Predictions on the second-class currents $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$ decays

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in collaboration with Rafel Escribano and Pablo Roig, PRD 94 (2016) 034008

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Universitat Autònoma de Barcelona

Outline

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   - Scalar Form Factor
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     - Dispersion relation: Omnès integral
     - Coupled channels
3. **Branching ratio predictions**
   - $\tau^- \to \pi^-\eta\nu_\tau$
   - $\tau^- \to \pi^-\eta'\nu_\tau$
   - $\eta^{(\prime)} \to \pi^+\ell^-\bar{\nu}_\ell \quad (\ell = e, \mu)$
4. **Conclusions**

Hadronic τ-decays

- Inclusive decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$
  - Fundamental SM parameters:

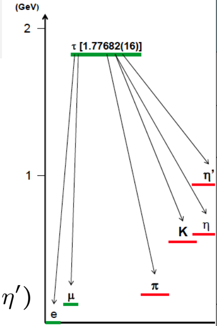
    $\alpha_s(m_\tau), m_s, |V_{us}|$

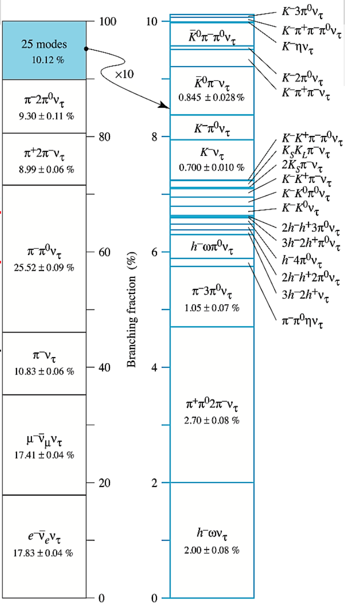
    (see Tau properties (III) and QCD (I))

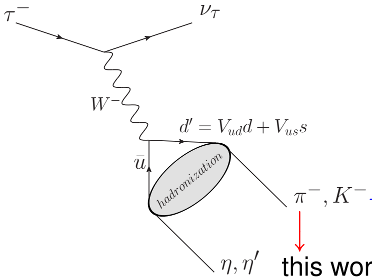
- Exclusive decays:
  - $\tau^- \to (PP, PPP, \ldots)\nu_\tau$, $(P = \pi, K, \eta')$

    Hadronization of QCD currents,

    Form Factors, resonance parameters (M_R, Γ_R)







$\to$ 11h05' Tau properties (II)

(Escribano, Gonzàlez-Solís and Roig JHEP 1310 (2013) 039)

(Escribano, Gonzàlez-Solís, Jamin and Roig JHEP 1409 (2014) 042)

this work (Escribano, Gonzàlez-Solís and Roig PRD 94 (2016) 034008)

$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$ decays

Motivations

- $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$ belong to the second-class current processes unobserved in Nature so far (Weinberg '58)

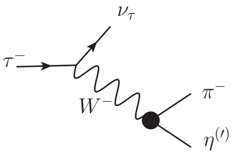
$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y}C|X\rangle = (-1)^I C|X\rangle$$
$$G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \quad \neq \quad G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

- It is an isospin violating process $(\underline{m_u \neq m_d}, e \neq 0)$
- Sensitive to the intermediate vector and scalar resonances $(\rho, \rho', a_0, a_0' ...)$ coupled to the $\bar{u}d$ operator

Purposes

- To describe the participating hadronic form factors
- To predict the decay spectra and to estimate the branching ratios
- To stimulate people from B-factories (Belle-II) to measure these decays

$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: Amplitude and decay width



$$\stackrel{q^2\ll M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{ud}\bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu(1-\underbrace{\cancel{\gamma^5}}_{0^-,1^+\nrightarrow 0^+,1^-})u|0\rangle$$

$$\leftrightarrow \quad \Delta_{PQ} = M_P^2 - M_Q^2$$

$$\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = \left[(p_{\eta^{(\prime)}}-p_\pi)^\mu + \frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{\pi\eta^{(\prime)}}F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0K^+}}{s}q^\mu C^S_{\pi^-\eta^{(\prime)}}F_0^{\pi^-\eta^{(\prime)}}(s)$$

$$\frac{d\Gamma\left(\tau^-\to\pi^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2M_\tau^3}{24\pi^3 s}S_{EW}|V_{ud}|^2|F_+^{\pi^-\eta^{(\prime)}}(0)|^2\left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1+\frac{2s}{M_\tau^2}\right)q^3_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s}q_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2\right\}$$

$$\widetilde{F}_{+,0}^{\pi^-\eta^{(\prime)}}(s) = \frac{F_{+,0}^{\pi^-\eta^{(\prime)}}(s)}{F_{+,0}^{\pi^-\eta^{(\prime)}}(0)}, \qquad F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{C^S_{\pi^-\eta^{(\prime)}}}{C^V_{\pi^-\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}}F_0^{\pi^-\eta^{(\prime)}}(0)$$

Framework and mixing

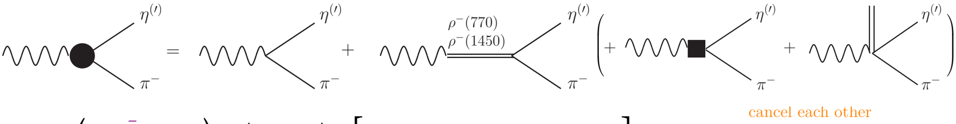
- **Chiral Perturbation Theory**: valid up to the first resonance $\sim \rho$ mass
- **Large-N_C**: to include η_0 singlet
- **Resonance Chiral Theory**: to access resonance regime
- π^0-η-η' mixing (P. Kroll, Mod. Phys. Lett. A20, 2667 (2005))

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon_{\pi\eta} \mathrm{c}\theta_{\eta\eta'} + \varepsilon_{\pi\eta'} \mathrm{s}\theta_{\eta\eta'} & \varepsilon_{\pi\eta'} \mathrm{c}\theta_{\eta\eta'} - \varepsilon_{\pi\eta} \mathrm{s}\theta_{\eta\eta'} \\ -\varepsilon_{\pi\eta} & \mathrm{c}\theta_{\eta\eta'} & -\mathrm{s}\theta_{\eta\eta'} \\ -\varepsilon_{\pi\eta'} & \mathrm{s}\theta_{\eta\eta'} & \mathrm{c}\theta_{\eta\eta'} \end{pmatrix} \cdot \begin{pmatrix} \pi_3 \\ \eta_8 \\ \eta_0 \end{pmatrix}$$

where $\varepsilon_{\pi\eta^{(\prime)}}$ and $\theta_{\eta\eta'}$ are the π^0-$\eta^{(\prime)}$ and η-η' mixing angles

Vector Form Factor: RChT

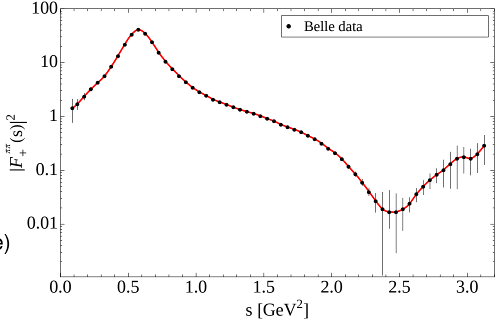
- The vector contribution current occurs via $\pi^0-\eta-\eta'$ mixing, so it is $\mathcal{O}(\varepsilon_{\pi\eta^{(\prime)}})$ and hence suppressed



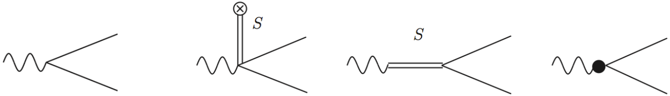
$$\begin{pmatrix} F_+^{\pi^-\eta}(s) \\ F_+^{\pi^-\eta'}(s) \end{pmatrix} = \underbrace{\begin{pmatrix} \varepsilon_{\pi\eta} \\ \varepsilon_{\pi\eta'} \end{pmatrix}}_{\text{suppresion}} \times \left[1 + \sum_{V=\rho,\rho',\rho''} \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} \right]$$

$\tau^- \to \pi^-\pi^0\nu_\tau$

Fujikawa et. al. PRD78 072006 (Belle)

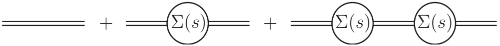


Scalar Form Factor: Breit-Wigner



$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}}\left[1 - \frac{8c_m(c_m-c_d)}{F^2}\frac{2m_K^2-m_\pi^2}{M_S^2} + \frac{4c_m}{F^2}\frac{(c_m-c_d)2m_\pi^2 + c_d\left(s+m_\pi^2-m_{\eta^{(\prime)}}^2\right)}{M_S^2-s}\right]$$

- Imposing $F_0^{\pi^-\eta^{(\prime)}}(s)$ to vanish for $s\to\infty \Rightarrow c_d - c_m = 0$ and $4c_dc_m = F^2$
- Resummation of self-energy insertions in propagator



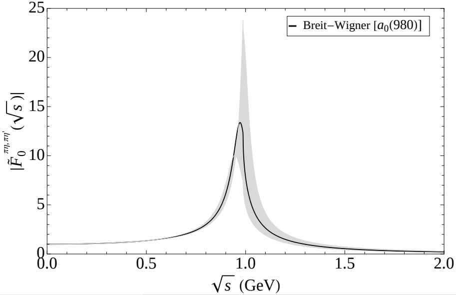
$$+\ldots = \frac{i}{s-M_{K^\star}^2+\Sigma(s)}$$

1 resonance: $M_S = 980(20)\,,\Gamma_S = 75(25)$

$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}}\frac{M_S^2+\Delta_{\pi^-\eta^{(\prime)}}}{M_S^2-s-iM_S\Gamma_S(s)}$$

$$c_0^{\pi^-\eta} = \cos\theta_P - \sqrt{2}\sin\theta_P$$

$$c_0^{\pi^-\eta'} = \cos\theta_P + \frac{1}{\sqrt{2}}\sin\theta_P$$



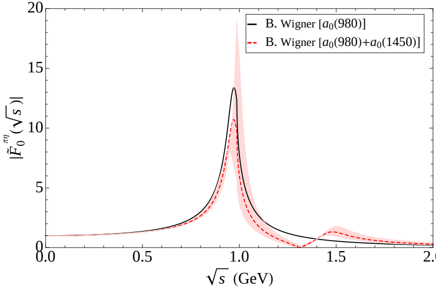
Scalar Form Factor: Breit-Wigner

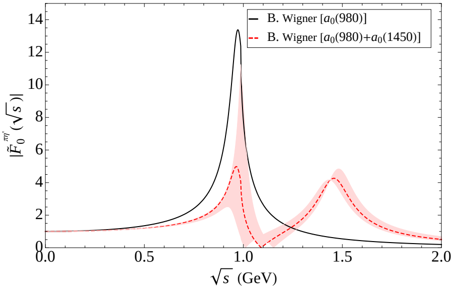
- $\pi - \eta - \eta'$ mixing: Next-to-leading order prediction in Res. ChPT

$$F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{C^S_{\pi^-\eta^{(\prime)}}}{C^V_{\pi^-\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}}F_0^{\pi^-\eta^{(\prime)}}(0)$$

$$\left.\begin{aligned} F_+^{\pi^-\eta^{(\prime)}}(0) &= \varepsilon_{\pi\eta^{(\prime)}} \\ F_0^{\pi^-\eta^{(\prime)}}(0) &= c_0^{\pi^-\eta^{(\prime)}}\frac{M_S^2+\Delta_{\pi^-\eta^{(\prime)}}}{M_S^2} \end{aligned}\right\} \quad \begin{aligned} \varepsilon_{\pi\eta} &= 9.8(3)\cdot 10^{-3} \\ \varepsilon_{\pi\eta'} &= 2.5(1.5)\cdot 10^{-4} \end{aligned}$$

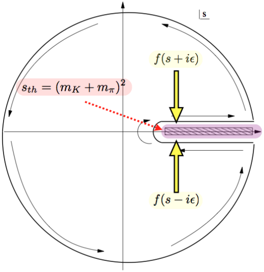
- Breit-Wigner with 2 resonances: $a_0(980)$ and $a_0(1450)$



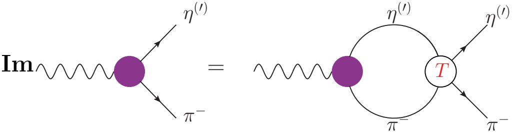


Scalar Form Factor: Dispersion relation

- Analyticity and elastic unitarity through a dispersion relation



$$F_+^{\pi^-\eta^{(\prime)}}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im} F_+^{\pi^-\eta^{(\prime)}}(s')}{s' - s - i\varepsilon}$$



$$\mathrm{Im} F_+^{\pi^-\eta^{(\prime)}}(s) = \sigma_{\pi^-\eta^{(\prime)}}(s) F_+^{\pi^-\eta^{(\prime)}} T^*(s) = F_+^{\pi^-\eta^{(\prime)}} \sin\delta^{\pi^-\eta^{(\prime)}}(s) e^{-i\delta^{\pi^-\eta^{(\prime)}}(s)}$$

- Watson theorem: phase of $F_+^{\pi^-\eta^{(\prime)}}(s)$ is $\delta^{\pi^-\eta^{(\prime)}}(s)$ in the elastic approx.
- Omnès solution (Omnès '58)

$$F_+^{\pi^-\eta^{(\prime)}}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right]$$

Scalar Form Factor: Omnès integral

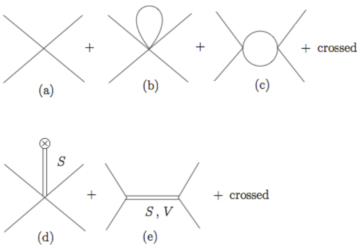
- Analyticity and elastic unitarity through the Omnès solution

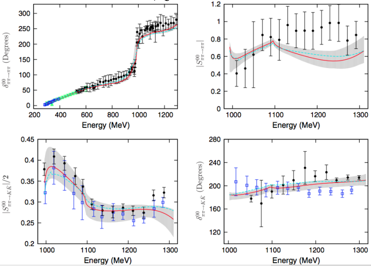
$$F_0^{\pi^-\eta^{(\prime)}}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right] = P(s)\Omega(s)$$

- Elastic unitarity: Form factor phase= $\delta_{\pi^-\eta^{(\prime)}}$ $2 \to 2$ elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan\frac{\mathrm{Im}\,t_{1,0}(s)}{\mathrm{Re}\,t_{1,0}(s)}, \quad t_{1,0}(s) = \frac{N_{1,0}(s)}{1+g(s)N_{1,0}(s)} = \frac{N(s)}{D(s)}$$

- $N_{1,0}$: $U(3)\times U(3)$ amplitudes in $R\chi$T (Guo-Oller: Phys.Rev. D84 (2011) 034005)

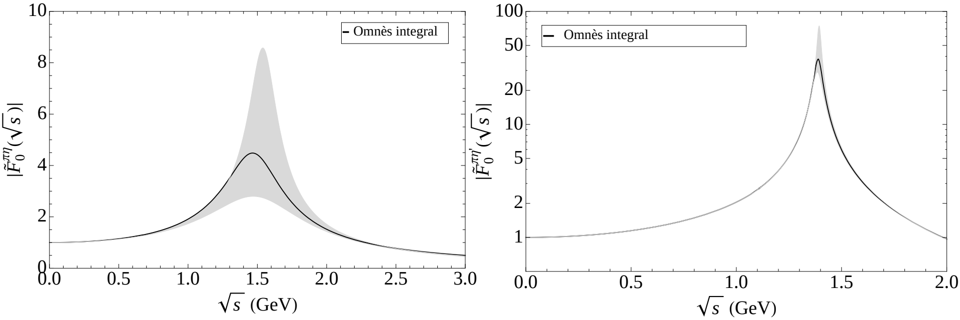




$$\tilde{c}_d = c_d/\sqrt{3}$$
$$\tilde{c}_m = c_m/\sqrt{3}$$
$$c_d = 19.8^{+2.0}_{-5.2}\ \mathrm{MeV}$$
$$c_m = 41.9^{+3.9}_{-9.2}\ \mathrm{MeV}$$
$$M_{a_0,S_8} = 1397^{+73}_{-61}\ \mathrm{MeV}$$
$$M_{S_1} = 1100^{+30}_{-63}\ \mathrm{MeV}$$

Scalar Form Factor: Omnès

- Assuming $F_0^{\pi^-\eta^{(\prime)}}(s)$ to behave as s^{-1} : $F_0^{\pi\eta^{(\prime)}}(s) = P(s)\Omega(s)$,
- $P(s)$ constant. Our choice: $P(s) = F_0^{Breit-Wigner}(0)$



Scalar Form Factor: Closed expression

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i,j} \frac{(1 - s/s_{p_i})}{(1 - s/s_{z_j})} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of $D(s)^{-1} = (1 + g(s)N(s))^{-1}$

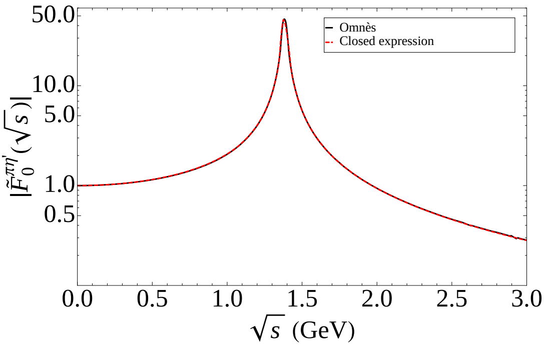
Iwamura, Kurihara, Takahashi ’77
Kamal ’79, Kamal, Cooper ’80
Jamin, Oller, Pich ’01

$s_0 = 0$

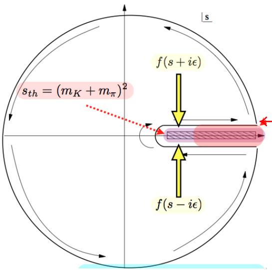
$F_0(s_0) = F_0^{\pi^-\eta',BW}(0) = 0.05$

$s_{z_1} = 1.397$ GeV

$N(s) = N_{\pi\eta' \to \pi\eta'}$

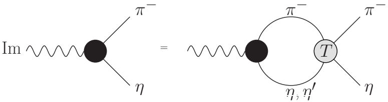


Scalar Form Factor: Coupled channels case

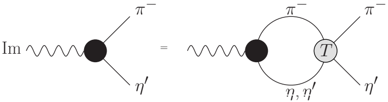


$$F_0^i(s) = \frac{1}{\pi}\sum_{j=1}^{2}\int_{s_i}^{\infty} ds' \frac{\Sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^\star}{(s'-s-i\varepsilon)}$$

Other cuts $(K\bar{K}, \pi\eta' ...)$



$$F_0^{\pi\eta}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta}(s')}{s'-s-i\varepsilon}$$



$$F_0^{\pi\eta'}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta'}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta'}(s')}{s'-s-i\varepsilon}$$

Scalar Form Factor: Coupled channels case (closed expression)

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i,j} \frac{(1 - s/s_{p_i})}{(1 - s/s_{z_j})} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of $\det D(s)^{-1}$

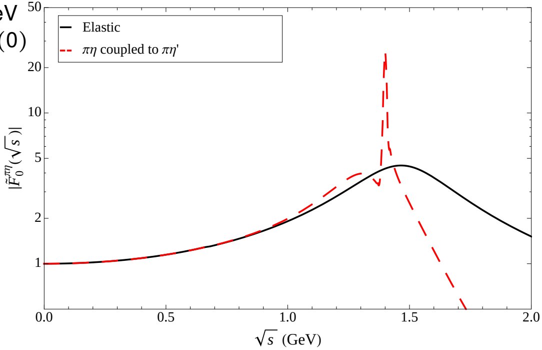
Iwamura, Kurihara, Takahashi PTF 58 (1977)
Kamal '79, Kamal, Cooper '80

$$F_0(s) = \begin{pmatrix} F_0^{\pi\eta}(s) \\ F_0^{\pi\eta'}(s) \end{pmatrix}, \quad s_{z_1} = 1.397 \text{ GeV}, \quad F_0(s_0) = F_0^{BW}(0)$$

$$D(s) = \mathbb{1} + g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 \\ 0 & g_{\pi\eta'} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$



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s_p and s_z: poles and zeros of $\det D(s)^{-1}$

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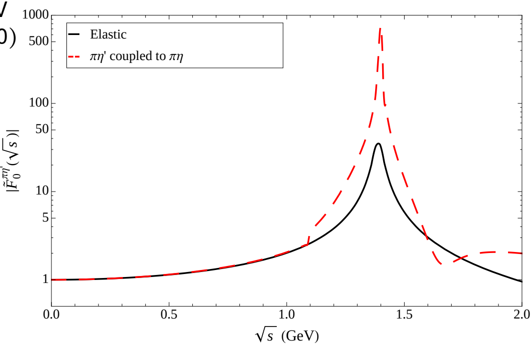
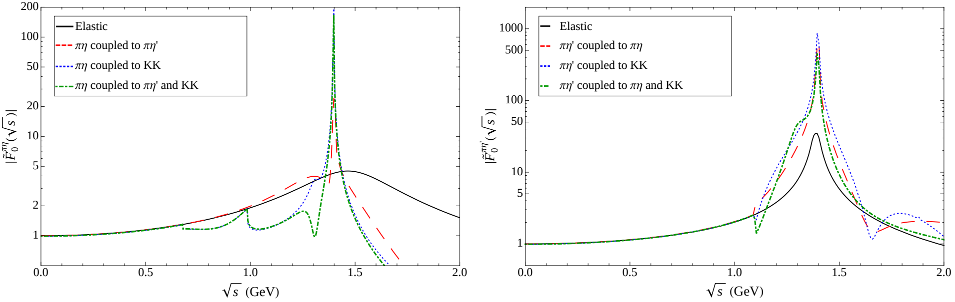
$$D(s) = \mathbb{1} + g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 \\ 0 & g_{\pi\eta'} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$

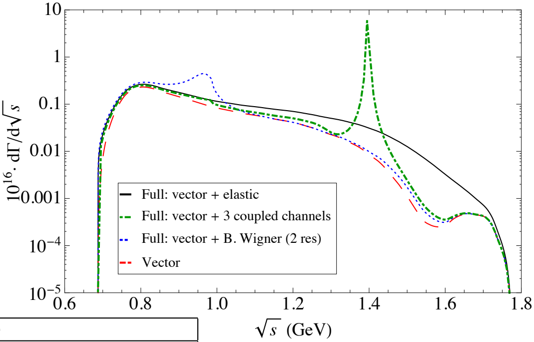


$\pi^-\eta^{(\prime)}$ Form Factors: recapitulate



- Vector Form Factor:
  - Driven by the $\pi^-\pi^0$ vector form factor
- Scalar Form Factor
  1. Breit-Wigner: with $a_0(980)$ and $a_0(1450)$ resonances
  2. Omnès solution: analyticity+elastic final state interactions
  3. Closed Form: coupled-channels

$\tau^- \to \pi^- \eta \nu_\tau$: Invariant mass distribution and Branching Ratio



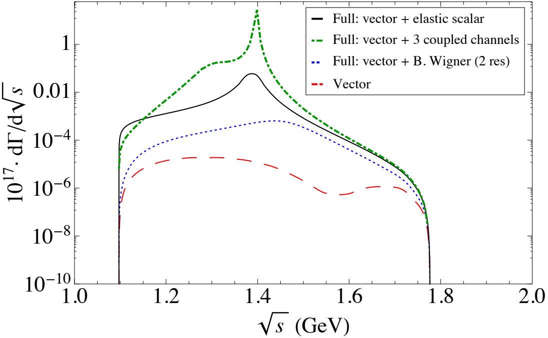
$BR_V \cdot 10^5$	$BR_S \cdot 10^5$	$BR \cdot 10^5$	Reference
0.25	1.60	1.85	Tisserant, Truong '82
0.12	1.38	1.50	Bramón, Narison, Pich '87
0.15	1.06	1.21	Neufeld, Rupertsberger '94
0.36	1.00	1.36	Nussinov, Soffer '08
[0.2, 0.6]	[0.2, 2.3]	[0.4, 2.9]	Paver, Riazuddin '10
0.44	0.04	0.48	Volkov, Kostunin '12
0.13	0.20	0.33	Descotes-Genon, Moussallam '14
0.26(2)	$0.72^{+0.46}_{-0.22}$	0.98(51)	Breit-Wigner [$a_0(980)$]
0.26(2)	$0.48^{+0.29}_{-0.14}$	0.74(32)	Breit-Wigner [$a_0(980) + a_0(1450)$]
0.26(2)	$0.10^{+0.02}_{-0.03}$	0.36(4)	Elastic Omnès solution
0.26(2)	0.15(9)	0.41(9)	2 coupled channels ($\pi^-\eta$ to $\pi^-\eta'$)
0.26(2)	1.86(11)	2.12(11)	2 coupled channels ($\pi^-\eta$ to K^-K^0)
0.26(2)	1.41(9)	1.67(9)	3 coupled channels

THIS WORK (last six rows)

$BR^{BaBar}_{exp} < 9.9 \cdot 10^{-5}$ 95%CL
(PRD 83 (2011) 032002)

$BR^{Belle}_{exp} < 7.3 \cdot 10^{-5}$ 90%CL
(PoS EPS -HEP2009, 374 (2009))

$\tau^- \to \pi^-\eta'\nu_\tau$: Invariant mass distribution and Branching Ratio



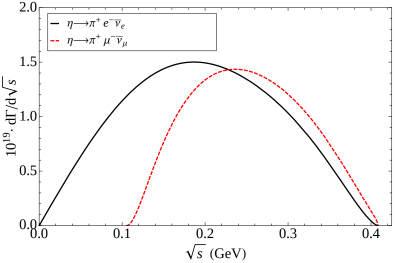
	BR_V	BR_S	BR	Reference
	$< 10^{-7}$	$[0.2, 1.3]\cdot 10^{-6}$	$[0.2, 1.4]\cdot 10^{-6}$	Nussinov, Soffer '99
	$[0.14, 3.4]\cdot 10^{-8}$	$[0.6, 1.8]\cdot 10^{-7}$	$[0.61, 2.1]\cdot 10^{-7}$	Paver, Riazuddin '11
	$1.11\cdot 10^{-8}$	$2.63\cdot 10^{-8}$	$3.74\cdot 10^{-8}$	Volkov, Kostunin '12
THIS WORK	$[0.3, 5.7]\cdot 10^{-10}$	$[2\cdot 10^{-11}, 7\cdot 10^{-10}]$	$[0.5\cdot 10^{-10}, 1.3\cdot 10^{-9}]$	Breit-Wigner (1 res)
	$[0.3, 5.7]\cdot 10^{-10}$	$[5\cdot 10^{-11}, 2\cdot 10^{-9}]$	$[0.8\cdot 10^{-10}, 2.6\cdot 10^{-9}]$	Breit-Wigner (2 res)
	$[0.3, 5.7]\cdot 10^{-10}$	$[2\cdot 10^{-9}, 4\cdot 10^{-8}]$	$[2.6\cdot 10^{-9}, 4\cdot 10^{-8}]$	Elastic Omnès solution
	$[0.3, 5.7]\cdot 10^{-10}$	$[2\cdot 10^{-7}, 2\cdot 10^{-6}]$	$[2\cdot 10^{-7}, 2\cdot 10^{-6}]$	2 cc ($\pi^-\eta'$ to $\pi^-\eta$)
	$[0.3, 5.7]\cdot 10^{-10}$	$[3\cdot 10^{-7}, 3\cdot 10^{-6}]$	$[3\cdot 10^{-7}, 3\cdot 10^{-6}]$	2 cc ($\pi^-\eta'$ to K^-K^0)
	$[0.3, 5.7]\cdot 10^{-10}$	$[1\cdot 10^{-7}, 1\cdot 10^{-6}]$	$[1\cdot 10^{-7}, 1\cdot 10^{-6}]$	3 coupled channels

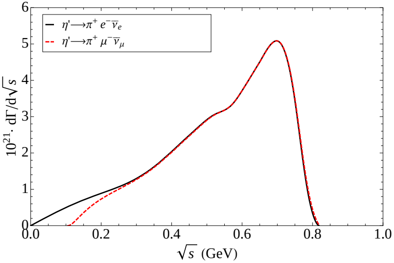
$BR_{exp}^{BaBar} < 4\cdot 10^{-6}$ (90% CL), (PRD 86, 092010 (2012))

Branching Ratio estimates: $\eta^{(\prime)} \to \pi^+ \ell^- \bar{\nu}_\ell \quad (\ell = e, \mu)$

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{G_F^2 |V_{ud} F_+(0)|^2 (C_V^{\pi\eta})^2 (s - m_l^2)^2}{24\pi^3 M_\eta^3 s}$$

$$\left\{ (2s + m_\ell^2) q_{\pi\eta}^3 |\widetilde{F}_+(s)|^2 + \frac{3}{4s} \Delta_{\pi\eta}^2 {m_\ell}^2 q_{\pi\eta} |\widetilde{F}_0(s)|^2 \right\}$$





Decay	Descotes-Genon, Moussallam '14	Our estimate
$\eta \to \pi^+ e^- \bar{\nu}_e + c.c.$	$\sim 1.40 \cdot 10^{-13}$	$0.6 \cdot 10^{-13}$
$\eta \to \pi^+ \mu^- \bar{\nu}_\mu + c.c.$	$1.02 \cdot 10^{-13}$	$0.4 \cdot 10^{-13}$
$\eta' \to \pi^+ e^- \bar{\nu}_\mu + c.c.$		$1.7 \cdot 10^{-17}$
$\eta' \to \pi^+ \mu^- \bar{\nu}_\mu + c.c.$		$1.7 \cdot 10^{-17}$

Outlook

- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$: second-class currents unseen in Nature
  - Isospin-violating processes and hence suppressed
- Form Factors:
  - Vector Form Factor: driven by $\tau^- \to \pi^- \pi^0 \nu_\tau$ data
  - Scalar Form Factor: Breit-Wigner, Omnès, coupled-channels
  - We predict $\tau^- \to \pi^- \eta \nu_\tau \sim 1.7 \cdot 10^{-5}$ and $\tau^- \to \pi^- \eta' \nu_\tau$ $\mathcal{O}(10^{-7} - 10^{-6})$
- We encourage experimental groups (Belle-II) to pursue this decay mode

Back-up

Hadronic Matrix Element

- Taking the divergence we obtain on the L.H.S

$$\langle 0|\partial_\mu(\bar{s}\gamma^\mu u)|K^+\eta^{(\prime)}\rangle = i(m_s - m_u)\langle 0|\bar{s}u|K^+\eta^{(\prime)}\rangle = i\Delta_{K\pi} C^S_{K^-\eta^{(\prime)}} F_0^{K^-\eta^{(\prime)}}(s) \qquad (1)$$

where $\Delta_{PQ} = M_P^2 - M_Q^2\,,\quad C^S_{K^-\eta} = 1/\sqrt{6}\,,\quad C^S_{K^-\eta'} = 2/\sqrt{3}$

- on the R.H.S (vector current not conserved)

$$iq_\mu\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle = iC^V_{K\eta^{(\prime)}}\left[(m^2_{\eta^{(\prime)}} - m^2_{K^-})F_+^{K^-\eta^{(\prime)}}(s) - sF_-^{K^-\eta^{(\prime)}}(s)\right] \qquad (2)$$

- Equating eqs. (1,2) allows us to relate $F_-^{K^-\eta^{(\prime)}}(s)$ with $F_0^{K^-\eta^{(\prime)}}(s)$ as

$$F_-^{K^-\eta^{(\prime)}}(s) = -\frac{\Delta_{K^-\eta^{(\prime)}}}{s}\left[\frac{C^S_{K\eta^{(\prime)}}}{C^V_{K\eta^{(\prime)}}}\frac{\Delta_{K\pi}}{\Delta_{K^-\eta^{(\prime)}}}F_0^{K^-\eta^{(\prime)}}(s) + F_+^{K^-\eta^{(\prime)}}(s)\right] \qquad (3)$$

- The hadronic matrix element finally reads ($q^\mu = (p_{\eta^{(\prime)}} + p_{K^-})^\mu +$ and $q^2 = s$)

$$\langle K^-\eta^{(\prime)}|\bar{s}\gamma^\mu u|0\rangle =$$

$$\left[(p_{\eta^{(\prime)}} - p_K)^\mu + \frac{\Delta_{K^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{K\eta^{(\prime)}}F_+^{K^-\eta^{(\prime)}}(s) + \frac{\Delta_{K\pi}}{s}q^\mu C^S_{K\eta^{(\prime)}}F_0^{K^-\eta^{(\prime)}}(s) \qquad (4)$$

Scalar Form Factor: Closed expression

Once subtracted dispersion relation

$$F(s+i\varepsilon) = F(s_0) + \frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\sigma(s')t_{IJ}^{*}(s')F(s')}{(s'-s_0)(s'-s-i\varepsilon)} = F(s_0) + \widetilde{F}(s+i\varepsilon)$$

$$\widetilde{F}(s+i\varepsilon) - \widetilde{F}(s-i\varepsilon) = 2i\sigma(s)t^{*}(s+i\varepsilon)F(s+i\varepsilon)$$
$$= 2i\sigma(s)t^{*}(s+i\varepsilon)[F(s_0) + \widetilde{F}(s+i\varepsilon)]$$

$$\left.\begin{array}{l} t = N/D \\ \mathrm{Im}\, t^{-1} = -\sigma(s) \\ \mathrm{Im} D(s) = -N\sigma(s) \end{array}\right\} \qquad \begin{array}{l} \widetilde{F}(s+i\varepsilon)D(s+i\varepsilon) - \widetilde{F}(s-i\varepsilon)D(s-i\varepsilon) \\ = -2i\,\mathrm{Im} D(s)F(s_0), \end{array}$$

$$\begin{aligned} \widetilde{F}(s+i\varepsilon) &= \frac{1}{D(s+i\varepsilon)}\frac{-(s-s_0)}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im} D(s')F(s_0)}{(s'-s_0)(s'-s)} \\ &= -D(s+i\varepsilon)^{-1}\left[D(s+i\varepsilon) - D(s_0)\right]F(s_0) \end{aligned}$$