Lattice calculation of |Vus| from inclusive strangeness changing τ decay

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@ Tau 2016, IHEP, Beijing, September 19, 2016

outline

- **Introduction**
  Inclusive tau decay experiment
  Finite energy sum rule and |Vus| determination

- **Lattice HVPs and tau decay**

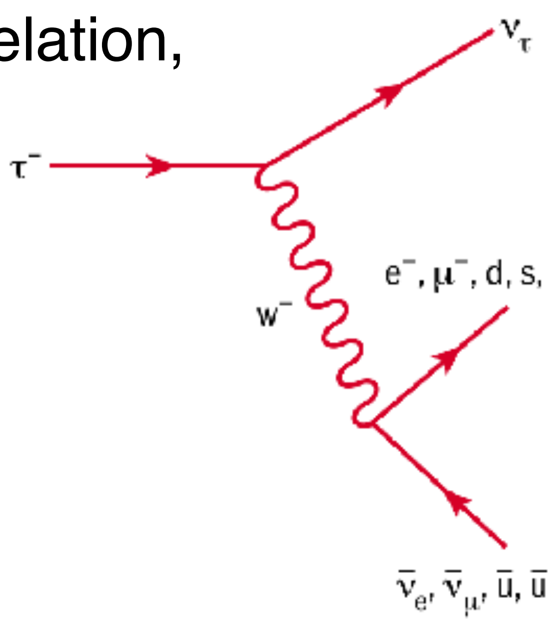
- **Result of |Vus|**

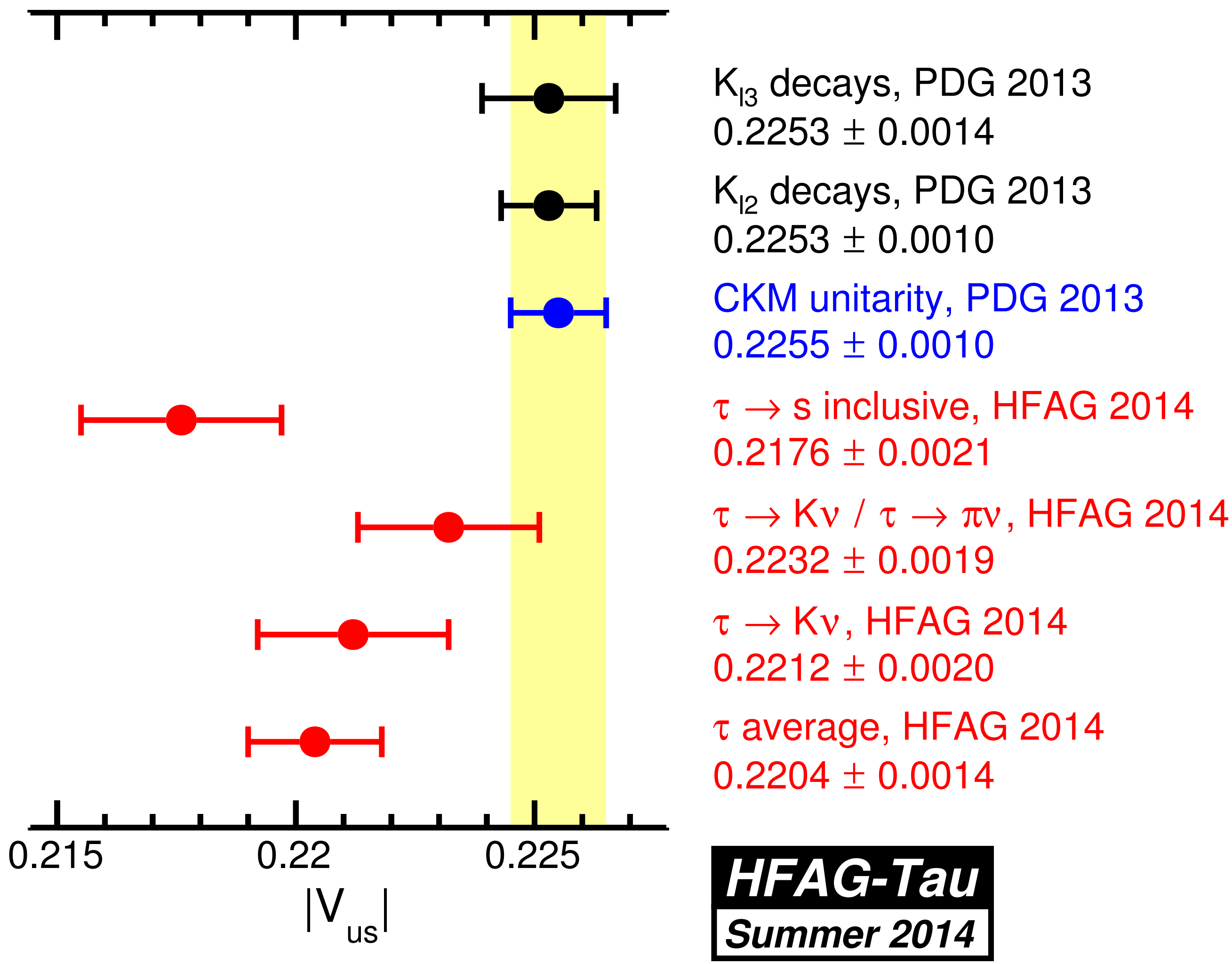
- **Summary**

Intruduction

- Lattice QCD calculation can apply to the exclusive modes:
  fπ, fK: K ->π
- How about inclusive hadronic decay?
  We use τ inclusive Kaon decay experiments -> IVusI determination

- Using optical theorem and dispersion relation,
  τ decay differential cross section
  (τ hadronic decay/τ leptonic decay)
  and the hadronic vacuum polarization
  (HVP) function are related.
  -> We can use lattice HVP calculations.





- |Vus| from inclusive τ decay -> 3 σ deviation from CKM unitarity
- pQCD and high order OPE -> problematic uncertainties?

This work

- We would like to propose an alternative method to calculate
  |Vus| from the inclusive τ decay.

- By combing both the lattice data and pQCD,
  we could expect more precise determination of |Vus|.
- As a result, pQCD uncertainty can be suppressed.

Conventional study

|Vus| determination
from finite energy sum rule

Finite energy sum rule

- The finite energy sum rule (FESR)

$$\int_0^{s_0} \omega(s)\rho(s)ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi(s)ds,$$

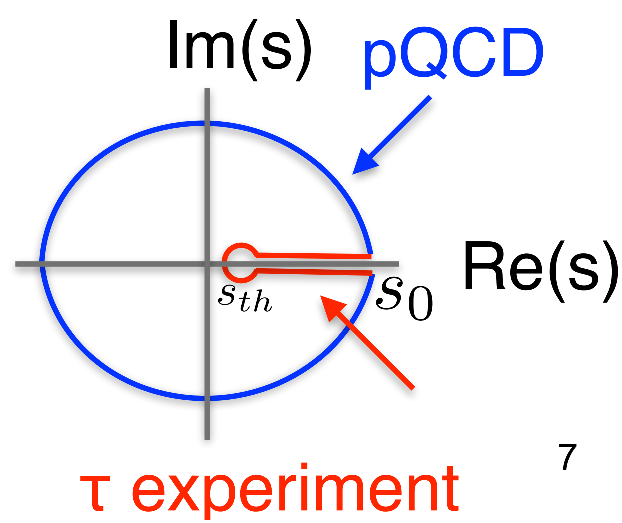
S0 … finite energy,

w(s) is an arbitrary **analytic** function with polynomial in s.

- LHS … ρ(s) is related to the experimental τ inclusive decays

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2|V_{us}|^2 S_{EW}}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^1(s)+\mathrm{Im}\Pi^0(s)\right]$$

$$\rho(s) \equiv |V_{us}|^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^1(s)+\mathrm{Im}\Pi^0(s)\right]$$



- RHS … Analytic calculation with perturbative QCD (pQCD) and OPE (s0 should be large enough = mτ^2)

Our strategy

- Using a different type of the weight function w(s) which has residues

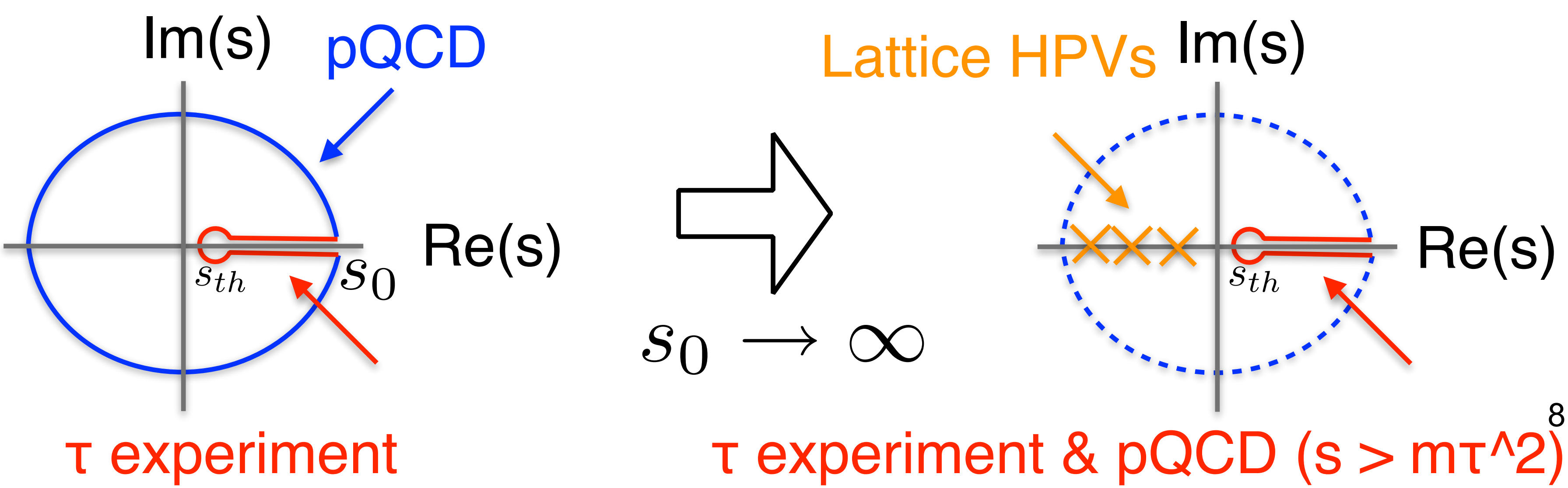
$$\omega(s) \;=\; \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$$

and taking S0 -> ∞,

$$\int_0^\infty \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right)$$

LHS … Experimental data and pQCD

RHS … Lattice HPVs Π(Q) at Euclidean momentum region



Comparison with conventional method

- Our formula

$$\int_0^\infty \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right)$$

Data

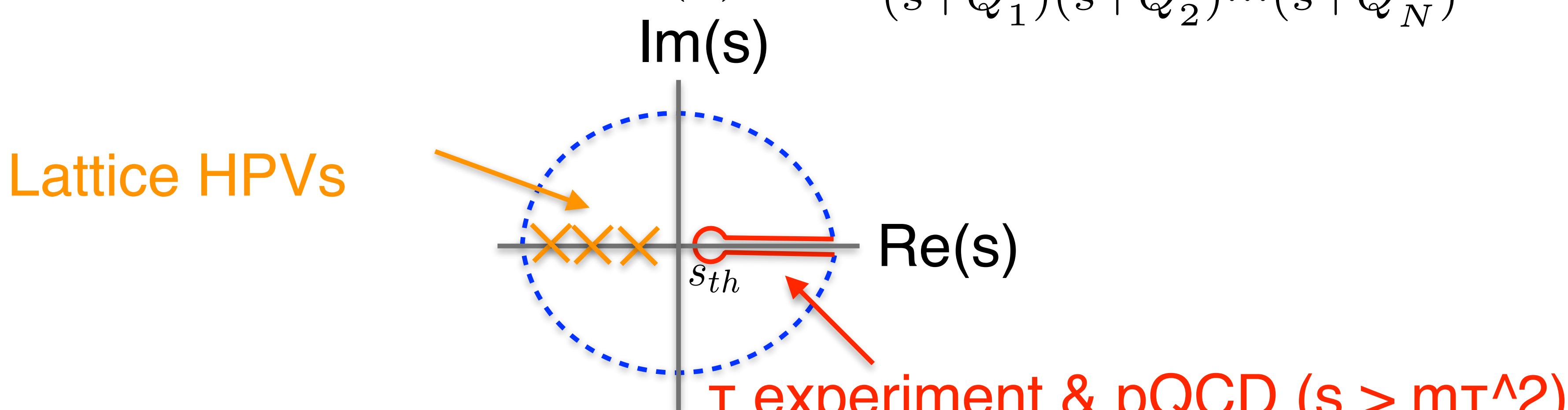
Finite Energy Sum rule … Tau exp. v.s. pQCD

Our method … Tau exp. v.s. pQCD & **Lattice data**

Weight function: ω(s)

Finite Energy Sum rule … ω(s) : polynomial in s

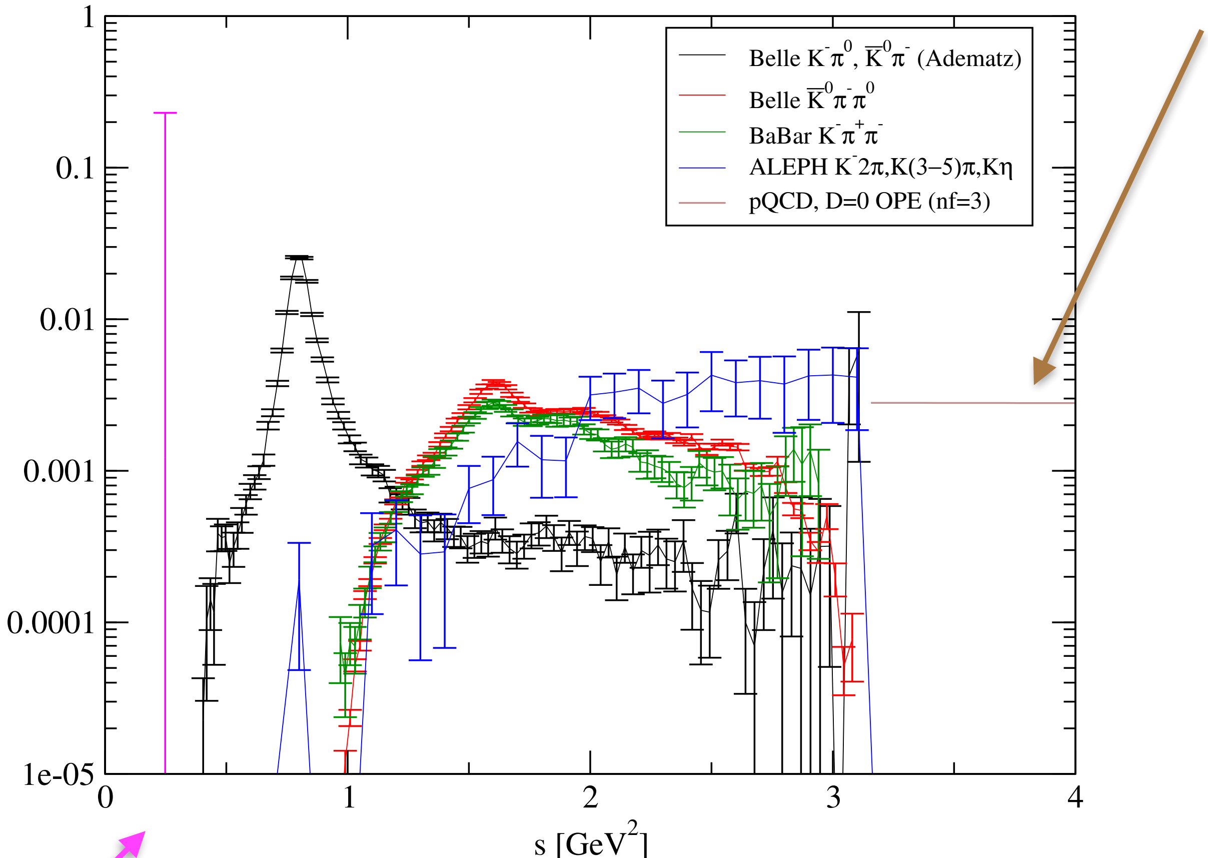
Our method … $\omega(s) = \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$



τ inclusive decay experiment

$$\rho(s) \equiv |V_{us}|^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^1(s) + \mathrm{Im}\Pi^0(s) \right]$$

To compare with experiments,
a conventional value of |Vus|=0.2253 is used



For K pole, we assume a delta function form, whose coefficient is obtained from the experimental value of K-> μ decay width

$$\delta(s - m_k^2)0.0012299(46) \ \sim 2f_k^2|V_{us}|^2$$

Advantage of Weight function

- we use pole-type weight function;

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s + Q_k^2)}, \quad (Q_k^2 > 0)$$

(Number of poles: N)

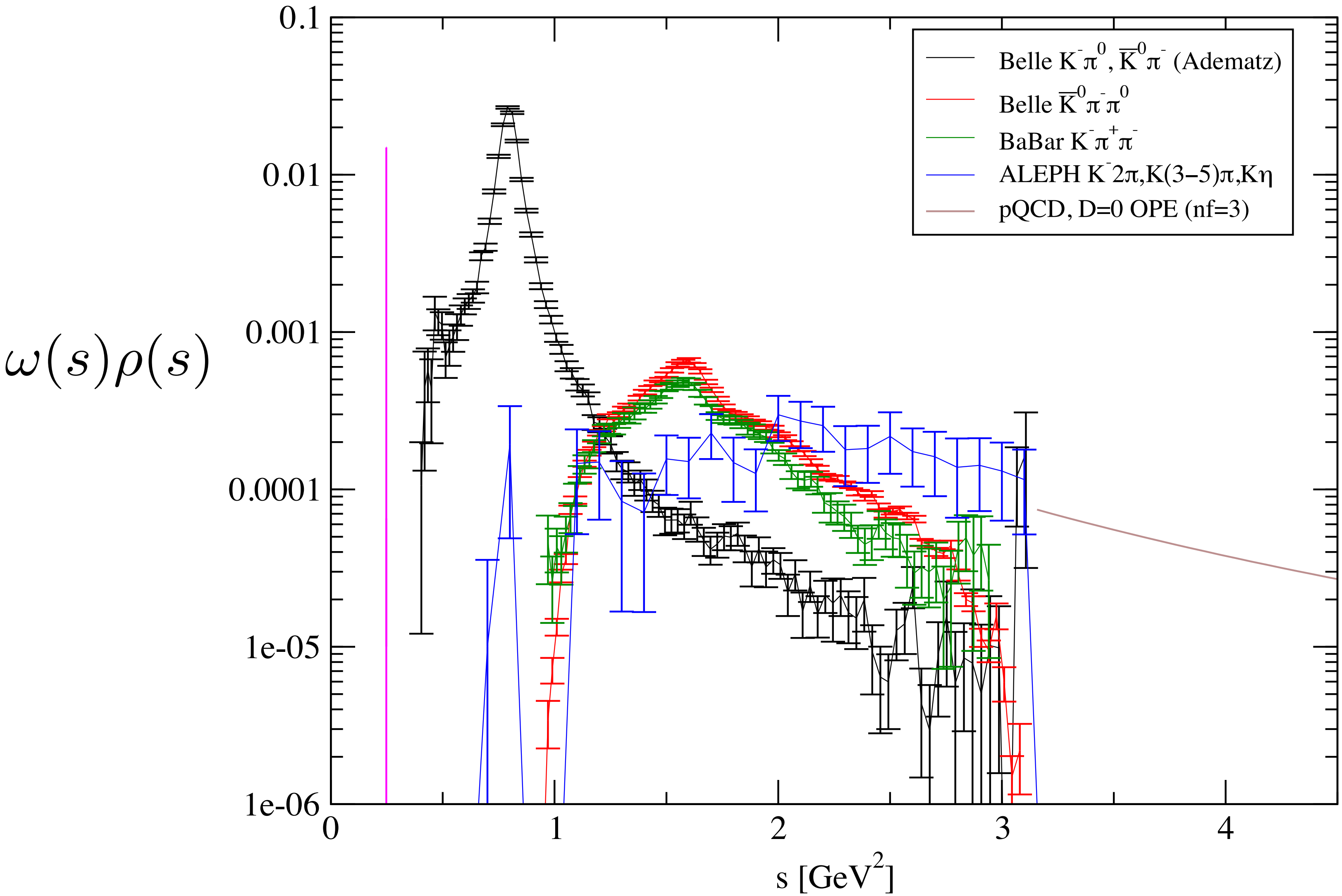
For convergence of contour integral,
a weight function with N ≧ 3 is required

This weight function can suppress

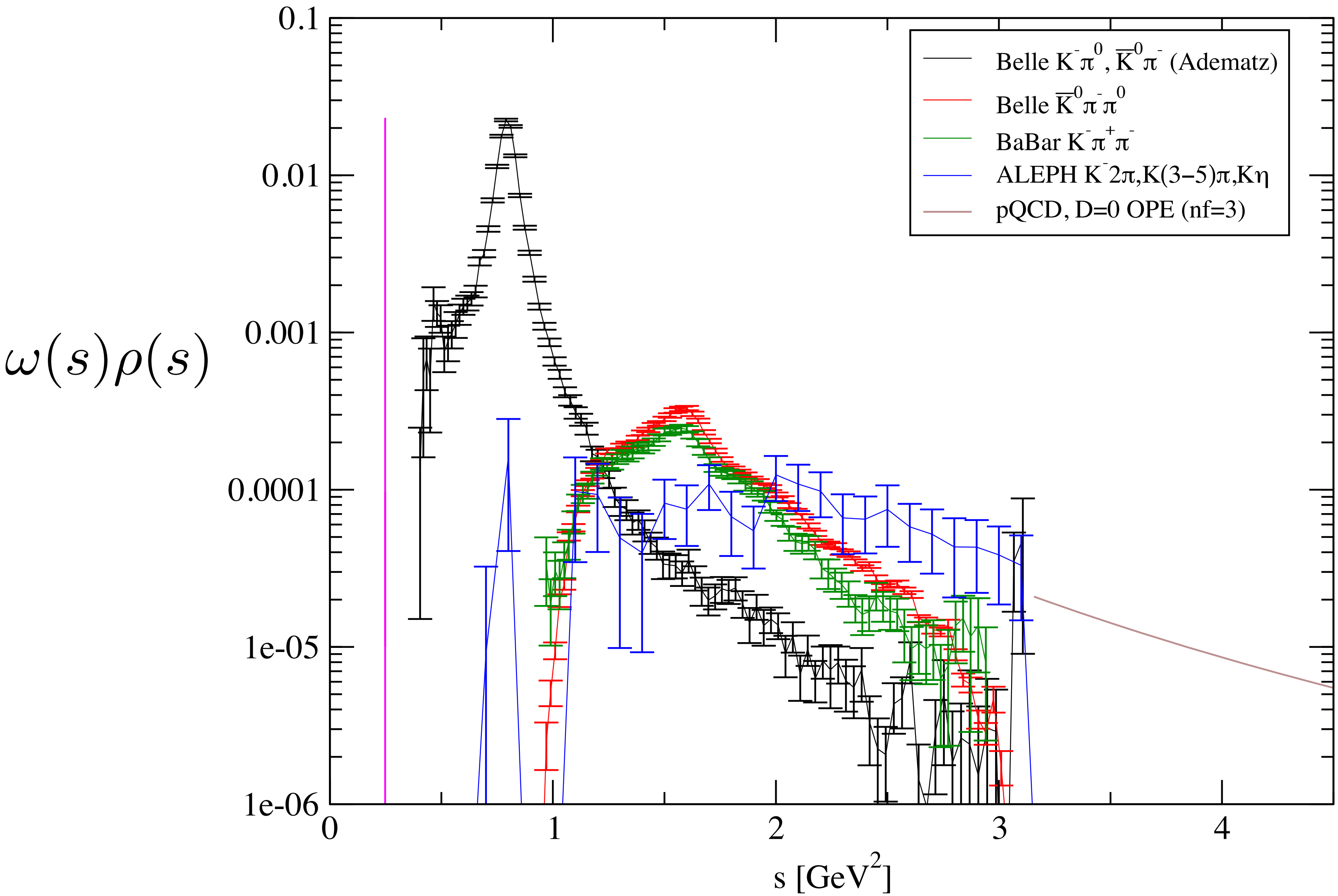
- larger error parts from higher multi hadron final states at s > mk^2
- contributions from pQCD at s > mτ^2

For lattice HVPs,
Q^2 values should not be too small to avoid finite size(time) effect,
and not to be large to avoid large discretization error.

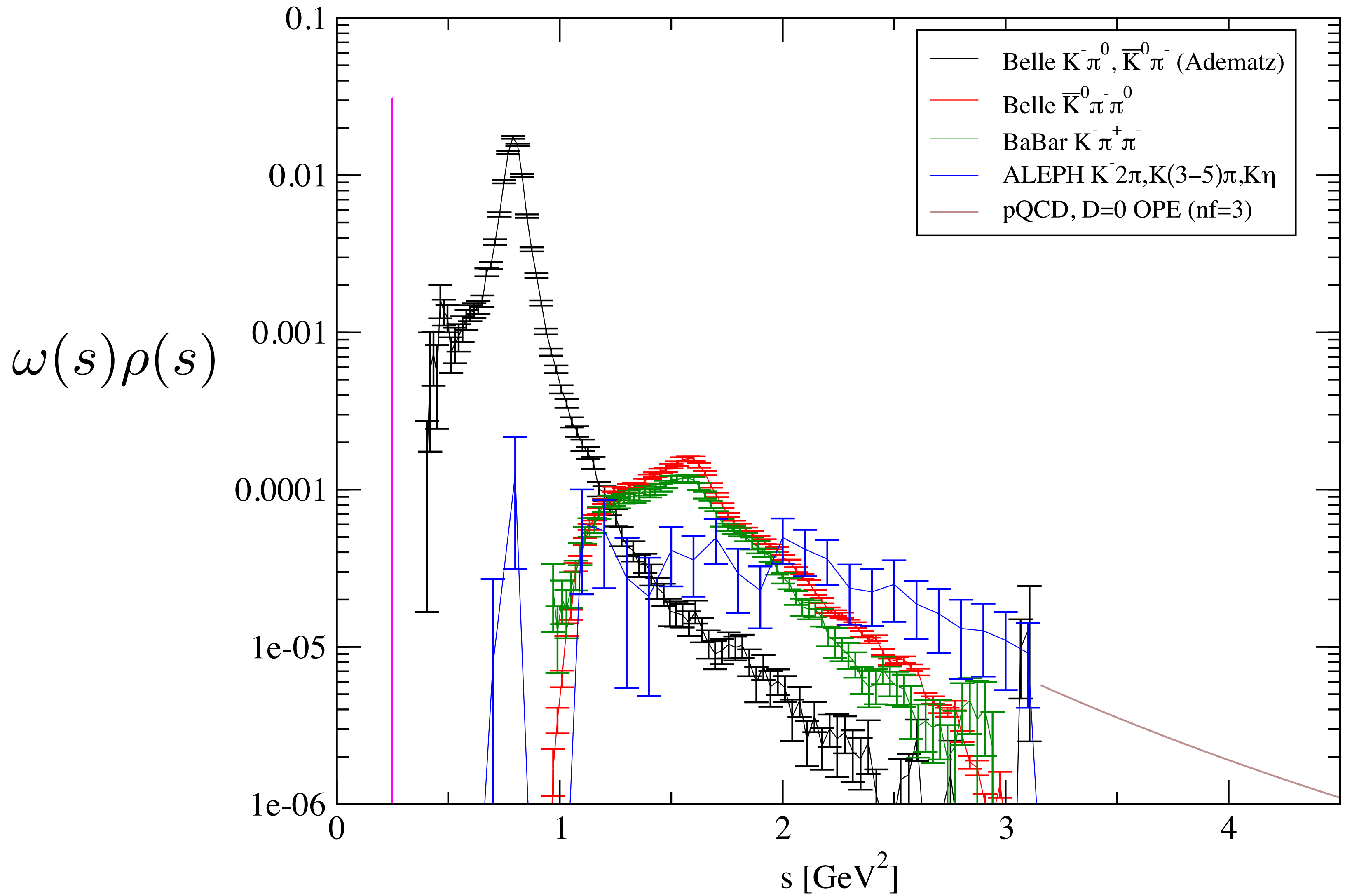
- example: N=3, $\{Q_1^2, Q_2^2, Q_3^2\} = \{0.1, 0.2, 0.3\}$



• example: N=4, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2\} = \{0.1, 0.2, 0.3, 0.4\}$



- example: N=5, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\} = \{0.1, 0.2, 0.3, 0.4, 0.5\}$



Lattice calculation

Lattice HVPs

HVPs from V/A current-current correlation functions with u s flavors,

we consider zero-spatial momentum

$$\Pi^{V/A}_{\mu\nu}(t) = \frac{1}{V}\sum_{\vec{x}} \langle J^{V/A}_{\mu}(\vec{x},t) J^{V/A}_{\nu}(\vec{x},0)\rangle$$

Spin =1, 0 components can be obtained in momentum space as

$$\Pi_{\mu\nu}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2),$$

On the lattice, those with subtraction of unphysical zero-mode can be obtained by discrete Fourier transformation,

(direct double subtraction, sine cardinal Fourier transformation.)

$$\hat{\Pi}(q^2) = \sum_{t=-T/2}^{t=T/2-1} \left(\frac{e^{i\tilde{q}t} - 1}{q^2} + \frac{t^2}{2}\right)\Pi(t)$$

$$\tilde{q}_\mu = 2\sin\left(q_\mu/2\right)$$

lattice QCD ensemble and parameters

2+1 flavor domain-wall fermion gauge ensemble generated by RBC-UKQCD

Vol.	a^{-1}[GeV]	m_π[GeV]	m_K[GeV]	stat.
$24^3 \times 64$	1.785(5)	0.340	0.533	450
		0.340	0.593	450
$32^3 \times 64$	2.383(9)	0.303	0.537	372
		0.303	0.579	372
		0.360	0.554	207
		0.360	0.596	207
$48^3 \times 96$	1.730(4)	0.139	0.499	88
		$0.135^\dagger$	$0.4937^\dagger$	5 PQ-correction, (88)
$64^3 \times 128$	2.359(7)	0.139	0.508	80

- Our main analysis is done on L=48 and 64, at almost physical quark mass region, L=5 fm.
- **PQ-correction**: partially quench (PQ) corrected HVP data at the physical point (†)

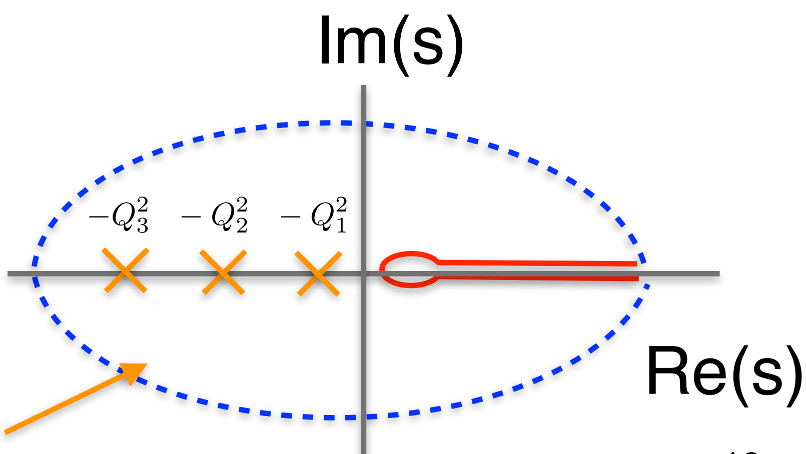
A systematic study of weight function dependence

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s + Q_k^2)}, \quad (Q_k^2 > 0)$$

- C (center value of weights),
- Δ (separation of the pole position),
- N (the number of the poles).

$$\{Q_1^2, Q_2^2, \cdots, Q_N^2\} = \{C - (N/2+1)\Delta, \cdots, C - \Delta, C, C + \Delta, \cdots, C + (N/2+1)\Delta\}$$

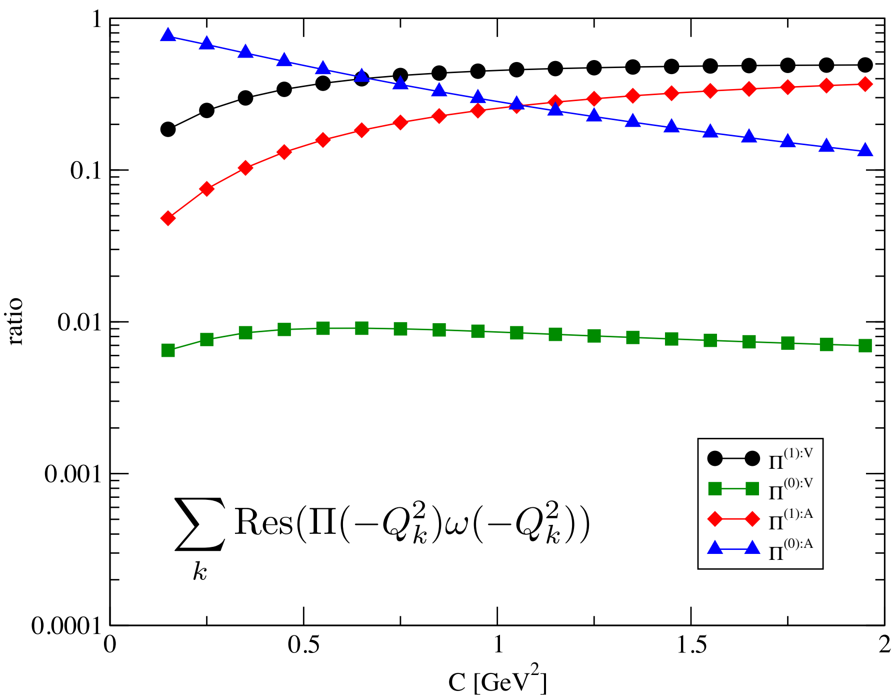
$$C = \frac{Q_1^2 + Q_2^2 + \cdots + Q_N^2}{N}$$



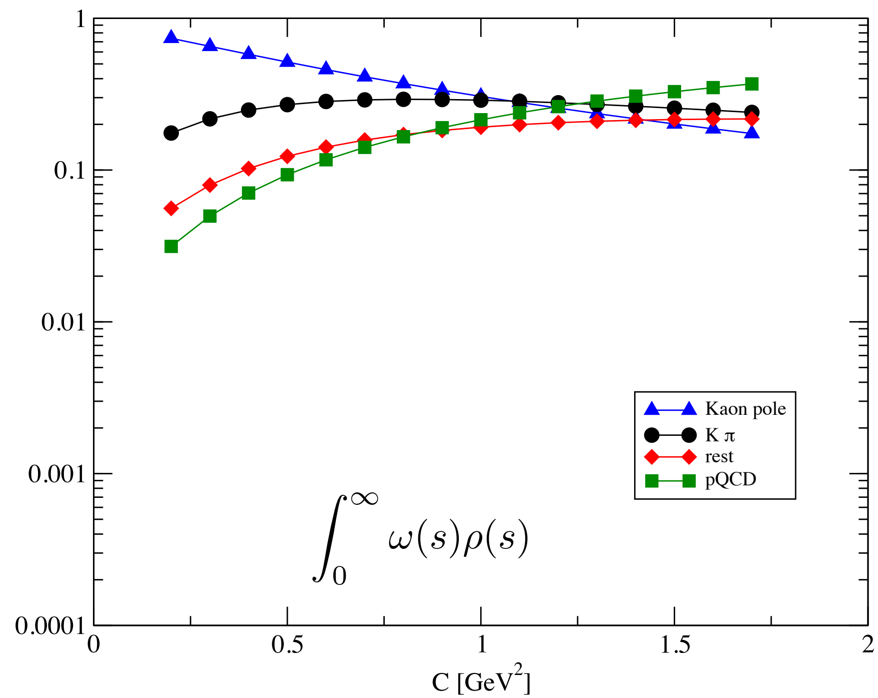
pole positions (N=3 case)

- N=3, Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$



Experiment



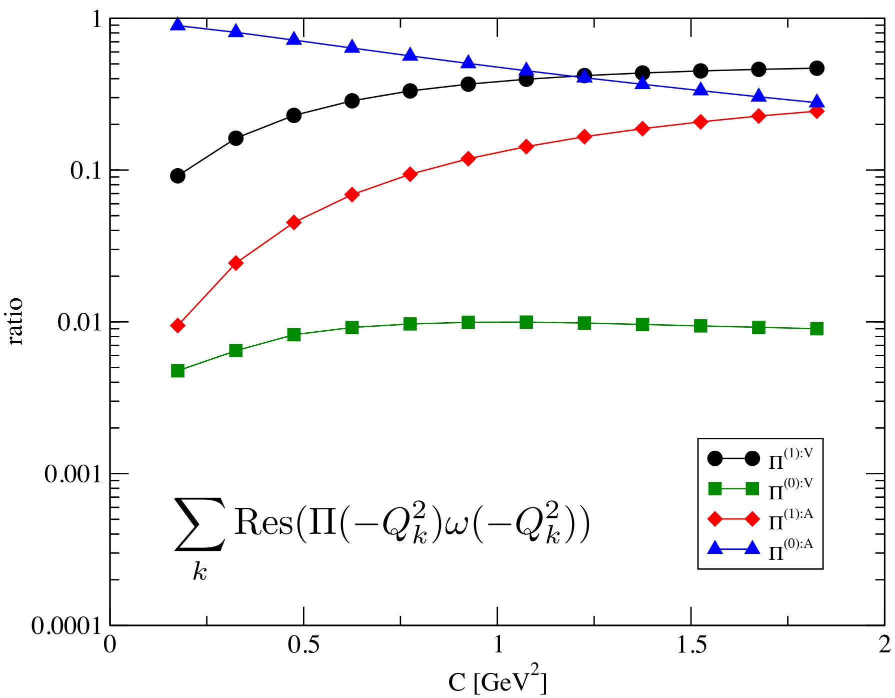
Left : Ratios of each contribution of V/A with spin=0, 1 to the total residue. (Lattice)

Right: Ratios of each decay modes to total cross section. (Experiments)

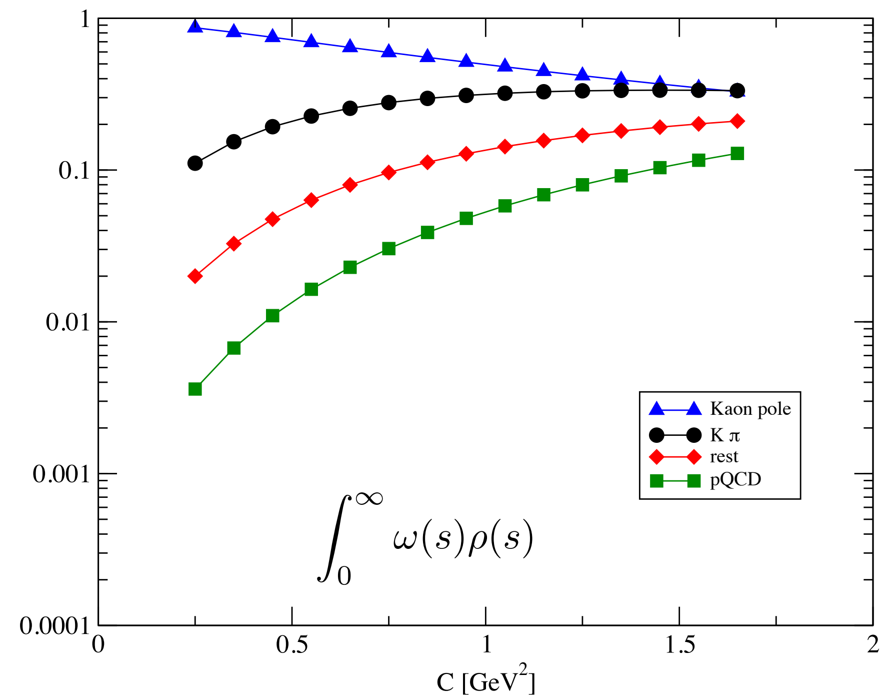
rest : multi π channels, K η

- N=4, Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$



Experiment



- For larger N with smaller Q^2, Kaon pole is the most dominant contribution.
- pQCD and rest modes are highly suppressed.

|Vus| from K pole

- |Vus| can be determined from K pole channel only (exclusive mode).
- Since τ -> K decay mode is dominated by axial spin = 0 channel,
  so we have

$$A_0 \ : |V_{us}^{A_0}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}}}{F_{lat}(\Pi^{(0):A})}}$$

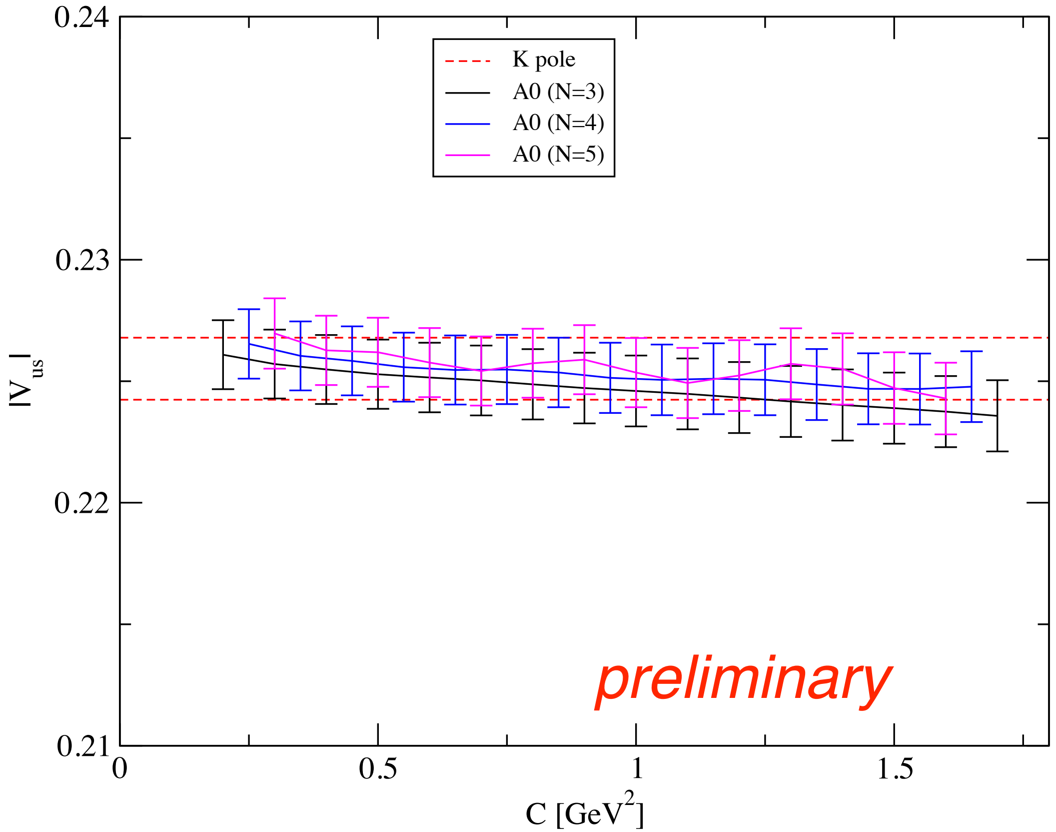
$$\rho_{exp}^{K-pole} = 0.0012299 \int_0^\infty ds \omega(s) \delta(s - m_K^2) = 0.0012299 \omega(m_K^2)$$

We use $f_k^2 |V_{us}|^2 = 0.0012299(46)$

obtained from the experimental value of K-> μ decay width

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}\left(\omega(-Q_k^2)\right) \Pi_{lat}(-Q_k^2)$$

$|V_{us}^{A_0}|$ from L=48 lattice at physical quark mass



N : number of poles

K pole: determined from fK (K decay constant)

|Vus| is universal and consistent with fK determination (mild dependence of C, N)

Our result suggests : A0 channels is dominated by K pole

(Excited mode contributions and lattice discretization error are small
in this momentum region.)

|Vus| from All channels

- A0 channel is dominated by K pole.

-> The kaon decay constant in the continuum limit can be used.

(well determined from lattice QCD)

we use $f_K^{phys} = 0.15551(83)[\mathrm{GeV}]$ [RBC/UKQCD, 2014]

- How about other channels?
- Lattice HVPs for A1, V1, V0 < - > multi hadron states & pQCD
- We take the continuum limit using the data L=48 and 64

$$V_1 + V_0 + A_1 + A_0 \ : |V_{us}^{V_1+V_0+A_1+A_0}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}} + \rho_{exp}^{others}}{(f_K^{phys})^2 \omega(m_K^2) + F_{lat}(\Pi_{\mathrm{others}}) - \rho_{\mathrm{pQCD}}}};$$

$$\rho_{exp}^{others} = |V_{us}|^2 \int_{s_{th}}^{m_\tau^2} ds \omega(s) \mathrm{Im}\Pi(\mathrm{s})$$

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}(\omega(-\mathrm{Q_k^2}))\Pi_{\mathrm{lat}}(-\mathrm{Q_k^2}) \qquad \rho_{pQCD} = \int_{m_\tau^2}^{\infty} ds \omega(s) \Pi_{OPE}(s)$$

Systematic error estimate

- **Higher order discretization error** of a^4 for V1+V0+A1,

$$\mathcal{O}(C^2a^4), \quad (a^{-1} = 2.37[\mathrm{GeV}])$$

- **Finite volume correction**

  1 loop ChPT analysis of current-current correlator on finite volume for Kπ channel (V1).

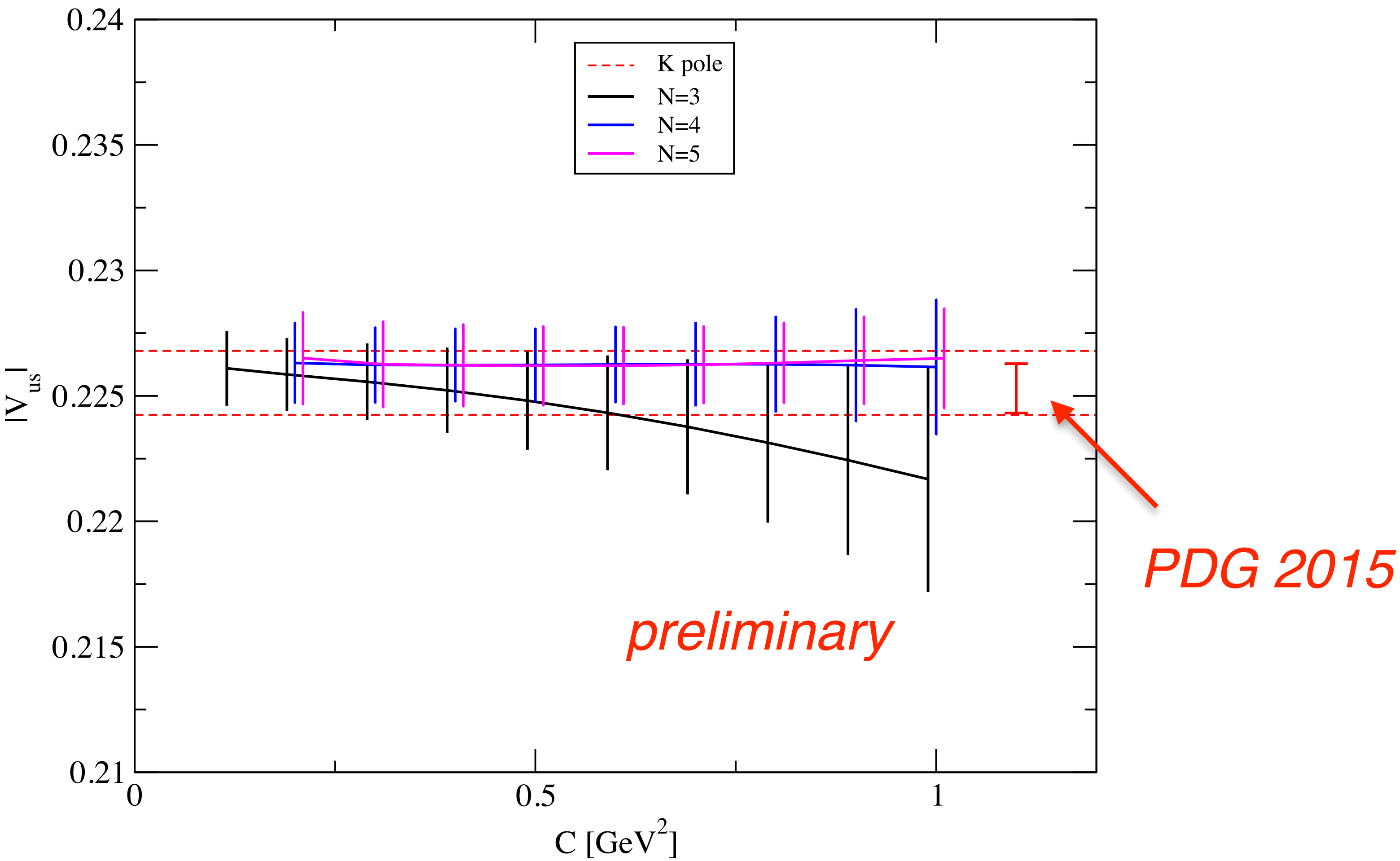
- **Isospin breaking effects**

We put 0.2 % for isospin breaking (EM) effect on V1+V0+A1.
Strong isospin breaking corrected Kπ experimental data used.
(Analysis with s-dependent isospin breaking effect is ongoing)
[Ref: Antonelli, Cirigliano, Lusiani, and Passemar, JHEP10(2013)070]

- **pQCD (OPE) uncertainty**

  2% for possible duality-violation effect

|Vus| for all channels

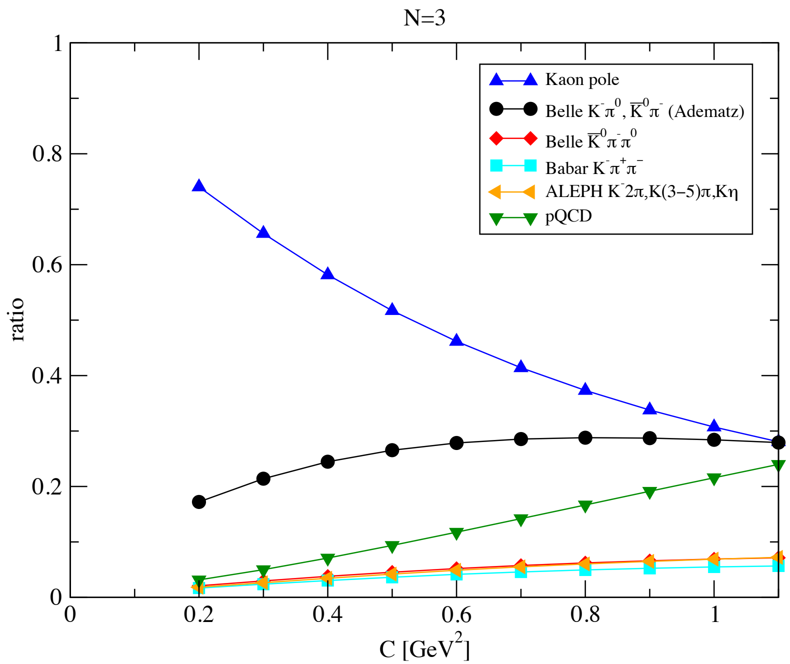


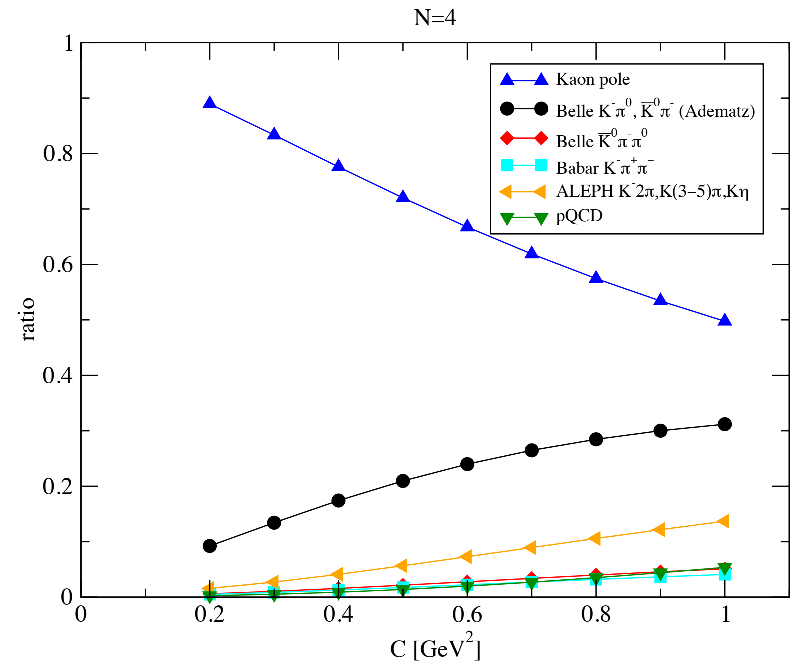
K pole: determined from fK (K decay constant)

For N=4, 5, full result (V1 + V0 + A1+A0) is stable against the change of C, which is consistent with K pole determination.

Ratio of contributions

- N=3
- N=4



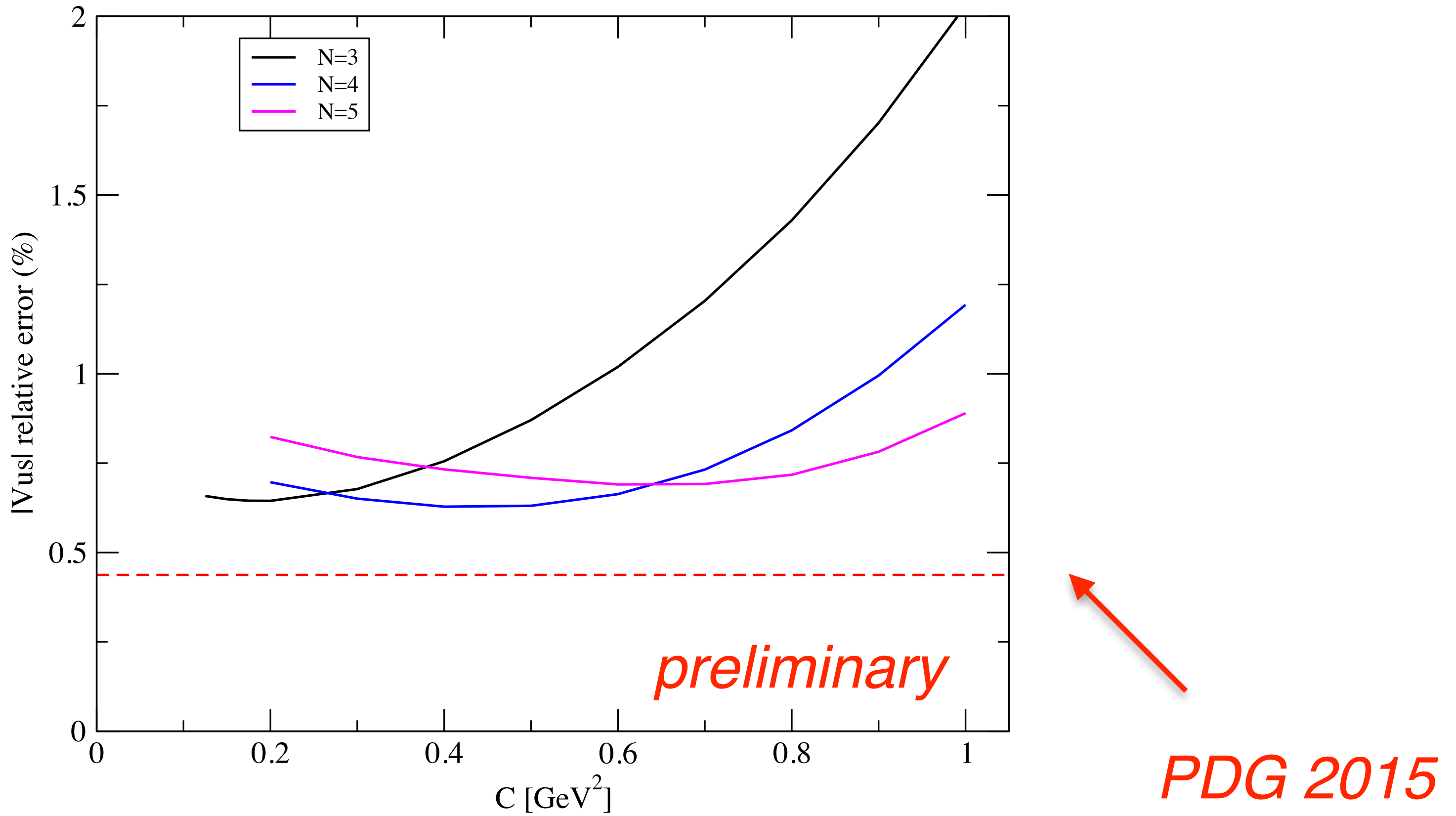


In large C region, perturbative QCD dominates spectral integral in both N.

N=3 : C ~ 0.5, 50 % : K, 30 % : Kπ, 20% : multi π & pQCD

N=4 : In small C ~ 0.2, 80%: K, 20 %: Kπ -> **K & Kπ dominant case**

|Vus| relative error



C and N dependence of error.

Minimum error can be found depending on the value of N,

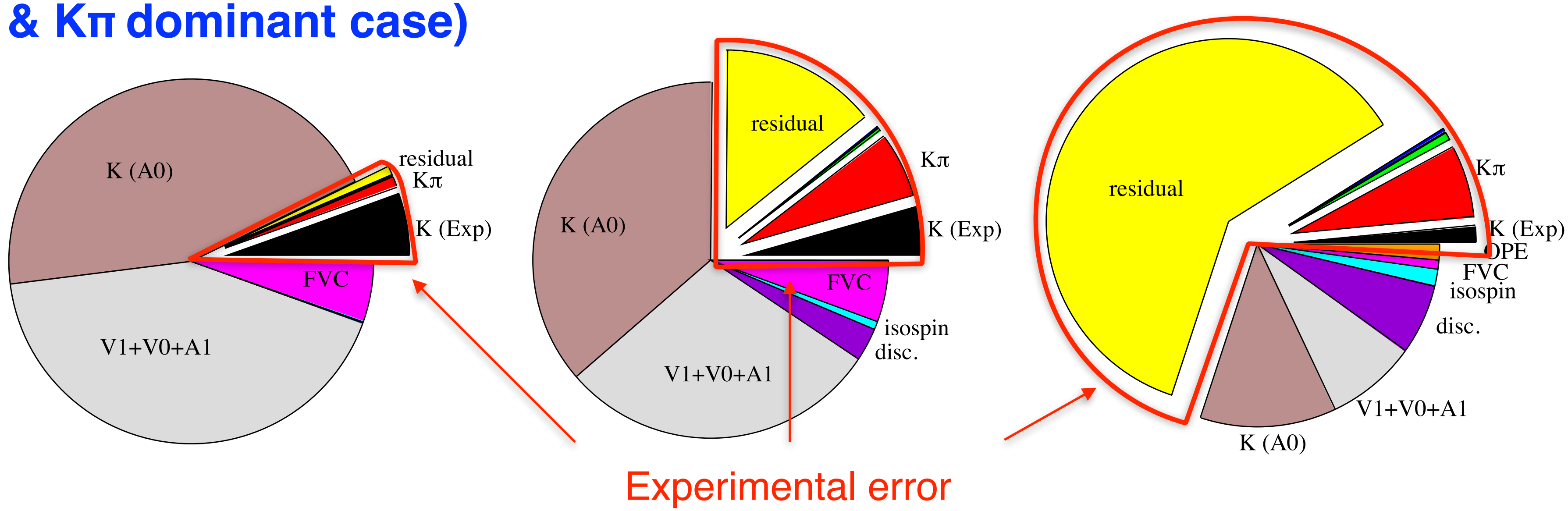
In the case of N=4, C ~ 0.5.

|Vus| error^2 budget

preliminary

$N = 4, C = 0.2$ | $N = 4, C = 0.5$ | $N = 3, C = 0.5$

(K & Kπ dominant case)



error^2	(%)
Exp.	6
K(A0)	45
V1+V0+A1	43
disc.	0
isospin	0
FVC	5
OPE	0

error^2	(%)
Exp.	25
K(A0)	36
V1+V0+A1	29
disc.	3
isospin	1
FVC	6
OPE	0

error^2	(%)
Exp.	70
K(A0)	12
V1+V0+A1	8
disc.	6
isospin	2
FVC	1
OPE	1

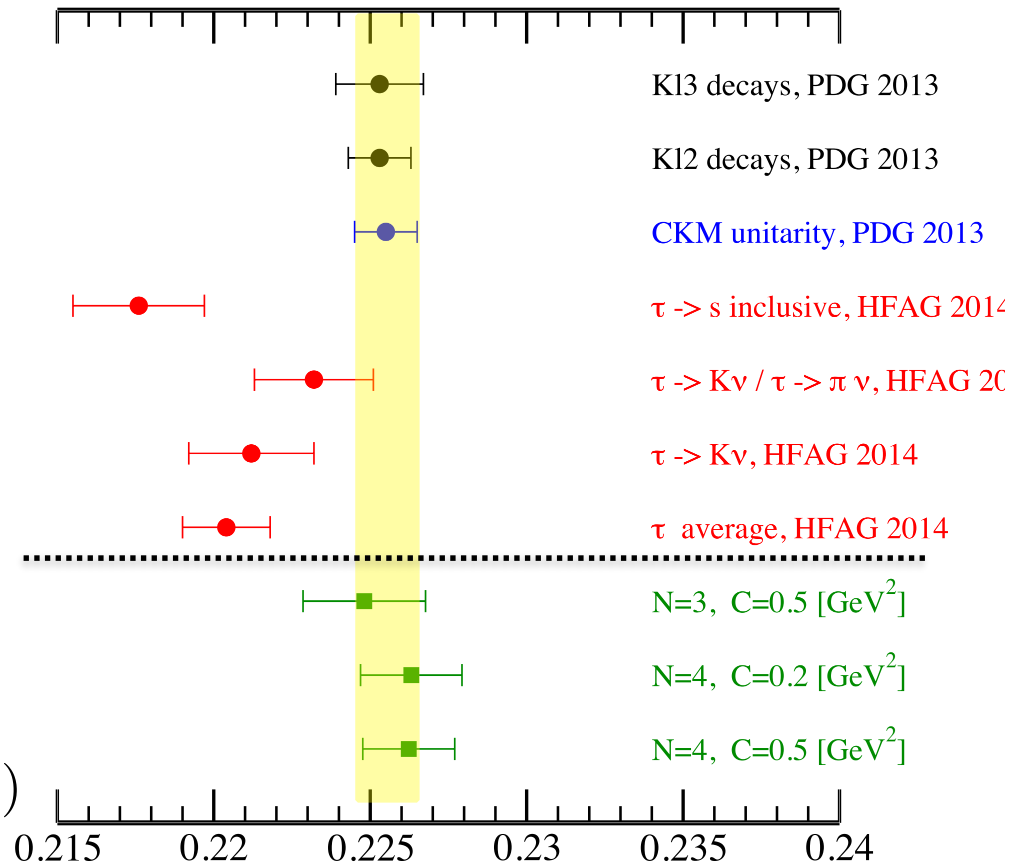
Result

very preliminary

Our result

for all channels

$$(V_1 + V_0 + A_1 + A_0)$$



This work

All our results (C<1, N=3,4) are consistent with each other and CKM unitarity constraint as well.

Summary

The dispersive relation between the inclusive τ decay experiments and the lattice vacuum polarizations, from which we can determine the CKM matrix element |Vus|.

We

- By introducing a weight function with poles at spacetime momenta and lattice QCD, we extend finite energy sum rule analysis to carry out a new type of |Vus| determination, which potentially brings a better precision.
- By changing the number and location of poles, N and C, we could adjust "**inclusiveness**", the impact of multi hadron states, apart from those from K pole and K-Pi, which gives us a new systematic analysis.
- For most accurate Vus, Large N and smaller C, is preferable, where the lattice error (error of f_K and stat error of A1+V1+V0) dominate in our current analysis.

Future Prospects:

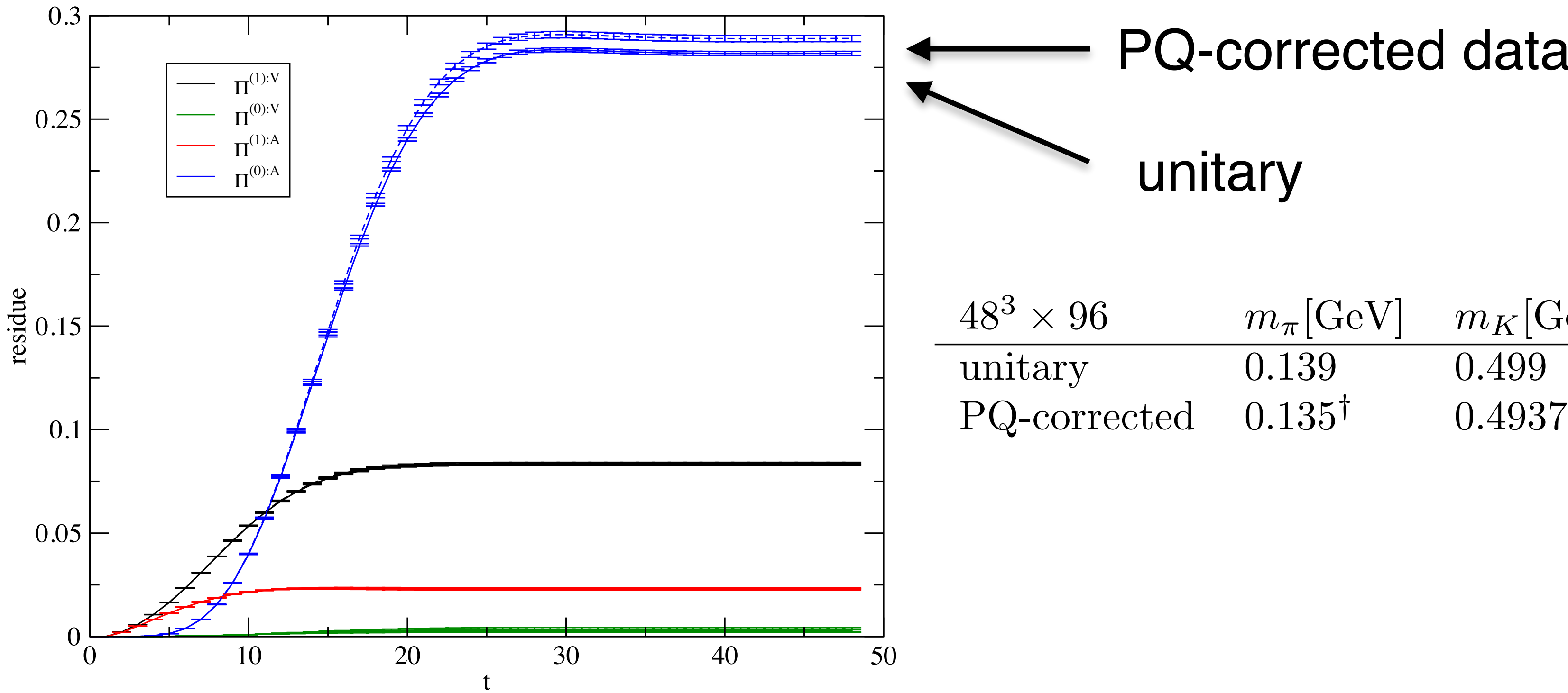
Improvement in both experiments and lattice QCD is possible.
Experiment -> multi hadron (high s) channels,
Lattice -> statistical error is dominant.

light u d current analysis is also possible -> Flavor breaking |Vus|
Other quantities such as α_had, HVP contribution to (g-2)μ

Thank you

Comparison of unitary and PQ-corrected data on L=48

Effective residue



$48^3 \times 96$	m_π[GeV]	m_K[GeV]
unitary	0.139	0.499
PQ-corrected	$0.135^\dagger$	$0.4937^\dagger$

$N = 3$, and $(Q_1^2, Q_2^2, Q_3^2) = (0.1, 0.2, 0.3)$.

Only A0 has visible difference (Kaon),
other channels are consistent with each other (quark mass effect is negligible for multi hadron states).