Lattice calculation of the lowest-order hadronic contribution to muon g-2

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Work done at -
University of Glasgow (HPQCD Collaboration*)

*Christine Davies, *P. G. de Oliveira, *J. Koponen, *G. P. Lepage

TAU2016, Beijing, China
September 19th, 2016

Relevance and Timeliness

$a_\mu^{exp} = 11\,659\,208.9\,(63) \times 10^{-10}$ **(0.54 ppm) (BNL E821)**

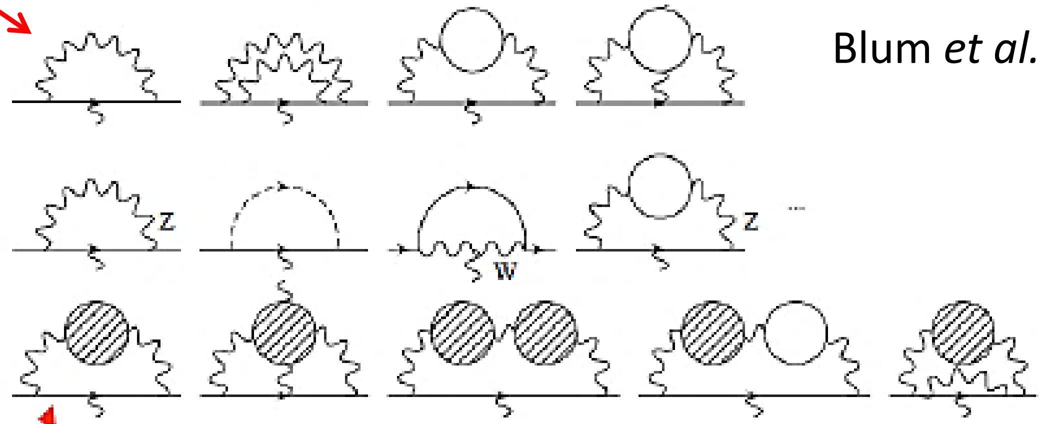
$a_\mu^{SM} = 11\,659\,182.8\,(49) \times 10^{-10}$ **(0.42 ppm)**

$a_\mu^{exp} - a_\mu^{SM} = 26.1(8) \times 10^{-10}$ 3+σ !

Expt. at FNAL E989 & E34 at J-PARC will reduce uncertainty to 0.16 ppm starting 2017.

Current theoretical status of muon g-2

$$\frac{\alpha_{QED}}{2\pi} = 0.00116$$

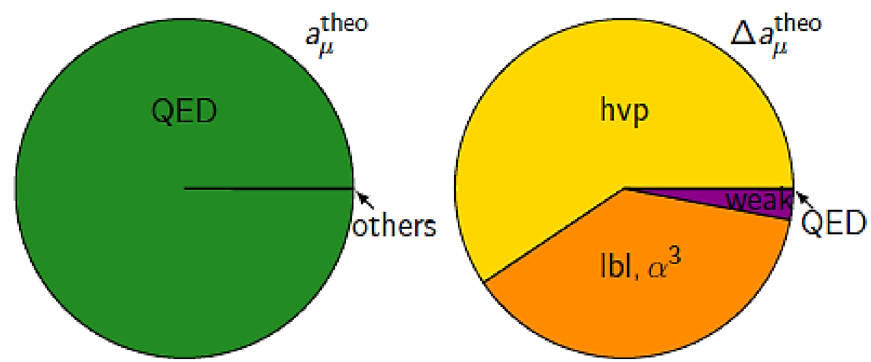


Blum *et al.* 1301.2607

QED, EW & QCD
Contributes;
QED dominates

LO Hadronic vacuum polarisation (HVP)

Contribution	Result ($\times 10^{-10}$)	Error
QED (leptons)	11658471.8	0.00 ppm
HVP(lo) [1]	692.3	0.36 ppm
HVP(ho)	-9.8	0.01 ppm
HLbL [2]	10.5	0.22 ppm
EW	15.4	0.02 ppm
Total SM	11659180.2	0.42 ppm



[1] Davier *et al.* Eur. Phys. J. C71 (2011) 1515
[2] Prades, de Rafael, Vainshtein, 0901.0306

Motivations for first-principle approach to HVP

Mostly no reliance on experimental inputs

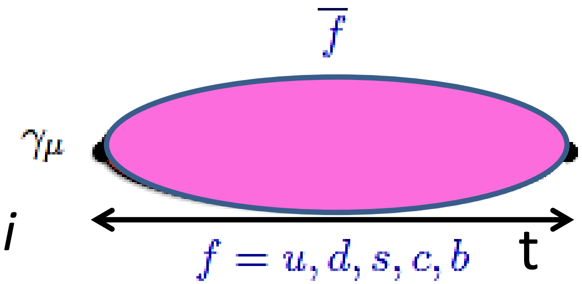
No model dependence (except chiral-continuum extrapolation)

Can lattice QCD deliver reliable estimates for the lowest-order HVP contribution with less than 0.5% uncertainty in coming years?

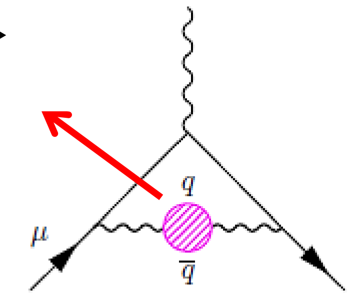
HVP from lattice QCD

In Euclidean space,

connected contribution for flavour *i*



(*Lautrup & de Rafael, Blum*)



Blum, hep-lat/0212018

$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i\,(q^2)$$

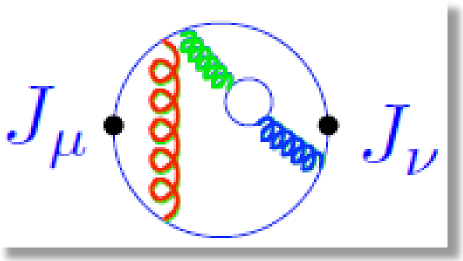
On lattice

$f(q^2)$ known, divergent function of q^2

Renormalised Vacuum Polarisation function: $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

Integrand strongly peaked at $q^2 \sim \frac{m_\mu^2}{4} \sim 0.003\ \mathrm{GeV}^2$

HPQCD method for HVP calculation



Calculate correlation function on lattice

$$\langle j^j(\vec{x},t)j^j(0)\rangle$$

Time moments of lattice current-current correlators

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^j(\vec{x},t)j^j(0)\rangle$$

$$= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)\Big|_{q^2=0}$$

$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j}\Pi_j$ with $\Pi_j = (-1)^{j+1}\frac{G_{2j+2}}{(2j+2)!}$

- Use Pade approximant (ratio of m/n polynomials) for better q^2 behaviour
- Allows us to reconstruct $\hat{\Pi}(q^2)$ and integrate .

B. Chakraborty et al., Phys. Rev. D ***89,*** *114501 (2014)*

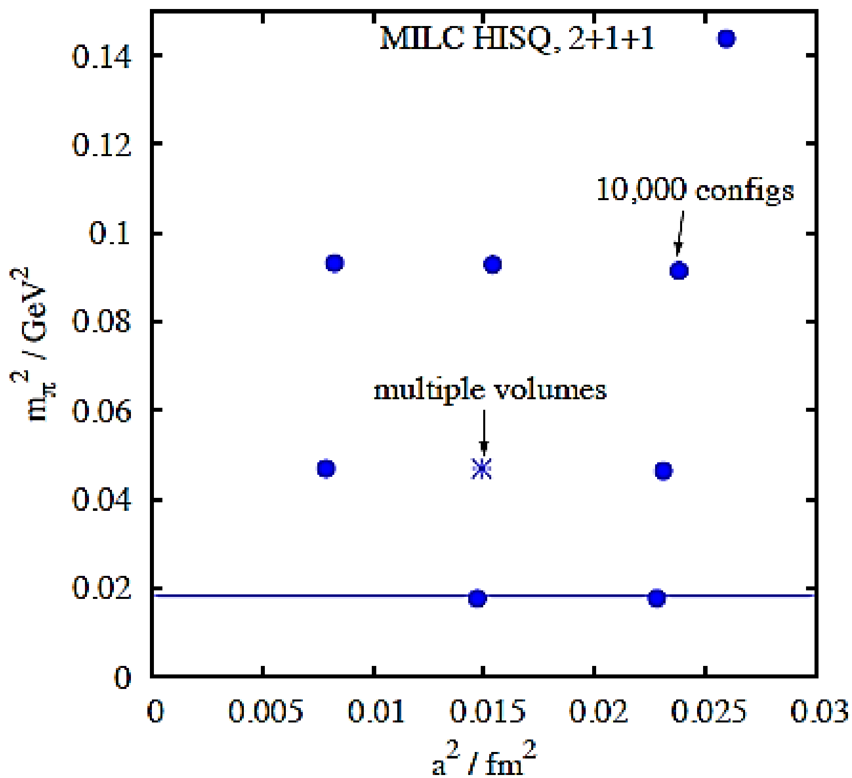
Analysis Ingredients

MILC configurations : up/down, strange, charm quarks in the sea

$m_u = m_d = m_l$

Multiple $m_{u/d}$
Including physical

Valence quark
Masses tuned
accurately



Lattice spacing
a ~ 0.09 -0.15 fm

Multiple volumes
~ (5-6 fm)^3

High statistics:
10,000 configurations
on coarsest lattice

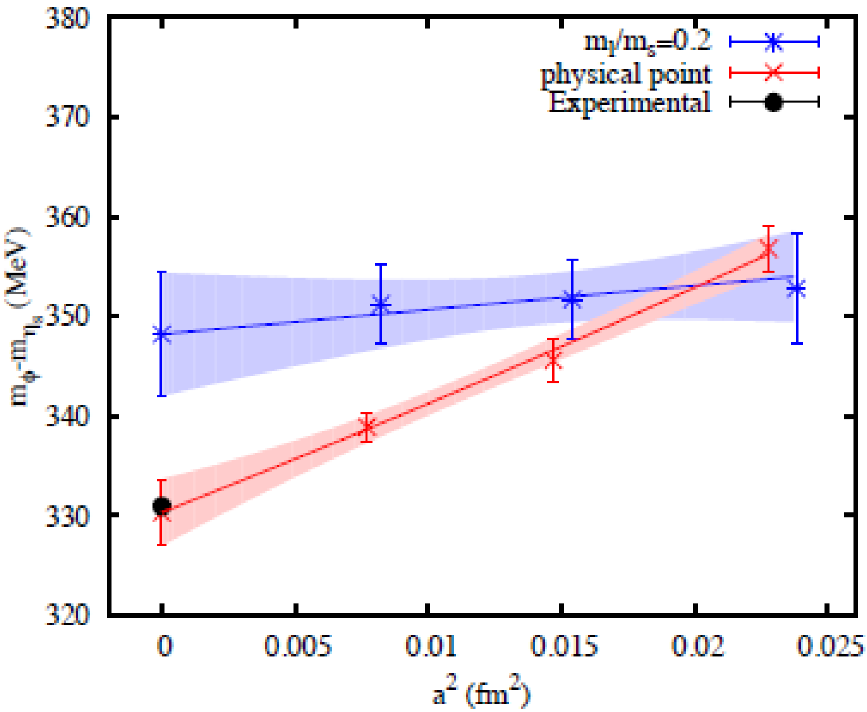
Vector current renormalisation Z_V
calculated nonperturbatively
with high precision

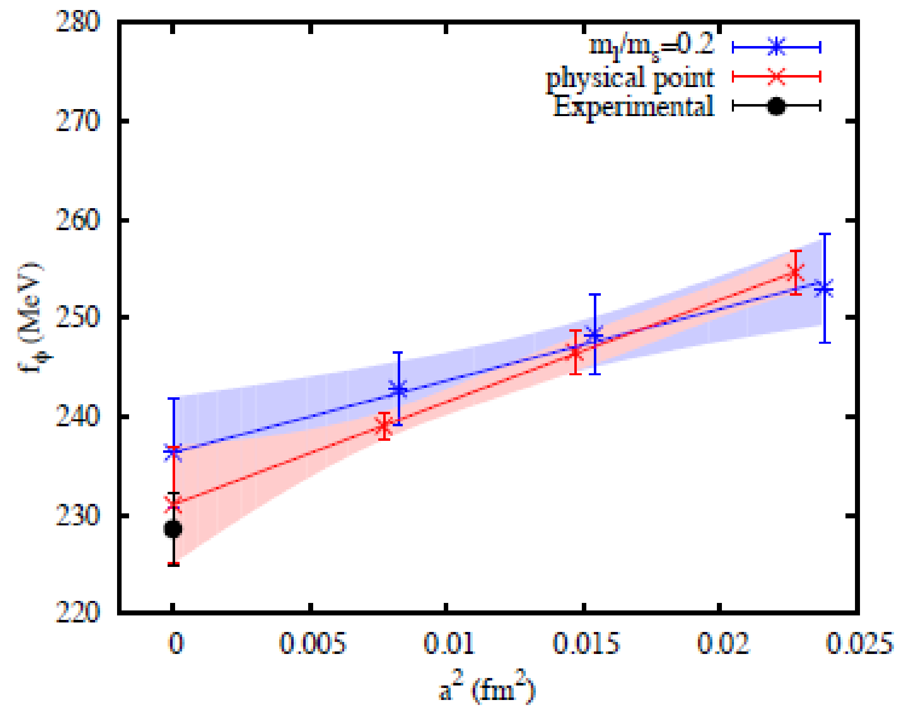
Check strange-strange vector correlators

At large time : Correlator $\sim Ae^{-mt}$

ϕ-meson mass

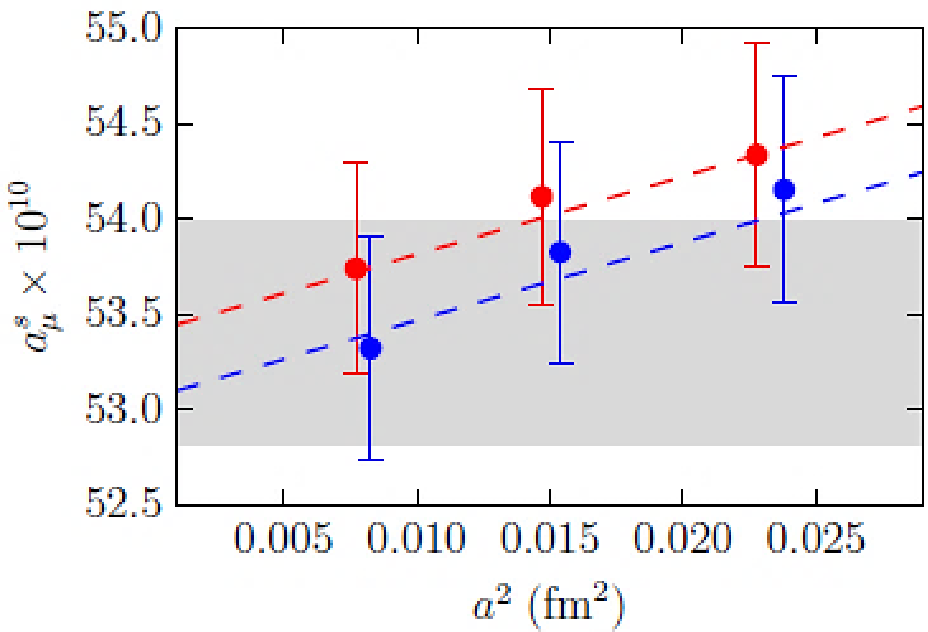
Related to decay constant
(annihilation amplitude)





Experimental $\Gamma(\phi \to e^+ + e^-)$
used for f_ϕ

Strange quark contribution to HVP



	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	1.1%

$$a_{\mu,lat}^s = a_\mu^s \times \left(1 + c_{a^2}\left(\frac{a\Lambda_{QCD}}{\pi}\right)^2 + c_{sea}\delta x_{sea} + c_{val}\delta x_{val}\right)$$

With $m_{u/d}^{lat} = m_{u/d}^{phys}$, after extrapolation to $a = 0$

$$a_\mu^s = 53.41(59) \times 10^{-10}$$

*B. Chakraborty et al., Phys. Rev. D **89**, 114501 (2014)*

Light quark (connected) contribution to HVP

Additional challenges:

1. Noisy data
2. Finite volume error
3. ππ contribution

New ingredient to handle noisy data:

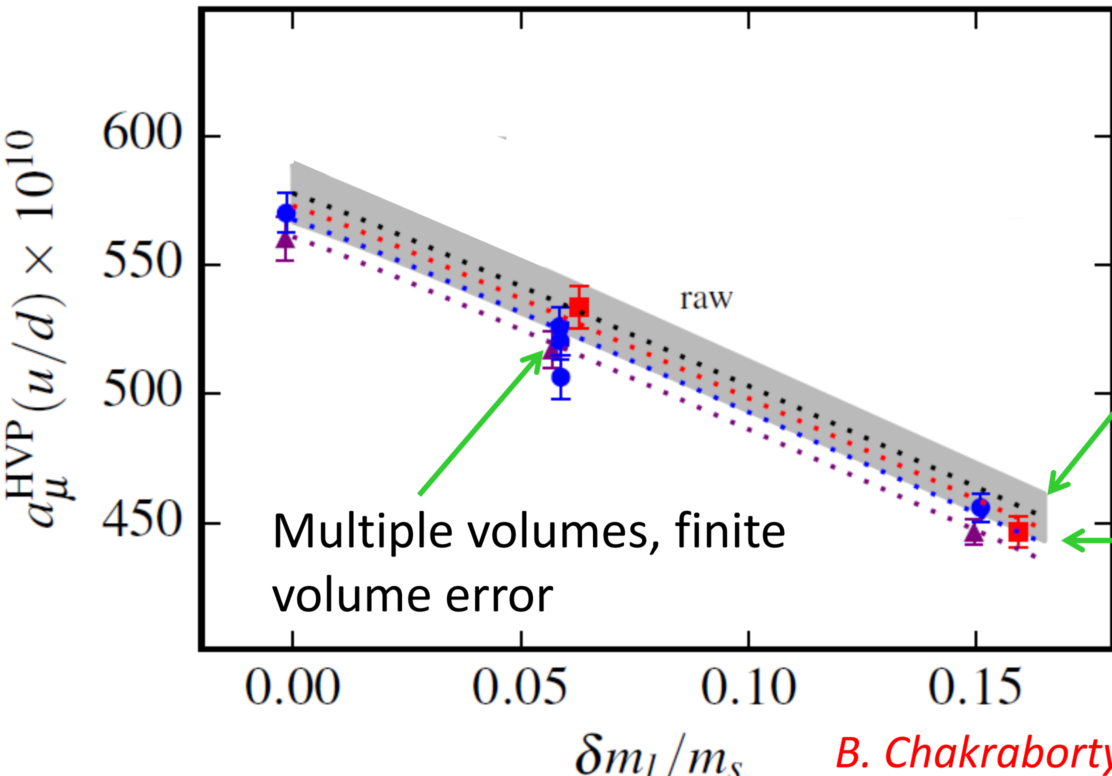
$$G(t) = \begin{cases} G_{data}(t), & t \leq t^* \\ G_{fit}(t), & t > t^* \end{cases}$$

From Monte Carlo

From multi-exponential fit

$t^* = 1.5fm = 6/\rho$

+ **Smearings at source-sink**



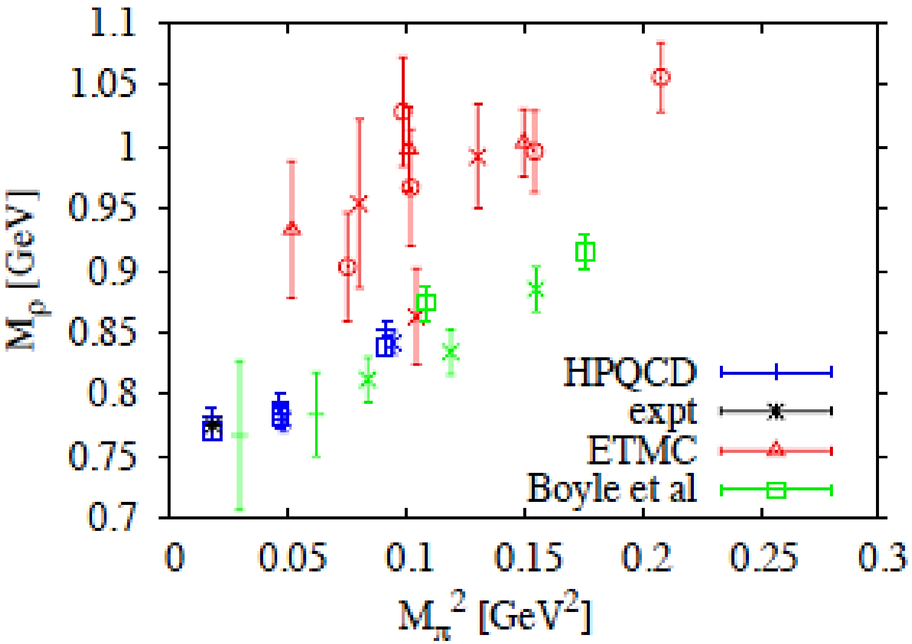
$m_u = m_d$

No corrections For Pion mass

Ensemble size 10,000

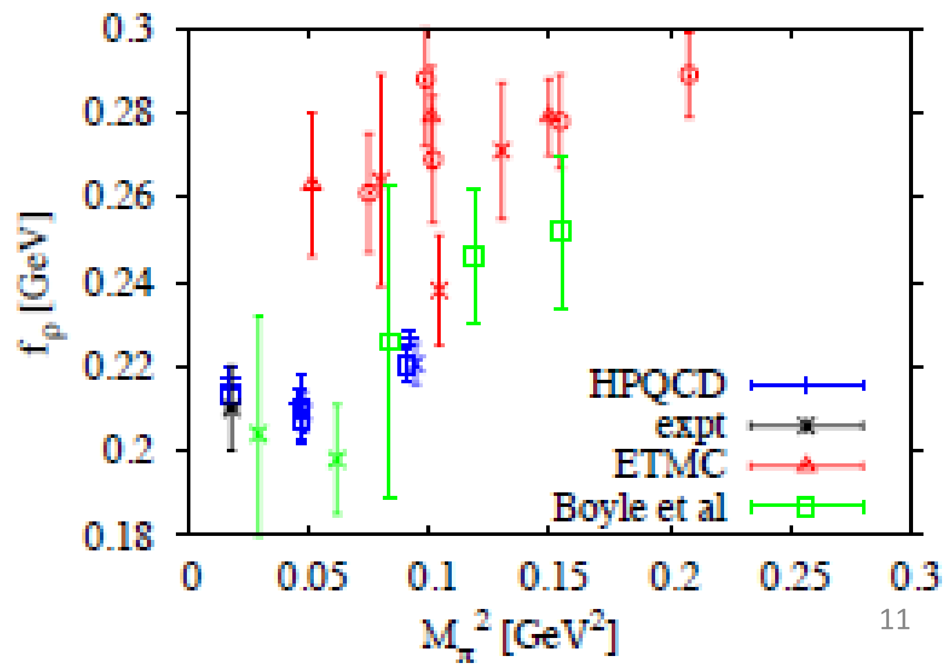
B. Chakraborty et. al, arXiv:1601.0371

Analysis of ρ meson parameters



Comparison with
ETMC (1308.4327)
And Boyle et al. (1107.1497)

HPQCD : a 0.09-0.15 fm
L 2.5-5.8 fm
ETMC : a 0.06-0.08 fm
L 2.5-2.9fm
Boyle et al : a 0.09-0.14 fm
L 2.7-4.6fm



Our approach: To correct Taylor coefficients

$$\hat{\Pi}_j^{latt} \to \left(\hat{\Pi}_j^{latt} - \hat{\Pi}_j^{latt}(\pi\pi)\right)\left[\frac{m_\rho^{2j,latt}}{m_\rho^{2j,expt}}\right] + \hat{\Pi}_j^{cont}(\pi\pi)$$

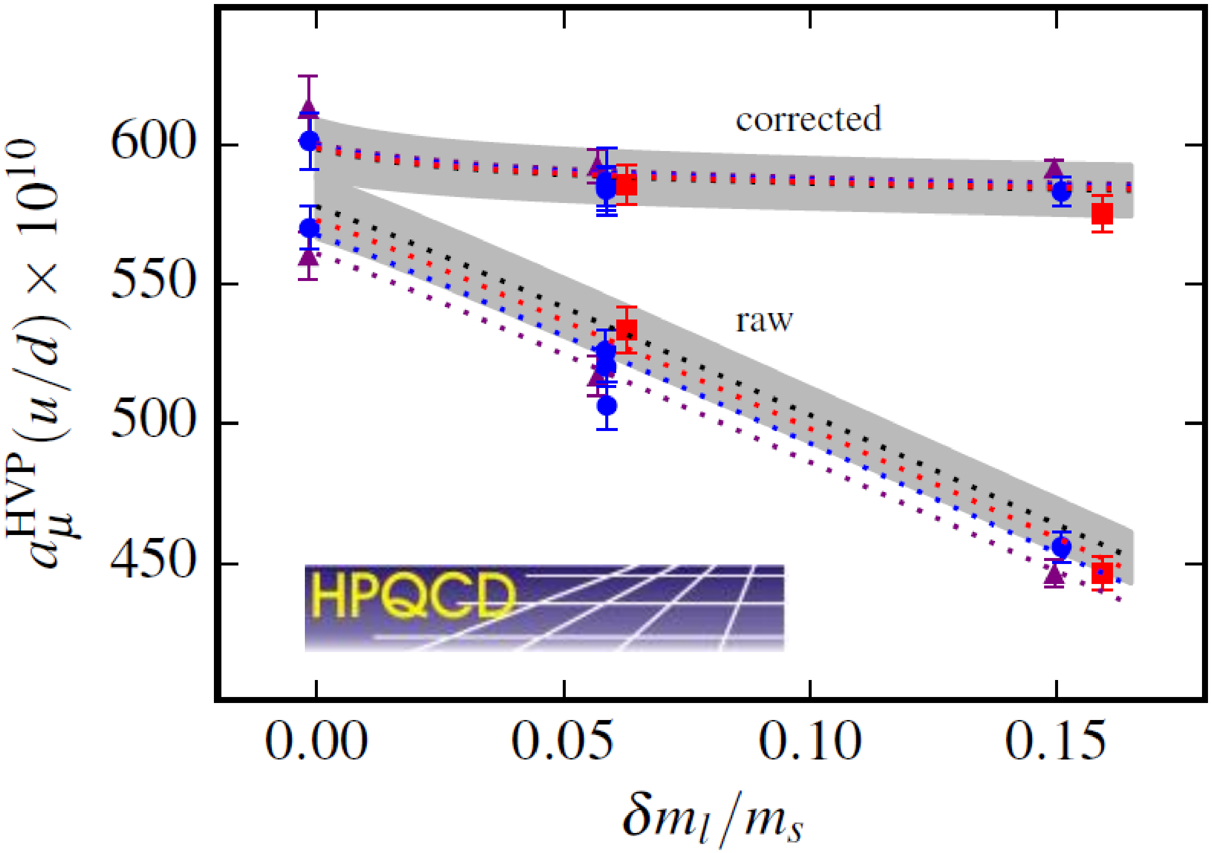
Remove $\pi\pi$ contribution on lattice using 1-loop ChPT; remove finite Volume error, taste-effect (Jegerlehner + Szafron 1101.2872)

Rescale with experimental m_ρ (elaboration of ETMC 1308.4327); removes light quark mass systematics

Restore $\pi\pi$ Contribution from continuum ChPT

7 fm lattice reduces finite volume error by 10% in continuum $\pi\pi$ contribution

B. Chakraborty et. al, arXiv:1601.0371



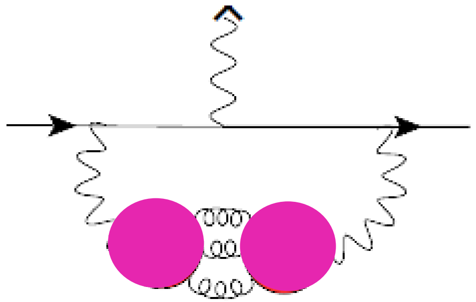
$$a_\mu^{u/d} = 598(11) \times 10^{-10}$$

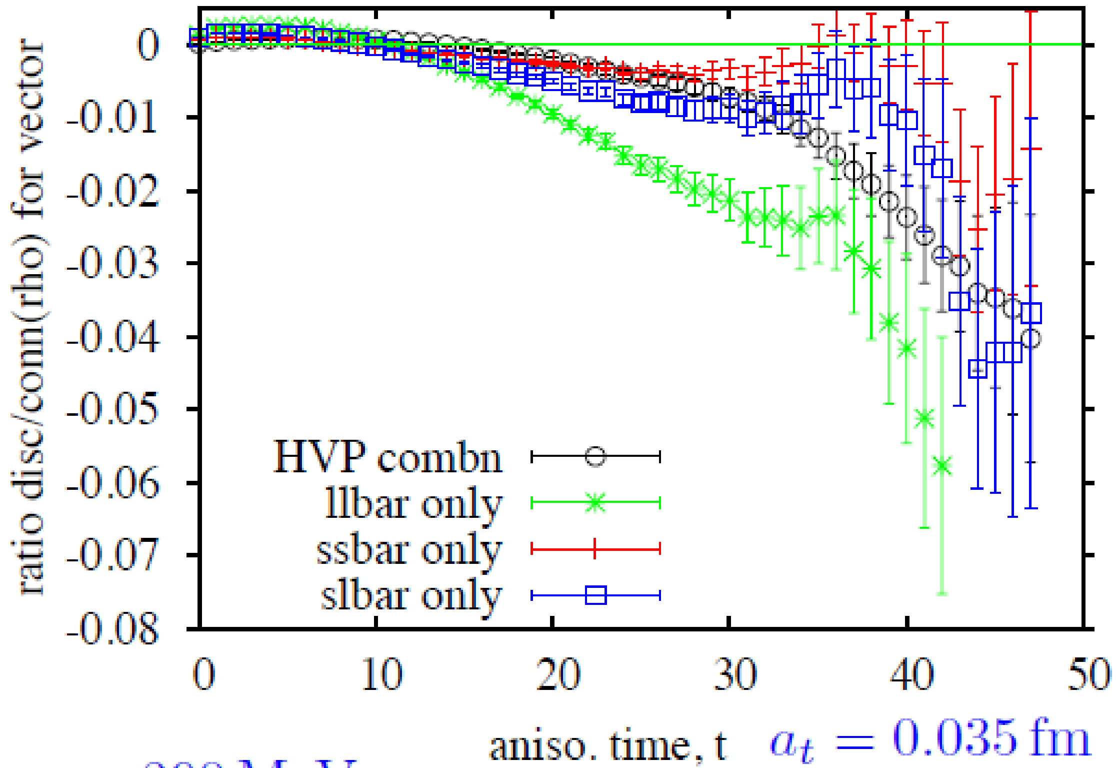
B. Chakraborty et. al, arXiv:1601.0371

Quark-line disconnected contribution

Vanishes if $m_u = m_d = m_s$

since, $\sum_i e_i = 0$





$a_t = 0.035$ fm

$m_\pi = 390$ MeV

HVP disconnected contribution :

- Negative
- 0.2(1.5)% of the connected

Hadron Spectrum + HPQCD

*B. Chakraborty et al., Phys. Rev. D **93,**7074509 (2016)*

HPQCD estimate for total $a_\mu HVP, LO$

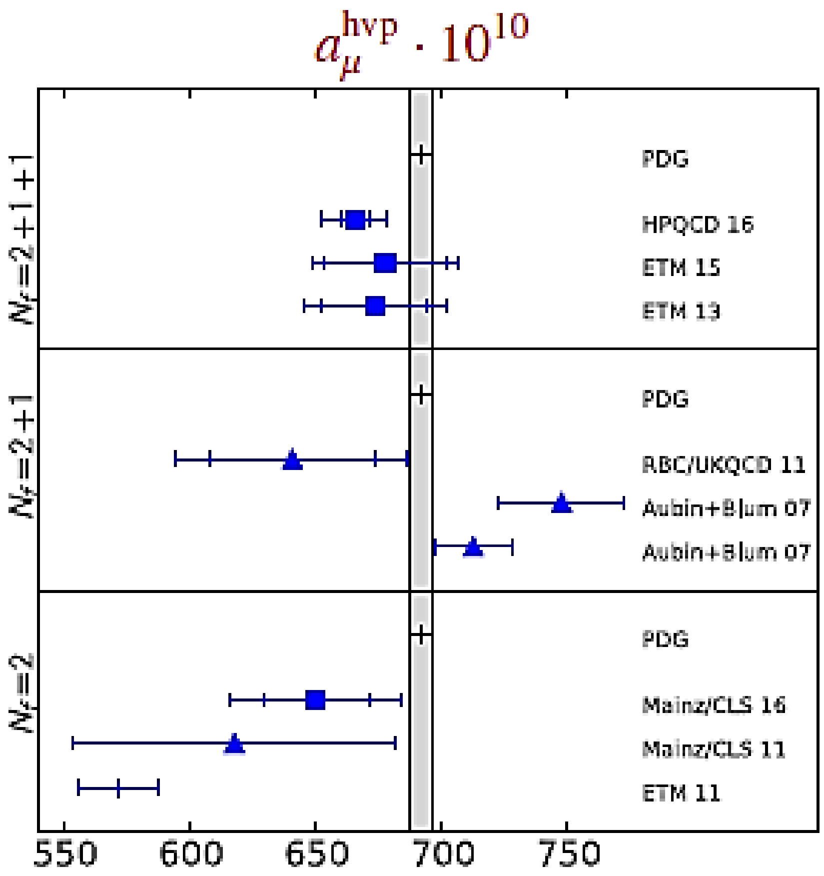
ETMC 1308.4327

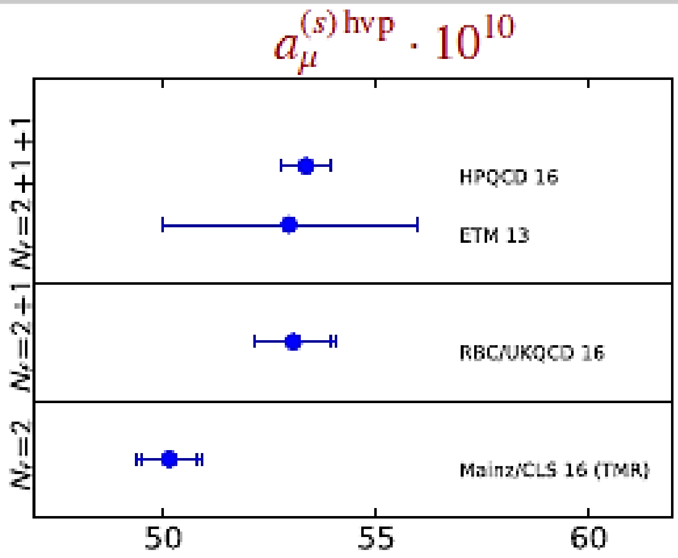
HPQCD: x 10^{-10}

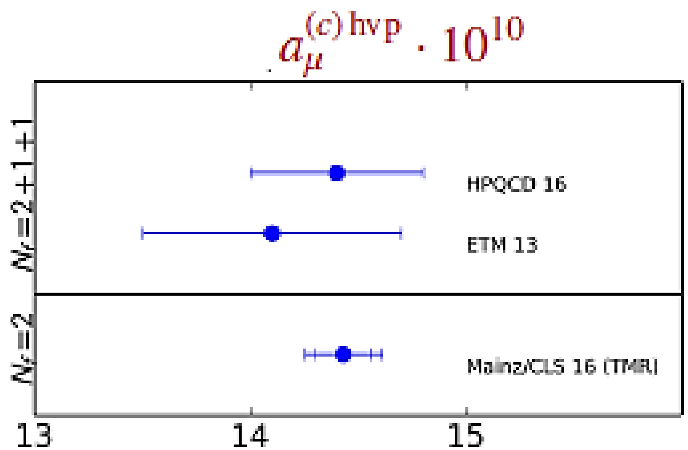
567(11)stat	light, connected	**598(11)**	inc. 1% QED and 1% isospin uncty
	strange connected	**53.4(6)**	1403.1778
	charm connected	**14.4(4)**	1403.1778, 1208.2855
	bottom connected	**0.27(4)**	1408.5768
	disconn. (estimate)	**0(9)**	Take 1.5% as uncty. Contrbn negative
674(28)	TOTAL	**666(6)(12)**	

B. Chakraborty et. al, arXiv:1601.0371

Comparison of different lattice results



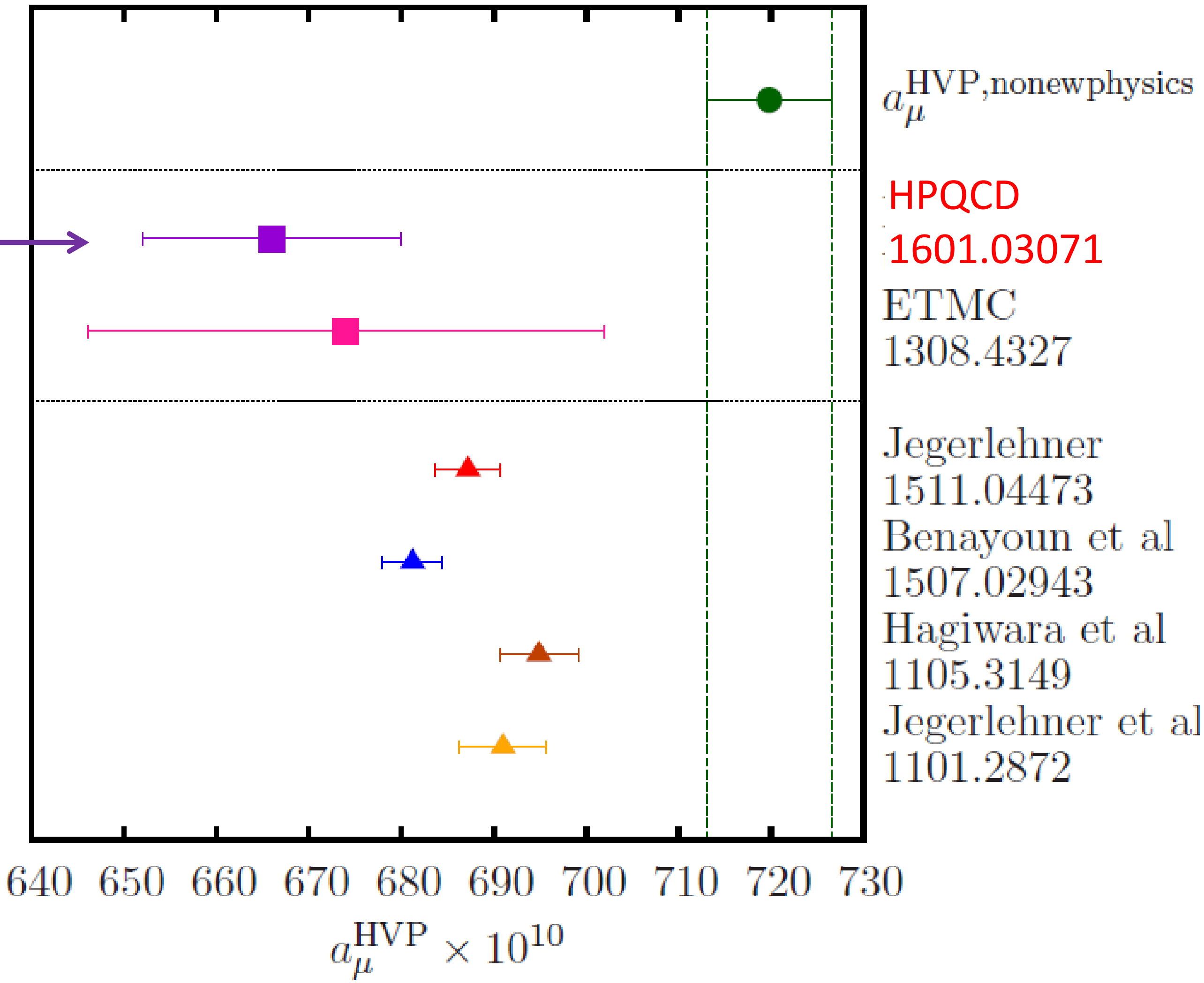




(Talk by Prof. H. Wittig in Lattice2016)

Lattice – Continuum comparison

3.5σ from no new physics



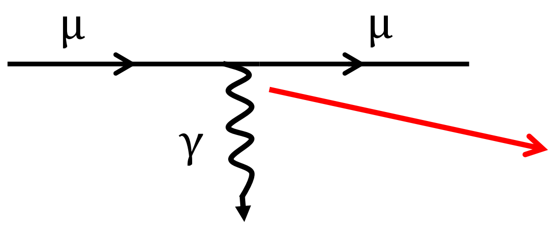
Lattice calculation using u/d,s,c sea quarks + physical valence quarks

In future : Increase statistics, QED and isospin effects, disconnected piece
More Calculations underway (Mainz, RBC/UKQCD, BMW …)

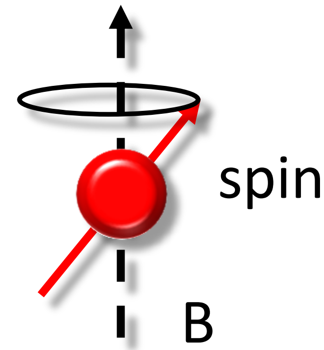
Back up

Relevance and Timeliness

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad a_\mu = \frac{g-2}{2}$$



g = 2



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$\boldsymbol{a_\mu^{exp}}$ **-** $\boldsymbol{a_\mu^{SM}} = \mathbf{26.1(8)} \times\ \mathbf{10^{-10}}$

3+σ !

Expt. at FNAL E989 & J-PARC will reduce uncertainty to 0.16 ppm starting 2017

Hadronic contributions

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$
$$= a_\mu^{HVP,LO} + a_\mu^{HVP,HO} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation:

$$a_\mu^{HLBL} = 10.5(2.6) \times 10^{-10}$$

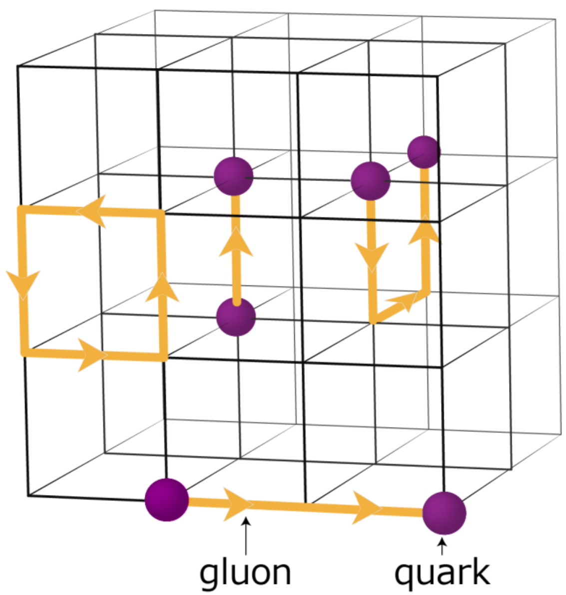
$$a_\mu^{HVP,HO} = -8.85(9) \times 10^{-10}$$

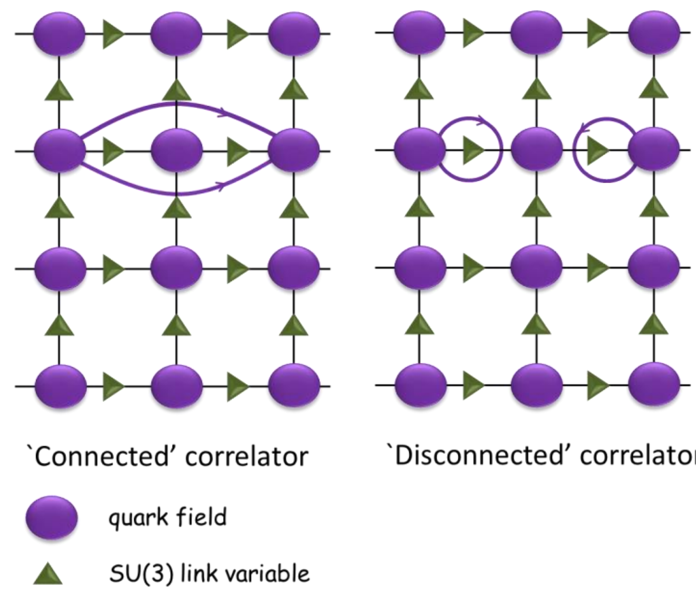
NLO+NNLO
Kurz *et al.*, 1403.6400

$$a_\mu^{HVP,no\,new\,physics} = 719.8(6.8) \times 10^{-10}$$

Lattice recipe for meson correlators

- Expectation values of observables : $\int DUD\psi D\bar{\psi} exp(-\int L_{QCD} d^4x)$
- 4-D space-time lattice
- Gauge configurations : gluons + sea quarks





- Discretise : $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma.\Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point correlation function : extract meson properties
- Corrections for lattice calculation

Computation at Darwin@Cambridge

Part of STFC's HPC facility

9600 Intel Sandybridge cores,
Infiniband interconnect,
fast switch, 2Pbytes storage

DiRAC

Under Dirac 2 : Approx. 4 million core hours used

Fit function

- The result for a_μ^s converge to within errors, by the [1,1] Padé approximant, and no spurious poles appear up to and including [2,2].
- we fit the $[2,2]$ results from each configuration set to a function of the form

$$a^s_{\mu,\mathrm{lat}} = a^s_\mu \times \left(1 + c_{a^2}\left(\frac{a\Lambda_{\mathrm{QCD}}}{\pi}\right)^2 + c_{\mathrm{sea}}\delta X_{\mathrm{sea}} + c_{\mathrm{val}}\delta X_{\mathrm{val}}\right)$$

where $\Lambda_{\mathrm{QCD}} = 0.5\,\mathrm{GeV}$ and

$$\delta X_{\mathrm{sea}} \equiv \sum_{q=u,d,s} \frac{m_q^{\mathrm{sea}} - m_q^{\mathrm{phys}}}{m_s^{\mathrm{phys}}}$$

$$\delta X_s \equiv \frac{m_s^{\mathrm{val}} - m_s^{\mathrm{phys}}}{m_s^{\mathrm{phys}}}.$$

- Discretization effects are handled by c_{a^2}.