QCD perturbative series in tau decays: scheme variations of the coupling

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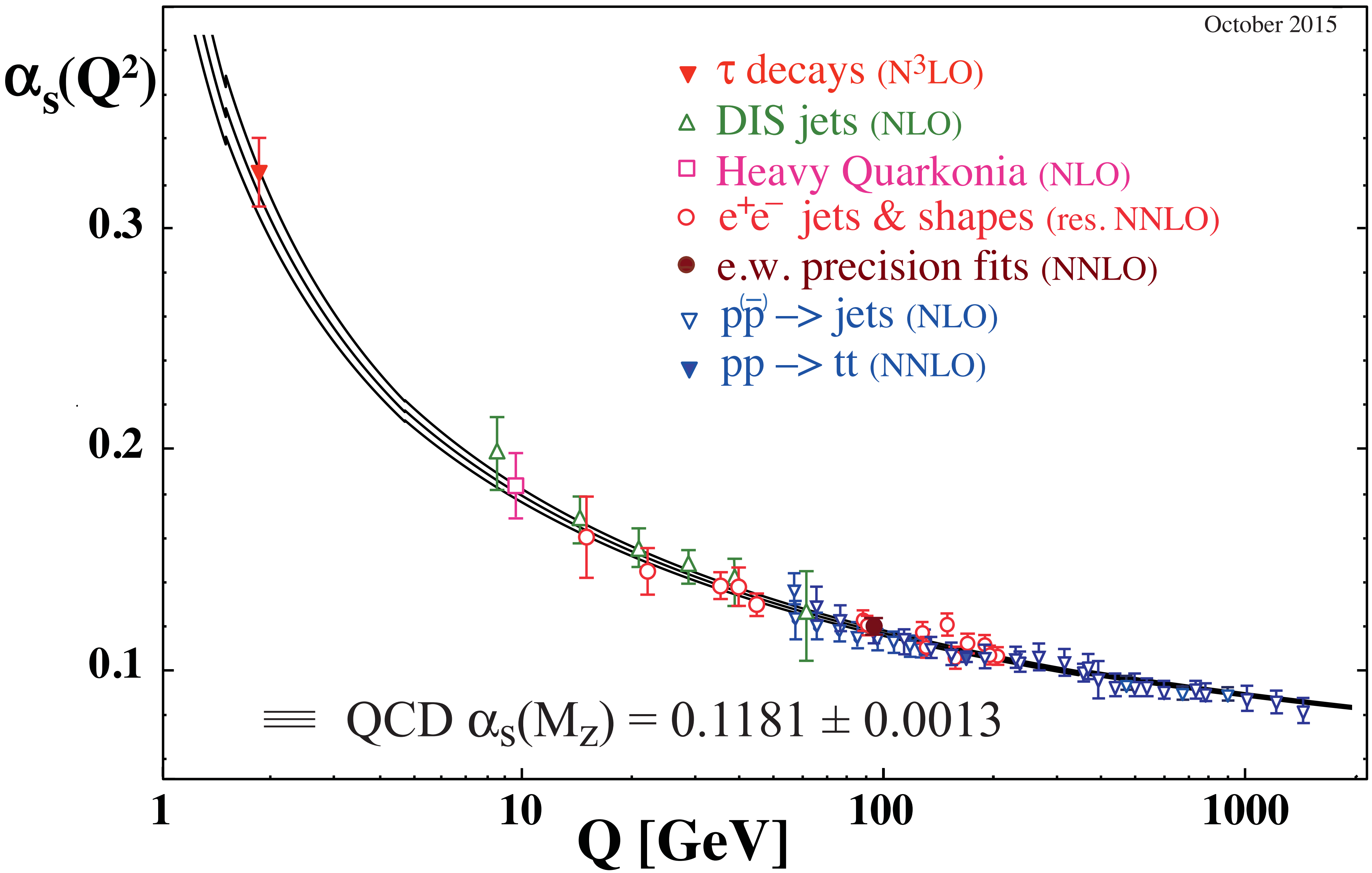
-- In coll. with Matthias Jamin and Ramon Miravitllas (UAB/IFAE, Spain)
[arXiv:1606.08659] (to appear in PRL)

Introduction

Strong coupling



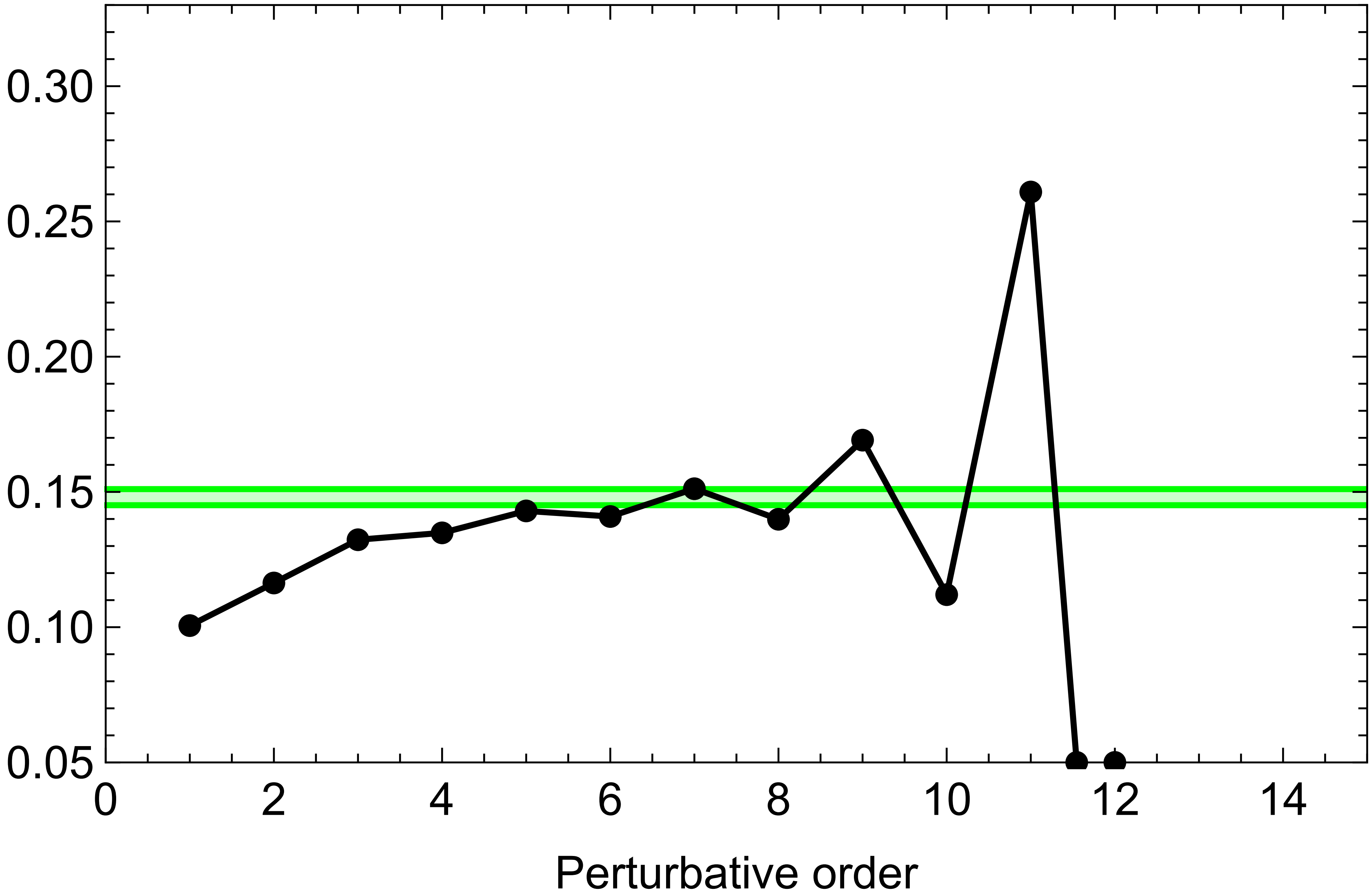
Not a physical observable: scheme an scale dependent

Key idea of the talk

Perturbative expansions in QFTs are (at best) asymptotic

Different schemes lead to different asymptotic series

freedom to exploit and look for the optimal series



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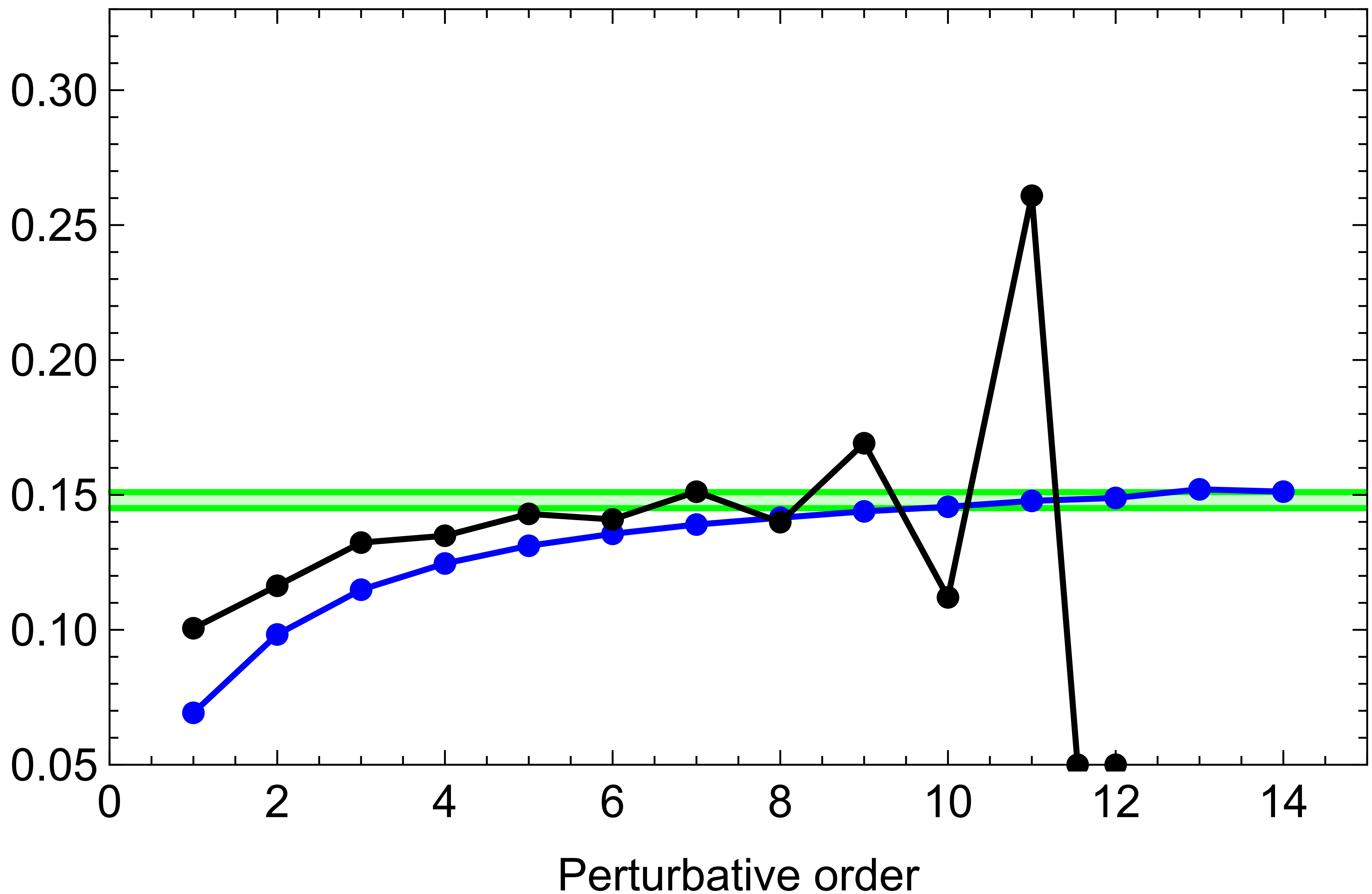

A few definitions

$$a_Q = \frac{\alpha_s}{\pi}$$

The scale evolution is given by the QCD beta function

$$-Q\,\frac{\mathrm{d}a_Q}{\mathrm{d}Q} \equiv \beta(a_Q) = \beta_1\,a_Q^2 + \beta_2\,a_Q^3 + \beta_3\,a_Q^4 + \beta_4 a_Q^5 + \beta_5 a_Q^6 \ldots$$

Baikov, Chetyrkin, Kühn '16
(mind the different conventions)

A scale invariant (but scheme dependent) QCD scale Λ can be defined as

$$\Lambda \equiv Q\,\mathrm{e}^{-\frac{1}{\beta_1 a_Q}}\,[a_Q]^{-\frac{\beta_2}{\beta_1^2}}\,\exp\left[\int_0^{a_Q}\frac{\mathrm{d}a}{\tilde{\beta}(a)}\right],$$

where we introduced the combination

$$\frac{1}{\tilde{\beta}(a)} \equiv \frac{1}{\beta(a)} - \frac{1}{\beta_1 a^2} + \frac{\beta_2}{\beta_1^2 a}$$

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See next talk by J. Khün!

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scheme independent

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$$\frac{1}{\tilde\beta(a)} \equiv \frac{1}{\beta(a)} - \frac{1}{\beta_1 a^2} + \frac{\beta_2}{\beta_1^2 a}$$

In another scheme one would have

$$\tilde a = a + c_1 a^2 + c_2 a^3 + c_4 a^4 + \cdots \longrightarrow \tilde\Lambda = \Lambda e^{c_1/\beta_1}$$

Celmaster & Gonsalves '79

Why varying the scheme?

Higher-loop calculations allow us to gain a better understanding of the series at intermediate orders.

↓

Scheme variations give us an extra handle on the perturbative series.

↓

Optimisation of the series in the spirit of asymptotic expansions.

↓

Applications (not limited to) tau decays (FOPT vs CIPT), Higgs decays, ….

Scheme variations

In the large-β_0 limit its convenient to redefine the coupling as

Beneke '99

$$\frac{1}{\hat{a}_Q} \equiv \beta_1\left(\ln\frac{Q}{\Lambda} + \frac{C}{2}\right) = \frac{1}{a_Q} + \frac{\beta_1}{2}C$$

We propose the following generalisation to the QCD coupling (C-scheme)

$$\boxed{\frac{1}{\hat{a}_Q} + \frac{\beta_2}{\beta_1}\ln\hat{a}_Q \equiv \beta_1\left(\ln\frac{Q}{\Lambda} + \frac{C}{2}\right) = \frac{1}{a_Q} + \frac{\beta_1}{2}C + \frac{\beta_2}{\beta_1}\ln a_Q - \beta_1\int\limits_0^{a_Q}\frac{\mathrm{d}a}{\tilde{\beta}(a)}}$$

$$\hat{a}_Q \equiv \hat{a}_Q(C)$$

Scheme variations

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free parameter

$$\hat{a}_Q \equiv \hat{a}_Q(C)$$

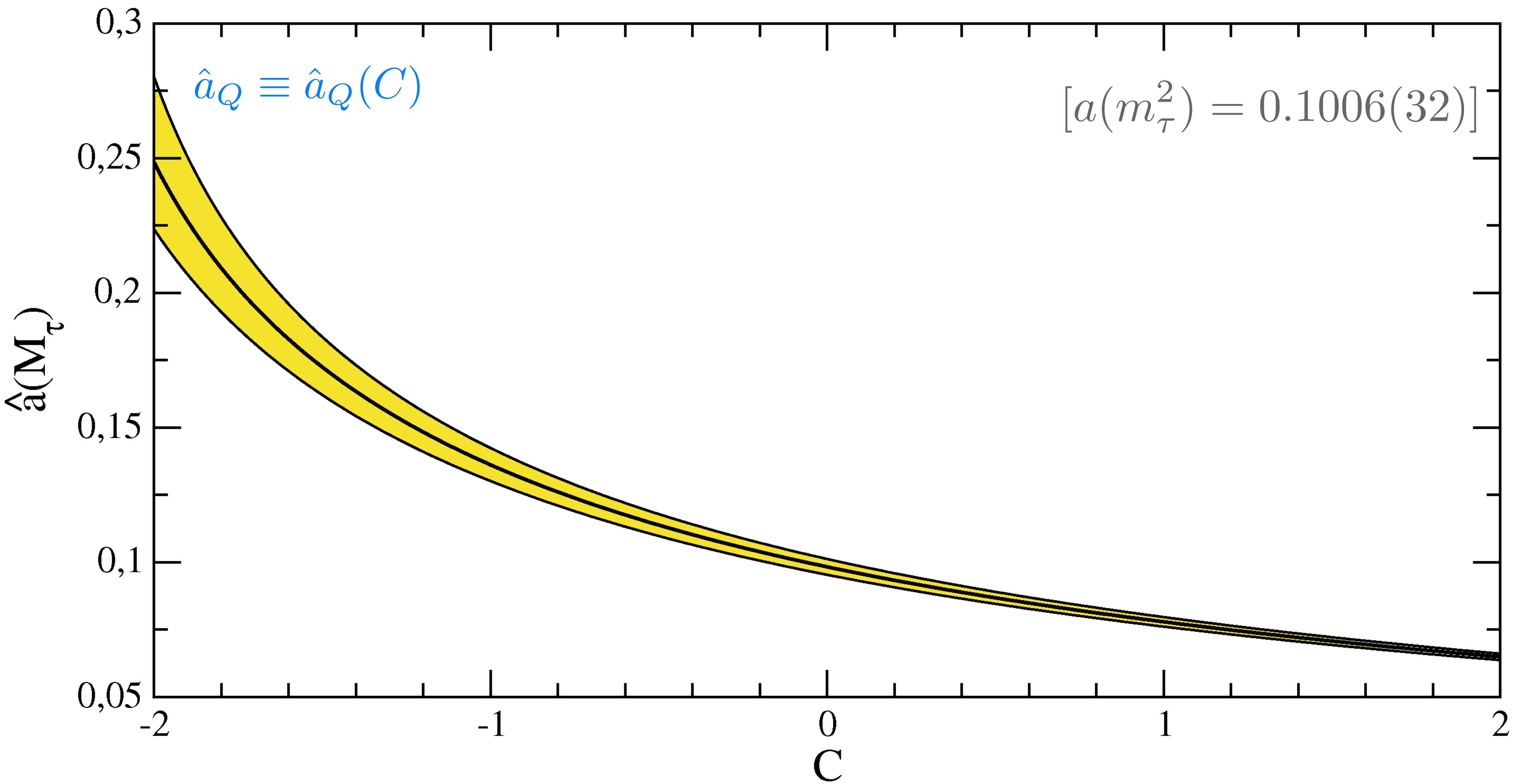
In this new scheme the beta function takes the simple form:

$$\boxed{-Q\frac{\mathrm{d}\hat{a}_Q}{\mathrm{d}Q} \equiv \hat{\beta}(\hat{a}_Q) = \frac{\beta_1\hat{a}_Q^2}{\left(1-\frac{\beta_2}{\beta_1}\hat{a}_Q\right)}\,.}$$

only scheme independent coefficients appear

The new coupling as a function of *C* for $\alpha_s(m_\tau^2) = 0.3160(10)$

PDG '16



(From the numerical solution, not a perturbative result.)

Relations between the different schemes

The perturbative relation between the two schemes up to fifth order is

$$\hat{a}(a) = a + c_1 a^2 + c_2 a^3 + c_3 a^4 + c_4 a^5 + \cdots$$

The first four coefficients read

$$c_1 = -\tfrac{9}{4}C$$

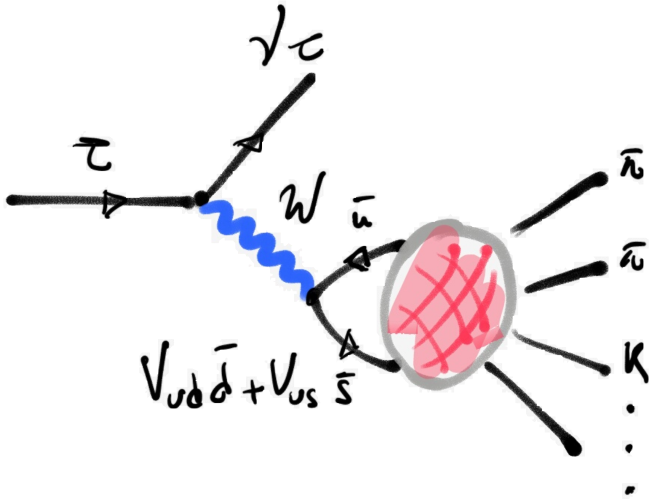
$$c_2 = -\left(\frac{3397}{2592} + 4C - \frac{81}{16}C^2\right)$$

$$c_3 = -\left(\frac{741103}{186624} + \frac{233}{192}C - \frac{45}{2}C^2 + \frac{729}{64}C^3 + \frac{445}{144}\zeta_3\right)$$

$$c_4 = -\left(\frac{727240925}{80621568} - \frac{869039}{41472}C - \frac{26673}{512}C^2 + \frac{351}{4}C^3 - \frac{6561}{256}C^4 - \frac{445}{32}\zeta_3 C + \frac{10375693}{373248}\zeta_3 - \frac{1335}{256}\zeta_4 - \frac{534385}{20736}\zeta_5\right)$$

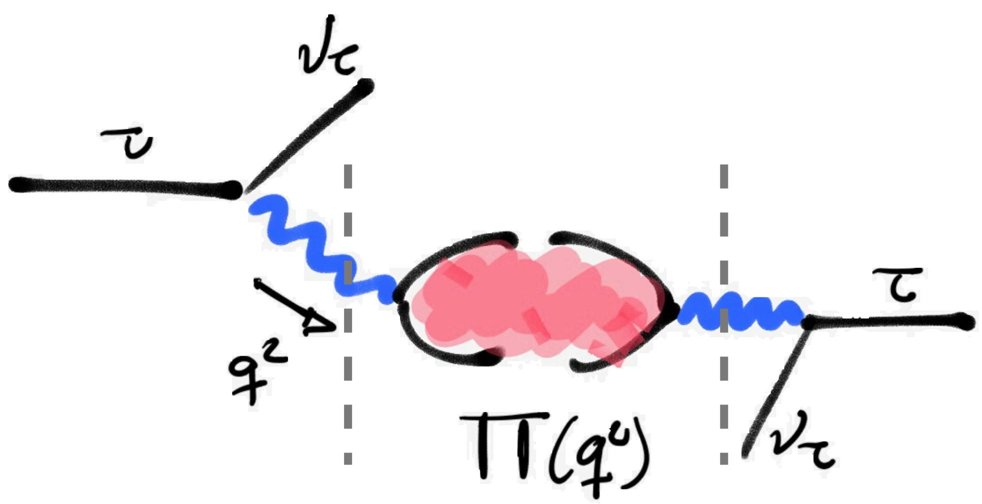
Application to tau decays

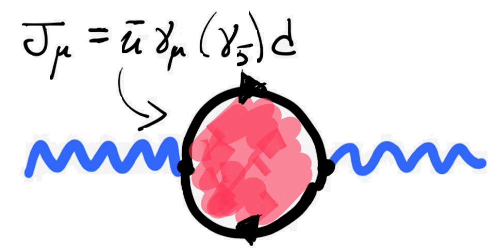
Applications to hadronic tau decays



$$R_\tau = \frac{\Gamma[\tau \to \text{hadrons}\, \nu_\tau]}{\Gamma[\tau \to e^- \bar{\nu}_e\, \nu_\tau]}$$

Theoretical computation: optical theorem + Cauchy integration

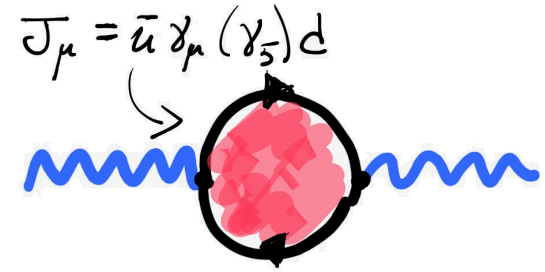




$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|\rangle$$

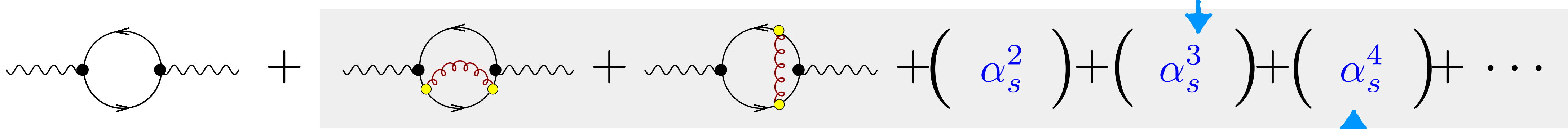
$$R_\tau^{(\mathrm{th})} = \frac{-1}{2\pi i} \oint_{|z|=m_\tau^2} dz\, w(z)\, \tilde{\Pi}(z)$$

Braaten, Narison, Pich '92



Perturbative QCD contribution to the Adler function

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91



Baikov, Chetyrkin, Kühn '08

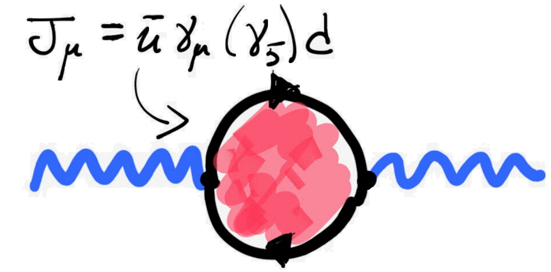
$\overline{\mathrm{MS}}$ result

$$\widehat{D}(a_Q) = \sum_{n=1}^{\infty} c_{n,1} a_Q^n = a_Q + 1.640\, a_Q^2 + 6.371\, a_Q^3 + 49.08\, a_Q^4 + \ldots$$

In the C-scheme we have

$$\widehat{D}(\hat{a}_Q) = \hat{a}_Q + (1.640 + 2.25C)\,\hat{a}_Q^2 + (7.682 + 11.38C + 5.063C^2)\,\hat{a}_Q^3 + (61.06 + 72.08C + 47.40C^2 + 11.39C^3)\,\hat{a}_Q^4 + \cdots$$

The Adler function in the C-scheme



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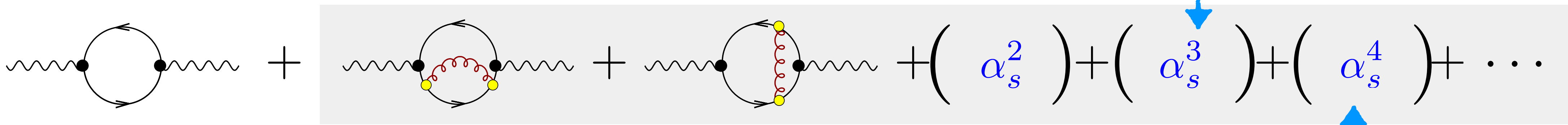

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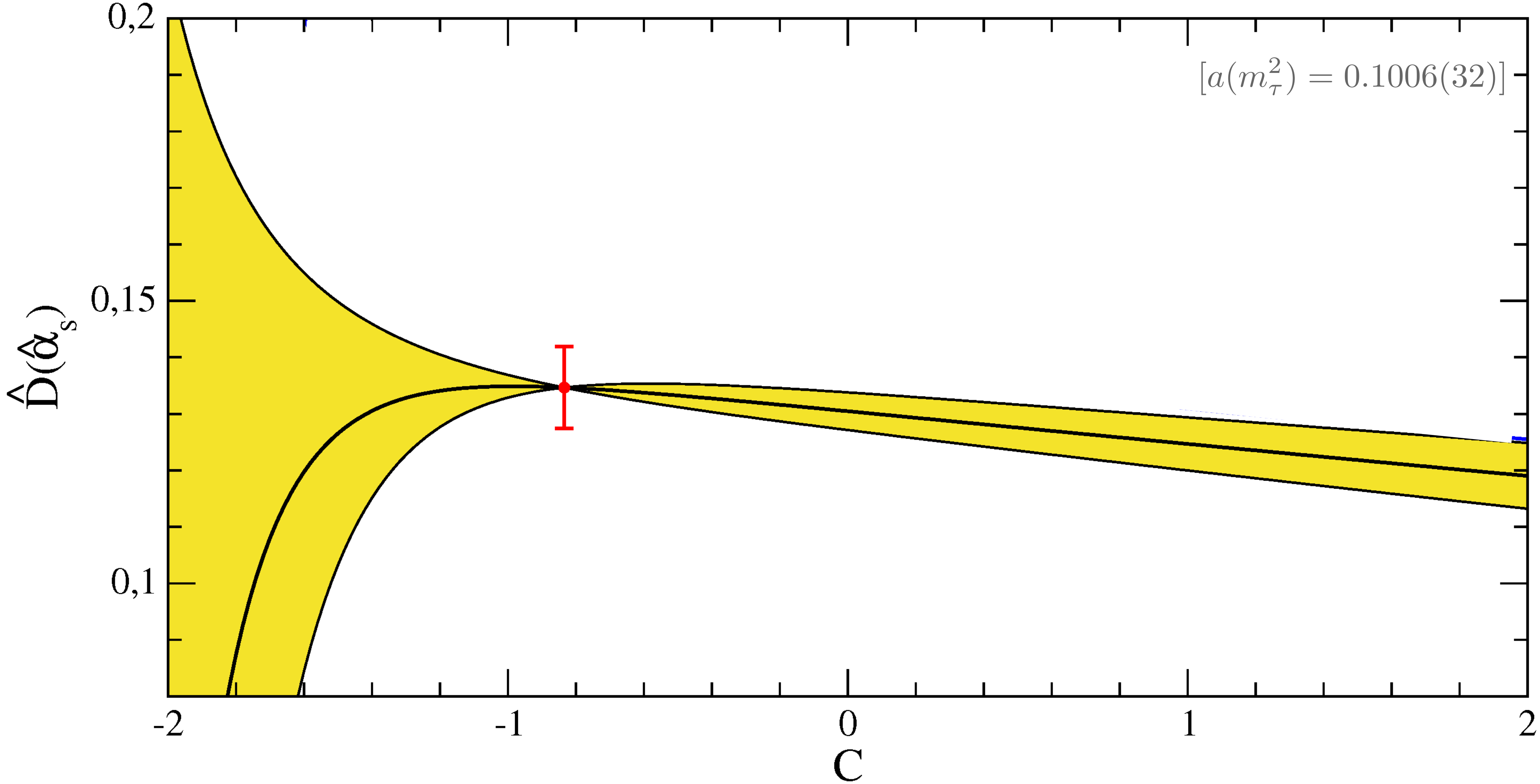
The Adler function in the C-scheme

Using the C dependence we are able to kill the fifth order contribution

$c_{5,1} = 283$

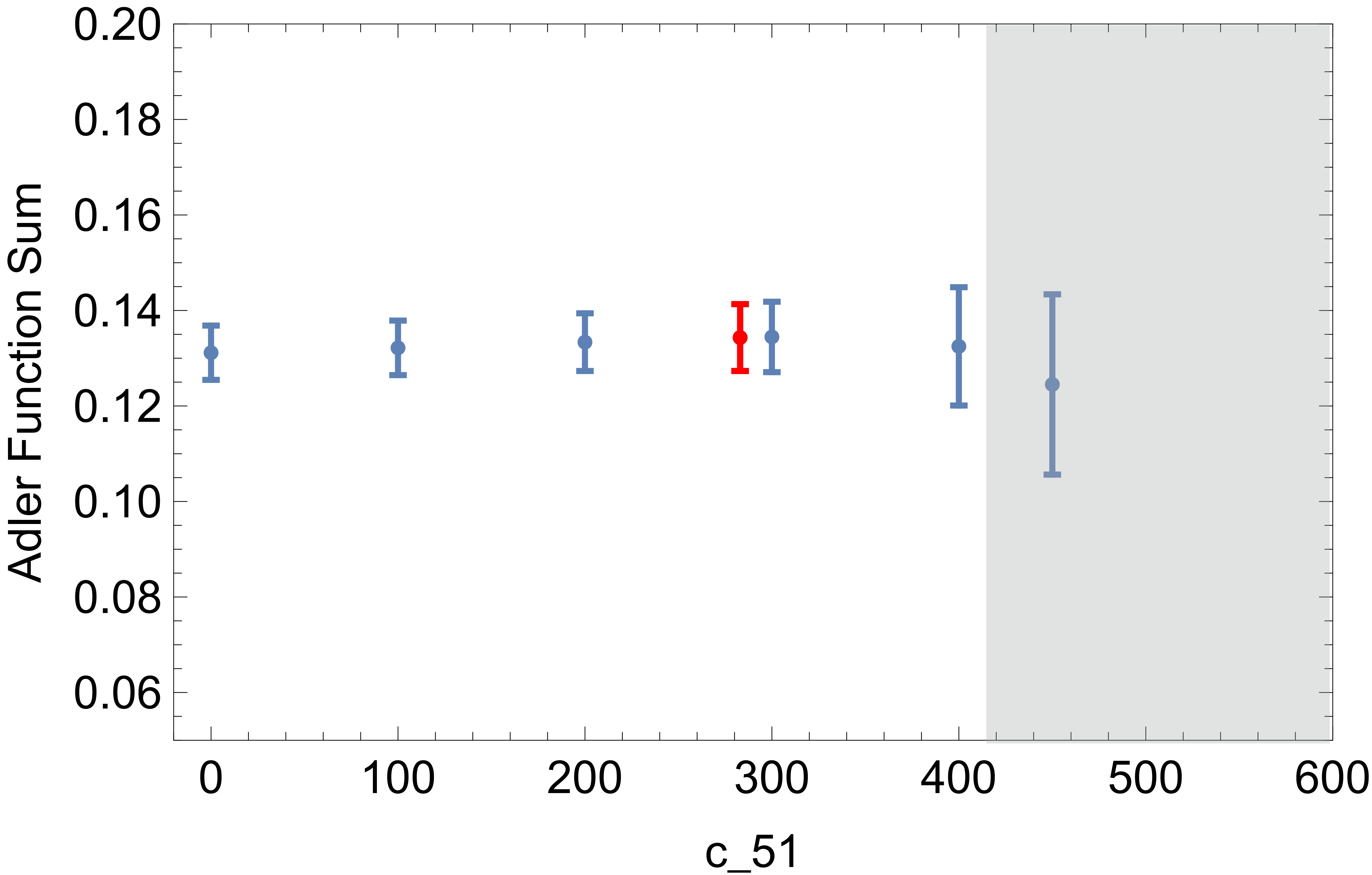
Beneke & Jamin '08



$$\widehat{D}(\hat{a}_{M_\tau}, C = -0.783) \; = \; 0.1343 \pm 0.0070 \pm 0.0067$$

$\mathcal{O}(\hat{a}^4)$ α_s

Dependence on the fifth order coefficient



(Shown only uncertainty due to truncation)

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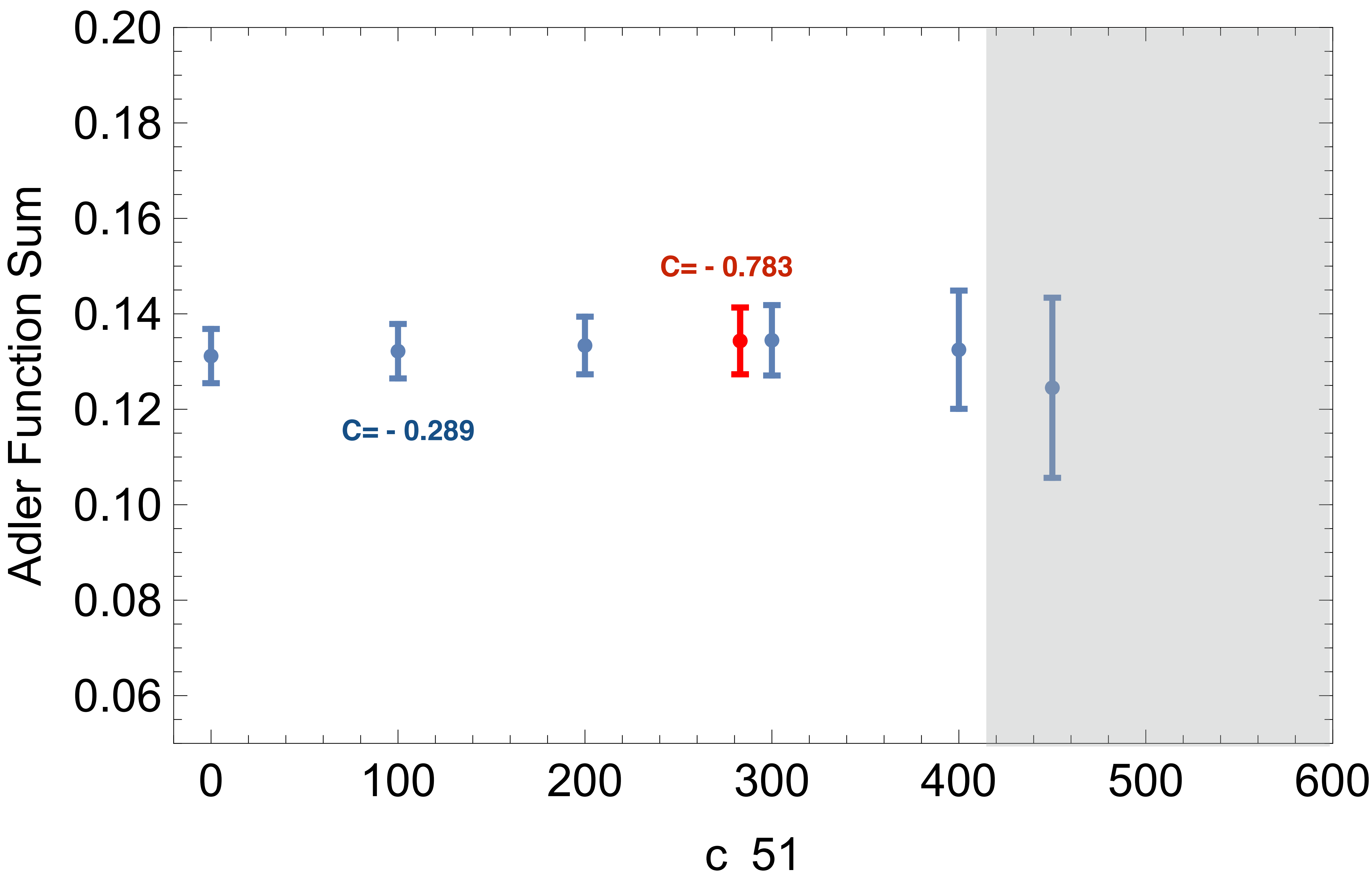

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Tau decays into hadrons

Perturbative contribution to the observable R_τ

$$R_\tau = \frac{\Gamma[\tau \to \mathrm{hadrons}\,\nu_\tau]}{\Gamma[\tau \to e^-\bar{\nu}_e\,\nu_\tau]} = N_c\, S_{\mathrm{EW}}\,(|V_{ud}|^2 + |V_{us}|^2)\,(1 + \delta^{(0)} + \cdots)$$

Prescriptions for the RG improvement

Fixed Order PT (FOPT)

$\mu = s_0$

$$\delta^{(0)}_{\mathrm{FO},w_i} = \sum_{n=1}^{\infty} a(s_0)^n \sum_{k=1}^{n} k\, c_{n,k}\, J^{\mathrm{FO},w_i}_{k-1}$$

Contour Improved PT (CIPT)

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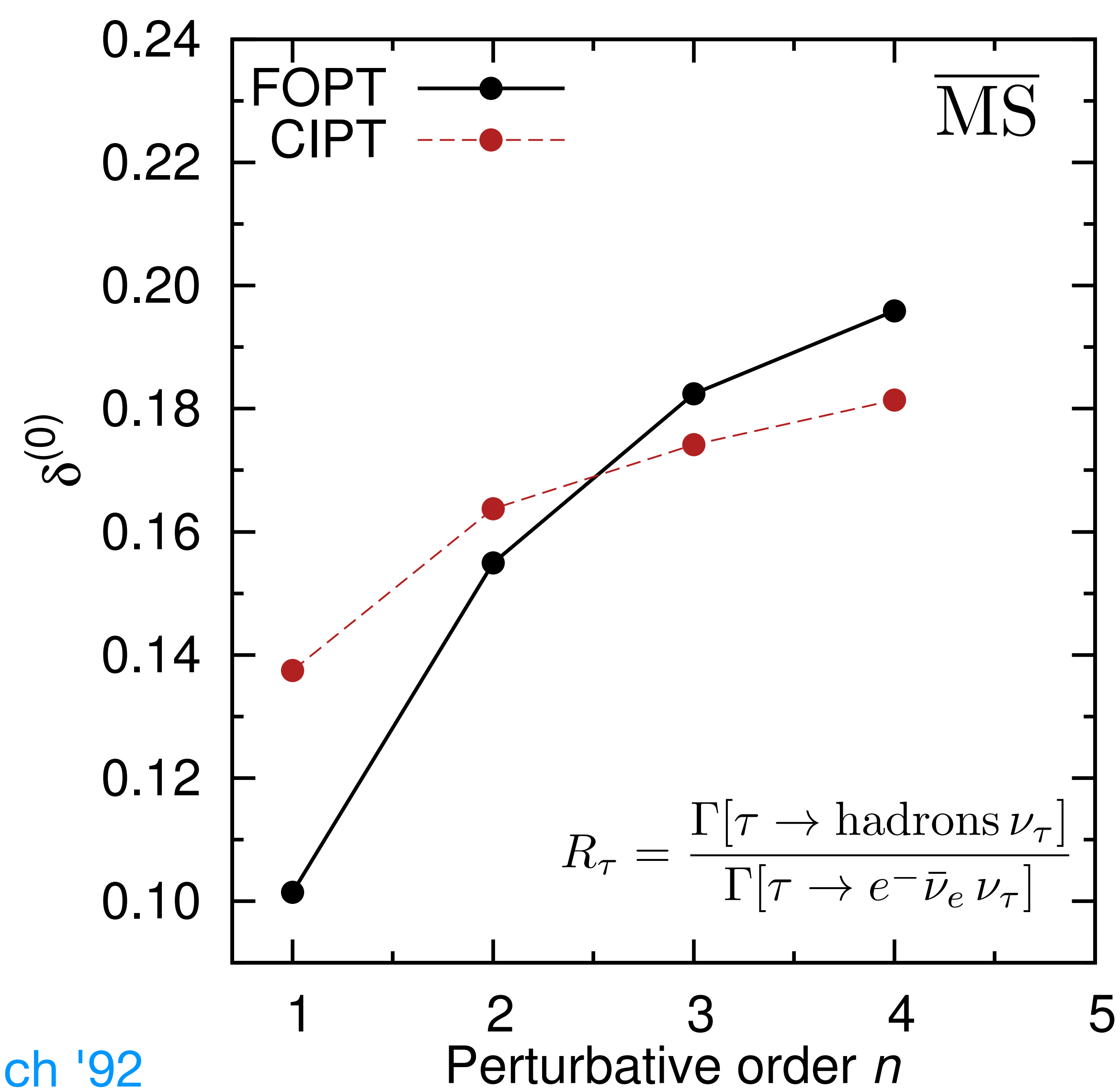
Le Diberder & Pich '92



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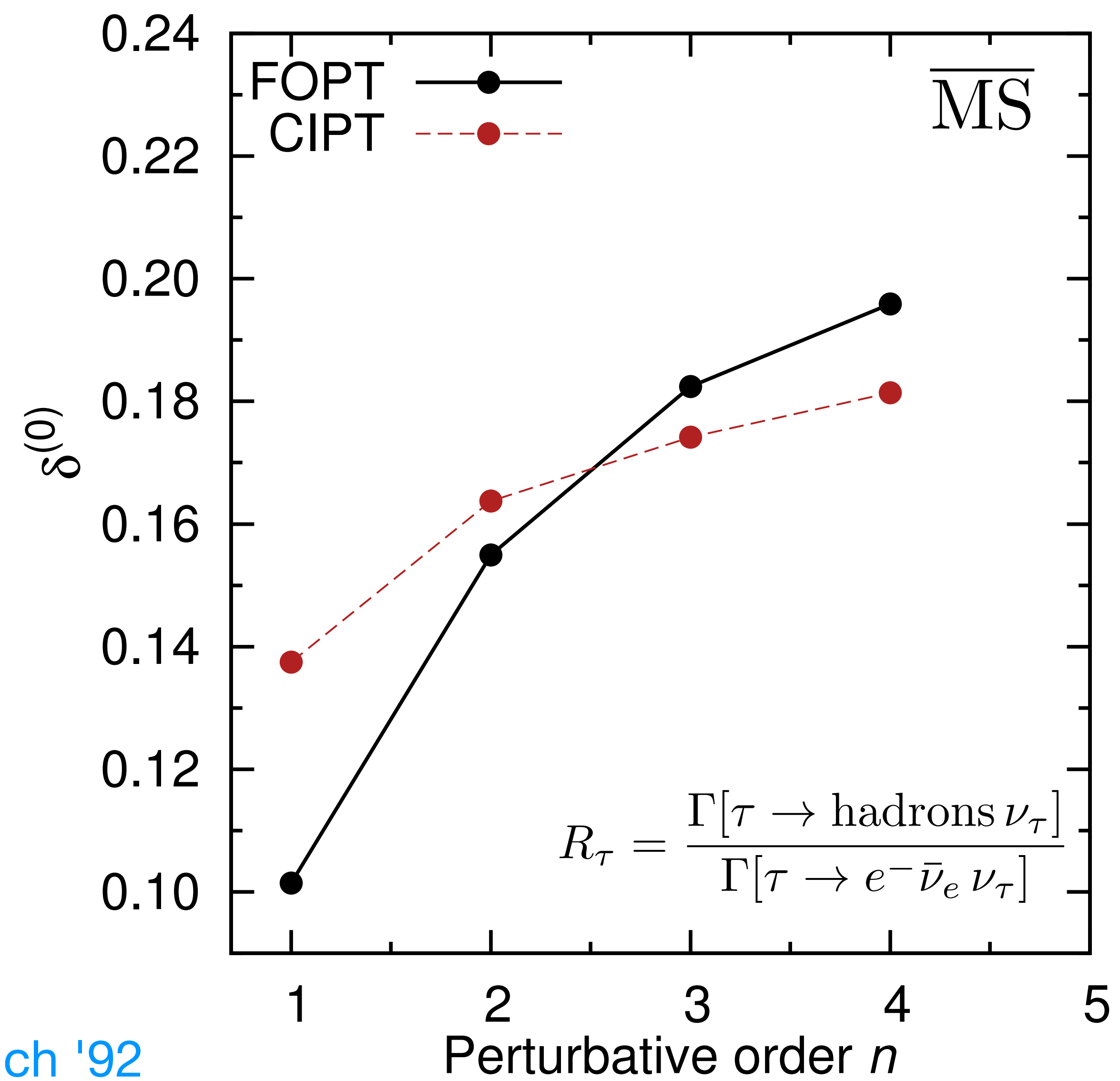

Le Diberder & Pich '92

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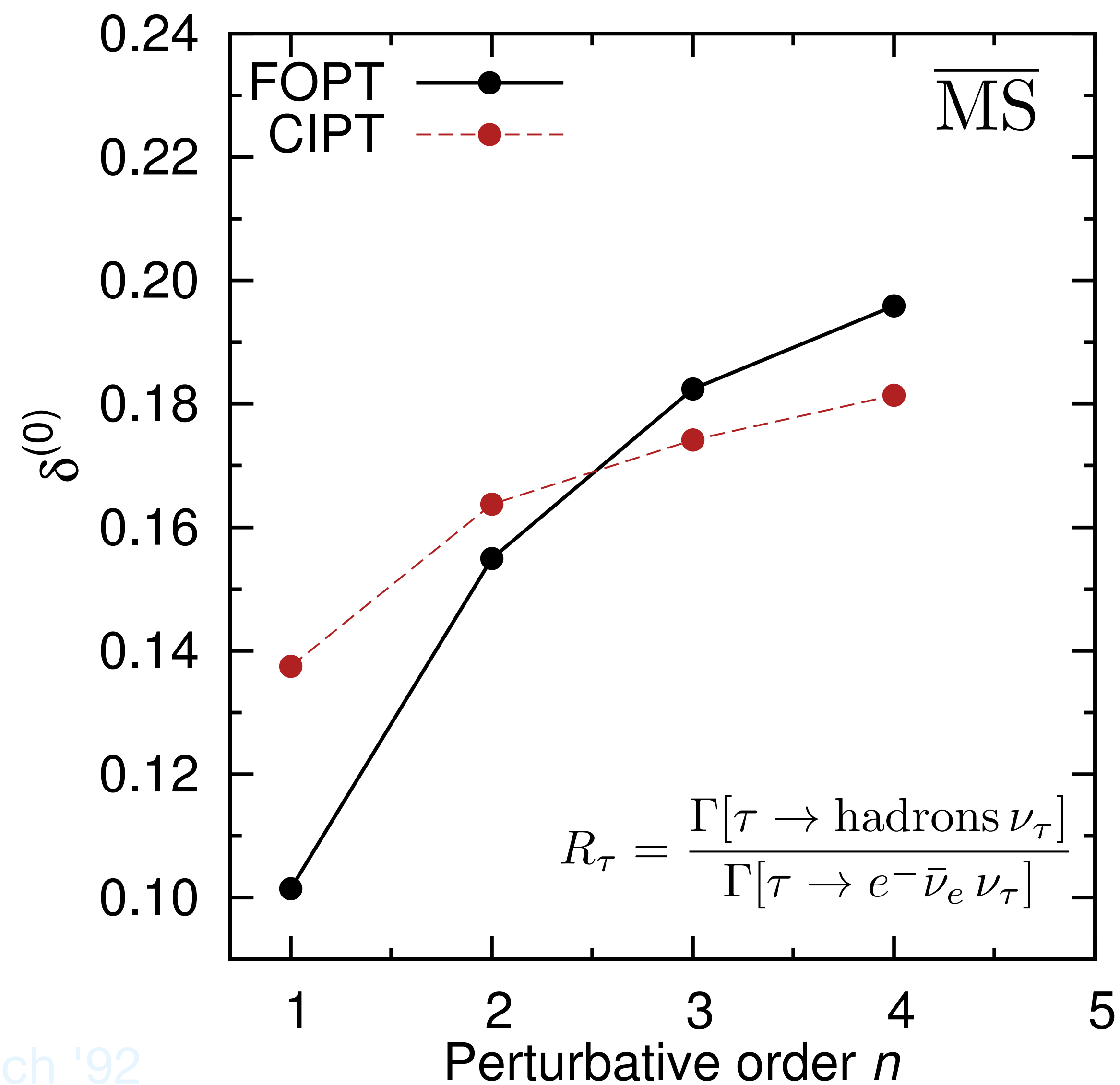

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$\overline{\rm MS}$ FOPT result:

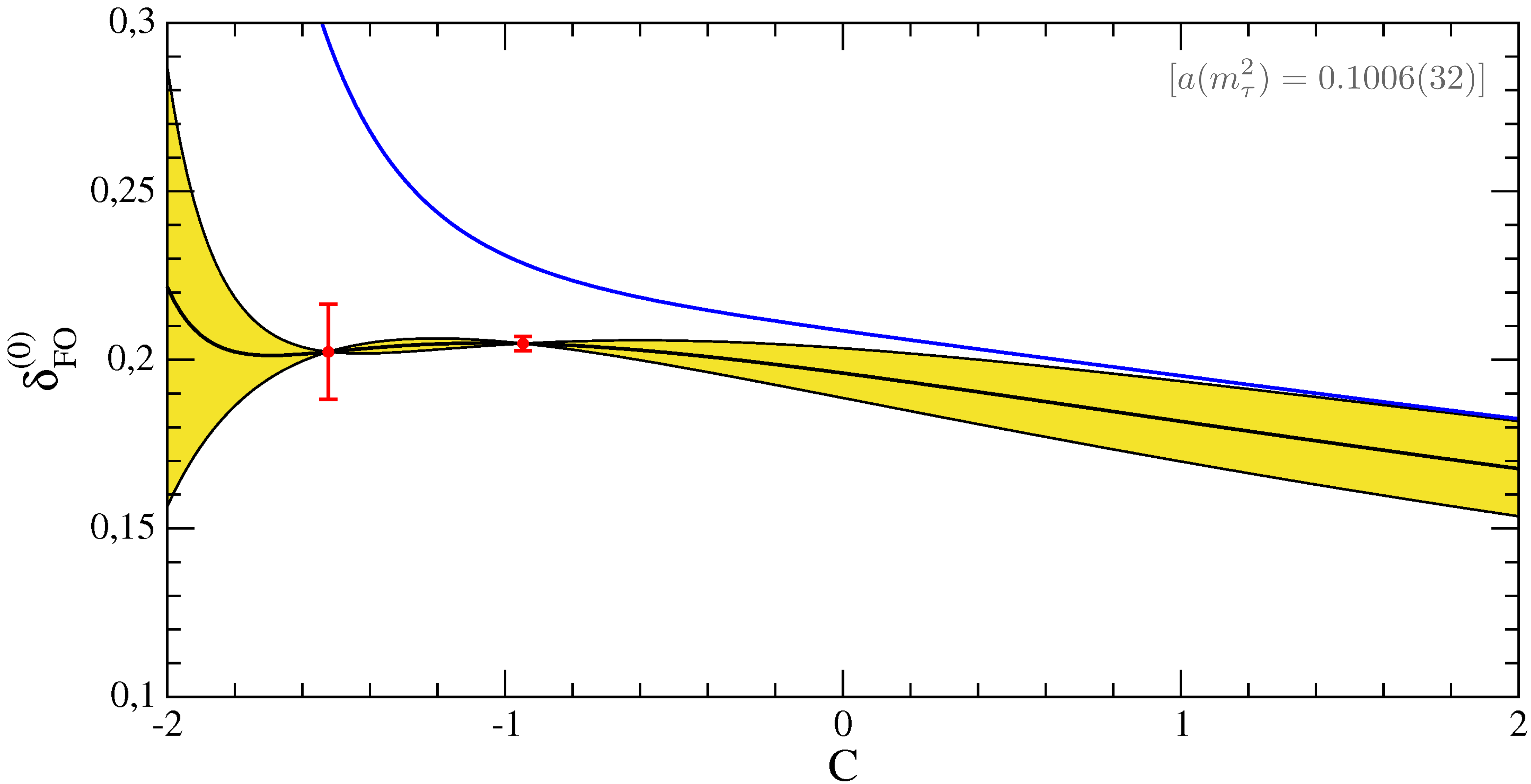
$$\delta^{(0)}_{\rm FO}(a_Q) \;=\; a_Q + 5.202\, a_Q^2 + 26.37\, a_Q^3 + 127.1\, a_Q^4 + \ldots$$

C-scheme FOPT result:

$$\boxed{\begin{aligned}\delta^{(0)}_{\rm FO}(\hat{a}_Q) \;=\; & \hat{a}_Q + (5.202 + 2.25C)\,\hat{a}_Q^2 + \;(27.68 + 27.41C + 5.063C^2)\,\hat{a}_Q^3 \\ & + \;(148.4 + 235.5C + 101.5C^2 + 11.39C^3)\,\hat{a}_Q^4 + \ldots\end{aligned}}$$

FOPT results

$c_{5,1} = 283$



$$\delta^{(0)}_{\rm FO}(\hat{a}_{M_\tau}, C = -0.882) \; = \; 0.2047 \pm 0.0034 \pm 0.0133$$

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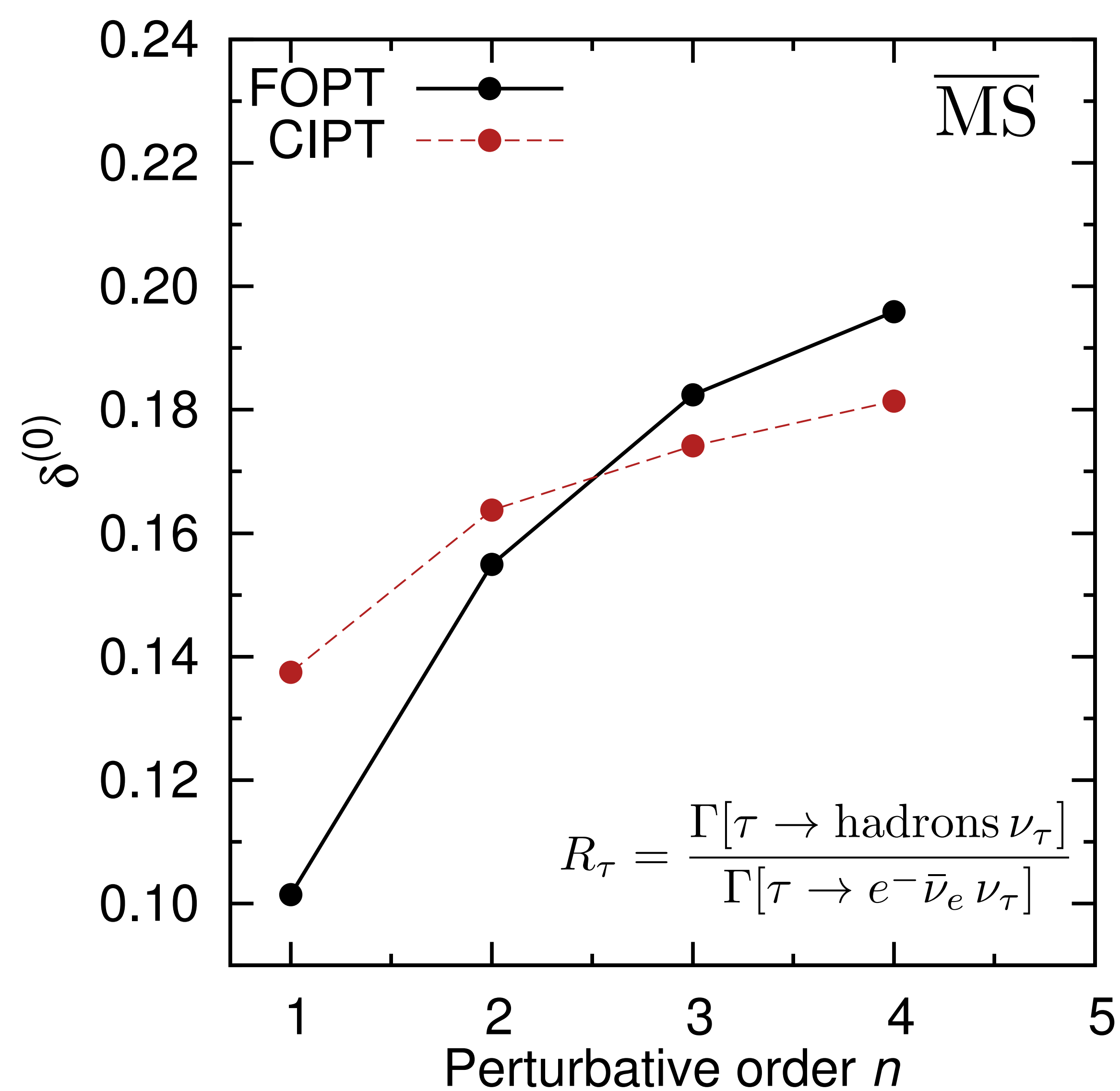

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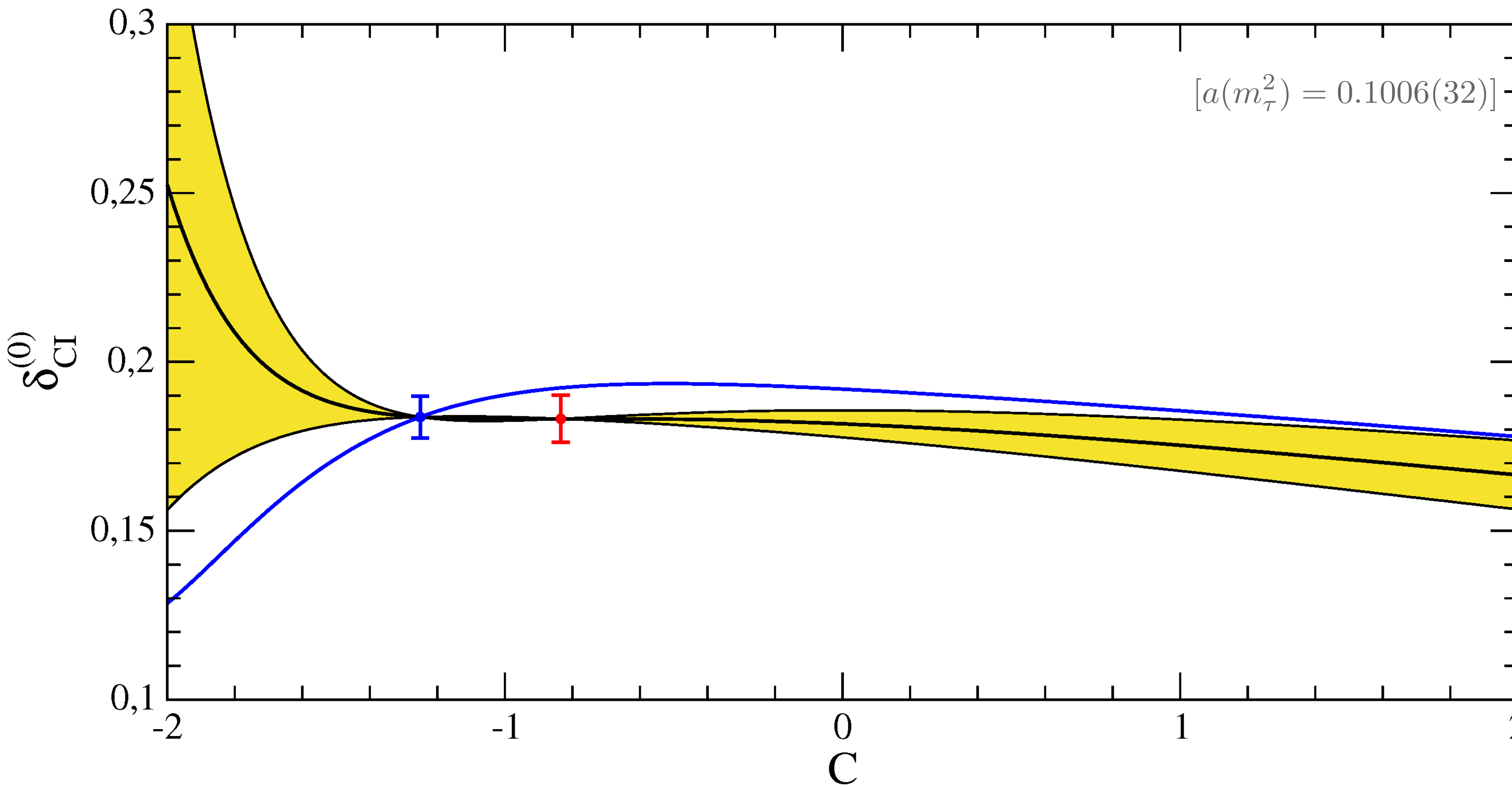

$$\delta^{(0)}_{\rm CI}(\hat{a}_{M_\tau}, C = -1.246) = 0.1840 \pm 0.0062 \pm 0.0084$$

Conclusions

Conclusions

- The C-scheme gives an extra handle on the pt. series: optimisation
- Can be applied to many different processes Jamin, Miravitllas '16
- Having the result for the fifth order coefficient would be excellent.
- Unfortunately, the CIPT vs FOPT problem persists

$$\delta^{(0)}_{\rm CI}(\hat{a}_{M_\tau}, C = -1.246) = 0.1840 \pm 0.0062 \pm 0.0084$$
$$\delta^{(0)}_{\rm FO}(\hat{a}_{M_\tau}, C = -0.882) = 0.2047 \pm 0.0034 \pm 0.0133$$

- Optimised results corroborate renormalon models of the Adler function

$$\widehat{D}(\hat{a}_{M_\tau}, C = -0.783) = 0.1343 \pm 0.0070 \pm 0.0067$$ [this work]

$$\widehat{D}(a_{M_\tau}) = 0.1354 \pm 0.0127 \pm 0.0058$$ [Renormalon based description]

Beneke & Jamin '08, DB, Beneke, Jamin '13

- Uncertainties tend to be smaller with more terms in the pt. series.

谢谢

Extra

Comparison

Comparison with $\overline{\mathrm{MS}}$

$[a(m_\tau^2) = 0.1006(32)]$

$$\widehat{D}(a_{M_\tau}) \;=\; 0.1316 \pm 0.0029 \pm 0.0060\,. \qquad \overline{\mathrm{MS}}$$

$$\widehat{D}(\hat{a}_{M_\tau}, C = -0.783) \;=\; 0.1343 \pm 0.0070 \pm 0.0067$$ [this work]

$$\widehat{D}(a_{M_\tau}) \;=\; 0.1354 \pm 0.0127 \pm 0.0058$$ [Renormalon based description]

Beneke & Jamin ’08, DB, Beneke, Jamin '13

$$\delta^{(0)}_{\mathrm{FO}}(a_{M_\tau}) \;=\; 0.1991 \pm 0.0061 \pm 0.0119\,. \quad \overline{\mathrm{MS}}$$

$$\delta^{(0)}_{\mathrm{BM}}(a_{M_\tau}) \;=\; 0.2047 \pm 0.0029 \pm 0.0130$$ [Renormalon based description]

$$\delta^{(0)}_{\mathrm{FO}}(\hat{a}_{M_\tau}, C = -0.882) \;=\; 0.2047 \pm 0.0034 \pm 0.0133$$

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Tau decays into hadrons: FOPT