transition form factors", S. J. Brodsky, F.-G. Cao, and G. F. de Teramond, PRD 84 (2011) 033001.

Evolution of pion DA



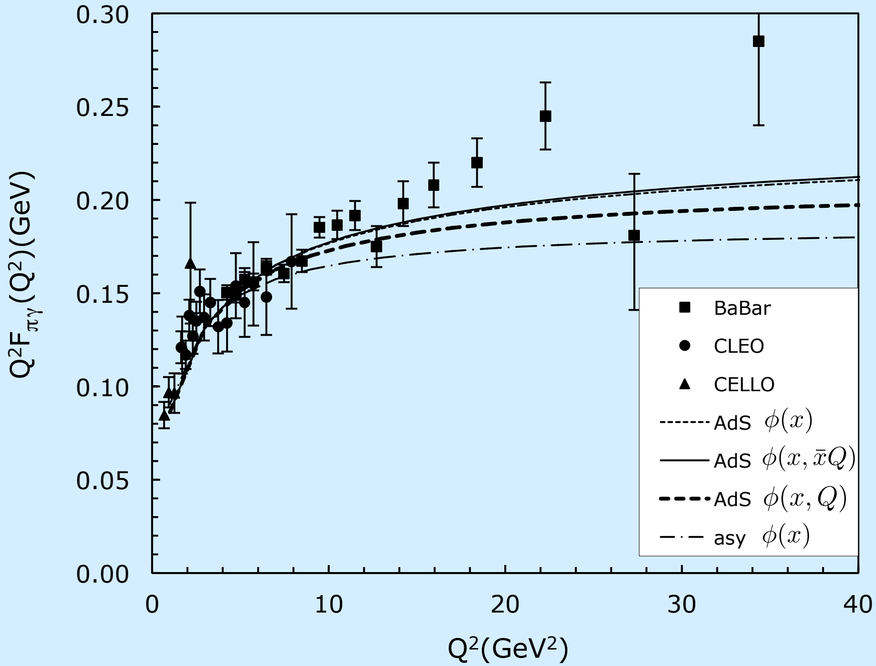
AdS/QCD form $\phi(x,\mu_0) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$



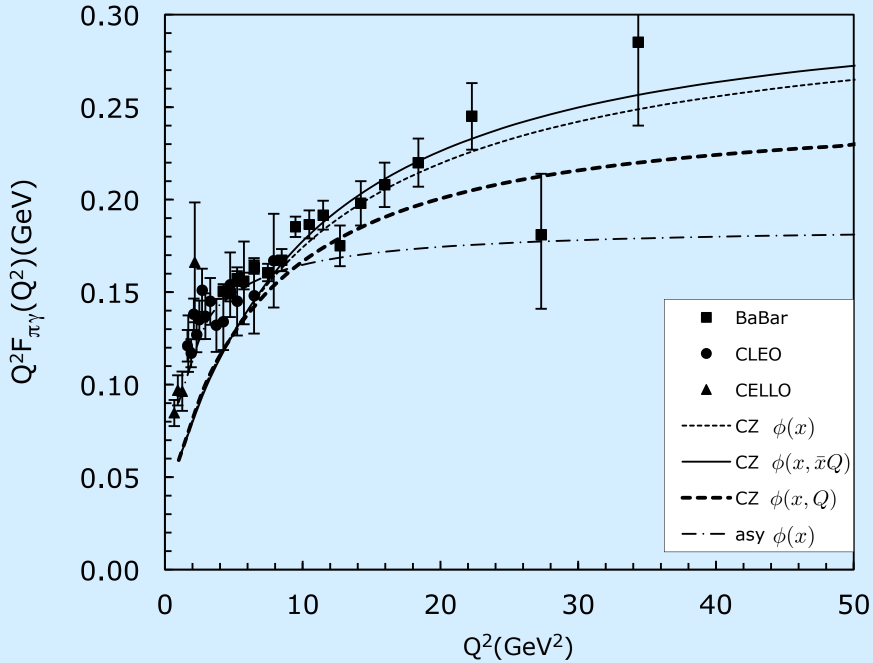
CZ form $\phi(x,\mu_0) = 5\sqrt{3} f_\pi x(1-x)(1-2x)^2$

$Q^2 = 1, 10, 100, 1000$ GeV^2, and asymptotic DA

Replacing $\phi(x,\bar{x}Q)$ with $\phi(x,Q)$ brings very small changes.

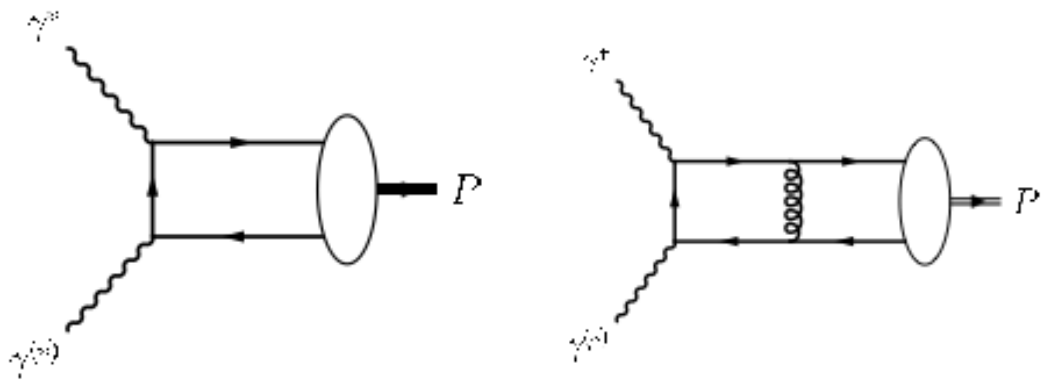


AdS model



CZ model

QCD corrections: NLO contributions



$$F_{\pi\gamma}(Q^2) \;=\; \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx\,\Phi_\pi(x,\mu_F)\,T_H^{\rm NLO}(x,Q^2,\mu_R) \qquad \text{for } \; Q^2\to\infty$$

$$T_H^{\rm NLO} \;=\; \frac{1}{xQ^2}\left\{1+C_F\frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{1}{2}\,\ln^2 x-\frac{x\ln x}{2(1-x)}-\frac{9}{2}+\left(\frac{3}{2}+\ln x\right)\ln\frac{Q^2}{\mu_R^2}\right]\right\}$$

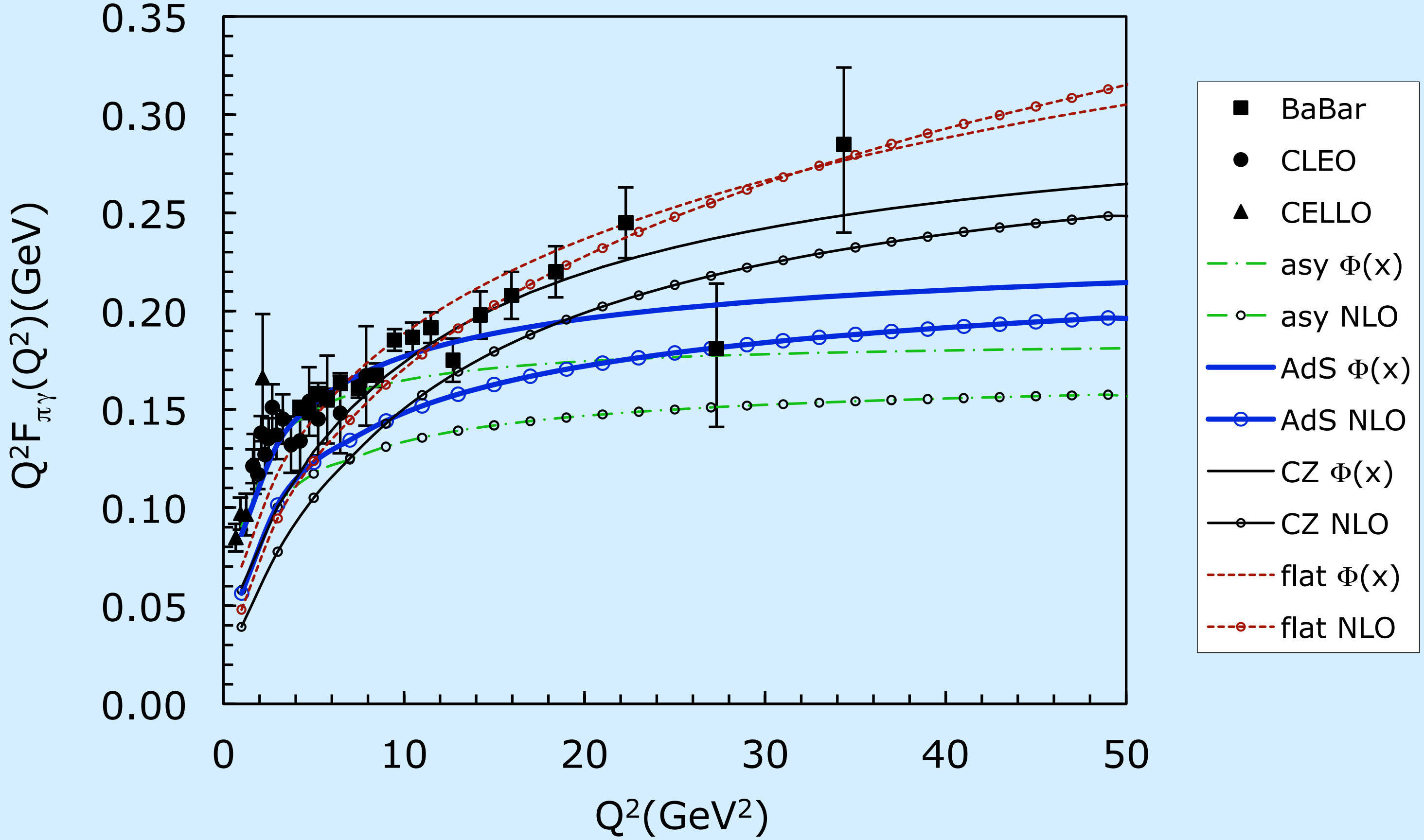
$$\Phi_\pi(x,\mu_F) \;=\; 6x(1-x)\left[1+\sum_{n=2,4,\ldots} a_n(\mu_0)\left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right)^{\gamma_n/\beta_0} C_n^{3/2}(2x-1)\right]$$

f_π pion decay constant; μ_F,μ_R,μ_0 factorization, renormalization, initial scale

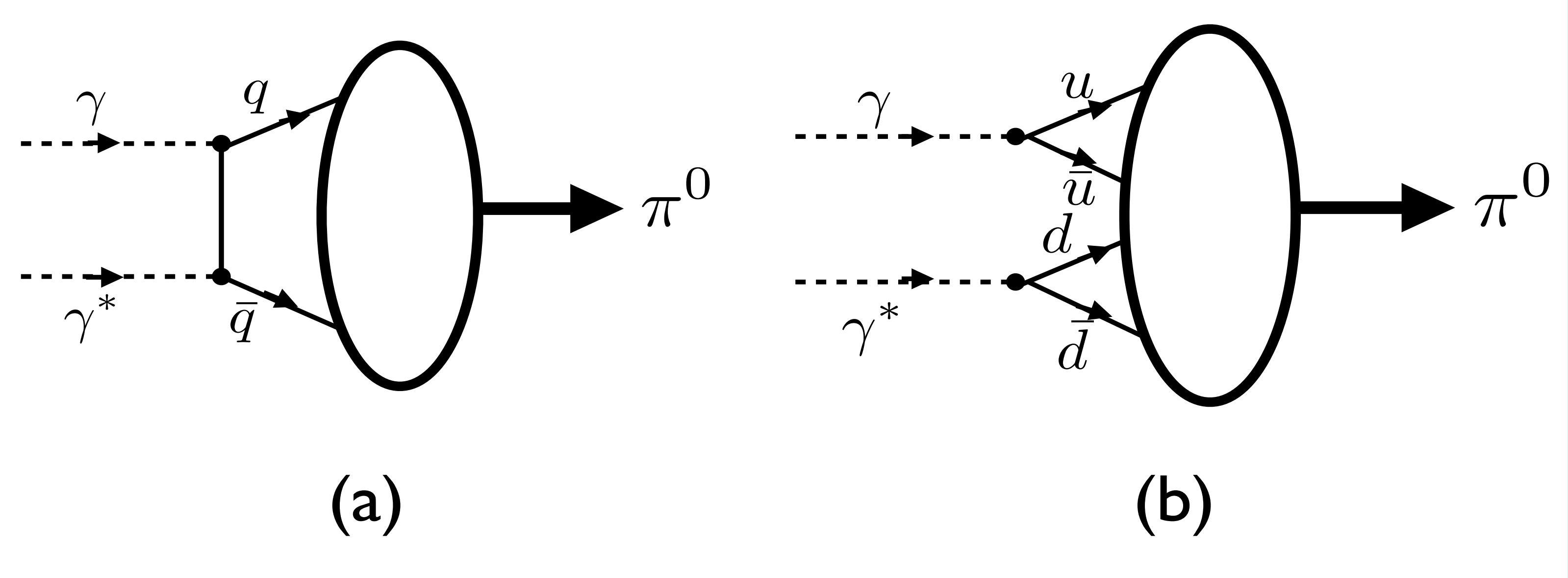
a_n embody soft physics convenient choice: $\mu_F=\mu_R=Q$ $\overline{MS}$ scheme

del Aguila-Chase (81); Braaten (83)

NLO contributions are about 10~20% for Q > a few GeV.



QCD corrections: higher Fock state contributions



Valence Fock state

Higher Fock states

Estimation of HFS contributions

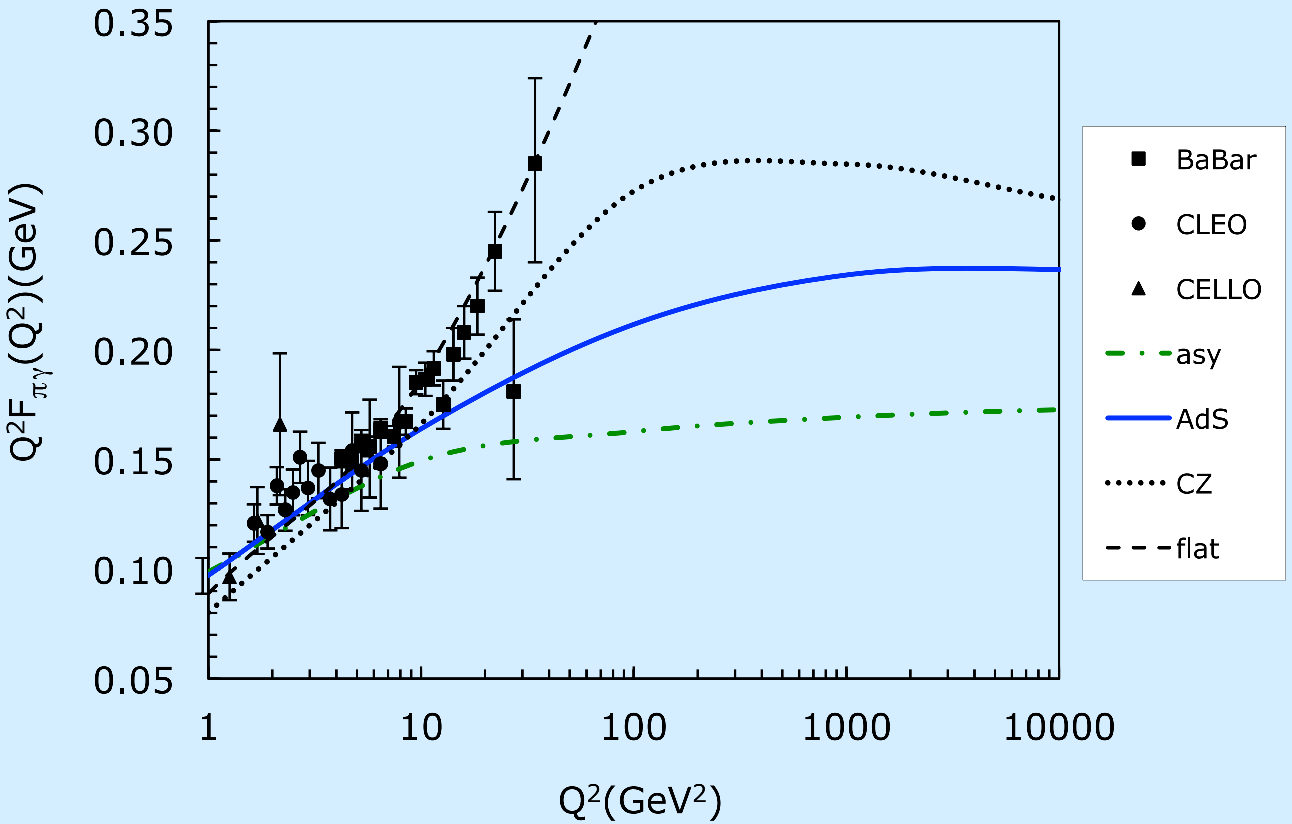
$$Q^2 F_{\pi\gamma}^{\mathrm{HFS}}\left(Q^2\right) = \frac{F_{\pi\gamma}(0)}{\left(1+Q^2/\Lambda^2\right)}$$

$P_{q\bar{q}} = 0.5;\ \Lambda = 1.1$ GeV

$<10\%$ for $Q^2 > 10\ \mathrm{GeV}^2$

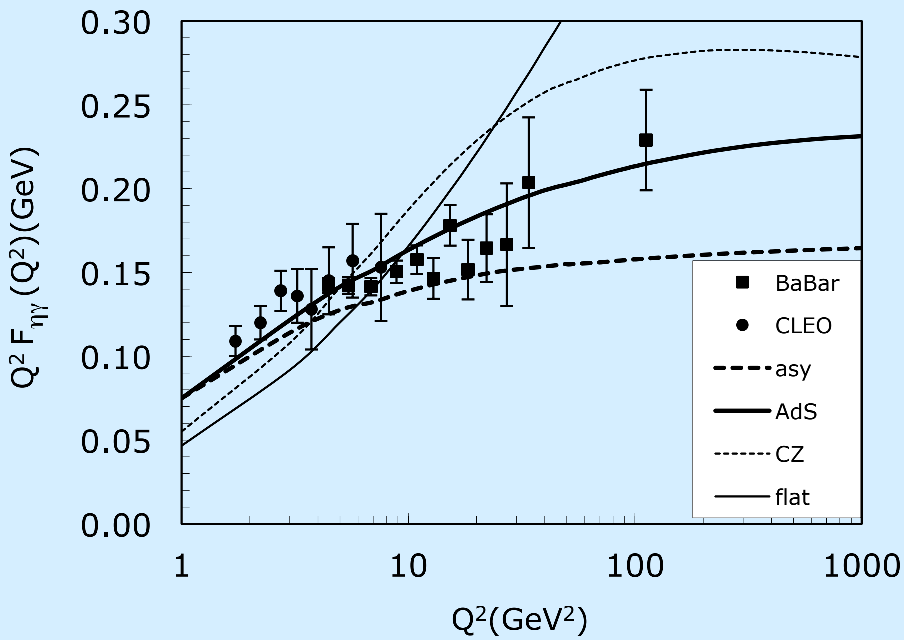
[X. G. Wu and T. Huang, PRD 82 (2010) 034024]

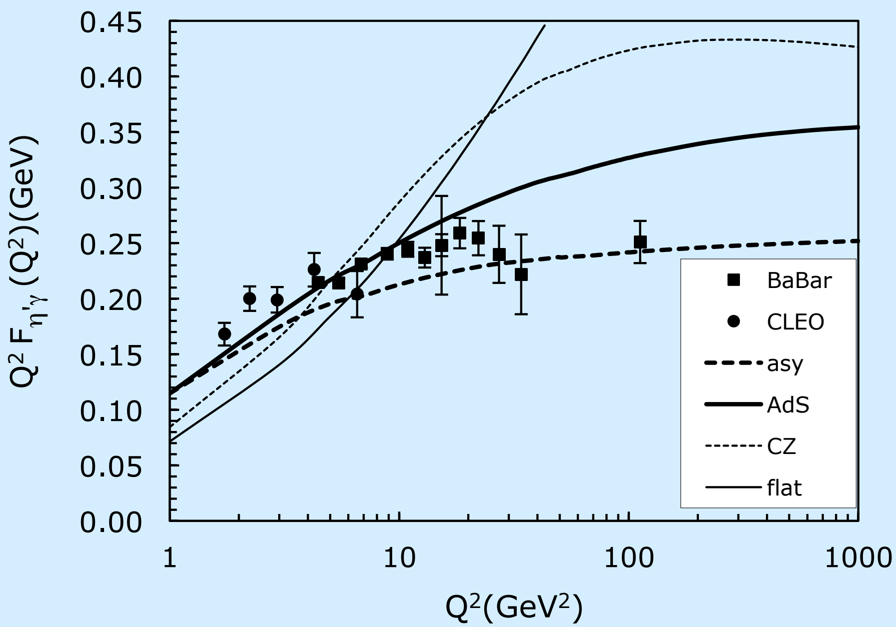
QCD prediction for the pion-photon TFF



eta-photon and eta'-photon TFFs

$$\begin{pmatrix} F_{\eta\gamma} \\ F_{\eta'\gamma} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} F_{\eta_8\gamma} \\ F_{\eta_1\gamma} \end{pmatrix}$$



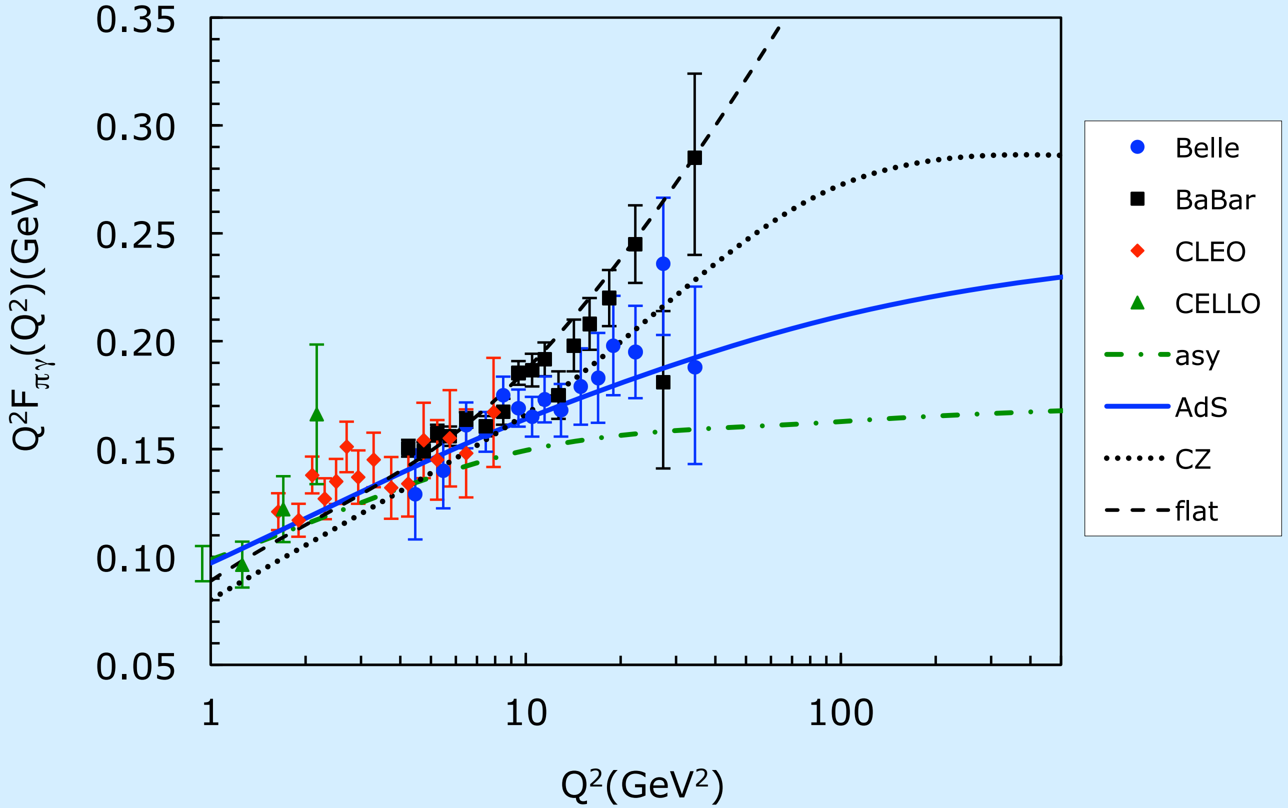


QCD calculations are in agreement with data for the eta- and eta'- photon transition form factors, but disagree with BaBar data for the pion-photon transition from factor.

Any inconsistence in the BaBar data?

Need new measurements!!

Belle data ring the bell !?



Belle data: PRD86 (2012) 092007

More data at large Q region will be able to distinguish these DA models.

Light-front holographic QCD prediction

An effective gravity theory on a higher dimensional anti-de Sitter (AdS) space

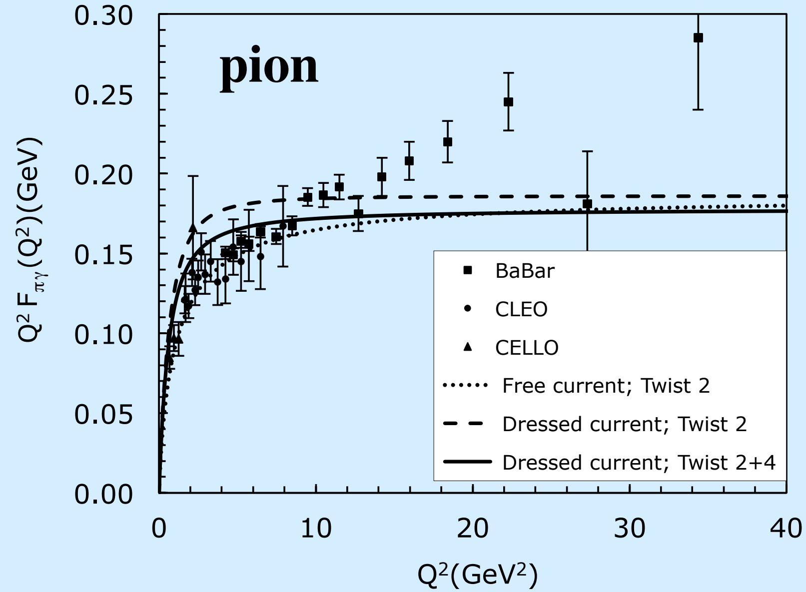
AdS/CFT correspondence

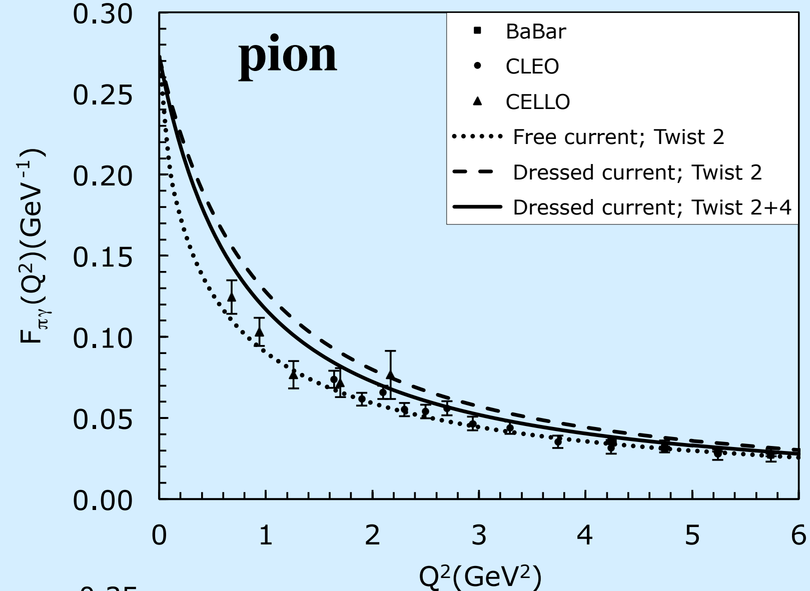
Conformal field theories in physical space-time

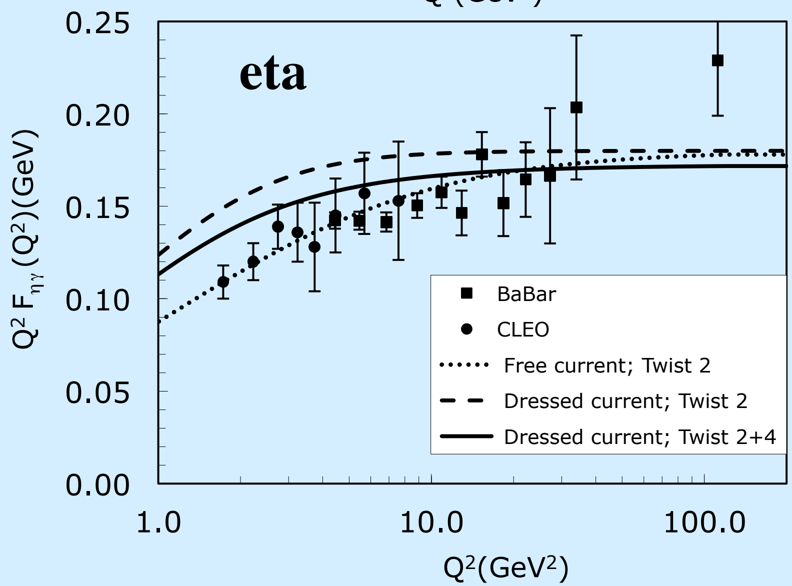
Semi-classical approximation for strongly-coupled QCD (insights into non-perturbative dynamics)

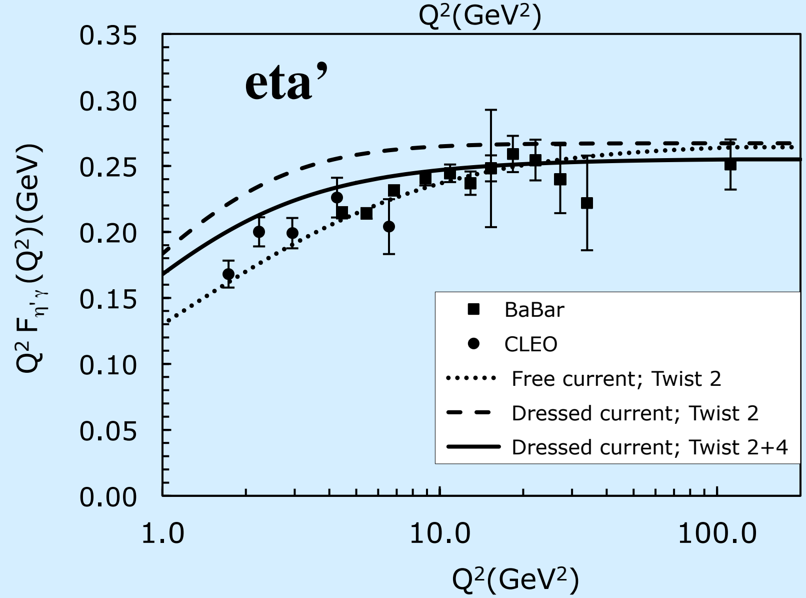
Matching the EM current matrix elements in AdS space to the Drell-Yan-West expression leads to light-front holographic QCD

Meson TFFs in Light-Front Holographic QCD





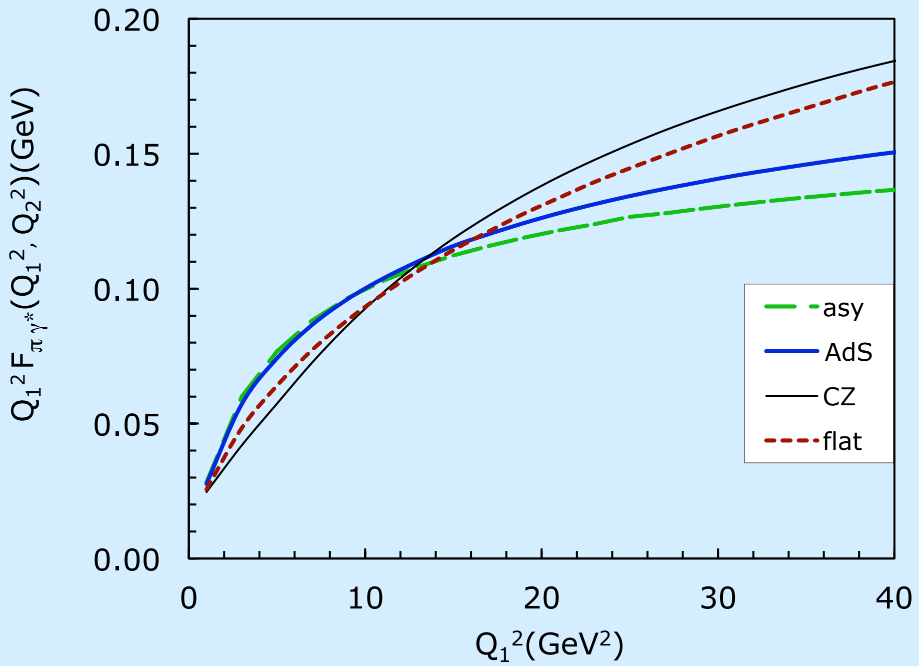




S. J. Brodsky, F.-G. Cao, and G. F. de Teramond, PRD 84 (2011) 075012

LF holographic QCD calculations are in agreement with data for the eta- and eta’-photon transition form factors, but disagree with BaBar data for the pion-photon transition from factor.

QCD prediction for the pion-virtual-photon TFF



$$T_H^{\gamma^*\gamma^*\to\pi}(x, Q_1, Q_2) = \frac{1}{\bar{x} Q_1 + x Q_2}$$

$$Q_1^2 F_{\pi\gamma^*}(Q_1, Q_2 = Q_1) \Rightarrow \frac{2}{3} f_\pi$$

$$F_{\pi\gamma^*}(Q_1 \to 0, Q_2 \to 0) \Rightarrow \frac{1}{4\pi^2 f_\pi}$$

$$Q_1^2 F_{\pi\gamma^*}(Q_1, Q_2 = 0) = Q_1^2 F_{\pi\gamma}(Q_1)$$

- Various parameterizations of TFF have been used in pervious calculations.

VMD: $$F_{\pi\gamma^*\gamma^*}\left(Q_1^2,Q_2^2\right)=\frac{1}{4\pi^2 f_\pi}\frac{M_V^2}{Q_1^2+M_V^2}\frac{M_V^2}{Q_2^2+M_V^2}$$

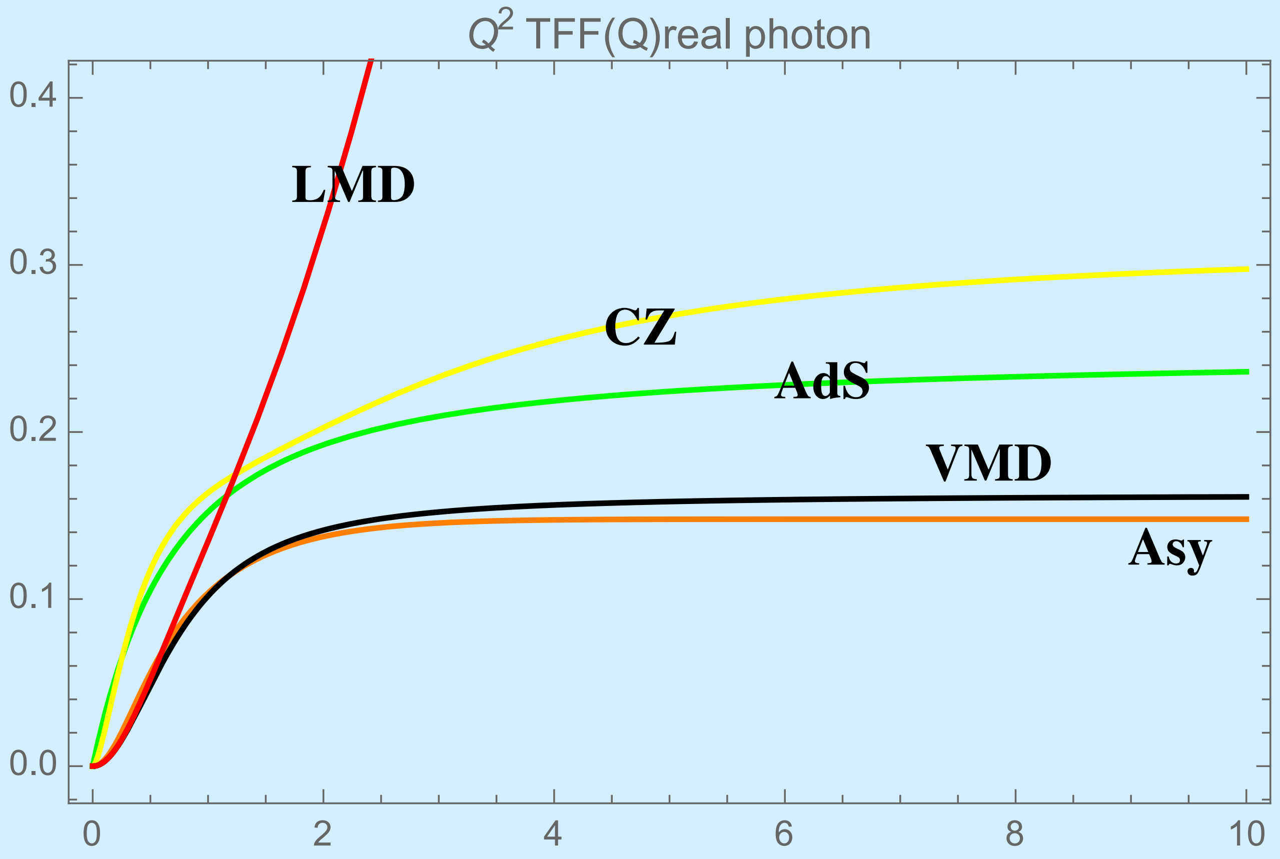
Does not have the correct large Q behavior!

LMD: $$F_{\pi\gamma^*\gamma^*}\left(Q_1^2,Q_2^2\right)=\frac{f_\pi}{3}\frac{Q_1^2+Q_2^2-\dfrac{M_V^4}{4\pi^2 f_\pi^2}}{\left(Q_1^2+M_V^2\right)\left(Q_2^2+M_V^2\right)}$$

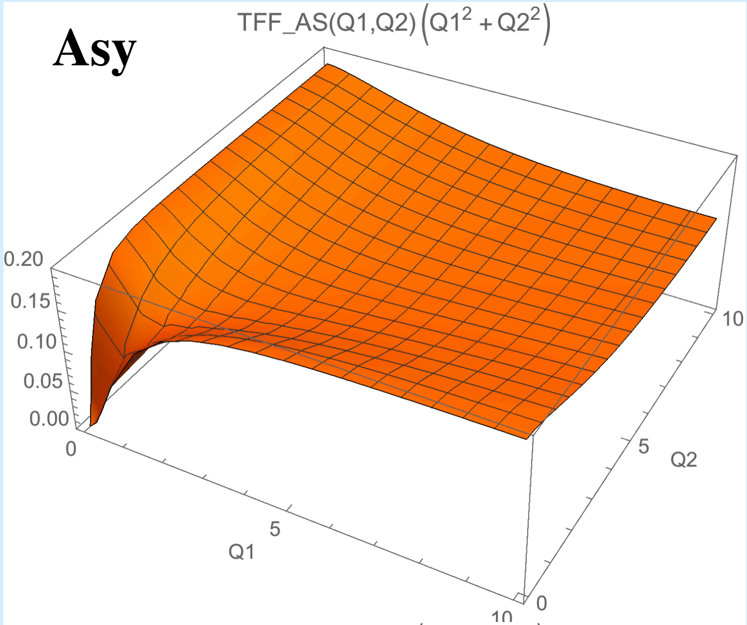
Does not reproduce the pion-real-photon TFF!

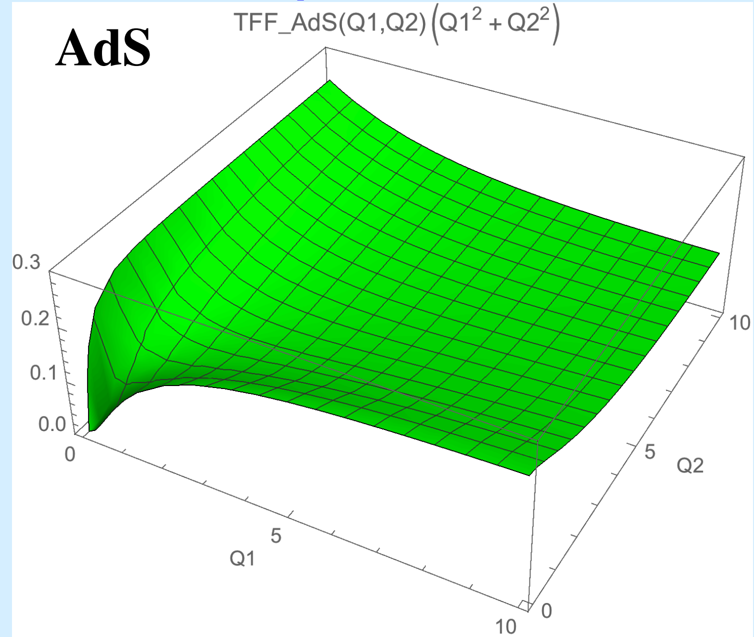
- Need QCD calculations for those TFFs.

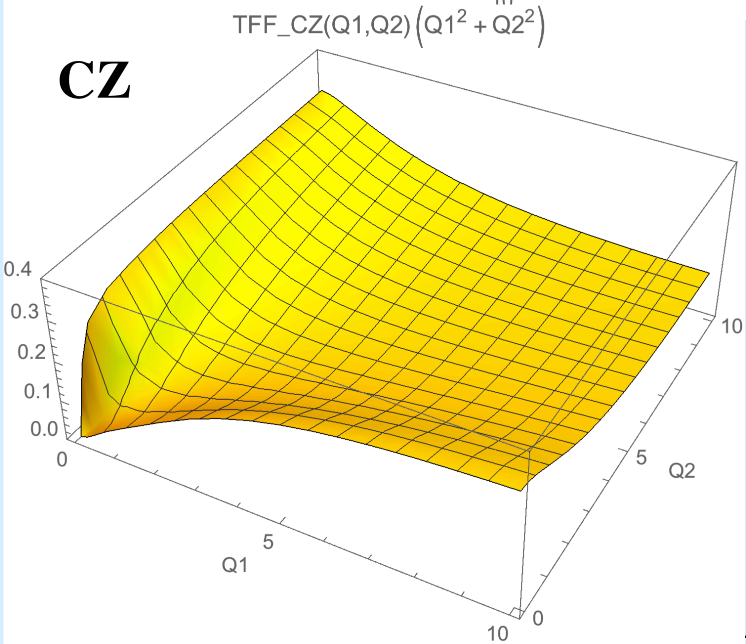
Transition form factors – real photon

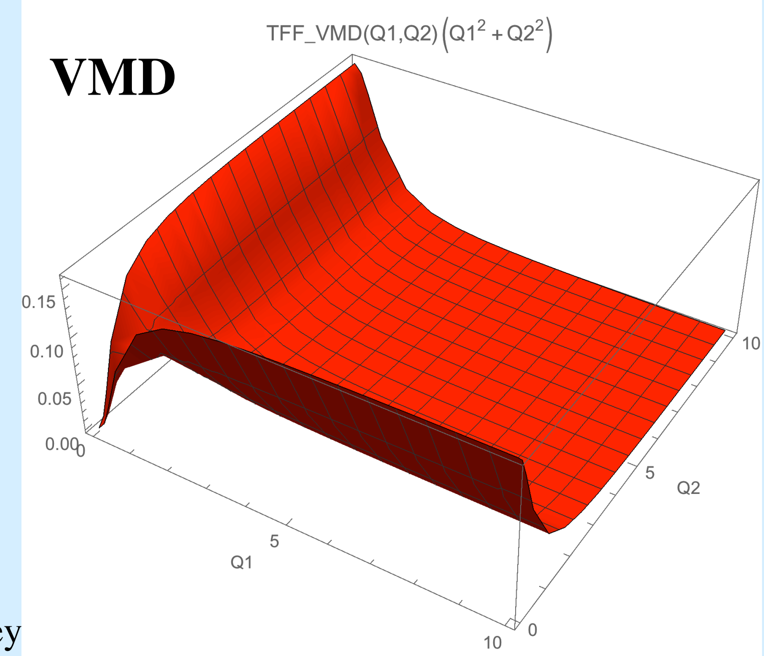


Transition form factors – virtual photon

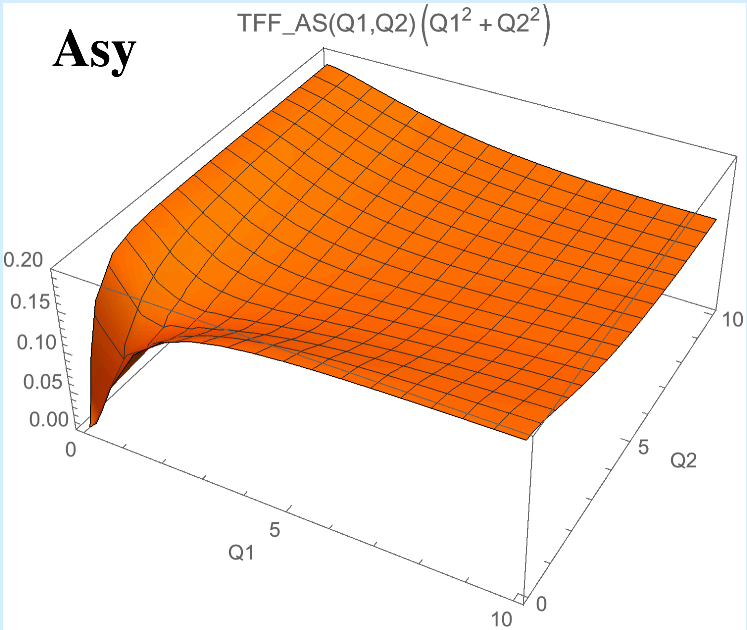


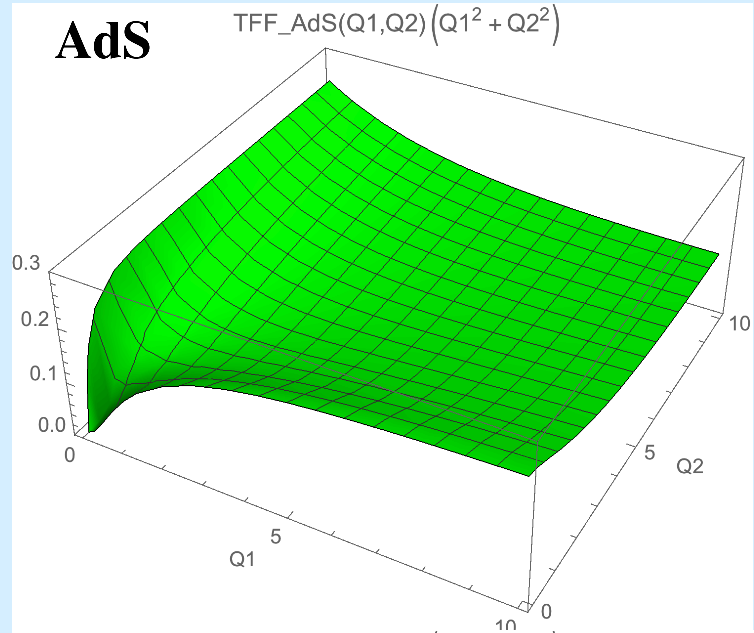


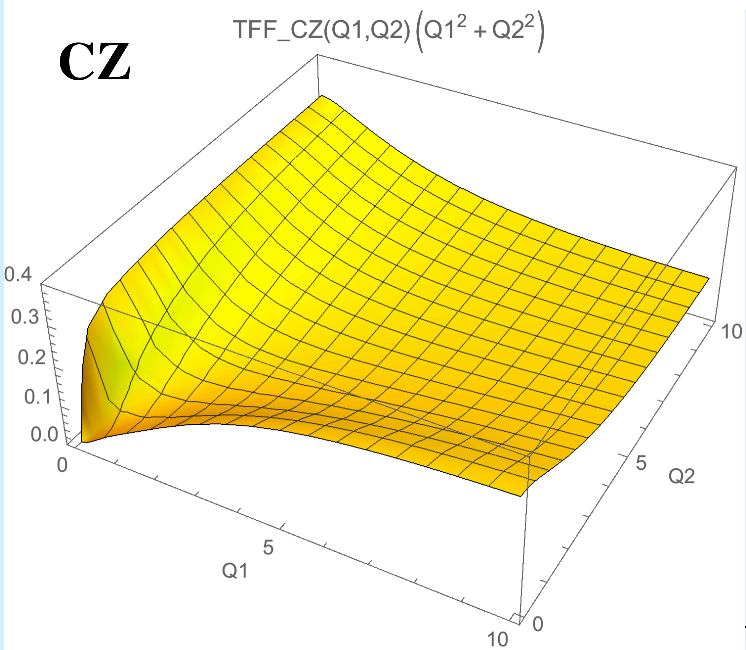


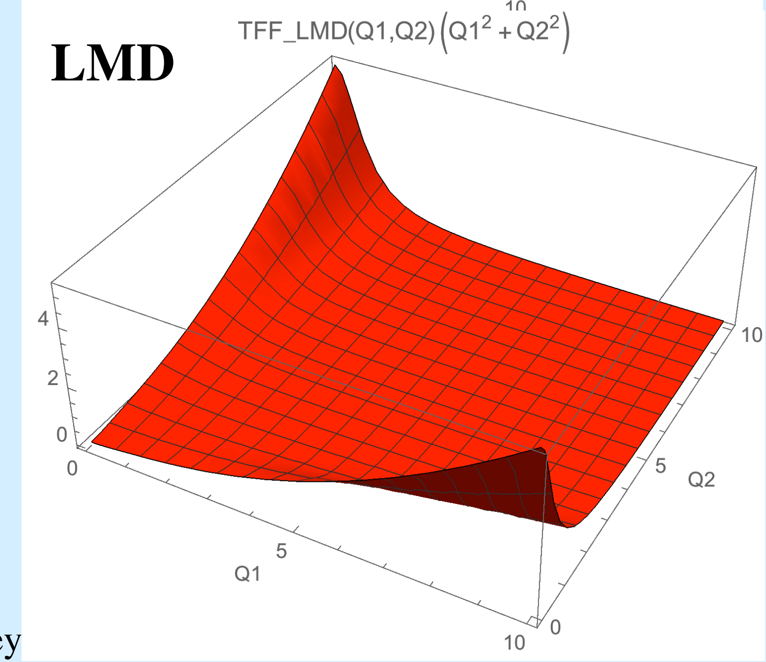


Transition form factors – virtual photon









Meson-real-photon TFFs

- The BaBar data for the pion TFF exhibit a rapid growth with the momentum transfer, while the Belle data do not exhibit such a trend.
- Perturbative QCD calculations and non-perturbative calculations with light-front holographic QCD show good agreement with all available data for the eta- and eta'-photon TFFs and all data for the pion-photon TFF except the data from the BaBar.

Pion-virtual-photon TFFs

- The VMD does not have the correct large Q behavior.
- The LMD does not reproduce the pion-real-photon TFF.

3. Evaluation of muon g-2

$$a_\mu^{\mathrm{HLxL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{2} T_i(Q_1,Q_2,\tau)\,\Pi_i(Q_1,Q_2,\tau)$$

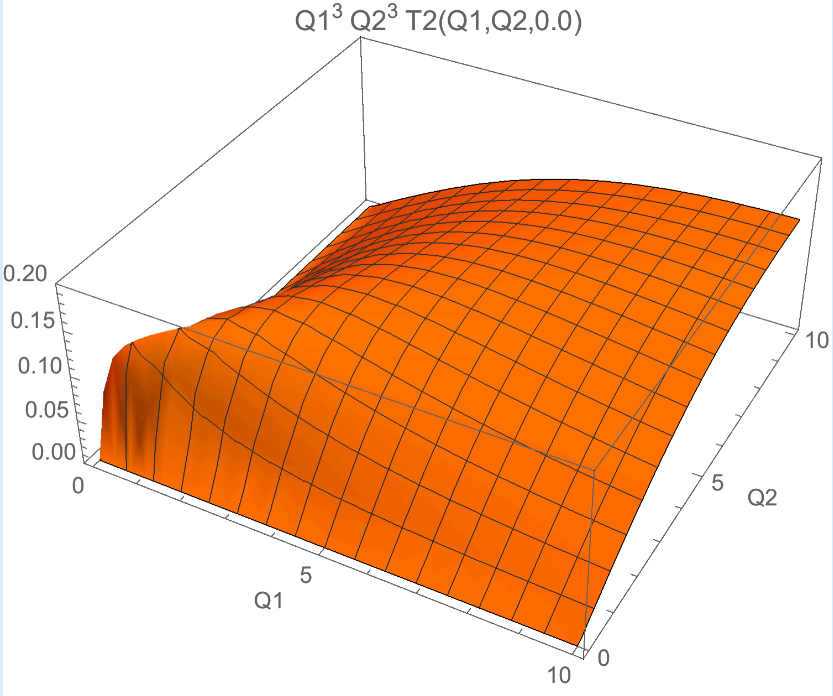
$$\Pi_1(Q_1,Q_2,\tau) = \frac{1}{s-M_\pi^2} F_{\pi\gamma^*\gamma^*}\left(-Q_1^2,-Q_2^2\right) F_{\pi\gamma\gamma^*}\left(-Q_3^2,0\right)$$

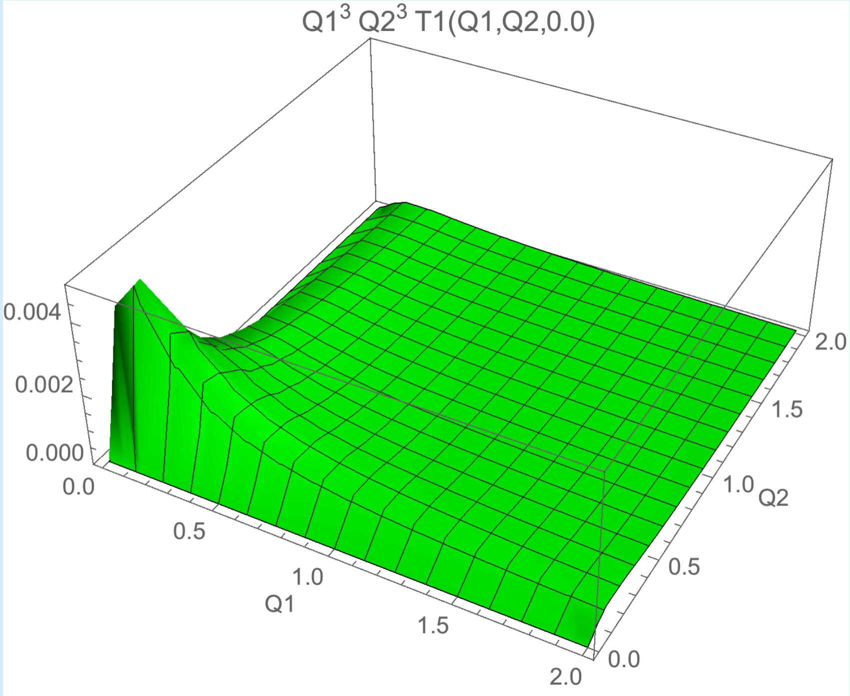
$$\Pi_2(Q_1,Q_2,\tau) = \frac{1}{t-M_\pi^2} F_{\pi\gamma^*\gamma^*}\left(-Q_1^2,-Q_3^2\right) F_{\pi\gamma\gamma^*}\left(-Q_2^2,0\right)$$

$$s = -Q_3^2 = -\left(Q_1^2 + 2Q_1 Q_2 \tau + Q_2^2\right), \quad t = -Q_2^2$$

τ is the cosine of the angle between the Eucliden momenta Q_1 and Q_2.

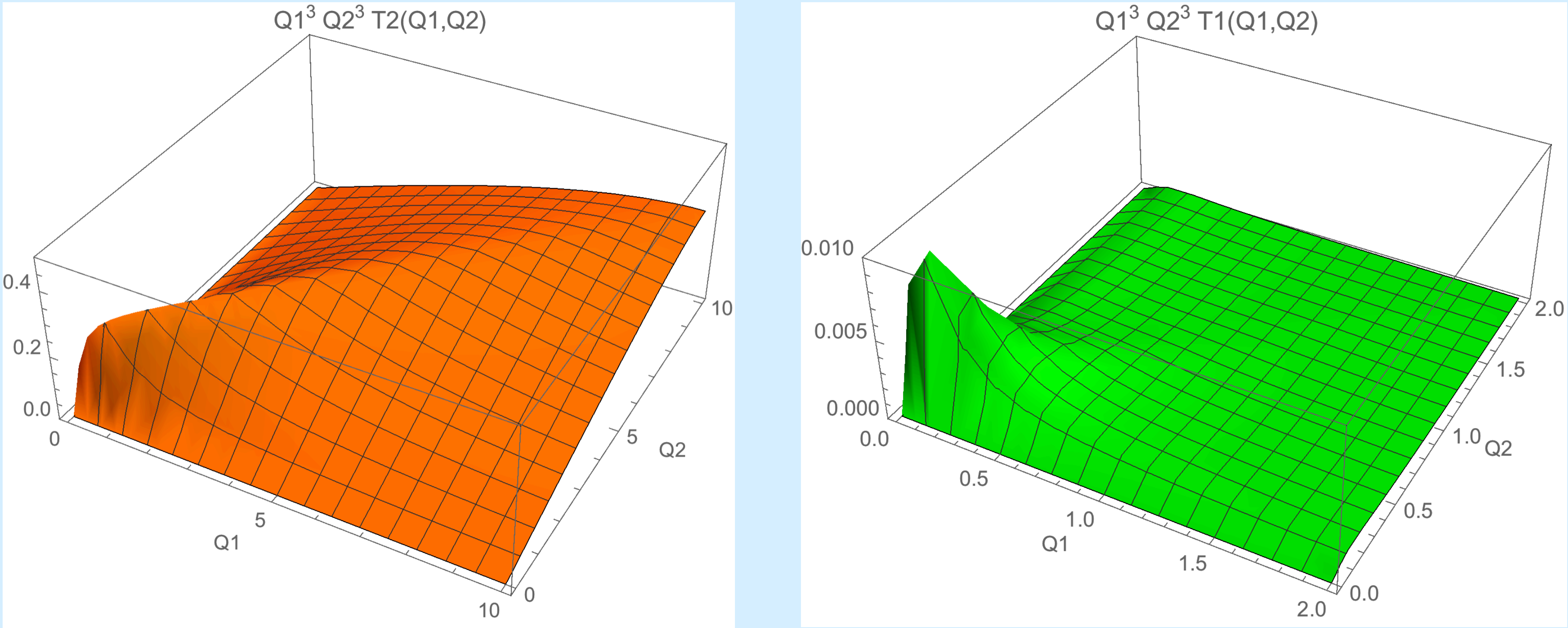
Integral kernels T2 and T1





T2 term provides the dominant contribution.

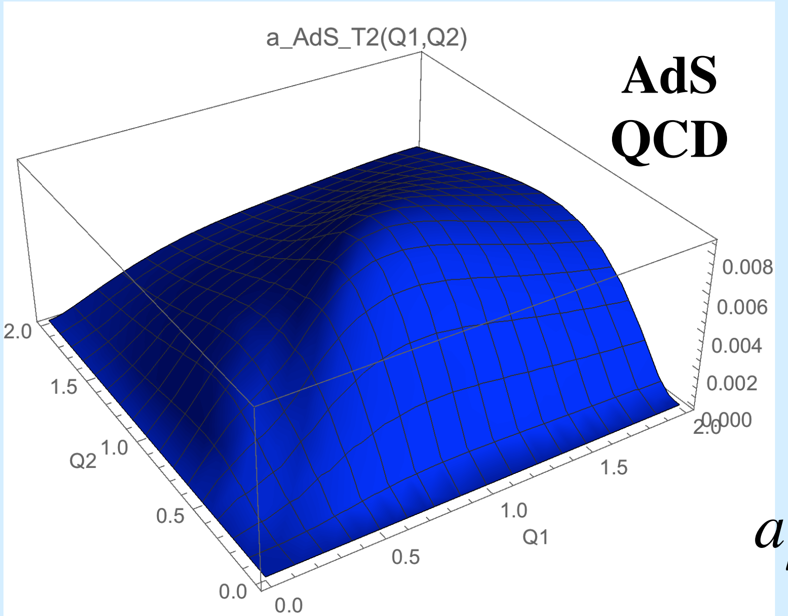
Integral kernels T2 and T1

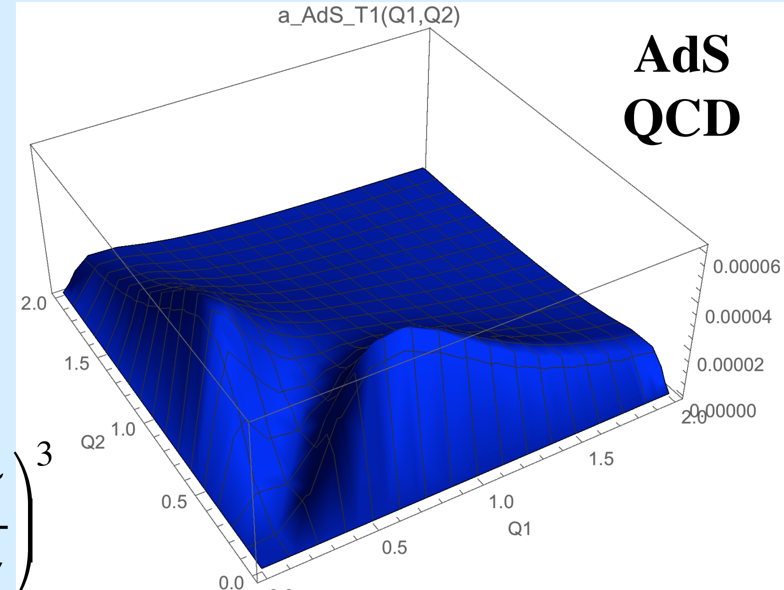


T1 and T2 integrated over the angle.
T2 term provides the dominant contribution.

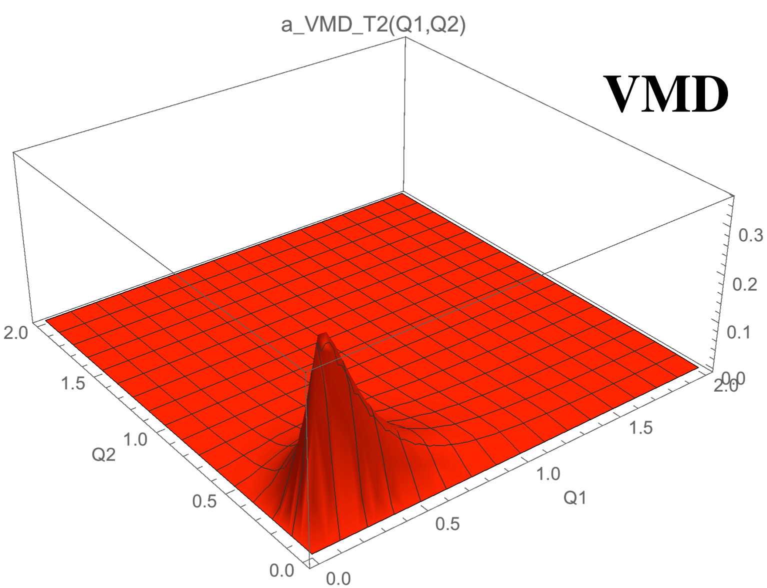
Contributions from T2 vs T1

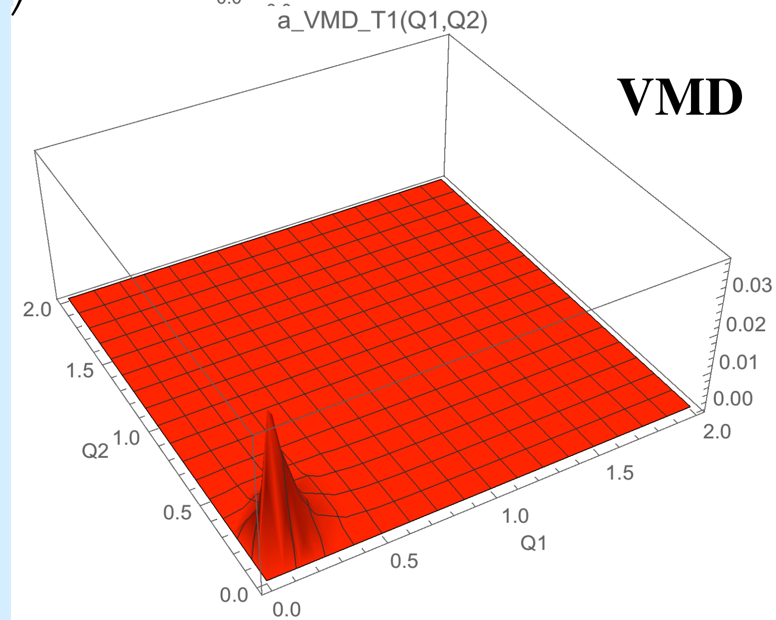
T2 **T1**





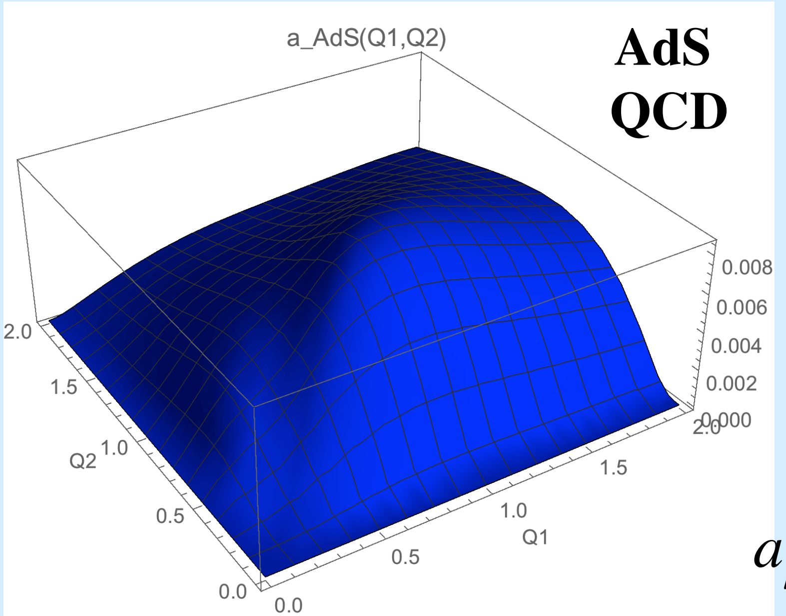
$$a_\mu^{\text{HLxL}} \times \left(\frac{\pi}{\alpha}\right)^3$$

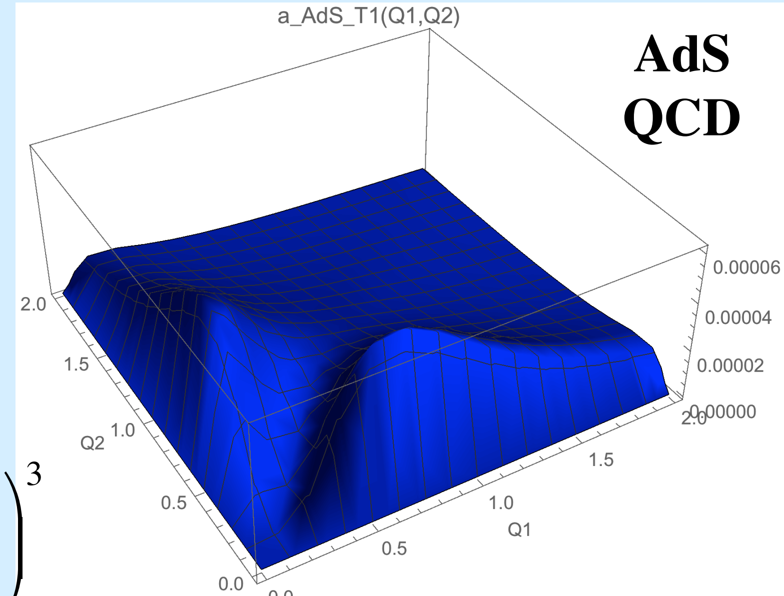




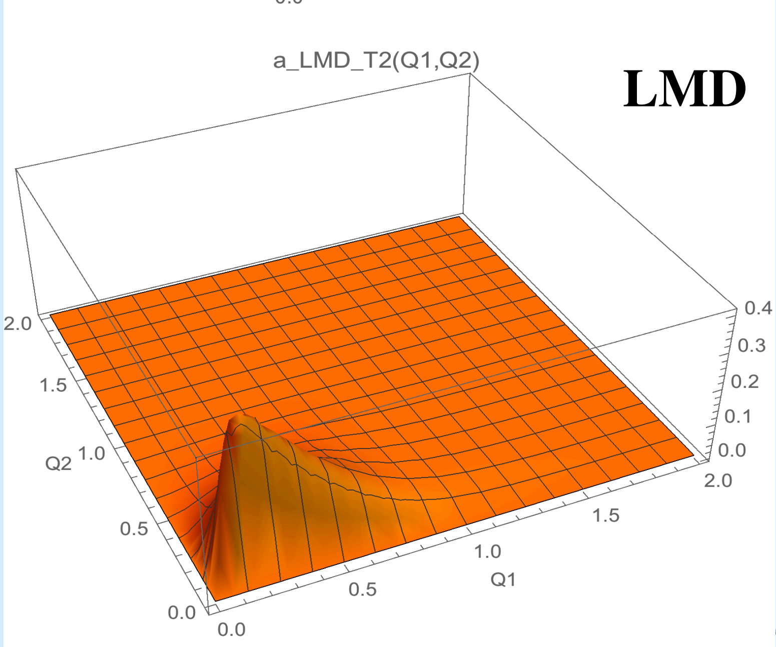
Contributions from T2 vs T1

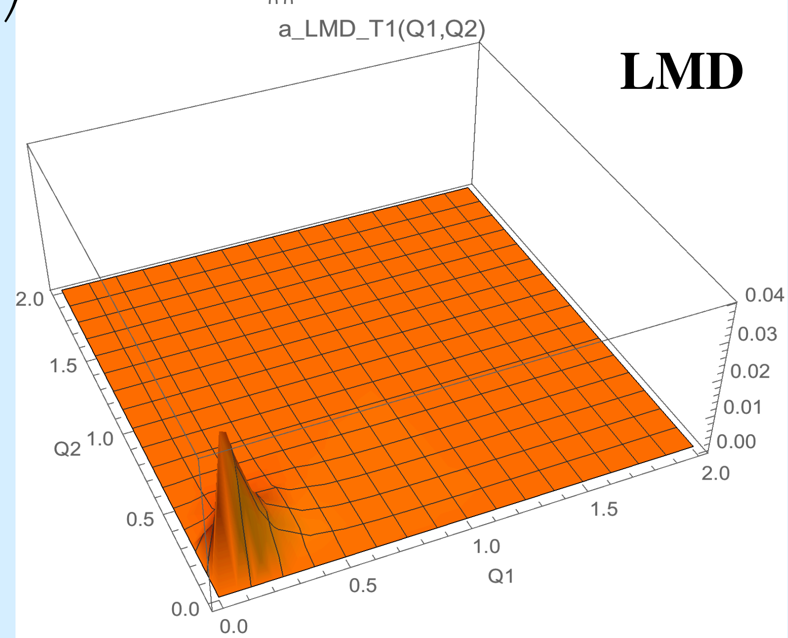
T2 **T1**





$$a_\mu^{\mathrm{HLxL}} \times \left(\frac{\pi}{\alpha}\right)^3$$

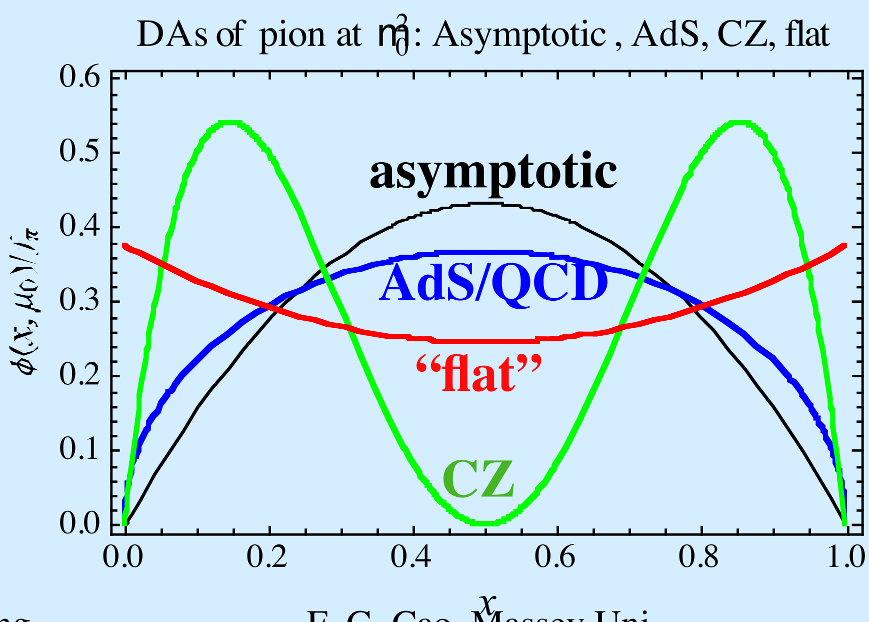




Results

a_μ^{HLxL} in the unit of 10^{-11}

	Pion	eta	eta'	Total
ASY	60.1	17.4	17.1	94.5
AdS	64.8	17.8	17.1	99.8
CZ	39.7	11.4	10.8	61.6
VMD LMD	56 73	13	12	81



4. Summary

- Hadronic LxL is one of the main sources of uncertainty in computing muon g-2.
- Calculations depend on the meson distribution amplitude.
- Numerical results 60 ~ 100 using three models of DA.
- Need better understanding of the DAs and/or other nonperturbative methods.