Determination of the QCD Coupling from ALEPH tau Decay Data

Antonio Rodríguez Sánchez

IFIC (UV-CSIC)

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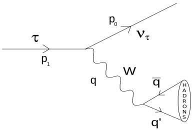
In collaboration with:

Antonio Pich

Spectral function from τ decays



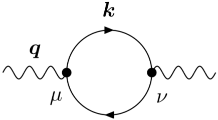
$$R_\tau \equiv \frac{\Gamma[\tau^- \to \nu_\tau \mathrm{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]}$$

Braaten-Narison-Pich '92

$$= 12\pi\, S_{\mathrm{EW}} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\, \Pi^{(1)}(s) + \mathrm{Im}\, \Pi^{(0)}(s)\right]$$

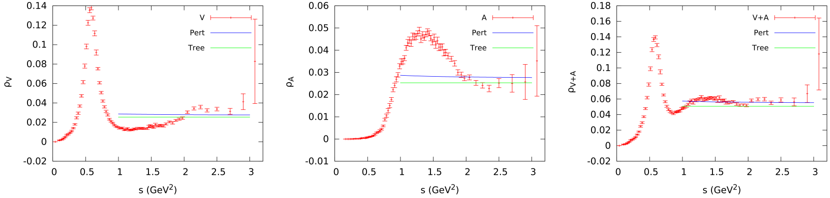
Two-point correlation function of quark currents

$$\Pi^{(J)}(s) \equiv \sum_{q=d,s} |V_{uq}|^2 \left(\Pi^{(J)}_{uq,V}(s) + \Pi^{(J)}_{uq,A}(s)\right)$$



Experimental spectral functions $\rho_{ud}(s) = \frac{1}{\pi}\, \mathrm{Im}\, \Pi_{ud}(s)$ from ALEPH

Davier et al., 1312.1501

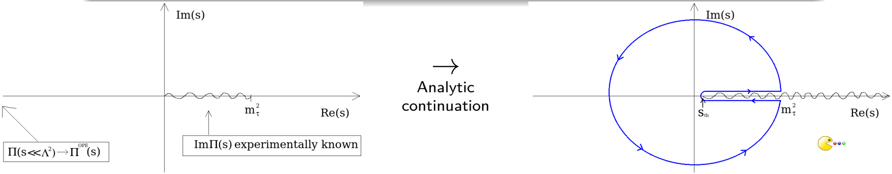


Theoretical Framework

OPE of the QCD correlator $\Pi^{(1+0)}(s)$ — Shifman-Vainshtein-Zakharov ('78)

$$\Pi^{\mathrm{OPE}}_{V/A}(s=-Q^2) = \sum_D \frac{1}{(Q^2)^{D/2}} \sum_{\dim \mathcal{O}=D} C_{D,V/A}(Q^2,\mu)\langle \mathcal{O}(\mu)\rangle \equiv \sum_D \frac{\mathcal{O}_{D,\,V/A}}{(Q^2)^{D/2}}; \qquad Q^2 \gg \Lambda^2$$

At $Q^2 \sim m_\tau^2$ **dominated by D = 0** (purely perturbative)



$$A^{\omega}_{V/A}(s_0) \equiv \underbrace{\int_{s_{\mathrm{th}}}^{s_0} \frac{ds}{s_0}\, \omega(s)\, \mathrm{Im}\,\Pi_{V/A}(s)}_{\text{Experimental}} = \underbrace{\frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi_{V/A}(s)}_{\text{Theoretical}}$$

Theoretical Framework: Duality Violations

$$A^{\omega}_{V/A}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi^{OPE}_{V/A}(s) + \Delta A^{\omega,\mathrm{DV}}_{V/A}(s_0)$$

Duality violations: Physical - OPE

$$\Delta A^{\omega,\mathrm{DV}}_{V/A}(s_0) \equiv \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi^{\mathrm{OPE}}_{V/A}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta\rho^{\mathrm{DV}}_{V/A}(s)$$

- Reduced with **pinched weight functions** (avoid the cut in the positive real axis):

  $$\omega(s_0) = 0,\ \omega'(s_0) = 0, \ldots$$ Le Diberder-Pich '92

- $\Delta A^{\omega,\mathrm{DV}}(s_0)$ **must decrease to 0 very fast** → Uncertainties based on stability under s_0
- More inclusive channels → lower DVs: $\Delta A^{\omega,\mathrm{DV}}_{V/A}(s_0) > \Delta A^{\omega,\mathrm{DV}}_{V+A}(s_0)$

Theoretical Framework: Perturbative contribution

Dominant contribution to $A^{\omega}_{V/A}(s_0 \sim m_\tau^2)$ $\rightarrow$ **Very sensitive to $\alpha_s(s_0 \sim m_\tau^2)$!**

$$A^{\omega}_{V/A}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi(s)$$

Adler function Adler '74

$$D(s) \equiv -s \frac{d\,\Pi^P(s)}{ds} = \frac{1}{4\pi^2} \sum_{n=0} \tilde{K}_n(\xi)\; a_s^n(-\xi^2 s)$$

Uncertainties

- $K_5 \in \{-125, 675\}$
- $\xi^2 \in \{0.5, 2\}$

$$2\frac{s}{a_s}\frac{da_s}{ds} = \sum_{n=1} \beta_n a_s^n(s)$$

FOPT

$$A^{\omega,P}(s_0) = \sum_i^m c_i' a_s^i(\xi^2 s_0)$$

CIPT Le Diberder-Pich '92

$$a_s(\xi^2 s_0\, e^{i\varphi}) \overset{\beta_{n>n_{max}}=0}{=} a_s(a_s(\xi^2 s_0), \varphi)$$

Theoretical Framework: Non-perturbative contribution

OPE of the QCD correlator $\Pi^{(1+0)}(s)$ — Shifman-Vainshtein-Zakharov ('78)

$$\Pi^{\rm OPE}_{V/A}(s=-Q^2) = \sum_D \frac{1}{(Q^2)^{D/2}} \sum_{\dim \mathcal{O}=D} C_{D,V/A}(Q^2,\mu)\langle\mathcal{O}(\mu)\rangle \equiv \sum_D \frac{\mathcal{O}_{D,\,V/A}}{(Q^2)^{D/2}}; \qquad Q^2 \gg \Lambda^2$$

$D > 0$ corrections are small at $Q^2 \sim m_\tau^2$ $\rightarrow$ Log dependence on Q^2 of $\mathcal{O}_{D,\,V/A}$ safely neglected

$$A^{\omega}_{V/A}(s_0) = \frac{i}{2}\oint_{|s|=s_0} \frac{ds}{s_0}\,\omega(s)\,\Pi_{V/A}(s)$$

$$A^{\omega,NP}_{V/A}(s_0) = \pi \sum_D a_{-1,\,D}\,\frac{\mathcal{O}_{D,\,V/A}}{s_0^{D/2}}$$

$$\omega(-s_0 x) = \sum_n a_{n,\,D}\, x^{n+D/2}$$

Results: ALEPH-like fits

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^{l} \left(1 + \frac{2s}{m_\tau^2}\right)$$

$$(k,l) = (0,0) \quad \rightarrow \quad \alpha_s(m_\tau^2), \mathcal{O}_{6\ V/A}, \mathcal{O}_{8\ V/A}$$
$$(k,l) = (1,0) \quad \rightarrow \quad \alpha_s(m_\tau^2), \langle a_s GG \rangle, \mathcal{O}_{6\ V/A}, \mathcal{O}_{8\ V/A}, \mathcal{O}_{10\ V/A}$$
$$(k,l) = (1,1) \quad \rightarrow \quad \alpha_s(m_\tau^2), \langle a_s GG \rangle, \mathcal{O}_{6\ V/A}, \mathcal{O}_{8\ V/A}, \mathcal{O}_{10\ V/A}, \mathcal{O}_{12\ V/A}$$
$$(k,l) = (1,2) \quad \rightarrow \quad \alpha_s(m_\tau^2), \mathcal{O}_{6\ V/A}, \mathcal{O}_{8\ V/A}, \mathcal{O}_{10\ V/A}, \mathcal{O}_{12\ V/A}, \mathcal{O}_{14\ V/A}$$
$$(k,l) = (1,3) \quad \rightarrow \quad \alpha_s(m_\tau^2), \mathcal{O}_{8\ V/A}, \mathcal{O}_{10\ V/A}, \mathcal{O}_{12\ V/A}, \mathcal{O}_{14\ V/A}, \mathcal{O}_{16\ V/A}$$

Fit $\alpha_s, \langle a_s GG \rangle, \mathcal{O}_6, \mathcal{O}_8$

$\mathcal{O}_{10\ldots16} = 0$

V+A

$$\alpha_s(m_\tau^2)^{FOPT} = 0.319\,^{+0.010}_{-0.006} \qquad \alpha_s(m_\tau^2)^{CIPT} = 0.339\,^{+0.011}_{-0.009}$$

Role of neglected power corrections?

Results: ALEPH-like fits

Role of neglected higher dimensional operators?

Make the same fit but including $\mathcal{O}_{10}$

1. Increased uncertainties
2. Good stability of $\alpha_s(m_\tau^2)$ with respect to the previous fit
3. Larger variation in condensates values

Take $V + A$ as reference and differences as additional uncertainties

$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.339\,^{+0.019}_{-0.017}$$
$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.319\,^{+0.017}_{-0.015}$$
$$\longrightarrow \alpha_s(m_\tau^2) = 0.329\,^{+0.020}_{-0.018}$$

Results: Strategy

Are there unaccounted systematic uncertainties?

Make new tests and $\alpha_s(m_\tau^2)$ determination with very different energies, moments and approaches:

- Completely different dependence on power corrections
- Different neglected higher-dimensional condensates in the fits
- Perturbative series are also different at every moment
- $\Delta A^{\omega,\mathrm{DV}}_{V/A}(s_0) = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0}\omega(s)\Delta\rho^{\mathrm{DV}}_{V/A}(s)$ changing s_0 and $\omega(s)$ (so that DV contributions are also very different)

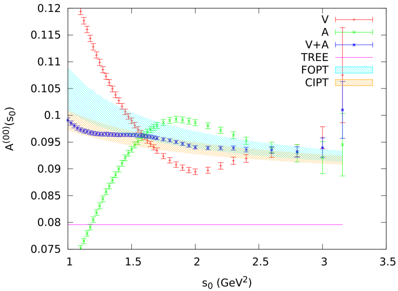
If all determinations and tests are in agreement, one can conclude that systematic uncertainties are properly estimated

Results: A simple test

$$A^{\omega}_{V/A}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi_{V/A}(s)$$

1. Take $\alpha_s(m_\tau^2) = 0.329\,^{+0.020}_{-0.018}$
2. Take $\omega(s) = 1$

- Not used in the previous fit
- Independent on dimensional operators
- Unprotected against DVs: larger than for the moments used



- Very good agreement at all channels at $s_0 \sim m_\tau^2$
- $V + A$ channel is much more s_0-stable than separated ones

Results: other fits, same result

Independent on $\mathcal{O}_{16}$

$$\omega'_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^{l} \xcancel{\left(1 + \frac{2s}{m_\tau^2}\right)}$$

$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.338\,^{+\,0.014}_{-\,0.012}$$

$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.319\,^{+\,0.013}_{-\,0.010}$$

$$\longrightarrow \alpha_s(m_\tau^2) = 0.329\,^{+\,0.016}_{-\,0.014}$$

Free from $D = 4$ OPE contributions (gluon condensate)

$$\omega^{(2,n)}\left(\frac{s}{m_\tau^2}\right) = \left(1 - \frac{s}{m_\tau^2}\right)^2 \sum_{k=0}^{n} (k+1) \left(\frac{s}{m_\tau^2}\right)^k ; \quad n = 1, \ldots 5$$

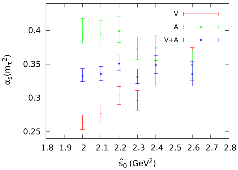
$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.336\,^{+\,0.018}_{-\,0.016}$$

$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.317\,^{+\,0.015}_{-\,0.013}$$

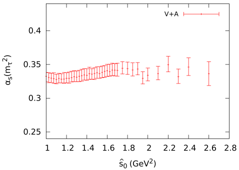
$$\longrightarrow \alpha_s(m_\tau^2) = 0.326\,^{+\,0.018}_{-\,0.016}$$

Playing with the s_0-dependence

$$\omega^{(20)} \equiv \left(1 - \frac{s}{s_0}\right)^2; \qquad s_0 > \hat{s}_0 \qquad \alpha_s(m_\tau^2),\ \langle a_s GG\rangle,\ \mathcal{O}_6 \qquad \text{CIPT}$$



$V + A$ stable beyond that region? .... $\rightarrow$



$$V + A \rightarrow \quad \begin{array}{l} \alpha_s(m_\tau^2)^{\text{CIPT}} = 0.335 \pm 0.014 \\ \alpha_s(m_\tau^2)^{\text{FOPT}} = 0.323 \pm 0.012 \end{array} \quad \rightarrow \alpha_s(m_\tau^2) = 0.329 \pm 0.013$$

Problems

- Much worse behaviour in separated V and A channels
- Very bad quality fit
- We are fitting the spectral function! Fitting m n-pinched $(A^{(n0)}(s_0))$ points equivalent to:

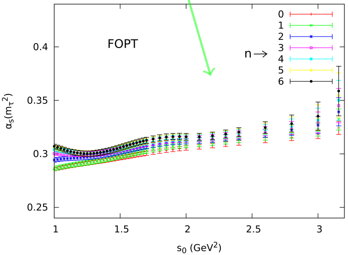
$$\left\{A^{(n,0)}(s_0),\ A^{(n-1,0)}(s_0),\ \cdots,\ A^{(0,0)}(s_0),\ \rho(s_0),\ \rho(s_0 + \Delta s_0),\ \ldots, \rho(s_0 + (m - n - 2)\Delta s_0)\right\}$$

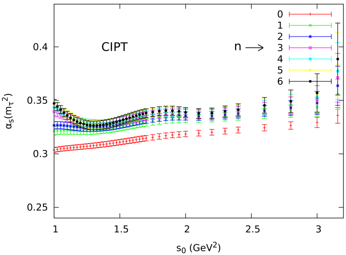
Results: an alternative approach

$$\omega_{1,n}(s) = 1 - \left(\frac{s}{s_0}\right)^{n+1}$$

$$A^{(1,n),\mathrm{NP}}_{V+A}(s_0) = (-1)^n\,\pi\,\frac{\mathcal{O}_{2n+4,V+A}}{s_0^{\,n+2}}$$

Power corrections are small at $s_0 \sim m_\tau^2$ for these moments





$\omega_a^{(1,n)}(x) = \left(1 - \left(\frac{s}{s_0}\right)^{n+1}\right) e^{-a\frac{s}{s_0}}$ → Reduction of DVs. Useful in the separated V and A channels

All condensates contribute to every moment... but for $a \sim 1$ the factorial supression is enough not to enhance its contribution

Results: an alternative approach

Uncertainties

- Perturbative $K_5 \in \{-125, 675\}$, $\xi \in \{0.5, 2\}$
- Experimental
- Non-perturbative: s_0-stability, differences among α_s from different moments

$$\alpha_s(m_\tau^2)^{A,\mathrm{CIPT}} = 0.325\,^{+0.018}_{-0.014}$$
$$\alpha_s(m_\tau^2)^{A,\mathrm{FOPT}} = 0.320\,^{+0.019}_{-0.016}$$

$$\alpha_s(m_\tau^2)^{V,\mathrm{CIPT}} = 0.326\,^{+0.021}_{-0.019}$$
$$\alpha_s(m_\tau^2)^{V,\mathrm{FOPT}} = 0.314\,^{+0.015}_{-0.011}$$

Doing the same in the $V+A$ channel (improvement in the s_0-stability is not clearly observed since DVs are tiny in this channel):

$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.328\,^{+0.014}_{-0.013}$$
$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.318\,^{+0.015}_{-0.012}$$
$$\longrightarrow \quad \alpha_s(m_\tau^2) = 0.323\,^{+0.015}_{-0.013}$$

Results: Modeling duality violations

$$\Delta A^{\omega(s)\,\mathrm{DV}}_{V/A}(s_0) \;=-\pi\int_{s_0}^{\infty}\frac{ds}{s_0}\omega(s)\Delta\rho^{\mathrm{DV}}_{V/A}(s)$$

Boito et al. 14'

Exponential × oscillatory function expected under some assumptions and approximations for $\Delta\rho^{\mathrm{DV}}_{V/A}(s)$ at high energies

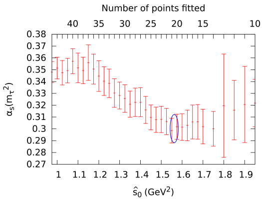
$$\Delta\rho^{DV}_{V/A} = e^{-\delta_{V/A}-\gamma s}\sin(\alpha_{V/A}+\beta_{V/A}s); \qquad s > \hat{s}_0$$

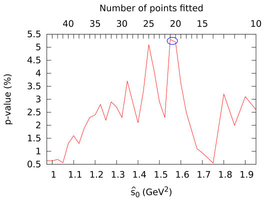
Ad-hoc functional form (assumed to be exactly true)

$\hat{s}_0$?

Uncertainties too large in A channel: not useful to obtain $\alpha_s(m_\tau^2)$

Fit $A^1_V(s_0) = \int^{s_0}\frac{ds}{s_0}\,\mathrm{Im}\,\Pi_V(s)$, $s_0 > \hat{s}_0$
(fitting the spectral function)





FOPT

- Instable just removing (or adding) 1 of ∼ 20 points!
- Bad p-value (even worse when it should be better!)

Results: Modeling duality violations

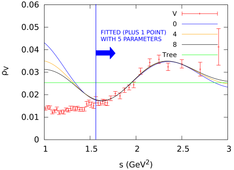
Ignoring instabilities, take as reference $\hat{s}_0 = 1.55$ GeV^2 Boito et al. '14

Simple generalization of previous ansatz

$$\Delta\rho_V^{\mathrm{DV}}(s) \;=\; s^{\lambda_V}\; e^{-(\delta_V+\gamma_V s)}\; \sin\left(\alpha_V + \beta_V s\right), \qquad s > \hat{s}_0$$

λ_V	$\alpha_s(m_\tau^2)$	p-value (%)
0	0.298 ± 0.010	5.3
4	0.306 ± 0.013	6.6
8	0.314 ± 0.015	7.7

Model-dependent approach



- Ignore the problems (p-value, instabilities, etc.) of the model, which make it very unlikely
- Take the minimum possible value for the strong coupling ($\hat{s}_0 = 1.55$ GeV^2 and $\lambda_V = 0$)
- $\alpha_s(m_\tau^2)^{\mathrm{FOPT}}_{\mathrm{MODEL,\ ALEPH}} = 0.298 \pm 0.010_{\mathrm{exp}}$ $\left(\alpha_s(m_\tau^2)^{\mathrm{FOPT}}_{\mathrm{MODEL,\ OPAL}} = 0.325 \pm 0.018_{\mathrm{exp}}\right)$

$\alpha_s(m_\tau^2)^{\mathrm{FOPT}}_{\mathrm{THIS\ WORK}} = 0.320 \pm 0.012_{\mathrm{total}}$ Not unlikely α_s values for an unlikely model

This particular model does not work as counterexample of unaccounted DVs in our model-independent framework, as expected

Conclusions

Purely perturbative contributions dominate uncertainties of $A^{\omega}_{V/A}(s_0 \sim m_\tau^2)$

Different strategies to extract $\alpha_s(m_\tau^2)$ from the ALEPH spectral function have been studied

Method	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments	$0.339\,^{+0.019}_{-0.017}$	$0.319\,^{+0.017}_{-0.015}$	$0.329\,^{+0.020}_{-0.018}$
Modified ALEPH moments	$0.338\,^{+0.014}_{-0.012}$	$0.319\,^{+0.013}_{-0.010}$	$0.329\,^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments	$0.336\,^{+0.018}_{-0.016}$	$0.317\,^{+0.015}_{-0.013}$	$0.326\,^{+0.018}_{-0.016}$
s_0 dependence	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform	$0.328\,^{+0.014}_{-0.013}$	$0.318\,^{+0.015}_{-0.012}$	$0.323\,^{+0.015}_{-0.013}$

$$\alpha_s(m_\tau^2)^{\mathrm{CIPT}} = 0.335 \pm 0.013$$

$$\alpha_s(m_\tau^2)^{\mathrm{FOPT}} = 0.320 \pm 0.012$$

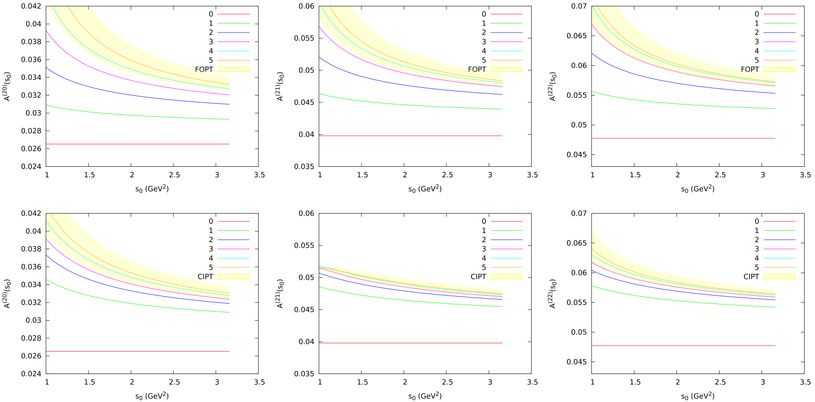
$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$

$$\alpha_s^{(n_f=5)}(M_Z^2) = 0.1197 \pm 0.0015$$

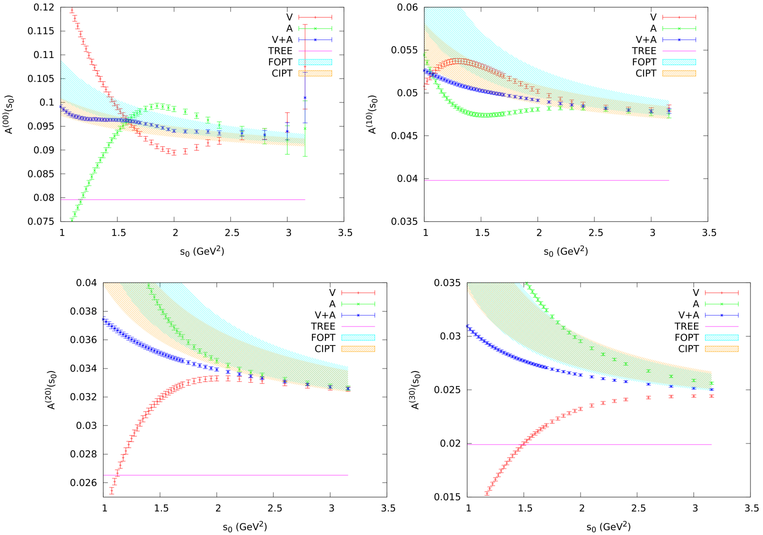
An improved understanding of higher-order perturbative corrections and more precise data would be needed to improve this $\alpha_s(m_\tau^2)$ determination

BACK-UP

Example of perturbative corrections at different orders



Comparing purely perturbative prediction with data



Data handling

$$\frac{1}{N}\,\frac{\Delta N^{(1)}_{V/A}(s_i)}{\Delta s_i} \approx \frac{1}{N}\,\frac{dN^{(1)}_{V/A}}{ds} = B_e\,\frac{dR^{(1)}_{\tau,V/A}}{ds}(s)$$

$$= \frac{12\pi}{m_\tau^2}\,B_e\,S_{\rm EW}\,|V_{ud}|^2\,\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\,\mathrm{Im}\,\Pi^{(1)}_{V/A}(s)$$

$$\frac{1}{N}\,\frac{\Delta N^{(0)}_{V/A}(s_i)}{\Delta s_i} \approx \frac{1}{N}\,\frac{dN^{(0)}_{V/A}}{ds} = B_e\,\frac{dR^{(0)}_{V/A}}{ds}(s)$$

$$= \frac{12\pi}{m_\tau^2}\,B_e\,S_{\rm EW}\,|V_{ud}|^2\,\left(1-\frac{s}{m_\tau^2}\right)^2\,\mathrm{Im}\,\Pi^{(0)}_{V/A}(s)$$

Data handling

$$A_V^\omega(s_0) = F \sum_{s_i}^{s_0-\frac{\Delta s_0}{2}} \frac{\Delta N_V(s_i)}{N}\, \omega_i(s_i, s_0)\, H(s_0, s_i)$$

$$A_A^\omega(s_0) = F \sum_{s_i}^{s_0-\frac{\Delta s_0}{2}} \frac{\Delta N_A(s_i)}{N}\, \omega_i(s_i, s_0)\, H(s_0, s_i) + F\, \frac{m_\tau^2}{s_0} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^{-2} B_\pi\, \omega_i(m_\pi^2, s_0)$$

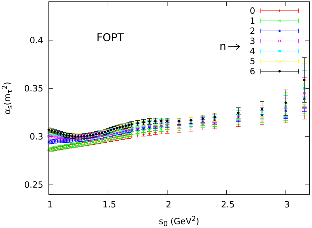
$$F = \left[12\pi\, S_{\mathrm{EW}}\, |V_{ud}|^2 B_e\right]^{-1}$$

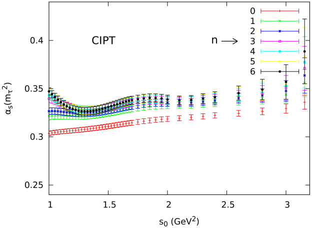
$$H(s_0, s_i) = \frac{m_\tau^2}{s_0} \left(1 - \frac{s_i}{m_\tau^2}\right)^{-2} \left(1 + \frac{2 s_i}{m_\tau^2}\right)^{-1}$$

Neglecting all non experimental uncertainties

$$\omega^{(1,n)}\left(\frac{s}{s_0}\right) = 1 - \left(\frac{s}{s_0}\right)^{n+1}$$

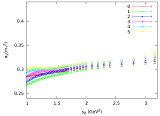
$$A^{(1,n),\mathrm{NP}}_{V+A}(s_0) = (-1)^n\,\pi\,\frac{\mathcal{O}_{2n+4,V+A}}{s_0^{\,n+2}}$$

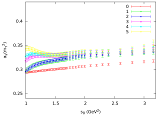




$$\omega^{(2,n)}\left(\frac{s}{s_0}\right) = \left(1-\left(\frac{s}{s_0}\right)\right)^2 \sum_{k=0}^{n}(k+1)\left(\frac{s}{s_0}\right)^k$$

$$A^{(2,n),\mathrm{NP}}_{V+A}(s_0) = (-1)^n\,\pi\left\{(n+2)\,\frac{\mathcal{O}_{2n+4,V+A}}{s_0^{n+2}} + (n+1)\,\frac{\mathcal{O}_{2n+6,V+A}}{s_0^{n+3}}\right\}$$





All perturbative and nonperturbative uncertainties neglected

Non perturbative contributions in the alternative approach

$$\omega_a^{(1,n)}\left(\frac{s}{s_0}\right) = \left(1-\left(\frac{s}{s_0}\right)^{n+1}\right)\, \mathrm{e}^{-a\left(\frac{s}{s_0}\right)}$$

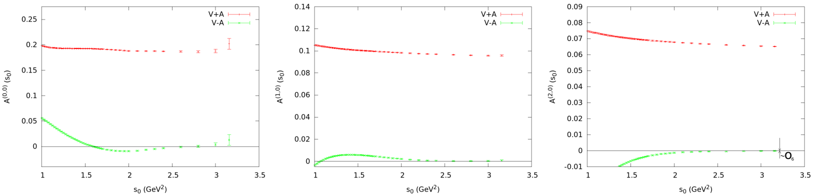
$$A_{V/A}^{\omega_a^{(1,n)},\mathrm{NP}}(s_0) = \pi \sum_D \frac{\mathcal{O}_{D,V/A}}{{s_0}^{D/2}} \frac{a^{\frac{D}{2}-1}}{(\frac{D}{2}-1)!} \left\{1 + \theta(D-4-2n)\, \frac{(-1)^n}{a^{n+1}}\, \frac{(\frac{D}{2}-1)!}{(\frac{D}{2}-n-2)!}\right\}$$

$$\Delta A_{V/A}^{\omega_a^{(1,n)},\mathrm{DV}}(s_0) = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \left(1-\left(\frac{s}{s_0}\right)^{n}\right) e^{-a\frac{s}{s_0}}\, \Delta\rho_{V/A}^{\mathrm{DV}}(s)$$

V+A vs V-A

Duality violation are larger in the separated channels → larger in $V + A$ than in $V - A$

Observables in $V - A$ channels are 100% nonperturbative → Perturbative uncertainties are 0



In order to extract $V - A$ condensates with good precision, experimental uncertainties are too large at $s_0 \sim m_\tau^2$ (more precise data could improve the situation!)

When one goes to lower energies, there are DV uncertainties in some channels. In that case, using an ansatz can be justified to estimate them

Is our QCD coupling determination an experimentally excluded subcase of the model

$$\Delta\rho_V^{\mathrm{DV}}(s) \ = \ s^{\lambda_V} \ e^{-(\delta_V+\gamma_V s)} \ \sin\left(\alpha_V + \beta_V s\right) \qquad\qquad s > 1.55\,\mathrm{GeV}^2$$

with $\exp^{-\delta_{V/A}} = 0$, so that it can be excluded by data? Let's see why not

$\Delta\rho_{DV}^{V/A} \neq 0$ at $s_0 \sim 1.55\,\mathrm{GeV}^2$ (especially important at separated channels for unprotected moments). Trivial because

- OPE is badly defined in the positive real axis
- $\mathrm{Im}\,\Pi^{OPE}$ is almost flat and hadronic spectrum is not at those energies

Since we have not assumed $\rho_{V/A}^{DV} \neq 0$ in any determination at $s \sim 1.55\mathrm{GeV}^2$ (not even at higher energies without testing possible DVs), our QCD determination is far from a subcase of the previous model.

Remember the strategy

- Go to higher energies (DVs reduced), use the V+A channel (DVs more reduced), try to avoid the hadronic cut (DVs even more reduced)
- Make lots of tests to see its effects, accounting them in a systematic uncertainty that absorbs the small DV effects from the different possible ρ_{DV}.