在 i 和 j 处各有一块 sextupole s_i s_j.

$$M = A_0^{-1} R_{0\to i} A_i e^{:f_i:} A_i^{-1} R_{i\to j} A_j e^{:f_j:} A_j^{-1} R_{j\to 0} A_0$$

$$= A_0^{-1} R_{0\to i} e^{:A_i f_i:} R_{i\to j} e^{:A_j f_j:} R_{j\to 0} A_0$$

$$\doteq A_0^{-1} R_{0\to i} e^{:A_i f_i:} R_{i\to j} R_{j\to 0} R_{j\to 0}^{-1} e^{:A_j f_j:} R_{j\to 0} A_0$$

$$= A_0^{-1} R_{0\to i} e^{:A_i f_i:} R_{i\to j\to 0} e^{:R_{j\to 0}^{-1} A_j f_j:} A_0$$

$$= A_0^{-1} R_{0\to i} R_{i\to j\to 0} e^{:R_{i\to j\to 0}^{-1} A_i f_i:} e^{:R_{j\to 0} A_j f_j:} A_0$$

$$= A_0^{-1} R_{2\pi\nu} e^{:g_i:} e^{:g_j:} A_0.$$

$$= A_0^{-1} R_{2\pi\nu} e^{:g_i + g_j + \frac{[g_i, g_j]}{2} \cdots} A_0.$$

$\cancel{A_i}g$, $A_i f_i = -\frac{1}{3} s_i \left(\left(\sqrt{\beta_{xi}}\, x + \eta_{xi}\delta\right)^3 - 3\cdot\left(\sqrt{\beta_{xi}}\, x + \eta_{xi}\delta\right)\left(\sqrt{\beta_{yi}}\, y + \eta_{yi}\delta\right)^2 \right)$

$$R_{i\to j} = \begin{pmatrix} \cos\mu_{ij} & \sin\mu_{ij} & & \\ -\sin\mu_{ij} & \cos\mu_{ij} & & \\ & & \cos\mu_{ij} & \sin\mu_{ij} \\ & & -\sin\mu_{ij} & \cos\mu_{ij} \end{pmatrix}$$

$$\mathcal{M}_f = \mathcal{R}_{2\pi\nu} e^{:g_i:} e^{:g_j:} = \mathcal{R}_{2\pi\nu} e^{:g_i + g_j + \frac{[g_i, g_j]}{2}:} \Omega$$

$$= \mathcal{R}_{2\pi\nu} e^{:g_i+g_j:} e^{:\frac{[g_i,g_j]}{2}:} \Omega'$$

$$\mathcal{R}_{2\pi\nu} e^{:h_4(\vec{J}):} = e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu} e^{:\overset{g_i+g_j}{\uparrow}{O_3}:} e^{:\overset{\frac{1}{2}[g_i,g_j]}{\uparrow}{O_4}:} e^{-:F_3:} e^{-:F_4:}$$

$$= e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu} e^{:O_3:} e^{:-F_3:} e^{:F_3:} e^{:O_4:} e^{-:F_3:} e^{-:F_4:}$$

$$= e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu} e^{:O_3:} e^{:-F_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$= e^{:F_4:} \mathcal{R}_{2\pi\nu} e^{:\mathcal{R}_{2\pi\nu}^{-1} F_3:} e^{:O_3:} e^{:-F_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$= e^{:F_4:} \mathcal{R}_{2\pi\nu} e^{:(\mathcal{R}_{2\pi\nu}^{-1} - I) F_3 + O_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$\boxed{\text{其中 } (\mathcal{R}_{2\pi\nu}^{-1} - I) F_3 + O_3 = h_3(\vec{J}) = 0}$$

$$\therefore = e^{:F_4:} \mathcal{R}_{2\pi\nu} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$\boxed{\text{其中 } F_4 = \frac{(e^{:F_3:} O_4 - h_4(\vec{J}))}{I - \mathcal{R}_{2\pi\nu}^{-1}}}$$

tune shift with amplitude:

$$\vec{\mu}(\vec{J}) = \vec{\mu} + \frac{\partial h_4(\vec{J})}{\partial \vec{J}} \cdots\cdots$$

interlived sextupole terms.

$$O_4 = \frac{1}{2}\left[g_i,\, g_j\right]$$

resonance basis:

$$\underset{x - ip_x}{\underset{\shortparallel}{h_x^+}} = \sqrt{2J_x}\, e^{+i\phi_x} \qquad \underset{x + ip_x}{\underset{\shortparallel}{h_x^-}} = \sqrt{2J_x}\, e^{-i\phi_x}.$$

$$\therefore\ x = \sqrt{2J_x}\cos\phi_x = \frac{1}{2}\left(h_x^+ + h_x^-\right)$$

$$p_x = -\frac{1}{2i}\left(h_x^+ - h_x^-\right) = -\sqrt{2J_x}\sin\phi_x$$

$$\mathcal{R}x = \frac{1}{2}\mathcal{R}\left(h_x^+ + h_x^-\right) = \frac{1}{2}\mathcal{R}\left(\sqrt{2J_x}\,e^{+i\phi_x} + \sqrt{2J_x}\,e^{-i\phi_x}\right).$$

$$= \frac{\sqrt{2J_x}}{2}\left(e^{i(\phi_x+\mu)} + e^{-i(\phi_x+\mu)}\right)$$

Only pick up ~~JQ~~ coefficients of J_x^2: $\left(\frac{\partial h_4(\vec{J})}{\partial J_x}\right)$.

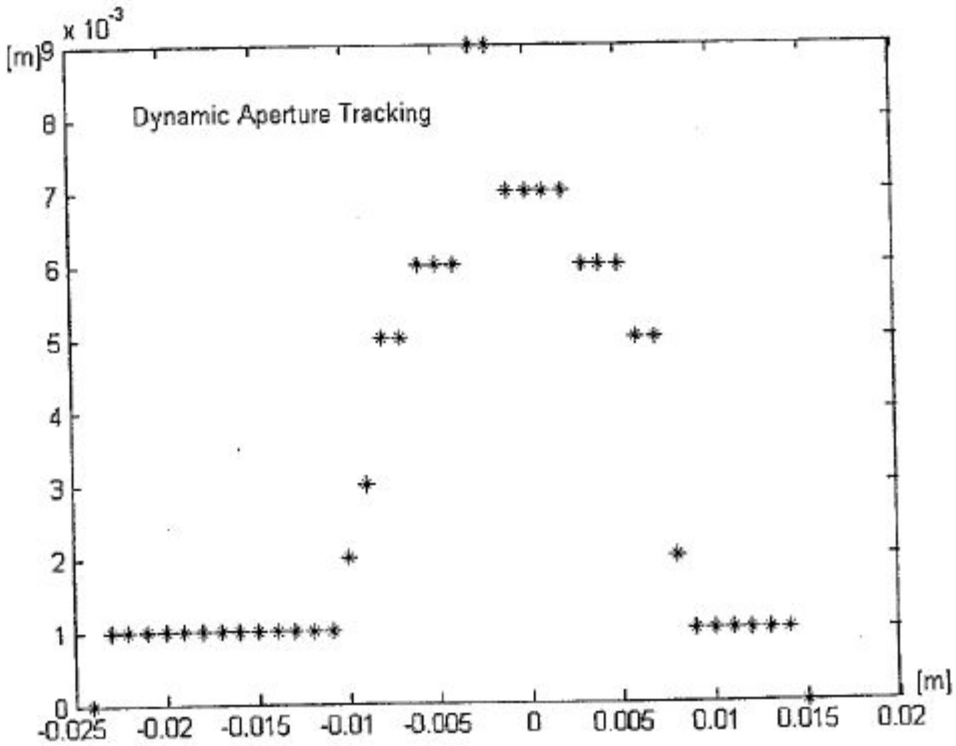
$$\boxed{\frac{\partial \nu_x}{\partial J_x} = \frac{1}{2\pi}\cdot\frac{1}{8}\cdot S_i S_j\, \beta_{xi}^{3/2}\beta_{xj}^{3/2}\left(\underline{3\cdot\sin(\mu_{ij}) + \sin(3\mu_{ij})}\right).}$$

$\Uparrow$

Goal of Optimization

Maybe something wrong.

$$
\begin{aligned}
\frac{\partial \nu_x}{\partial J_x} &= -\frac{1}{16\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\beta_{xj}^{3/2}\beta_{xk}^{3/2} \\
&\quad\times\left[\frac{3\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}+\frac{\cos\left(\left|3\mu_{j\to k,x}\right|-3\pi\nu_x\right)}{\sin\left(3\pi\nu_x\right)}\right], \\
\frac{\partial \nu_x}{\partial J_y} &= \frac{\partial \nu_y}{\partial J_x} \\
&= \frac{1}{8\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\sqrt{\beta_{xj}\beta_{xk}}\beta_{yj}\left[\frac{2\beta_{xk}\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}\right. \\
&\quad-\frac{\beta_{yk}\cos\left[|\mu_{j\to k,x}+2\mu_{j\to k,y}|-\pi\left(\nu_x+2\nu_y\right)\right]}{\sin\pi\left(\nu_x+2\nu_y\right)} \\
&\quad\left.+\frac{\beta_{yk}\cos\left[|\mu_{j\to k,x}-2\mu_{j\to k,y}|-\pi\left(\nu_x-2\nu_y\right)\right]}{\sin\pi\left(\nu_x-2\nu_y\right)}\right], \\
\frac{\partial \nu_y}{\partial J_y} &= -\frac{1}{16\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\sqrt{\beta_{xj}\beta_{xk}}\beta_{yj}\beta_{yk}\left[\frac{4\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}\right. \\
&\quad+\frac{\cos\left[|\mu_{j\to k,x}+2\mu_{j\to k,y}|-\pi\left(\nu_x+2\nu_y\right)\right]}{\sin\pi\left(\nu_x+2\nu_y\right)} \\
&\quad\left.+\frac{\cos\left[|\mu_{j\to k,x}-2\mu_{j\to k,y}|-\pi\left(\nu_x-2\nu_y\right)\right]}{\sin\pi\left(\nu_x-2\nu_y\right)}\right]
\end{aligned}
\tag{119}
$$



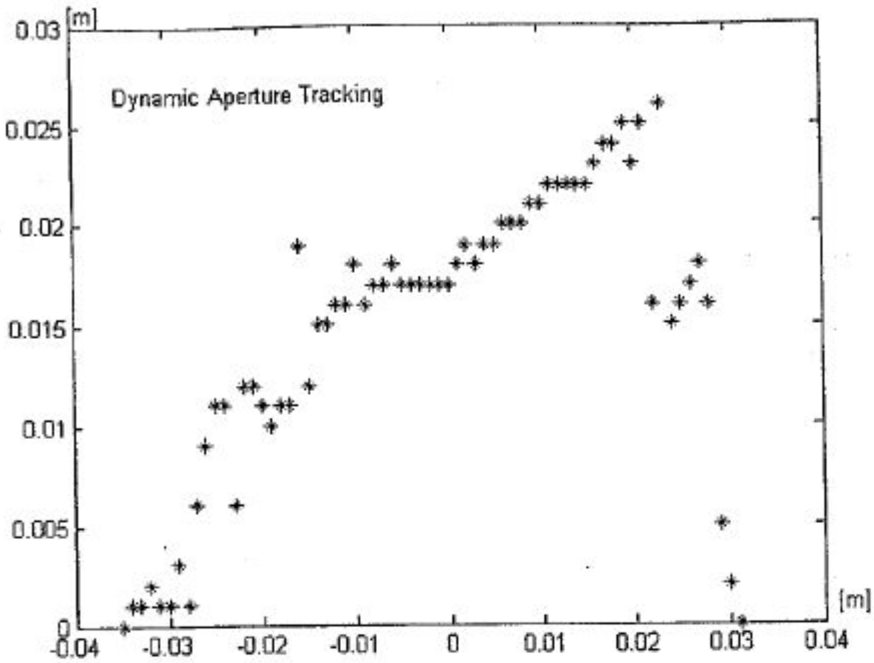


图 3-13 进一步优化后动力学孔径跟踪的结果 4（对应各磁铁族的强度为：