INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Introduction to Tau 2016

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The 14th International Workshop on Tau Lepton Physics
IHEP, Beijing, September 19, 2016

Outline :

1. Introduction and Motivation:
   Tau lepton as a laboratory to explore the Standard Model and possible extensions

2. Leptonic Tau decays

3. Hadronic Tau decays

4. Lepton Flavour Violation

5. Taus at the LHC

6. Conclusion and outlook

   NB: several very interesting topics not covered:
   CP violation, g-2, EDMs, neutrinos, etc…

1. Introduction and Motivation

1.1 The triumph of the Standard Model

- New era in particle physics :
  ⇨ (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*

- Key results at LHC after run I + beginning of run II
  - *The Higgs boson* (last missing piece of the SM) has been found:
    ⇨ it looks very standard
  - The Higgs boson is "*light*" (m_h ~ 125 GeV → not the heaviest SM particle)
  - *No* "*mass-gap*" above the SM spectrum (i.e. no unambiguous sign of NP up to ~ 1 TeV)

- *Was this unexpected*?
  Not really! ⇨ *Consistent* with (pre-LHC) indications coming from indirect NP searches (*EWPO* + *flavour physcs*)

1.2 Quest for New Physics

- *Shall we continue to test the Standard Model and search for New Physics?*

  Yes! ⇨ Despite its phenomenological successes, the SM has some *deep unsolved* problems:
  - hierarchy problem
  - flavour pattern
  - dark-matter, etc.…

  - Strong interaction not so well understood: confinement, etc

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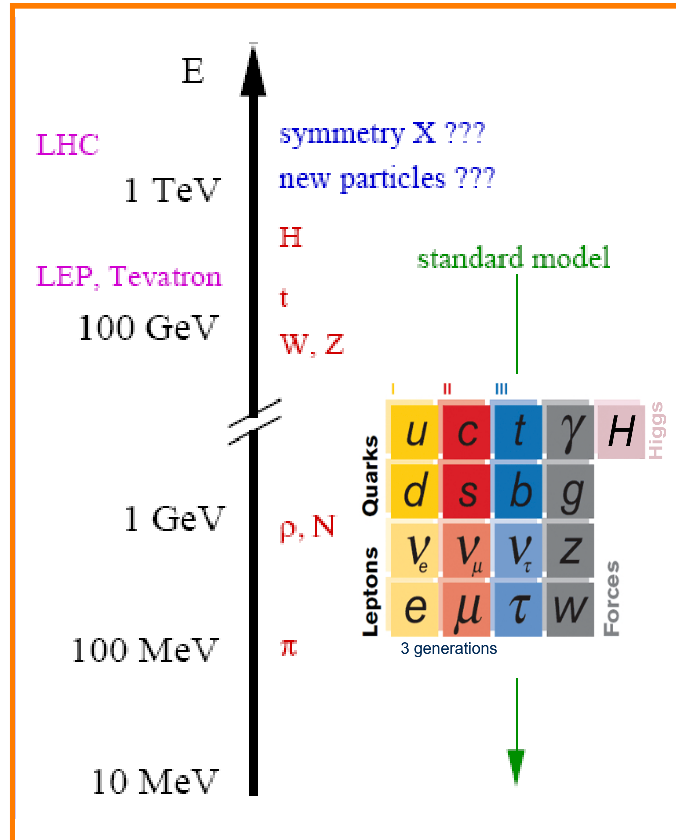

- Consider the SM as as an *effective theory*, i.e. the limit *–in the accessible range of energies and effective couplings–* of a more fundamental theory, with
  - new degrees of freedom
  - new symmetries

1.2 Quest for New Physics

- *Where do we look?* Everywhere!

  ⇒ search for New Physics with a *broad search strategy* given the lack of clear indications on the SM-EFT boundaries (*both in terms of energies and effective couplings*)

Where is the tail?

Y. Grossman@KEKFF'14

⇒ Key unique role of *Tau physics*

1.3 τ lepton as a unique probe of new physics

- Unique probe of *Lepton Universaity* and *Charged Lepton Flavour Violation*
  No SM background
  Indirect probe of flavor-violating NP occurring at energies not directly accessible at accelerators:
  - Kaon physics: $\boxed{\dfrac{s\bar{d}s\bar{d}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^5 \text{ TeV}}$
    [ε_K]
  - Tau physics: $\boxed{\dfrac{\tau\bar{\mu}f\bar{f}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^2 \text{ TeV}}$
    [$\tau \to \mu\gamma$]

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  Ex: h → ττ, LFV in h → τμ

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  Ex: $h \rightarrow \tau\tau$, LFV in $h \rightarrow \tau\mu$

- Studying its hadronic decays: *inclusive* & *exclusive*
  - Unique probe of some of the *fundamental SM parameters*

    ⇨ α_S, $|V_{us}|$, m_s

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  - Ideal set-up for the "R&D" of theory tools about *non perturbative* & *perturbative dynamics*: OPE, Chiral Perturbation Theory, Resonances, large N_c, dispersion relations lattice QCD, etc…

    ⇨ improve our understanding of the SM and QCD at low energy

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  - Inputs for the *muon g-2*

1.3 τ lepton as a unique probe of new physics

- A lot of progress in tau physics since its discovery on all the items described before ⇨ important experimental efforts from *LEP*, *CLEO, B factories: Babar, Belle, BES, VEPP-2M, LHCb, neutrino experiments*,…

  ⇨ More to come from *LHCb, BES, VEPP-2M, Belle II, CMS, ATLAS*

Experiment	Number of τ pairs
LEP	~3x10^5
CLEO	~1x10^7
BaBar	~5x10^8
Belle	~9x10^8
Belle II	~10^{12}

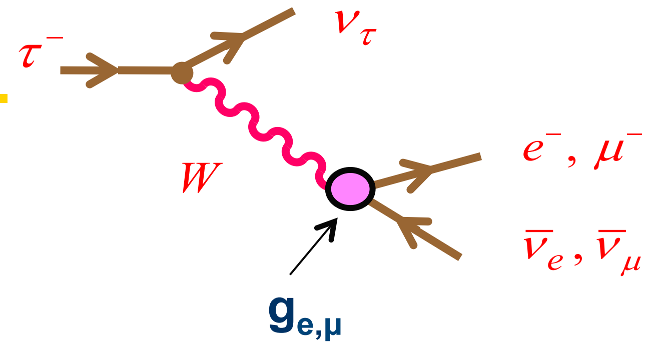
- But τ physics has still potential "*unexplored frontiers*"

  ⇨ deserve future exp. & th. efforts

- In the following, some selected examples and the conference will give more!

2. Leptonic τ-decays

2.1 Lepton Universality



- The leptonic decay width:

$$\Gamma(\tau \to \nu_\tau l \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2/m_\tau^2)\left(1+\delta_{RC}\right)$$

Experimental inputs:

$\Gamma(\tau_{l3})$ Rates with well-determined treatment of radiative decays

- Branching ratios
- Tau lifetimes

Inputs from theory: *Marciano'88*

δ_{RC} Radiative corrections

A. Lusiani@FCCP'15

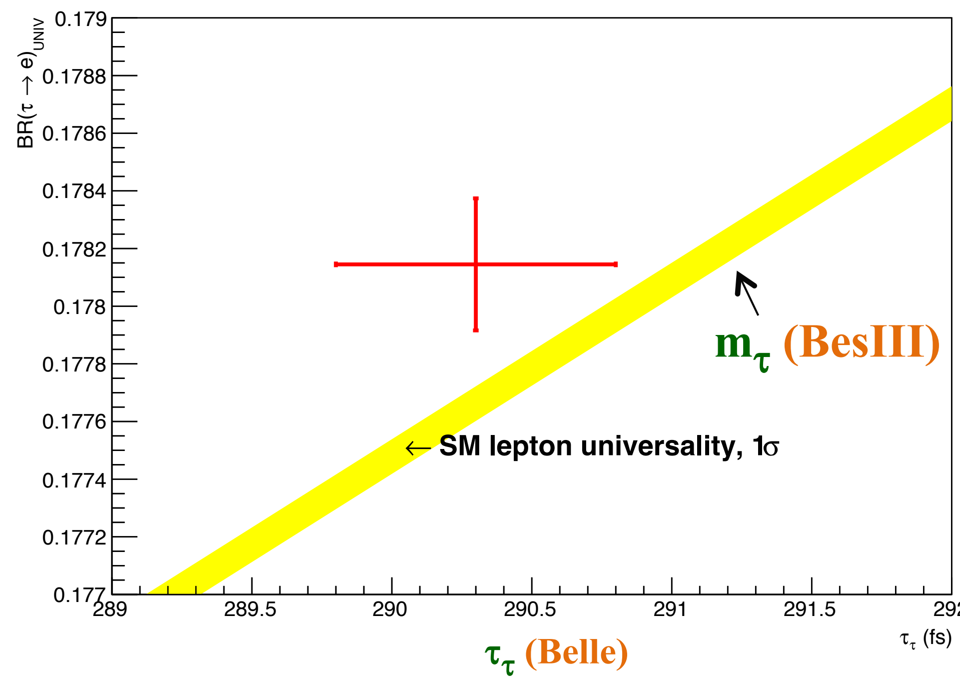
- Test of μ/e universality:

$$\left(B_\mu/B_e\right)_{\exp} = 0.9761 \pm 0.0028$$

Non-BF: 0.9725 ± 0.0039

BaBar '10: 0.9796 ± 0.0039

$$\Rightarrow \boxed{B_e^{univ} = (17.818 \pm 0.0022)}$$



2.1 Lepton Universality

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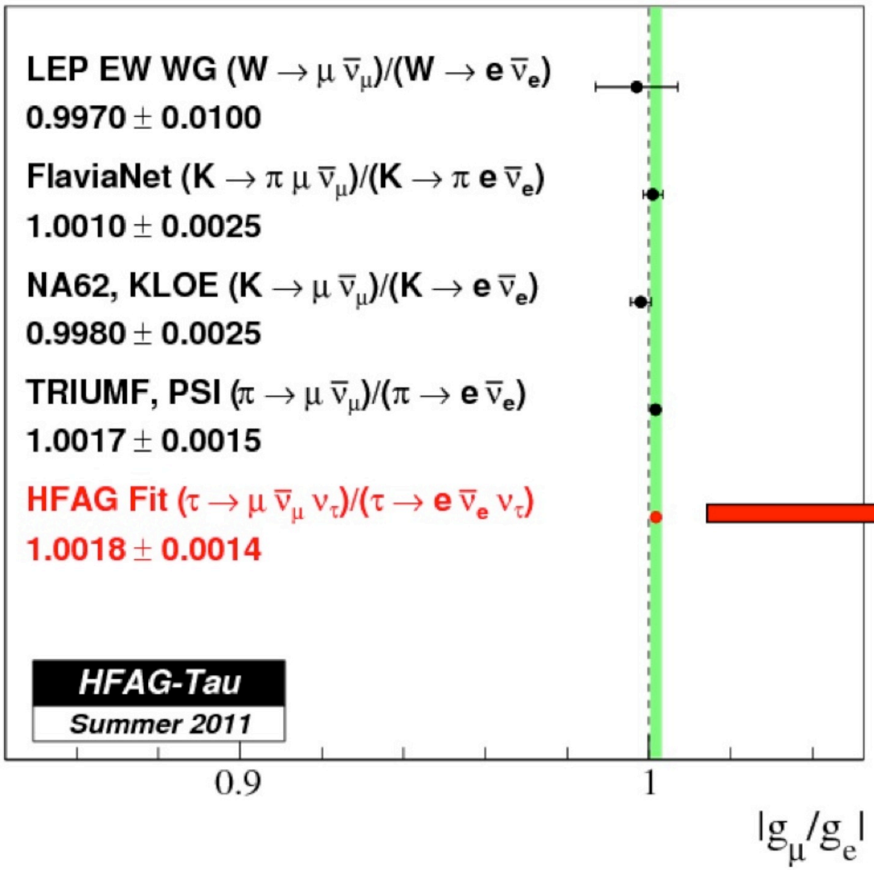

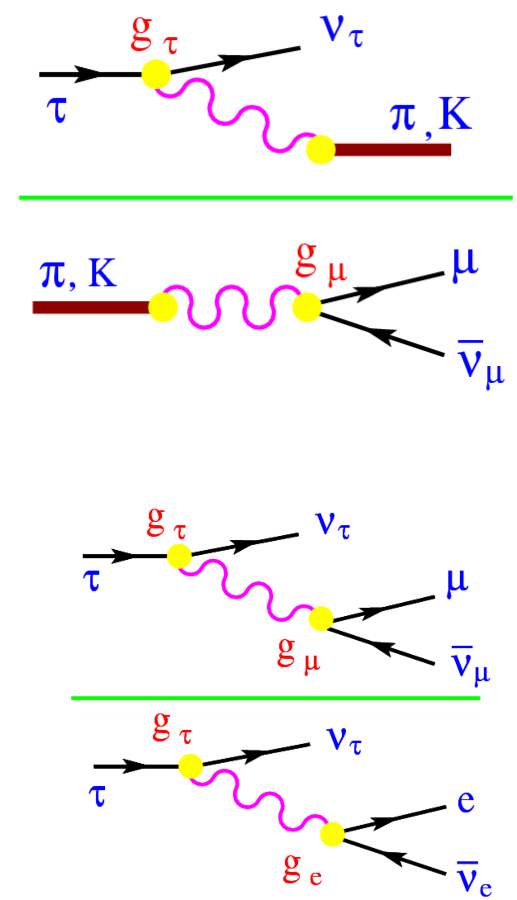

- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)

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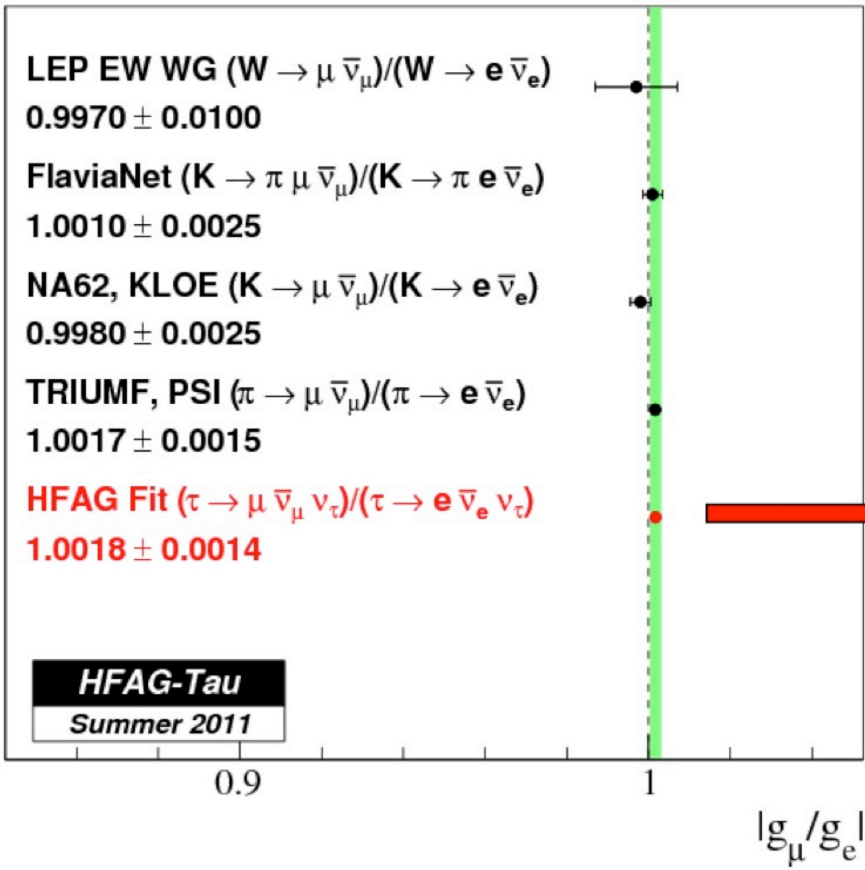
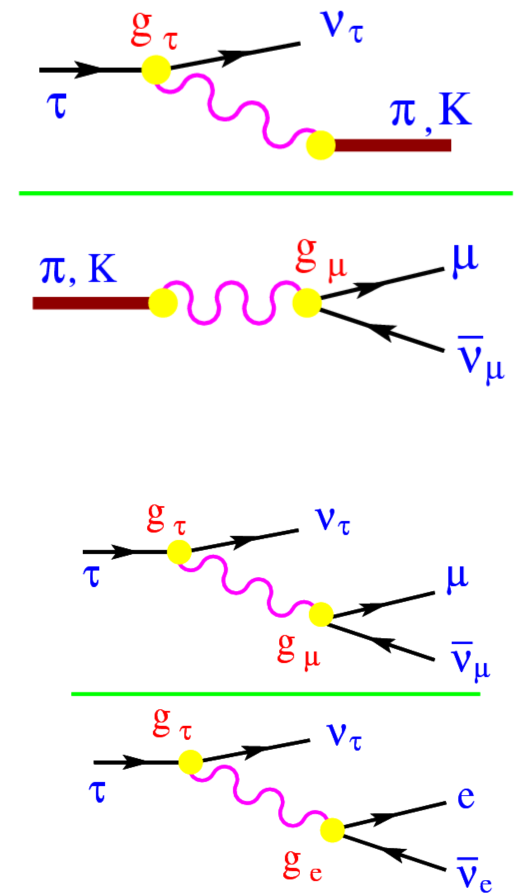




- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)
- What about the *third family*?

2.1 Lepton Universality

- What about the *third family*?

A. Pich@KEKFF'15

based on HFAG'14

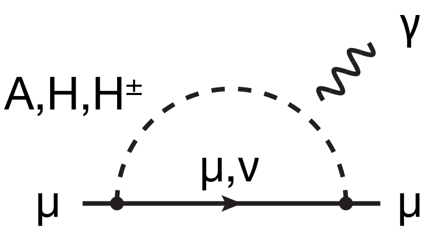
	$\left\| g_\tau / g_\mu \right\|$
$B_{\tau\to e} \;\; \tau_\mu / \tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.9962 ± 0.0027
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.9858 ± 0.0070
$B_{W\to\tau} / B_{W\to\mu}$	1.034 ± 0.013

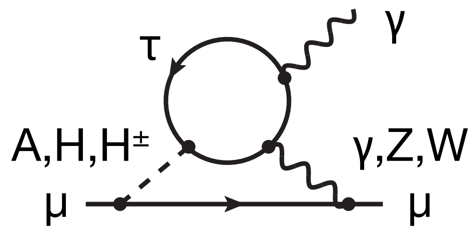
	$\left\| g_\tau / g_e \right\|$
$B_{\tau\to\mu} \;\; \tau_\mu / \tau_\tau$	1.0029 ± 0.0015
$B_{W\to\tau} / B_{W\to e}$	1.031 ± 0.013

- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays

2.2 Lepton universality & NP

- The lepton universality tests give strong constraints on type-X (lepton-specific) 2HDMs ⇨ Model favoured to explain the g-2 discrepancy





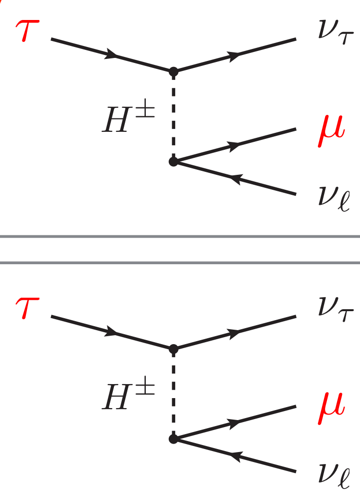
M. Endo@b2tip'15

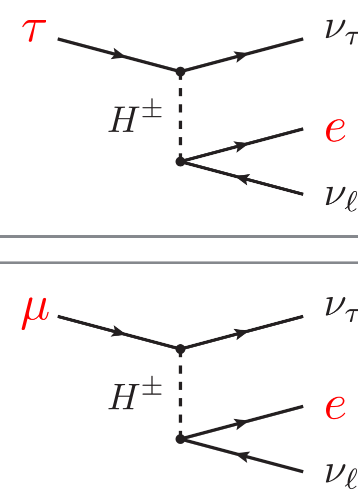
Barr-Zee contribution with A enhances muon g-2

⇨ Contribution to LU : Large Negligible

$$\frac{g_\mu}{g_e} = \frac{1+\Delta_{\tau\to\mu}}{1+\Delta_{\tau\to e}} = 1.0018 \pm 0.0014$$

$$\frac{g_\tau}{g_e} = \frac{1+\Delta_{\tau\to\mu}}{1+\Delta_{\mu\to e}} = 1.0029 \pm 0.0015$$





[HFAG'14]

Note: $Y_{\tau,\mu} \gg Y_e$

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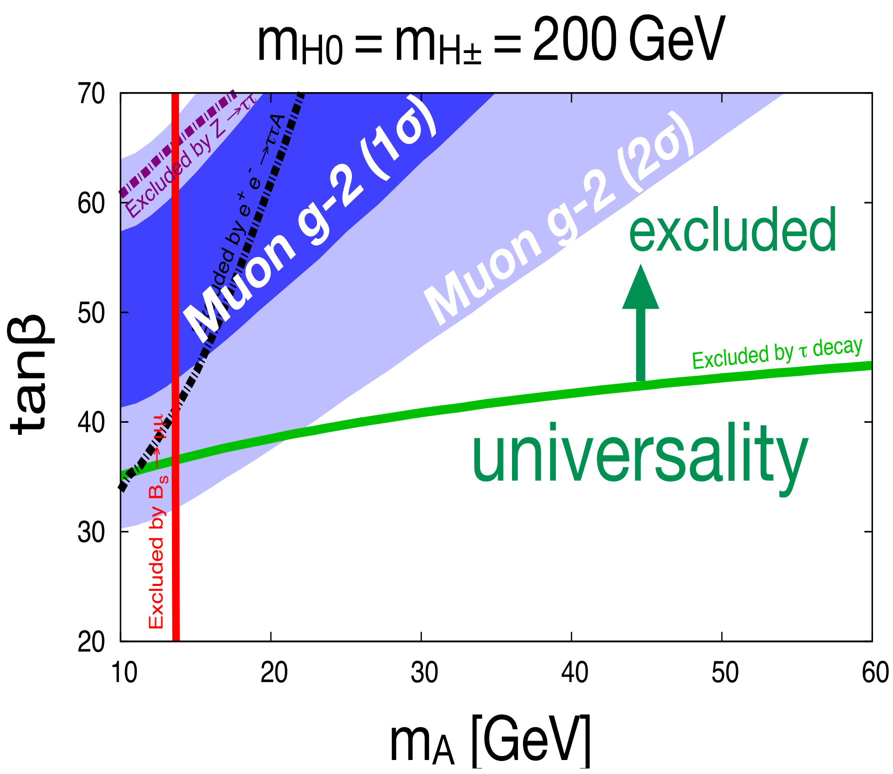
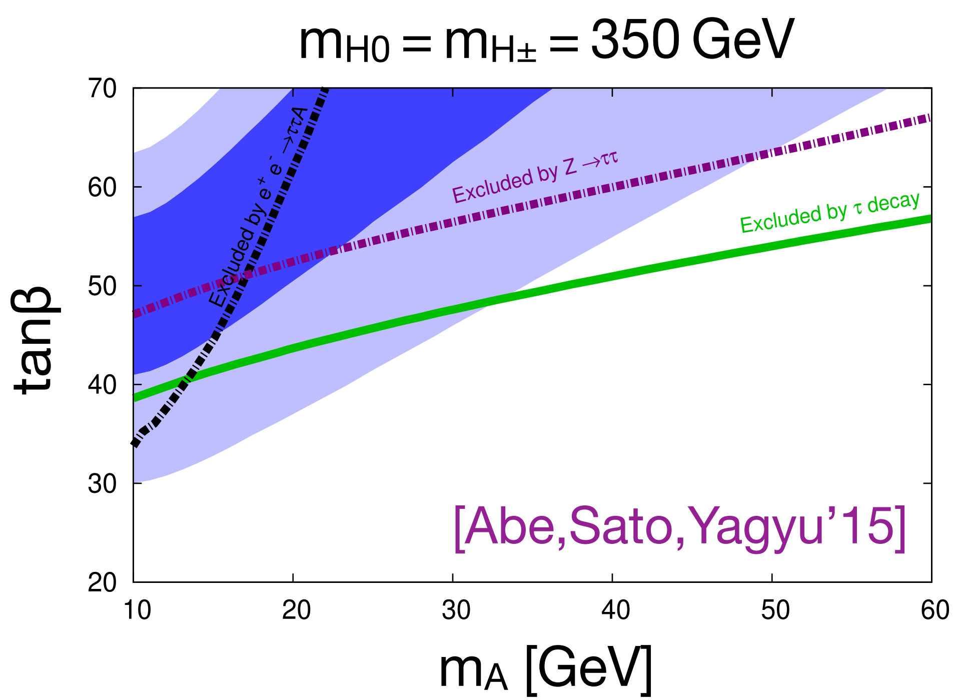
M. Endo@b2tip'15

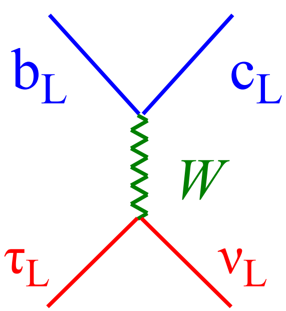


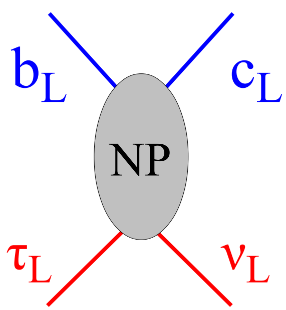


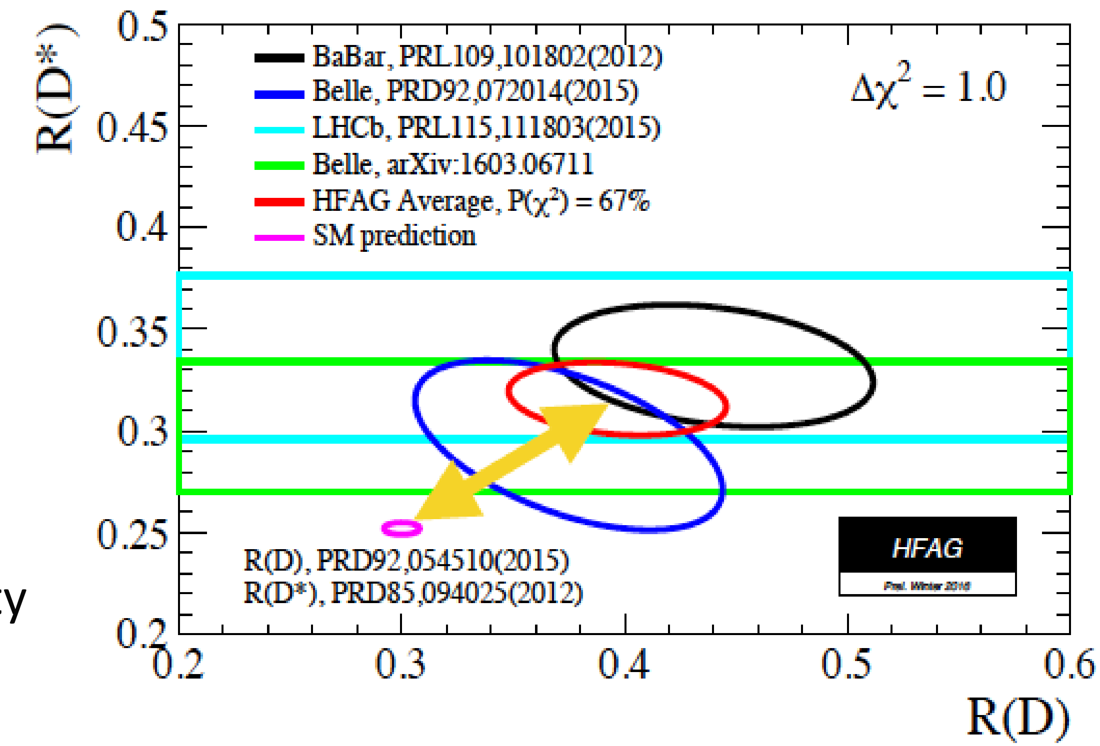
2.2 Lepton universality & NP

- A renewed interest in possible violations of LFU has been triggered by two very different sets of observations in B physics:
  1. LFU test in b → c charged currents: τ vs. light leptons (μ, e) :

$$\boxed{R(X) = \frac{\Gamma(B \to X \tau \bar{\nu})}{\Gamma(B \to X \ell \bar{\nu})}}$$







SM prediction solid: f.f. uncertainty cancels (to a good extent...) in the ratio

Consistent results by 3 different exps ⇨ 4σ excess over SM (combining D and D*)

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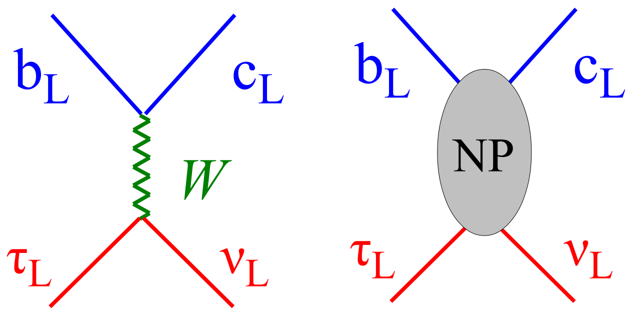

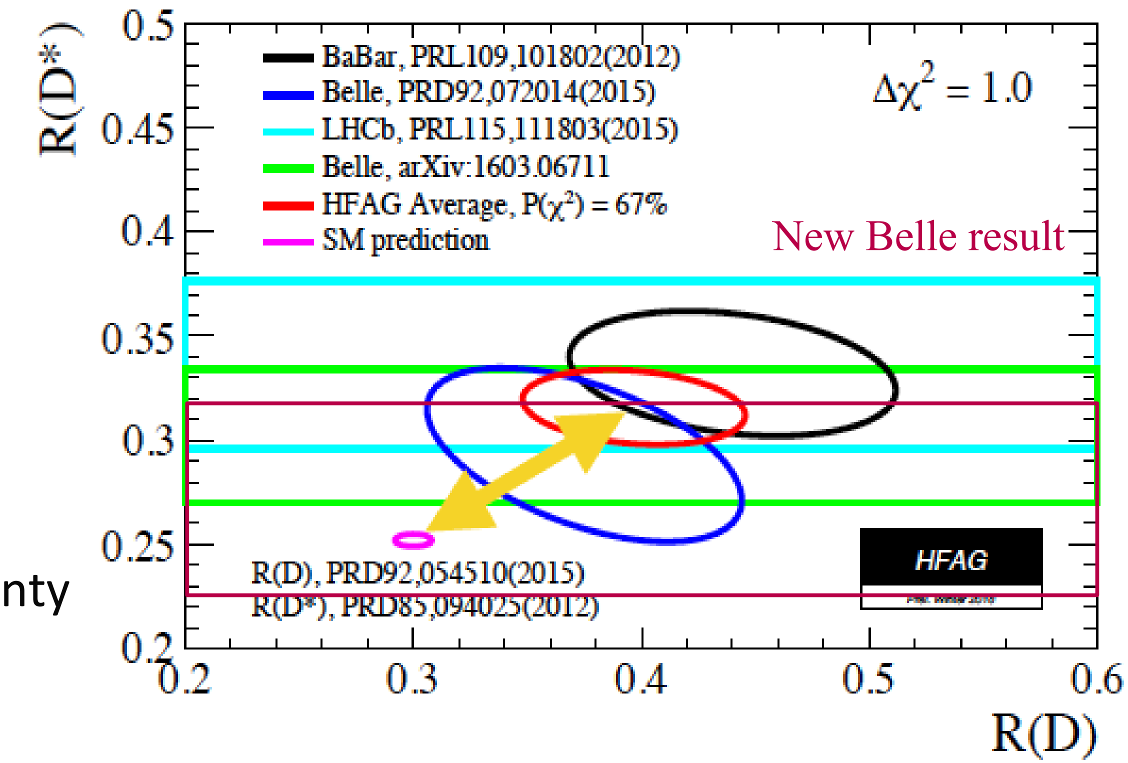

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⇨ New results from LHCb on Friday, see talk by *Kristof De Bruyn*

2.2 Lepton universality & NP

- A renewed interest in possible violations of LFU has been triggered by two very different sets of observations in B physics:
  1. LFU test in b → c charged currents: τ vs. light leptons (μ, e) :
  2. LFU test in b → s neutral currents: μ vs. e :

$$R_K = \frac{\mathrm{Br}[B^+ \to K^+ \mu^+ \mu^-]_{[1,6]}}{\mathrm{Br}[B^+ \to K^+ e^+ e^-]_{[1,6]}} = 0.745 \cdot (1 \pm 13\%) \quad \text{vs} \quad R_K^{\mathrm{SM}} = 1.003 \pm 0.0001$$

2.6σ deviation from the SM

- This has triggered intense theoretical activities:
  D & D* channels are well consistent with a universal enhancement (~15%) of the SM $b_L \to c_L\ \tau_L\ \nu_L$ amplitude (*RH or scalar amplitudes disfavored*)

  Natural to conceive NP models where LFU is violated more in processes involving *3rd gen. quarks & leptons* (↔ *hierarchy in Yukawa coupl.*)

See talk by *Dr. Xinqiang Li*

- Angular distribution could help? ⇨ Belle II

2.3 Leptonic decays & NP

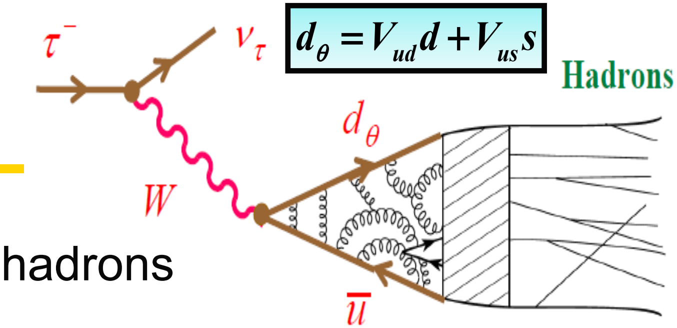
- For constraints on the *Lorentz structure*:

  ⇒ Michel parameters

Important activity in Belle ⇒ see talks by *Denis Epifanov*, *Nobuhiro Shimizu*

3. Hadronic τ-decays

3.1 Introduction



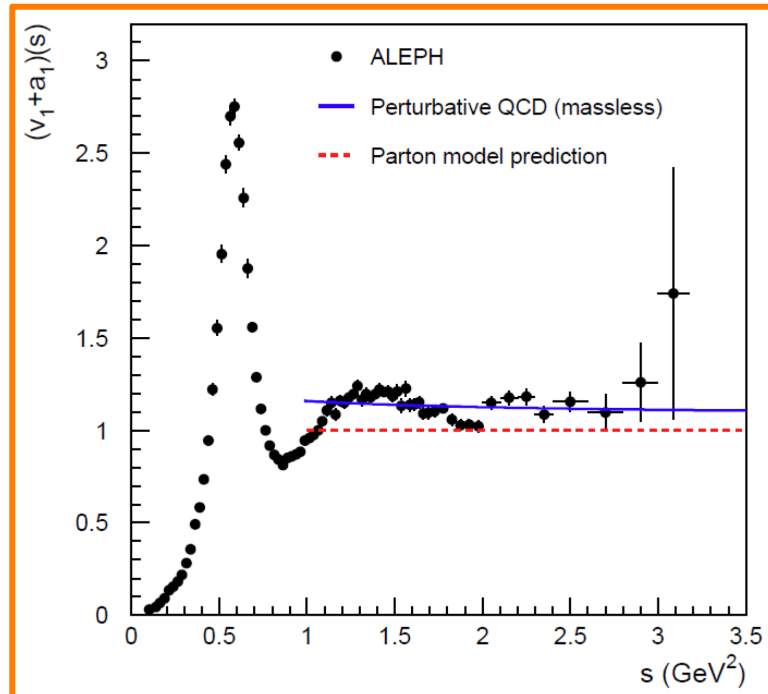
- Tau, the only lepton heavy enough to decay into hadrons
- $\boxed{m_\tau \sim 1.77\,\text{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*
- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $\left(\alpha_S(m_\tau), |V_{us}|, m_s\right)$

Davier et al'13

- We consider $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S=0}\right)}$

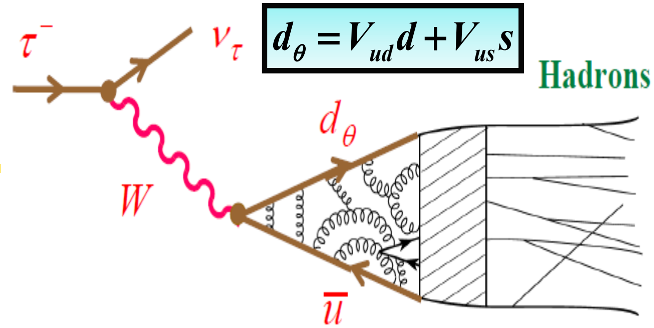
  $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S\neq 0}\right)}$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!



- Observable studied: $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$

3.2 Theory

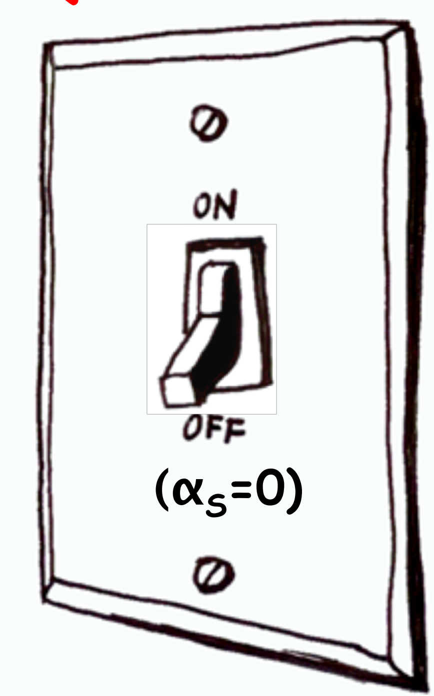


- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^S \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- $$\boxed{\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} = \frac{R_\tau^S}{R_\tau^{NS}}} \Rightarrow \left|V_{us}\right|$$

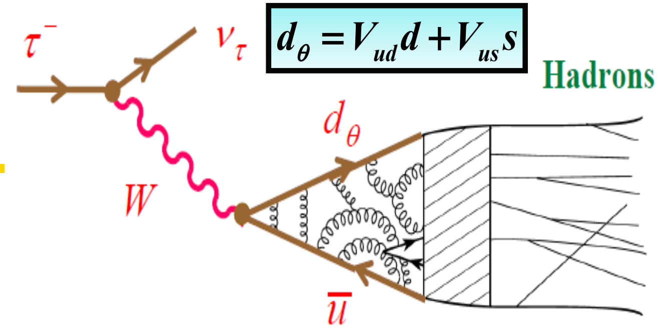
QCD switch



$(\alpha_S=0)$

Figure from M. González Alonso'13

3.2 Theory



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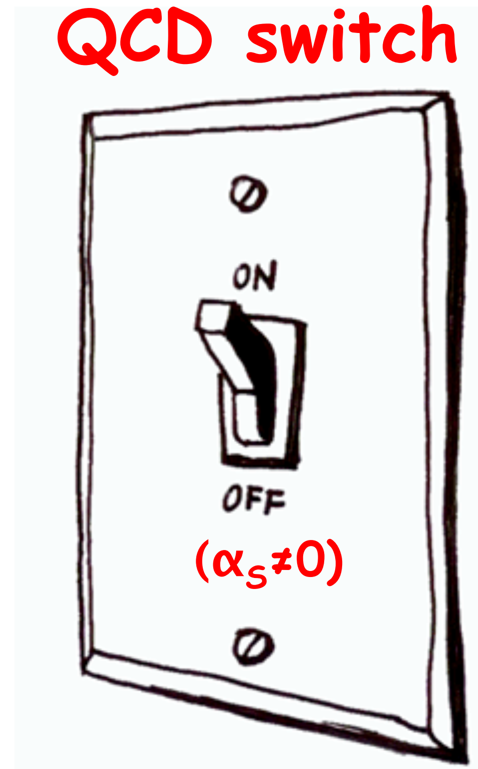
- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- Experimentally: $$R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$$



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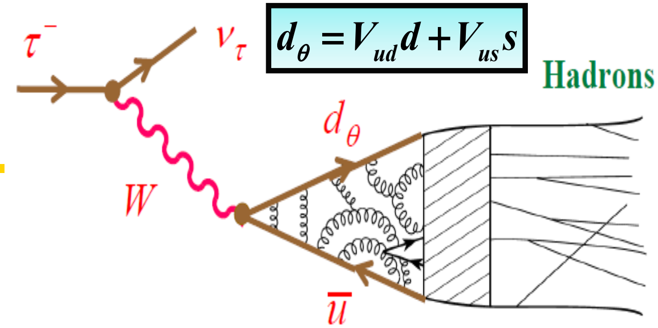
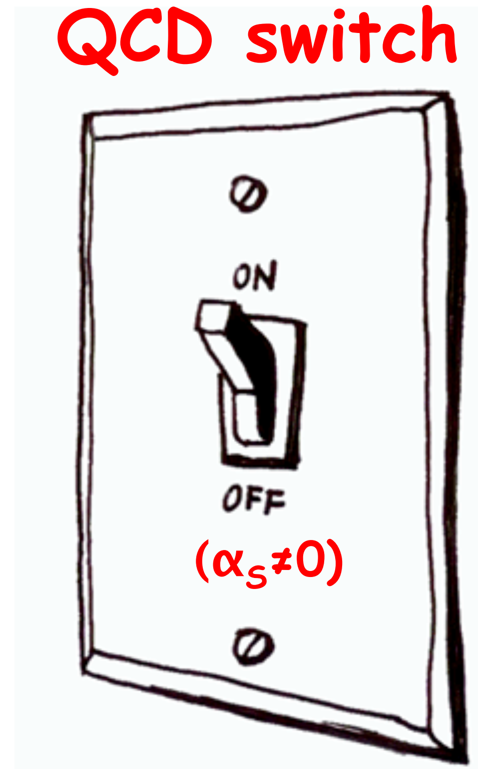


- $\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} \approx N_C}$ parton model prediction

- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- Experimentally: $\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

- Due to *QCD corrections*: $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$



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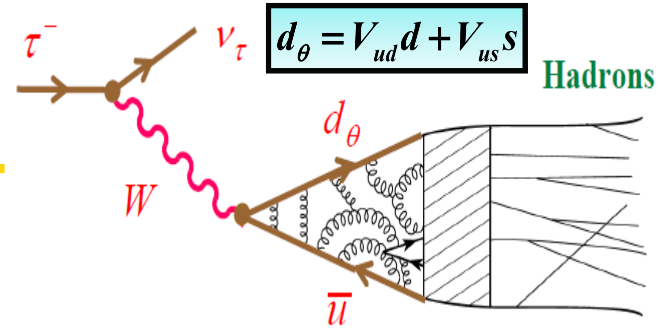
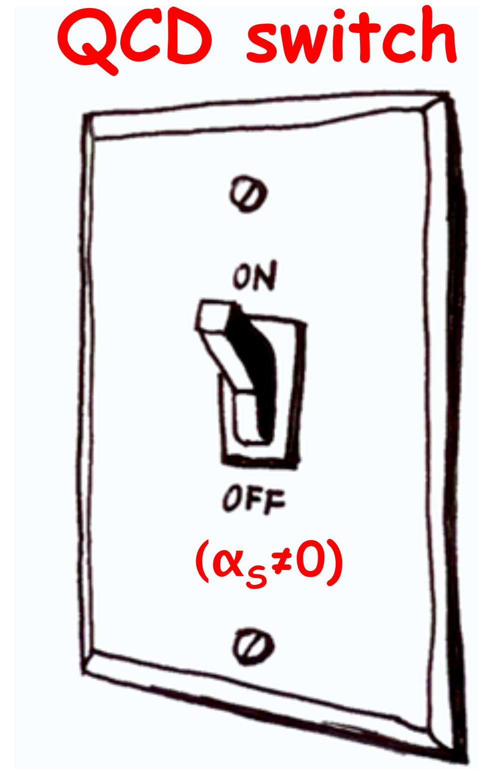


- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$ naïve QCD prediction



- Extraction of the strong coupling constant :

$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_s) \Rightarrow \alpha_S$$

measured calculated

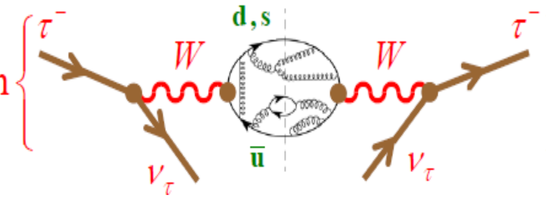
- Determination of V_{us} : $\boxed{\dfrac{|V_{us}|^2}{|V_{ud}|^2} = \dfrac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$

- Main difficulty: compute the QCD corrections with the best accuracy

3.3 Calculation of the QCD corrections

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$$



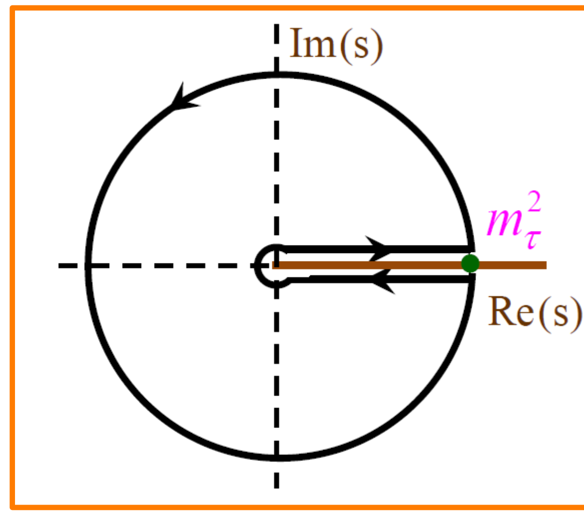
$$\boxed{R_\tau(m_\tau^2)=12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon)+\mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$\boxed{R_\tau(m_\tau^2)=6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]}$$



- We are now at sufficient energy to use OPE:

$$\boxed{\Pi^{(J)}(s)=\sum_{D=0,2,4\ldots}\frac{1}{(-s)^{D/2}}\sum_{\dim O=D}\mathrm{C}^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}$$

Wilson coefficients | Operators

μ: separation scale between short and long distances

3.3 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + ... \approx 20\%$ $\quad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

3.3 Calculation of the QCD corrections

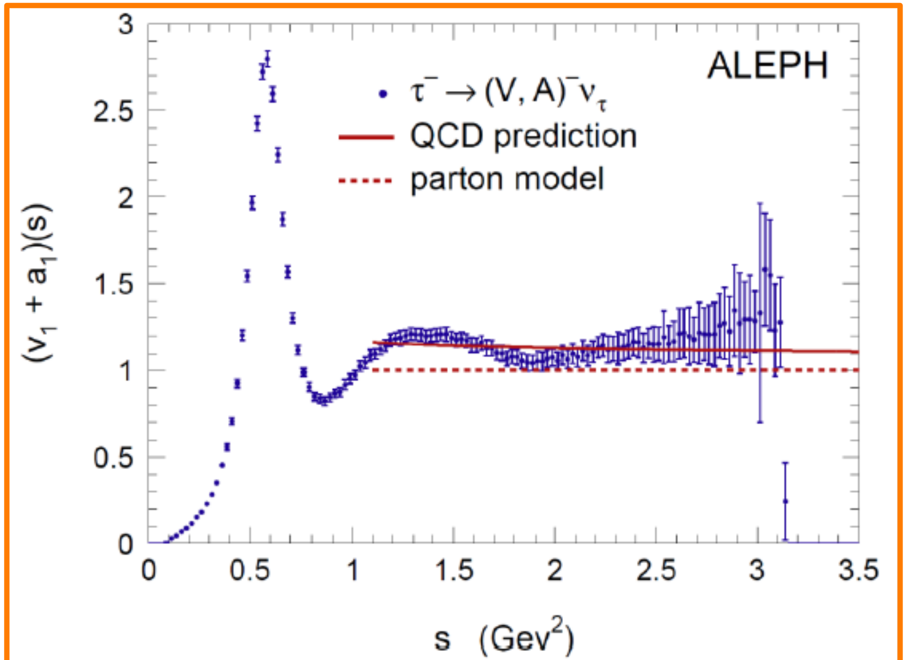
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

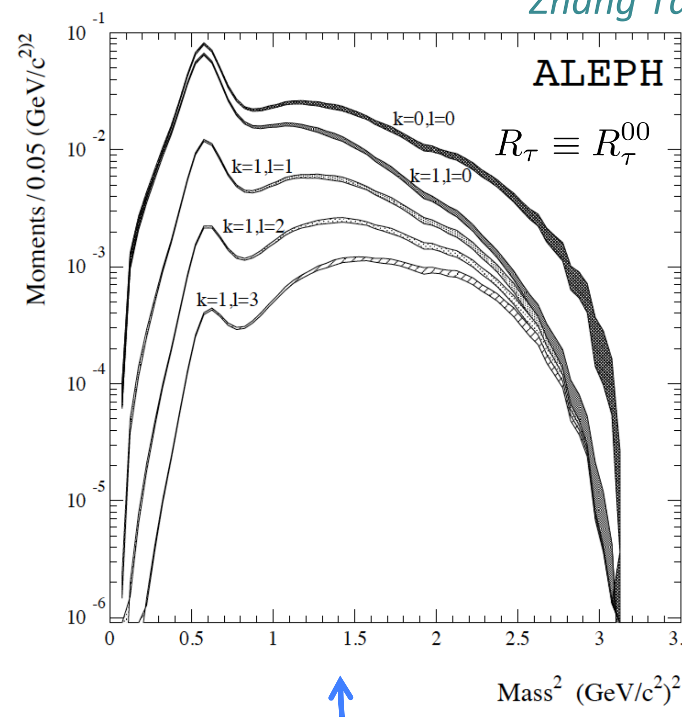
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

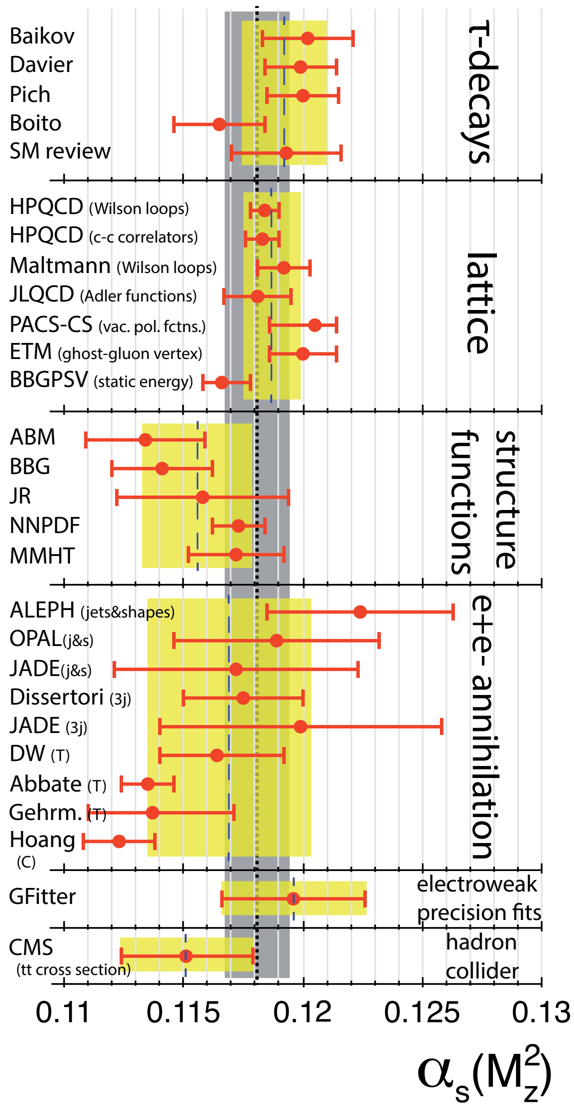


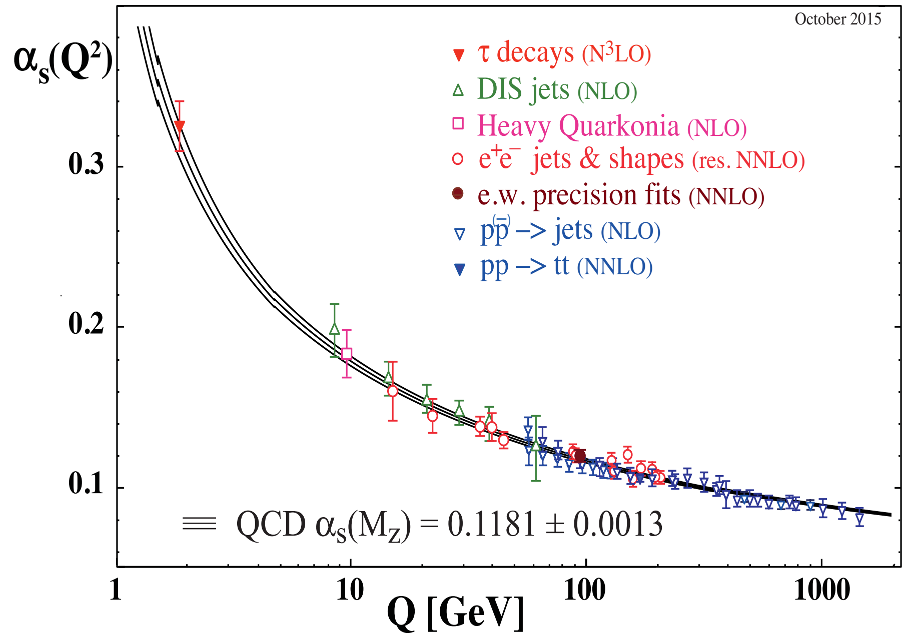
Zhang'Tau14



3.4 Extraction of α_s

Bethke, Dissertori, Salam, PDG'15





- *Extraction of α_s* from hadronic τ very interesting : Moderate precision at the τ mass ⇨ very good precision at the Z mass

- Beautiful test of the QCD running

3.4 Extraction of α_s

- Several delicate points:
  - How to compute the perturbative part: CIPT vs. FOPT?
  - How to estimate the non perturbative contribution? Where do we truncate the expansion, what is the role of higher order condensates?
  - Which weights should we use?
  - What about duality violations?

⇨ A MITP topical workshop in Mainz: March 7-12, 2016

Determination of the fundamental parameters of QCD

A session on Tuesday afternoon

- New data on spectral functions needed to help to answer some of these questions

3.4 Inclusive determination of V_{us}

- With QCD on: $\boxed{\dfrac{|V_{us}|^2}{|V_{ud}|^2} = \dfrac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$

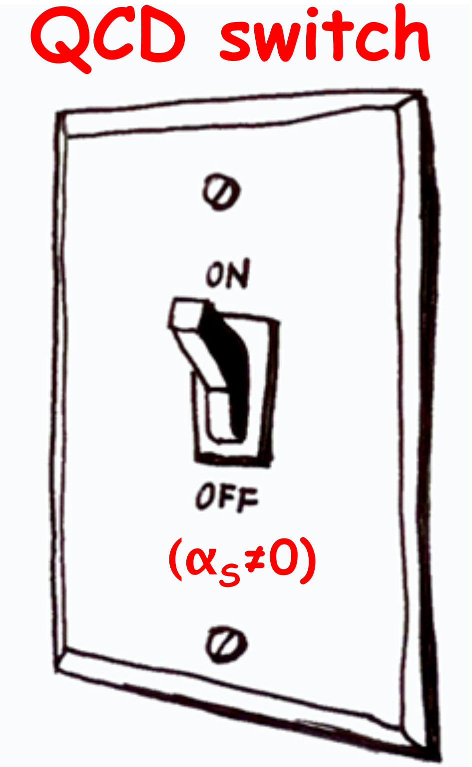
- Use OPE: $\boxed{R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1+\delta_P+\delta_{NP}^{ud}\right)}$

$$\boxed{R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1+\delta_P+\delta_{NP}^{us}\right)}$$

- $\boxed{\delta R_\tau \equiv \dfrac{R_{\tau,NS}}{|V_{ud}|^2} - \dfrac{R_{\tau,S}}{|V_{us}|^2}}$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0239(30)$$ *Gamiz et al'07, Maltman'11*



$$\boxed{|V_{us}|^2 = \dfrac{R_{\tau,S}}{\dfrac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

HFAG'14

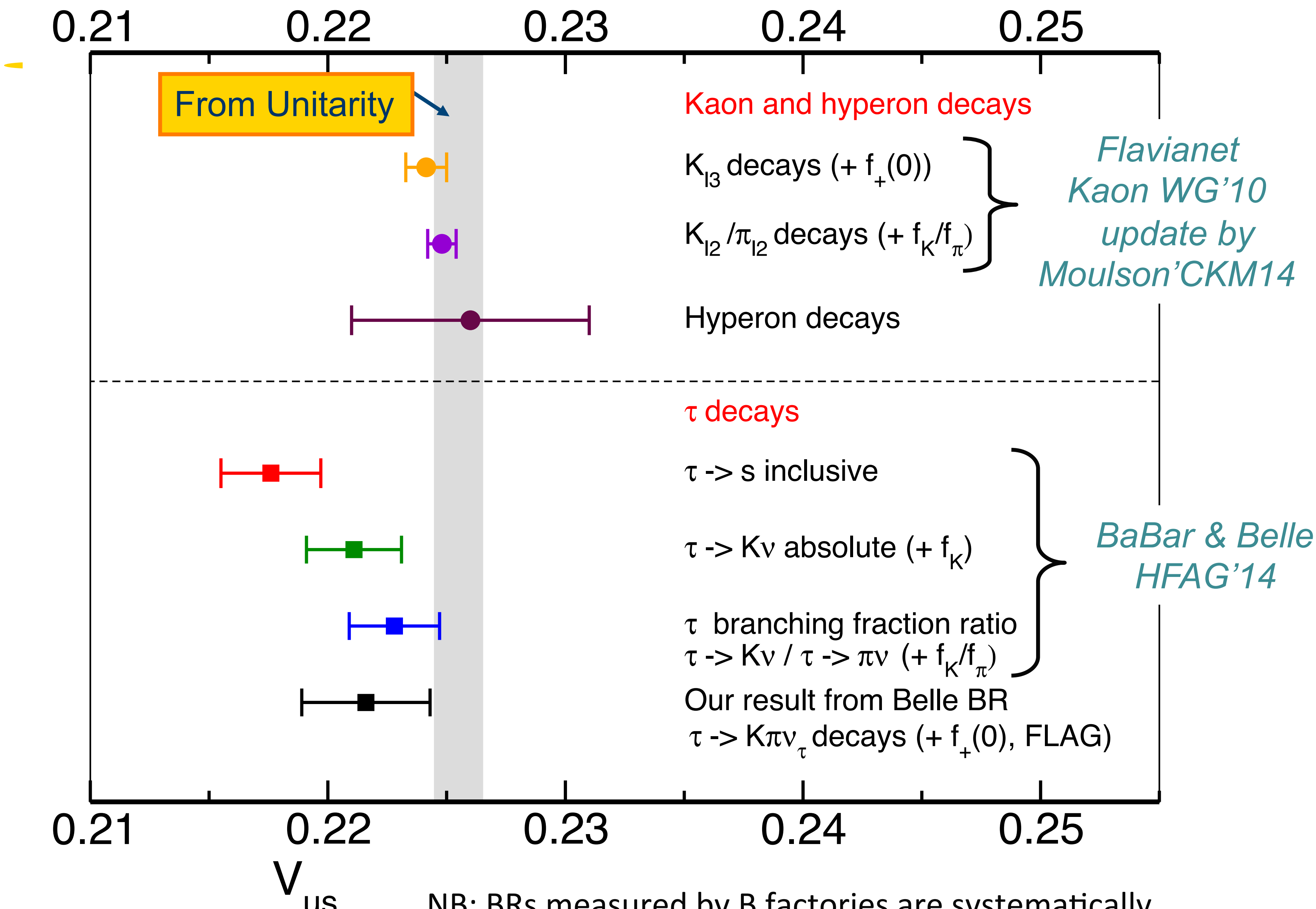
$R_{\tau,S} = 0.1615(28)$

$R_{\tau,NS} = 3.4650(84)$

$|V_{ud}| = 0.97425(22)$

$\Rightarrow$ $\boxed{|V_{us}| = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$

3.4σ away from unitarity!



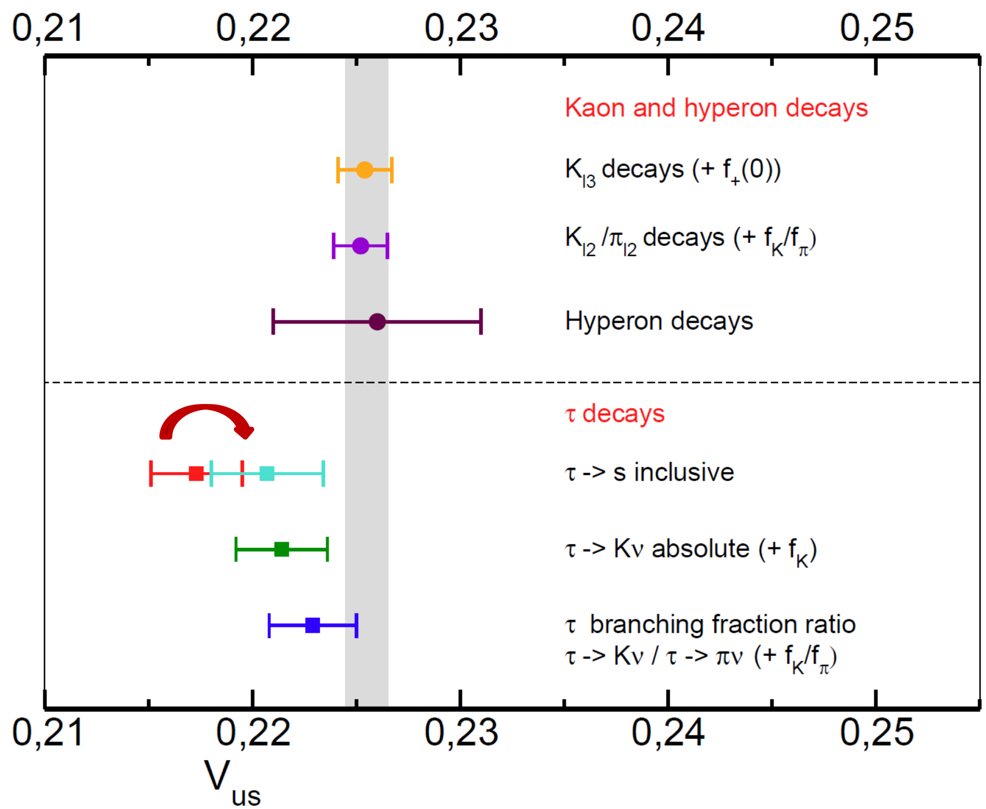
NB: BRs measured by B factories are systematically smaller than previous measurements

3.5 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	→ $(0.713 \pm 0.003)\%$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	→ $(0.471 \pm 0.018)\%$
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	→ $(0.857 \pm 0.030)\%$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	→ $(2.967 \pm 0.060)\%$

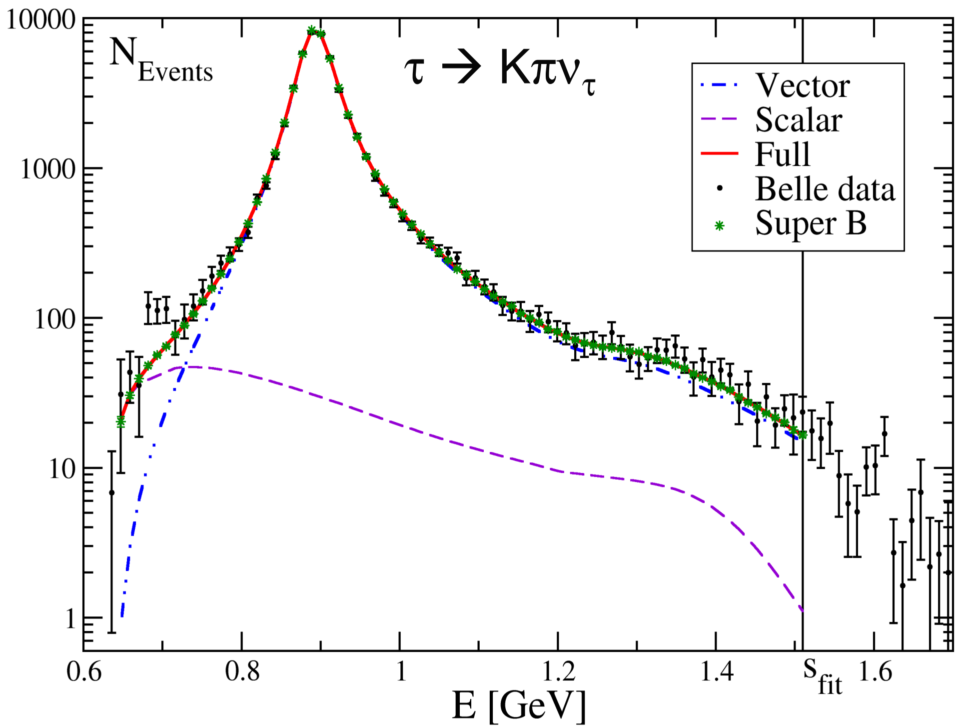
- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us} seem to have been resolved !
  - ⇨ See talk by *K. Maltman*
  - See also talks in V_{us} session
- Crucial input: $\tau \to K\pi\nu_\tau$ Br + spectrum
  - ⇨ need new data



3.6 Exclusive Tau decays

- Invariant mass spectra: constraints on FF very important for testing QCD dynamics and the SM and new physics:

$K\pi$ form factors : ⇨ V_{us}



Jamin, Pich, Portolés'08

Boito, Escribano, Jamin'09,'10

Bernard, Boito, E.P'11

Bernard'13,

Escribano, González-Solis, Jamin, Roig'14

See talk by *S. González-Solis, R. Escribano*

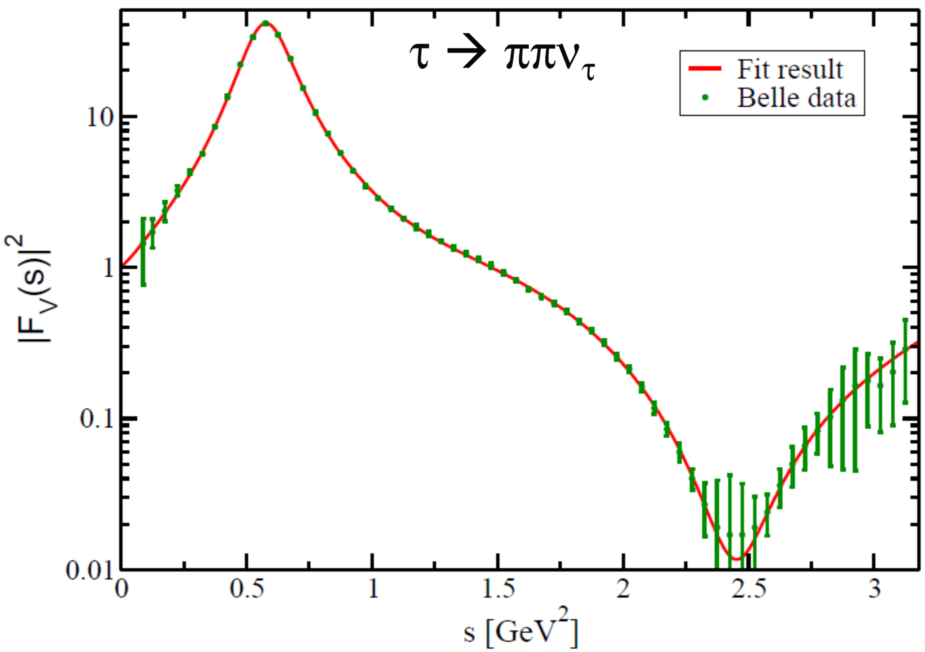
3.6 Exclusive Tau decays

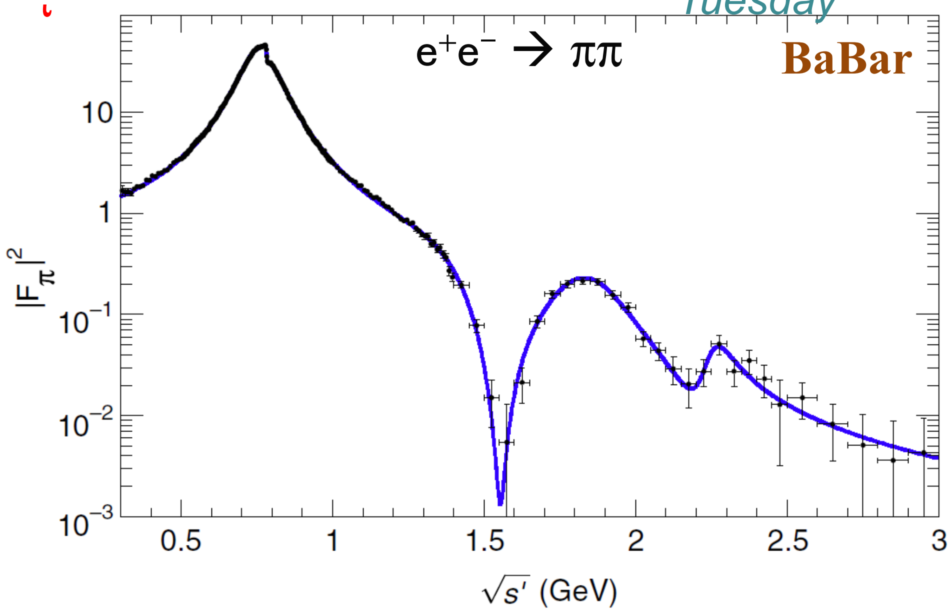
- Invariant mass spectra: constraints on FF very important for testing QCD dynamics and the SM and new physics:

$\pi\pi$ form factors ⇨ g-2 of the muon, LFV hadronic tau decays, proton radius etc

Gómez Dumm – Roig'13 *Cirigliano, Celis, E.P.'14*

See talks on *Monday* & *Tuesday*





- 3 body tau spectra also important: e.g. $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to K\pi\pi\nu_\tau$, $\tau \to \eta^{(\prime)}\pi\pi\nu_\tau$

  ⇨ in this case Dalitz plots needed, see talk by *Z. Was*

e.g.: Gómez Dumm & Roig'12
Was et al.

4. Charged Lepton-Flavour Violation

4.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

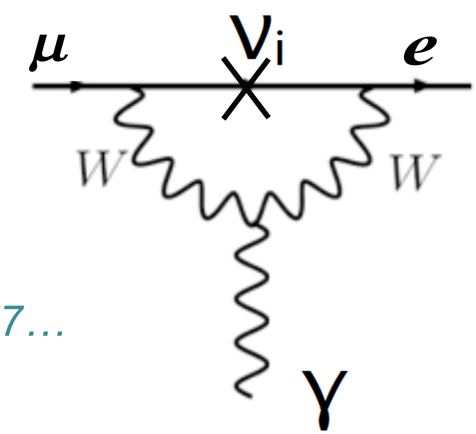
- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

E.g.: $\boxed{\mu \to e\gamma}$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$



- Extremely *clean probe of beyond SM physics*

4.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

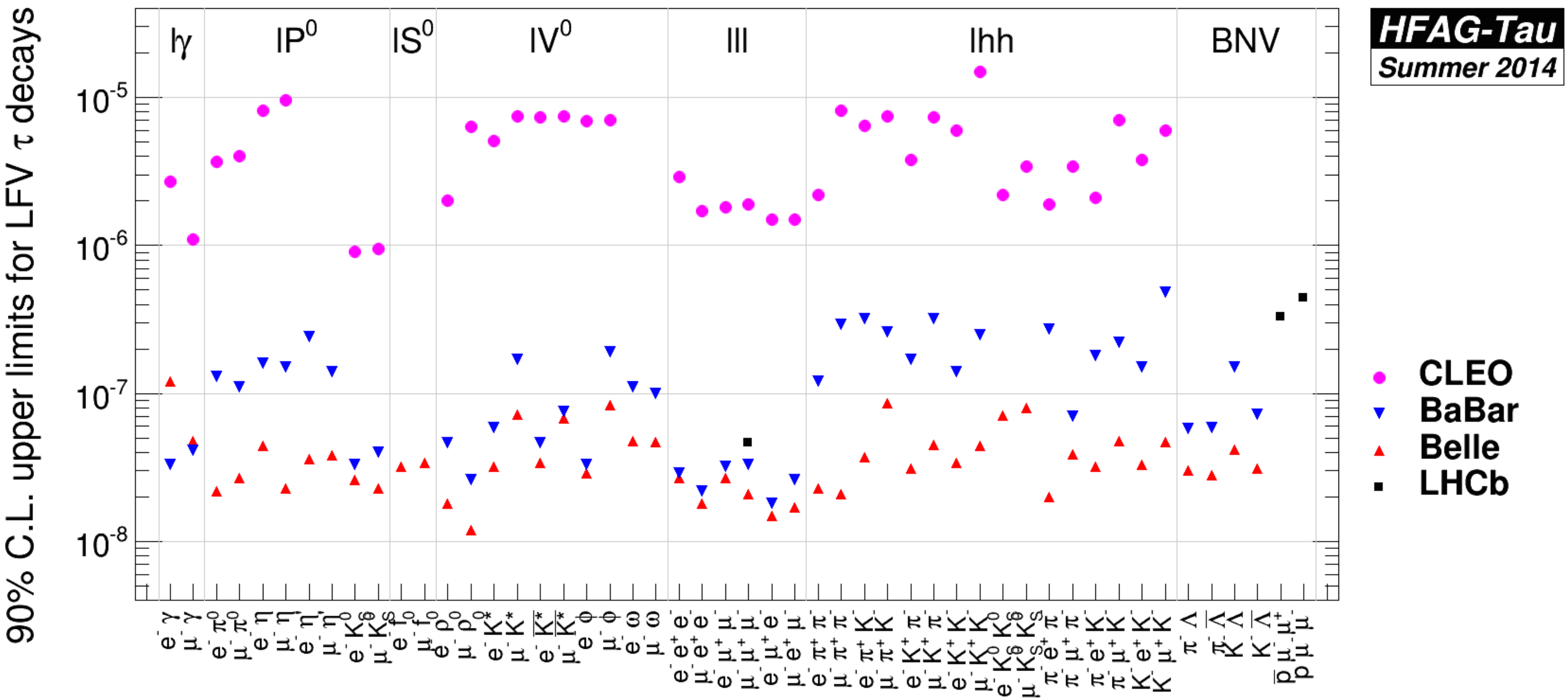
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent

- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

4.2 CLFV processes: tau decays

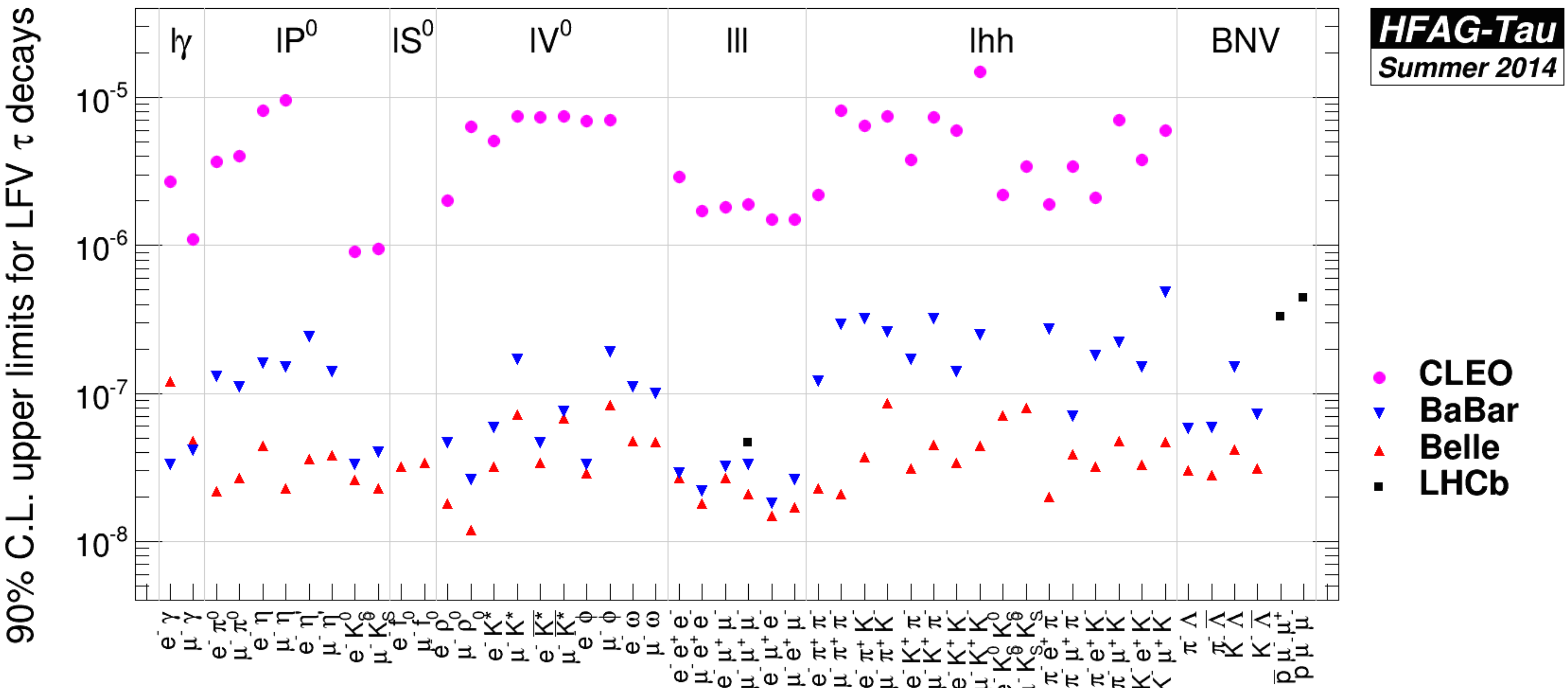
- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P},\ldots$



- 48 LFV modes studied at Belle and BaBar

4.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ↖ $P,\ S,\ V,\ P\bar{P}, \ldots$



- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

See talks on Thursday afternoon

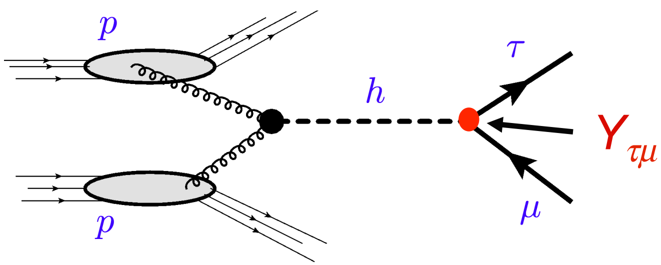
4.5 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

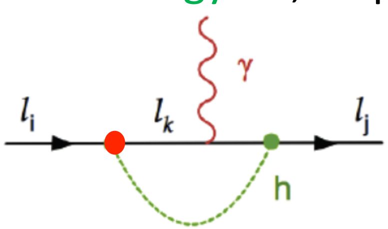
Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

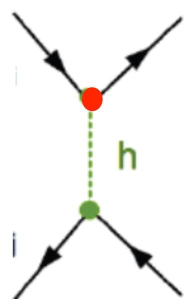
- High energy : LHC

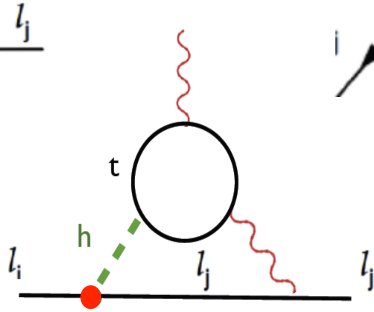


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







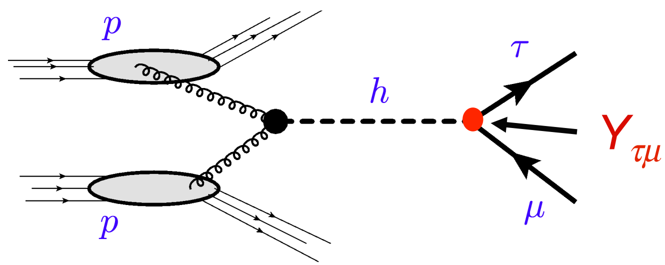
4.5 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H}$$ $\Rightarrow$ $$-Y_{ij}\left(\bar{f}_L^i f_R^j\right)h$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

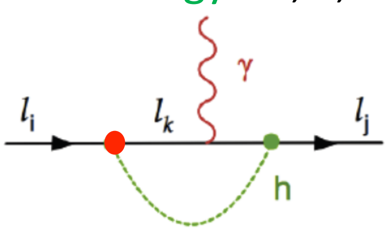
- High energy : LHC

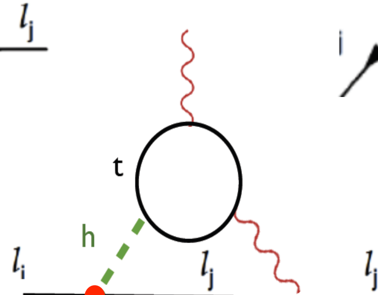


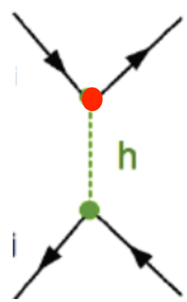
Hadronic part treated with perturbative QCD

Reverse the process

- Low energy : D, S, G operators







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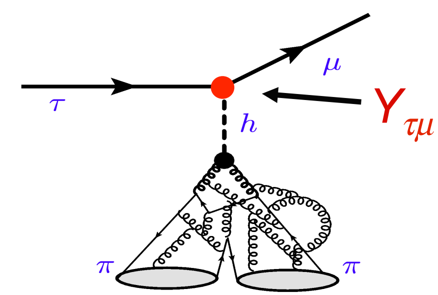
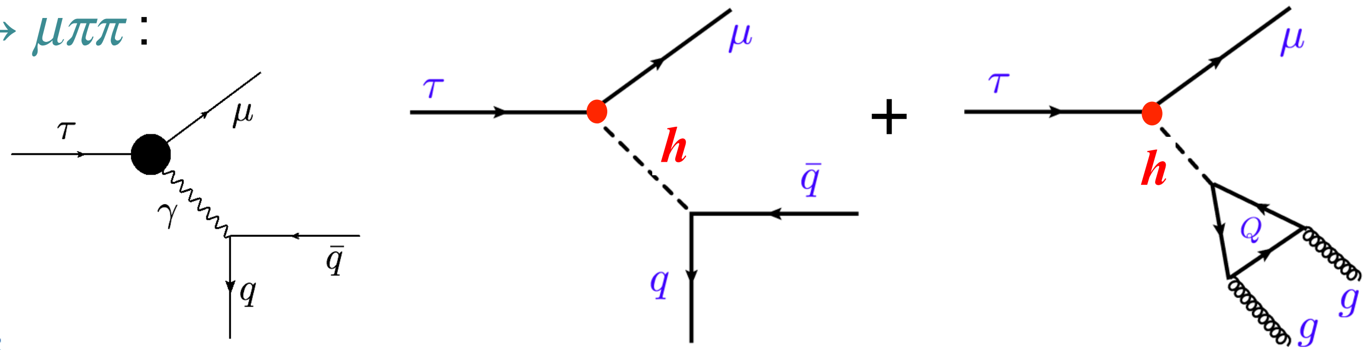


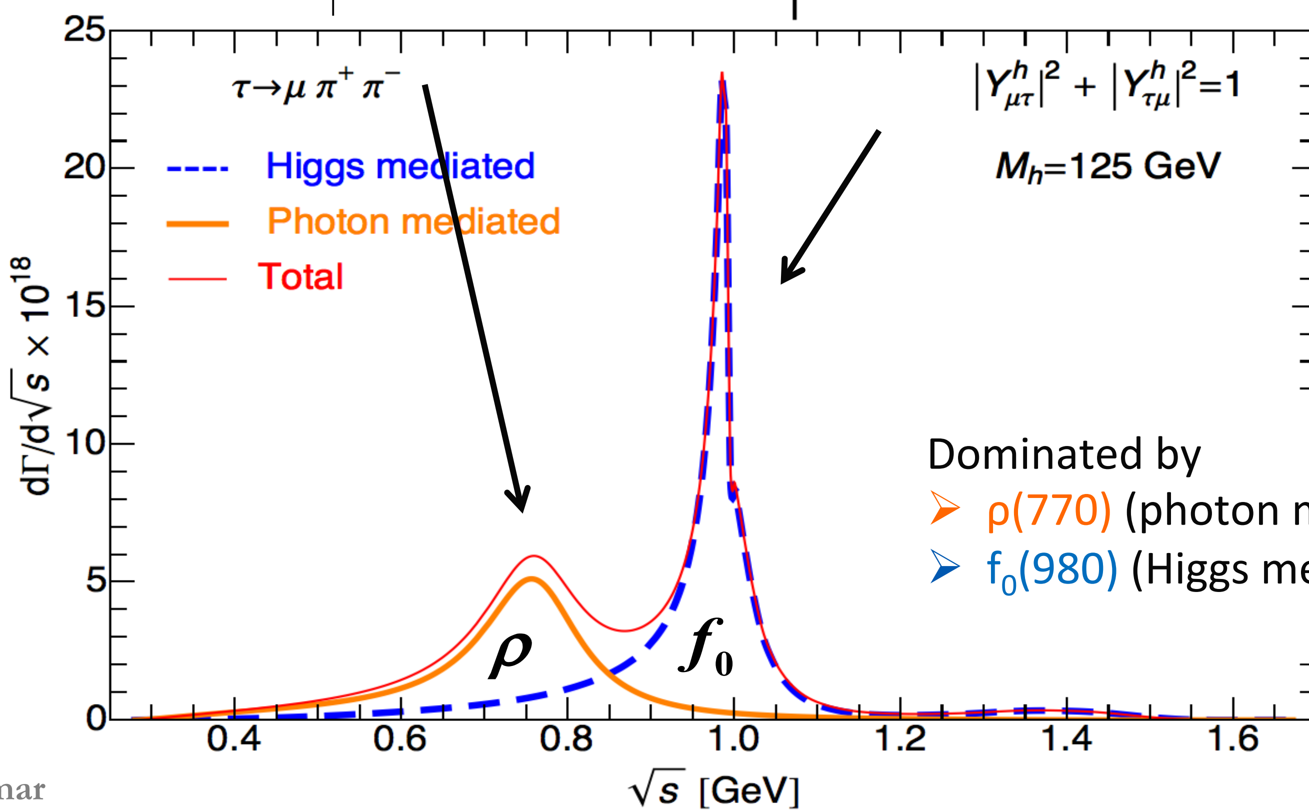
Hadronic part treated with non-perturbative QCD

4.6 Constraints in the τμ sector

Cirigliano, Celis, E.P.'14

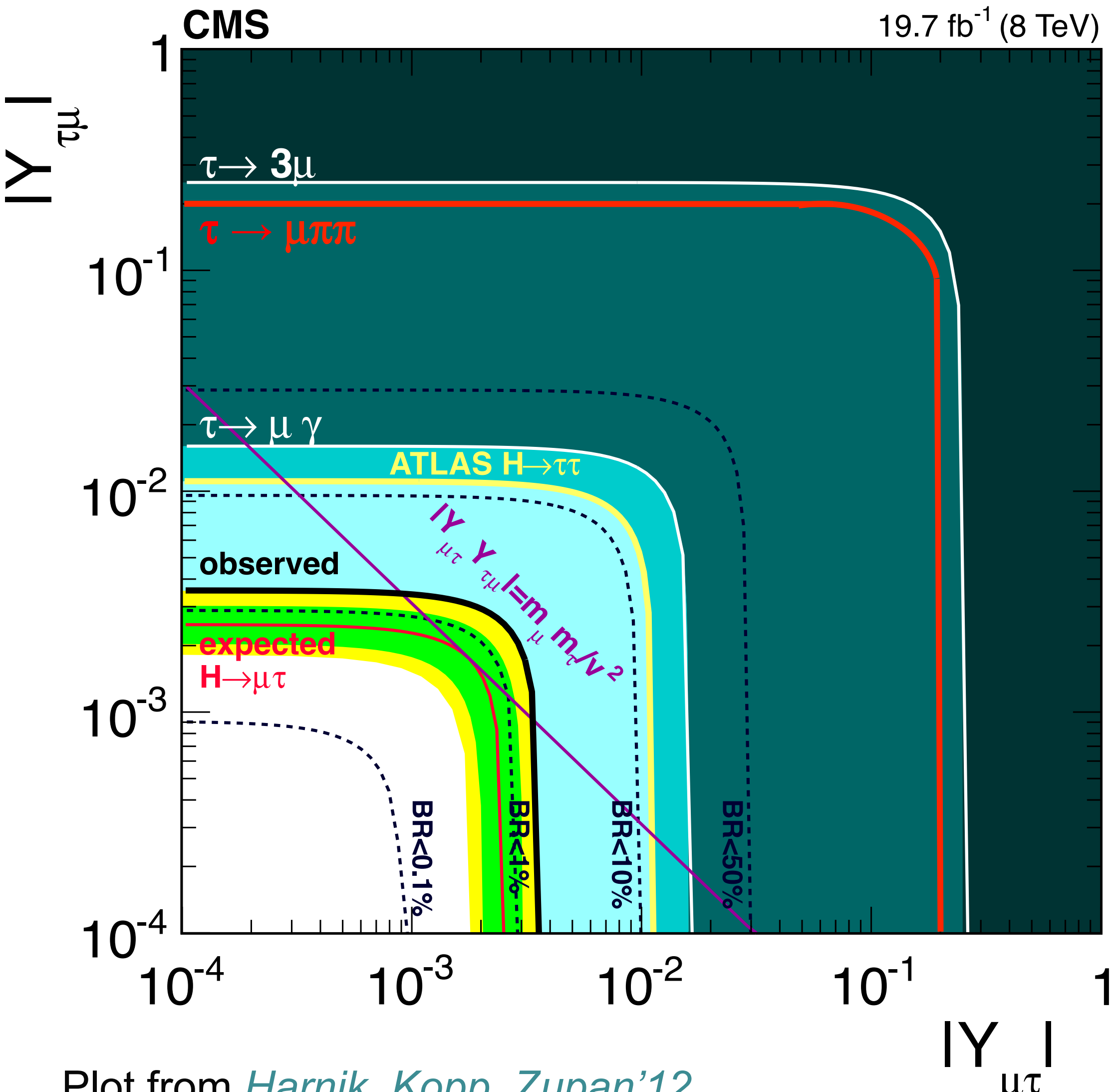
- At low energy
  - $\tau \rightarrow \mu\pi\pi$:





Dominated by
- ρ(770) (photon mediated)
- $f_0(980)$ (Higgs mediated)

4.6 Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12*
updated *by CMS'15*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ⇨ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ⇨ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

⇨ $\boxed{BR(\tau \to \mu\gamma) < 2.2\times 10^{-9}}$

$\boxed{BR(\tau \to \mu\pi\pi) < 1.5\times 10^{-11}}$

4.7 Hint of New Physics in $h \to \tau\mu$?

$$BR\left(h \to \tau\mu\right) = \left(0.84^{+0.39}_{-0.37}\right)\%$$

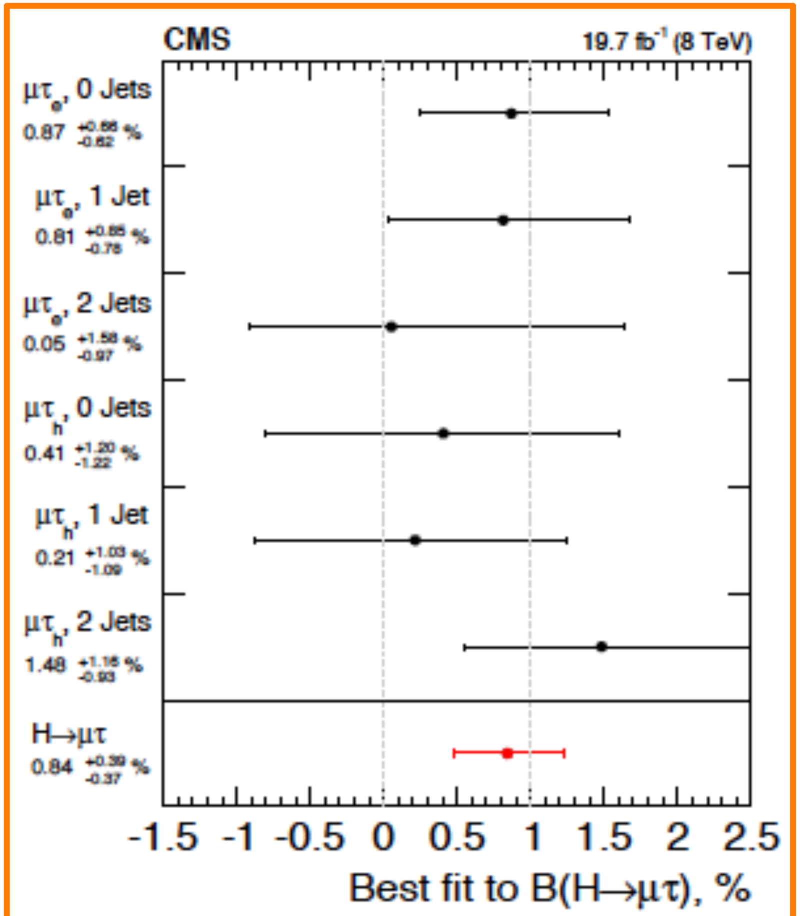
@2.4σ

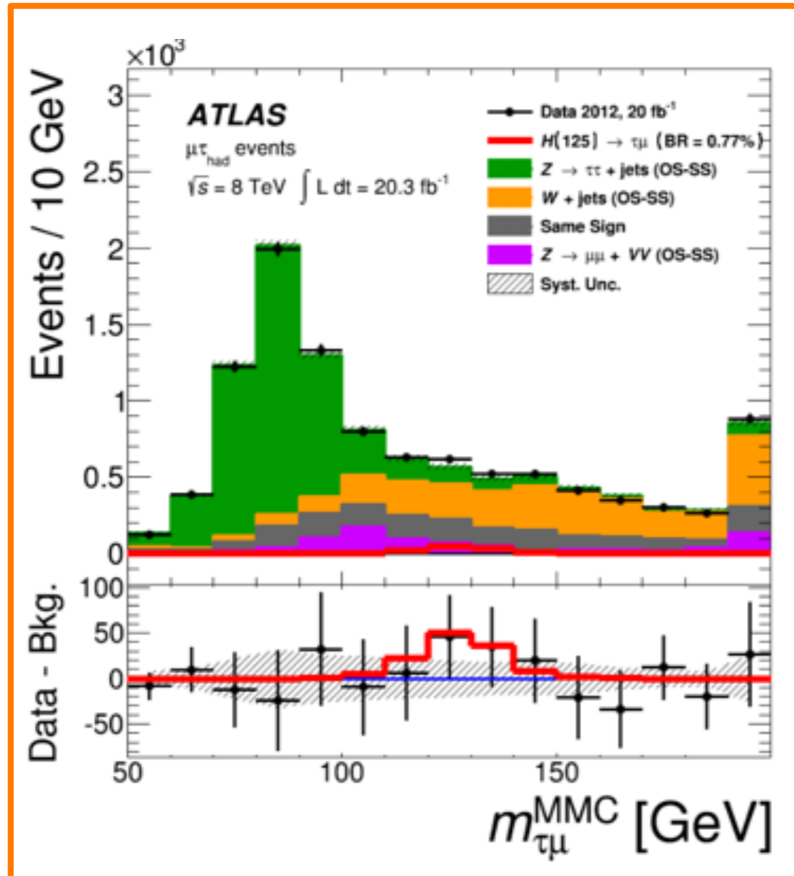
CMS'15

$$BR\left(h \to \tau\mu\right) = \left(0.53 \pm 0.51\right)\%$$

@1σ

ATLAS'15





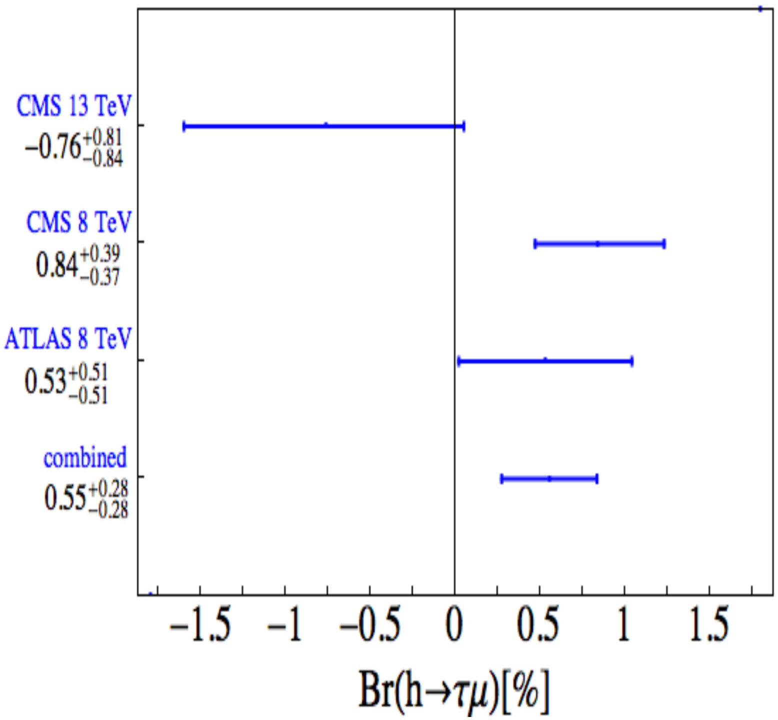
$$BR\left(h \to \tau\mu\right) = \left(-0.76^{+0.81}_{-0.84}\right)\%$$

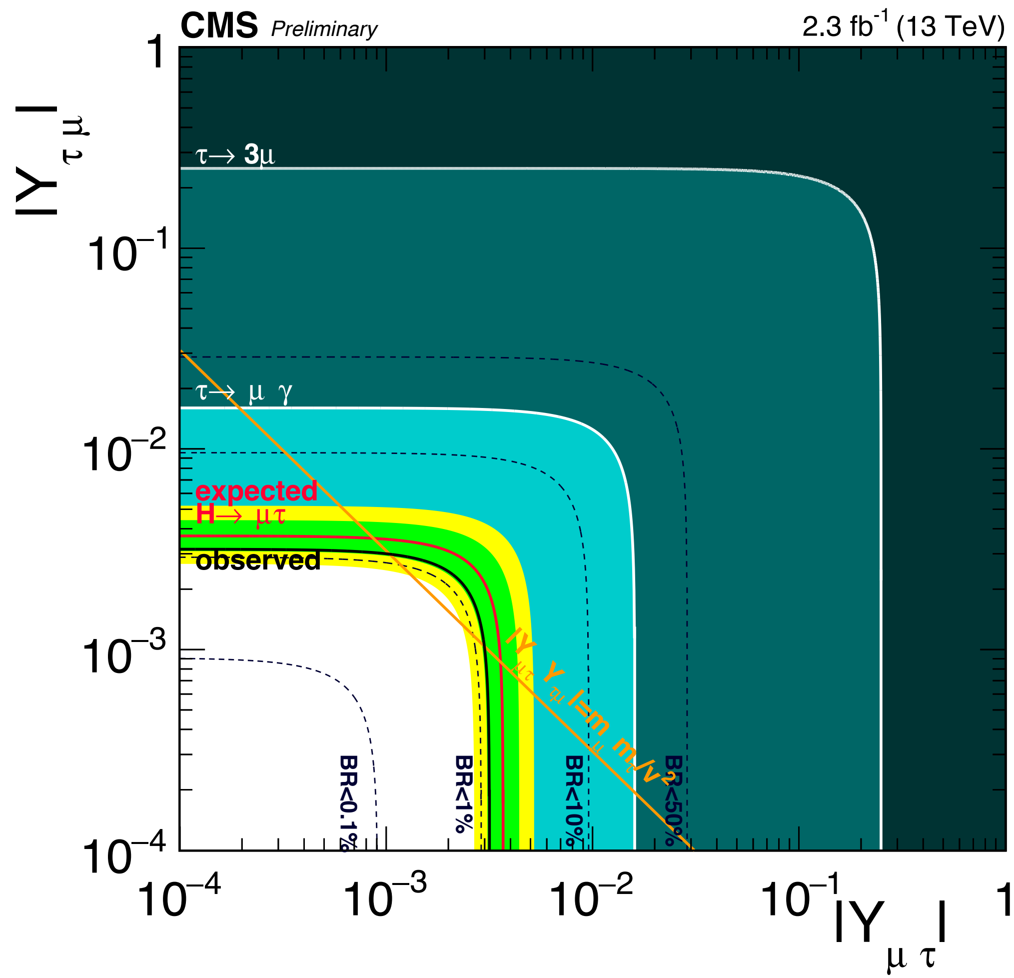
13 TeV@CMS

M. Cepeda@Higgs Tasting'16

4.7 Hint of New Physics in h → τμ?

CMS'16



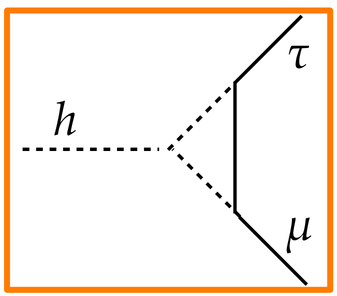


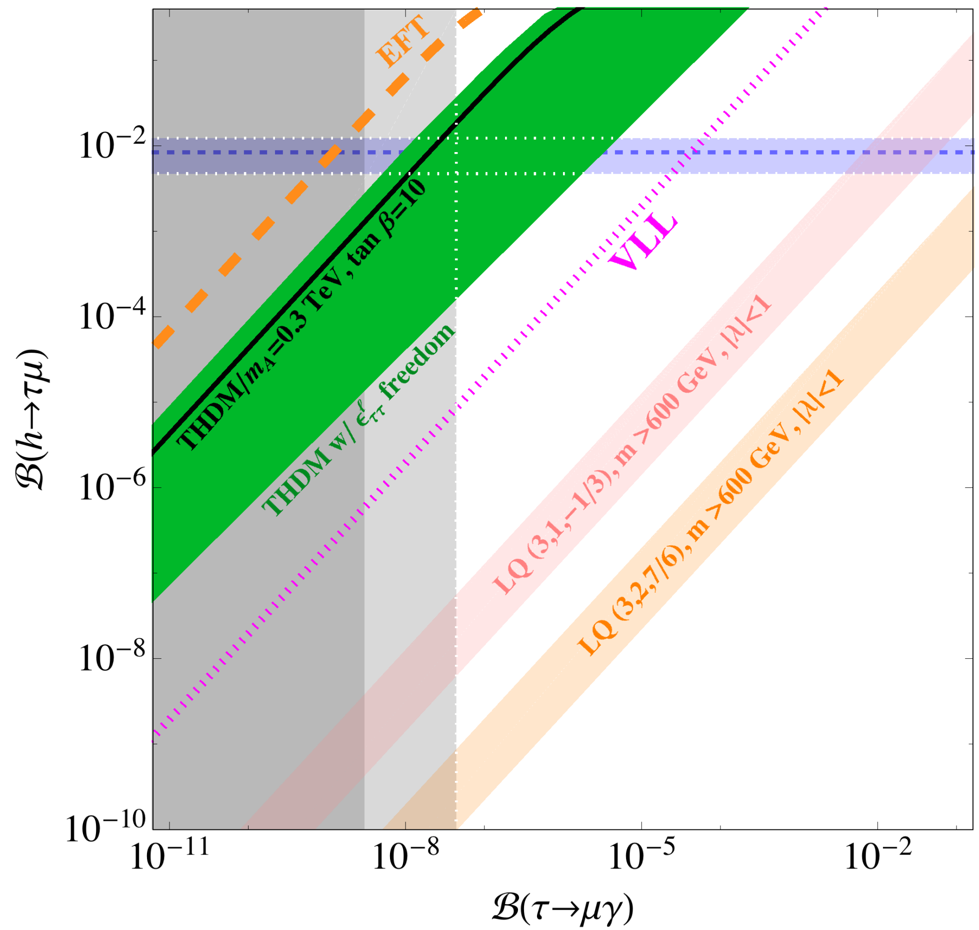
See talks on *Friday*

4.8 Interplay between LHC & Low Energy

Dorsner et al.'15

- If real what type of NP?
- If $h \to \tau\mu$ due to loop corrections:
  - extra charged particles necessary
  - $\tau \to \mu\gamma$ too large





- $h \to \tau\mu$ possible to explain if extra scalar doublet:
  ⇨ *2HDM of type III*
- Need other sources of EWSB: *2HDMs, technicolour models* — *Altmannshofer et al.'15*
- Constraints from $\tau \to \mu\gamma$ important!

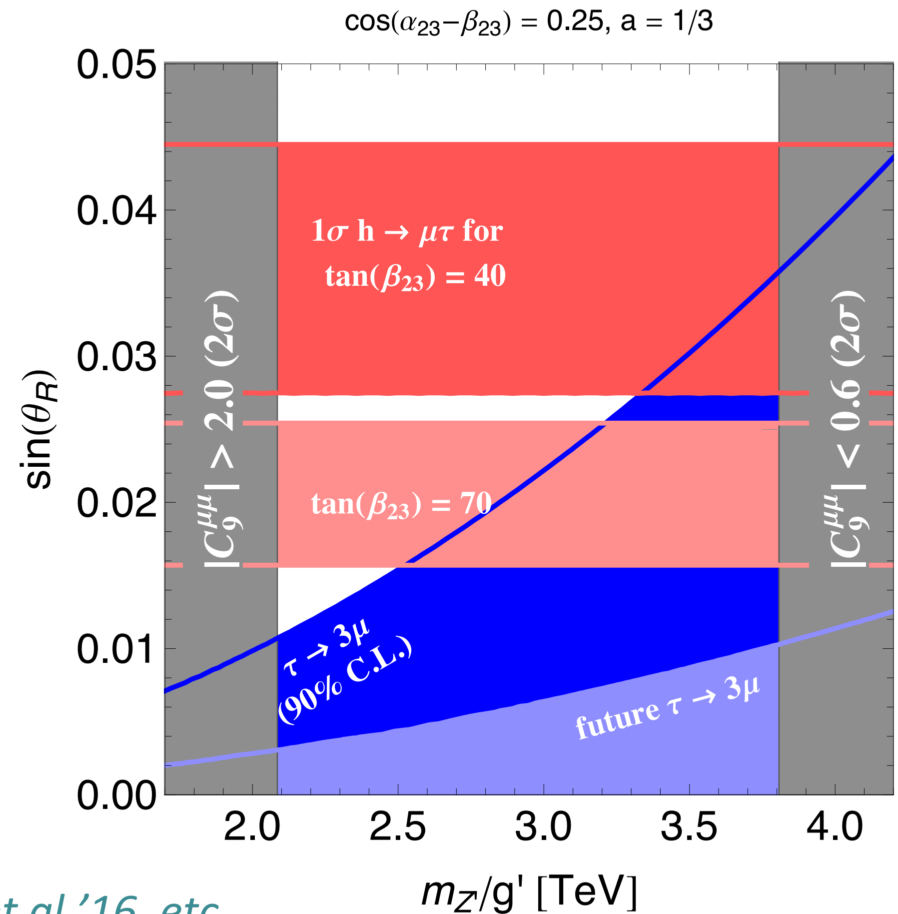
4.8 Interplay between LHC & Low Energy

- *2HDMs* with gauged $L_\mu - L_\tau$
  ⇨ Z', explain anomalies for
  - $h \to \tau\mu$
  - $B \to K^*\mu\mu$
  - $R_K = B \to K\mu\mu / B \to Kee$

- Constraints from $\tau \to 3\mu$ *crucial* ⇨ *Belle II, LHC*

- See also, e.g.:
  Aristizabal-Sierra & Vicente'14,
  Lima et al'15,
  Omhura, Senaha, Tobe '15
  Altmannshofer et al.'15
  Bauer and Neubert'16, Buschmann et al.'16, etc…
  ⇨ *See talk by P. Roig*

Altmannshofer & Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15



5. Taus at LHC

- Tau: Largest Higgs-Lepton coupling: 4^{th} Higgs BR

See talks on Wednesday

ATLAS'15

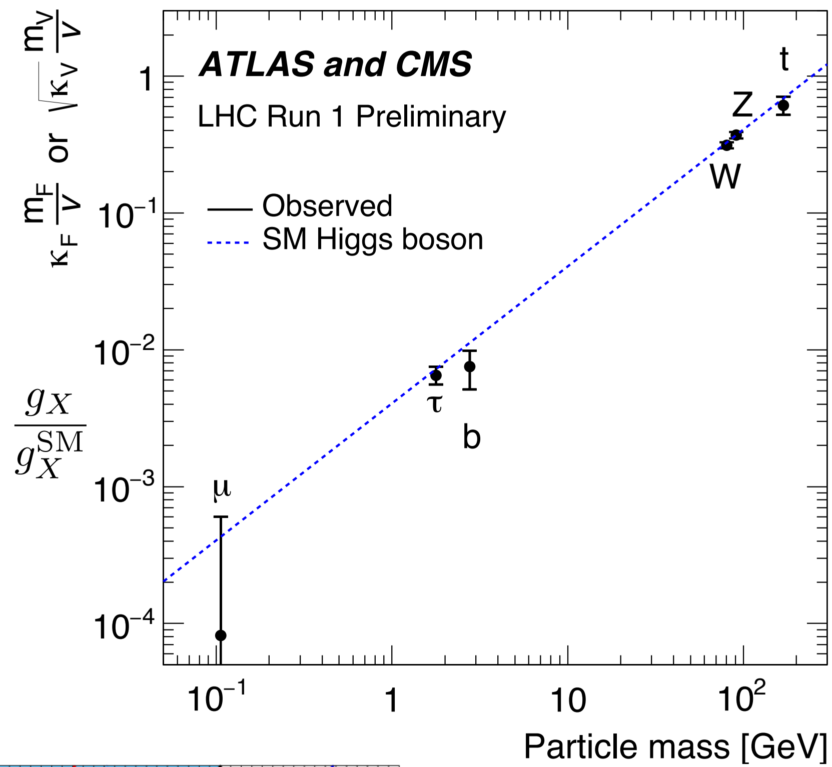
- Excellent signature to probe NP:
  Difficult to identify light objects ($Z, W^{\pm}$) with only Jets
  QCD Jets orders of magnitude larger
  Must rely on *leptons*

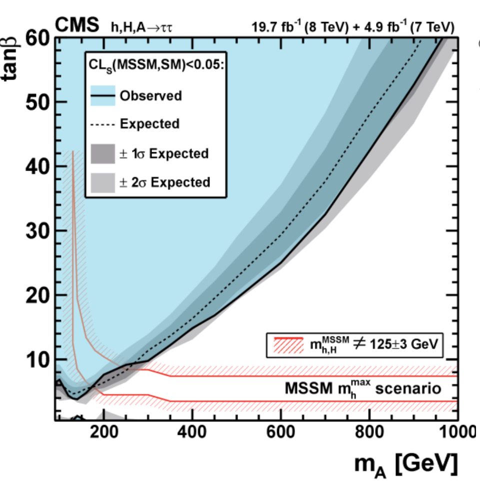
- LHC produces high-momenta τ's
  Tightly collimated decay products (mini-jet like)
  Momentum reconstruction possible

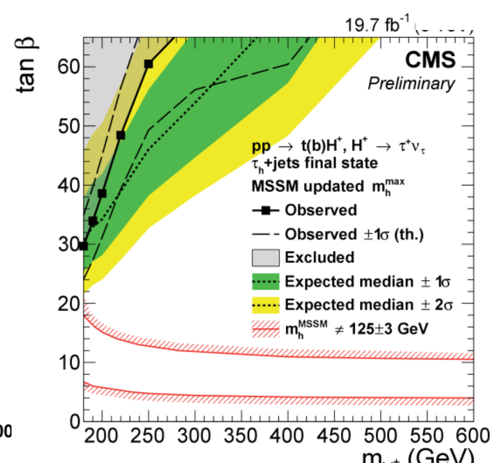
- Low multiplicity, Good tagging efficiency

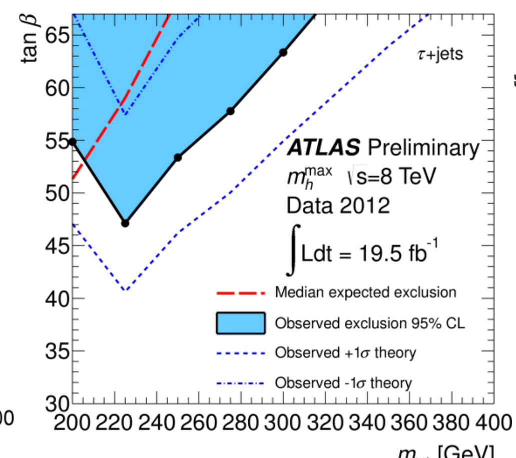
- Polarization information

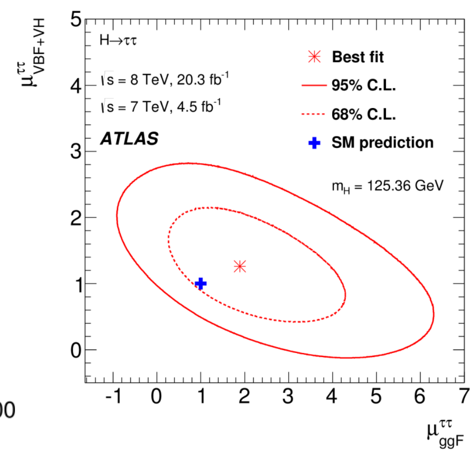
$$\kappa_X \equiv \frac{g_X}{g_X^{\mathrm{SM}}}$$











6. Conclusion and outlook

Conclusion and outlook

- Tau physics is a very rich field: test QCD and EW, neutrino physics, etc..
- Several interesting anomalies: LFU, Vus, Higgs LFV, CPV in $\tau \to K\pi\nu_\tau$, g-2
- Important experimental activities: Belle, BaBar, LHCb, BESIII, VEPP
- Intense theoretical activities : QCD, new physics
- A lot of *very interesting physics* remains to be done in the tau sector!

⇨ I hope Tau 2016 gives a chance to dicuss some of it

7. Back-up

Conclusion and outlook

- Leptonic Universality hints

- Hadronic τ-decays very interesting to study
  - Very precise determination of α_s
  - Extraction of V_{us}

- Charged LFV are a very important probe of new physics

- Several topics extremely interesting to study that I did not address:
  - Michel parameters
  - CPV asymmetry in $\tau \to K\pi\nu_\tau$
  - EDM and g-2 of the tau
  - Neutrino physics

- A lot of *very interesting physics* remains to be done in the tau sector!

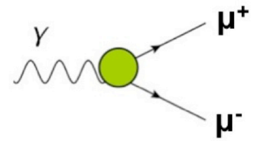
⇨ I hope Tau 2016 gives a chance to dicuss some of it

5. LFC processes: anomalous magnetic moment of the muon

5.1 Introduction

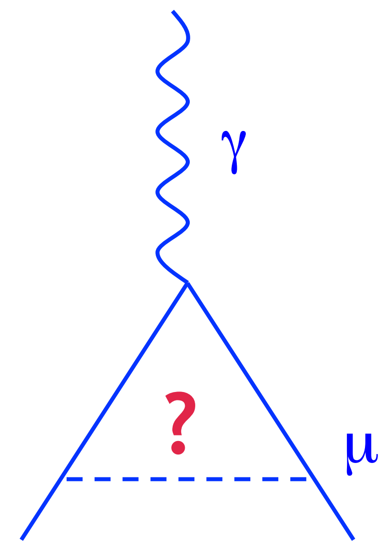
$$a_\mu = \frac{(g-2)_\mu}{2}$$

Anomalous magnetic moment



- The gyromagnetic factor of the muon is modified by loop contribution

- We can also study a_e with better experimental precision but if new physics heavy then more sensitivity in a_μ



$$a_\ell^{NP}(\Lambda_{NP}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{NP}^2}\right) \Rightarrow \frac{a_\mu^{NP}}{a_e^{NP}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 43{,}000$$

a_τ even more sensitive but insufficient experimental accuracy *Eidelman, Giacomini, Ignatov, Passera'07*

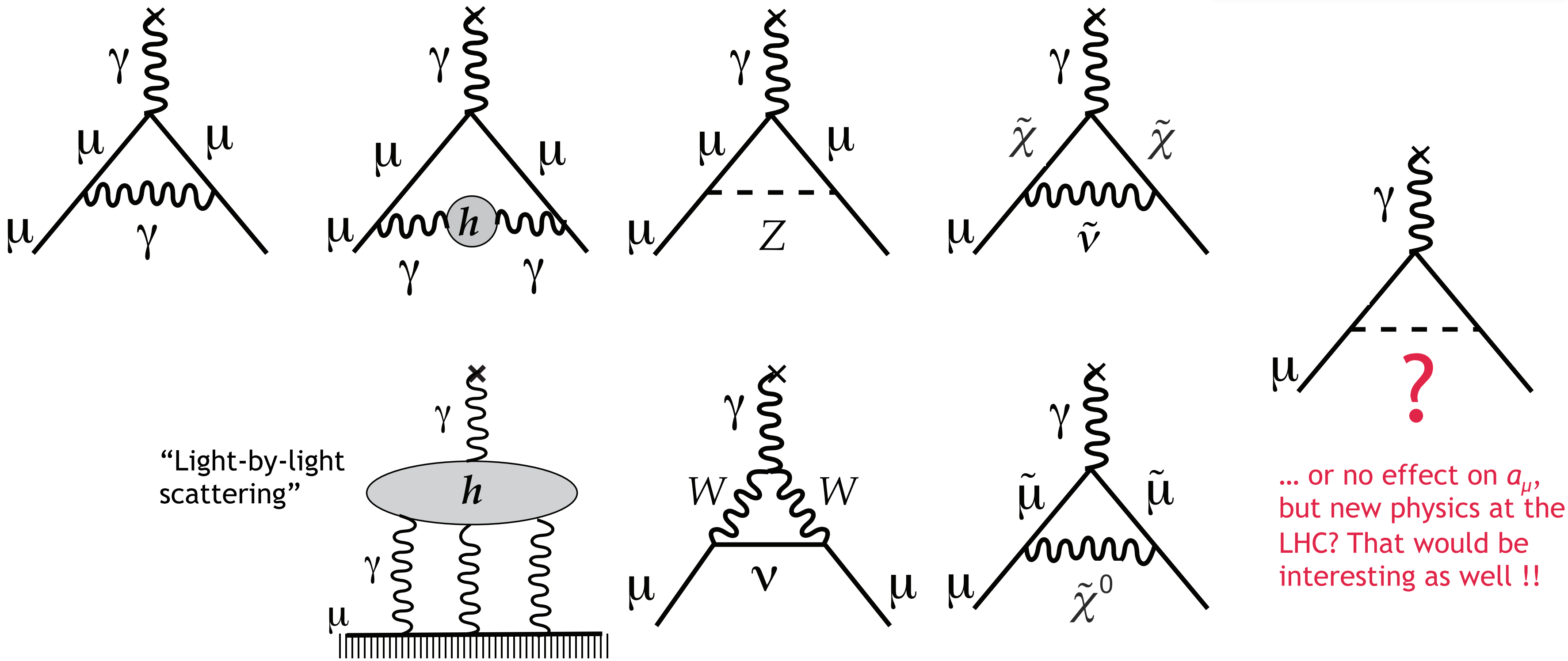
- But a_e important if NP is light
  ⇒ Important constraints on NP scenarios

Giudice, Paradisi, Passera'12

5.2 Contribution to $(g-2)_\mu$

Hoecker'11

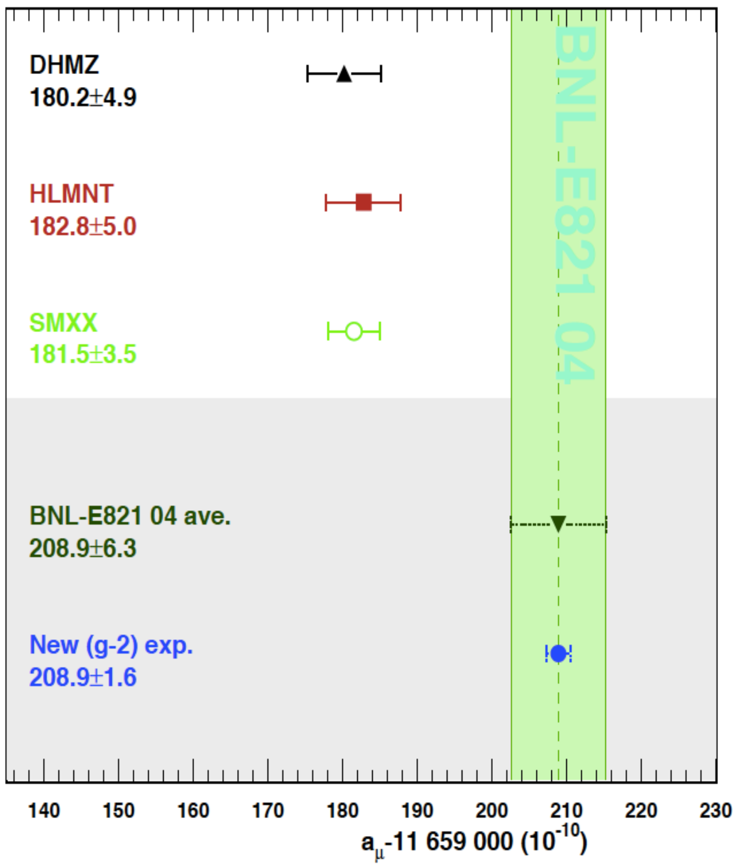
QED | Hadronic | Weak | SUSY… ? | … or some unknown type of new physics ?



Need to compute the SM prediction with high precision! ⇨ *Not so easy!*

5.3 Confronting measurement and prediction

Blum et al.'13

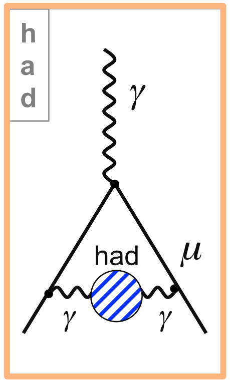


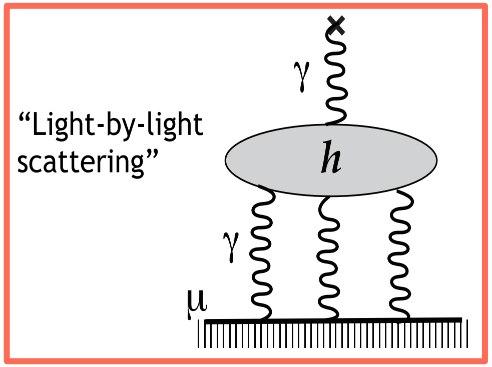
$$a_\mu|_{\text{exp}} = 0.001\ 165\ 920\ 80(54)(33)[63]$$
$$a_\mu|_{\text{theo}} = 0.001\ 165\ 918\ 40(59)$$
$$a_\mu|_{\text{exp}} - a_\mu|_{\text{theo}} = 240 \times 10^{-11} \qquad 2.9\sigma \text{ diff.}$$

Theoretical Prediction: *Lafferty, summary talk@Tau2014*

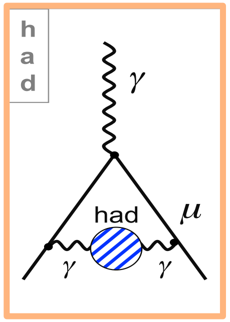
Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	690.8 ± 4.7
HVP(higher order) (*)	-10.0 ± 0.02
HLBL	11.6 ± 4.0
EW	15.4 ± 0.1
Total	11659179.7 ± 6.2

Jegerlehner and Nyffeler '09



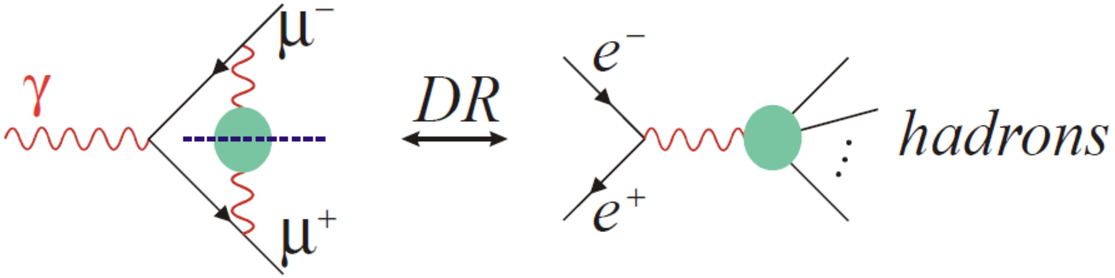


5.4 Towards a model independent determination of HVP and LBL



- Hadronic contribution cannot be computed from first principles due to low-energy hadronic effects

- Use analyticity + unitarity ⇒ real part of photon polarisation function from *dispersion relation* over *total hadronic cross section data*



$$R_V(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

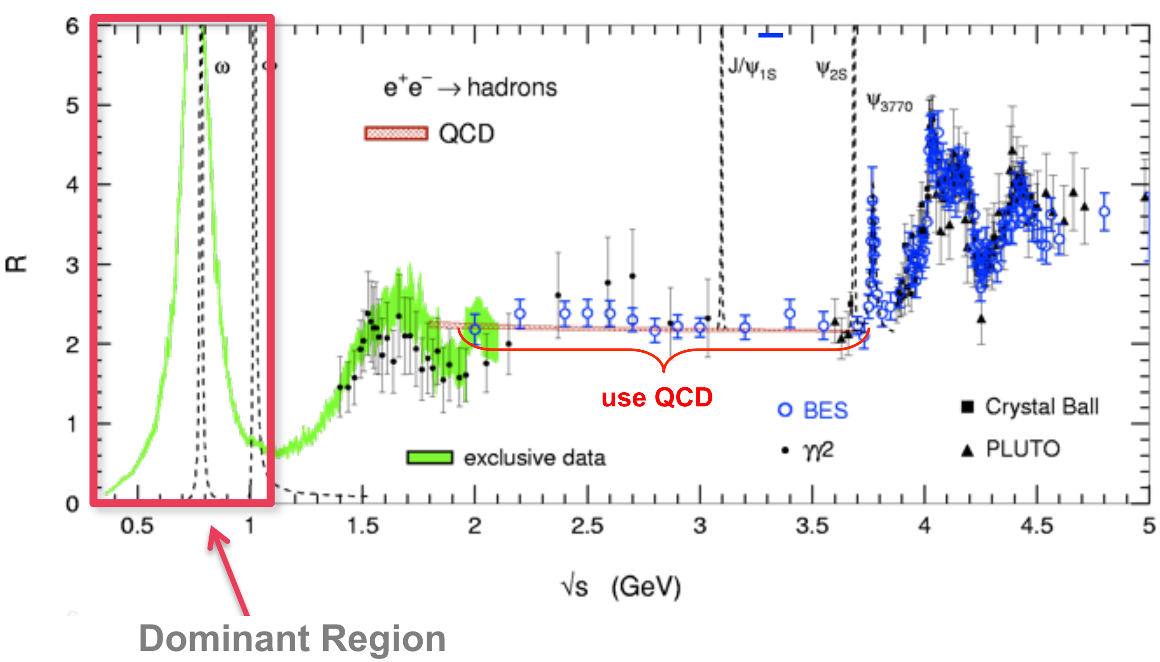
- Leading order hadronic vacuum polarization :

$$a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_V(s)$$

- Low energy contribution dominates : ~75% comes from $s < (1\ \mathrm{GeV})^2$
  ⇒ ππ contribution extracted from data

5.4 Towards a model independent determination of HVP and LBL

- Huge 20-years effort by experimentalists and theorists to reduce error on lowest-order hadronic part
  - Improved e^+e^- cross section data from Novisibirsk (Russia)
  - More use of perturbative QCD
  - Technique of "*radiative return*" allows to use data from Φ and *B* factories
  - Isospin symmetry allows us to also use τ hadronic spectral functions



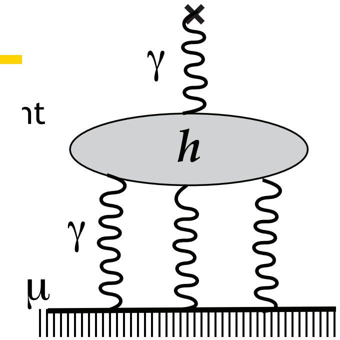
But still some progress need to be done

- Inconsistencies τ vs. e+e-: *Isospin corrections?*
- Inconsistencies between ISR and direct data: *Radiative corrections?*
- Lattice Calculation?

New data expected from VEPP, KLOE2, BES-III?

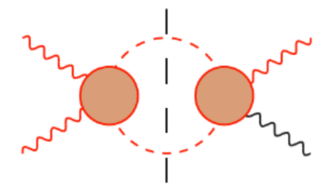
⇒ Belle II

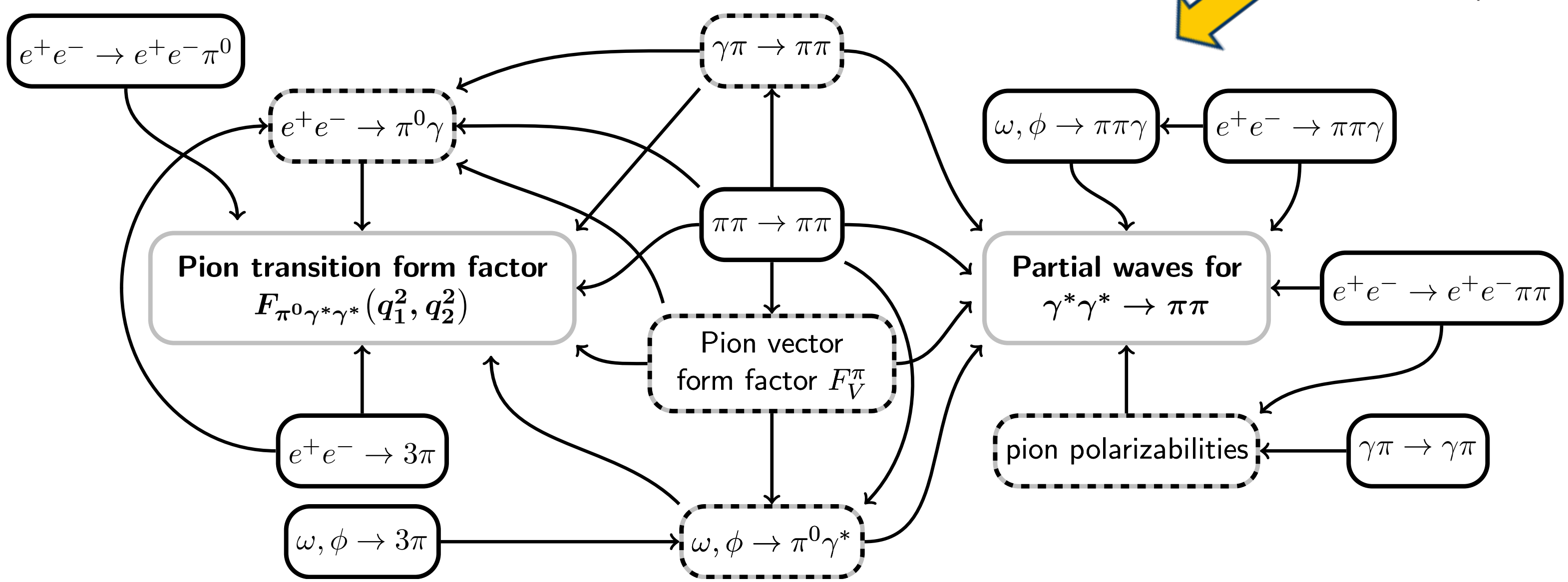
5.4 Towards a model independent determination of HVP and LBL



- For light-by-light scattering: until recently it was believed that dispersion relation approach not possible (4-point function)
  only model dependent estimates

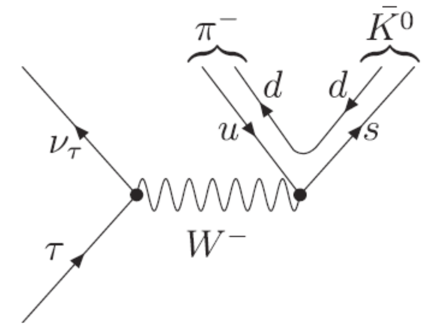
- But recent progress from Bern group: *Colangelo, Hoferichter, Procura, Stoffer'14*
  Data driven estimate possible using *dispersion relations!*





5. CPV in tau decays

5.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$= |p|^2 - |q|^2 \approx (0.36 \pm 0.01)\%$$ in the SM

Bigi & Sanda'05

Grossman & Nir'11

$$|K_S^0\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$

$$|K_L^0\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

$$\langle K_L | K_S \rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}(\varepsilon_K)$$

- Experimental measurement : *BaBar'11*

$$A_{Q\,\exp} = (-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}})\%$$ ⇨ 2.8σ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

Grossman & Nir'11

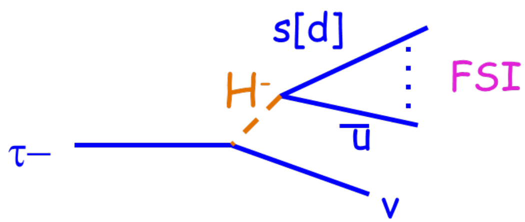
$$A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)} = (-0.54 \pm 0.14)\%$$

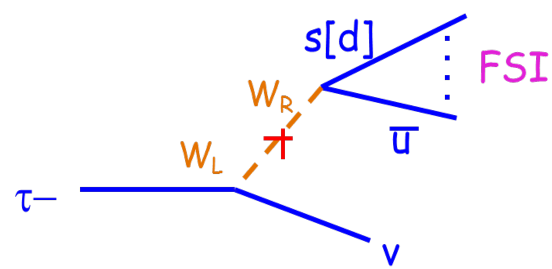
Belle, Babar, CLOE, FOCUS

5.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14*)?

Bigi'Tau12





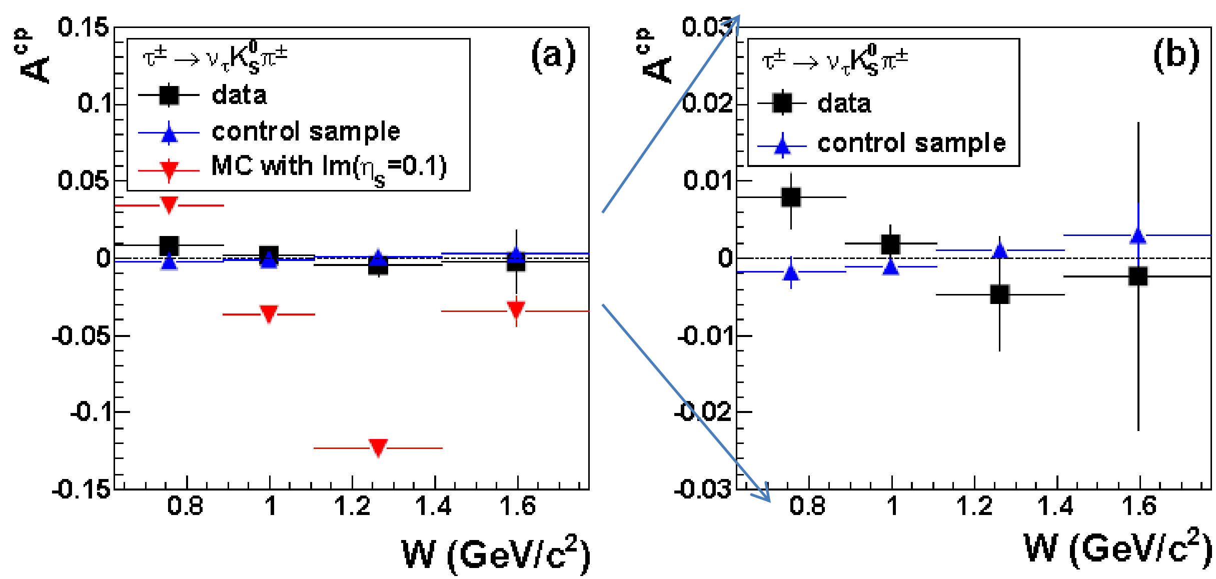
Very difficult to explain!

- Problem with this measurement? ⇨ It would be great to have other experimental measurements from *Belle, BES III or Tau-Charm factory*

Belle'11

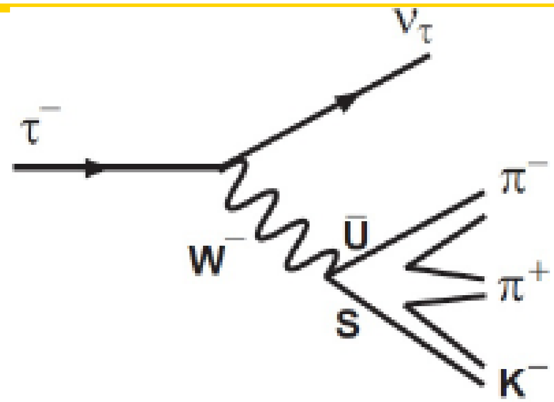
- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*

  ⇨ Belle does not see any asymmetry at the 0.2 - 0.3% level



5.2 Three body CP asymmetries

- Ex: $\tau \to K\pi\pi\nu_\tau$



- A variety of CPV observables can be studied :
  $\tau \to K\pi\pi\nu_\tau, \tau \to \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,….

  e.g., Choi, Hagiwara and Tanabashi'98
  Kiers, Little, Datta, London et al.,'08
  Mileo, Kiers and, Szynkman'14

  Same principle as in charm, *see Bevan'15*

  Difficulty : Treatement of the hadronic part
  Hadronic final state interactions have to be taken into account!
  Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies

Lepton universality - HFAG 2016 prelim.

Standard Model for leptons λ, $\rho = e, \mu, \tau$ (Marciano 1988)

$$\Gamma[\lambda \to \nu_\lambda \rho \overline{\nu}_\rho(\gamma)] = \Gamma_{\lambda\rho} = \Gamma_\lambda B_{\lambda\rho} = \frac{B_{\lambda\rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(\frac{m_\rho^2}{m_\lambda^2}\right) r_W^\lambda r_\gamma^\lambda \,,$$

where

$$G_\lambda = \frac{g_\lambda^2}{4\sqrt{2}M_W^2} \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad f_{\lambda\rho} = f\left(\frac{m_\rho^2}{m_\lambda^2}\right)$$

$$r_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} \qquad r_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$

Tests of lepton universality from ratios of above partial widths:

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{B_{\tau e}}{B_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau e} r_W^\tau r_\gamma^\tau}} = 1.0012 \pm 0.0015 = \sqrt{\frac{B_{\tau e}}{B_{\tau e}^{\text{SM}}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{B_{\tau\mu}}{B_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau\mu} r_W^\tau r_\gamma^\tau}} = 1.0030 \pm 0.0014 = \sqrt{\frac{B_{\tau\mu}}{B_{\tau\mu}^{\text{SM}}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{B_{\tau\mu}}{B_{\tau e}} \frac{f_{\tau e}}{f_{\tau\mu}}} = 1.0019 \pm 0.0014$$

- precision: **0.20—0.23%** pre-B-Factories $\Rightarrow$ **0.14—0.15%** today
  thanks essentially to the Belle tau lifetime measurement, PRL 112 (2014) 031801
- $r_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$ and $r_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$ (Marciano 1988), M_W from PDG 2013

Universality improved $B(\tau \to e\nu\bar{\nu})$ and R_{had} - HFAG 2016 prelim.

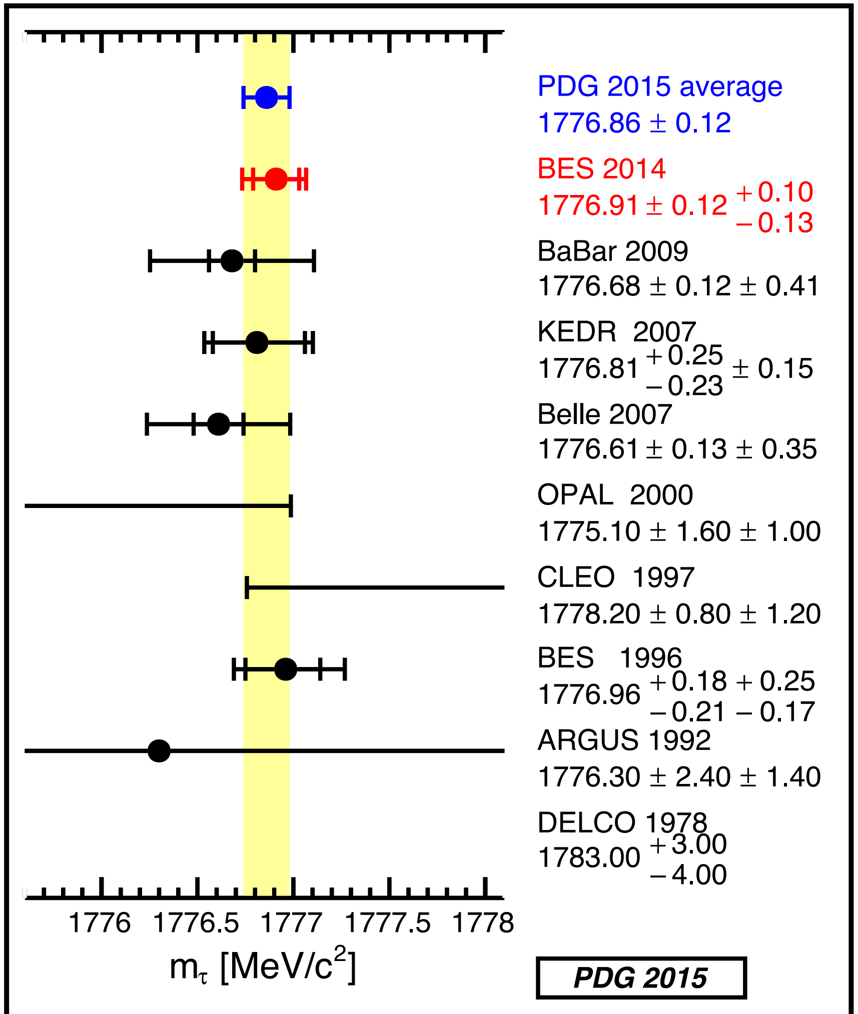
Universality improved $B(\tau \to e\nu\bar{\nu})$

- (M. Davier, 2005): assume SM lepton universality to improve $B_e = B(\tau \to e\bar{\nu}_e\nu_\tau)$ fit B_e using three determinations:
  - $B_e = B_e$
  - $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$
  - $B_e = B(\mu \to e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$ [above we have: $B(\mu \to e\bar{\nu}_e\nu_\mu) = 1$]
- $\boldsymbol{B_e^{\text{univ}} = (17.818 \pm 0.022)\%}$ HFAG-PDG 2016 prelim. fit

$R_{\text{had}} = \Gamma(\tau \to \text{hadrons})/\Gamma_{\text{univ}}(\tau \to e\nu\bar{\nu})$

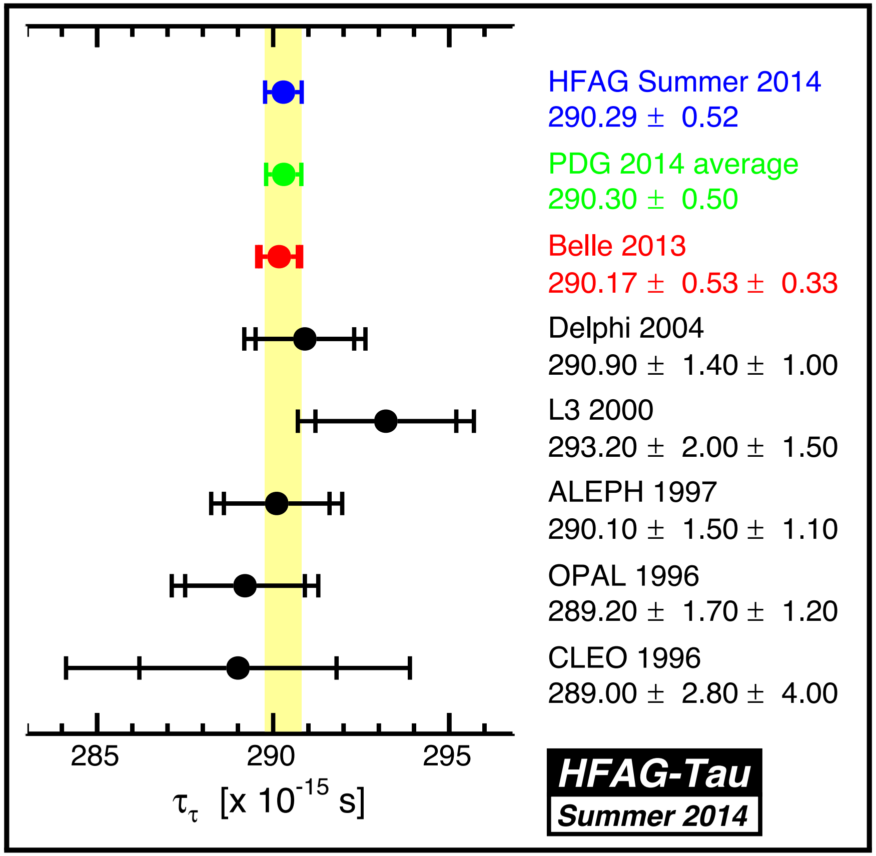
- $$R_{\text{had}} = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma_{\text{univ}}(\tau \to e\nu\bar{\nu})} = \frac{B_{\text{hadrons}}}{B_e^{\text{univ}}} = \frac{1 - B_e^{\text{univ}} - f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2) \cdot B_e^{\text{univ}}}{B_e^{\text{univ}}}$$
  - two different determinations, second one not "contaminated" by hadronic BFs
- $\boldsymbol{R_{\text{had}} = 3.6359 \pm 0.0074}$ HFAG-PDG 2016 prelim. fit
- $\boldsymbol{R_{\text{had}}(\text{leptonic BFs only}) = 3.6397 \pm 0.0070}$ HFAG-PDG 2016 prelim. fit

Tau mass



- most precise measurements by **e^+e^- colliders at $\tau^+\tau^-$ threshold**
  - few events but very significant

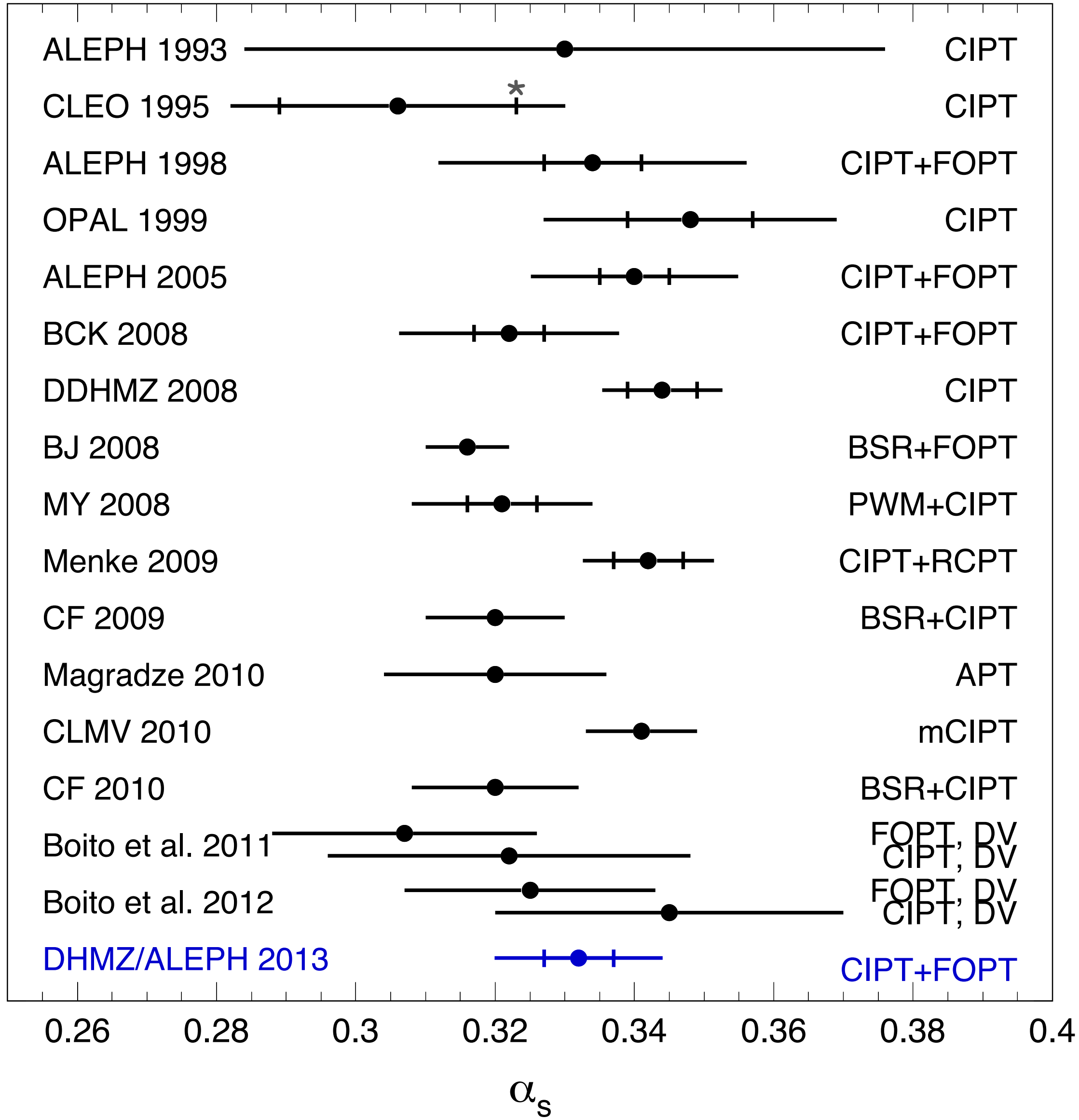
Tau lifetime



- LEP experiments, many methods
  - impact parameter sum (IPS)
  - momentum dependent impact parameter sum (MIPS
  - 3D impact parameter sum (3DIP)
  - impact parameter difference (IPD)
  - decay length (DL)
- Belle
  - 3-prong vs. 3-prong decay length
  - largest syst. error: alignment

Comparison with Other Determinations

- Baikov, Chetyrkin, Köhn, PRL 101 (2008) 012002, [0801.1821]
- Beneke, Jamin, JHEP 0809 (2008) 044, [0806.3156]
- Maltman, Yavin, PRD78 (2008) 094020, [0807.0650]
- Menke, 0904.1796
- Caprini, Fischer, EPJC64 (2009) 35, [0906.5211]
- Magradze, Few Body Syst. 48 (2010) 143, Erratum-ibid. 53 (2012) 365, [1005.2674]
- Cvetic, Loewe, Martinez, Valenzuela, PRD82 (2010) 093007, [1005.4444]
- Caprini, Fischer, Rom.J.Phys. 55 (2010) 527, [1012.1132]
- Boito et al., PRD84 (2011) 113006, [1110.1127]; PRD85 (2012) 093015, [1203.3146]
- Beneke, Boito, Jamin, JHEP 1301 (2013) 125, [1210.8038]



* experimental uncertainty when available is shown as inner error bar

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

- $R_{\tau,V+A}(s_0) = N_C\, S_{EW}\, |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}\right)$

  $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part (m_q=0)

$$\boxed{-s\frac{d}{ds}\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_S(-s)}{\pi}\right)^n}$$

$K_0 = K_1 = 1,\ K_2 = 1.63982,\ K_3 = 6.37101$

$K_4 = 49.07570$ Baikov, Chetyrkin, Kühn'08

$$\Rightarrow\ \boxed{\delta_P = \sum_{n=1} K_n A^n(\alpha_S) = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots}$$

with $A^n(\alpha_S) = \frac{1}{2\pi i}\oint_{|s|=m_\tau^2} \frac{ds}{s}\left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8}\right)\left(\frac{\alpha_S(-s)}{\pi}\right)^n$ $\qquad a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

$$\Rightarrow\ \boxed{\delta_P \approx 20\%} \qquad \boxed{\left(\delta_P = 0.2066 \pm 0.0070\right)}$$ Davier et al'08

CIPT vs. FOPT

- $$\boxed{-s\frac{d}{ds}\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_S(\text{-}s)}{\pi}\right)^n}$$

$$\boxed{\delta_P = \underbrace{\sum_{n=1} K_n A^n(\alpha_S)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n \left(\frac{\alpha_S(\text{-}s)}{\pi}\right)^n}_{\text{FOPT}}}$$

$$r_n = K_n + g_n$$

$$A^n(\alpha_S) = \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\left(1-2x+2x^3-x^4\right)\left(\frac{\alpha_S(-xm_\tau^2)}{\pi}\right)^n = a_\tau + \ldots$$

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

Pich Tau'10

LeDiberder & Pich''92

➡ The dominant corrections come from the contour integration

Large running of α_S along the circle $s = m_\tau^2 e^{i\varphi}$, $\varphi \in [0, 2\pi]$

Perturbative $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$K_0 = K_1 = 1$, $K_2 = 1.63982$, $K_3 = 6.37101$, $K_4 = 49.07570$ **Baikov-Chetyrkin-Kühn '08**

$$\delta_{\mathrm{P}} = \sum_{n=1} K_n \; A^{(n)}(\alpha_s) = a_\tau + 5.20\; a_\tau^2 + 26\; a_\tau^3 + 127\; a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6   **[additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]**

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\;a(-s)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n\;A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n\;a_\tau^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a_\tau(-x\,m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

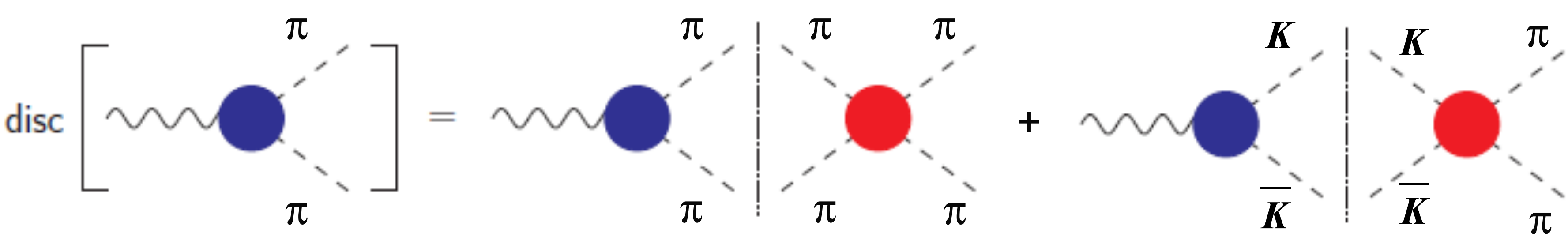
Large running of a_s along the circle $s = m_\tau^2\, e^{i\varphi}$, $\varphi \in [0\,,2\pi]$

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs up to $\sqrt{s}\sim1.4$ GeV ⇨ *Coupled channel analysis*
  Inputs: I=0, S-wave $\pi\pi$ and $K\bar{K}$ data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:

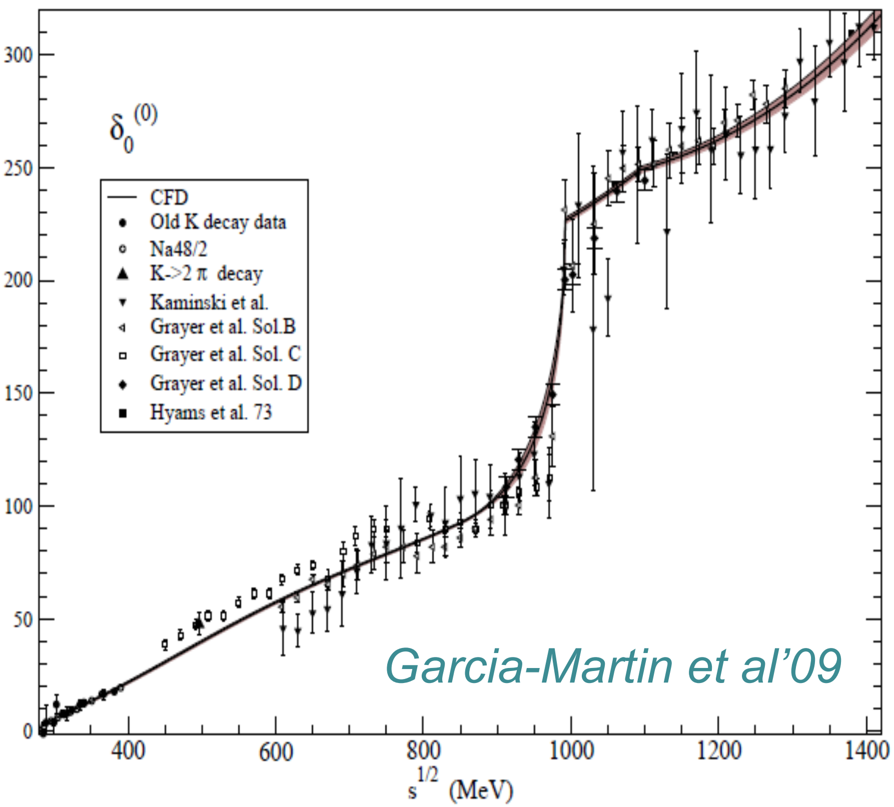


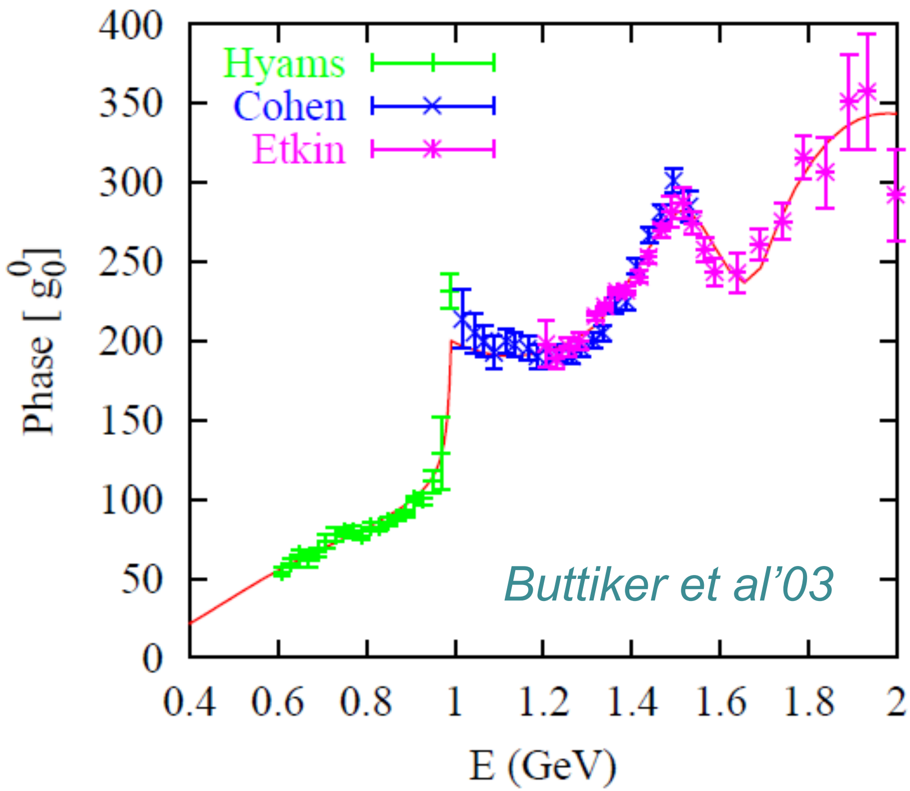
⇨ $$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$ $\qquad n = \pi\pi, K\bar{K}$

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\boxed{\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T_{nm}^{*}\sigma_m(s)X_m^{(N)}\right\}} \longrightarrow \boxed{\mathrm{Re}X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-s}\mathrm{Im}X_n^{(N+1)}}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

  Feynman-Hellmann theorem: ⇨

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$\begin{aligned} M^2_{\pi^+} &= (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2) \\ M^2_{K^+} &= (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2) \\ M^2_{K^0} &= (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2) \end{aligned} \Rightarrow$$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!

  $\Rightarrow$ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

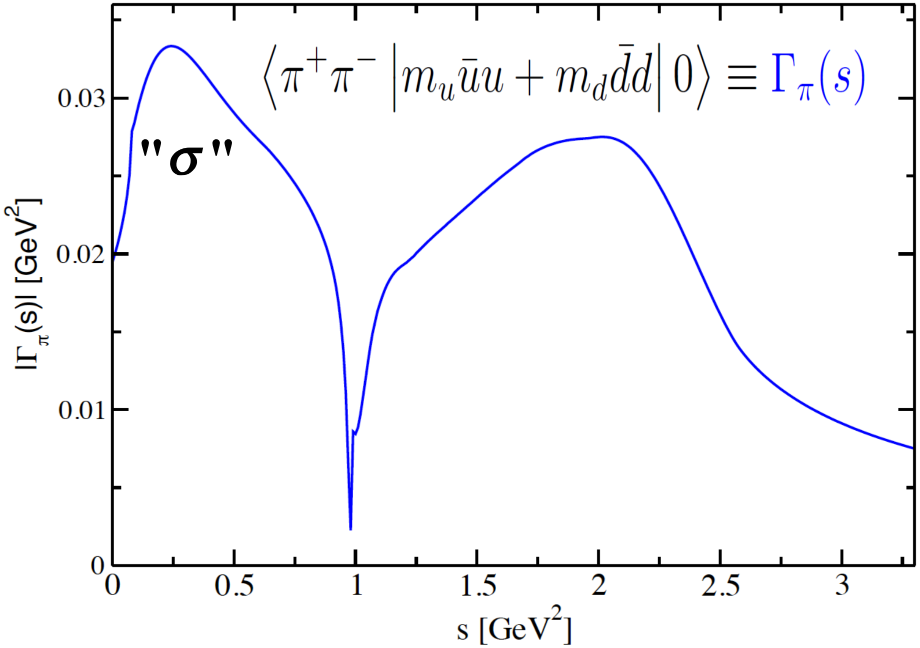
- General solution

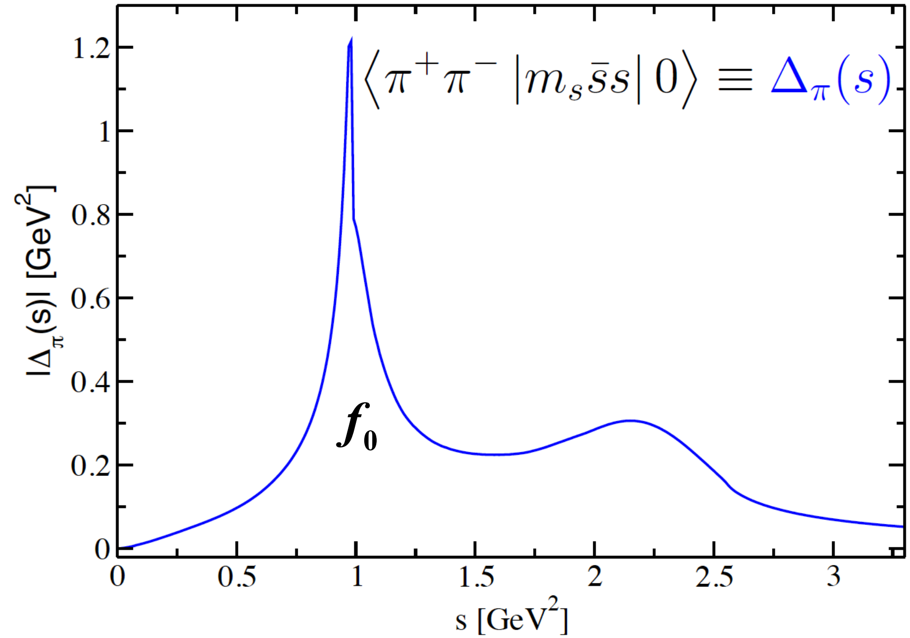
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

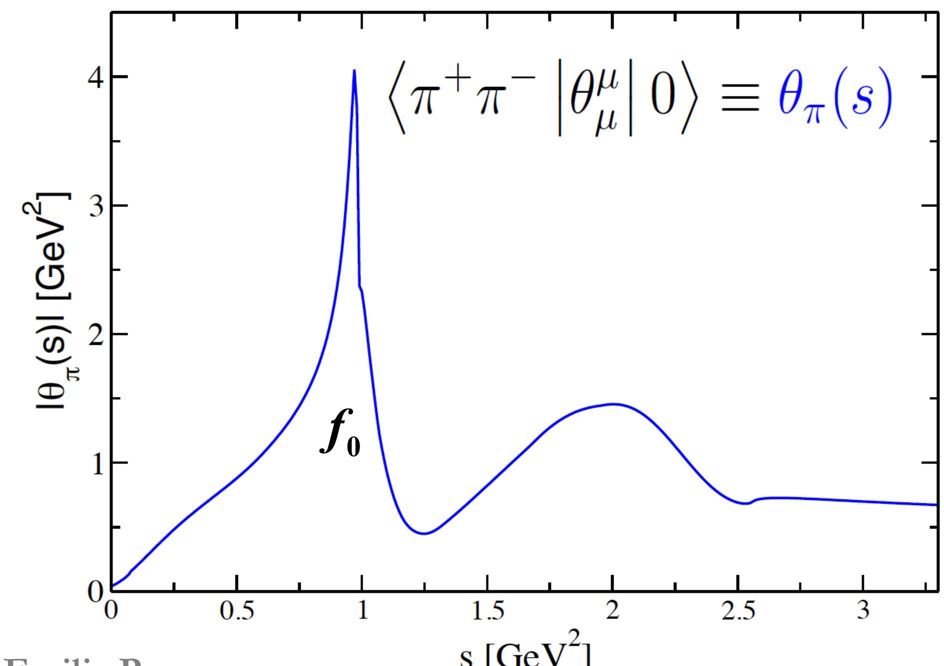
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

  ⇨ Relax the constraints and match to ChPT

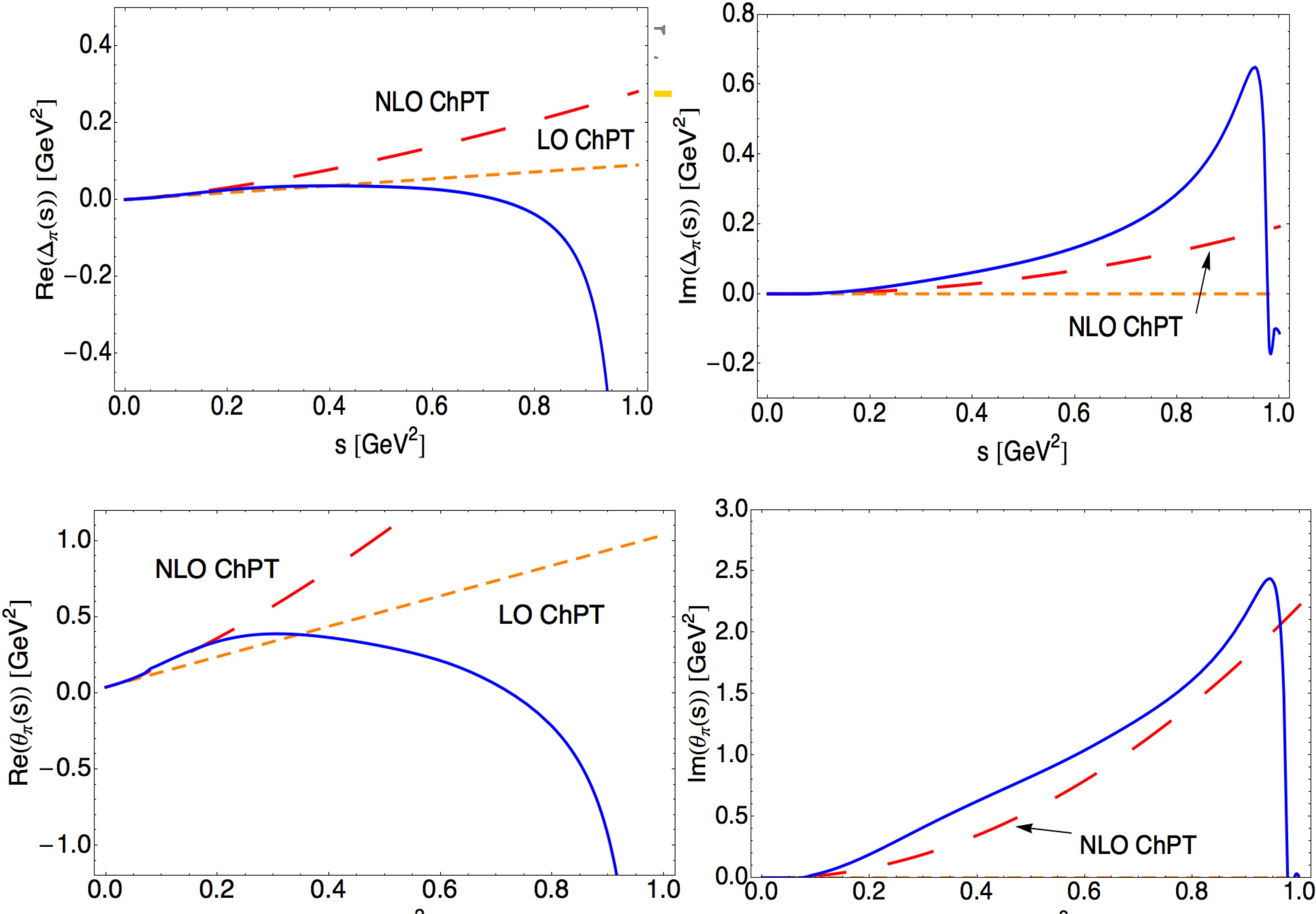
$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$







Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data


NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]

3.4.3 Determination of $F_V(s)$

- Vector form factor

  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

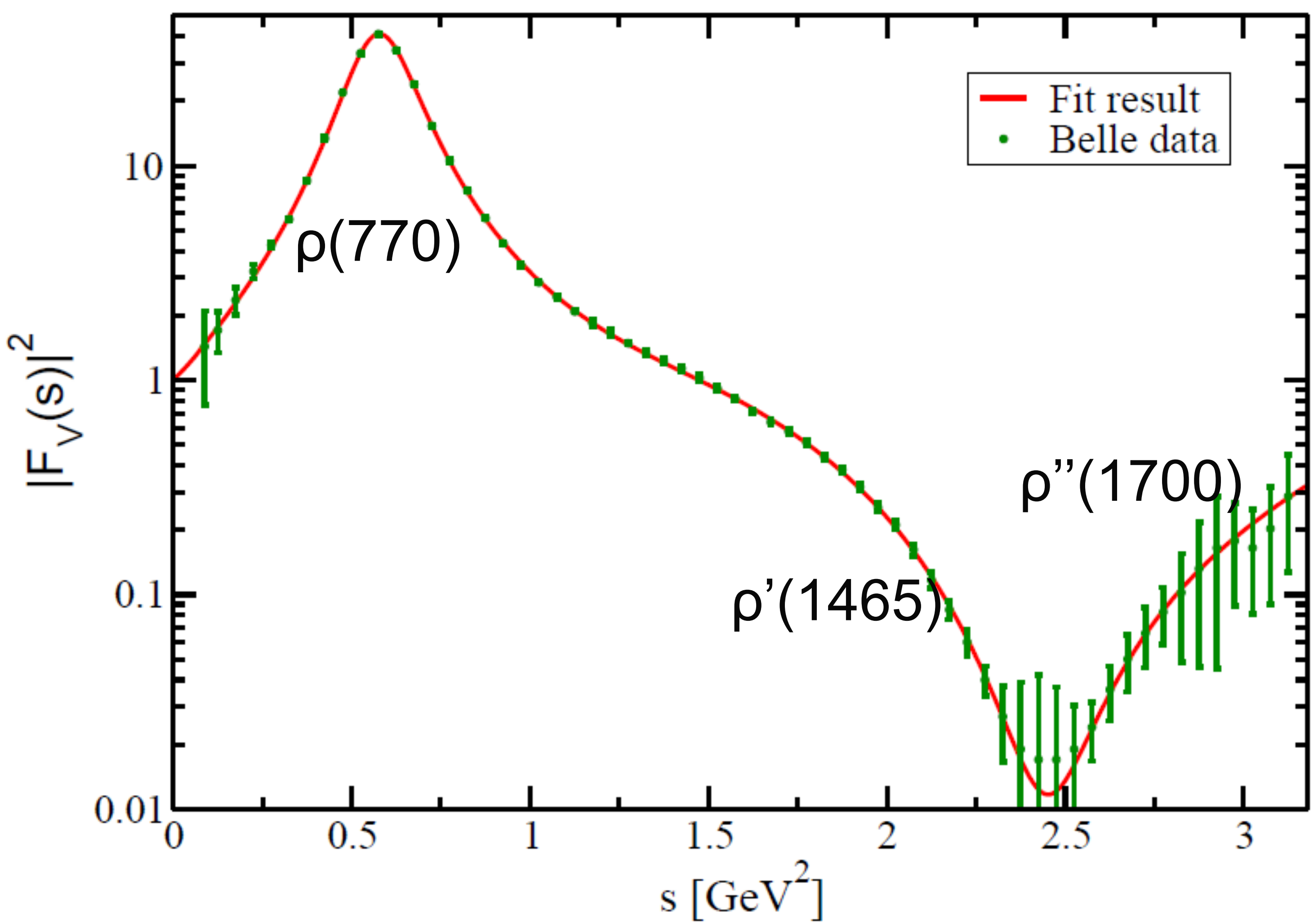
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{\left(s' - s - i\varepsilon\right)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

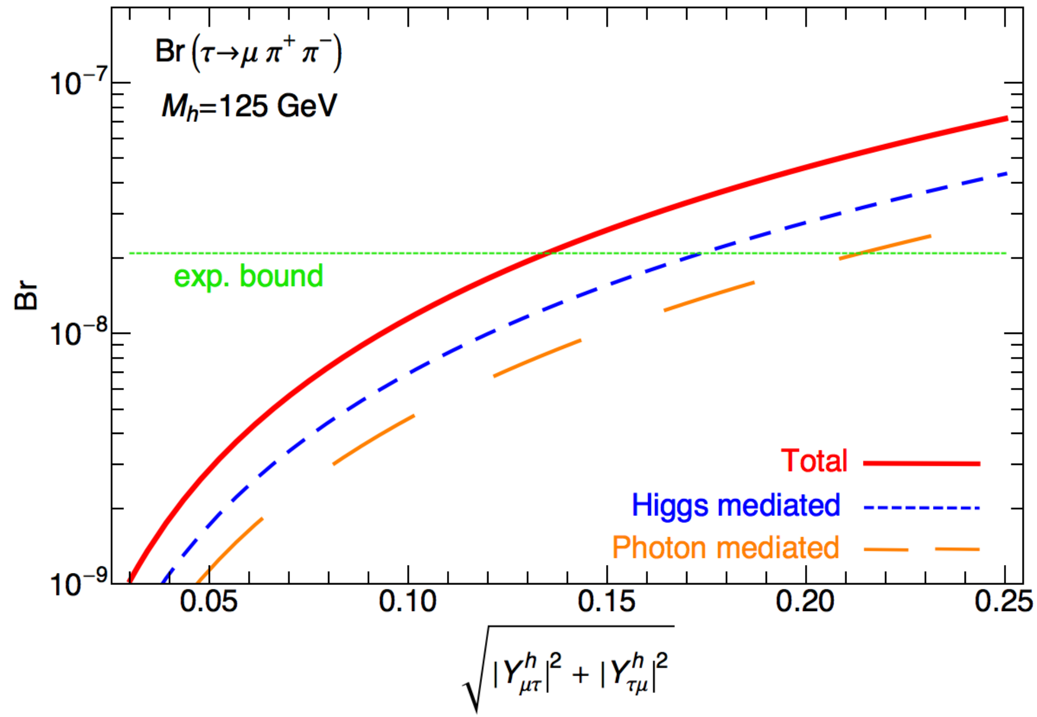
  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

3.4.3 Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

Less stringent but more robust handle on LFV Higgs couplings

Process	(BR × 10^8) 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	< 1.4 × 10^3 [87]	< 6.3	Scalar, Gluon

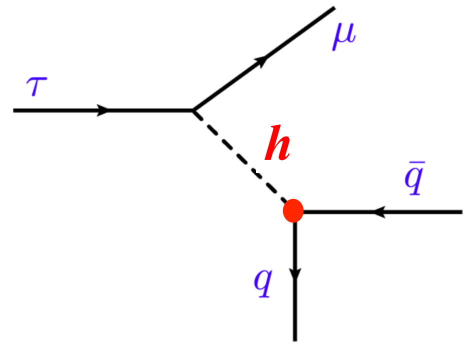
?

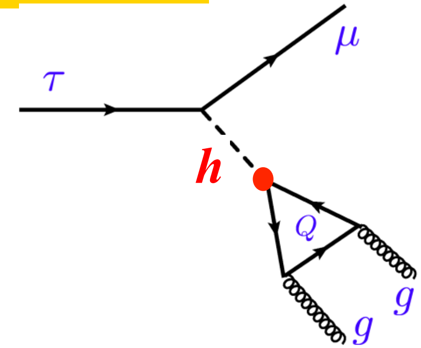
Belle'08'11'12 except last from *CLEO'97*

3.5 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL





- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!

- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

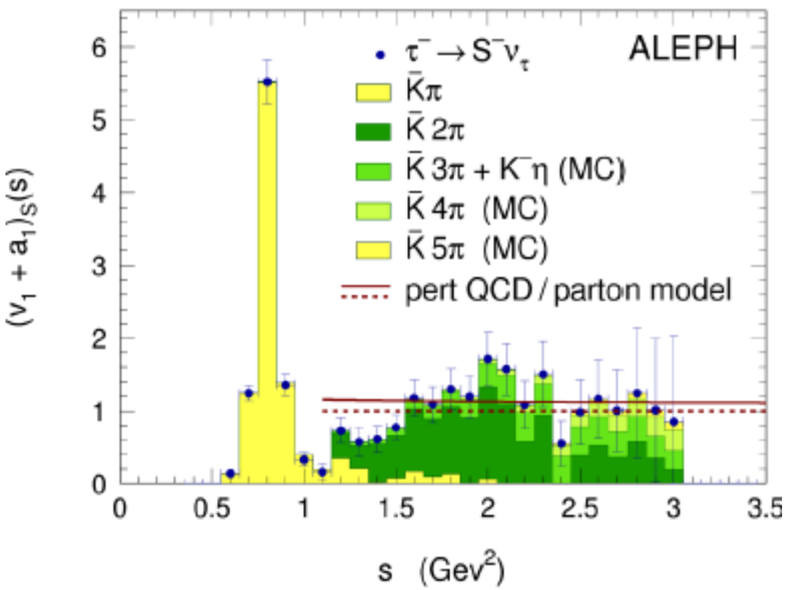
below present experimental limits!

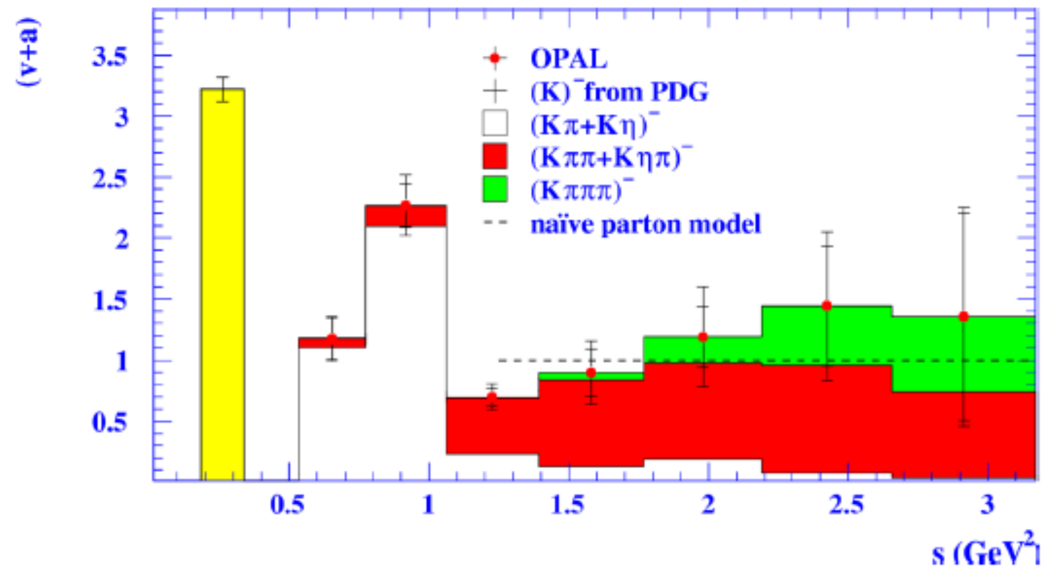
- If discovered ⇨ among other things *upper limit* on $Y_{u,d,s}$*!*

⇨ Interplay between high-energy and low-energy constraints!

3.6 Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller $\tau \rightarrow$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$\boxed{R_\tau^S\Big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_\tau^S\Big|_{\text{new}} = 0.1615(28)}$$

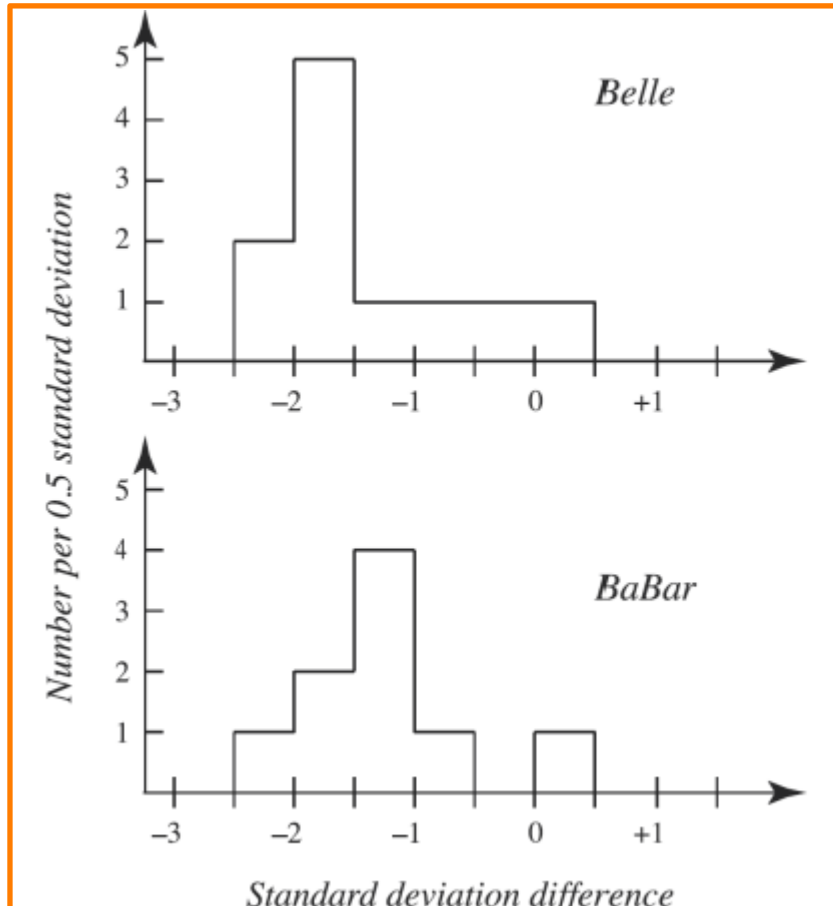
$$\boxed{\left|V_{us}\right|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{\left|V_{us}\right|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

3.6 Prospects : τ strange Brs

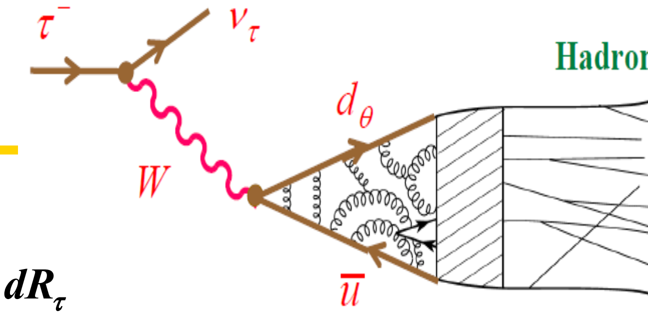
- *PDG 2014*: « Eigtheen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference (#σ)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	+1.4
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	−2.9
$K^-K^+\pi^-\nu_\tau$	−2.9
$K^-K^+K^-\nu_\tau$	−5.4
$\eta\ K^-\nu_\tau$	−1.0
$\phi\ K^-\nu_\tau$	−1.3



2.2 Experimental situation



- Observable studied $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$ and $\frac{dR_\tau}{ds}$

- Decomposition as a function of observed and separated final states

$$\boxed{R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}}$$

Zhang'Tau14

$$\upsilon_1 / a_1 \left[\tau^- \to V^- / A^- \nu_\tau\right] \propto \frac{\text{BR}\left[\tau^- \to V^- / A^- \nu_\tau\right]}{\text{BR}\left[\tau^- \to e^- \bar{\nu}_e \nu_\tau\right]} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \frac{m_\tau^2}{\left(1 - s/m_\tau^2\right)^2 \left(1 + 2s/m_\tau^2\right)}$$

Vector/Axial-vector spectral functions — branching fractions — mass spectrum — kinematic factor

2.2 Lepton universality & NP

- A renewed interest in possible violations of LFU has been triggered by two very different sets of observations in B physics:
  1. LFU test in b → s neutral currents: μ vs. e :

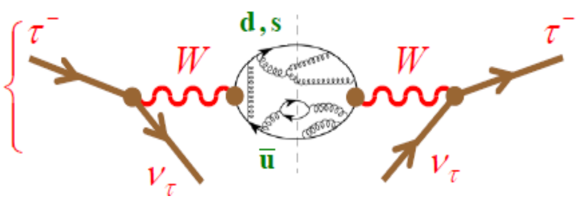
$$R_K = \frac{\mathrm{Br}[B^+ \to K^+\mu^+\mu^-]_{[1,6]}}{\mathrm{Br}[B^+ \to K^+e^+e^-]_{[1,6]}} = 0.745 \cdot (1 \pm 13\%) \qquad \text{vs} \qquad R_K^{\mathrm{SM}} = 1.003 \pm 0.0001$$

2.6σ deviation from the SM

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$\Gamma_{\tau \to \nu_\tau + \mathrm{had}} \sim \mathrm{Im} \left\{ \right.$  $\left. \right\}$

$$\mathrm{Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi} v_1/a_1(s)$$

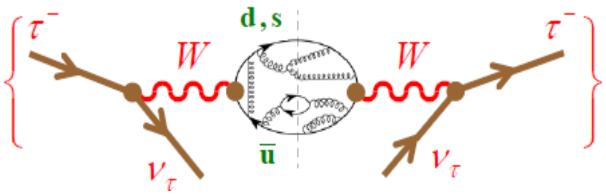
$$\Rightarrow \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\Pi^{(0)}(s+i\varepsilon) \right]}$$

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

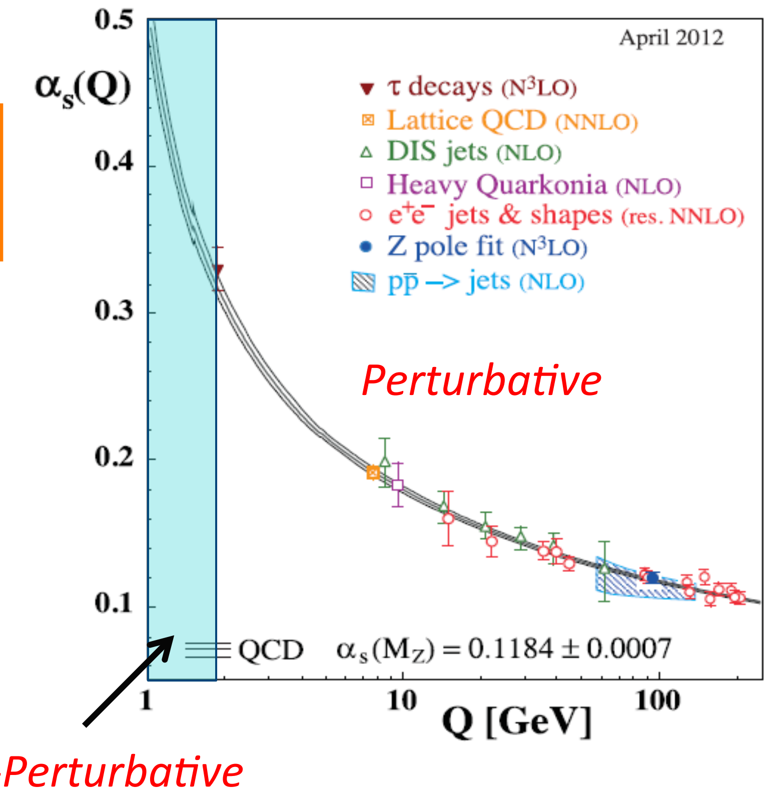
$$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$$



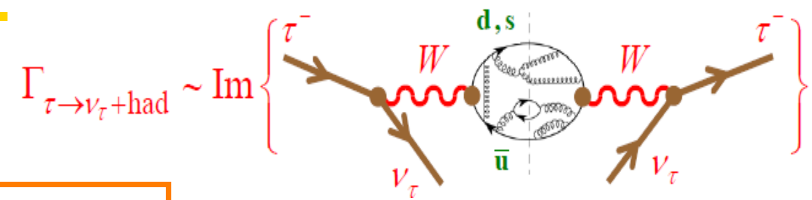
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- We are in the *non-perturbative* region: we do not know how to compute!

- Trick: use the analytical properties of Π!



2.4 Calculation of the QCD corrections



Braaten, Narison, Pich'92

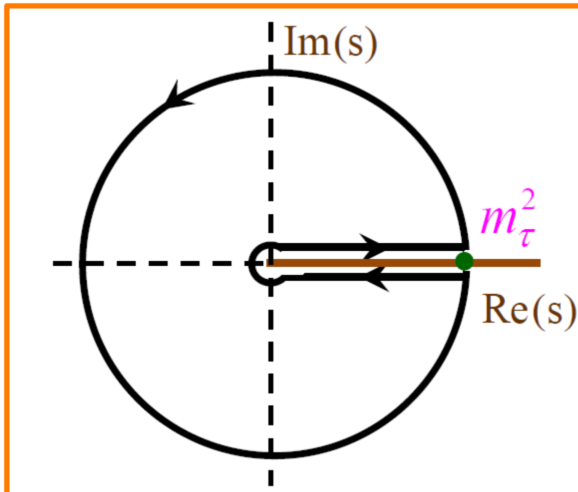
- Calculation of R_τ:

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathrm{C}^{(J)}(s,\mu)\, \langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C \; S_{EW} \left(1 + \delta_P + \delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\boxed{\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%}$ $\quad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R$_\tau$:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\; S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\; a_\tau^2 + 26\; a_\tau^3 + 127\; a_\tau^4 + \ldots \approx 20\%$ $\quad a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\; \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\; \left(\propto m_s\right)$

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

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Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ (\propto m_u, m_d)$ but not for $R_\tau^{S}\ (\propto m_s)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

2.4 Calculation of the QCD corrections

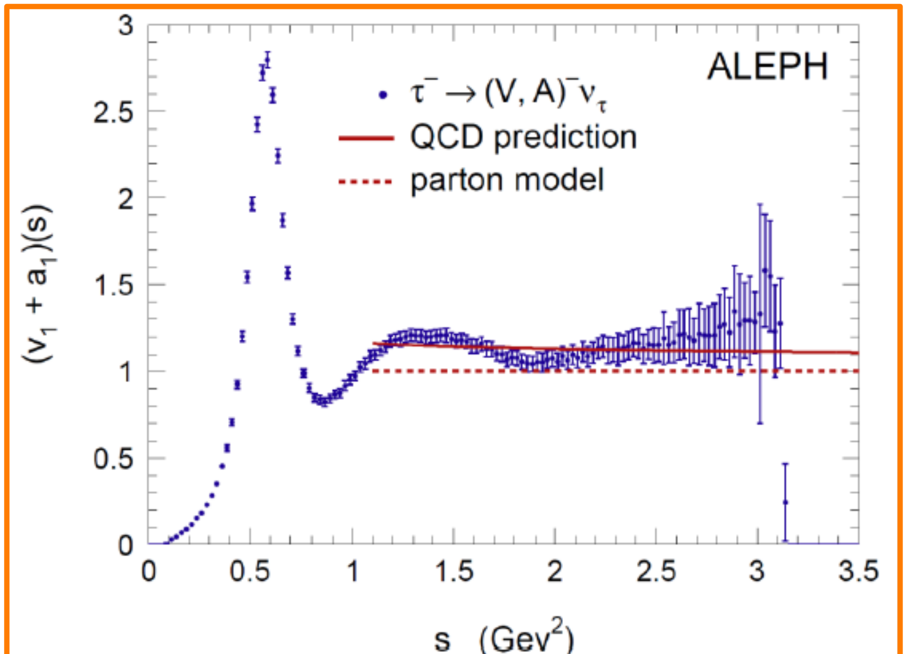
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

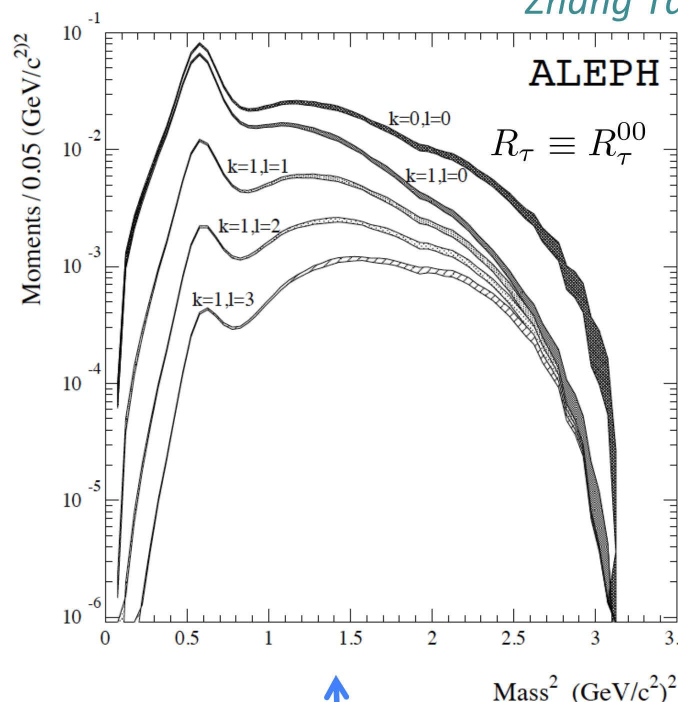
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R^{k\ell}_{\tau,U}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

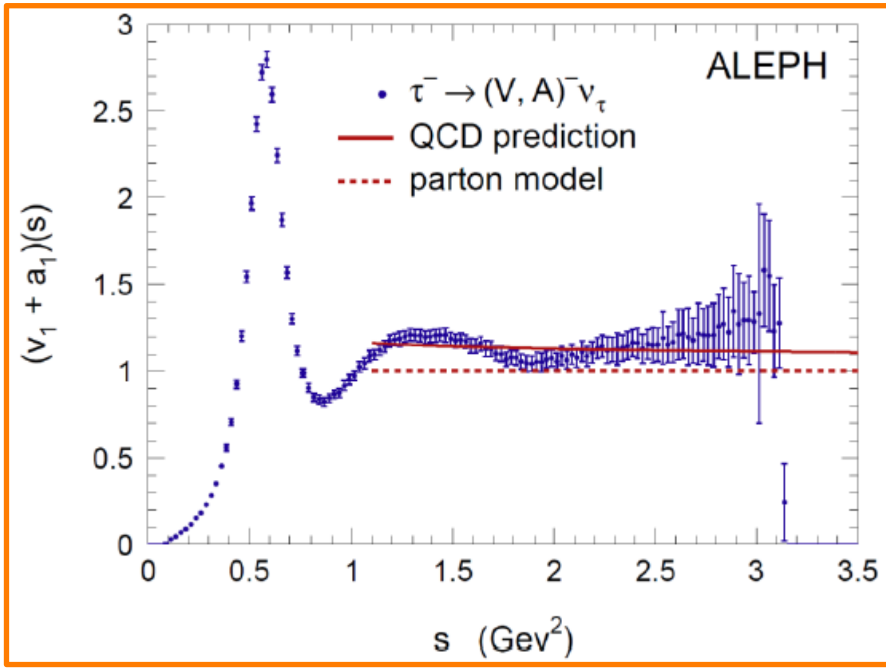
- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C \; S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$
- Perturbative part (D=0): $\delta_P \approx 20\%$
- D=2: quark mass corrections, *neglected*
- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  Use of weighted distributions

$$\Rightarrow \boxed{\delta_{NP} = -0.0064 \pm 0.0013}$$

Davier et al'14



- Small unknown NP part $\left(\delta_{NP} : 3\% \; \delta_P\right)$ very precise extraction of α_S !

2.5 Results and determination of α_s

Pich'Tau14

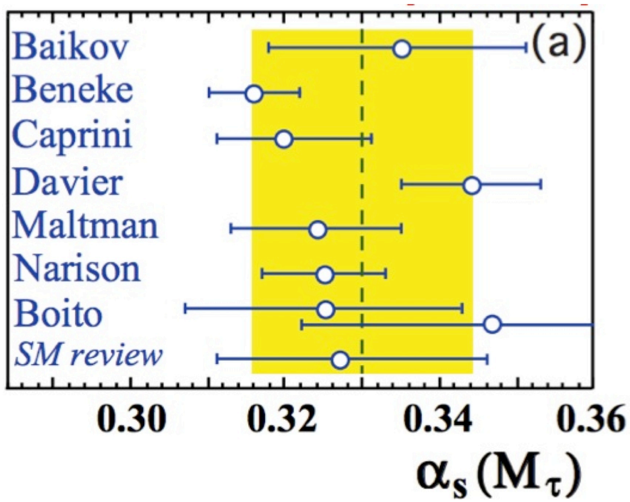
Reference	Method	δ_{NP}	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT		0.1998 (43)	0.332 (16)	0.1202 (19)
Davier et al'14	**CIPT, FOPT**	**– 0.0064 (13)**	**–**	**0.332 (12)**	**0.1199 (15)**
Beneke-Jamin	BSR + FOPT	– 0.007 (3)	0.2042 (50)	0.316 (06)	0.1180 (08)
Maltman-Yavin	PWM + CIPT	+ 0.012 (18)	–	0.321 (13)	0.1187 (16)
Menke	CIPT, FOPT		0.2042 (50)	0.342 (11)	0.1213 (12)
Narison	CIPT, FOPT		–	0.324 (08)	0.1192 (10)
Caprini-Fischer	BSR + CIPT		0.2037 (54)	0.322 (16)	–
Abbas et al	IFOPT		0.2037 (54)	0.338 (10)	
Cvetič et al	β_{exp} + CIPT		0.2040 (40)	0.341 (08)	0.1211 (10)
Boito et al	CIPT, DV	– 0.002 (12)	–	0.347 (25)	0.1216 (27)
	FOPT, DV	– 0.004 (12)		0.325 (18)	0.1191 (22)
Pich'14	CIPT	– 0.0064 (13)	0.2014 (31)	0.342 (13)	0.1213 (14)
	FOPT			0.320 (14)	0.1187 (17)
Pich'14	**CIPT, FOPT**	**– 0.0064 (13)**	**0.2014 (31)**	**0.332 (13)**	**0.1202 (15)**

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
IFOPT Improved FOPT

β_{exp}: Expansion in derivatives of α_s (β function)
PWM: Pinched-weight moments
CIPTm: Modified CIPT (conformal mapping)
DV: Duality violation (OPAL only)

2.5 Results and determination of α_s

PDG'13



- $$\boxed{\alpha_s\left(m_\tau^2\right) = 0.332 \pm 0.013}$$

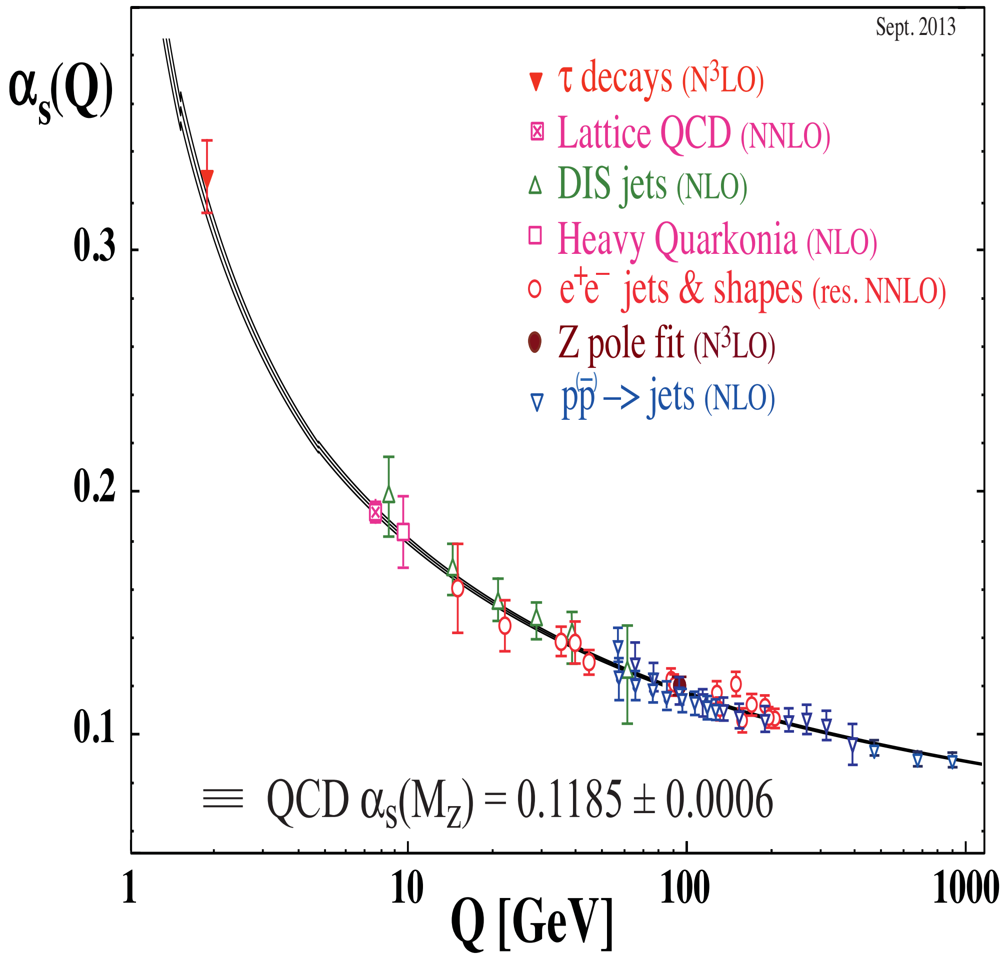
$$\Downarrow$$

Pich'Tau14

$$\boxed{\alpha_s\left(M_Z^2\right) = 0.1202 \pm 0.0015}$$

to be compared to

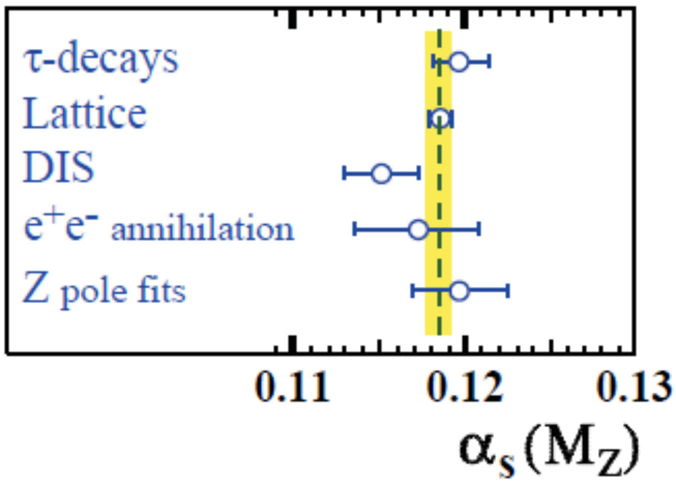
$$\boxed{\alpha_s\left(M_Z^2\right)_{Z\ \mathrm{width}} = 0.1197 \pm 0.0028}$$

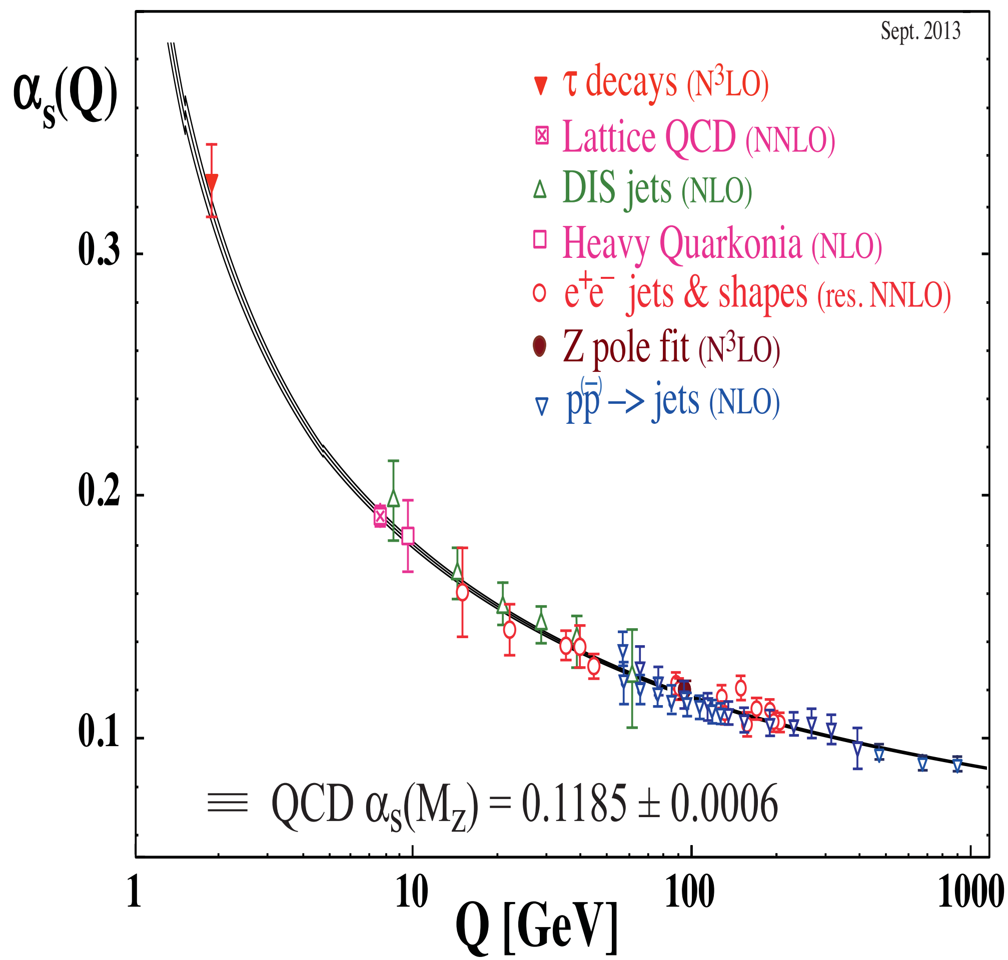


- Impressive test of the running of α_S!

2.5 Results and determination of α_s

PDG'13





- *Extraction of α_s* from hadronic τ decays very *competitive*!

- If new data room for *improvement*!
  - Study of duality violation effects
  - Improve precision on non-perturbative determination : higher order condensates, etc
  - New physics?

3.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

3.6 Model discriminating of BRs

- Studies in specific models

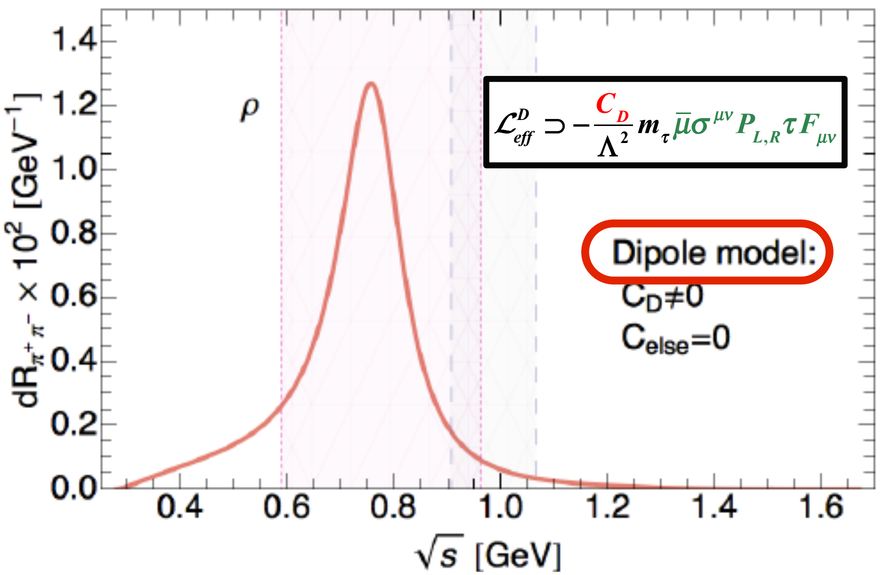
Buras et al.'10

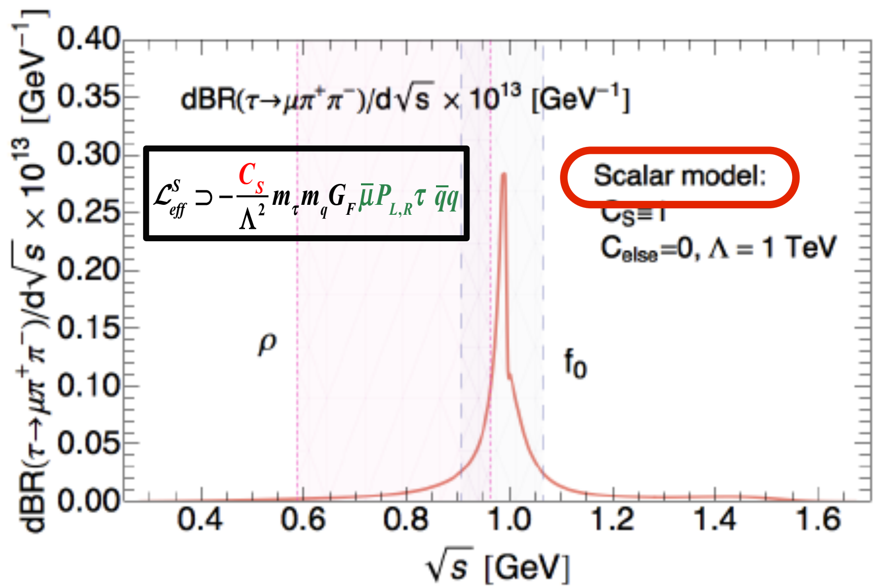
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 0.1$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.02\ldots 0.04$	$0.03\ldots 1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots 2$	~ 5	$0.3\ldots 0.5$	$1.5\ldots 2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots 1.6$	~ 0.2	$5\ldots 10$	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots 0.15$	$10^{-12}\ldots 26$

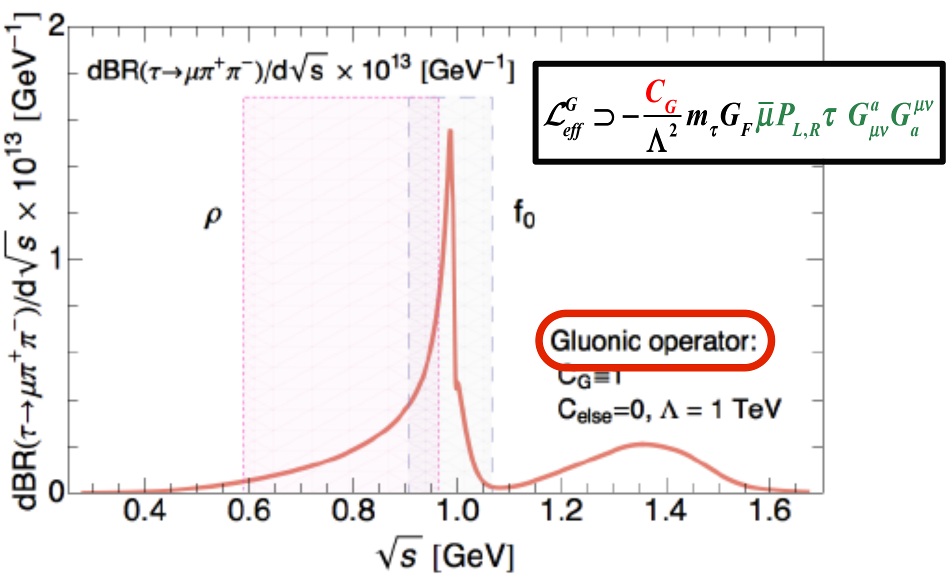
⇒ Disentangle the *underlying dynamics* of NP

3.7 Model discriminating of Spectra: $\tau\rightarrow\mu\pi\pi$

Celis, Cirigliano, E.P.'14







Very different distributions according to the *final hadronic state!*

NB: See also Dalitz plot analyses for $\tau \rightarrow \mu\mu\mu$

Dassinger et al.'07

4.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\,\Gamma P_{L,R}\tau\ \bar{q}\Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\ G_{\mu\nu}^a G_a^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\,\Gamma P_{L,R}\tau\ \bar{\mu}\,\Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

4.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

4.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. f_η, $f_{\eta'}$)

4.4 Model discriminating power of Tau processes

- Summary table: *Celis, Cirigliano, E.P.'14*

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- Recent progress in $\tau \to \mu(e)\pi\pi$ using *dispersive techniques* *Daub et al'13* *Celis, Cirigliano, E.P.'14*

- Hadronic part: $\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} |0\rangle = \left(\textit{Lorentz}\ \mathbf{struct.}\right)_\mu^i F_i(s)}$ with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave ππ and KK scattering data as input

$$n = \pi\pi, K\bar{K} \qquad \boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$