



Consideration of crab waist scheme in CEPC

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Head on & crab waist

	Head-on	Crab waist(θ -half crossing angle)
Beam-beam parameter	$\xi_x = \frac{N_e r_e \beta_x}{2\pi\gamma \sigma_x (\sigma_x + \sigma_y)},$ $\xi_y = \frac{N_e r_e \beta_y}{2\pi\gamma \sigma_y (\sigma_x + \sigma_y)},$	Piwinski's angle: $\phi = \frac{\sigma_z \tan \theta}{\sigma_x},$ $\sigma_{x,eff} = \sigma_x \sqrt{1 + \phi^2},$ $\xi_x = \frac{N_e r_e \beta_x}{2\pi\gamma \sigma_{x,eff} (\sigma_{x,eff} + \sigma_y)},$ $\xi_y = \frac{N_e r_e \beta_y}{2\pi\gamma \sigma_y (\sigma_{x,eff} + \sigma_y)},$
Hour glass factor	$F_h = \frac{\beta_y}{\sqrt{\pi} \sigma_s} e^{\frac{\beta_y^2}{2\sigma_s^2}} K_0\left(\frac{\beta_y^2}{2\sigma_s^2}\right)$	$\sigma_{s,eff} = \frac{\sigma_x}{\theta},$ $F_h = \frac{\beta_y}{\sqrt{\pi} \sigma_{s,eff}} e^{\frac{\beta_y^2}{2\sigma_{s,eff}^2}} K_0\left(\frac{\beta_y^2}{2\sigma_{s,eff}^2}\right)$
luminosity	$L = \frac{N_e^2 n_b f_0}{4\pi \sigma_x \sigma_y} F_h$	$L = \frac{N_e^2 n_b f_0}{4\pi \sigma_{x,eff} \sigma_y} F_h$

Consideration at high energy(H)

Collision scheme	Head-on(Pre-CDR)	Crossing angle
Beam energy [GeV]	120	120
Current[mA]	16.6	16.6
No.bunches	50	50
Ne[10 ¹¹]	3.79	3.79
$\beta_{IP}(x/y)[m]$	0.8/0.0012	0.8/0.0012
$\varepsilon_x[nm]$	6.12	6.12
$\varepsilon_y[pm]$	18.4	18.4
$\sigma_{s,tot}[mm]$	2.65	2.65
$\theta[mrad](\text{half crossing angle})$	0	15
Piwinski's angle ϕ	0	0.56<1
ξ_x	0.118	0.118
ξ_y	0.083	0.083
F(crossing angle factor)	1	0.86
L/IP[10 ³⁴]	2	1.37

60° FODO → 90° FODO : $\varepsilon_x \rightarrow \frac{1}{3} \varepsilon_x, \alpha_p \rightarrow \frac{4}{9} \alpha_p,$
 $\phi \rightarrow 0.44 < 1$, crab waist scheme can not be realized.

Consideration at low energy(Z)

- Scale from Higgs energy(90° FODO), we got ε_x and σ_s at low energy(Z):

$$\varepsilon_x \propto \gamma^2, \quad \sigma_s = \frac{\alpha_p R \sigma_e}{v_s} \propto \gamma^{3/2},$$

$$\varepsilon_x = 0.29\text{nm}, \quad \sigma_s = 0.27\text{mm}.$$

So Piwinski's angle: $\phi = 0.27$

- $V_{RF} \downarrow, \sigma_s \uparrow \Rightarrow$ Piwinski's angle $\uparrow$:

$$V_{RF} = 0.094\text{GV时}, \quad \sigma_s = 3\text{mm}, \quad \phi = 3$$

Parameters at low energy(Z) head-on vs. crab waist

Collision scheme	Head-on	Crab waist
Beam energy [GeV]	45.5	45.5
Current[mA]	4.37	4.37
P[MW]	0.27	0.27
No.bunches	2000	200
Ne[10 ⁹]	2.47	24.7
$\beta_{IP}(x/y)[m]$	0.8/0.0012	0.8/0.0012
$\varepsilon_x[nm]$	0.29	0.29
$\varepsilon_y[pm]$	0.88	0.88
$\sigma_s[mm]$	0.27	3
$\theta[mrad](\text{half crossing angle})$	0	15
Piwinski's angle ϕ	0	3
ξ_x	0.04	0.04
ξ_y	0.03	0.09
L/IP[10 ³³]	1	3

Conclusions

- At high energy(H), we can not use crab waist for present lattice.
- At low energies(Z)crab waist scheme can provide higher luminosity than head-on collision.



The tentative parameters

parameter	FCC-ee crab waist (2 IPs)			
	Z (IPAC'15)	Z (this design)	t (IPAC'15)	t (this design)
E_{beam} [GeV]	45.5	45.6	175	←
current [mA]	1450	←	6.6	←
$P_{\text{SR,tot}}$ [MW]	100	95	100	94
no. bunches	45154	←	51	←
N_b [10^{11}]	0.66	←	2.6	←
ϵ_x [nm]	0.13	0.15	2	1.83
ϵ_y [pm]	1.0	1.0	2	1.83
β_x^* [m]	0.5	1	0.5	1
β_y^* [mm]	1	1	1	2
RF frequency [MHz]	400			
RF voltage [GV]	0.4	0.08	11	9.6
circumference [km]	100	99.938	100	99.938
mom. comp. [10^{-5}]	0.5	0.936	0.5	0.936
synchrotron tune	-0.03	-0.018	-0.07	-0.0856
$\sigma_{z,\text{SR}}$ [mm]	1	3.4	2.31	2.4
$\sigma_{z,\text{tot}}$ [mm] (w	2.8		2.83	
$\sigma_{\delta,\text{SR}}$ [%]	0.037	0.041	0.202	0.138
$\sigma_{\delta,\text{tot}}$ [%] (w beamstr.)	0.127		0.248	
θ_c [mrad]	30			
Piwinski angle	5.3	←	1.8	←
L^* [m]	2	2.2	2	2.2
beam-beam param.	0.07		0.06	
beam-beam param.	0.18		0.12	
luminosity/IP [10^{34}]	247		2.6	

FCC-ee Parameters

Version 2.0 (2014-09-05)	LEP2	FCC-ee Z	FCC-ee W	FCC-ee H	FCC-ee tt
Circumference [km]	26.7	100			
Bending radius [km]	3.1	11			
Beam energy [GeV]	104	45.5	80	120	175
Beam current [mA]	3.04	1450	152	30	6.6
Bunches / beam	4	16700	4490	1360	98
Bunch population [10^{11}]	4.2	1.8	0.7	0.46	1.4
Beam size at IP s* [mm]					
- Horizontal	182	121	41	22	45
- Vertical	3.2	0.25	0.084	0.044	0.045
Energy loss / turn [GeV]	3.34	0.03	0.33	1.67	7.55
SR power / beam [MW]	11	50			
Total RF voltage [GV]	3.5	2.5	4	5.5	11
RF frequency [MHz]	352	800			
Luminosity / IP [$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	0.012	28.0	12.0	6.0	1.8
Luminosity lifetime [min] ⁽¹⁾	434	298	73	29	21

⁽¹⁾ Luminosity lifetime corresponds to 4 IPs.