New ISR Cross Section Results on $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\eta$ from *BABAR*

Konrad Griessinger
on behalf of the *BABAR* Collaboration

Institute for Nuclear Physics
Mainz University

International Workshop on Tau Lepton Physics,
September 2016

Outline

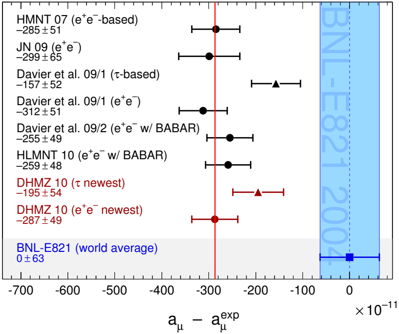
1. Introduction
2. Cross section $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$
3. Cross section $e^+e^- \to \pi^+\pi^-\eta$
4. Summary

The contributions to a_μ and its uncertainty

$$\vec{\mu} = g\frac{e}{2m}\vec{s}$$
$$(g_\mu - 2)/2 =: a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{hadronic}}$$

Interaction	Contribution $[\cdot 10^{-11}]$	Uncertainty $[\cdot 10^{-11}]$
QED [1]	116 584 718.951	0.080
EW [7]	153.6	1
hadronic VP [5, 11]	6837	43
hadronic LbL [10, 2]	119	41
total theory	116 591 828	60
E821 experiment [12]	116 592 089	63
deviation exp-theo	261	87

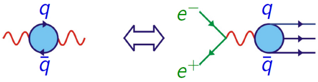
Discrepancy between SM prediction and direct measurement from Eur.Phys.J., C71:1515, 2011 [5].



Just a fluctuation?

3σ effect, thus reduction of uncertainties necessary!

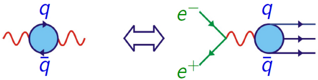
Connection between a_μ and σ_{had}



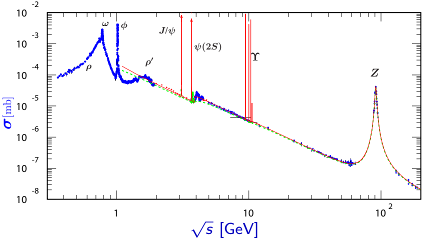
$$a_\mu^{\mathrm{had}} \approx \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} K_\mu(s) \cdot \sigma_{e^+e^-\to\mathrm{had}}(s)\,\mathrm{d}s$$

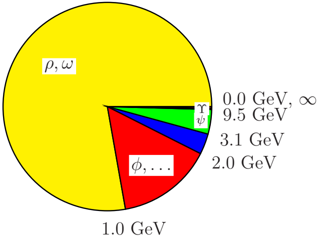
Kernel function ($K_\mu(s)$) — cross section ($\sigma_{e^+e^-\to\mathrm{had}}(s)$)

Connection between a_μ and $\sigma_{\rm had}$



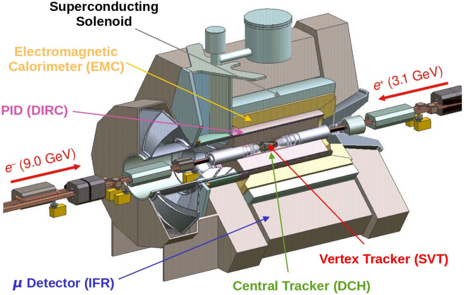
$$a_\mu^{\rm had} \approx \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} K_\mu(s)\cdot\sigma_{e^+e^-\to{\rm had}}(s){\rm d}s$$





$\sigma_{\rm had}$ (left) [12] and relative contributions to a_μ^{had} (right) [9].

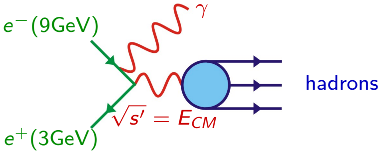
The *BABAR* Experiment

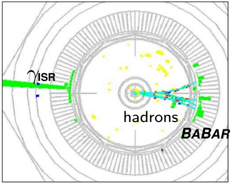


Experimental specifications

Energy: $\sqrt{s} \approx 10.58\,\text{GeV}$ $(E_{e^-} \approx 9.0\,\text{GeV}, E_{e^+} \approx 3.1\,\text{GeV})$,
Luminosity: $\mathcal{L} \approx 500\,\text{fb}^{-1}$ $(\Upsilon(4S))$

Initial State Radiation (ISR) events at *BABAR*

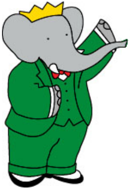


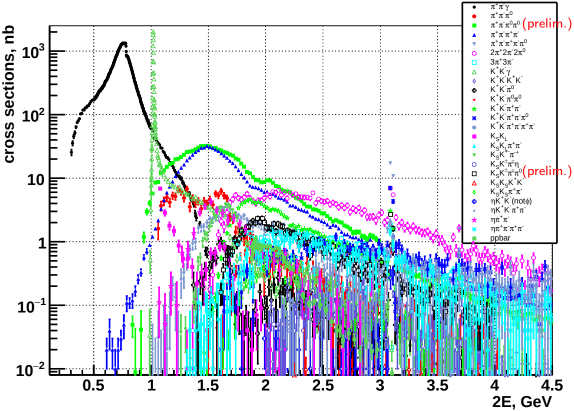


ISR selection

- Detected high energy photon: $E_\gamma > 3\text{GeV}$
  $\rightarrow$ defines E_{CM} & provides strong background rejection
- Event topology: γ_{ISR} back-to-back to hadrons
  $\rightarrow$ high acceptance
- Kinematic fit including γ_{ISR}
  $\rightarrow$ very good energy resolution $(4 - 15\text{MeV})$
- e^+e^--boost into the laboratory reference frame
  $\rightarrow$ high efficiency at production threshold of hadronic system
- Continuous measurement from threshold to $\sim$5GeV
  $\rightarrow$ provides common, consistent systematic uncertainties

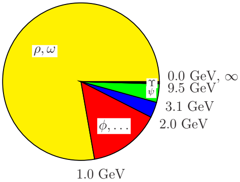
Most important channels

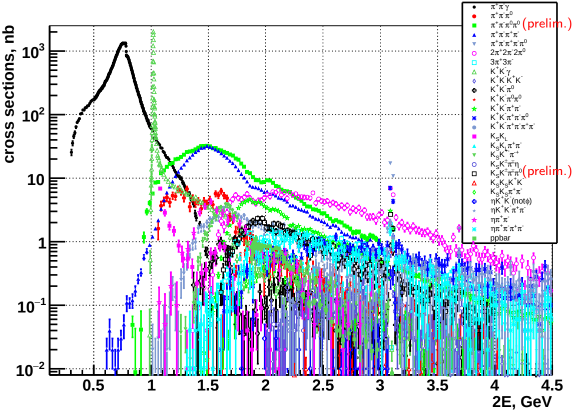




Cross Sections of the single channels measured at *BABAR* (Courtesy of F. Ignatov).

Most important channels

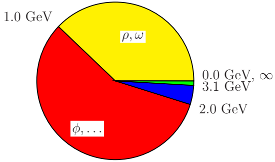


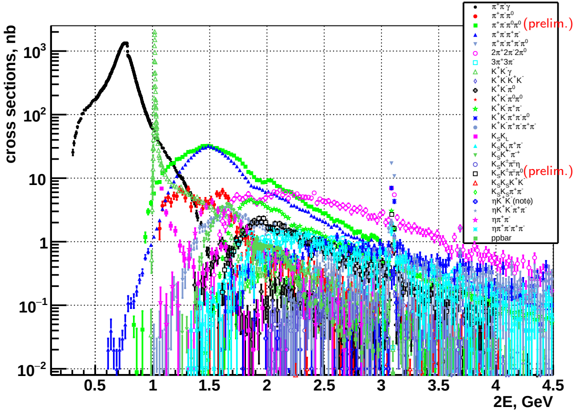


Right panel: Cross Sect. of single channels (Courtesy of F. Ignatov).

Left panel: Relative contributions to a_μ^{had} (from Nuovo Cim., C034S1:31-40, 2011 [9]).

Most important channels





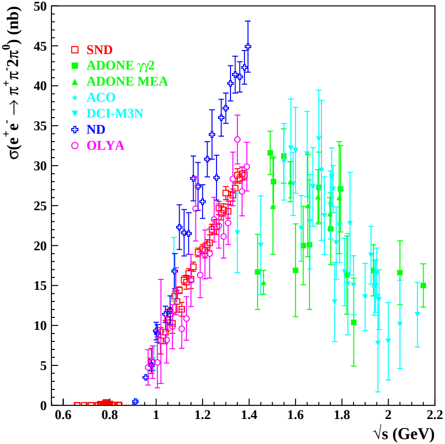
Right panel: Cross Sect. of single channels (Courtesy of F. Ignatov).

Left panel: Relative contributions to δa_{μ}^{had} (from Nuovo Cim., C034S1:31-40, 2011 [9]).

$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$$

PRELIMINARY

$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ world data set before *BABAR*

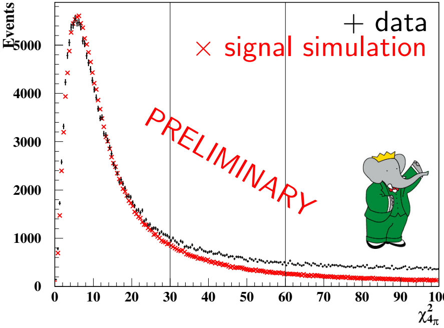


- limited precision
- big disagreement between experiments
- small energy ranges

Event Selection

Full $\Upsilon(4S)$ on peak data set of 454.4 fb^{-1}

$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{\rm ISR}$$



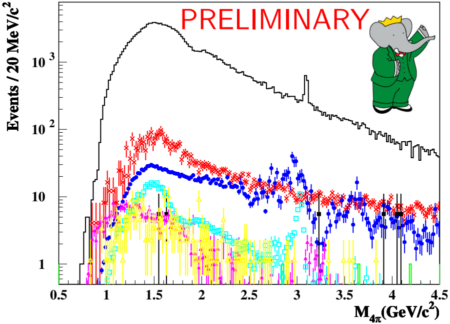
Main Selection Requirements

- exactly 2 charged tracks
- ≥ 5 photons
- $E_\gamma^{\rm lab} > 0.05$ GeV
- $|M_{\pi^0}^{\rm reco} - M_{\pi^0}^{\rm PDG}| < 0.03$ GeV
- $E_{\gamma_{\rm ISR}} > 3$ GeV
- 6C kinematic fit: $\chi^2_{2\pi 2\pi^0\gamma} < 30$
- reject other hypotheses
- Muon and Kaon PID

Background subtraction

$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{\rm ISR}$$

Simulated background channels:

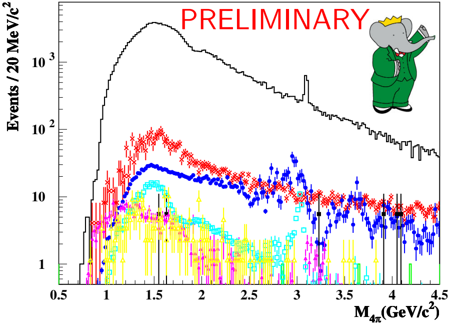


$q\bar{q}$, 3π, $4\pi 2\pi^0$, $K_S K\pi$, $K^+K^-2\pi^0$, $\tau\tau$, $\pi^+\pi^-3\pi^0$

Background subtraction

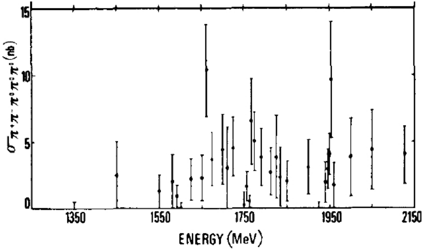
$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{\text{ISR}}$$

Simulated background channels:



$q\bar{q}$, 3π, $4\pi 2\pi^0$, $K_S K\pi$, $K^+K^-2\pi^0$, $\tau\tau$, $\pi^+\pi^-3\pi^0$

Main issue: background from $e^+e^- \to \pi^+\pi^-3\pi^0$

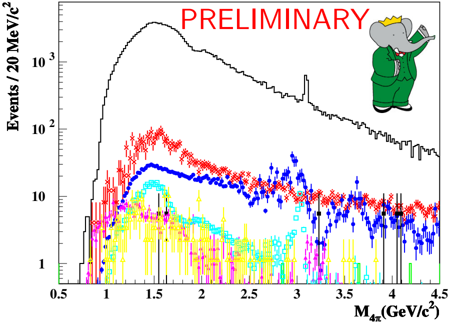


Only little data [3] and no full simulation available

Background subtraction

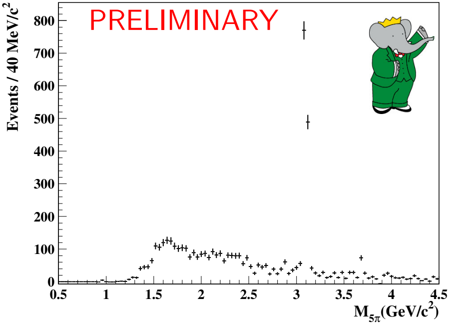
$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{\rm ISR}$$

Simulated background channels:



$q\bar{q}$, 3π, $4\pi2\pi^0$, $K_S K\pi$, $K^+K^-2\pi^0$, $\tau\tau$, $\pi^+\pi^-3\pi^0$

Main issue: background from $e^+e^- \to \pi^+\pi^-3\pi^0$

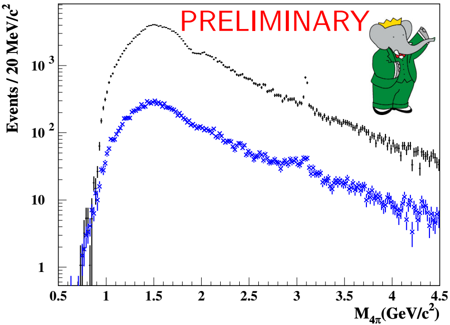


$\Rightarrow$ dedicated *BABAR* measurement

$\Rightarrow$ adjust simulation

Background subtraction: cross check

Sideband bkg subtraction

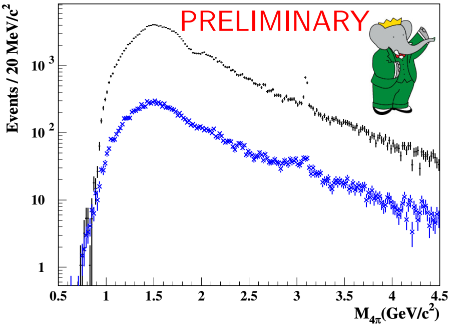


$$N_{1\mathrm{s}} = \frac{\beta}{\beta - \alpha} \cdot N_1 - \frac{1}{\beta - \alpha} \cdot N_2$$

$$\alpha := \frac{N_{2\mathrm{s}}}{N_{1\mathrm{s}}} , \qquad \beta := \frac{N_{2\mathrm{b}}}{N_{1\mathrm{b}}}$$

Background subtraction: cross check

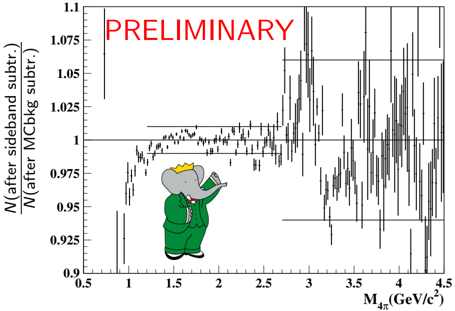
Sideband bkg subtraction



$$N_{1s} = \frac{\beta}{\beta-\alpha}\cdot N_1 - \frac{1}{\beta-\alpha}\cdot N_2$$

$$\alpha := \frac{N_{2s}}{N_{1s}}\ , \qquad \beta := \frac{N_{2b}}{N_{1b}}$$

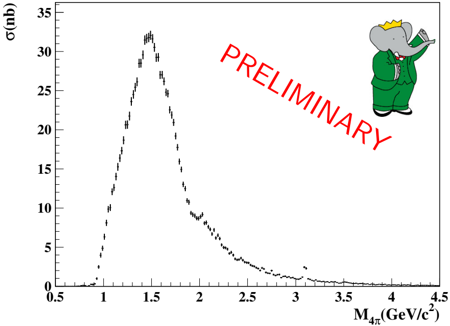
Comparison of both methods



Less than 1 % discrepancy in the peak region around 1.5 GeV/c^2

Resulting cross section

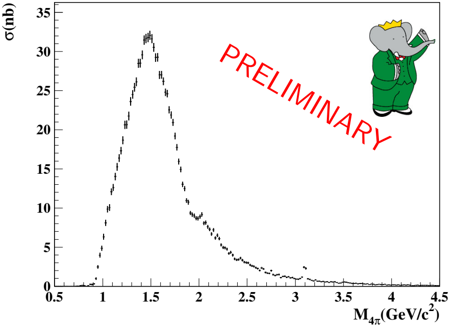
$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$$

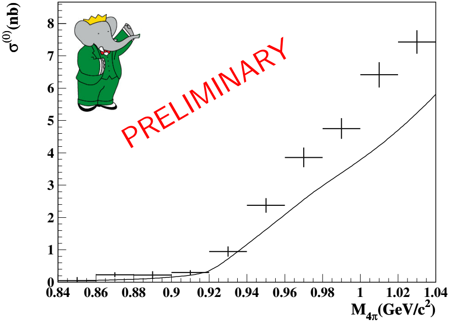


E_{CM}(GeV)	Syst. unc.
1.2 – 2.7	3.1%
2.7 – 3.2	6.7%
> 3.2	7.1%

Resulting cross section

$$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$$



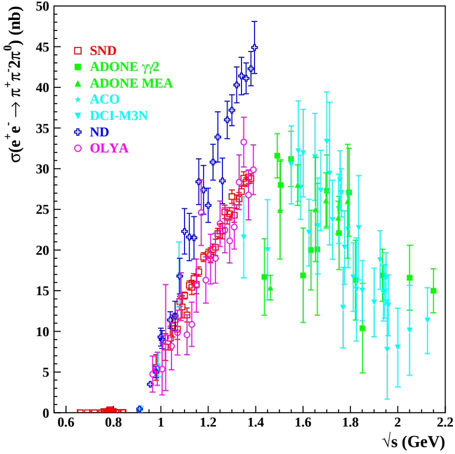


$E_{\rm CM}$(GeV)	Syst. unc.
1.2 – 2.7	3.1%
2.7 – 3.2	6.7%
> 3.2	7.1%

Comparison to Chiral Pert. Theo.

(Eur.Phys.J., C24:535–545, 2002 [6])

Contribution of $\pi^+\pi^-2\pi^0$ to $g_\mu - 2$

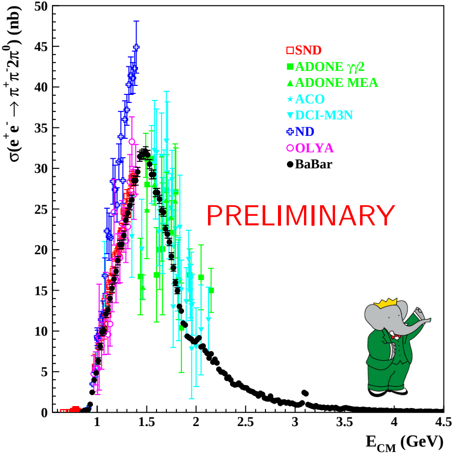


$$a_\mu^{\mathrm{had}} = \frac{1}{4\pi^3}\int_{m^2_{\pi^0}}^{\infty} \frac{\sqrt{1-\frac{4m_e^2}{s}}}{1+\frac{2m_e^2}{s}} K_\mu(s)\sigma(s)\mathrm{d}s$$

Before *BABAR* (Eur.Phys.J.,C31:503,2003) [4]

$$a_\mu(1.02 < \sqrt{s} < 1.8\,\mathrm{GeV}) = (16.76 \pm 1.31 \pm 0.20_{\mathrm{rad}}) \times 10^{-10}$$

Contribution of $\pi^+\pi^-2\pi^0$ to $g_\mu - 2$



$$a_\mu^{\mathrm{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \frac{\sqrt{1-\frac{4m_e^2}{s}}}{1+\frac{2m_e^2}{s}} K_\mu(s)\sigma(s)\mathrm{d}s$$

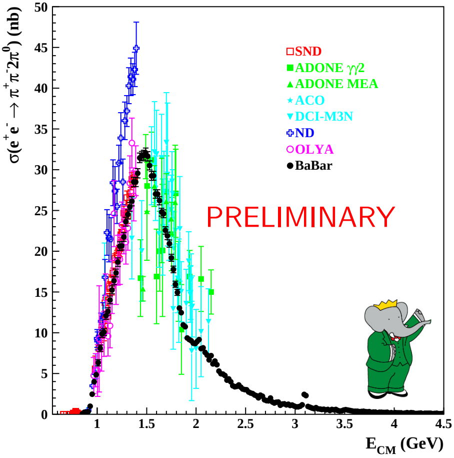
Before *BABAR* (Eur.Phys.J.,C31:503,2003) [4]

$a_\mu(1.02 < \sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(16.76 \pm 1.31 \pm 0.20_{\mathrm{rad}}) \times 10^{-10}$

New result in the same energy range

$a_\mu(1.02 < \sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(17.4 \pm 0.1_{\mathrm{stat}} \pm 0.6_{\mathrm{syst}}) \times 10^{-10}$

Contribution of $\pi^+\pi^-2\pi^0$ to $g_\mu - 2$



$$a_\mu^{\mathrm{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \frac{\sqrt{1-\frac{4m_e^2}{s}}}{1+\frac{2m_e^2}{s}} K_\mu(s)\sigma(s)\mathrm{d}s$$

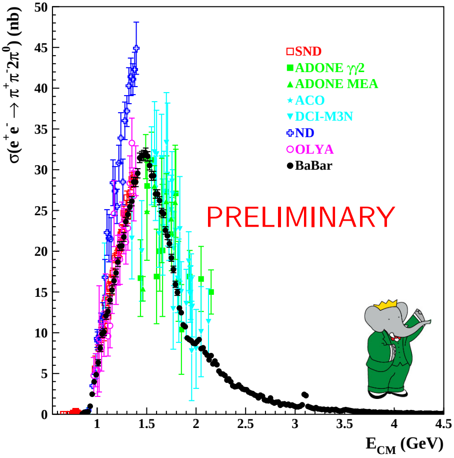
New result starting at lower limit

$a_\mu(0.85 < \sqrt{s} < 1.8\,\mathrm{GeV}) = (17.9 \pm 0.1_{\mathrm{stat}} \pm 0.6_{\mathrm{syst}}) \times 10^{-10}$

New result in a wider energy range

$a_\mu(0.85 < \sqrt{s} < 3.0\,\mathrm{GeV}) = (21.8 \pm 0.1_{\mathrm{stat}} \pm 0.7_{\mathrm{syst}}) \times 10^{-10}$

Contribution of $\pi^+\pi^-2\pi^0$ to $\Delta\alpha(M_Z^2)$



$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}$$

$$\Delta\alpha(q^2) = \frac{1}{4\pi^2\alpha} \oint \frac{\sqrt{1 - \frac{4m_e^2}{s}}}{1 + \frac{2m_e^2}{s}} \frac{\sigma^{(0)}(s)}{1 - \frac{s}{q^2}} \mathrm{d}s$$

New result in a wider energy range

$$\Delta\alpha(0.85 < \sqrt{s} < 1.8\,\text{GeV}) = (4.44 \pm 0.02_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-4}$$

$0.85\,\text{GeV} \le E_{\text{CM}} \le 3.0\,\text{GeV}$

$$\Delta\alpha(0.85 < \sqrt{s} < 3.0\,\text{GeV}) = (6.58 \pm 0.02_{\text{stat}} \pm 0.22_{\text{syst}}) \times 10^{-4}$$

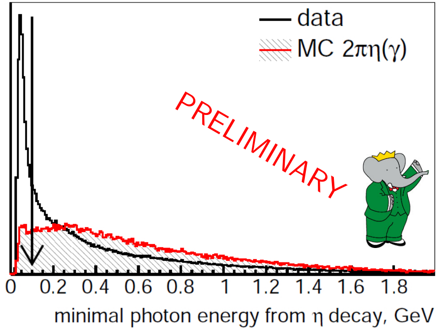
$e^+e^- \to \pi^+\pi^-\eta$

PRELIMINARY

Event Selection

Full $\Upsilon(4S)$ on peak data set of 468.1 fb^{-1}

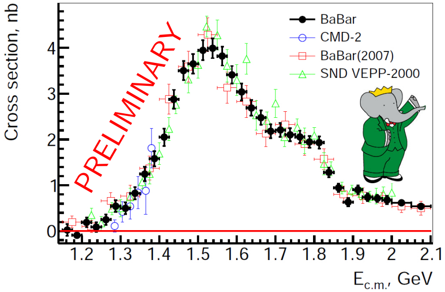
$$e^+e^- \to \pi^+\pi^-\eta\gamma_{\rm ISR}$$
$$(\eta \to \gamma\gamma)$$

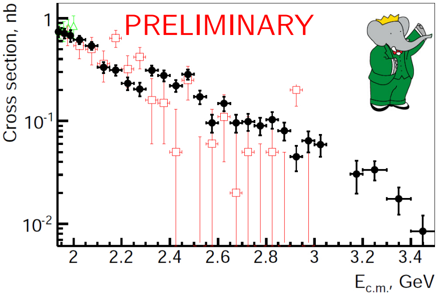


Main Selection Requirements

- at least 2 charged tracks
- ≥ 3 photons
- $E_\gamma^{\rm lab} > 0.1$ GeV
- $0.44 < M_\eta^{\rm reco} < 0.64$ GeV/c^2
- $E_{\gamma\rm ISR} > 3$ GeV

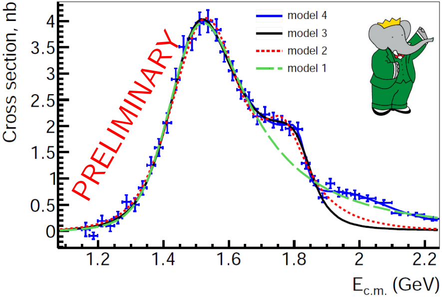
Cross section $e^+e^- \to \pi^+\pi^-\eta$

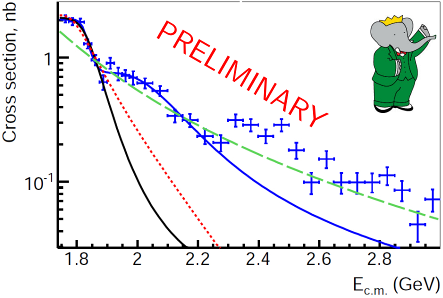




- Most accurate $\sigma(e^+e^- \to \pi^+\pi^-\eta)$ measurement to date
- First measurement up to 3.5 GeV
- Especially above 1.6 GeV more precise than previous data

Fits to the cross section $e^+e^- \to \pi^+\pi^-\eta$



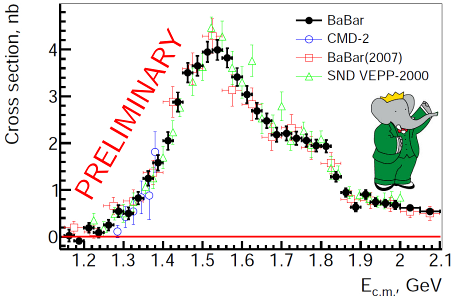


- Model 1: $\rho(770) - \rho(1450)$, fit: $E_{\mathrm{CM}} < 1.7$ GeV
- Model 2: $\rho(770) - \rho(1450) - \rho(1700)$, fit: $E_{\mathrm{CM}} < 1.9$ GeV
- Model 3: $\rho(770) - \rho(1450) + \rho(1700)$, fit: $E_{\mathrm{CM}} < 1.9$ GeV
- Model 4: $\rho(770) - \rho(1450) + \rho(1700) + \rho(2150)$, fit: $E_{\mathrm{CM}} < 2.2$ GeV

"+": relative phase 0°, "-": relative phase 180°

Contribution of $\pi^+\pi^-\eta$ to $g_\mu - 2$

$$a_\mu^{\mathrm{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} \frac{\sqrt{1-\frac{4m_e^2}{s}}}{1+\frac{2m_e^2}{s}} K_\mu(s)\sigma(s)\mathrm{d}s$$



HLMNT 2011 [8]

$a_\mu(\sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(0.88 \pm 0.10) \times 10^{-10}$

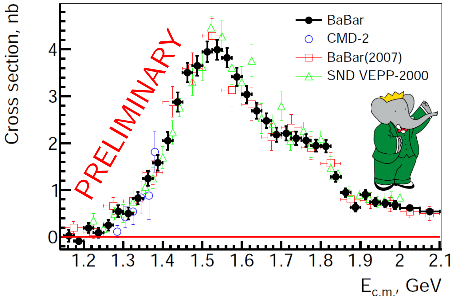
DHMZ 2011 [5]

$a_\mu(\sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(1.15 \pm 0.06_{\mathrm{stat}} \pm 0.08_{\mathrm{syst}}) \times 10^{-10}$

$\Rightarrow$ discrepancy?

Contribution of $\pi^+\pi^-\eta$ to $g_\mu - 2$

$$a_\mu^{\mathrm{had}} = \frac{1}{4\pi^3}\int_{m^2_{\pi^0}}^{\infty} \frac{\sqrt{1-\frac{4m_e^2}{s}}}{1+\frac{2m_e^2}{s}} K_\mu(s)\sigma(s)\mathrm{d}s$$



HLMNT 2011 [8]

$a_\mu(\sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(0.88 \pm 0.10) \times 10^{-10}$

DHMZ 2011 [5]

$a_\mu(\sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(1.15 \pm 0.06_{\mathrm{stat}} \pm 0.08_{\mathrm{syst}}) \times 10^{-10}$

New result

$a_\mu(\sqrt{s} < 1.8\,\mathrm{GeV}) =$
$(1.19 \pm 0.02_{\mathrm{stat}} \pm 0.06_{\mathrm{syst}}) \times 10^{-10}$

Summary

- ISR physics has proven to be a very productive field even years after the end of data taking at the B-factories
- Precision measurements of hadronic cross sections have greatly improved a_μ^{SM} & more hadronic final states in preparation
- New results from *BABAR*:
  - ⋆ $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$
  - ⋆ $e^+e^- \to \pi^+\pi^-\eta$
  - ⋆ For more *BABAR* ISR results see W. Gradl's presentation on Friday

Thank you!

Any questions?

Backup slides

References I

[1] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio.
Complete tenth-order qed contribution to the muon $g-2$.
Phys. Rev. Lett., 109:111808, Sep 2012.

[2] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera, and P. Stoffer.
Remarks on higher-order hadronic corrections to the muon $g-2$.
Phys.Lett., B735:90–91, 2014.

[3] G. Cosme, B. Dudelzak, B. Grelaud, B. Jean-Marie, S. Jullian, et al.
Hadronic Cross-Sections Study in e^+e^- Collisions from 1.350GeV to 2.125GeV.
Nucl.Phys., B152:215, 1979.

References II

[4] M. Davier, S. Eidelman, A. Höcker, and Z. Zhang.
Updated estimate of the muon magnetic moment using revised results from e^+e^- annihilation.
Eur. Phys. J., C31:503–510, 2003.

[5] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang.
Reevaluation of the Hadronic Contributions to the Muon $g-2$ and to $\alpha(M_Z)$.
Eur.Phys.J., C71:1515, 2011.

[6] G. Ecker and R. Unterdorfer.
Four-Pion Production in e^+e^- Annihilation.
Eur. Phys. J., C24:535–545, 2002.

References III

[7] C. Gnendiger, D. Stöckinger, and H. Stöckinger-Kim.
The electroweak contributions to $(g-2)_\mu$ after the Higgs boson mass measurement.
Phys.Rev., D88(5):053005, 2013.

[8] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner.
$(g-2)_\mu$ and $\alpha(M_Z^2)$ re-evaluated using new precise data.
J. Phys., G38:085003, 2011.

[9] F. Jegerlehner.
Electroweak effective couplings for future precision experiments.
Nuovo Cim., C034S1:31–40, 2011.

[10] F. Jegerlehner and A. Nyffeler.
The Muon g-2.
Phys.Rept., 477:1–110, 2009.

References IV

[11] A. Kurz, T. Liu, P. Marquard, and M. Steinhauser.
Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order.
Phys.Lett., B734:144–147, 2014.

[12] K. Olive et al.
Review of Particle Physics.
Chin.Phys., C38:090001, 2014.