Update of the Hadronic Vacuum Polarization Contribution to a_μ

Michel Davier
in collaboration with
A. Hoecker, B. Malaescu, Z. Zhang

Outline

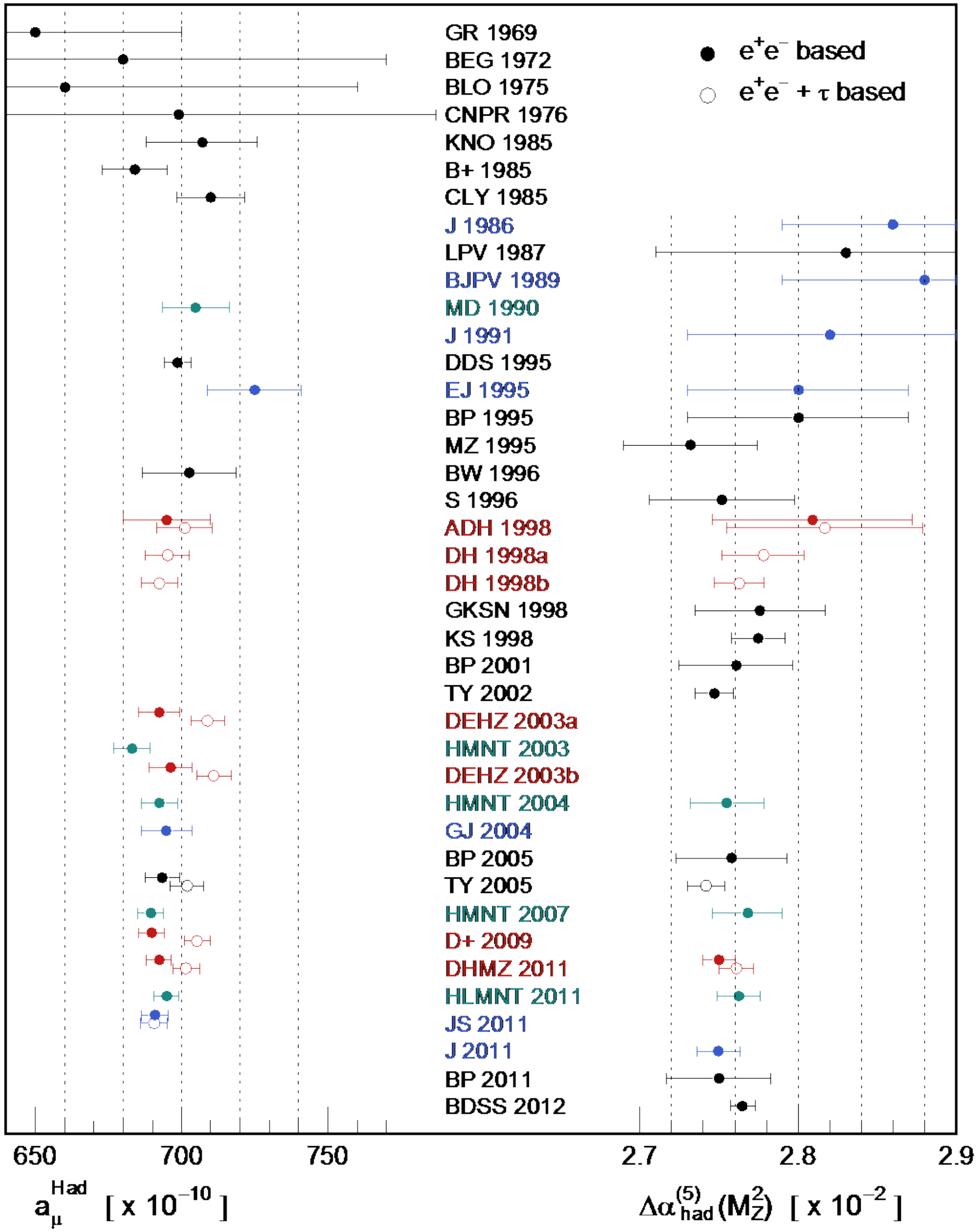
Introduction

Precise data now available since 2010 update

New evaluation of hadronic VP contribution to a_μ

Conclusion & perspectives

Introduction



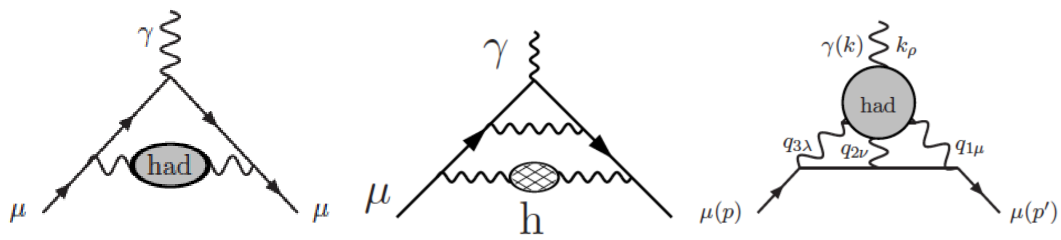
LO Hadronic Vacuum Polarization (HVP) being the most uncertain part for a_μ & $\Delta\alpha$ has been in focus over the last 5 decades.

The precision is steadily improving thanks to

- more precise/complete e+e- annihilation (& tau) data
- state of the art techniques for data interpolation, combination and error correlation treatment

Davier-Hoecker-Malaescu-Zhang, in "*Standard Theory Essays in the 60th Anniversary of CERN*"

Hadronic Contribution a_μ^{had}



Leading-Order Higher-Order Light-By-Light

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}$$
$$\simeq 700(\sim 7) - 9.79(0.09) + 10.5(2.6)$$
$$(\rightarrow\sim 4) \quad - 8.63\ (0.09)$$

Had NLO: Hagiwara et al., 2007
2011
Had NNLO: Kurz et al., 2014

Had LBL: Prades-de Rafael-Vainshtein, 2008

[In this talk all a_μ^{had} values are given in units of 10^{-10}]

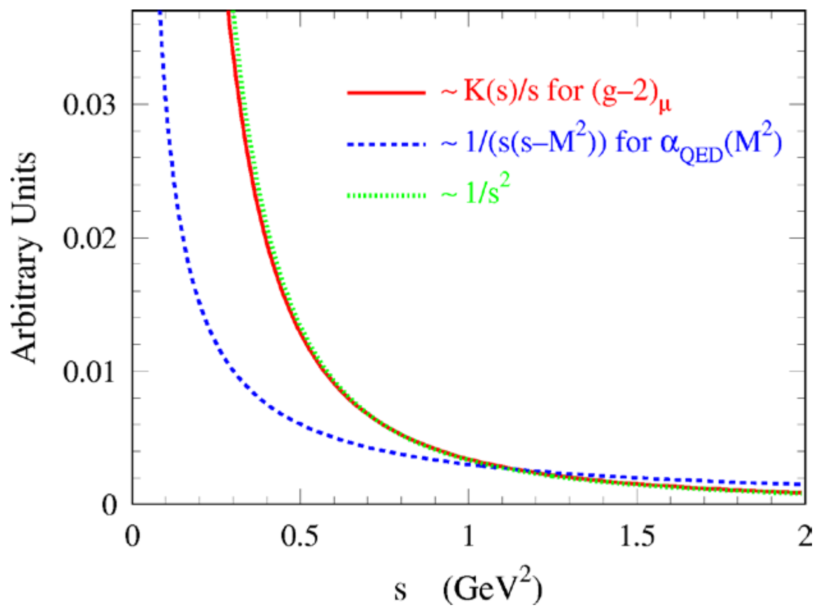
- Hadronic (q & g) loop contributions cannot reliably be calculated from perturbative QCD (pQCD)
  - → Use dispersion relations with e+e- annihilation data
  - ➔ Essentially model independent
- There are also lattice and model-based attempts

LO Hadronic Contribution a_μ^{had}

- We focus here on methods which provide precise & model independent estimate
- Using analyticity and unitarity one can write a dispersion relation based on low energy e^+e^- data to calculate the dominant LO contributions:

$$a_\mu^{\rm had} = \frac{\alpha^2(0)}{3\pi^2}\int_{4\pi_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)\,, \qquad R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Dispersion relation: Bouchiat and Michel, 1961



➔ The QED kernel K(s) has such an s dependence that low energy data contribute most

Brodsky, de Rafael, 1968

Our group contribution to LO Hadronic a_μ^{had}

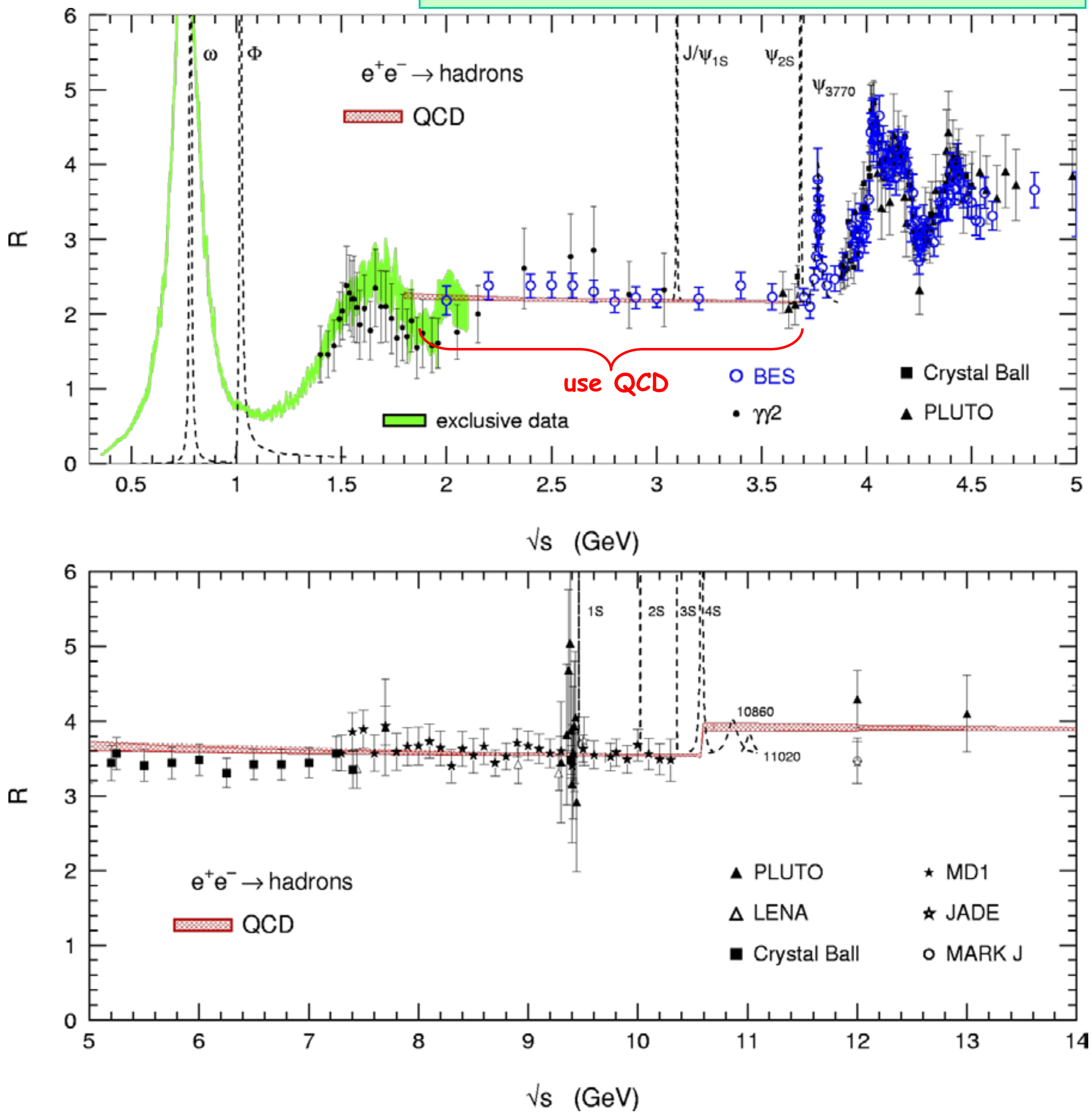
The dispersive approach follows the availability of trustful experimental data

- Use data on e+e- $\rightarrow$ hadrons and on $\tau \rightarrow \nu$ hadrons (CVC+isospin breaking), more precise then Alemany-Davier-Hoecker 1997
- Detailed QCD studies of τ decays (ALEPH) and tests of quark-hadron duality
  $\Rightarrow$ substitute pQCD above 1.8 GeV to less precise data Davier-Hoecker 1998,98
- Update with new data from VEPP-2M Davier-Eidelman-Hoecker-Zhang 2003,03
- Detailed study of isospin-breaking effects when using τ spectral functions
  Davier-Hoecker-Lopez-Malaescu-Mo-Toledo-Wang-Yuan-Zhang 2010
- Improvement of statistical and systematic tools (HVPTools) and update with new BABAR $\pi+\pi-$ data Davier-Hoecker-Malaescu-Yuan-Zhang 2010
- Global update Davier-Hoecker-Malaescu-Zhang 2011

- New update today, taking advantage of more complete data from BABAR, KLOE, BESIII, CMD3 and SND at VEPP-2000, KEDR

Input e^+e^- Data in Combination with pQCD

Davier-Hoecker-Zhang, RMP 78 (2006) 1043



- $[\pi^0\gamma\text{-}1.8\text{GeV}]$
  - sum about 22→37 exclusive channels
  - estimate unmeasured channels using isospin relations
- [1.8-3.7] GeV
  - good agreement between data and pQCD calculation; previous extensive QCD tests with τ data
    → use 4-loop pQCD
  - J/ψ, ψ(2s): Breit-Wigner integrals
- [3.7-5] GeV
  charm particle thresholds

  → use data
- >5GeV
  use 4-loop pQCD calculation

Combination procedure (1)

The integration of data points belonging to different experiments, with different within-experiment and inter-experiment correlated systematic errors, and with different data densities requires a **careful treatment**

It is mandatory to test the accurateness of the integration procedure in terms of central value and error using representative models with known truth.

DHMZ approach:

- Quadratic interpolation of the data points/bins for each experiment
- Local weighted average between interpolations performed in infinitesimal bins (1 MeV)
- Full covariance matrices: correlations between data points of an experiment (systematic errors), between experiments and channels (VP, luminosity, …)
- Consistent error propagation using pseudo experiments (toys)
- Possible bias tested in 2π channel using a GS model: negligible for quadratic interpolation, but not for linear model (trapezoidal rule)

Combination procedure (2)

Taking most important e+ e- → π+ π- channel as an example

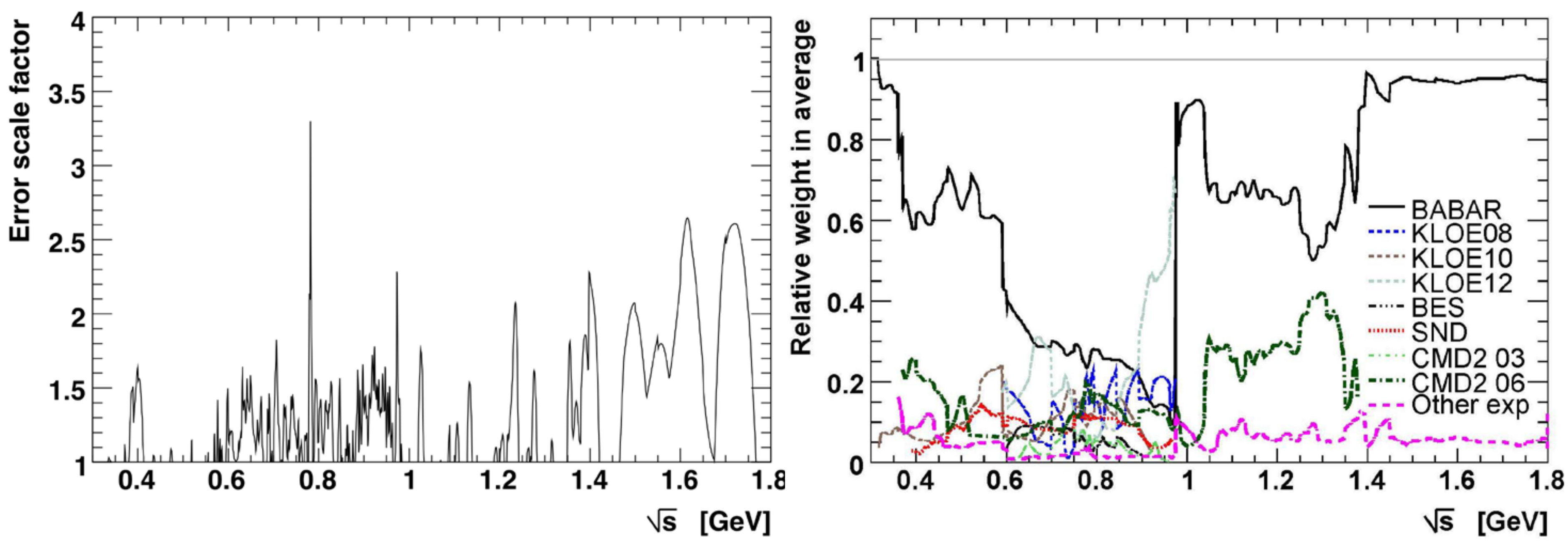
Incompatibilities between data points lead to error rescaling

Performed using PDG prescription

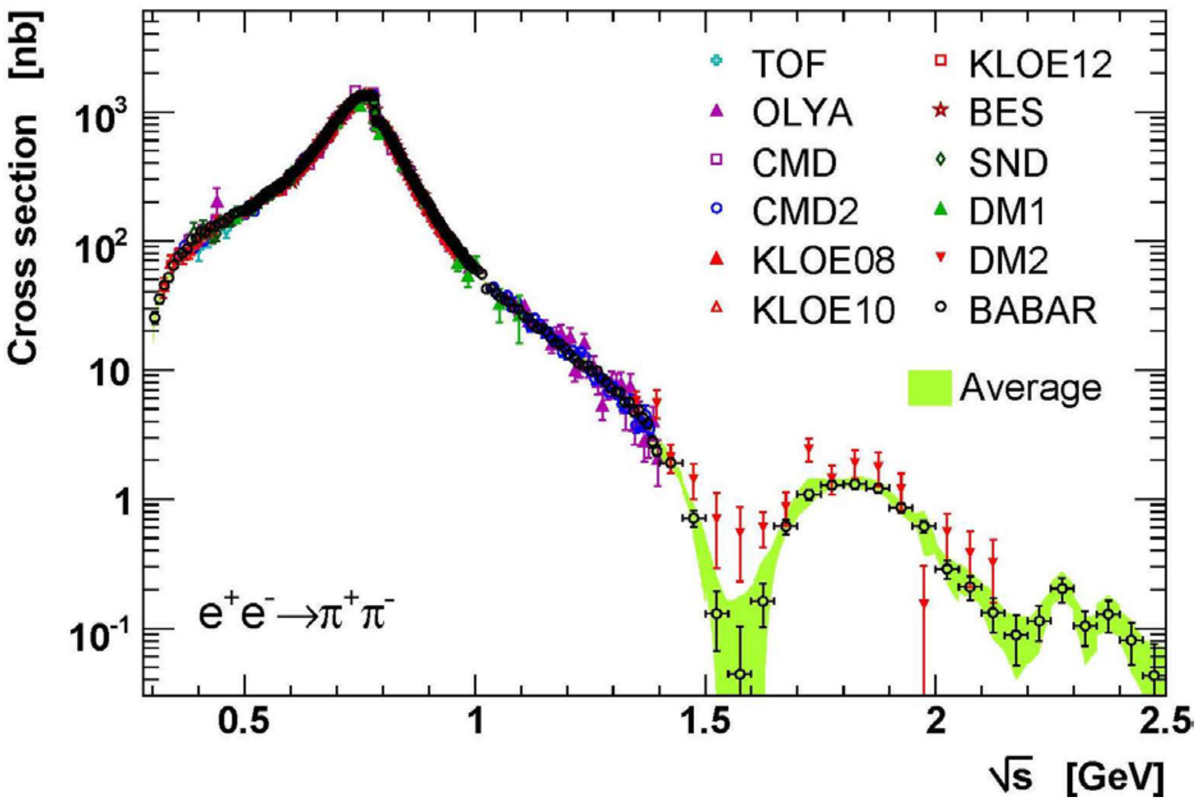
Major limiting factor for further improving the precision

Weights of experiments in average versus mass

BABAR dominates everywhere, except for KLOE 08 between 0.85 and 0.93 GeV



A Summary of $\pi^+\pi^-$ Input Data



2 main types of experiments:

1) Energy scan:

CMD(2)	(δ_{sy}: 0.8%)	2003
SND	(δ_{sy}: 1.5%)	2003
DM1, OLYA, TOF, DM2		

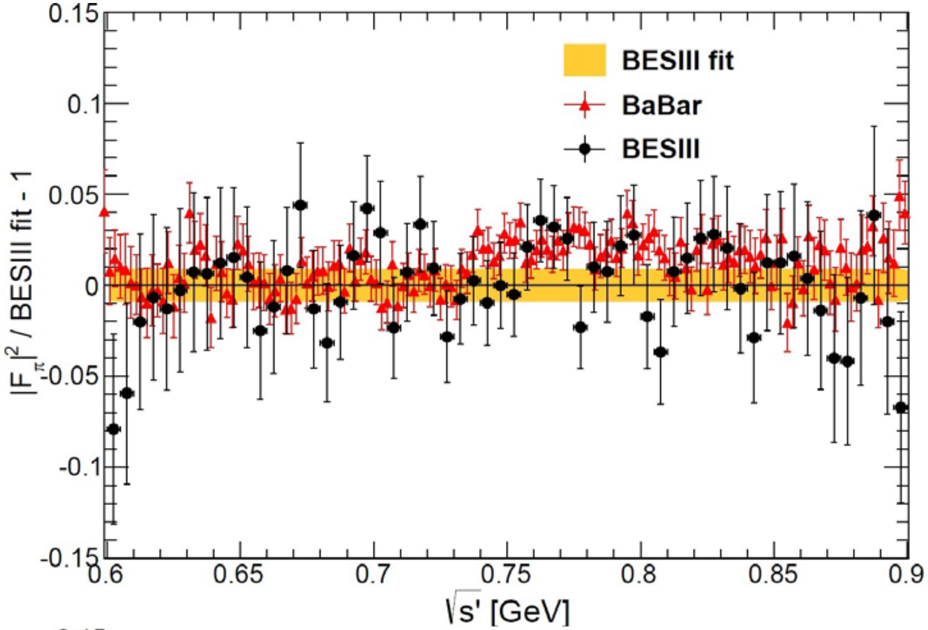
2) ISR based experiments:

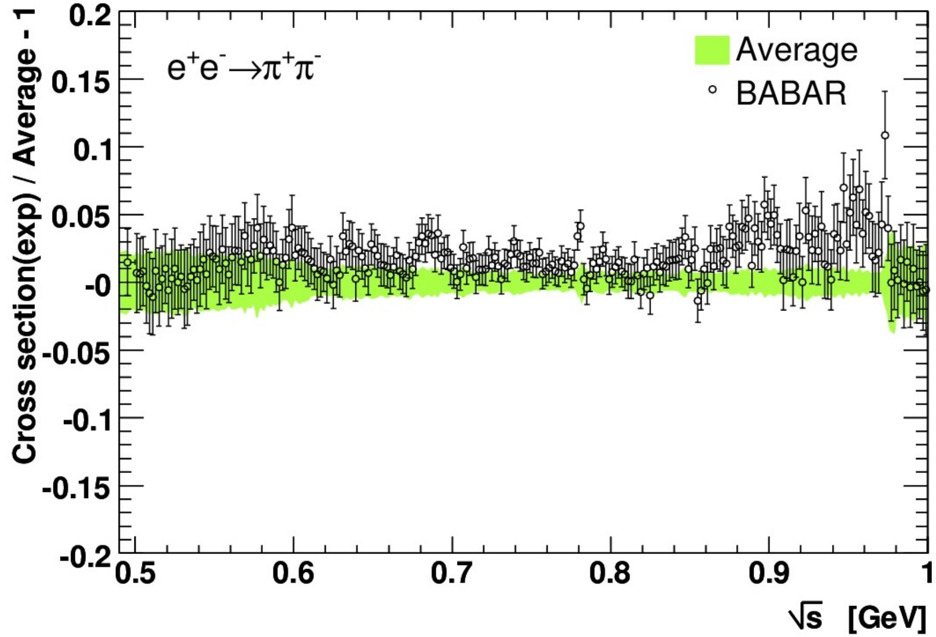
BABAR	(δ_{sy}: 0.5%)	2010
BESIII	(δ_{sy}: 0.9%)	2015
KLOE	(δ_{sy}: 0.8%)	2008
	(δ_{sy}: 1.4%)	2010
	(δ_{sy}: 0.8%)	2012
		correlated/2008

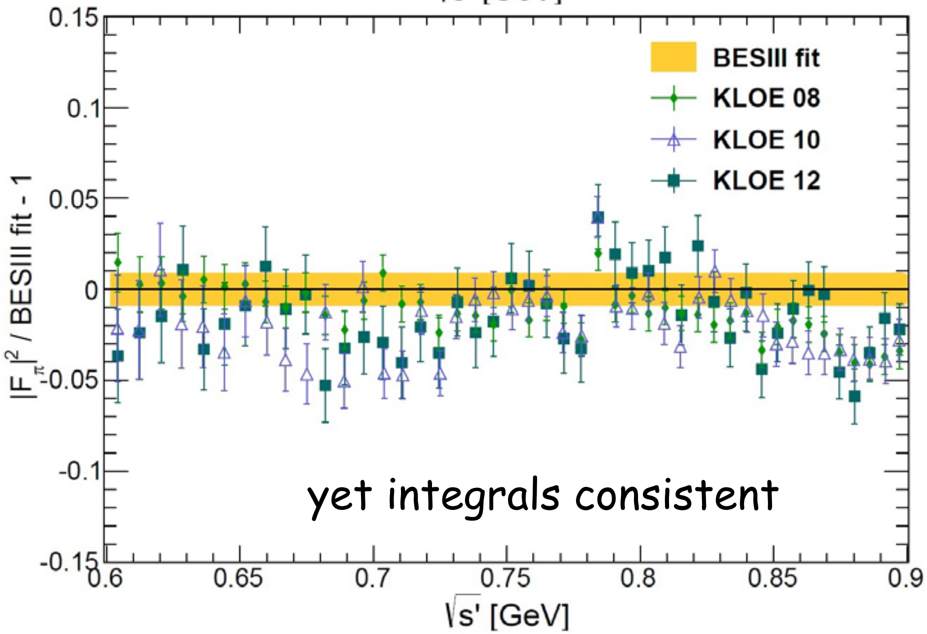
➔ BABAR covers the full mass range with high precision

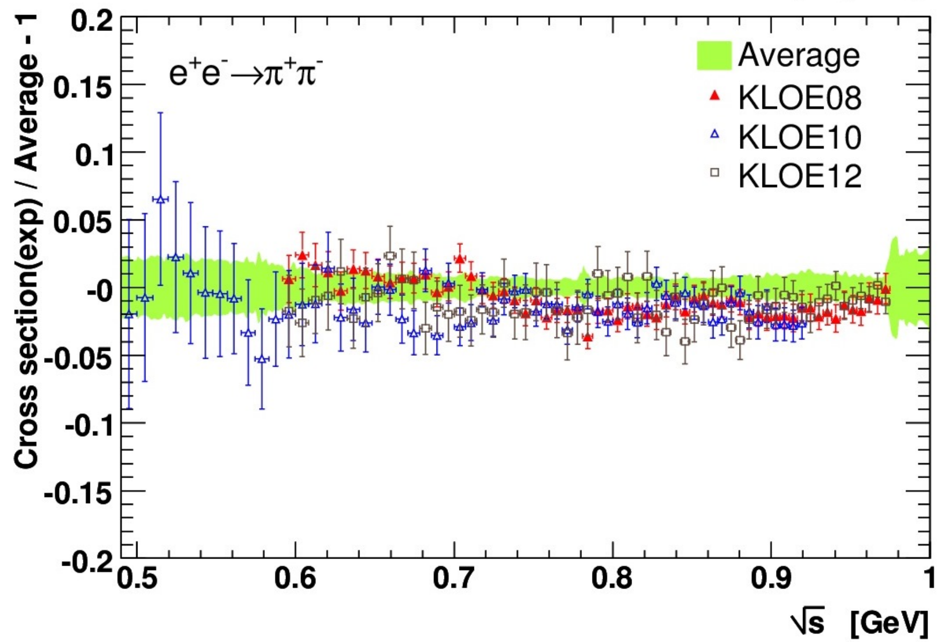
➔ Looks good, but discrepancy observed between BABAR and KLOE, a bit reduced with KLOE12

Close Comparison of $\pi^+\pi^-$ Cross Sections









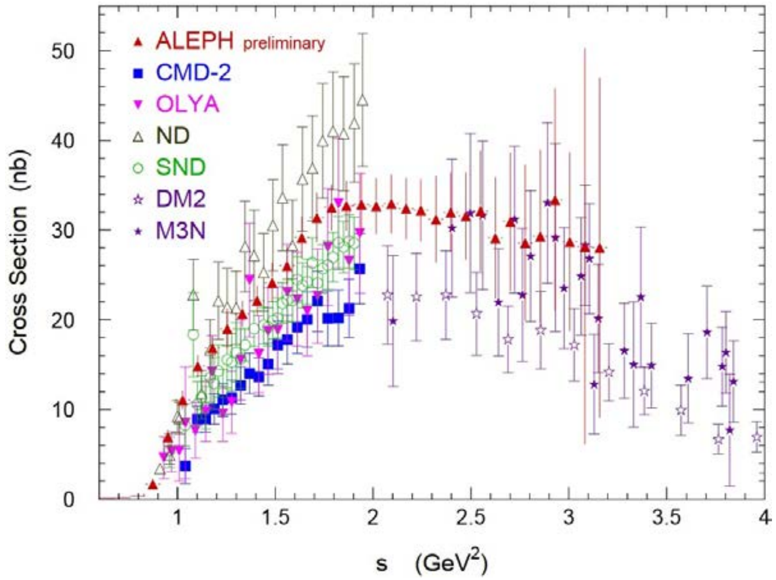
The $\pi^+\pi^-$ Channel: impact on $a_\mu^{had\ LO}$

- 2003: dominated by VEPP-2M data $508.2 \pm 5.2 \pm 2.7$ (5.9)
- 2010: including ISR KLOE 2008 and BABAR data
  $508.4 \pm 1.3 \pm 2.6$ (2.9)
- 2010: including KLOE 2010 $507.8 \pm 1.2 \pm 2.5 \pm 0.6$ (2.9)
  stat syst corr syst (tot)
- 2016: with KLOE 2012 and BESIII $506.9 \pm 1.1 \pm 2.2 \pm 0.7$ (2.5)

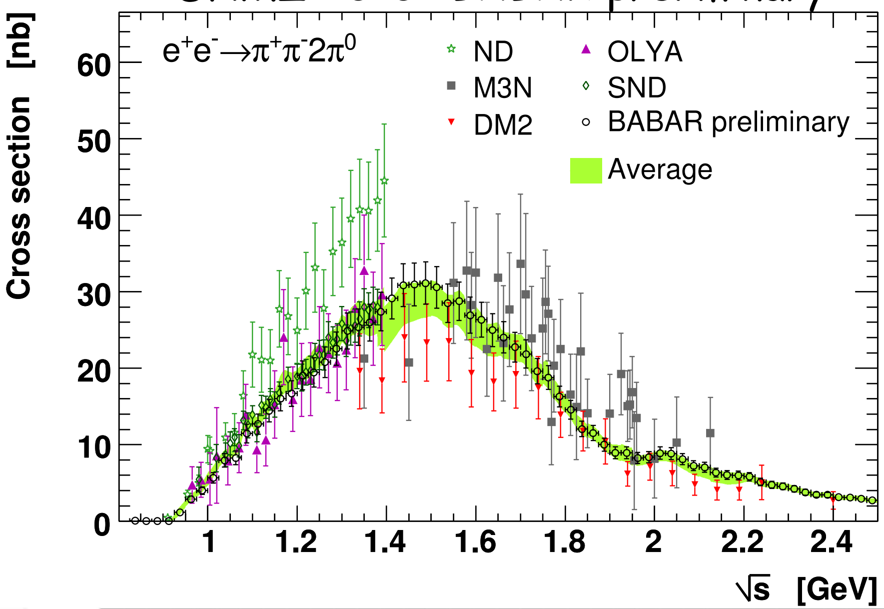
- Recall estimate 2013 using τ decays (ALEPH+OPAL+CLEO+Belle) $\nu\pi^\pm\pi^0$
  $516.2 \pm 2.9 \pm 2.0_{IB}$ (3.5) ($+2.2\sigma$)
- Some IB corrections still under debate (γ-ρ mixing)
- Because of the impressive progress of e+e- data, τ input relatively less precise and less reliable due to the IB uncertainties. Interesting by itself, but not to evaluate precise HVP contributions.

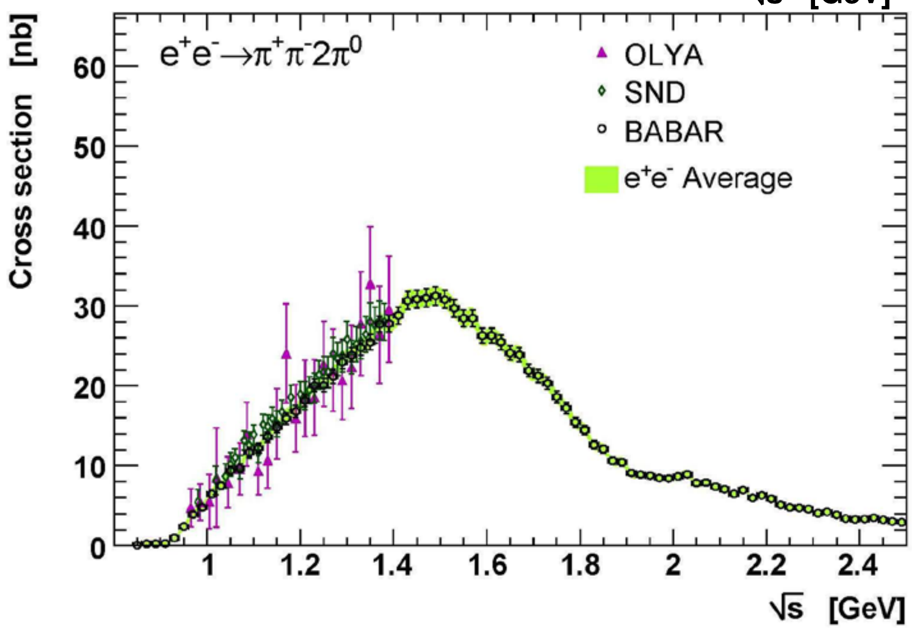
The $\pi^+\pi^-2\pi^0$ Channel: new data from BABAR

DEHZ 2003



DHMZ 2010 +BABAR preliminary

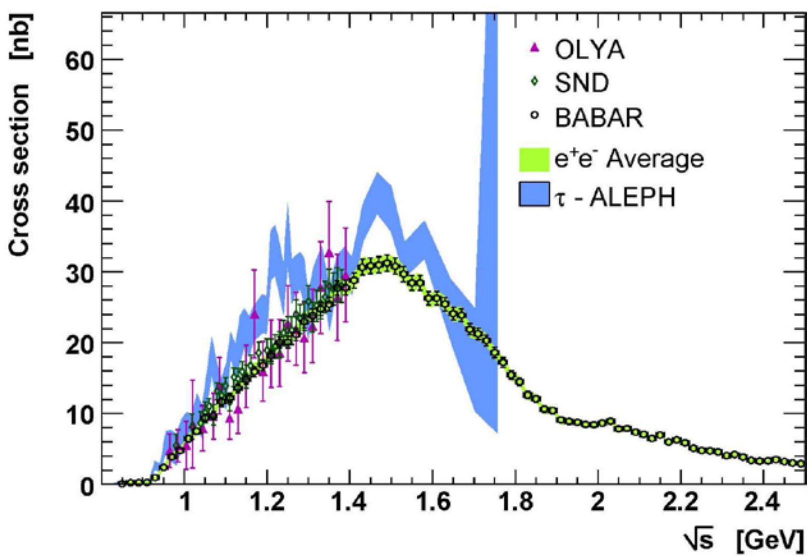




- August 2016: final BABAR results
- Full statistics (x2)
- Improved analysis
- Smaller systematic uncertainty (3.1%)
- Talk at Tau2016 (K. Griessinger)

The $\pi^+\pi^-2\pi^0$ Channel: impact on $a_\mu^{had\ LO}$

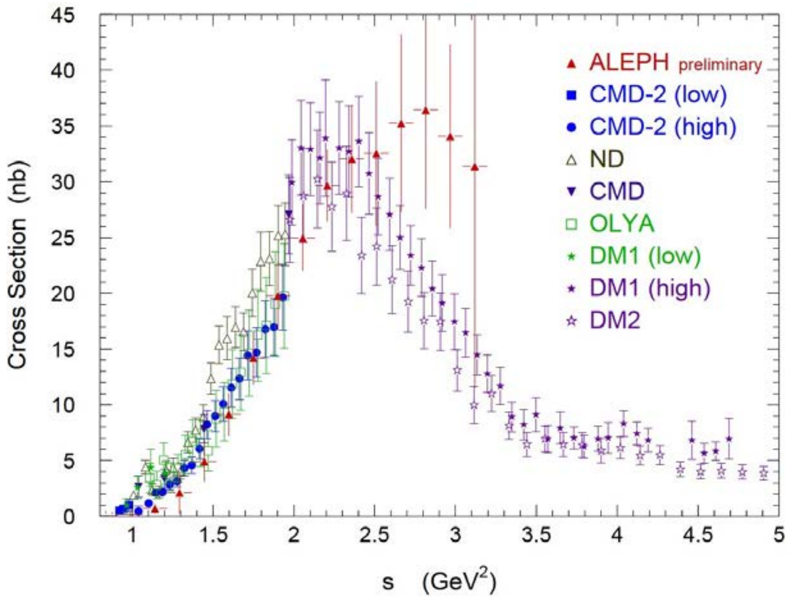
- 2003: VEPP-2M data $16.76 \pm 1.31 \pm 0.20$ (1.33)
- 2010: including preliminary ISR BABAR data with enlarged systematic uncertainty (A. Petzold, EPS-HEP 2007)
  $18.01 \pm 0.14 \pm 1.17 \pm 0.40$ (1.24)
- 2016: with BABAR with full statistics (preliminary)
  $18.16 \pm 0.06 \pm 0.49 \pm 0.27$ (0.56)
- Recall estimate using τ decays (ALEPH) $\nu\pi^\pm\pi^+\pi^-\pi^0$ and $\nu\pi^\pm\pi^0\pi^0\pi^0$
  $21.02 \pm 1.16 \pm 0.40_{IB}$ (1.22) ($+2.1\sigma$)



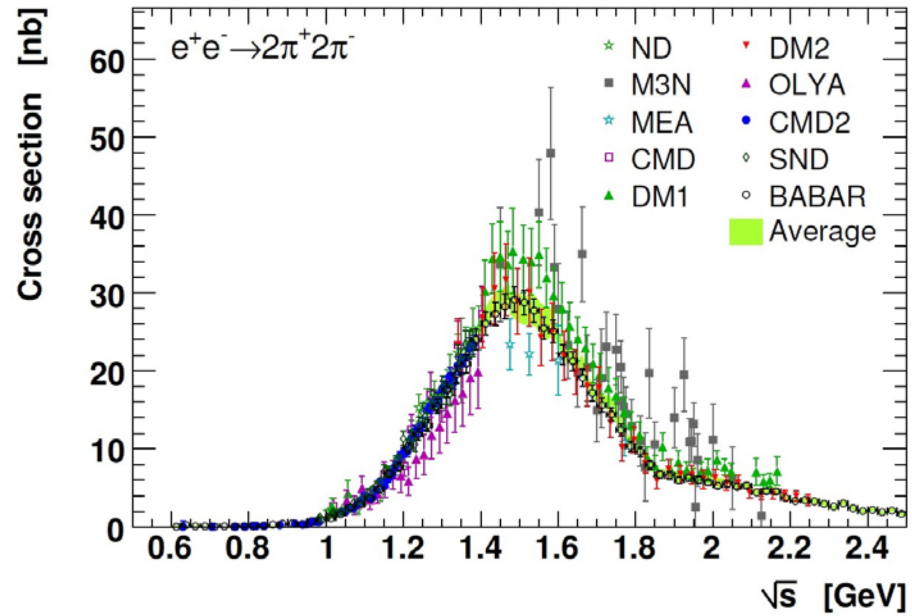
B factories ???

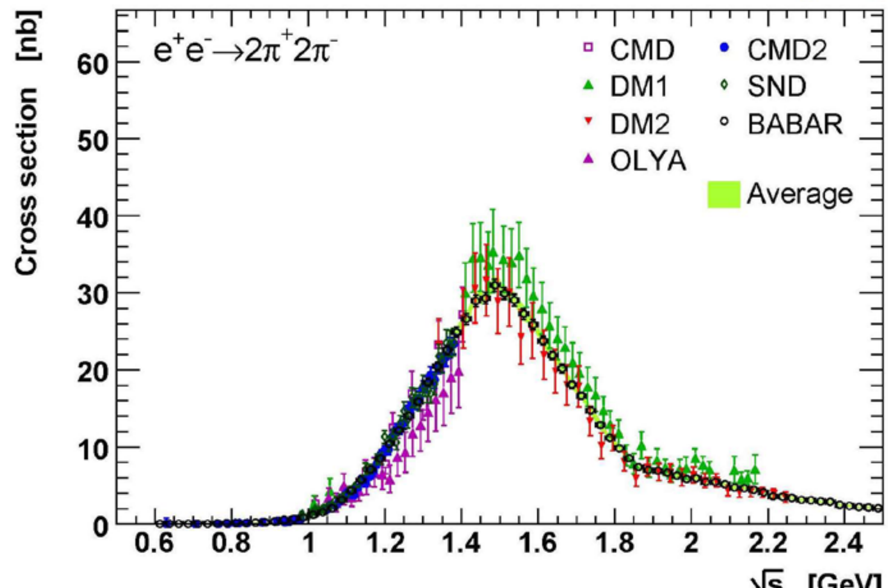
The 2π+2π- Channel: new data from BABAR

DEHZ 2003



DHMZ 2010 +BABAR 2005

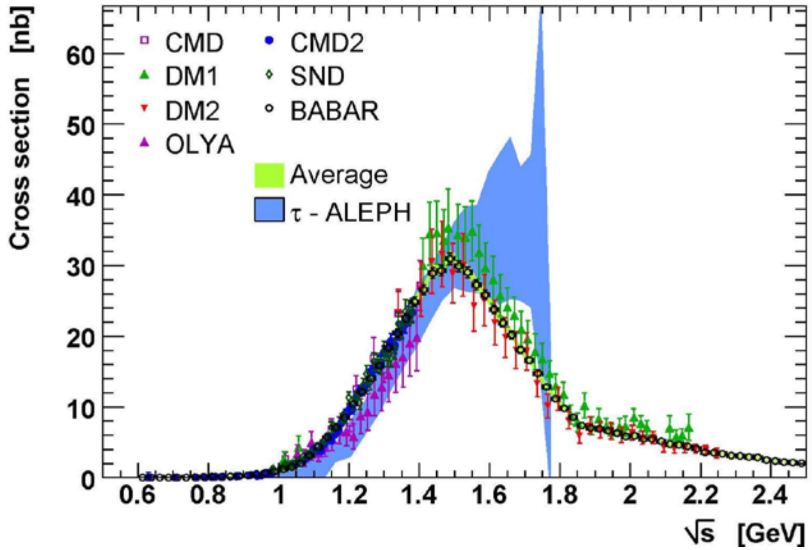




- 2012: final BABAR results
- Full statistics (x5)
- Improved analysis
- Smaller systematic uncertainty (2.4%)

The $2\pi^+2\pi^-$ Channel: impact on $a_\mu^{had\ LO}$

- 2003: VEPP-2M data $14.21 \pm 0.87 \pm 0.23$ (0.90)
- 2010: including BABAR (partial statistics)
  $13.35 \pm 0.10 \pm 0.43 \pm 0.29$ (0.53)
- 2016: with final BABAR with full statistics
  $13.70 \pm 0.03 \pm 0.28 \pm 0.13$ (0.31)
- Recall estimate using τ decays (ALEPH) to $\nu\pi^\pm\pi^0\pi^0\pi^0$
  $12.79 \pm 0.65 \pm 0.35_{IB}$ (0.74) (-1.1σ)

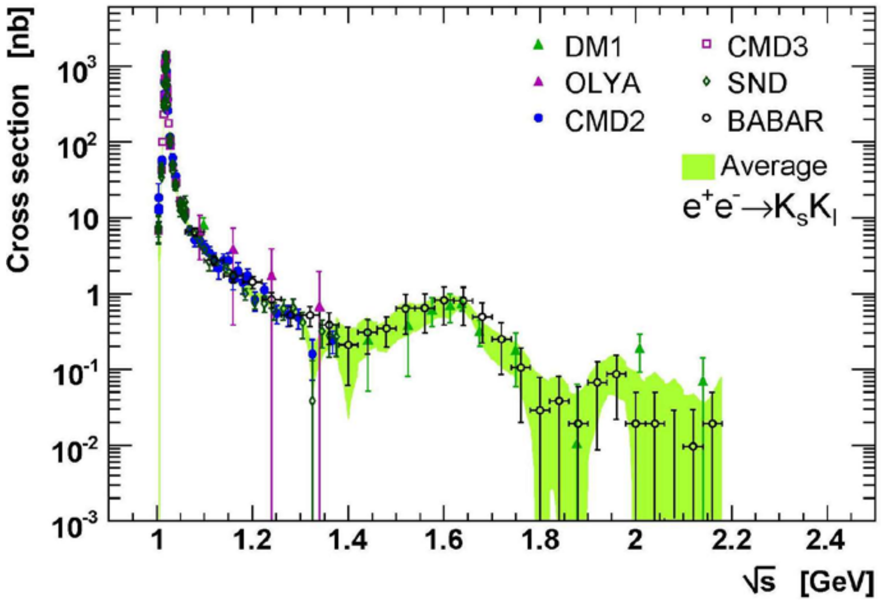


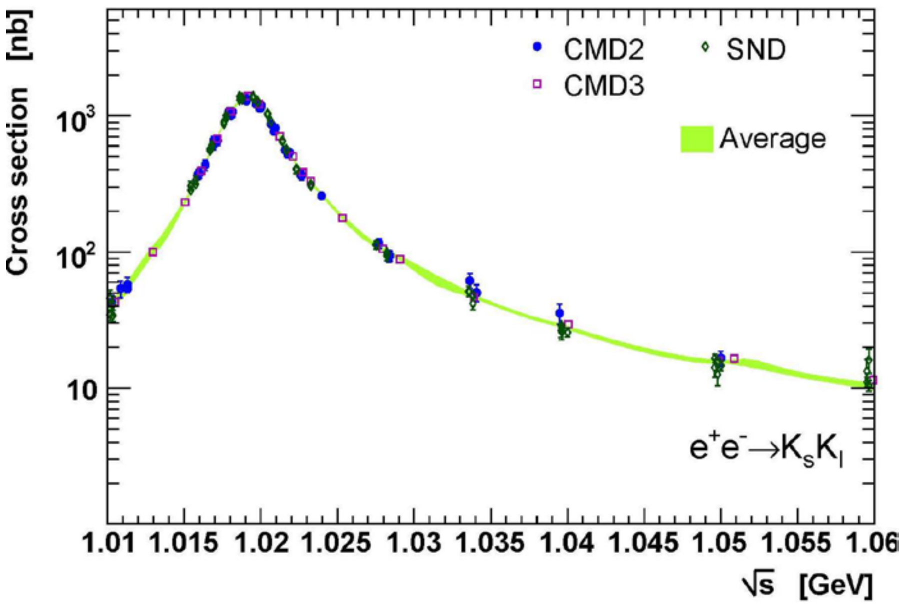
Since the τ estimates for 4π and $3\pi\pi0$ are anticorrelated because of $\tau\rightarrow\pi3\pi0$ it makes sense to combine them

e+e-	31.86 ± 0.64	
τ	33.81 ± 1.53	($+1.2\sigma$)

$\Rightarrow$ consistent, but τ value much less precise and affected by IB systematics

New Data on KsKl Channel: BABAR + CMD3





- BABAR 2015: with Kl + Ks detection from threshold to 2.2 GeV
- CMD3 2016: Ks detection ϕ region
- Compatibility of measurements in the ϕ region

peak integral $\propto \Gamma^{\phi}{}_{ee}\, BR_{KsKl}$ (keV)

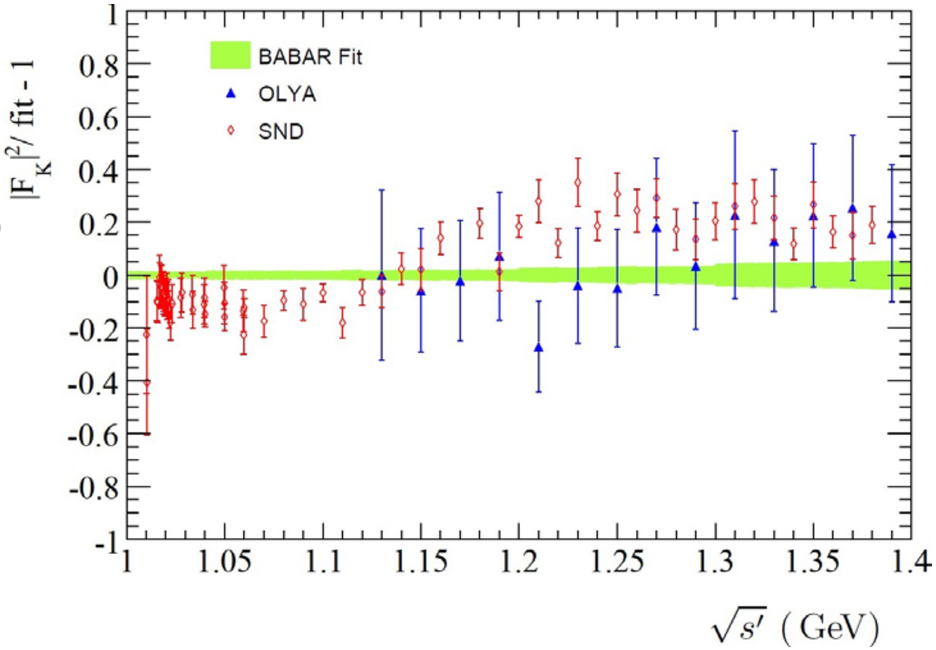
CMD2	0.427 ± 0.008
SND	0.438 ± 0.015
BABAR	0.420 ± 0.013
CMD3	0.428 ± 0.010

- $a_\mu^{had\ LO}$

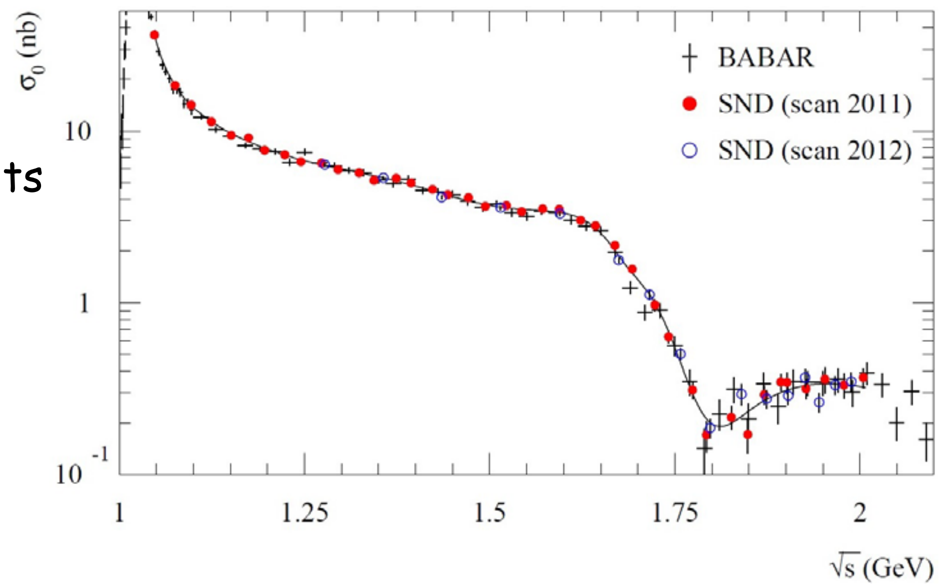
2010	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$
2016	$12.81 \pm 0.06 \pm 0.18 \pm 0.15$

New data on K^+K^- Channel: BABAR + SND

- BABAR 2013
- Mass spectrum from threshold to 5 GeV
- Low systematic uncertainty (0.7% - 3.7%)
- Discrepancy with SND at VEPP-2M



- SND 2016
- New data at VEPP-2000
- Disagreement with previous SND results
- Agreement with BABAR



Clouds over the $\phi \to K^+K^-$ contribution

- Discrepancy between BABAR and CMD2/SND (VEPP2M) for the $\phi \to K+K-$ cross section
- BABAR (syst=0.7%) higher by 5.1% wrt CMD2 (syst=2.2%) and by 9.6% wrt SND (syst=7.1%)
- Remark: difficult measurement in scan mode (very low energy kaons), not the case with ISR method (kaons boosted)

- Including the BABAR data in the new evaluation raises significantly the K+K- contribution to $a_\mu^{had\ LO}$ from $21.63 \pm 0.27 \pm 0.58 \pm 0.36$
  to $22.67 \pm 0.25 \pm 0.32 \pm 0.15$

- CMD3 is close to publish new data with 'large' peak cross section, claiming to have solved the long-standing isospin-breaking between K+K- and KsKl decays (preliminary results in arxiv:1603.03230)
- KsKl results already published (2016), but K+K- still being scrutinized
- Puzzling K+K- increase: ~11% wrt CMD2 (~5% wrt BABAR) !

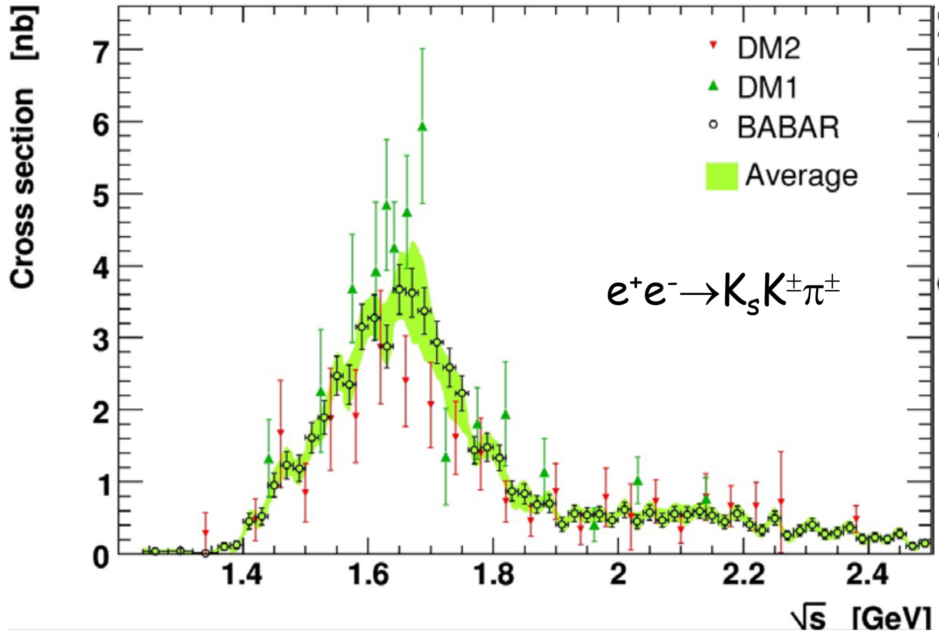
- To be watched carefully….

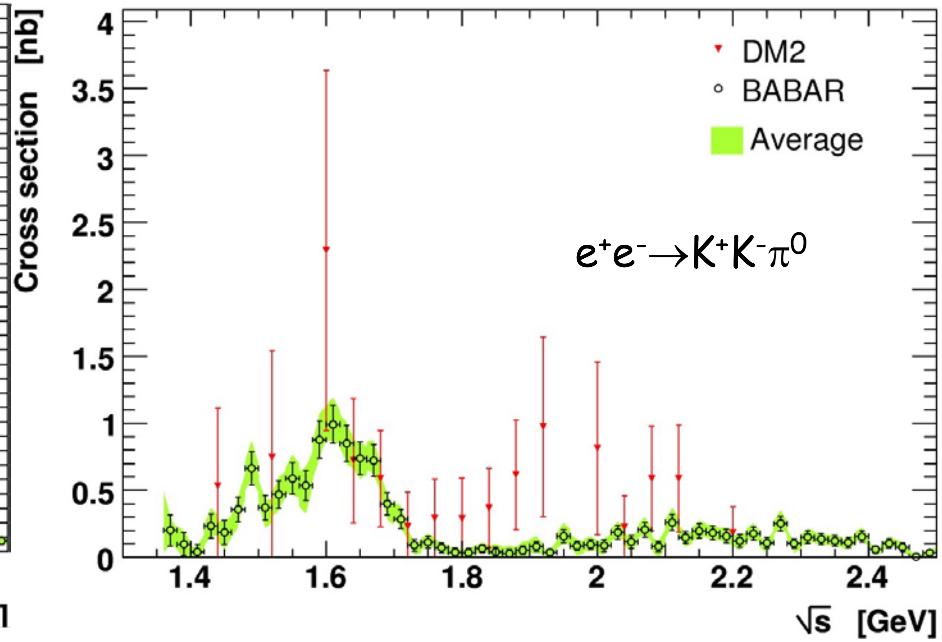
Data on K Kbar π now complete (BABAR)

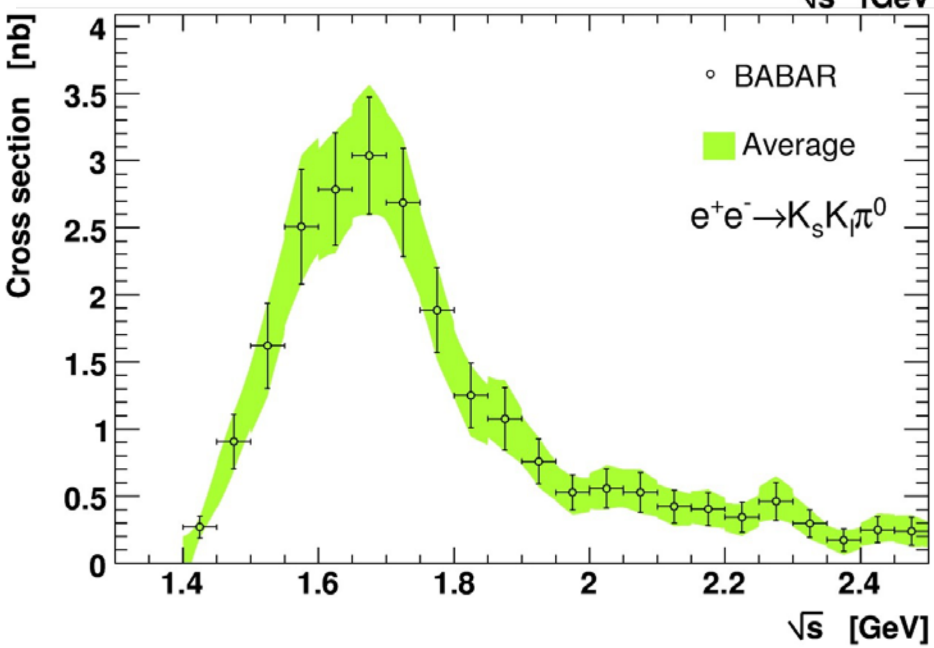
- 2010: only $K_sK^{\pm}\pi^{\pm}$ and $K^+K^-\pi^0$ measured (BABAR)
- Below 2 GeV $K^*(890)\pi$ dominance observed (with small $\phi\pi\pi$ contribution)
- Isospin relations used to estimate the full $KK\pi$ contribution
- 2016: last mode measured by BABAR: $K_sK_l\,\pi^0$ (talk here by W. Gradl)
- All channels measured
  $\Rightarrow$ no need to resort to estimate using isospin relations, only $K_s \to K_l$
- Precision on $a_\mu^{had\ LO}$ increased by ~20%
- Consistency with isospin estimate

2010:	2.39 ± 0.16	$K_sK^{\pm}\pi^{\pm}$ + isospin
2016:	2.45 ± 0.15	sum of the 3 channels

Data on K Kbar π now complete (BABAR)



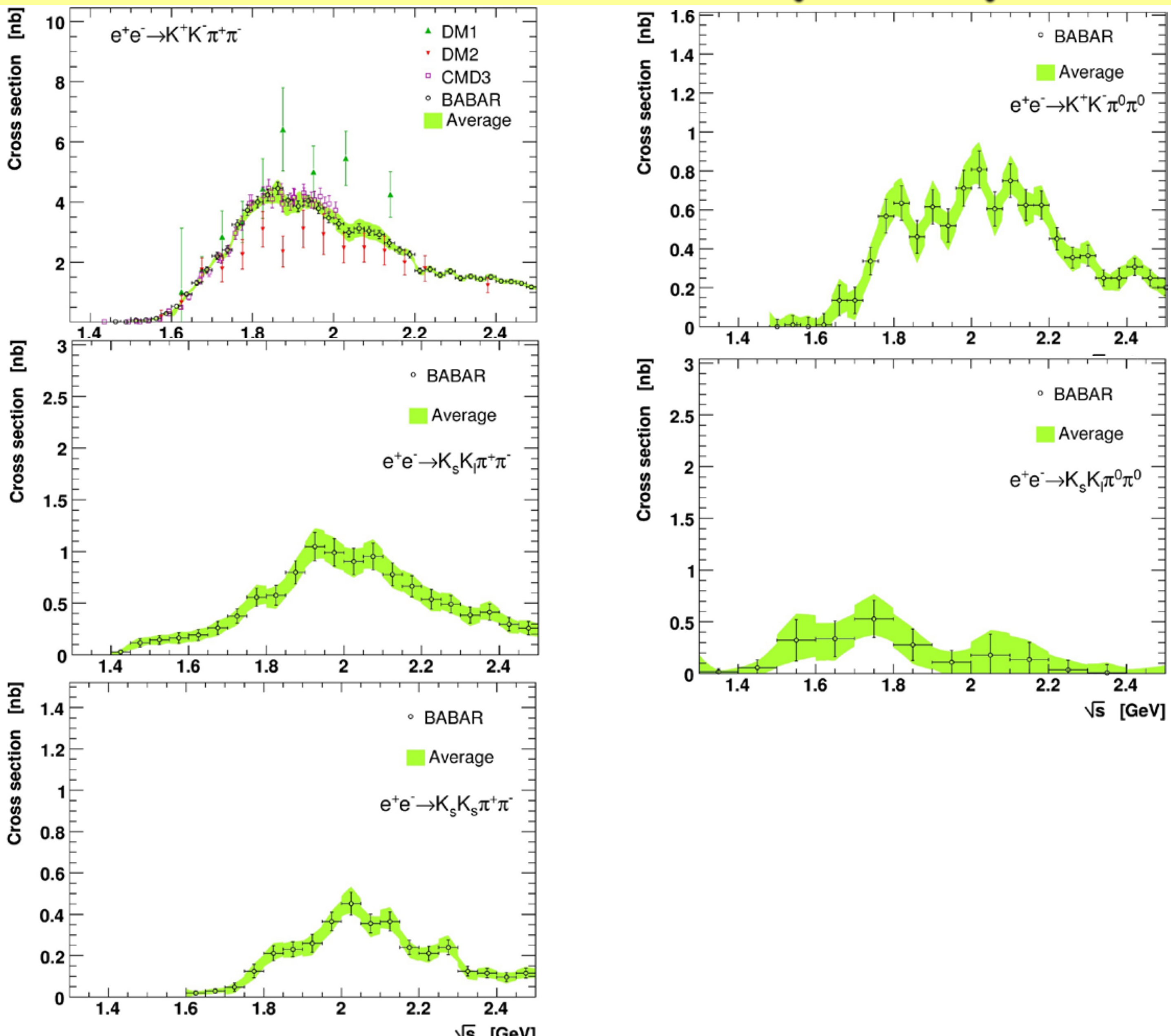




Data on K Kbar $\pi\pi$ nearly complete (BABAR)

- 2010: only $K^+K^-\pi^+\pi^-$ and $K^+K^-\pi^0\pi^0$ measured (BABAR)
- Below 2 GeV K*(890)Kπ dominance observed (with smaller KKρ and $\phi\pi\pi$ contributions)
- Isospin relations used + assumption on $(KK)^0\rho^0$ and $(KK)^{\pm}\rho^{\pm}$ (with 100% uncertainty) to estimate the full KK$\pi\pi$ contribution
- 2016: new modes measured by BABAR: $K_sK_l\ \pi^+\pi^-$, $K_sK_s\ \pi^+\pi^-$, $K_sK_l\ \pi^0\pi^0$
  $K_sK^{\pm}\pi^{\pm}\pi^0$ available shortly
  full data statistics for all modes
  also data from CMD3 in agreement with BABAR
- All channels measured
  $\Rightarrow$ no need to resort to estimates using isospin relations, only $K_s \rightarrow K_l$
- Precision on $a_\mu^{had\ LO}$ will be increased by ~ 6 (0.39 $\rightarrow$ 0.06)
- Keep 2010 prescription before public release of the last channel

Data on K Kbar ππ nearly complete (BABAR)



a_μ Tau 2016 preliminary

$a_\mu^{\text{had LO}}$

DEHZ 2003 $696.3 \pm 6.2_{exp} \pm 3.6_{rad}$ (7.1_{tot})

DHMZ 2011 $692.3 \pm 1.4_{stat} \pm 3.1_{syst} \pm 2.4_{corrsyst} \pm 0.2_{\psi} \pm 0.3_{QCD}$ (4.2_{tot})

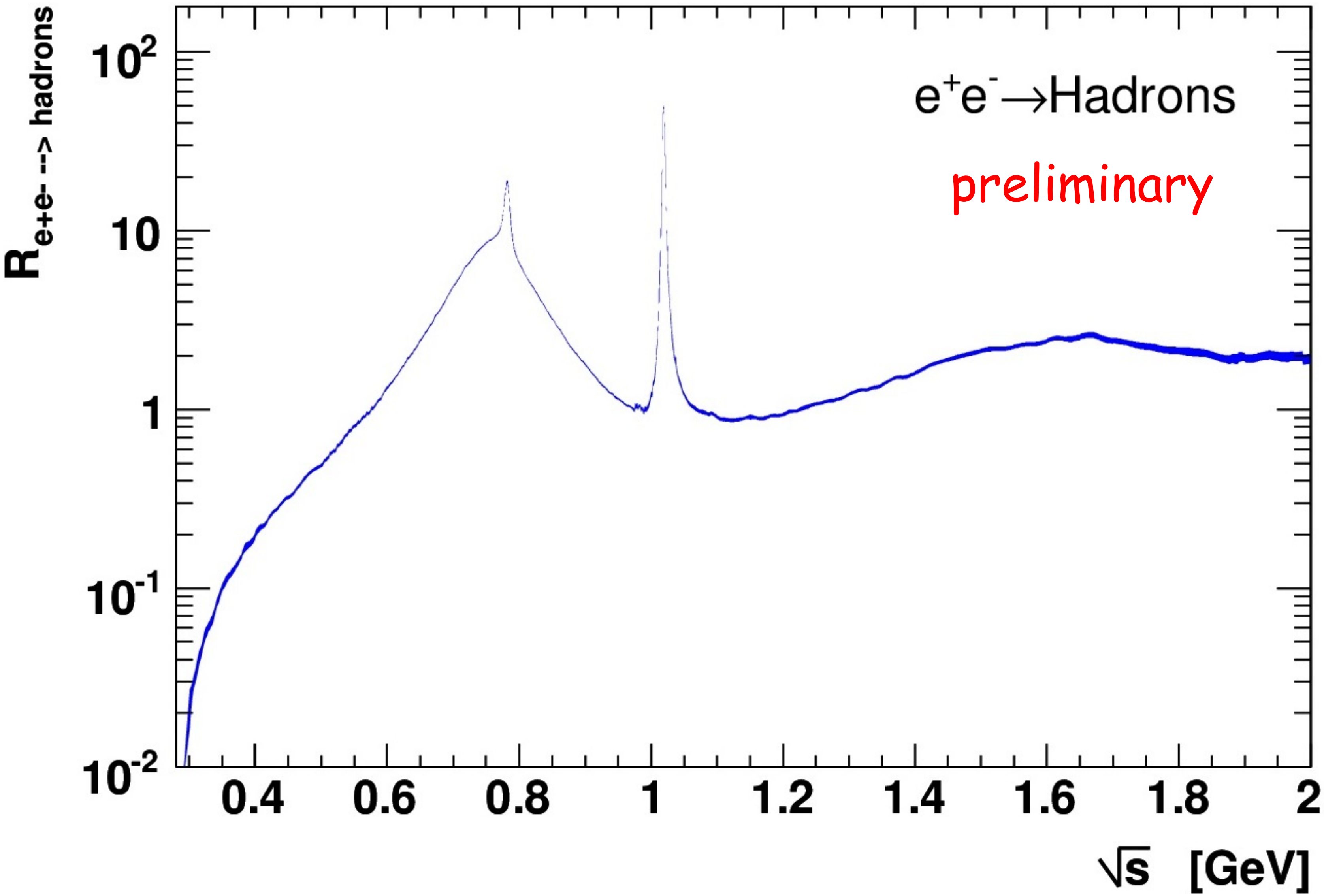
DHMZ 2016 $692.8 \pm 1.2_{stat} \pm 2.6_{syst} \pm 1.6_{corrsyst} \pm 0.1_{\psi} \pm 0.3_{QCD}$ (3.3_{tot})

a_μ

QED	11658471.885	+- 0.004
EW	15.4	+- 0.1
had LBL	10.5	+- 2.6
had LO	692.8	+- 3.3
had NLO	-9.87	+- 0.09
had NNLO	1.24	+- 0.01
prediction	11659181.9	+- 4.2
exp BNL	11659208.9	+- 6.3

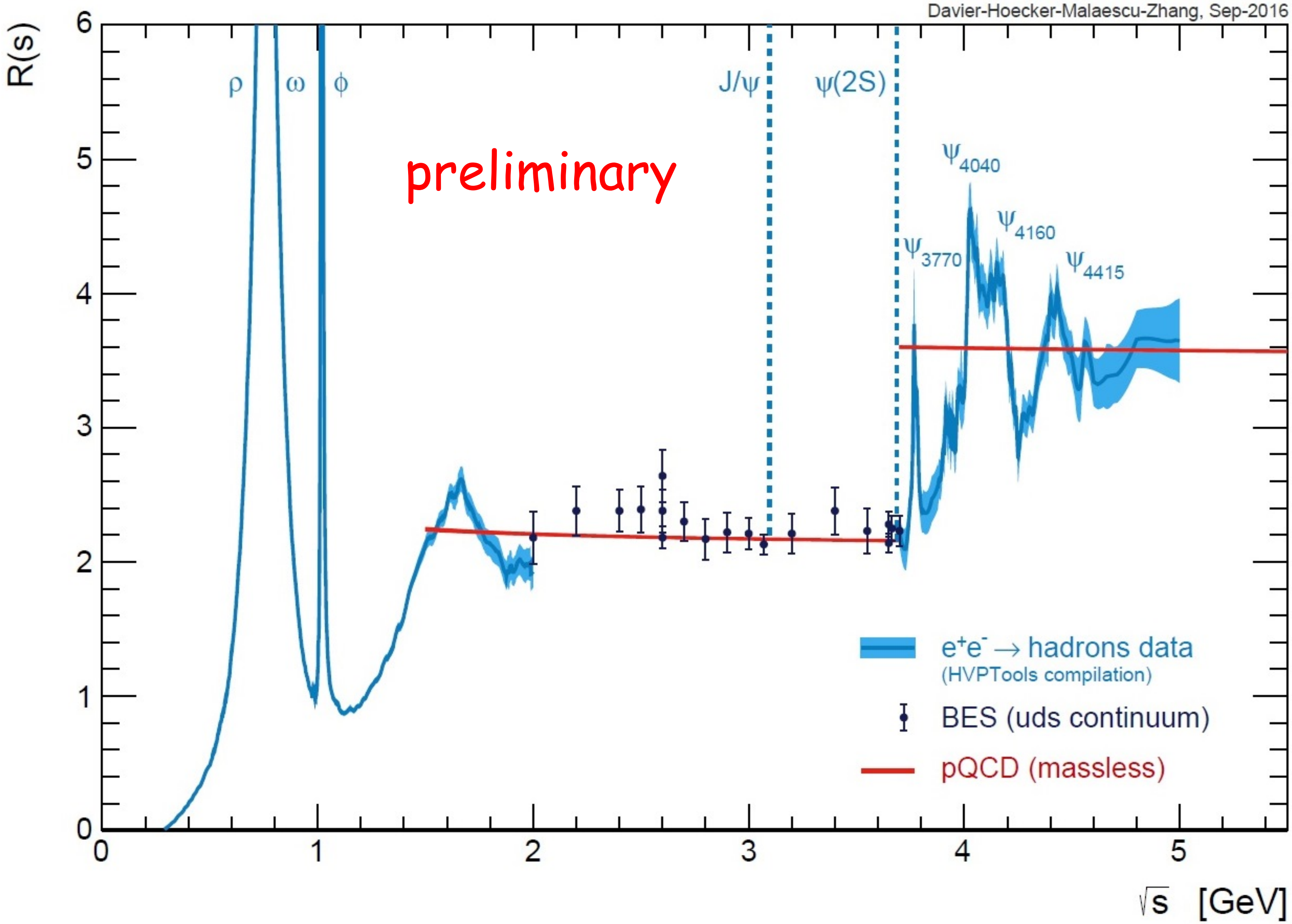
deviation	27.0	+- 7.6	3.6σ

R(s) 2016



$R_{e+e- \to hadrons}$
10^2
10
1
10^{-1}
10^{-2}
$e^+e^- \to$Hadrons
preliminary
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$\sqrt{s}$ [GeV]

R(s) 2016


Davier-Hoecker-Malaescu-Zhang, Sep-2016
R(s)
ρ
ω
ϕ
J/ψ
ψ(2S)
preliminary
ψ4040
ψ4160
ψ3770
ψ4415
e+e- → hadrons data
(HVPTools compilation)
BES (uds continuum)
pQCD (massless)
√s [GeV]
0
1
2
3
4
5
6

Perspectives

- KLOE-BABAR discrepancy for $\pi^+\pi^-$ still unresolved
- New higher precision $\pi^+\pi^-$ measurements needed:
  CMD3 in progress
  BABAR new independent analysis underway
  0.3% systematic uncertainty looks reachable
- $\pi^+\pi^-\pi^0$ could be improved for ω and ϕ resonances
- K+K- results from CMD2/3 to be scrutinized and understood
- Important to cross-compare BABAR ISR and forthcoming CMD3/SND scan results in 1-2 GeV range; so far excellent agreement
- More at BESIII and Belle-II ?
- A factor of 2 in precision has been obtained in the last 10 years; now comparable (3.3) to estimated systematic uncertainty of had-LBL (2.6)

- Lattice calculations are progressing (talk by B. Chakraborty)

- New approach: obtaining $a_\mu^{had\ LO}$ from a dispersion integral in the spacelike region by an ultrahigh precision measurement of Bhabha differential cross section to obtain $\Delta\alpha_{had}(Q^2)$ Carloni Calame et al. 2015
  or μ e elastic scattering (Venanzoni et al., workshop 'Physics beyond colliders', CERN, Sept. 7 2016)

Conclusion

- More precise/complete e+e- input data +
  Dedicated data treatment and uncertainty correlation studies

  ➔ steadily improving precision for the LO HVP evaluation
  based on dispersion relations (20% since 2011)

- The long standing e+e-/tau discrepancy nearly resolved, but
  additional systematics from isospin-breaking corrections

- Improvements are expected in the next years with
  - new $\pi^+\pi^-$ analyses underway (BABAR, CMD3)
  - collecting more data on multihadronic channels at VEPP-2000

- Precision of dispersive approach for had LO should be at the same level as had LBL or better when FNAL experiment gets its final result $\Rightarrow$ if unchanged, deviation from SM will be at $>7\sigma$