

Strong isospin breaking at production of light scalars

N.N. Achasov and G.N. Shestakov

Laboratory of Theoretical Physics

Sobolev Institute for Mathematics

Novosibirsk, 630090, Russia

Abstract

It is discussed breaking the isotopic symmetry as **the tool** of studying the production and nature of light scalar mesons.

Based on

1. N.N. Achasov, S.A. Devyanin and G.N. Shestakov.

The $S^* - \delta^0 / f_0(980) - a_0(980)$ mixing as the threshold phenomenon.

Phys. Lett. B 88 (1979) 367–371.

2. N.N. Achasov, S.A. Devyanin and G.N. Shestakov.

Mixing of the scalar $S^* / f_0(980)$ and $\delta^0 / a_0(980)$ mesons and related phenomena.

Sov. J. Nucl. Phys. 33 (1981) 715.

Based on

3. N.N. Achasov and N.N. Shestakov.

Mechanisms of the reaction $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$ at high energies.

[Phys.Rev. D 56 \(1997\) 212–220.](#)

4. N.N. Achasov and A.V. Kiselev.

Once more on the **mixing** of the $a_0(980)$ and $f_0(980)$ mesons.

[Phys. Lett. B 534 \(2002\) 83.](#)

5. N.N. Achasov and G.N. Shestakov.

Proposed search for the $a_0^0(980) - f_0(980)$ mixing in polarization phenomena.

[Phys. Rev. Lett. 92 \(2004\) 182001.](#)

Based on

6. N.N. Achasov and G.N. Shestakov.

Manifestation of the $a_0^0(980) - f_0(980)$ mixing in the reaction $\pi^- p \rightarrow \eta \pi^0 n$ on a polarized target.

[Phys. Rev. D 70 \(2004\) 074015.](#)

7. N.N. Achasov, A.A. Kozhevnikov, and G.N. Shestakov.

Isospin breaking decay $\eta(1405) \rightarrow f_0(980) \pi^0 \rightarrow 3\pi$.

[Phys. Rev. D 92 \(2015\) 036003.](#)

8. N.N. Achasov, A.A. Kozhevnikov, and G.N. Shestakov.

Mechanisms of the isospin-breaking decay

$f_1(1285) \rightarrow f_0(980) \pi^0 \rightarrow 3\pi$.

[Phys. Rev. D 93 \(2016\) 114027.](#)

Introduction

The thirty seven years ago we discovered theoretically a threshold phenomenon known as the mixing of $a_0^0(980)$ and $f_0(980)$ resonances that breaks the isotopic invariance considerably, since the effect $\sim \sqrt{2(M_{K^0} - M_{K^+})/M_{K^0}} \approx 0,13$ in the module of the amplitude. This effect appears as the narrow, $2(M_{K^0} - M_{K^+}) \approx 8 \text{ MeV}$, resonant structure between the $K^+ K^-$ and $K^0 \bar{K}^0$ thresholds, $a_0^0(980) \rightarrow K \bar{K} \rightarrow f_0(980)$ and vice versa.

N.N. Achasov , S.A. Devyanin and G.N. Shestakov,
Phys. Lett. B 88, 367 (1979).

Nowadays this phenomenon is discovered experimentally and studied with the help of detectors

VES in Protvino and BES III in Beijing

in the processes:

Introduction

V. Dorofeev et al., Eur. Phys. J. A 38, 149 (2008), ibid 47, 68 (2011).

$$\pi^- N \rightarrow \pi^- f_1(1285) N \rightarrow \pi^- f_0(980) \pi^0 N \rightarrow \pi^- \pi^+ \pi^- \pi^0 N,$$

M. Ablikim et al., Phys. Rev. D 83, 032003 (2011),

Phys. Rev. Lett. 108, 182001 (2012), Phys. Rev. D 92, 012007 (2015).

$$J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \eta \pi^0,$$

$$\chi_{c1}(1P) \rightarrow a_0(980) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0,$$

$$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma f_0(980) \pi^0 \rightarrow \gamma 3\pi,$$

$$J/\psi \rightarrow \phi f_0(980) \pi^0 \rightarrow \phi 3\pi,$$

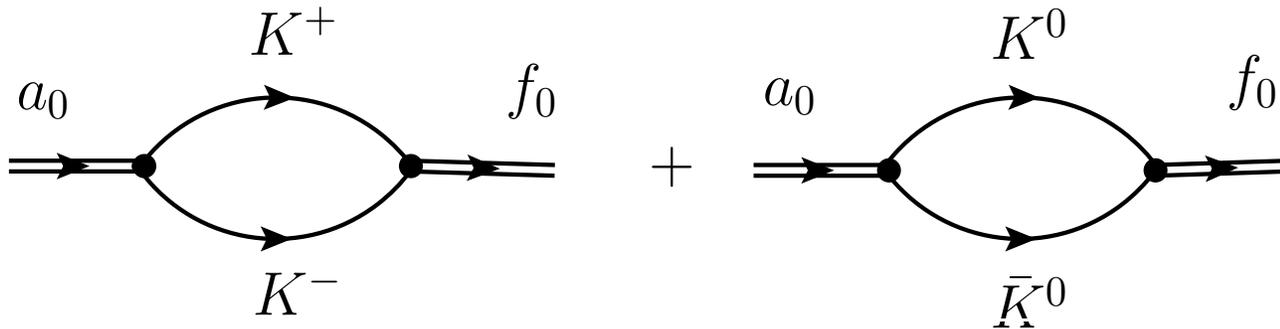
$$J/\psi \rightarrow \phi f_1(1285) \rightarrow \phi f_0(980) \pi^0 \rightarrow \phi 3\pi.$$

Introduction

It has become clear,
N.N. Achasov, A.A. Kozhevnikov, and G.N. Shestakov,
Phys. Rev. D 92, 036003 (2015) and Phys. Rev. D 93, 114027 (2016),
that the similar effect can appear not only due to
the $a_0^0(980) - f_0(980)$ mixing, but also for any mechanism of the
production of the $K\bar{K}$ pairs in the S wave,
 $X \rightarrow K\bar{K} \rightarrow f_0(980)/a_0(980)^a$.
Thus **a new tool** to study the mechanism of production and nature
of light scalars is emerged.

^a Each such mechanism reproduces both the narrow resonant peak and sharp jump of a phase of amplitude between the K^+K^- and $K^0\bar{K}^0$ thresholds.

The $a_0^0(980)$ - $f_0(980)$ mixing



$$\Pi_{a_0 f_0}(m) = \frac{g_{a_0 K^+ K^-} g_{f_0 K^+ K^-}}{16\pi} \left[i \left(\rho_{K^+ K^-}(m) - \rho_{K^0 \bar{K}^0}(m) \right) - \frac{\rho_{K^+ K^-}(m)}{\pi} \ln \frac{1 + \rho_{K^+ K^-}(m)}{1 - \rho_{K^+ K^-}(m)} + \frac{\rho_{K^0 \bar{K}^0}(m)}{\pi} \ln \frac{1 + \rho_{K^0 \bar{K}^0}(m)}{1 - \rho_{K^0 \bar{K}^0}(m)} \right]$$

$$\approx \frac{g_{a_0 K^+ K^-} g_{f_0 K^+ K^-}}{16\pi} \left[i \left(\rho_{K^+ K^-}(m) - \rho_{K^0 \bar{K}^0}(m) \right) \right],$$

where $m \geq 2m_{K^0}$, in the region $0 \leq m \leq 2m_K$,

The $a_0^0(980)$ - $f_0(980)$ mixing

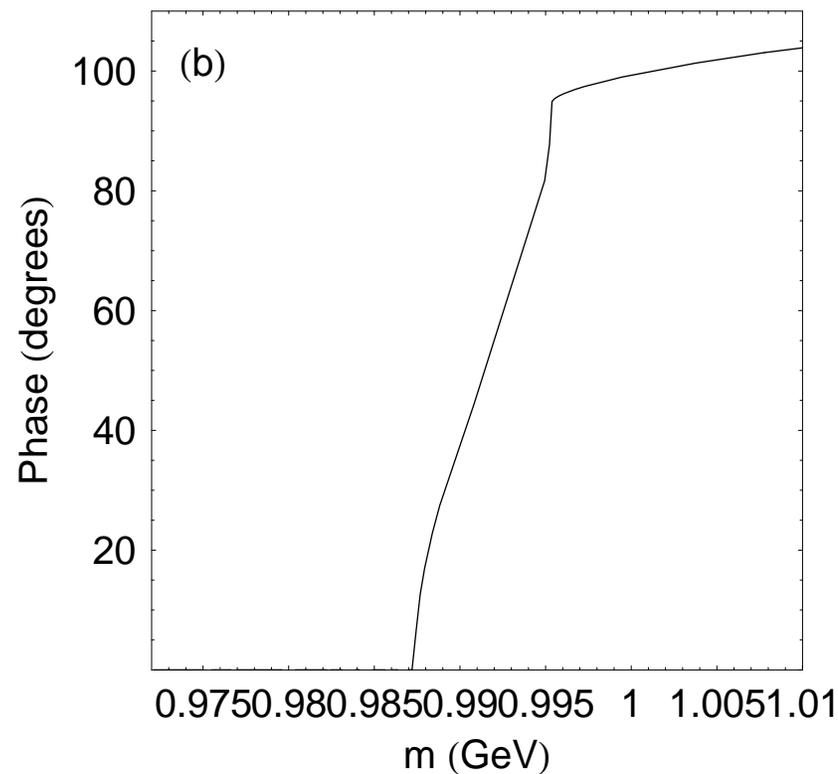
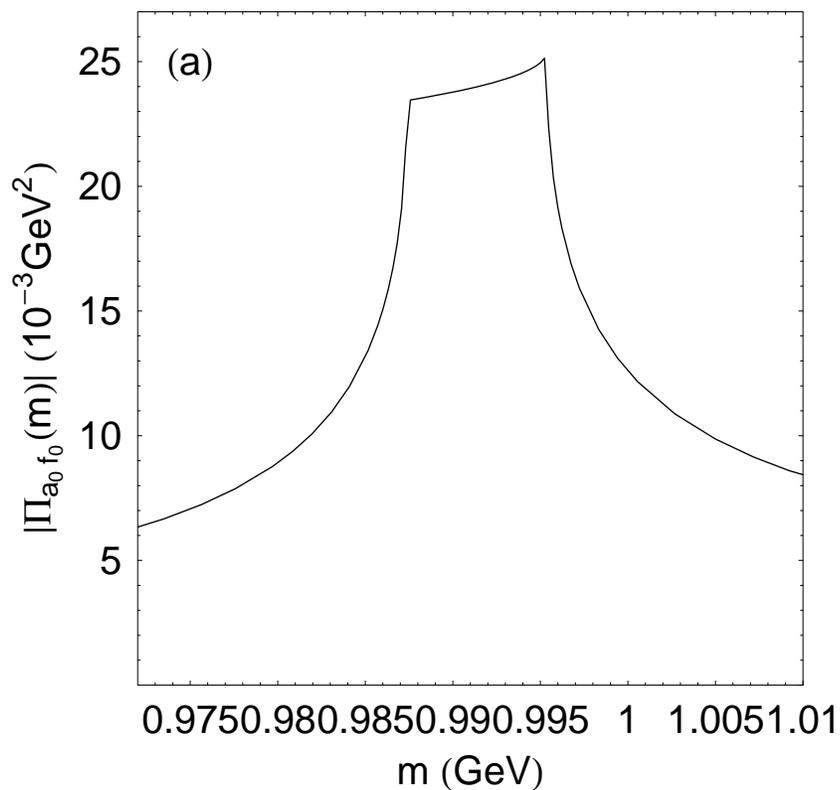
$\rho_{K\bar{K}}(m) = \sqrt{1 - 4m_K^2/m^2}$ should be replaced by $i|\rho_{K\bar{K}}(m)|$. In the region between the K^+K^- and $K^0\bar{K}^0$ thresholds, which is the 8 MeV wide,

$$\begin{aligned} |\Pi_{a_0 f_0}(m)| &\approx \frac{|g_{a_0 K^+ K^-} - g_{f_0 K^+ K^-}|}{16\pi} \sqrt{\frac{2(m_{K^0} - m_{K^+})}{m_{K^0}}} \\ &\approx 0.127 |g_{a_0 K^+ K^-} - g_{f_0 K^+ K^-}| / 16\pi \simeq 0.03 \text{ GeV}^2 \\ &\approx m_K \sqrt{m_{K^0}^2 - m_{K^+}^2} \approx m_K^{3/2} \sqrt{m_d - m_u}. \end{aligned}$$

Note that

$$|\Pi_{\rho^0 \omega}| \approx |\Pi_{\pi^0 \eta}| \approx 0.003 \text{ GeV}^2 \approx (m_d - m_u) \times 1 \text{ GeV}.$$

The $a_0^0(980)$ - $f_0(980)$ mixing

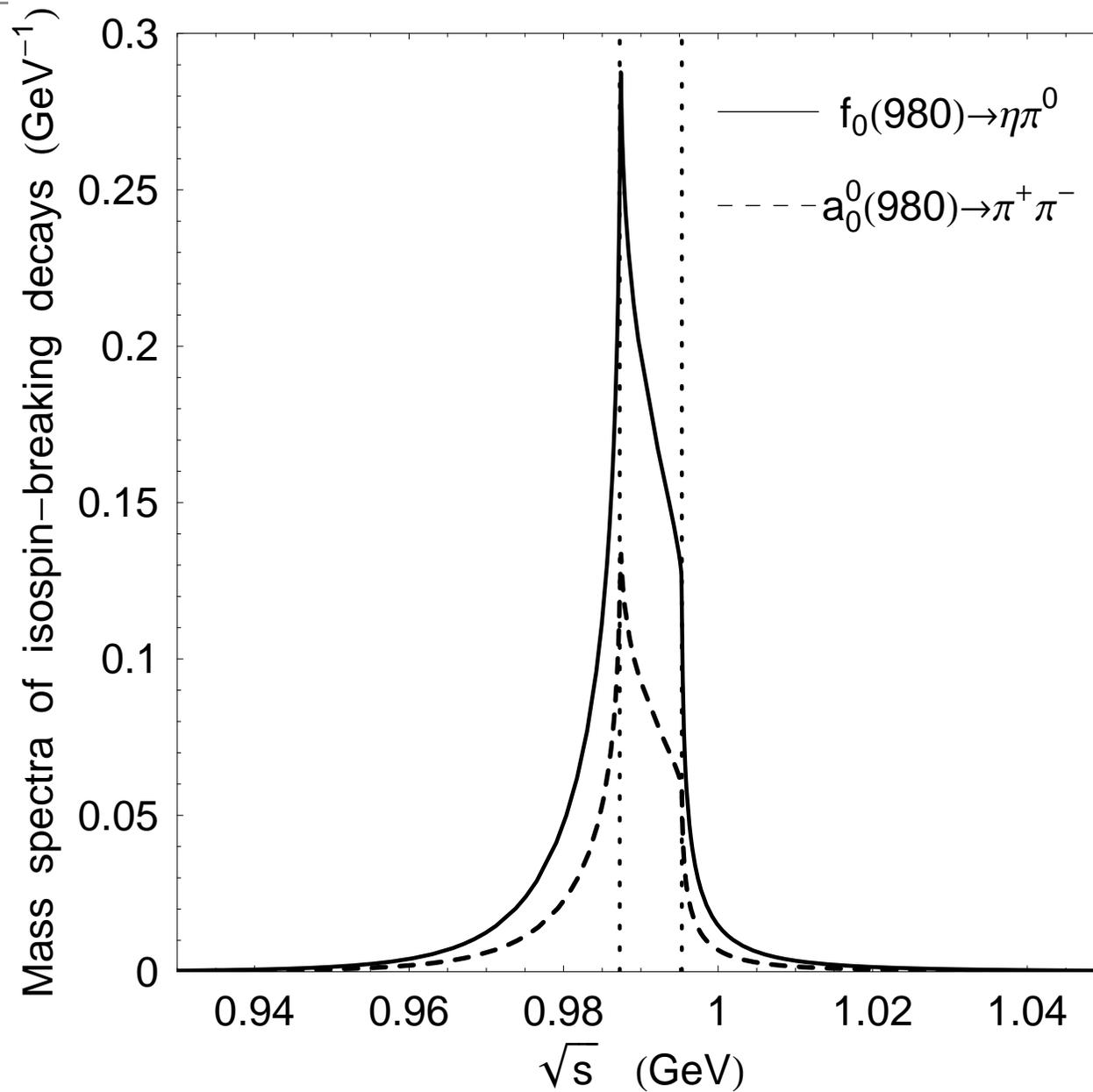


The $a_0^0(980)$ - $f_0(980)$ mixing

$$BR(f_0(980) \rightarrow K\bar{K} \rightarrow a_0^0(980) \rightarrow \eta\pi^0) = \int \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m) - \Pi_{a_0^0 f_0}^2(m)} \right|^2 \frac{2m^2\Gamma_{a_0^0 \rightarrow \eta\pi^0}(m)}{\pi} dm \approx 0.3\%$$

$$BR(a_0^0(980) \rightarrow K\bar{K} \rightarrow f_0(980) \rightarrow \pi\pi) = \int \left| \frac{\Pi_{a_0^0 f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m) - \Pi_{a_0^0 f_0}^2(m)} \right|^2 \frac{2m^2\Gamma_{f_0 \rightarrow \pi\pi}(m)}{\pi} dm \approx 0.2\%$$

Mass spectra



Polarization Phenomena

The phase jump suggest the idea to study the $a_0^0(980)$ - $f_0(980)$ mixing in polarization phenomena. If a process amplitude with a spin configuration is dominated by the $a_0^0(980)$ - $f_0(980)$ mixing then a spin asymmetry of a cross section jumps near the $K \bar{K}$ thresholds. An example is

$$\pi^- p_{\uparrow} \rightarrow (a_0^0(980) + f_0(980)) n \rightarrow a_0^0(980) n \rightarrow \eta \pi^0 n.$$

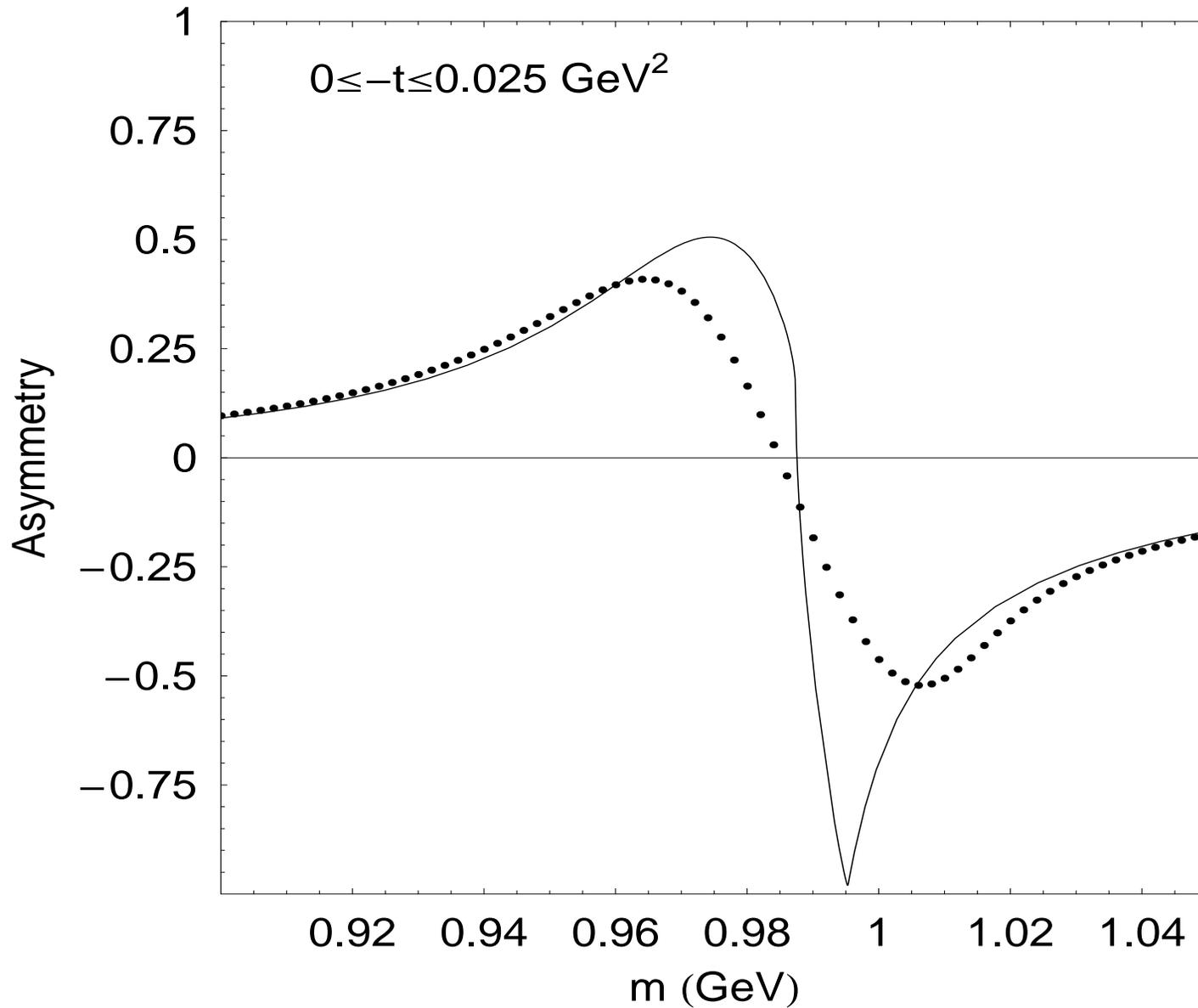
$$\frac{d^3 \sigma}{dt dm d\psi} = \frac{1}{2\pi} [|M_{+++}|^2 + |M_{+-}|^2 + 2 \Im(M_{+++} M_{+-}^*) P \cos \psi]$$

The dimensionless normalized spin asymmetry

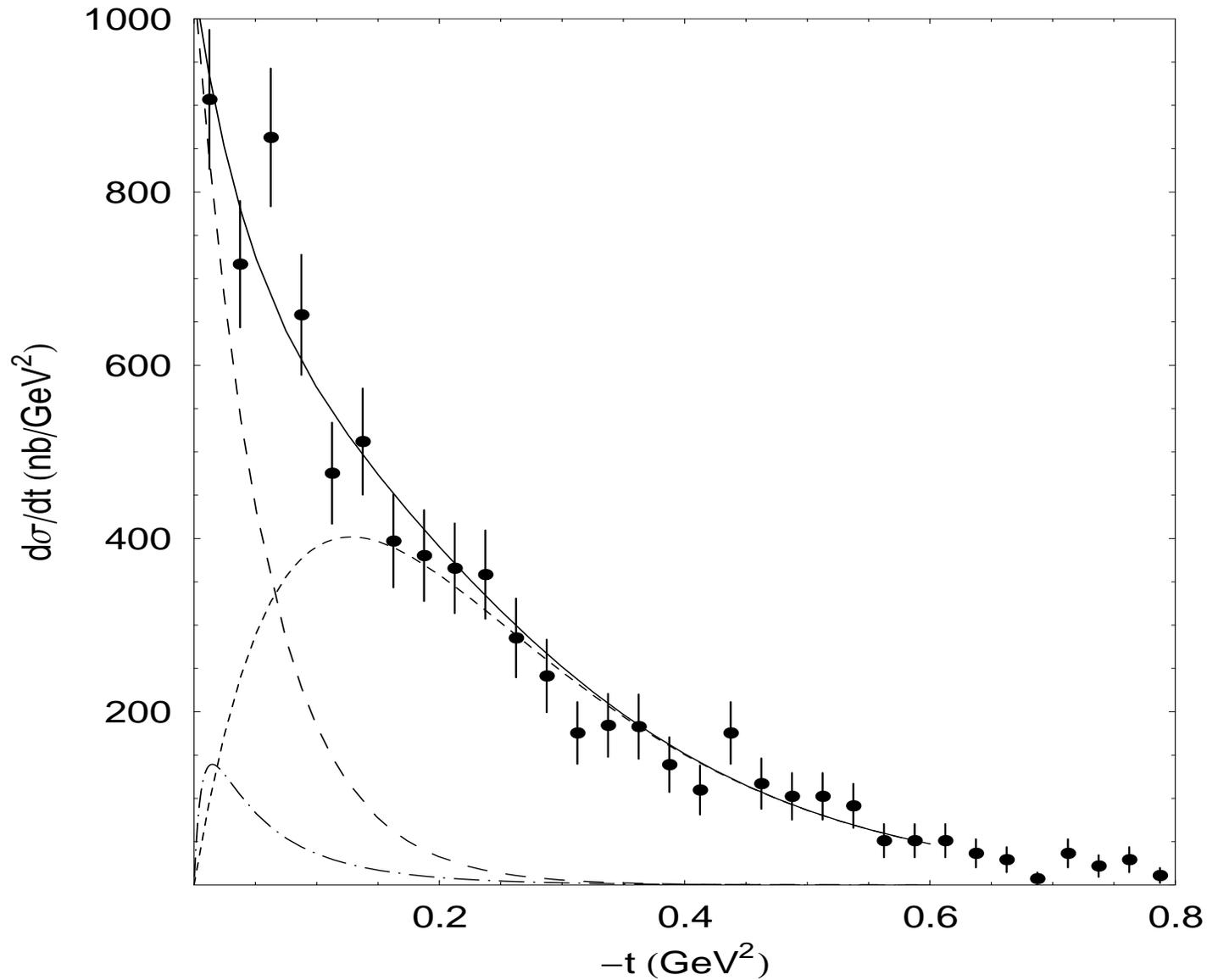
$$A(t, m) = 2 \Im(M_{+++} M_{+-}^*) / [|M_{+++}|^2 + |M_{+-}|^2],$$

$$-1 \leq A(t, m) \leq 1.$$

Spin Assymetry



Cross Sections



$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

Estimated are the contributions of the following mechanisms responsible for the decay

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0:$$

1) the contribution of the $a_0^0(980) - f_0(980)$ mixing,

$$f_1(1285) \rightarrow a_0(980)\pi^0 \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0,$$

2) the contribution of the transition

$$f_1(1285) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0,$$

arising due to the pointlike decay $f_1(1285) \rightarrow K\bar{K}\pi^0$,

3) the contribution of the transition

$$f_1(1285) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0, \text{ where } K^* = K^*(892), \text{ and}$$

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

4) the contribution of the transition

$$f_1(1285) \rightarrow (K_0^* \bar{K} + \bar{K}_0^* K) \rightarrow (K^+ K^- + K^0 \bar{K}^0)\pi^0 \rightarrow f_0(980) \rightarrow \pi^+ \pi^- \pi^0,$$

where $K_0^* = K_0^*(800)$ (or κ) and $K_0^*(1430)$.

These mechanisms break the conservation of the isospin due to **the nonzero mass difference of the K^+ and K^0 mesons.**

They result in the appearance of the narrow resonance structure in the $\pi^+ \pi^-$ mass spectrum in the region of the $K \bar{K}$ thresholds, with the width $\approx 2m_{K^0} - 2m_{K^+} \approx 8$ MeV.

The observation of such a structure in experiment is the direct indication on the $K \bar{K}$ loop mechanism of the breaking of the isotopic invariance.

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

We point out that existing data should be more precise, and it is difficult to explain them using the single specific mechanism from those listed above.

Taking the decay $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ as the example, we discuss the general approach to the description of the $K\bar{K}$ loop mechanism of the breaking of isotopic invariance.

1) The matter is that

the $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \eta \pi^0$ and $\chi_{c1}(1P) \rightarrow a_0^0(980)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ decays are described by the $a_0^0(980)$ - $f_0(980)$ mixing well enough:

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

$$\frac{BR(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \eta \pi^0)}{BR(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi \pi)}$$

$$BR(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi \pi)$$

$$= (0.60 \pm 0.20(stat.) \pm 0.12(sys.) \pm 0.26(para.))\%$$

$$\approx \frac{BR(f_0(980) \rightarrow K\bar{K} \rightarrow a_0^0(980) \rightarrow \eta \pi^0)}{BR(f_0(980) \rightarrow \pi \pi)},$$

$$\frac{BR(\chi_{c1}(1P) \rightarrow a_0^0(980)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(\chi_{c1} \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)}$$

$$BR(\chi_{c1} \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)$$

$$= (0.31 \pm 0.16(stat.) \pm 0.14(sys.) \pm 0.03(para.))\%$$

$$\approx \frac{BR(a_0^0(980) \rightarrow K\bar{K} \rightarrow f_0(980) \rightarrow \pi^+\pi^-)}{BR(a_0^0(980) \rightarrow \eta\pi^0)}.$$

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

As for the $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$ decay, its description requires the **"terrible"** $a_0^0(980)$ - $f_0(980)$ mixing:

$$\frac{\text{BR}(f_1(1285) \rightarrow a_0^0(980)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{\text{BR}(f_1(1285) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)}$$

$$= (2.5 \pm 0.9)\%$$

$$\approx \frac{\text{BR}(a_0^0(980) \rightarrow K\bar{K} \rightarrow f_0(980) \rightarrow \pi^+\pi^-)}{\text{BR}(a_0^0(980) \rightarrow \eta\pi^0)},$$

and, as a result, the inconvenient coupling constants of the scalar mesons with the pseudo-scalar mesons in the many cases:

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

$$\frac{g_{f_0\pi^+\pi^-}^2}{4\pi} = 1.2 \text{ GeV}^2, \quad \frac{g_{f_0K^+K^-}^2}{4\pi} = 5.7 \text{ GeV}^2,$$

$$\frac{g_{a_0^0\eta\pi^0}^2}{4\pi} = 1.9 \text{ GeV}^2, \quad \frac{g_{a_0^0K^+K^-}^2}{4\pi} = 9.9 \text{ GeV}^2.$$

For example, due to the very strong coupling of $a_0^0(980)$ with the $K\bar{K}$ channel, the width of the $a_0^0(980)$ resonance in the $\eta\pi^0$ mass spectrum turns out to be near 15 MeV.

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

2) The pointlike decay $f_1(1285) \rightarrow K\bar{K}\pi^0$ gives

$$\frac{BR(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(f_1(1285) \rightarrow K\bar{K}\pi)} = 0.0022$$

instead of the experimental value

$$\frac{BR(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(f_1(1285) \rightarrow K\bar{K}\pi)} = 0.033 \pm 0.010.$$

The $\pi^+\pi^-$ mass spectrum in the decay

$$f_1(1285) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$$

looks similar to the curves in the $a_0(980)$ - $f_0(980)$ mixing.

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

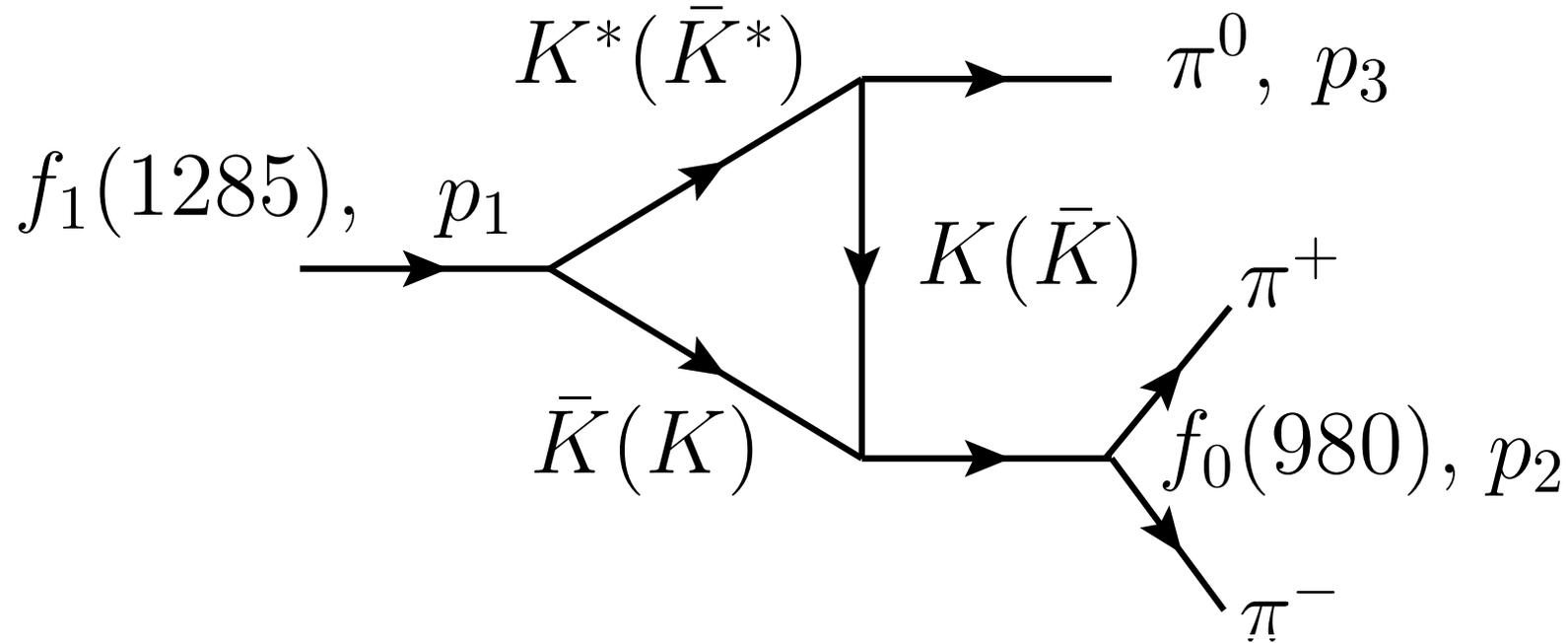
However, it is clear that the pointlike mechanism of the decay $f_1(1285) \rightarrow K\bar{K}\pi$ cannot by itself provide the considerable probability of the $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ transition.

3) The transition $f_1(1285) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ gives the shape of the $\pi^+\pi^-$ spectrum practically coincides with the corresponding spectrum caused by the $a_0^0(980) - f_0(980)$ mixing, but its

$$\text{BR}(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0) \approx 0.0255\%$$

is much less than the experimental value

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$



$$\text{BR}(f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0) = (0.30 \pm 0.09)\%.$$

$$f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$$

So, the $f_1(1285) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ transition mechanism alone is also insufficient to understand the experimental data.

4) The variant $f_1(1285) \rightarrow (K_0^*(800)\bar{K} + \bar{K}_0^*(800)K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ is rejected by the spectra in the $f_1(1285) \rightarrow K\bar{K}\pi$.

As for $f_1(1285) \rightarrow (K_0^*(1430)\bar{K} + \bar{K}_0^*(1430)K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$, it provides the results similar to $f_1(1285) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow (K^+K^- + K^0\bar{K}^0)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ and consequently cannot describe the data alone.

The consistency condition

$$\begin{aligned} \mathcal{M}_{f_1(1285) \rightarrow f_0(980)\pi^0}(s) = & g_{f_0 K^+ K^-} \{ A(s) \\ & \times i[\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)] + B(s)[\rho_{K^+ K^-}^2(s) \\ & - \rho_{K^0 \bar{K}^0}^2(s)] + O[\rho_{K^+ K^-}^3(s) - \rho_{K^0 \bar{K}^0}^3(s)] + \dots \}. \end{aligned}$$

With a good accuracy

$$\mathcal{M}_{f_1(1285) \rightarrow f_0(980)\pi^0}(s) = g_{f_0 K^+ K^-} A(s) i[\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)].$$

The amplitude $A(s)$ contains the information about all possible mechanisms of production of the $K \bar{K}$ system with isospin $I = 1$ in S wave in the process $f_1(1285) \rightarrow K \bar{K} \pi$.

The consistency condition

From the data on $f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ one can extract the information about $|A(s)|^2$ in the region of the K^+K^- and $K^0\bar{K}^0$ thresholds.

$$\frac{d\Gamma_{f_1(1285) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0}(s)}{d\sqrt{s}} = \frac{1}{16\pi} |\mathcal{M}_{f_1 \rightarrow f_0\pi^0}(s)|^2 p^3(s) \frac{2s\Gamma_{f_0 \rightarrow \pi^+\pi^-}(s)}{\pi |D_{f_0}(s)|^2}.$$

$$\frac{d\Gamma_{f_1 \rightarrow K^+K^-\pi^0}}{d\sqrt{s}} = \frac{2\sqrt{s}}{\pi} \rho_{K^+K^-}(s) p^3(s) |A(s)|^2.$$

$$\Gamma_{f_1 \rightarrow f_0\pi^0 \rightarrow \pi^+\pi^-\pi^0} = |A(4m_{K^+}^2)|^2 2.59 \times 10^{-6} \text{ GeV}^5.$$

$$J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0$$

According to BESIII, the mass and width of the $\eta(1405)$ peak in the $\pi^+\pi^-\pi^0$ channel are 1409.0 ± 1.7 MeV and 48.3 ± 5.2 MeV, respectively, while the branching ratio is

$$\begin{aligned} BR(J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0) \\ = (1.50 \pm 0.11 \pm 0.11) \cdot 10^{-5}. \end{aligned}$$

In addition, the BESIII gives the ratio

$$\frac{BR(\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{BR(\eta(1405) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} = (17.9 \pm 4.2)\%,$$

that rules out practically the explanation of the discovered effect by means of the $a_0(980)$ - $f_0(980)$ mixing.

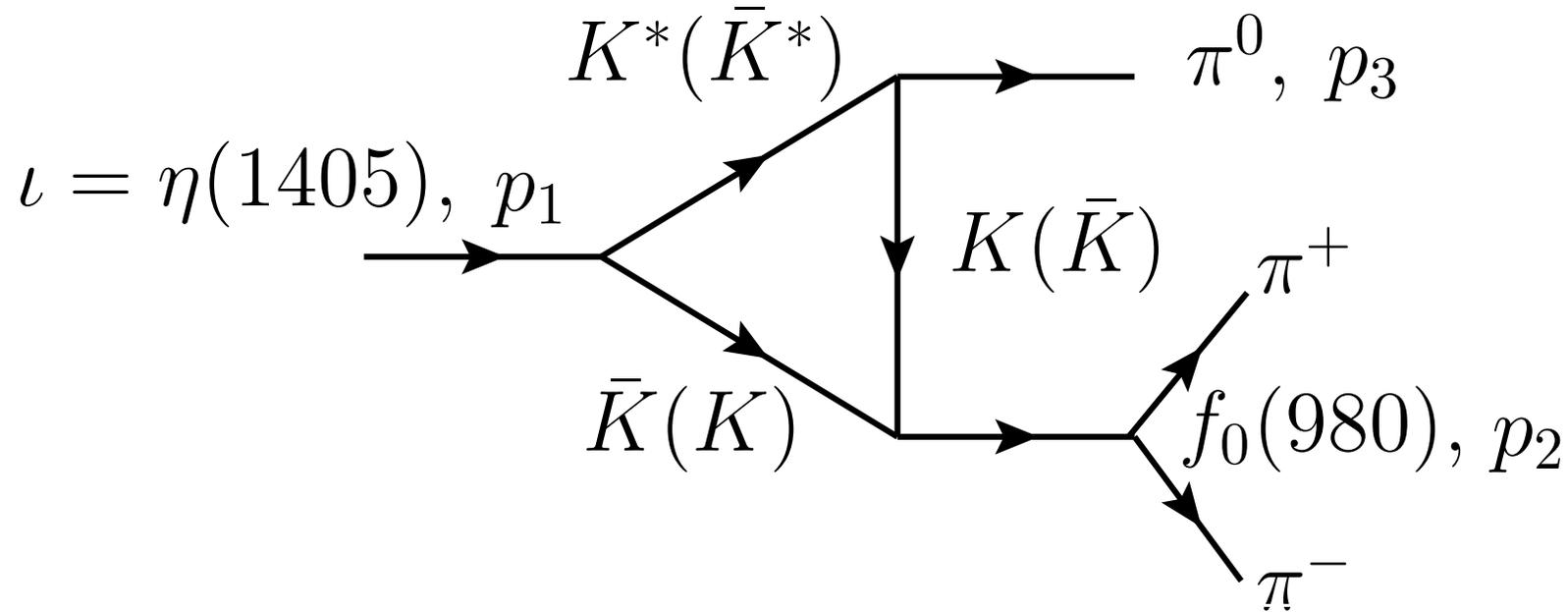
$$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma f_0(980) \pi^0 \rightarrow \gamma \pi^+ \pi^- \pi^0$$

We discuss the possibility of the theoretical explanation of the large breaking of isotopic invariance in the decay $\eta(1405) \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0$ by means of the anomalous Landau thresholds (the logarithmic triangle singularities) which are in the transition

$$\eta(1405) \rightarrow (K^* \bar{K} + \bar{K}^* K) \rightarrow (K^+ K^- + K^0 \bar{K}^0) \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow \pi^+ \pi^- \pi^0$$

and show that the account of the finite width of the K^* ($\Gamma_{K^* \rightarrow K \pi} \approx 50 \text{ MeV}$) smoothes the logarithmic singularities in the amplitude and results in the suppression of the calculated decay width of $\eta(1405) \rightarrow f_0(980) \pi^0 \rightarrow 3\pi$ by the factor of 6 – 8 as compared with the case of $\Gamma_{K^* \rightarrow K \pi} = 0$.

$$\eta(1405) \rightarrow K^* \bar{K} + c.c. \rightarrow K \bar{K} \pi^0 \rightarrow f_0(980) \pi^0 \rightarrow 3\pi$$



In the region of the $\eta(1405)$ resonance all intermediate particles in the loop of triangle diagram can lie on their mass shells. That is, in the hypothetical case of the stable K^* meson the logarithmic singularity appears in the imaginary part of the triangle diagram.

$$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma f_0(980) \pi^0 \rightarrow \gamma \pi^+ \pi^- \pi^0$$

The accounting of the finite width of the K^* resonance, i.e., the averaging of the amplitude over the resonance Breit-Wigner distribution in accord with the spectral Källén-Lehmann representation for the propagator of the unstable K^* meson, smoothes the logarithmic singularities of the amplitude and hence makes the compensation of the contributions of the $K^{*+} K^- + K^{*-} K^+$ and $K^{*0} \bar{K}^0 + \bar{K}^{*0} K^0$ intermediate states more strong.

This results in both the diminishing of the calculated width of the decay $\eta(1405) \rightarrow \pi^+ \pi^- \pi^0$ by a number of times in comparison with the case of $\Gamma_{K^* \rightarrow K \pi} = 0$, and in the concentration of the main effect of the isospin breaking in the domain of the $\pi^+ \pi^-$ invariant mass between the $K \bar{K}$ thresholds.

$$J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0$$

Assuming the dominance of the
 $\eta(1405) \rightarrow K^*\bar{K} + c.c. \rightarrow K\bar{K}\pi^0$ decay, one obtains

$$\text{BR}(J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma 3\pi) \approx 1.12 \cdot 10^{-5},$$

that reasonably agrees with experiment.

$$J/\psi \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0$$

CONCLUSION

We also analyze the difficulties related with the assumption of the dominance of the $\eta(1405) \rightarrow (K^*\bar{K} + \bar{K}^*K) \rightarrow K\bar{K}\pi$ decay mechanism and discuss the possible dynamics of the decay $\eta(1405) \rightarrow \eta\pi\pi$.

The decisive improvement of the experimental data on the $K\bar{K}$, $K\pi$, $\eta\pi$, and $\pi\pi$ mass spectra in the decay of the resonance structure $\eta(1405/1475)$ to $K\bar{K}\pi$ and $\eta\pi\pi$, and on the shape of the resonance peaks themselves in the $K\bar{K}\pi$ and $\eta\pi\pi$ decay channels is necessary for the further establishing the $\eta(1405) \rightarrow 3\pi$ decay mechanism.

Acknowledgments

The present work is partially supported by the Russian Foundation for Basic Research Grant No. 16-02-00065 and the Presidium of the Russian Academy of Sciences project No. 0314-2015-0011.

THANK YOU