



WHIZARD NLO



Jürgen R. Reuter, DESY





1) Introduction to WHIZARD

↪ cf. Wolfgang Kilian's talk



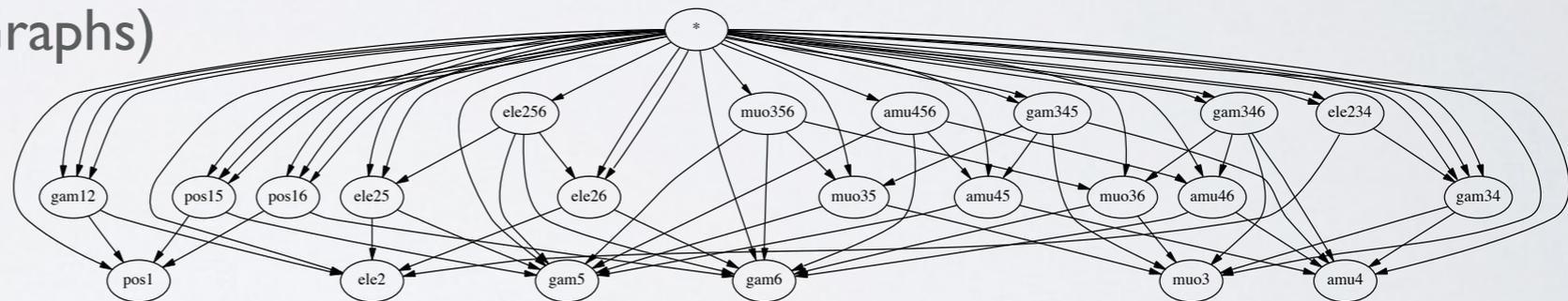
WHIZARD v2.3.0 (27±2 July 2016) <http://whizard.hepforge.org> <whizard@desy.de>

WHIZARD Team: *Wolfgang Kilian, Thorsten Ohl, JRR, Simon Braß/Bijan Chokoufé/C. Fleper/Marco Sekulla/Soyoung Shim/Christian Weiss/Florian Staub/Zhijie Zhao + 1 Master*

EPJ C71 (2011) 1742

- Universal event generator for lepton and hadron colliders
- Modular package:
 - **Phase space parameterization** (resonances, collinear emission, Coulomb etc.)
 - **O'Mega optimized matrix element generator** (recursiveness via Directed

Acyclical Graphs)



- **VAMP**: adaptive multi-channel Monte Carlo integrator
- **CIRCEI/2**: generator/simulation tool for lepton collider beam spectra
- **Lepton beam ISR** [Kuraev/Fadin, 2003; Skrzypek/Jadach, 1991](#)
- **Color flow formalism** [Stelzer/Willenbrock, 2003; Kilian/Ohl/JRR/Speckner, 2011](#)

WHIZARD: Installation and Run



- Download: <http://www.hepforge.org/archive/whizard/whizard-2.3.0.tar.gz>
- Unpack it, intended to be installed in `/usr/local` (or locally)
- Create `build` directory and do `./configure`
- `make`, [`make check`], `make install`
- Working directory: create SINDARIN steering file `<input>.sin`
- Working directory: run `whizard <input>.sin`
- Supported event formats: LHA, StdHep, LHEF (i-iii), HepMC, LCIO, div.ASCII
- Interfaces to external packages for **Feynman rules, hadronization, event formats, analysis, jet clustering etc.**: FastJet, GoSam, GuineaPig(++), HepMC, HOPPET, LCIO, LHAPDF(4/5/6), LoopTools, OpenLoops, PYTHIA6, [PYTHIA8], StdHep [internal]

```

PASS: hep_events.run
PASS: eio_data.run
PASS: eio_base.run
PASS: eio_raw.run
PASS: eio_checkpoints.run
PASS: eio_lhef.run
PASS: process_libraries.run
PASS: eio_ascii.run
PASS: eio_weights.run
PASS: eio_dump.run
PASS: iterations.run
PASS: eio_stdhep.run
PASS: rt_data.run
PASS: process_configurations.run
PASS: dispatch.run
PASS: event_streams.run
PASS: integrations.run
PASS: prclib_interfaces.run
PASS: shower.run
PASS: jets.run
PASS: hepmc.run
PASS: eio_hepmc.run
PASS: lcio.run
PASS: eio_lcio.run
SKIP: sf_lhapdf5.run
PASS: sf_lhapdf6.run
PASS: simulations.run
PASS: compilations.run
PASS: ttv_formfactors.run
PASS: phs_wood_vis.run
PASS: commands.run
PASS: integrations_history.run
PASS: prc_omega_diags.run
PASS: compilations_static.run
PASS: prc_omega.run
=====
Testsuite summary for WHIZARD 2.3.0
=====
# TOTAL: 110
# PASS: 109
# SKIP: 1
# XFAIL: 0
# FAIL: 0
# XPASS: 0
# ERROR: 0
=====

```

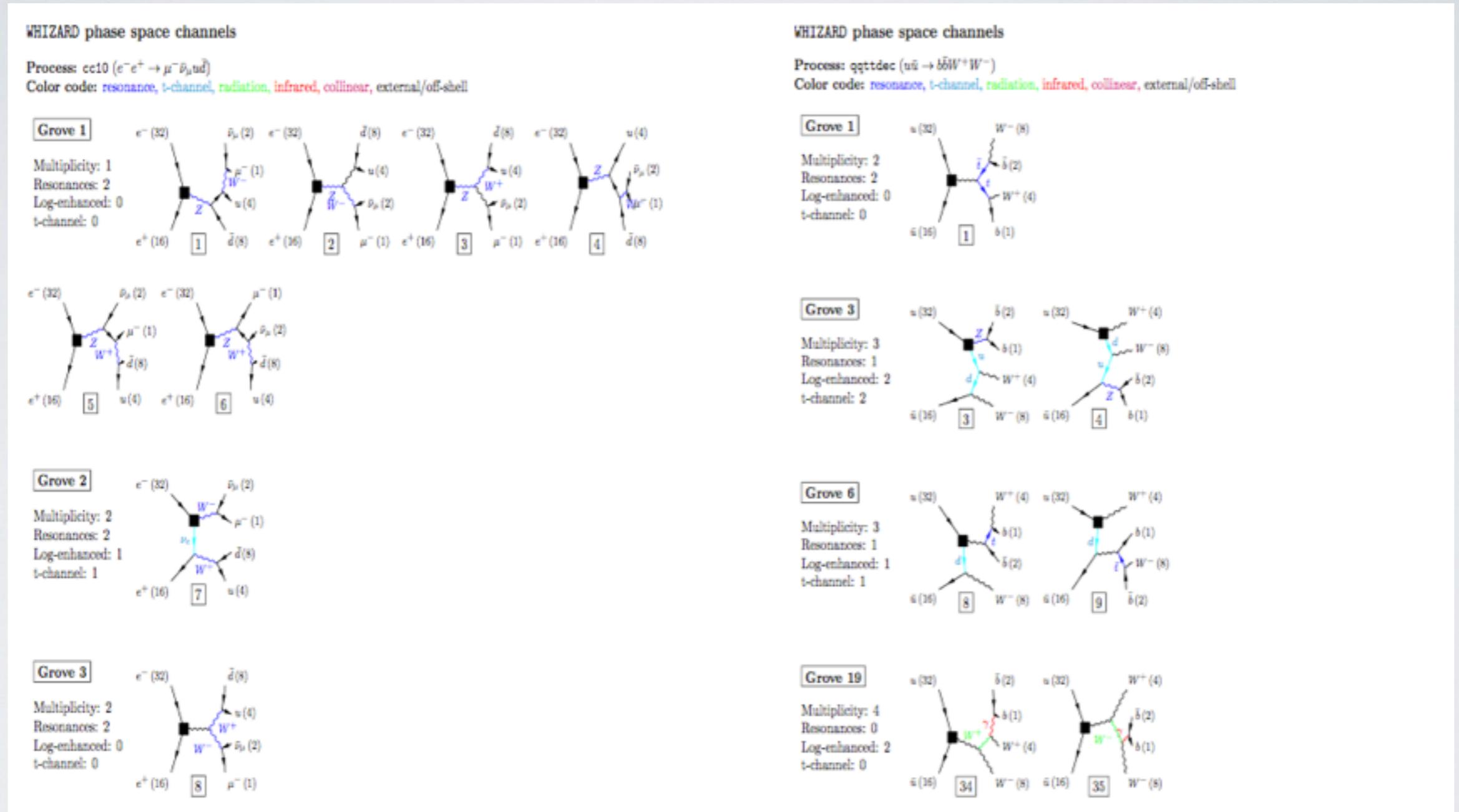
```

PASS: lhpdf6.run
PASS: pythia6_1.run
PASS: pythia6_3.run
PASS: pythia6_2.run
PASS: tauola_1.run
PASS: pythia6_4.run
PASS: tauola_2.run
PASS: mim_matching_isr.run
PASS: analyze_3.run
PASS: user_prc_threshold_1.run
PASS: mim_pythia6_isr.run
PASS: static_1.run
PASS: static_2.run
PASS: user_prc_threshold_2.run
=====
Testsuite summary for WHIZARD 2.3.0
=====
# TOTAL: 204
# PASS: 200
# SKIP: 1
# XFAIL: 3
# FAIL: 0
# XPASS: 0
# ERROR: 0
=====

```

Phase Space Setup

WHIZARD algorithm: heuristics to classify phase-space topology, adaptive multi-channel mapping \implies resonant, t-channel, radiation, infrared, collinear, external/off-shell



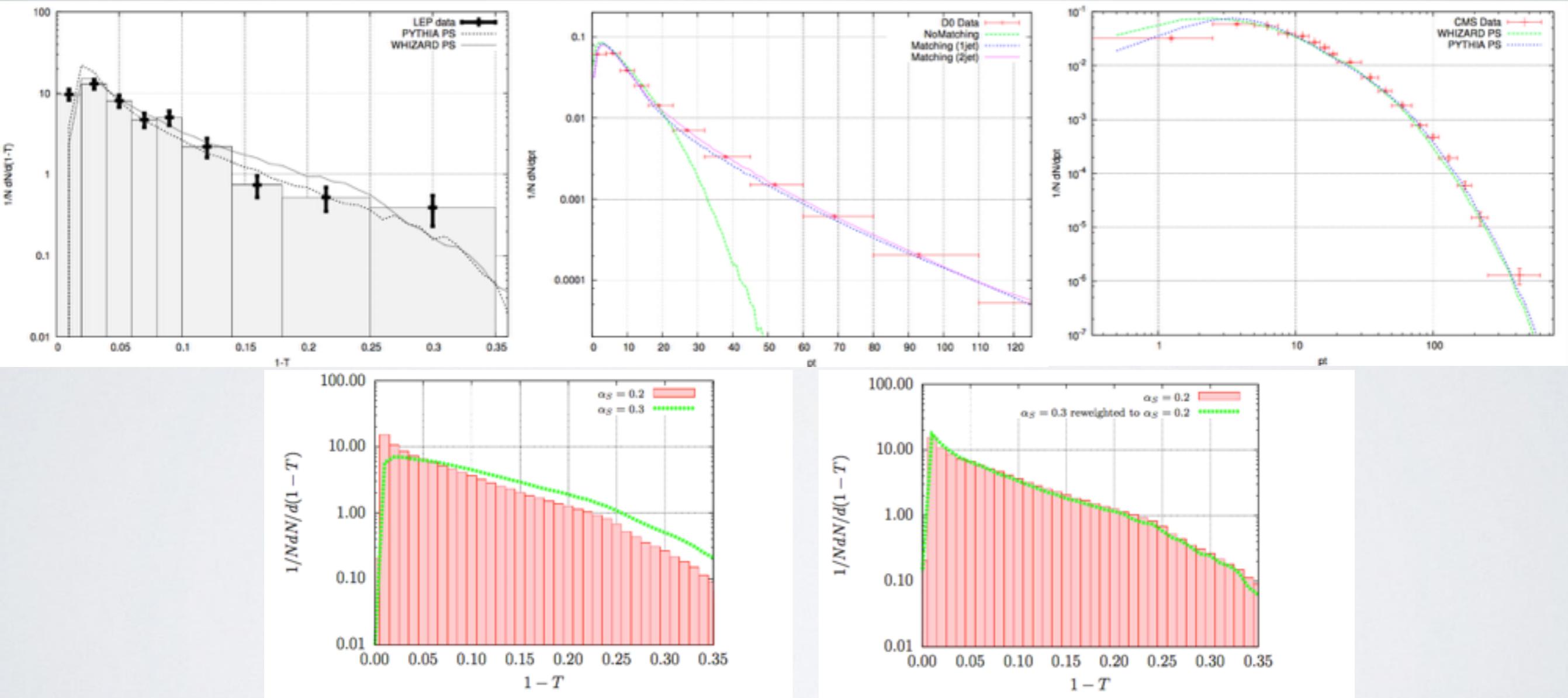
Complicated processes: **factorization into production and decay** with the unstable option



WHIZARD Parton Shower

- ▶ Two independent implementations: kT-ordered QCD and Analytic QCD shower
- ▶ Analytic shower: no shower veto \Rightarrow exact shower history known, allows reweighting

Kilian/JRR/Schmidt/Wiesler, JHEP 1204 013 (2012)

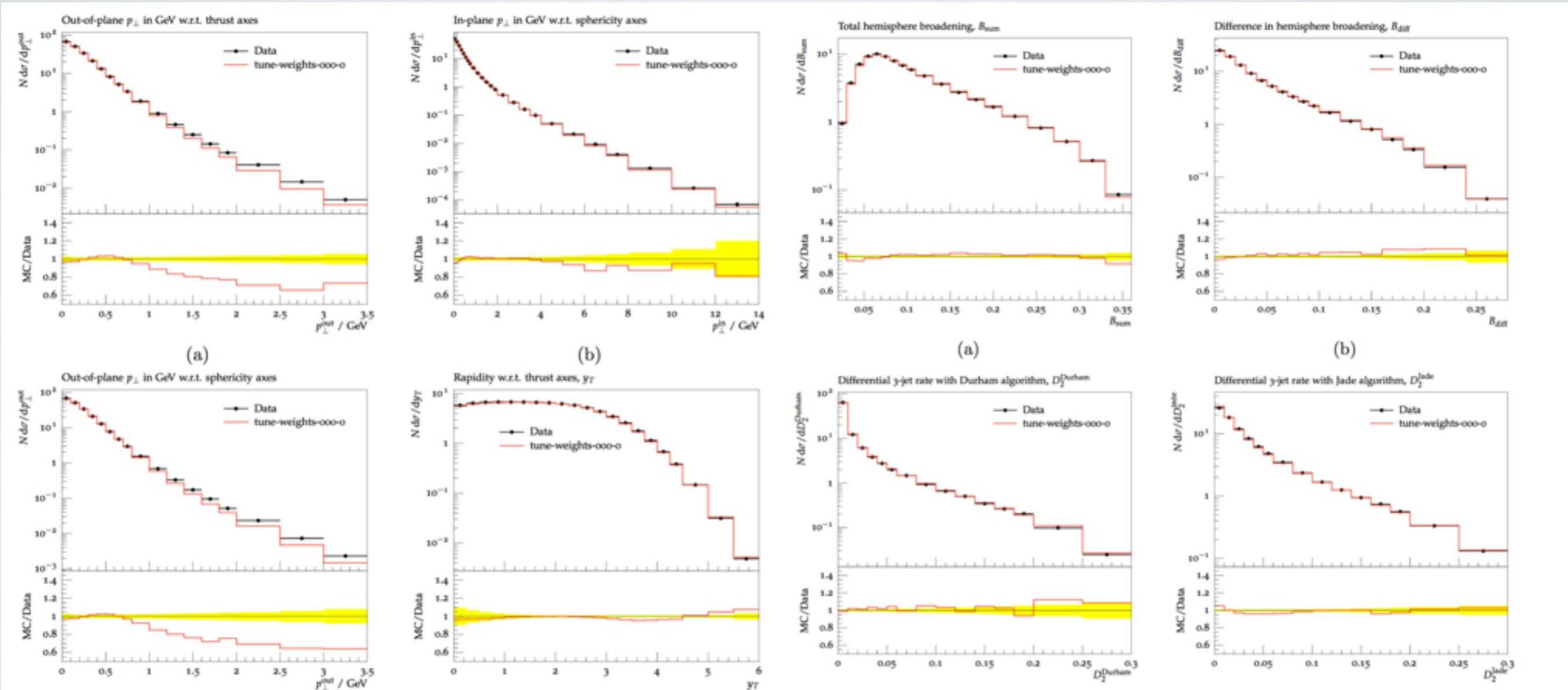


- ▶ Technical overhaul of the shower / merging part
- ▶ Plans: implement GKS matching, QED shower (also interleaved, infrastructure ready)



Tuning of the WHIZARD Parton Shower

- ▶ First tunes of both kT-ordered QCD and Analytic QCD shower [Chokoufe/Englert/JRR, 2015](#)
- ▶ Di- and Multijet data from LEP as given in RIVET analysis
- ▶ Usage of the PROFESSOR tool for determining the best fit [Buckley et al., 2009](#)





2) Automation of Fixed order NLO (QCD) in WHIZARD



NLO Development in WHIZARD

- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

Binoth Les Houches Interface (BLHA): Workflow

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
2. OLP generates code (Born/virtual interference), WHIZARD reads contract
3. NLO matrix element library loaded into WHIZARD



- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

Binoth Les Houches Interface (BLHA): Workflow

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
2. OLP generates code (Born/virtual interference), WHIZARD reads contract
3. NLO matrix element library loaded into WHIZARD

Working NLO interfaces to:

(first focus on QCD corrections)

- ★ GoSam [G. Cullen et al.] ↪ Talk by Nicolas Greiner
- ★ OpenLoops [F. Cascioli et al.]

WHIZARD v2.3.0 contains beta version

QCD corrections (massless and massive emitters)

```
alpha_power = 2
alphas_power = 0

process eett = e1,E1 => t, tbar
  { nlo_calculation = "full" }
```



NLO Development in WHIZARD

- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

Binoth Les Houches Interface (BLHA): Workflow

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
2. OLP generates code (Born/virtual interference), WHIZARD reads contract
3. NLO matrix element library loaded into WHIZARD

Working NLO interfaces to:

(first focus on QCD corrections)

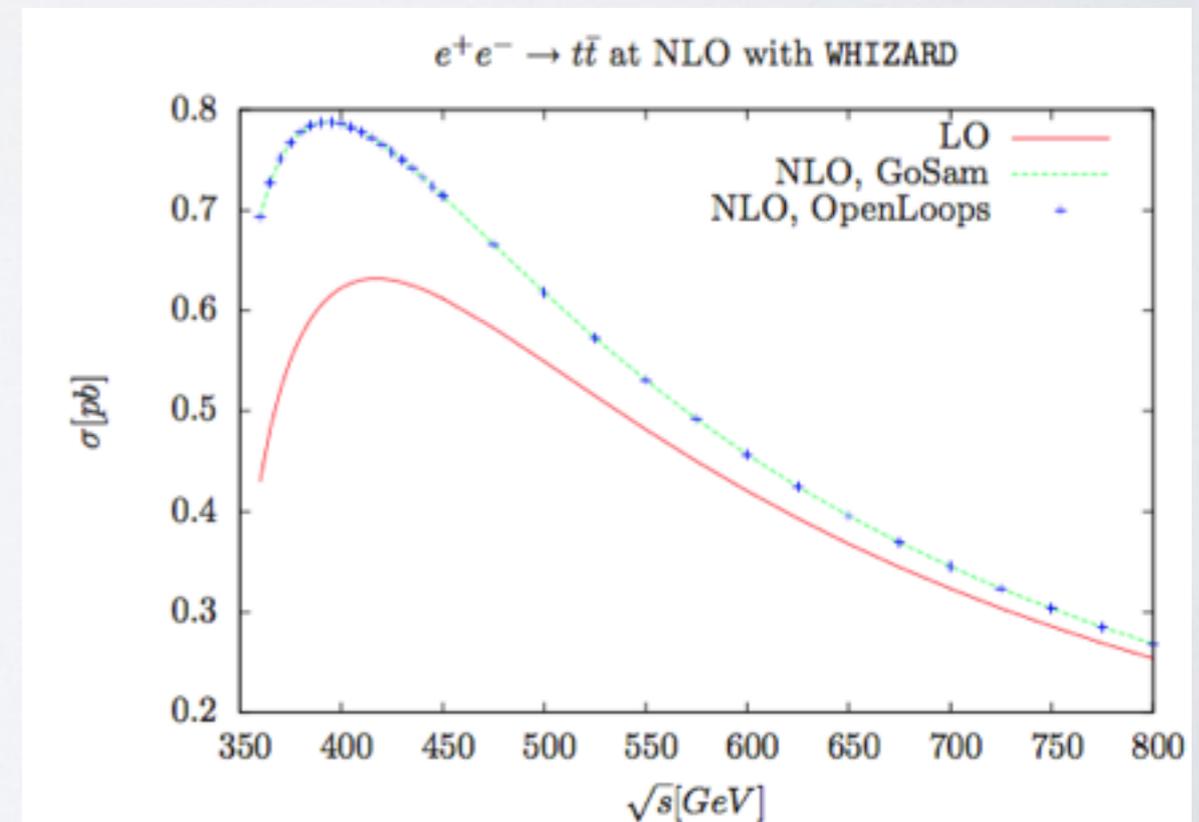
- ★ GoSam [G. Cullen et al.] ↪ Talk by Nicolas Greiner
- ★ OpenLoops [F. Cascioli et al.]

WHIZARD v2.3.0 contains beta version

QCD corrections (massless and massive emitters)

```
alpha_power = 2
alphas_power = 0

process eett = e1,E1 => t, tbar
  { nlo_calculation = "full" }
```





NLO Development in WHIZARD

- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

Binoth Les Houches Interface (BLHA): Workflow

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
2. OLP generates code (Born/virtual interference), WHIZARD reads contract
3. NLO matrix element library loaded into WHIZARD

Working NLO interfaces to:

(first focus on QCD corrections)

★ GoSam

[G. Cullen et al.]

↪ Talk by Nicolas Greiner

★ OpenLoops

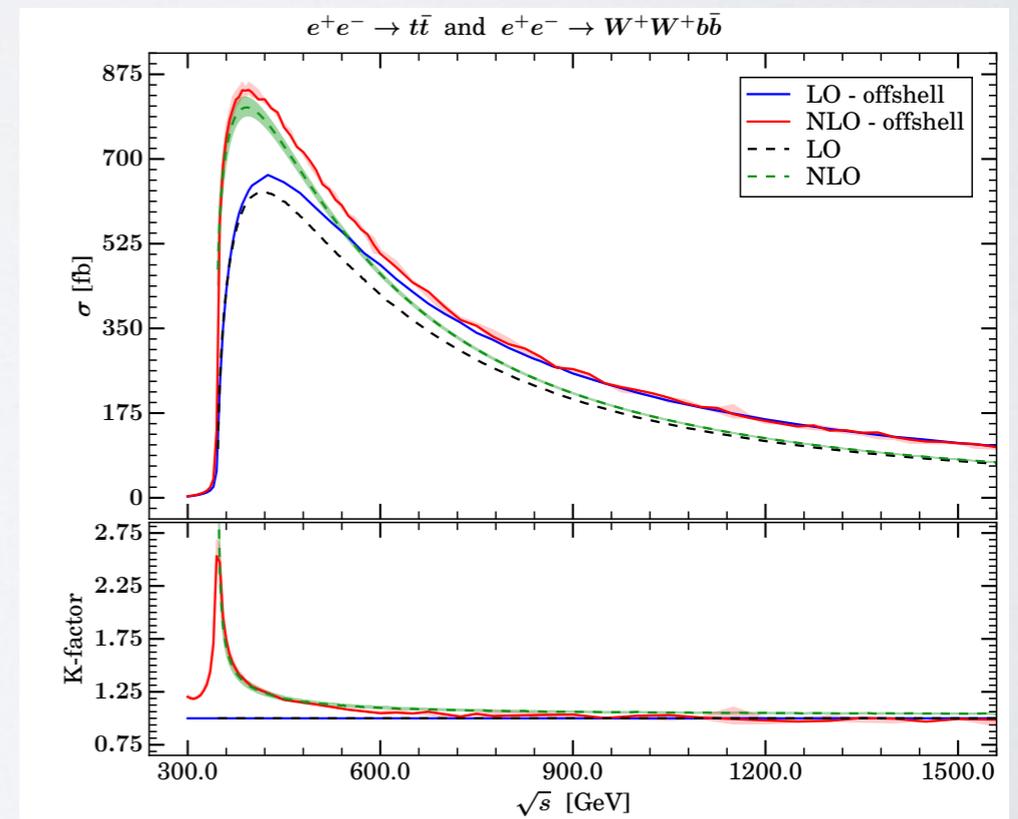
[F. Cascioli et al.]

WHIZARD v2.3.0 contains beta version

QCD corrections (massless and massive emitters)

```
alpha_power = 2
alphas_power = 0

process eett = e1,E1 => t, tbar
  { nlo_calculation = "full" }
```





NLO Development in WHIZARD

- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

Binoth Les Houches Interface (BLHA): Workflow

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
2. OLP generates code (Born/virtual interference), WHIZARD reads contract
3. NLO matrix element library loaded into WHIZARD

Working NLO interfaces to:

(first focus on QCD corrections)

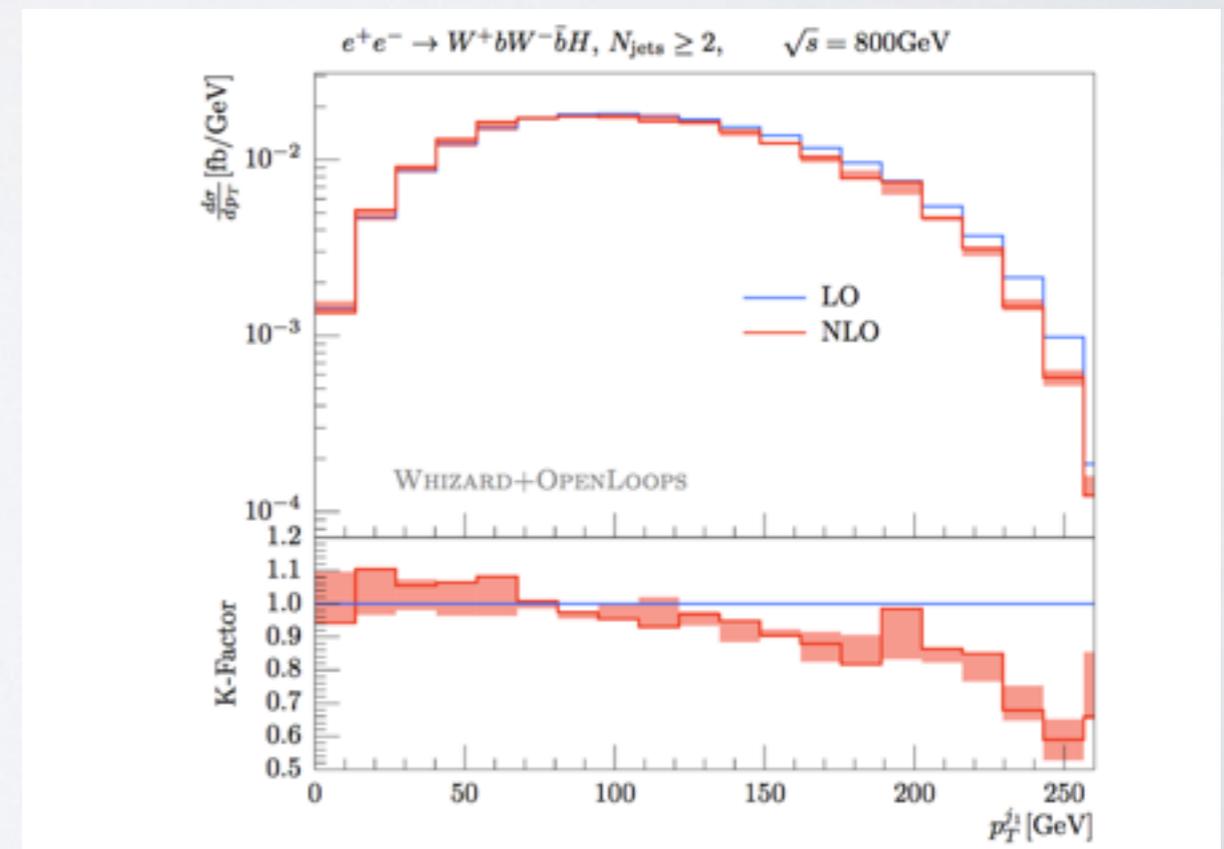
- ★ GoSam [G. Cullen et al.] ↪ Talk by Nicolas Greiner
- ★ OpenLoops [F. Cascioli et al.]

WHIZARD v2.3.0 contains beta version

QCD corrections (massless and massive emitters)

```
alpha_power = 2
alphas_power = 0

process eett = e1,E1 => t, tbar
  { nlo_calculation = "full" }
```





FKS Subtraction (Frixione/Kunszt/Signer)

Subtraction formalism to make real and virtual contributions separately finite

$$d\sigma^{\text{NLO}} = \underbrace{\int_{n+1} (d\sigma^R - d\sigma^S)}_{\text{finite}} + \underbrace{\int_{n+1} d\sigma^S + \int_n d\sigma^V}_{\text{finite}}$$

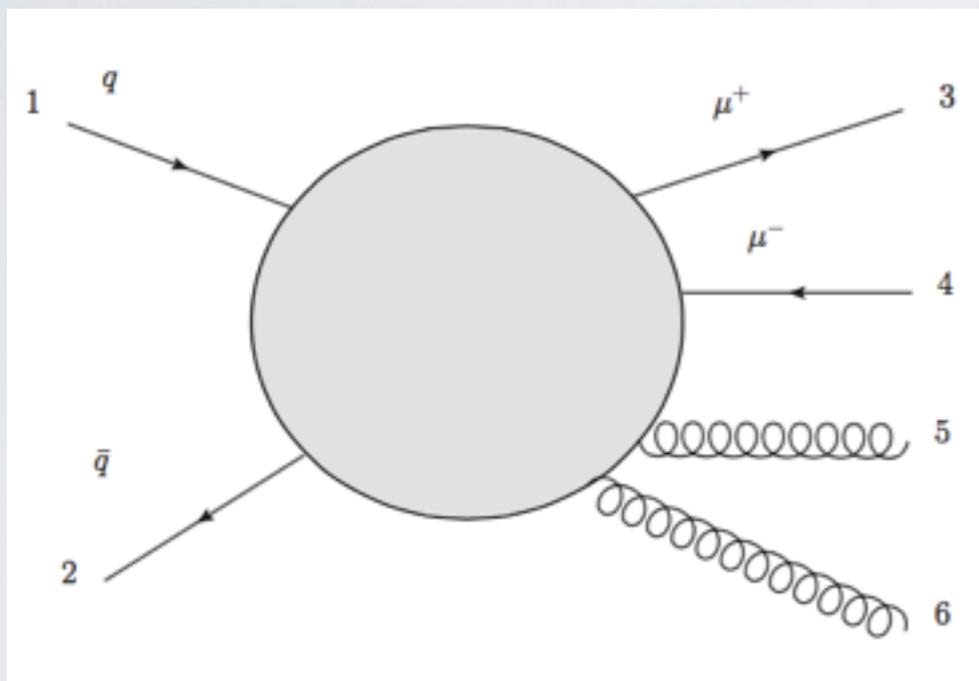


FKS Subtraction (Frixione/Kunszt/Signer)

Subtraction formalism to make real and virtual contributions separately finite

$$d\sigma^{\text{NLO}} = \underbrace{\int_{n+1} (d\sigma^R - d\sigma^S)}_{\text{finite}} + \underbrace{\int_{n+1} d\sigma^S + \int_n d\sigma^V}_{\text{finite}}$$

Automated subtraction terms in WHIZARD, algorithm:



- * Find all singular pairs

$$\mathcal{I} = \{(1, 5), (1, 6), (2, 5), (2, 6), (5, 6)\}$$

- * Partition phase space according to singular regions

$$\mathbb{1} = \sum_{\alpha \in \mathcal{I}} S_{\alpha}(\Phi)$$

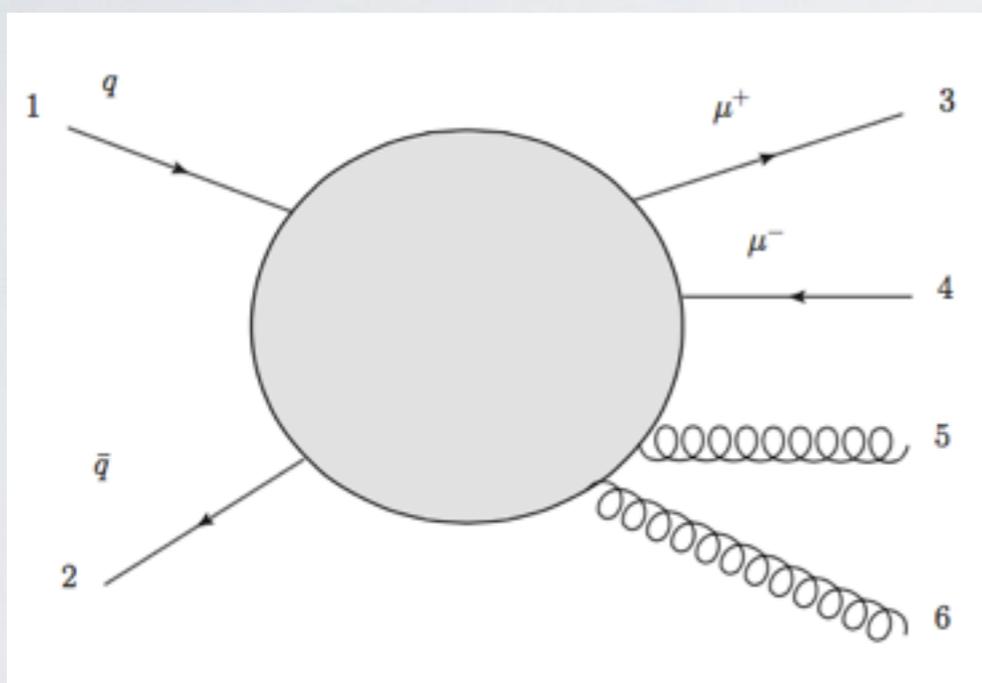
- * Generate subtraction terms for singular regions



Subtraction formalism to make real and virtual contributions separately finite

$$d\sigma^{\text{NLO}} = \underbrace{\int_{n+1} (d\sigma^R - d\sigma^S)}_{\text{finite}} + \underbrace{\int_{n+1} d\sigma^S + \int_n d\sigma^V}_{\text{finite}}$$

Automated subtraction terms in WHIZARD, algorithm:



- * Find all singular pairs

$$\mathcal{I} = \{(1, 5), (1, 6), (2, 5), (2, 6), (5, 6)\}$$

- * Partition phase space according to singular regions

$$\mathbb{1} = \sum_{\alpha \in \mathcal{I}} S_{\alpha}(\Phi)$$

- * Generate subtraction terms for singular regions

Soft subtraction involves color-correlated matrix elements:

$$\mathcal{B}_{kl} \sim - \sum_{\text{color spin}} \mathcal{A}^{(n)} \vec{Q}(\mathcal{I}_k) \cdot \vec{Q}(\mathcal{I}_l) \mathcal{A}^{(n)*},$$

Collinear subtraction involves spin-correlated matrix elements:

$$\mathcal{B}_{+-} \sim \text{Re} \left\{ \frac{\langle k_{\text{em}} k_{\text{rad}} \rangle}{[k_{\text{em}} k_{\text{rad}}]} \sum_{\text{color spin}} \mathcal{A}_+^{(n)} \mathcal{A}_-^{(n)*} \right\}$$



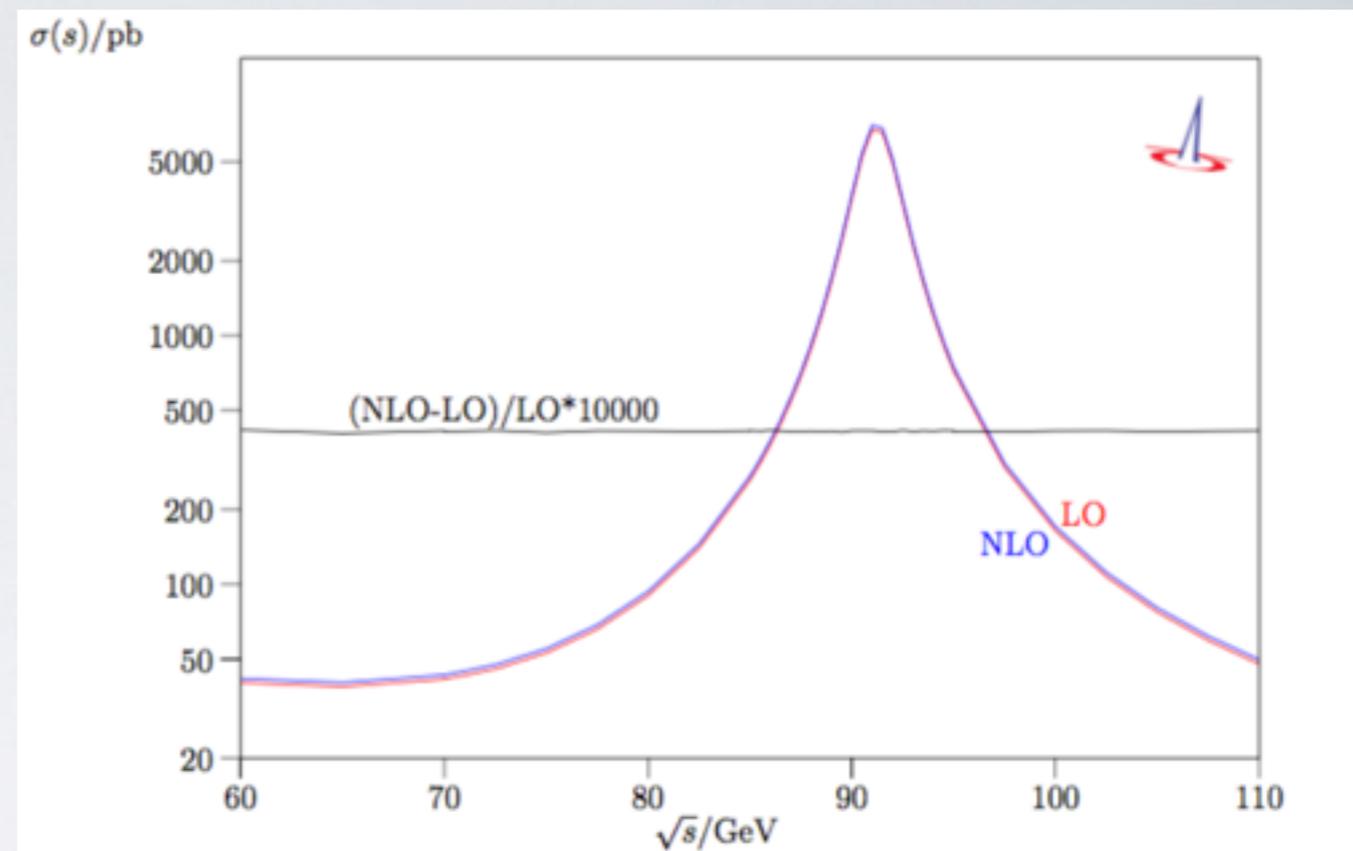
Simplest benchmark process:

$$e^+e^- \rightarrow q\bar{q} \quad \text{with} \quad (\sigma^{\text{NLO}} - \sigma^{\text{LO}}) / \sigma^{\text{LO}} = \alpha_s / \pi$$

Plot for total cross section for fixed strong coupling constant

Excerpt of validated QCD NLO processes

- $e^+e^- \rightarrow q\bar{q}$
- $e^+e^- \rightarrow q\bar{q}g$
- $e^+e^- \rightarrow \ell^+\ell^-q\bar{q}$
- $e^+e^- \rightarrow \ell^+\nu_\ell q\bar{q}$
- $e^+e^- \rightarrow t\bar{t}$
- $e^+e^- \rightarrow tW^-b$
- $e^+e^- \rightarrow W^+W^-b\bar{b}$
- $e^+e^- \rightarrow t\bar{t}H$



- Cross-checks with MG5_aMC@NLO, Sherpa, MUNICH
- Phase space integration performs great (V, R, S)
- Plan to also support NJet [Badger et al.]



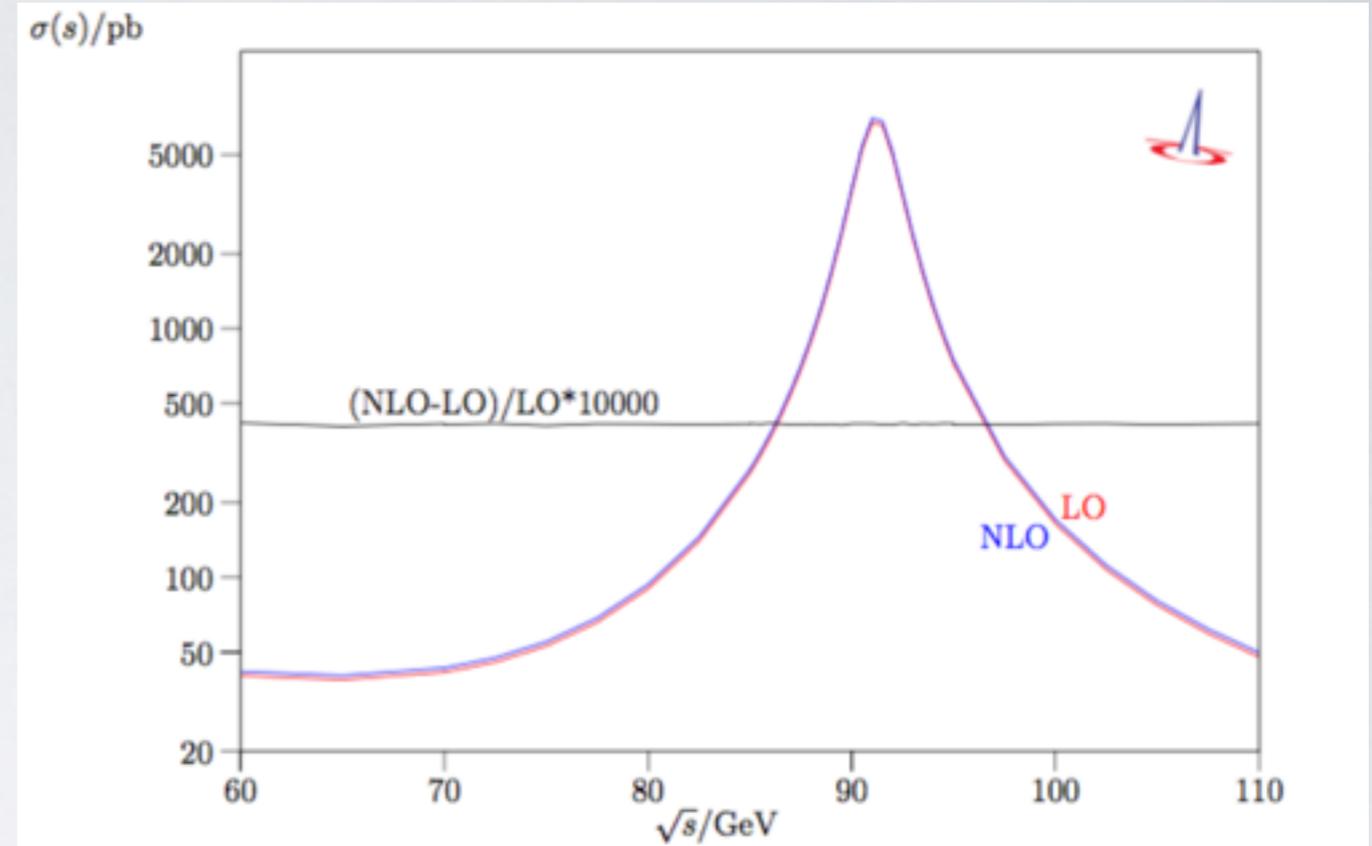
Simplest benchmark process:

$$e^+e^- \rightarrow q\bar{q} \quad \text{with} \quad (\sigma^{\text{NLO}} - \sigma^{\text{LO}}) / \sigma^{\text{LO}} = \alpha_s / \pi$$

Plot for total cross section for fixed strong coupling constant

Excerpt of validated QCD NLO processes

- $e^+e^- \rightarrow q\bar{q}$
- $e^+e^- \rightarrow q\bar{q}g$
- $e^+e^- \rightarrow \ell^+\ell^-q\bar{q}$
- $e^+e^- \rightarrow \ell^+\nu_\ell q\bar{q}$
- $e^+e^- \rightarrow t\bar{t}$
- $e^+e^- \rightarrow tW^-b$
- $e^+e^- \rightarrow W^+W^-b\bar{b}$
- $e^+e^- \rightarrow t\bar{t}H$



- Cross-checks with MG5_aMC@NLO, Sherpa, MUNICH
- Phase space integration performs great (V, R, S)
- Plan to also support NJet [Badger et al.]

- ◆ QCD NLO infrastructure in pp complete
- ◆ First attempts on electroweak corrections, interfacing the RECOLA code [Denner et al.]



Resonance mappings for NLO processes

12/31

- Amplitudes (except for pure QCD/QED) contain **resonances** (Z, W, H, t)
- **In general: resonance masses *not* respected by modified kinematics of subtraction terms**
- Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms



Resonance mappings for NLO processes

12/31

- Amplitudes (except for pure QCD/QED) contain **resonances** (Z, W, H, t)
- In general: resonance masses *not* respected by modified kinematics of subtraction terms**
- Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms
- Algorithm to include resonance histories** [[Ježo/Nason, 1509.09071](#)]
- Avoids double logarithms in the resonances' width
- Most important for narrow resonances ($H \rightarrow bb$)
- Separate treatment of Born and real terms,**
soft mismatch [, collinear mismatch]



Resonance mappings for NLO processes

Amplitudes (except for pure QCD/QED) contain **resonances** (Z, W, H, t)

In general: resonance masses *not* respected by modified kinematics of subtraction terms

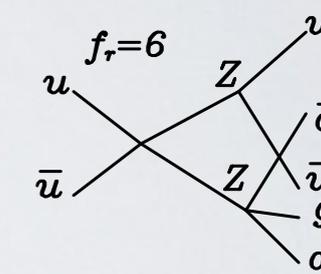
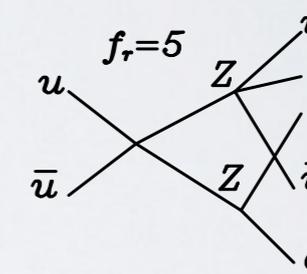
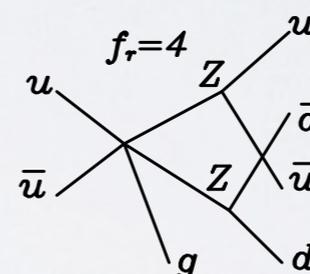
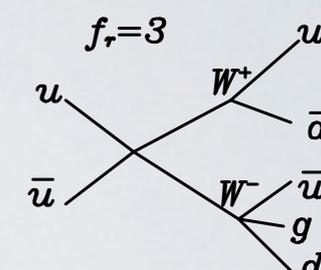
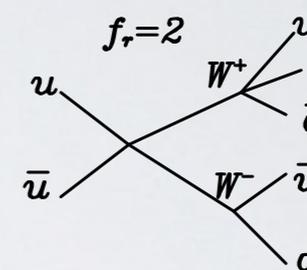
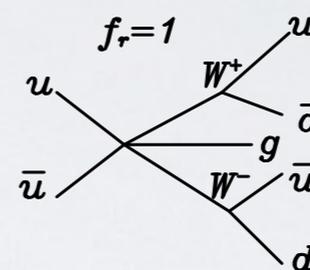
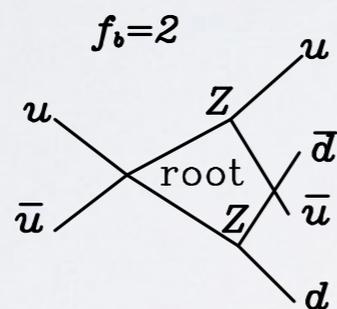
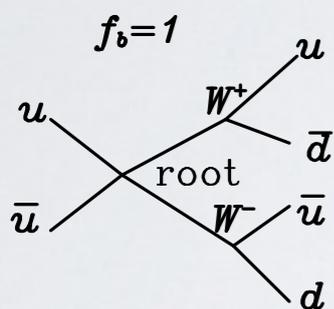
Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms

Algorithm to include resonance histories [Ježo/Nason, 1509.09071]

Avoids double logarithms in the resonances' width

Most important for narrow resonances ($H \rightarrow bb$)

Separate treatment of Born and real terms, soft mismatch [, collinear mismatch]





Resonance mappings for NLO processes

Amplitudes (except for pure QCD/QED) contain **resonances** (Z, W, H, t)

In general: **resonance masses not respected by modified kinematics of subtraction terms**

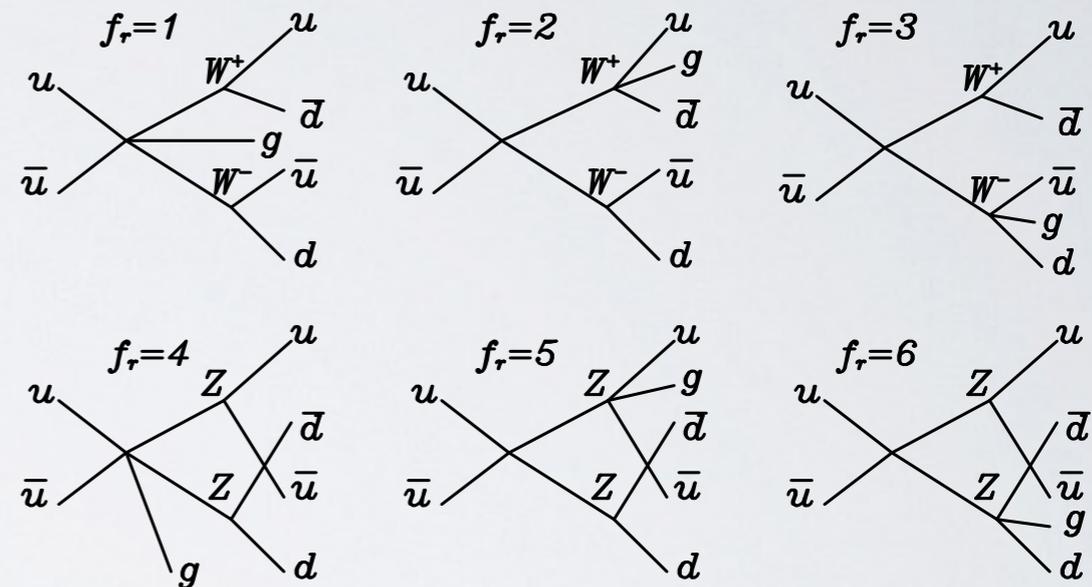
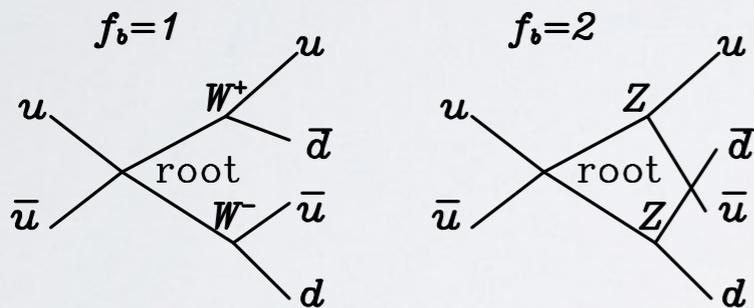
Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms

Algorithm to include resonance histories [Ježo/Nason, 1509.09071]

Avoids double logarithms in the resonances' width

Most important for narrow resonances ($H \rightarrow bb$)

Separate treatment of Born and real terms,
soft mismatch [, collinear mismatch]



WHIZARD complete automatic implementation: example $e^+ e^- \rightarrow \mu\mu bb$ (ZZ, ZH histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS



Resonance mappings for NLO processes

Amplitudes (except for pure QCD/QED) contain **resonances** (Z, W, H, t)

In general: **resonance masses not respected by modified kinematics of subtraction terms**

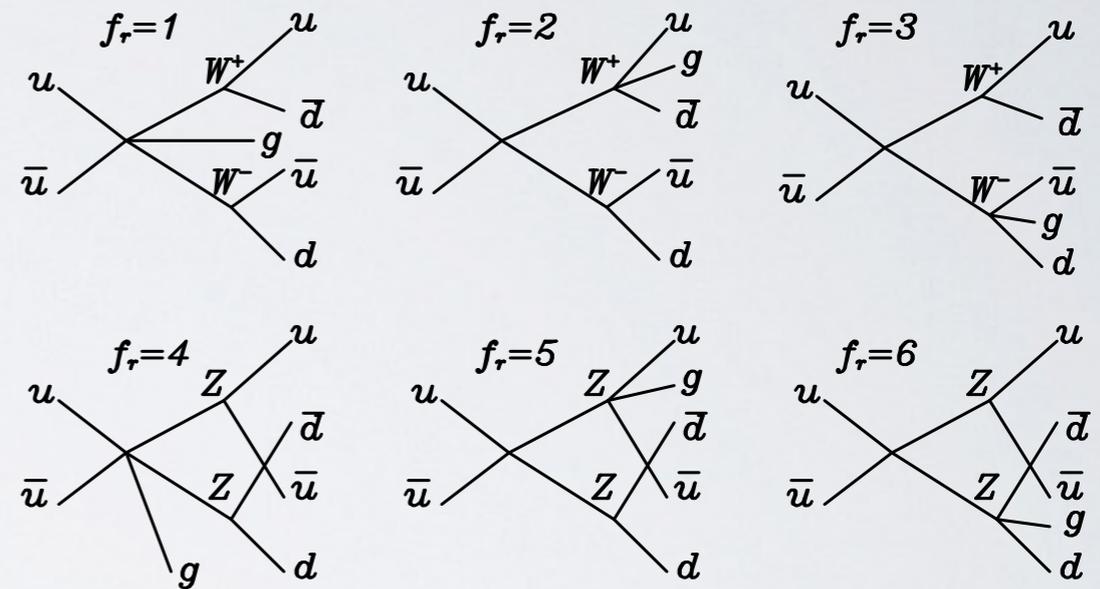
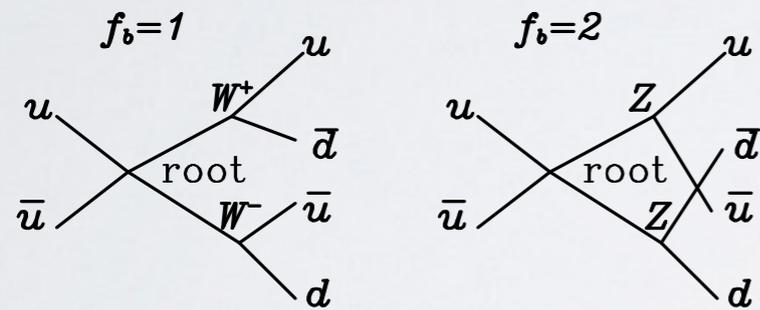
Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms

Algorithm to include resonance histories [Ježo/Nason, 1509.09071]

Avoids double logarithms in the resonances' width

Most important for narrow resonances ($H \rightarrow bb$)

Separate treatment of Born and real terms, soft mismatch [, collinear mismatch]



WHIZARD complete automatic implementation: example $e^+ e^- \rightarrow \mu\mu bb$ (ZZ, ZH histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	2.9057032E+00	8.35E-02	2.87	3.15*	7.90		
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

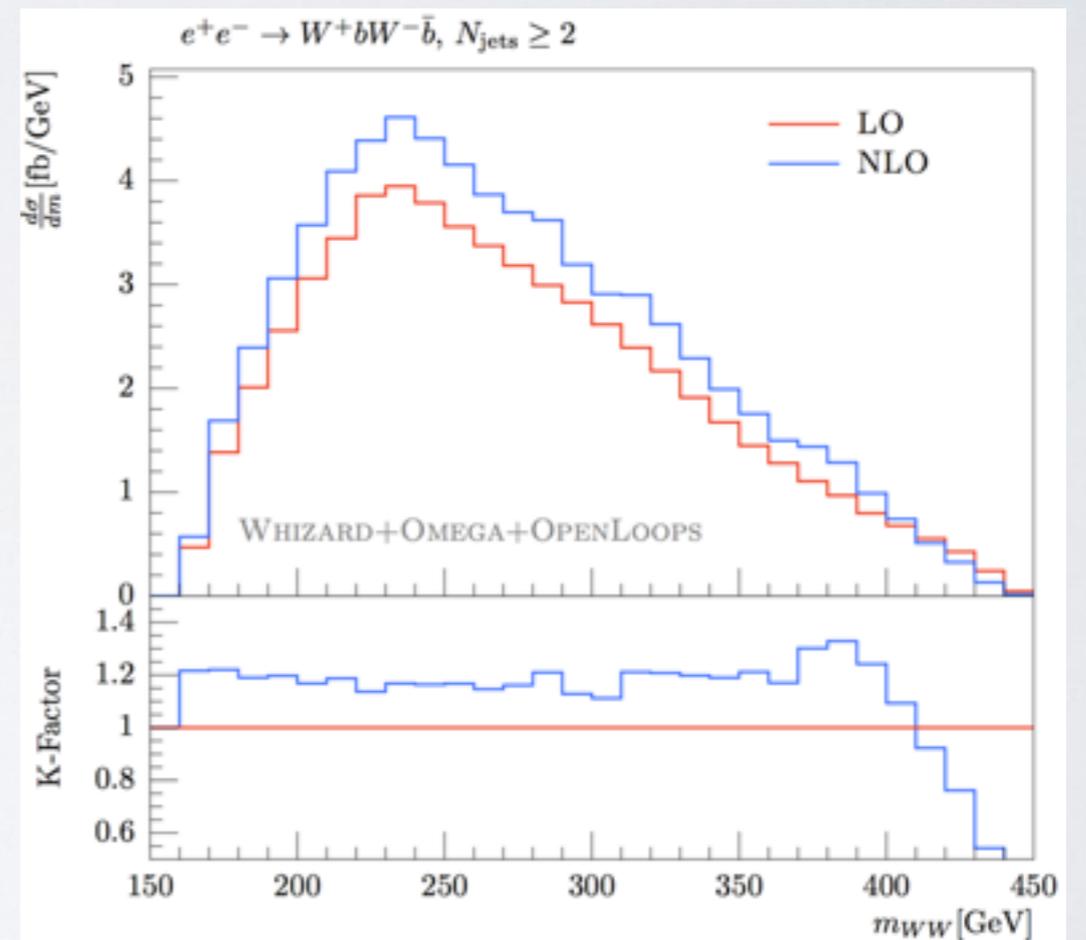
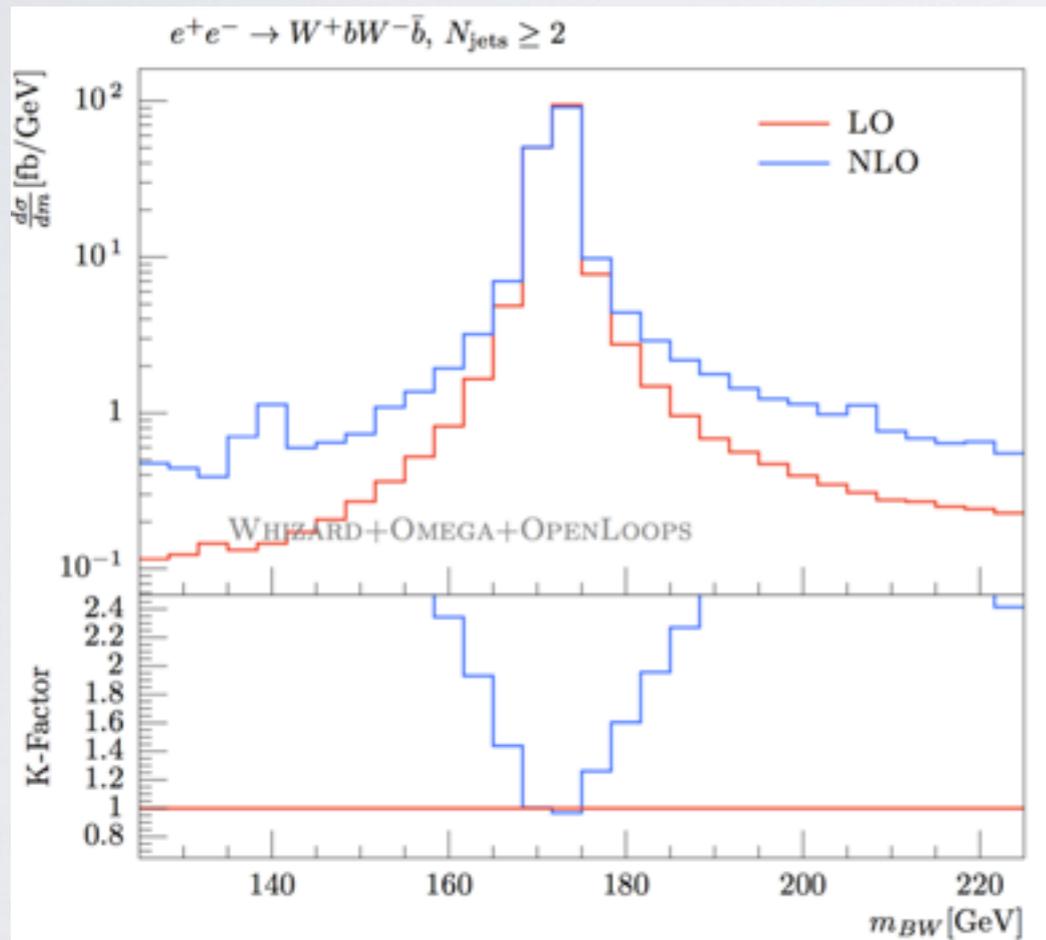
FKS with resonance mappings





NLO Fixed-Order Events

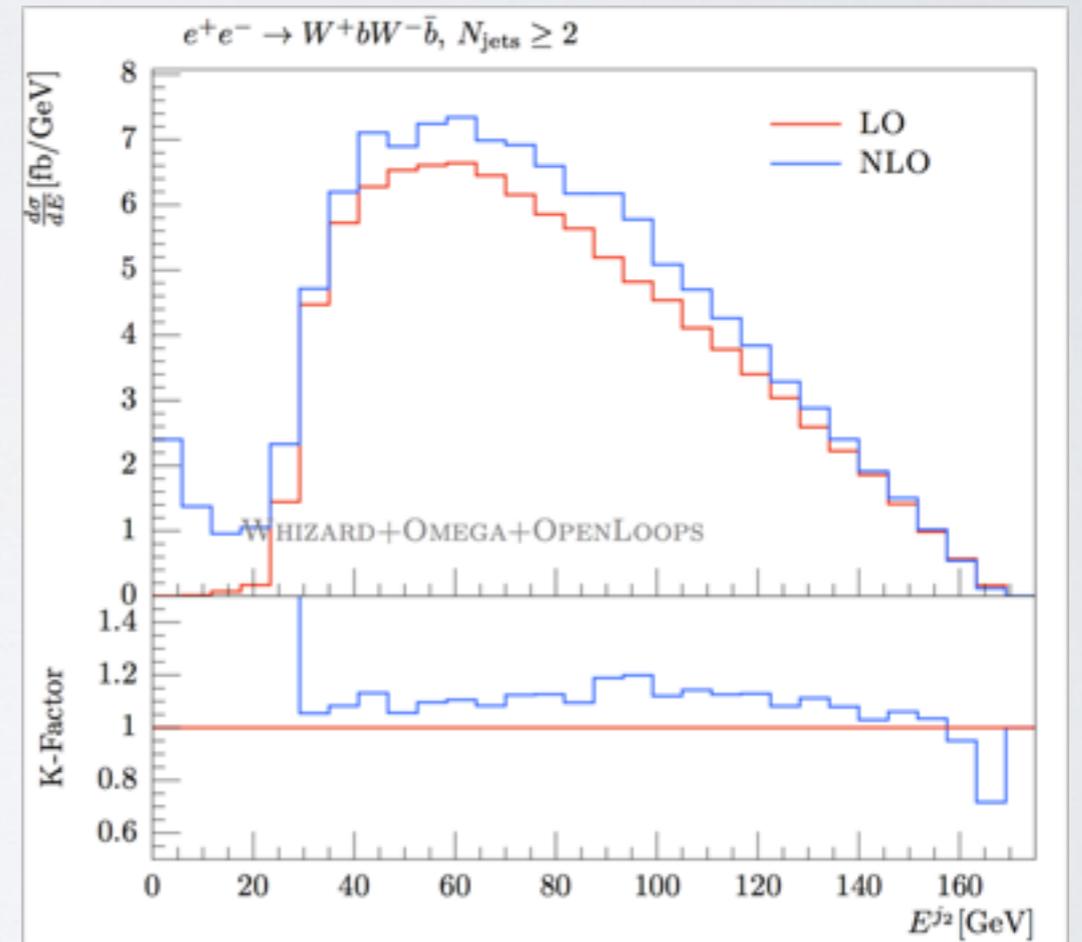
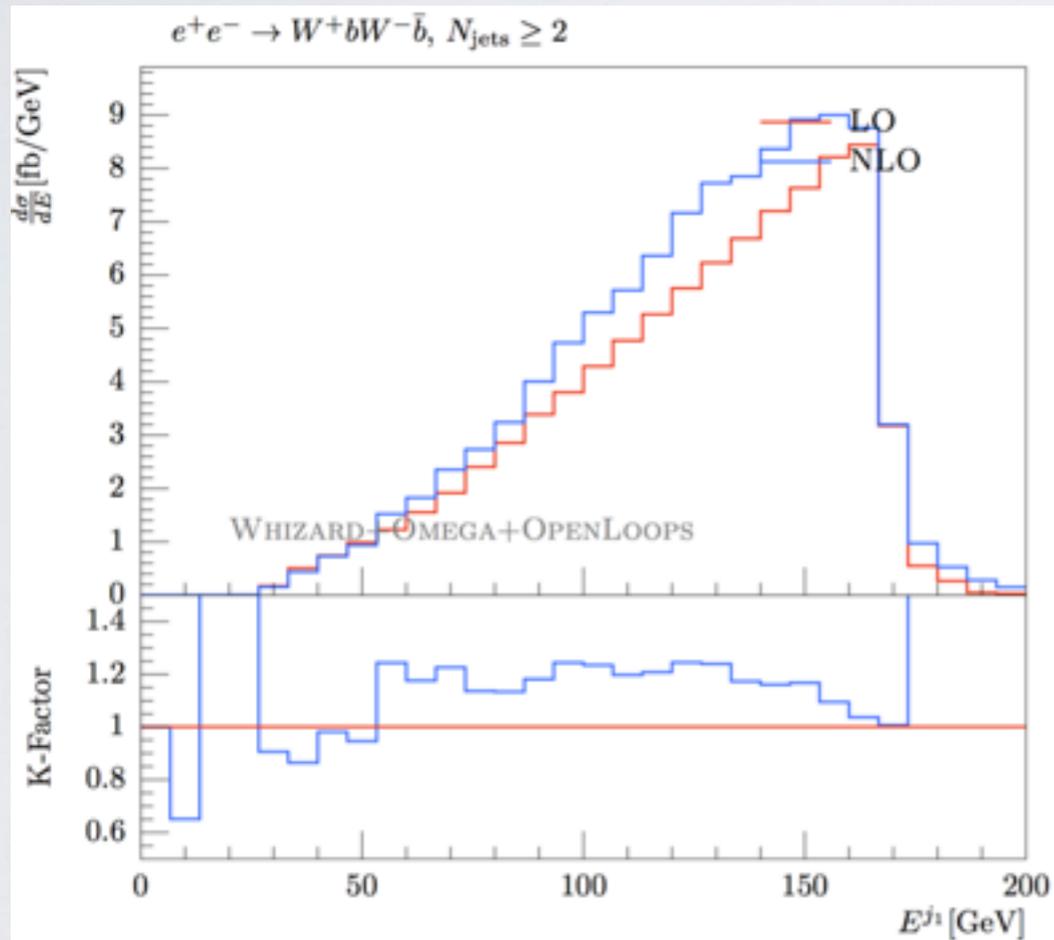
- Add weights of real emission events to weight of Born kinematics using the FKS mapping
- Output weighted events in WHIZARD (e.g. using HepMC), then analysis with Rivet
- Example process: $e^+e^- \rightarrow W^+W^-b\bar{b}$





NLO Fixed-Order Events

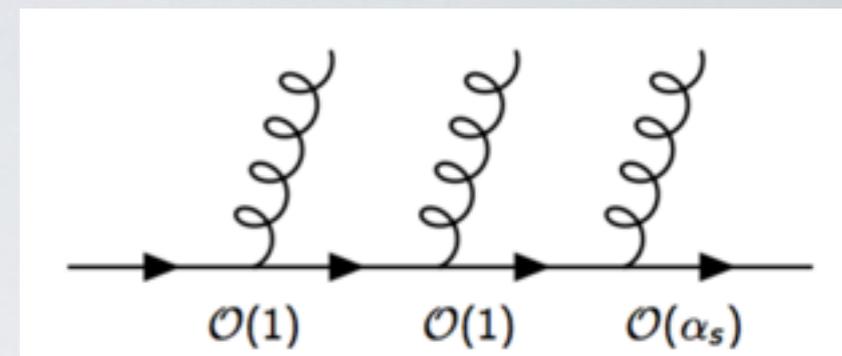
- Add weights of real emission events to weight of Born kinematics using the FKS mapping
- Output weighted events in WHIZARD (e.g. using HepMC), then analysis with Rivet
- Example process: $e^+e^- \rightarrow W^+W^-b\bar{b}$





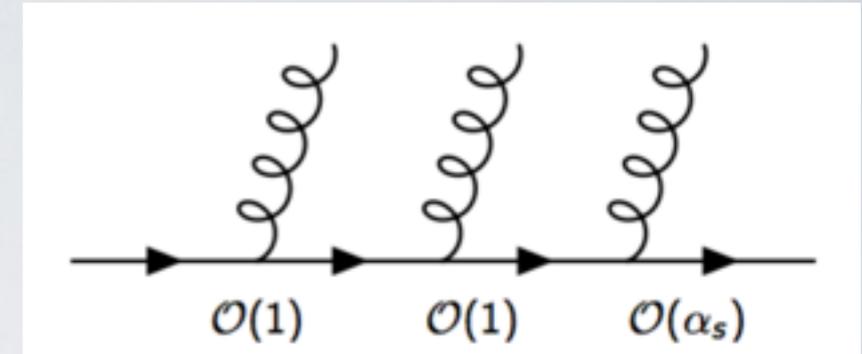
Automated POWHEG Matching in WHIZARD

- Soft gluon emissions before hard emission generate large logs
- Perturbative α_s : $|\mathcal{M}_{\text{soft}}|^2 \sim \frac{1}{k_T^2} \rightarrow \log \frac{k_T^{\text{max}}}{k_T^{\text{min}}}$
- Consistent matching of NLO matrix element with shower
- **POWHEG method**: hardest emission first [Nason et al.]





- **Soft gluon emissions before hard emission generate large logs**
- Perturbative α_s : $|\mathcal{M}_{\text{soft}}|^2 \sim \frac{1}{k_T^2} \rightarrow \log \frac{k_T^{\text{max}}}{k_T^{\text{min}}}$
- Consistent matching of NLO matrix element with shower
- **POWHEG method**: hardest emission first [Nason et al.]



- **Complete NLO events**

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_{\text{rad}} R(\Phi_{n+1})$$

- POWHEG generate events according to the formula:

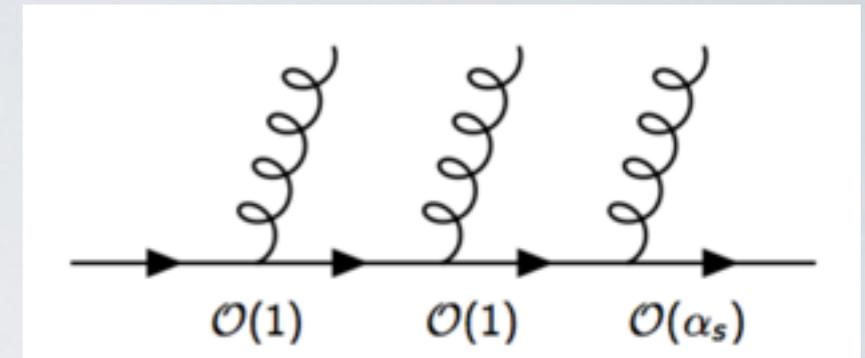
$$d\sigma = \bar{B}(\Phi_n) \left[\Delta_R^{\text{NLO}}(k_T^{\text{min}}) + \Delta_R^{\text{NLO}}(k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\text{rad}} \right]$$

- **Uses the modified Sudakov form factor:**

$$\Delta_R^{\text{NLO}}(k_T) = \exp \left[- \int d\Phi_{\text{rad}} \frac{R(\Phi_{n+1})}{B(\Phi_n)} \theta(k_T(\Phi_{n+1}) - k_T) \right]$$



- **Soft gluon emissions before hard emission generate large logs**
- Perturbative α_s : $|\mathcal{M}_{\text{soft}}|^2 \sim \frac{1}{k_T^2} \rightarrow \log \frac{k_T^{\text{max}}}{k_T^{\text{min}}}$
- Consistent matching of NLO matrix element with shower
- **POWHEG method**: hardest emission first [Nason et al.]



- **Complete NLO events**

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_{\text{rad}} R(\Phi_{n+1})$$

- POWHEG generate events according to the formula:

$$d\sigma = \bar{B}(\Phi_n) \left[\Delta_R^{\text{NLO}}(k_T^{\text{min}}) + \Delta_R^{\text{NLO}}(k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\text{rad}} \right]$$

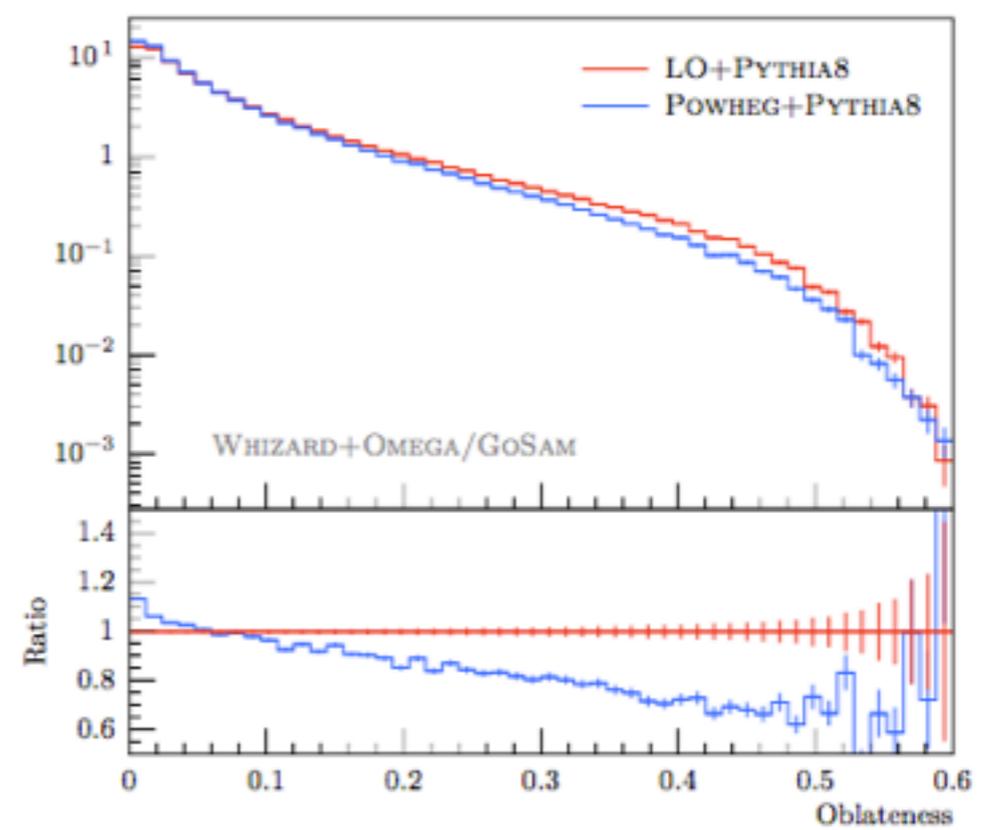
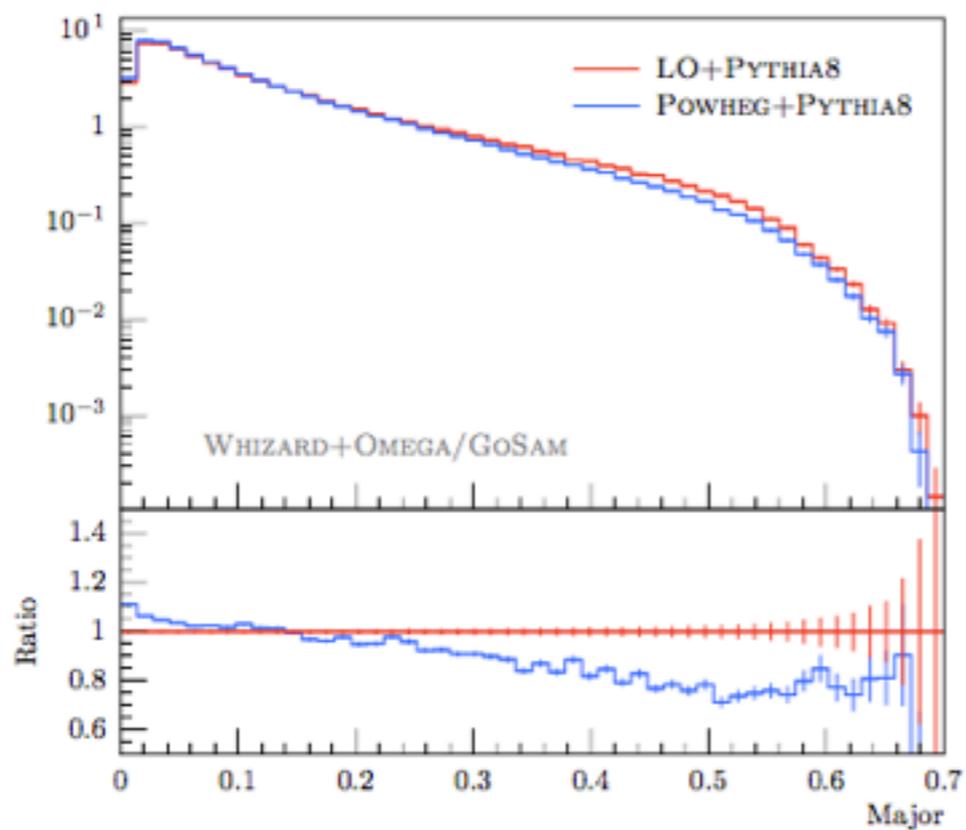
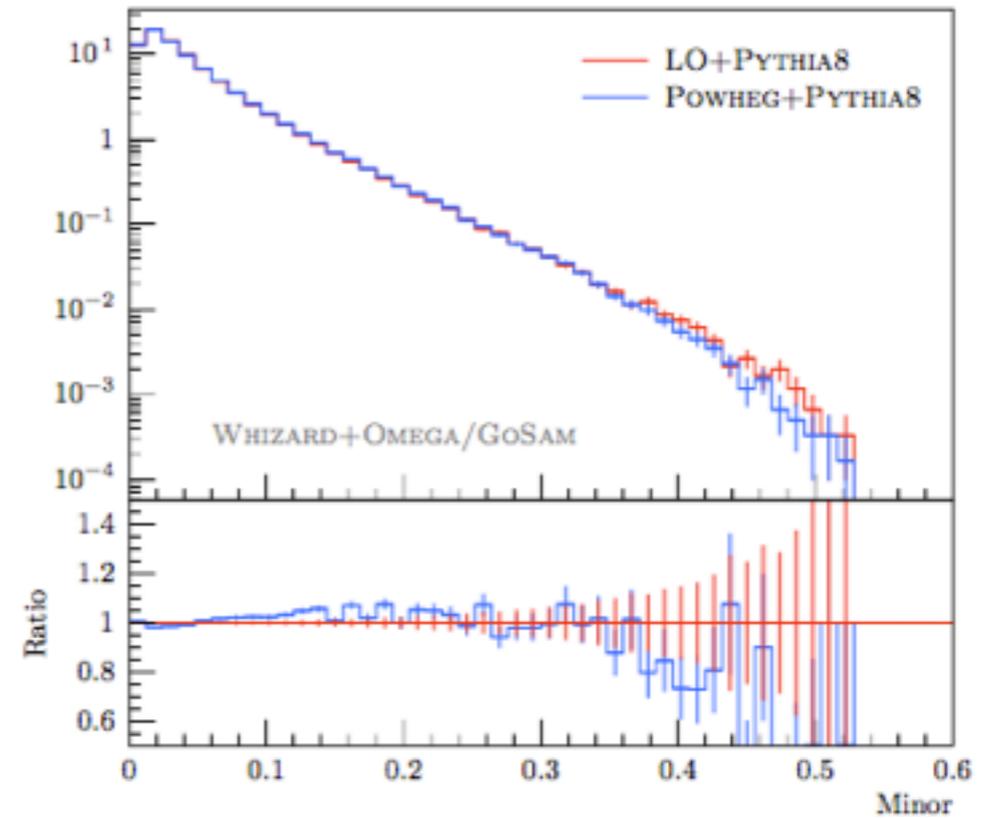
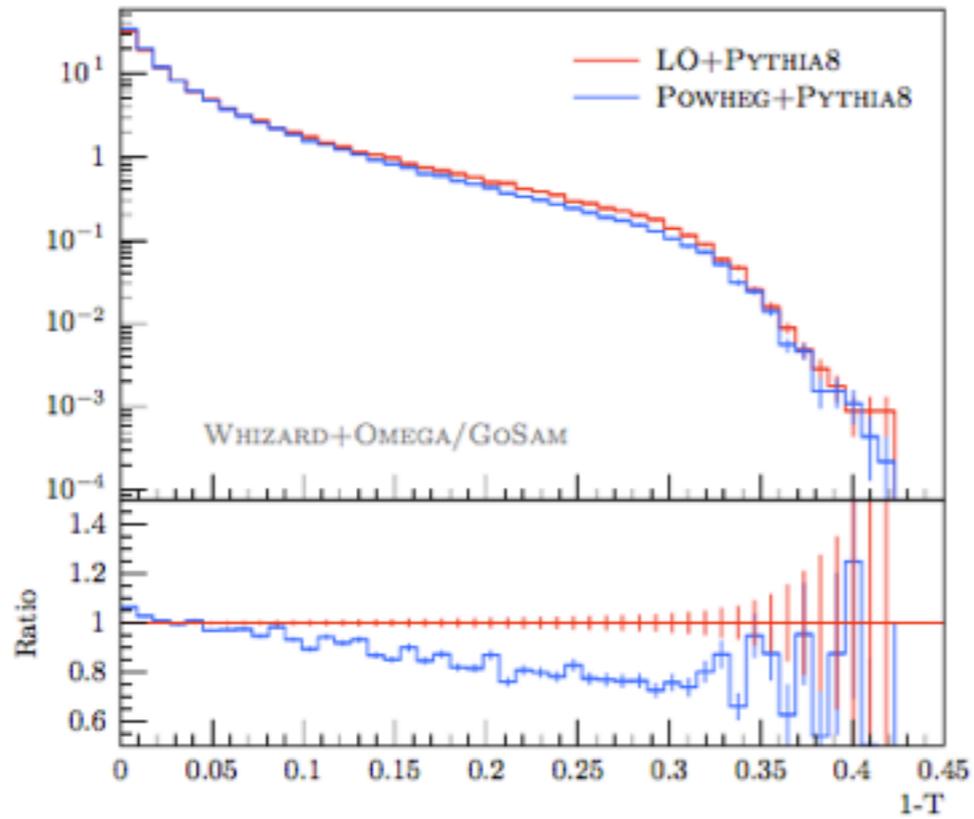
- **Uses the modified Sudakov form factor:**

$$\Delta_R^{\text{NLO}}(k_T) = \exp \left[- \int d\Phi_{\text{rad}} \frac{R(\Phi_{n+1})}{B(\Phi_n)} \theta(k_T(\Phi_{n+1}) - k_T) \right]$$

- Hardest emission: k_T^{max} ; shower with **imposing a veto**
- $\bar{B} < 0$ if virtual and real terms larger than Born: shouldn't happen in perturbative regions
- Reweighting such that $\bar{B} > 0$ for all events
- **POWHEG: Positive Weight Hardest Emission Generator** own implementation in WHIZARD



POWHEG Matching, example: e^+e^- to dijets



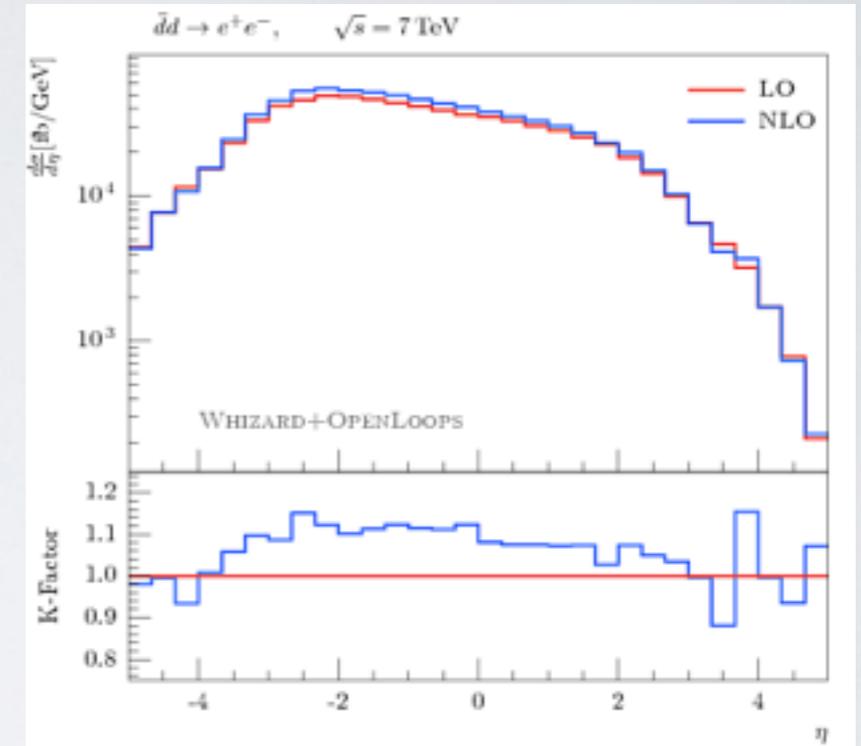
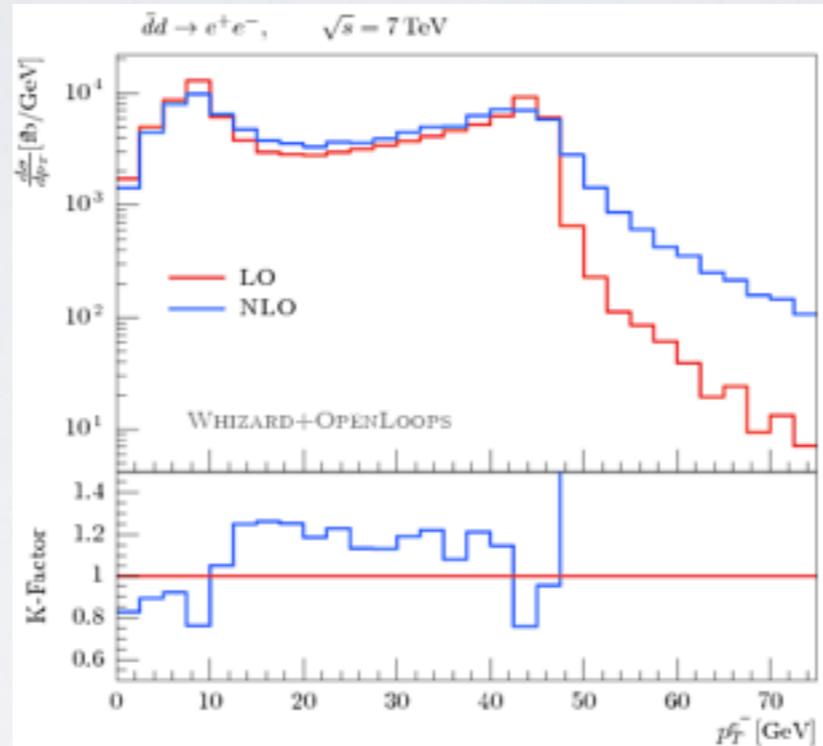
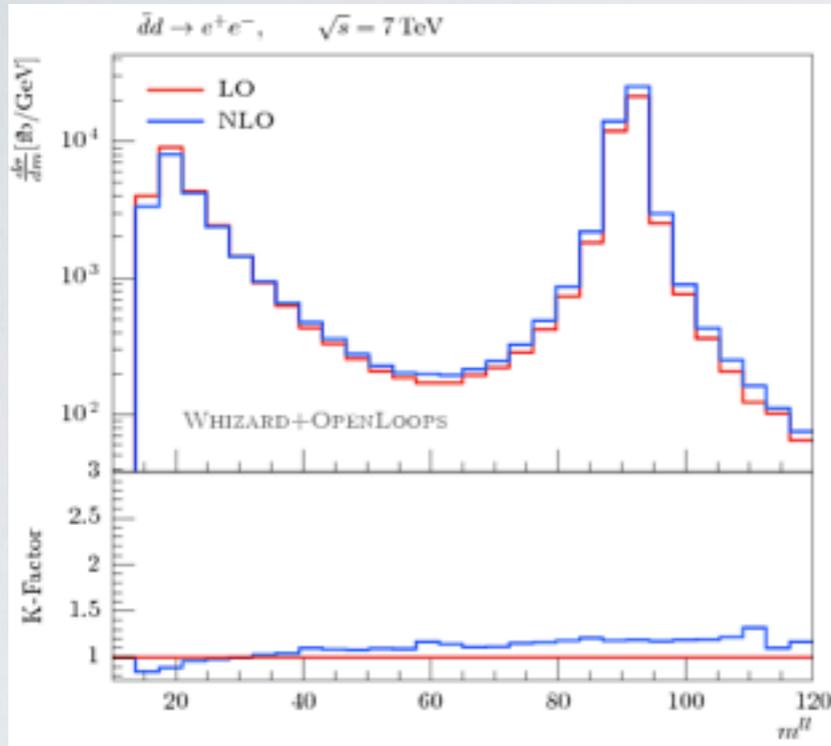


- Simplest hadron collider processes: $pp \rightarrow (Z \rightarrow ll) + X$, $pp \rightarrow (W \rightarrow lv) + X$, $pp \rightarrow ZZ + X$
- Standard candle processes



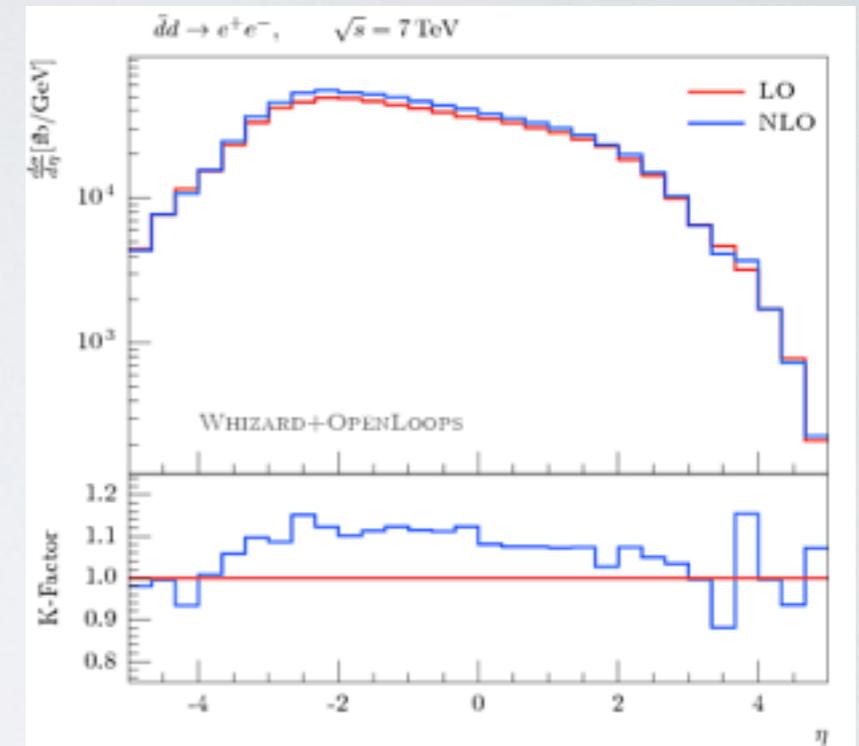
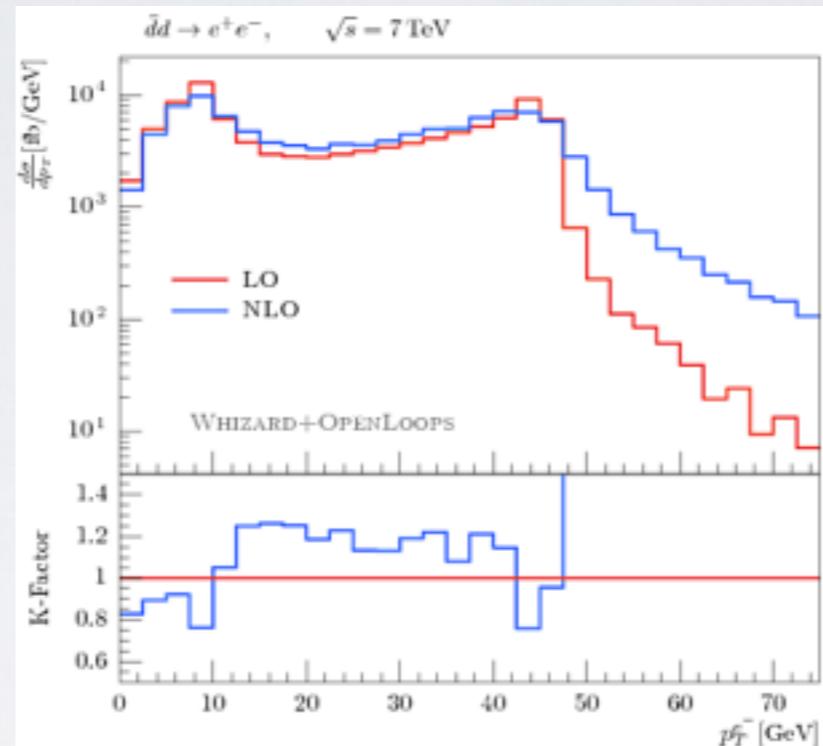
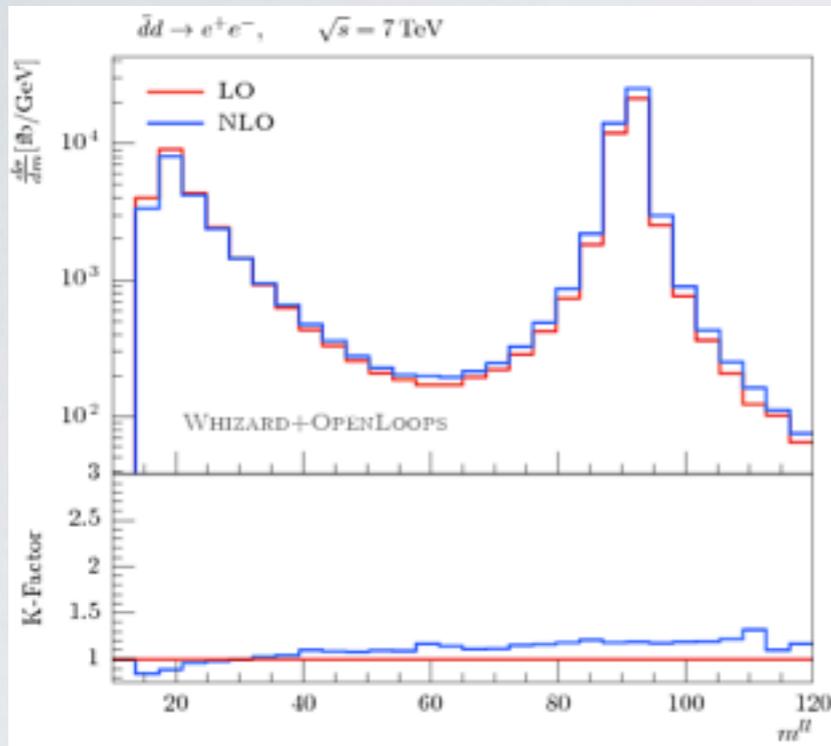
Prime WHIZARD LHC example: Drell-Yan

- Simplest hadron collider processes: $pp \rightarrow (Z \rightarrow ll) + X$, $pp \rightarrow (W \rightarrow lv) + X$, $pp \rightarrow ZZ + X$
- Standard candle processes





- Simplest hadron collider processes: $pp \rightarrow (Z \rightarrow ll) + X$, $pp \rightarrow (W \rightarrow lv) + X$, $pp \rightarrow ZZ + X$
- Standard candle processes



Open points / next steps:

- Flavor sums in fixed-order event generation
- Color in initial and final state (already validated for top decay)
- Gluons in the initial state
- Next processes: $pp \rightarrow Zj + X$, $pp \rightarrow tt + X$, $pp \rightarrow jj + X$
- automated POWHEG matching for hadron collider



Lepton colliders: tt and ttH (on- & off-shell)

- Paradigm processes at lepton colliders: **precision determination of m_t and Y_t**
- Major background for EW measurements (VW and VBS); any [most] BSM searches
- Investigate processes of increasing complexity: $2 \rightarrow 2$ to $2 \rightarrow 4$ to $2 \rightarrow 6$

$e^+e^- \rightarrow$	$n_{\text{loop diags}}$	Max. prop.	n_{hel}
$t\bar{t}$	2	3	16
$W^+W^-b\bar{b}$	157	5	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+$	830	5	16
$t\bar{t}H$	17	4	16
$bW^+\bar{b}W^-H$	1548	6	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+ H$	7436	6	16

On-Shell process: $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [[Jersak/Laermann/Zerwas, 1982](#)]
- NNLO QCD [[Chetyrkin/Kühn/Steinhauser, 1996](#); [Harlander/Steinhauser, 1998](#)]
- NLO EW [[Beenakker/von der Marck/Hollik, 1991](#); [Beenakker/Denner/Kraft, 1993](#)]
- Threshold enhancement [[Fadin/Khoze, 1987](#); [Strassler/Peskin, 1991](#); [Jezabek/Kühn/Teubner, 1992](#); [Sumino et al., 1992](#)]



- Paradigm processes at lepton colliders: **precision determination of m_t and Y_t**
- Major background for EW measurements (VVV and VBS); any [most] BSM searches
- Investigate processes of increasing complexity: $2 \rightarrow 2$ to $2 \rightarrow 4$ to $2 \rightarrow 6$

$e^+e^- \rightarrow$	$n_{\text{loop diags}}$	Max. prop.	n_{hel}
$t\bar{t}$	2	3	16
$W^+W^-b\bar{b}$	157	5	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+$	830	5	16
$t\bar{t}H$	17	4	16
$bW^+\bar{b}W^-H$	1548	6	144
$b\bar{b}\bar{\nu}_e e^- \nu_\mu \mu^+ H$	7436	6	16

Off-Shell process: $e^+e^- \rightarrow W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] ✗
- NLO QCD in MC [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanastasiou, 2015; Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, *in prep.*]

On-Shell process: $e^+e^- \rightarrow t\bar{t}$

- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]



Lepton colliders: tt and ttH (on- & off-shell)

- Paradigm processes at lepton colliders: **precision determination of m_t and Y_t**
- Major background for EW measurements (VVV and VBS); any [most] BSM searches
- Investigate processes of increasing complexity: $2 \rightarrow 2$ to $2 \rightarrow 4$ to $2 \rightarrow 6$

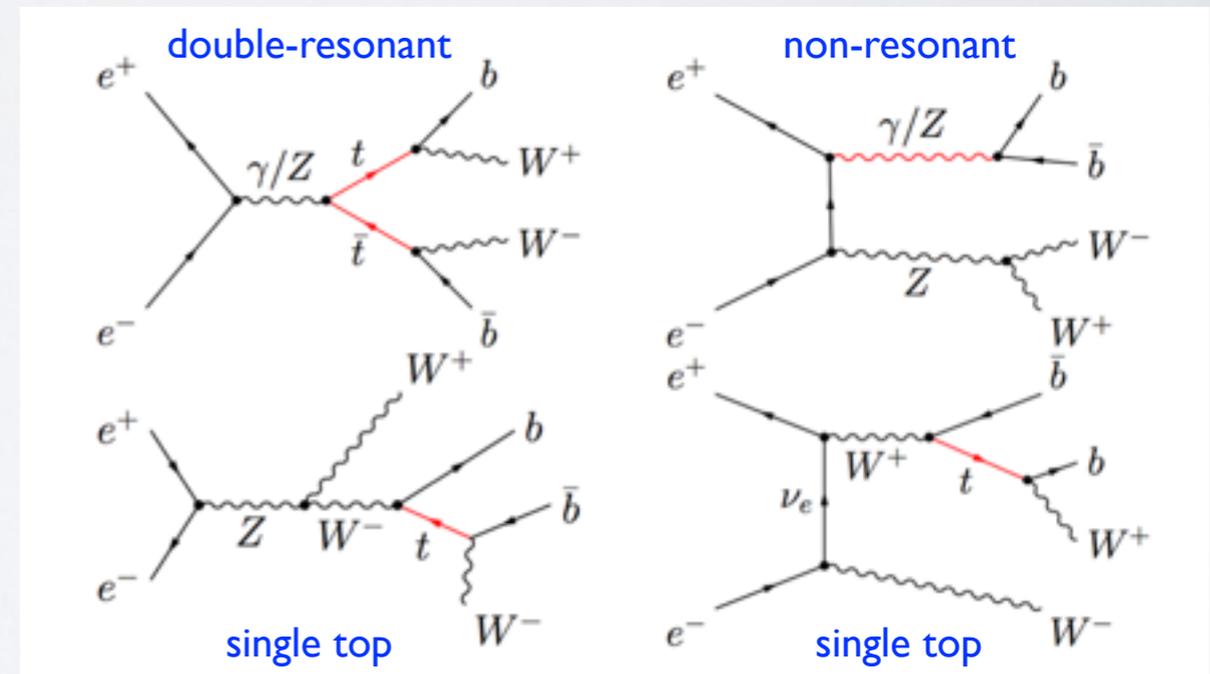
$e^+e^- \rightarrow$	$n_{\text{loop diags}}$	Max. prop.	n_{hel}
$t\bar{t}$	2	3	16
$W^+W^-b\bar{b}$	157	5	144
$b\bar{b}\nu_e e^- \nu_\mu \mu^+$	830	5	16
$t\bar{t}H$	17	4	16
$bW^+\bar{b}W^-H$	1548	6	144
$b\bar{b}\nu_e e^- \nu_\mu \mu^+ H$	7436	6	16

Off-Shell process: $e^+e^- \rightarrow W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] ✗
- NLO QCD in MC [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanastasiou, 2015; Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, *in prep.*]

On-Shell process: $e^+e^- \rightarrow t\bar{t}$

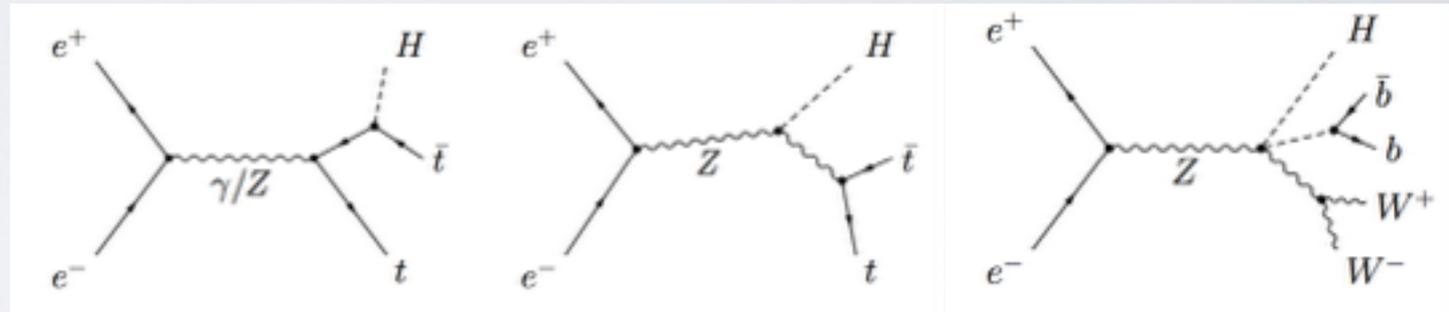
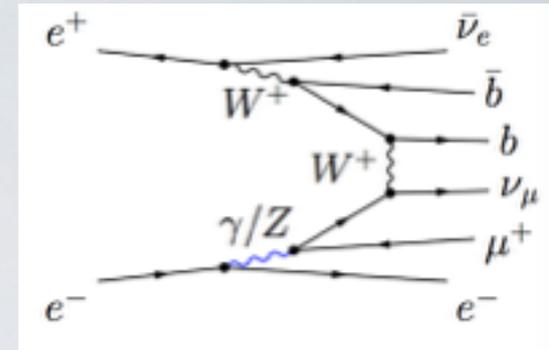
- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]





Lepton colliders: tt and ttH (on- & off-shell)

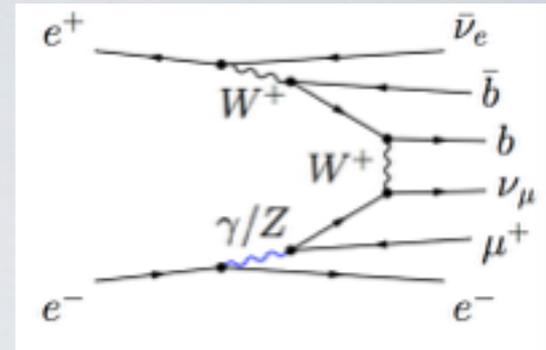
- Cross checks for $2 \rightarrow 2$ and $2 \rightarrow 4$ processes with Sherpa and Munich
- Using massive b quarks: no cuts necessary for $e^+e^- \rightarrow W^+W^-bb$
- Full process $e^+e^- \rightarrow \mu^+\nu_\mu e^-\bar{\nu}_e bb$ exhibits collinear singularity:
- ttH production: 8% contamination from Higgsstrahlung
- Contribution from quartic SM vertices



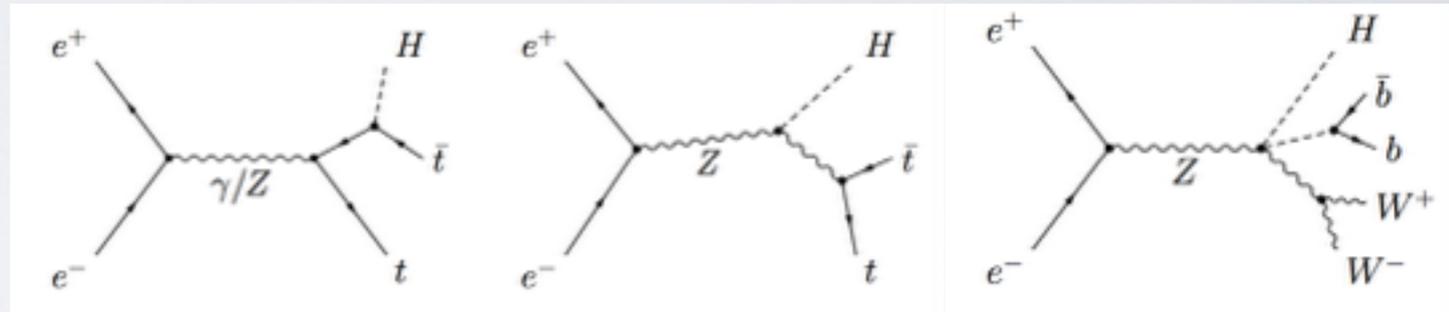


Lepton colliders: tt and ttH (on- & off-shell)

- Cross checks for $2 \rightarrow 2$ and $2 \rightarrow 4$ processes with Sherpa and Munich
- Using massive b quarks: no cuts necessary for $e^+e^- \rightarrow W^+W^-bb$
- Full process $e^+e^- \rightarrow \mu^+\nu_\mu e^-\bar{\nu}_e bb$ exhibits collinear singularity:
- ttH production: 8% contamination from Higgsstrahlung
- Contribution from quartic SM vertices



INPUT PARAMETERS:



$$m_Z = 91.1876 \text{ GeV},$$

$$m_b = 4.2 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV}$$

$$m_t = 173.2 \text{ GeV}.$$

$$\Gamma_Z^{\text{LO}} = 2.4409 \text{ GeV},$$

$$\Gamma_W^{\text{LO}} = 2.0454 \text{ GeV},$$

$$\Gamma_Z^{\text{NLO}} = 2.5060 \text{ GeV},$$

$$\Gamma_W^{\text{NLO}} = 2.0978 \text{ GeV}.$$

$$\Gamma_{t \rightarrow Wb}^{\text{LO}} = 1.4986 \text{ GeV},$$

$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{LO}} = 1.4757 \text{ GeV},$$

$$\Gamma_{t \rightarrow Wb}^{\text{NLO}} = 1.3681 \text{ GeV},$$

$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{NLO}} = 1.3475 \text{ GeV}.$$

$$m_H = 125 \text{ GeV}$$

$$\Gamma_H = 0.000431 \text{ GeV}$$

complex mass scheme:

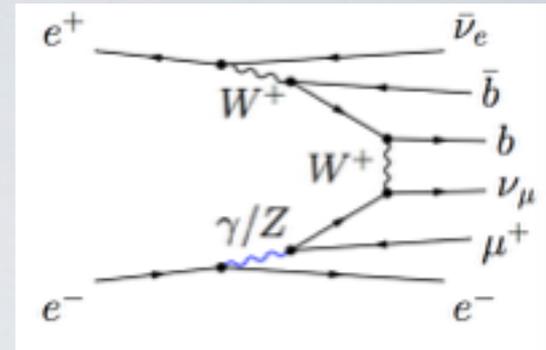
$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_W^2}{\mu_Z^2}$$

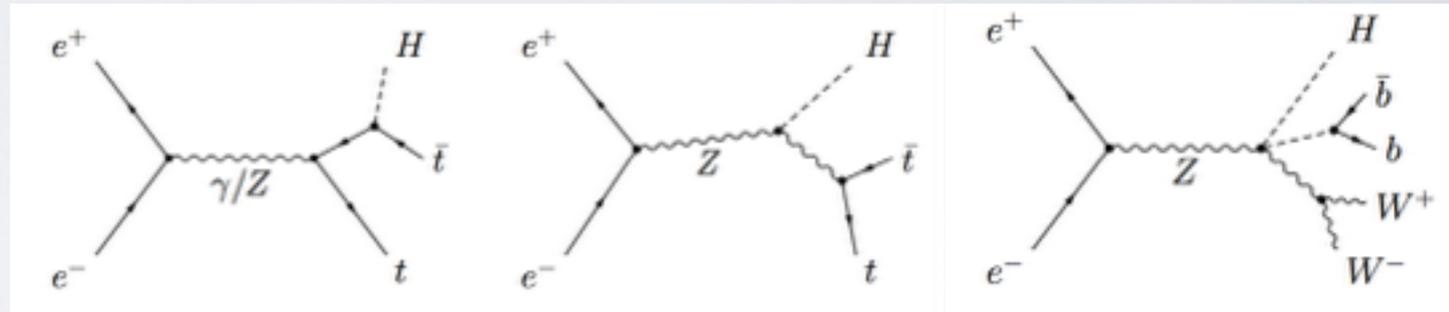


Lepton colliders: tt and ttH (on- & off-shell)

- Cross checks for $2 \rightarrow 2$ and $2 \rightarrow 4$ processes with Sherpa and Munich
- Using massive b quarks: no cuts necessary for $e^+e^- \rightarrow W^+W^-bb$
- Full process $e^+e^- \rightarrow \mu^+\nu_\mu e^- \bar{\nu}_e bb$ exhibits collinear singularity:
- ttH production: 8% contamination from Higgsstrahlung
- Contribution from quartic SM vertices



INPUT PARAMETERS:



$$m_Z = 91.1876 \text{ GeV},$$

$$m_b = 4.2 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV}$$

$$m_t = 173.2 \text{ GeV}.$$

$$\Gamma_Z^{\text{LO}} = 2.4409 \text{ GeV},$$

$$\Gamma_W^{\text{LO}} = 2.0454 \text{ GeV},$$

$$\Gamma_Z^{\text{NLO}} = 2.5060 \text{ GeV},$$

$$\Gamma_W^{\text{NLO}} = 2.0978 \text{ GeV}.$$

$$\Gamma_{t \rightarrow Wb}^{\text{LO}} = 1.4986 \text{ GeV},$$

$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{LO}} = 1.4757 \text{ GeV},$$

$$\Gamma_{t \rightarrow Wb}^{\text{NLO}} = 1.3681 \text{ GeV},$$

$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{NLO}} = 1.3475 \text{ GeV}.$$

complex mass scheme:

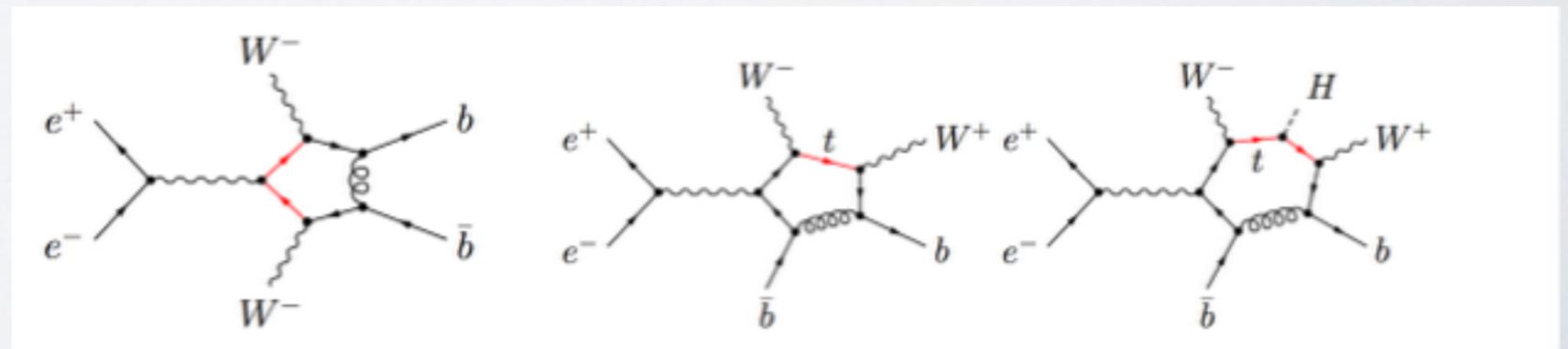
$$m_H = 125 \text{ GeV}$$

$$\Gamma_H = 0.000431 \text{ GeV}$$

$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

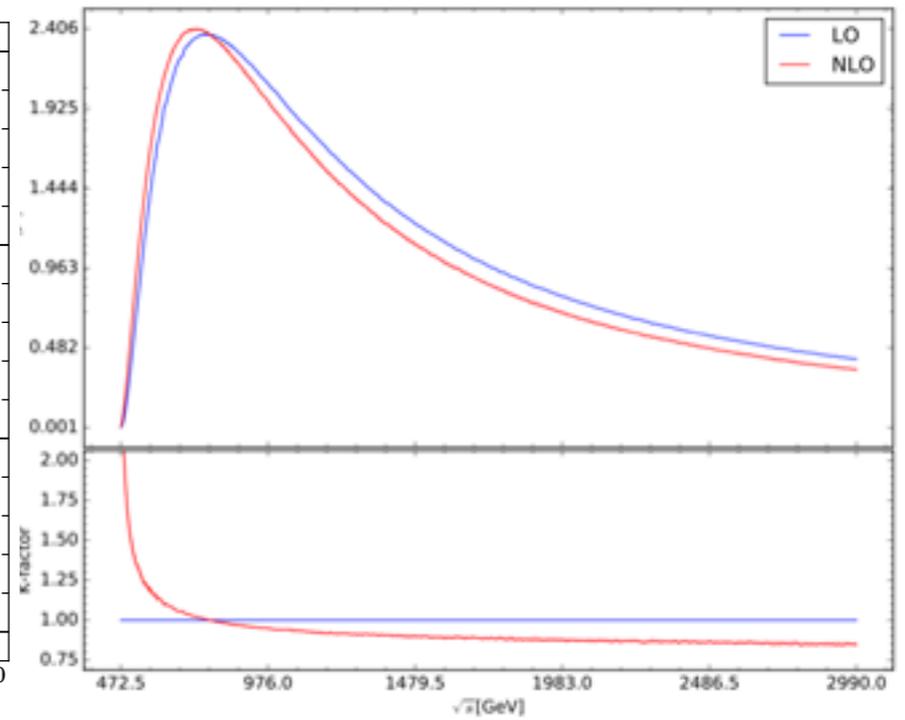
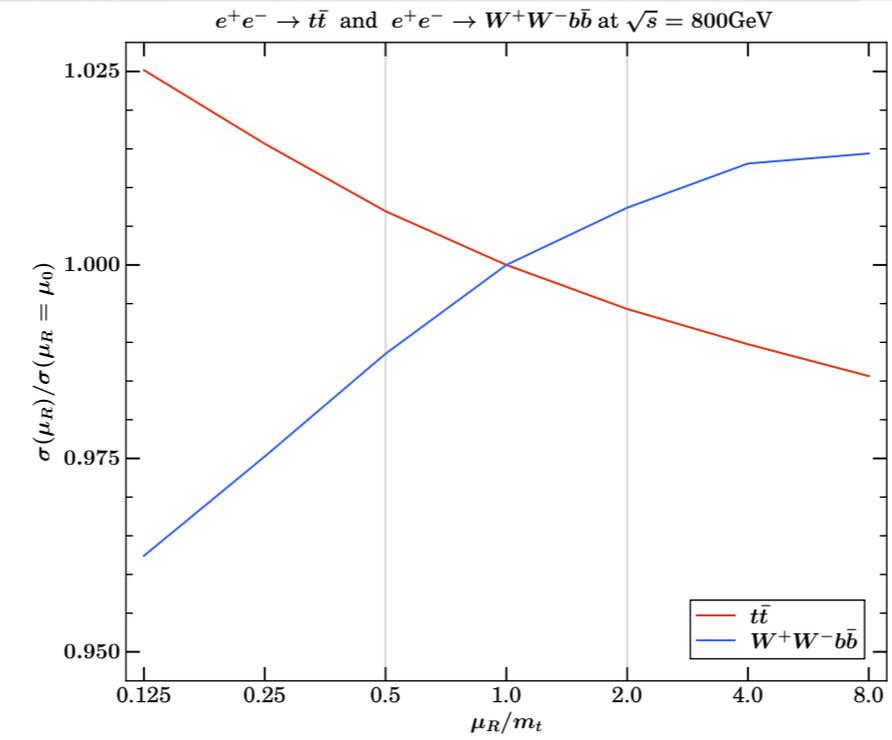
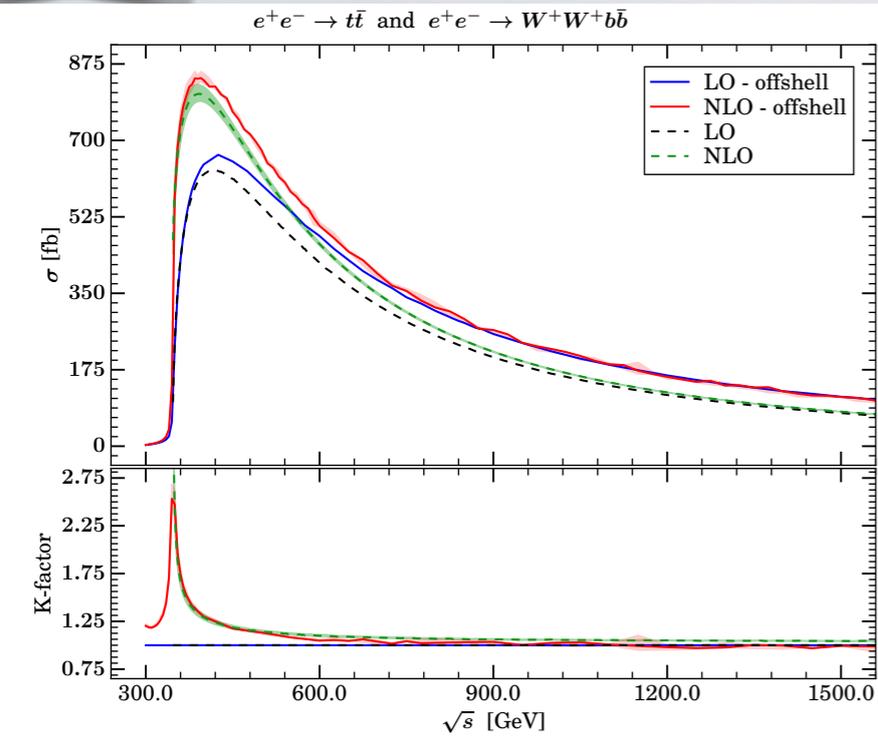
$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_W^2}{\mu_Z^2}$$

- Typical pentagon diagrams:



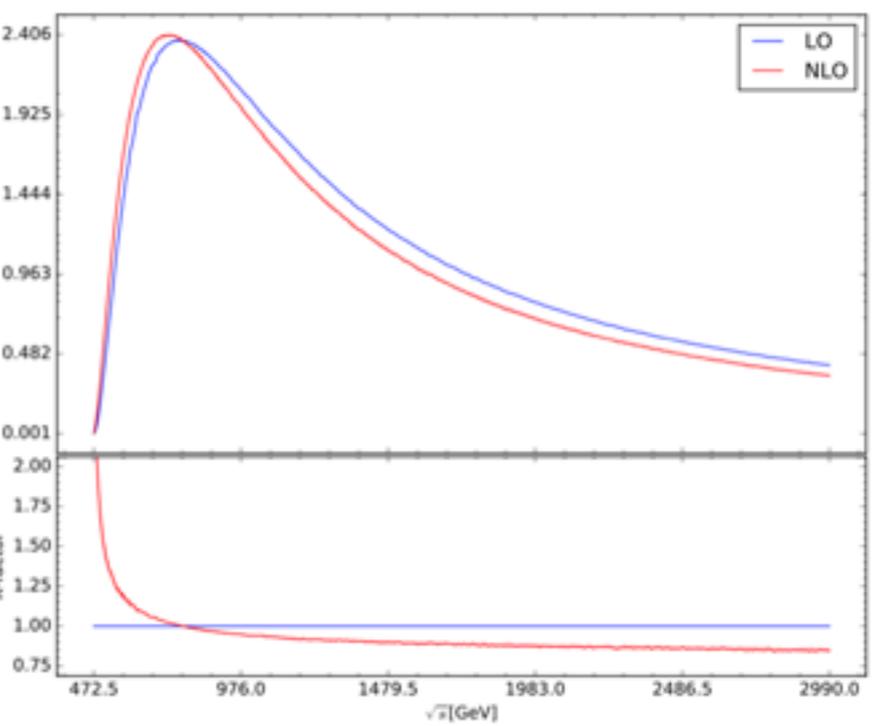
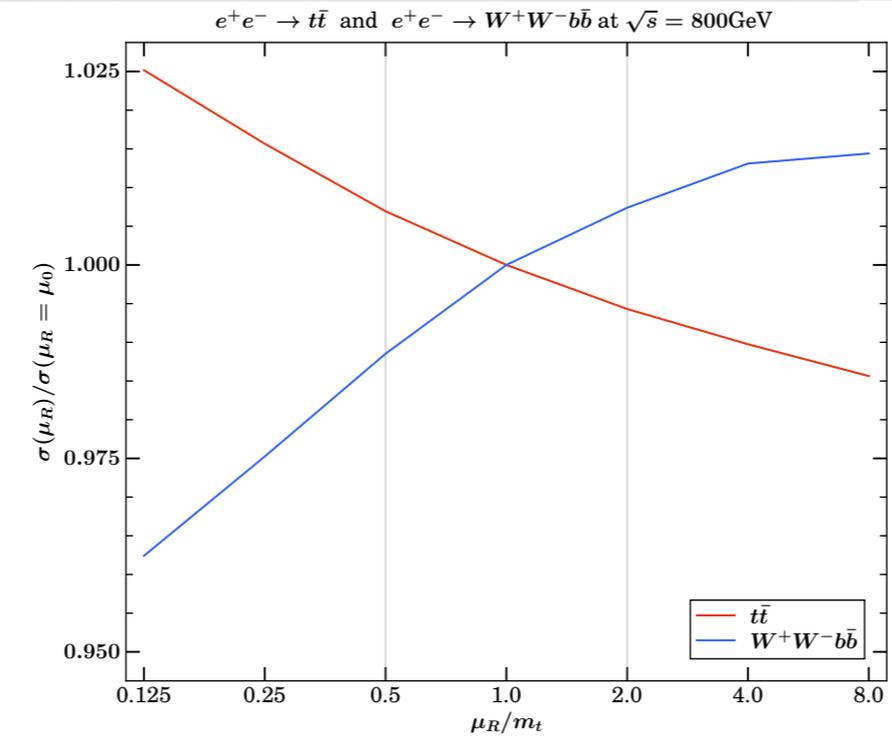
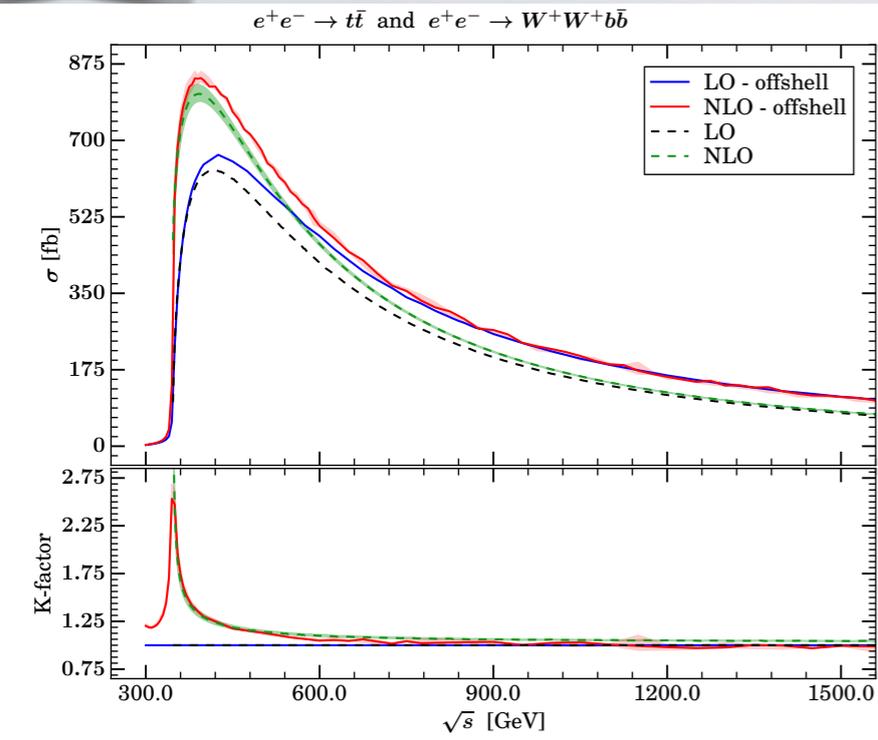


NLO QCD Results for $t\bar{t}$ and $t\bar{t}H$





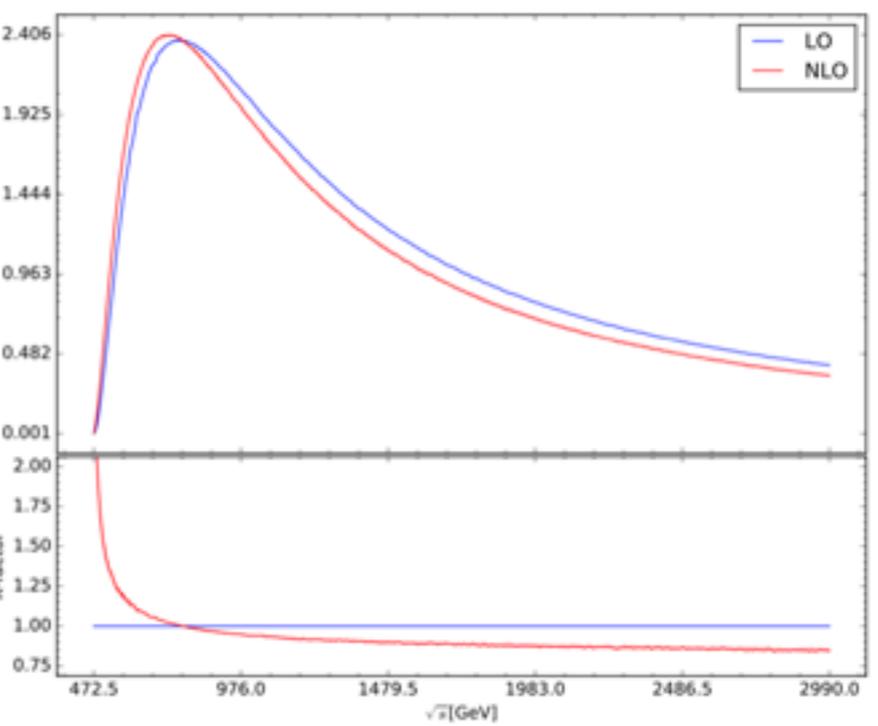
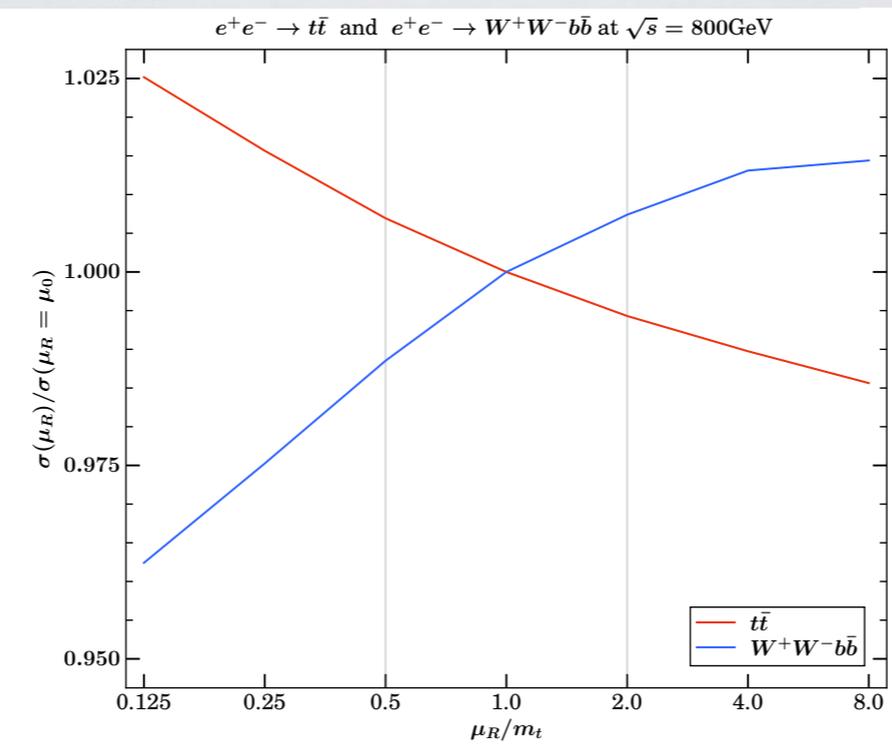
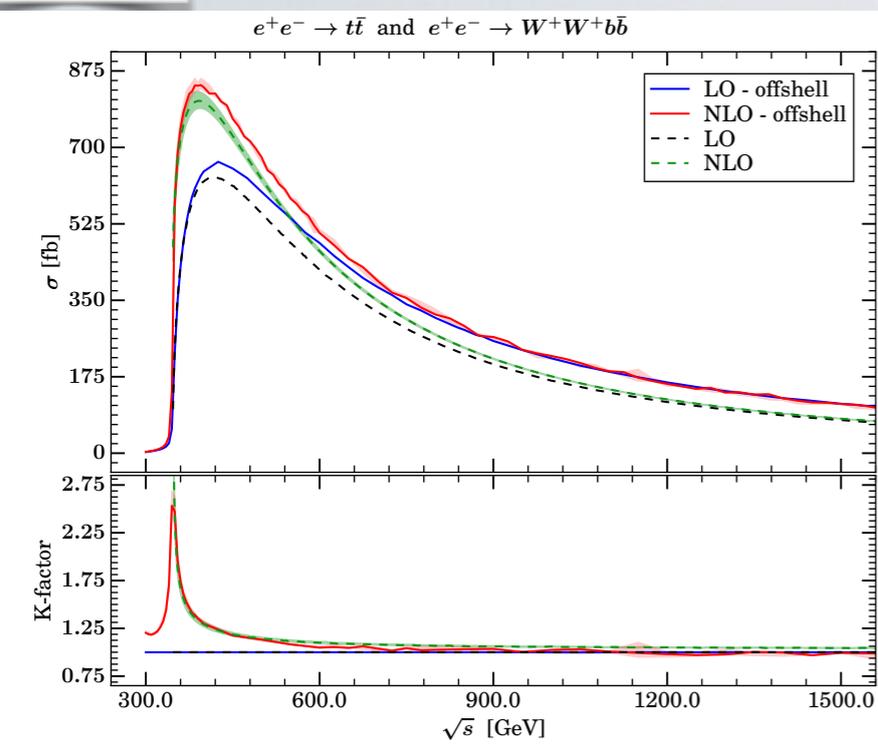
NLO QCD Results for $t\bar{t}$ and $t\bar{t}H$



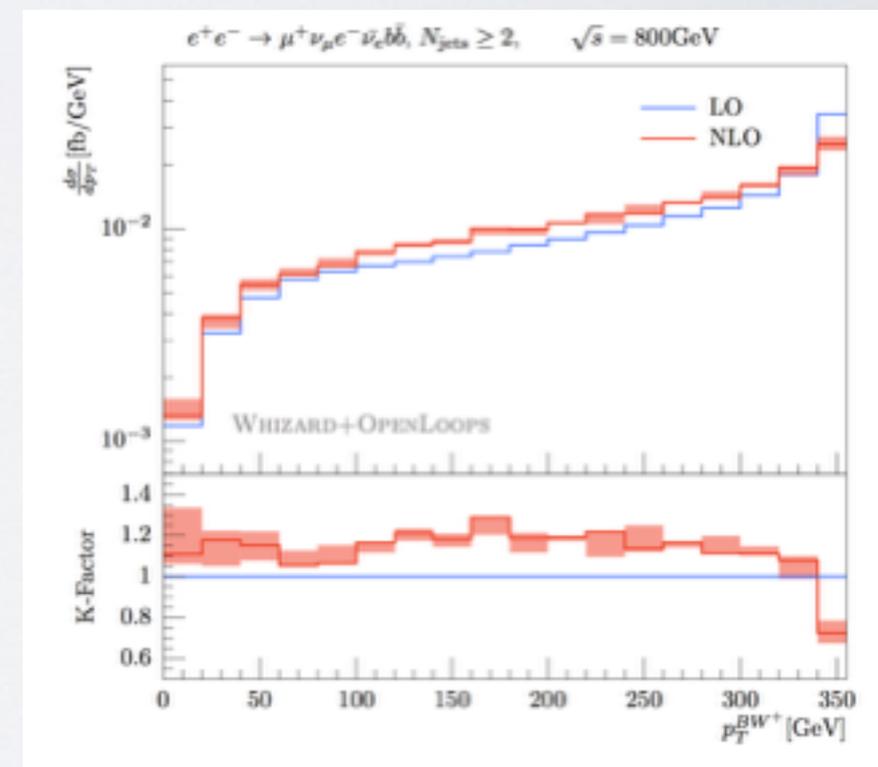
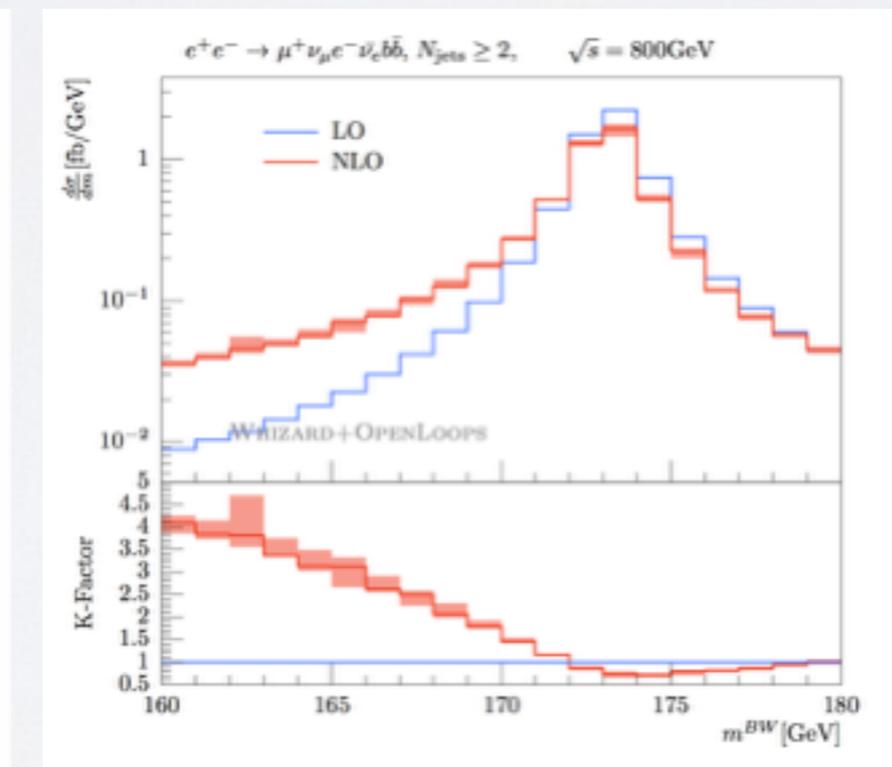
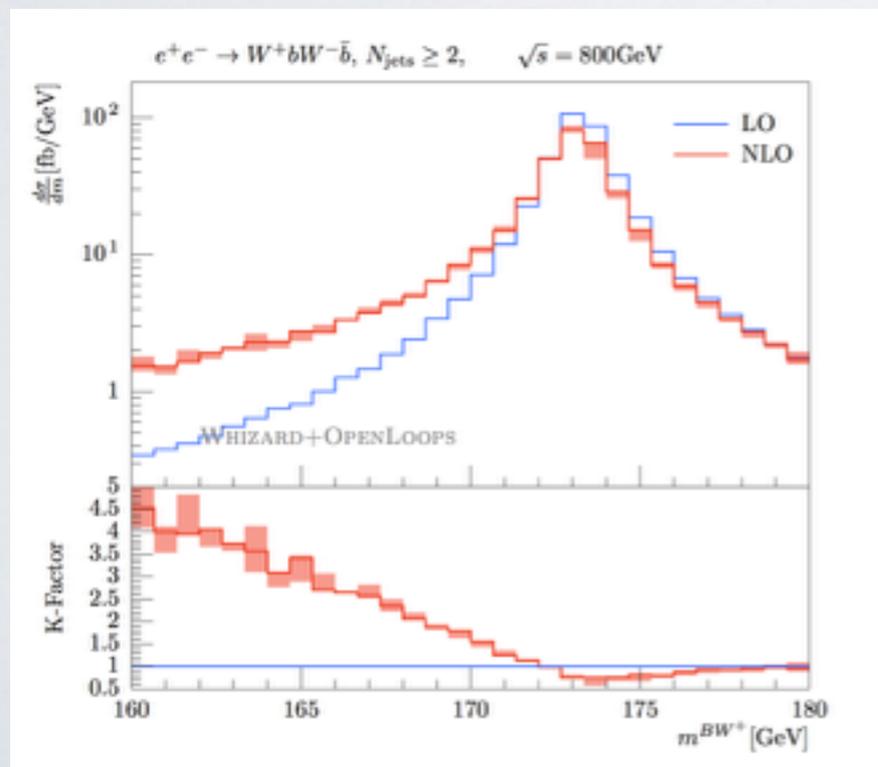
Choose $\sqrt{s} = 800 \text{ GeV}$ because its the maximum of the $t\bar{t}H$ cross section



NLO QCD Results for tt and ttH

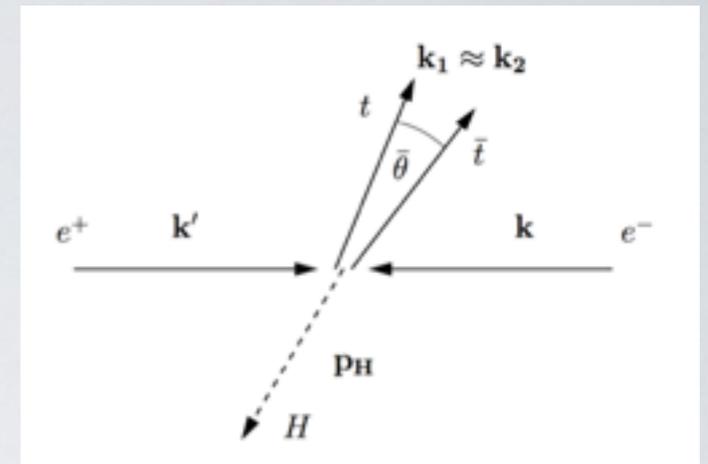
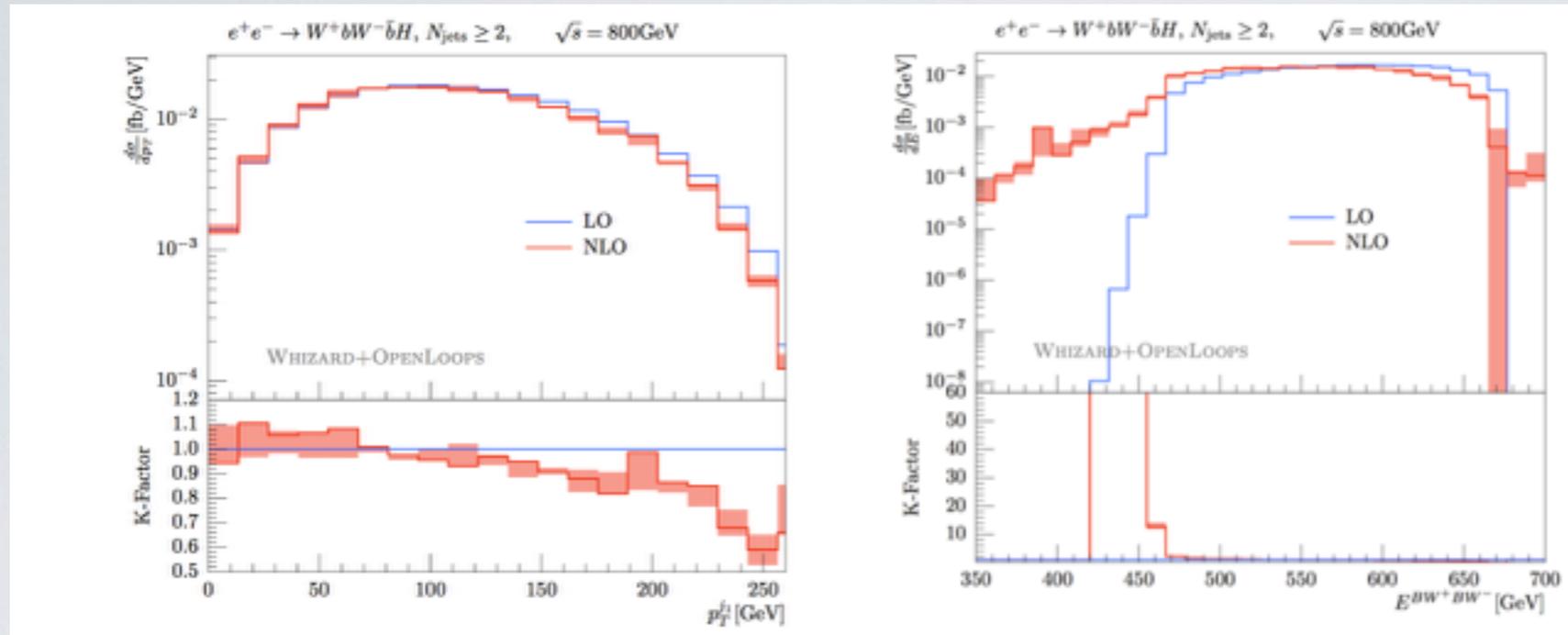


Choose $\sqrt{s} = 800$ GeV because its the maximum of the ttH cross section





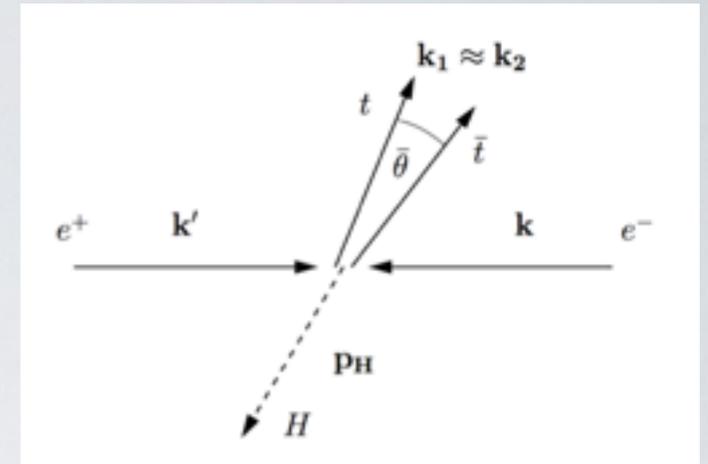
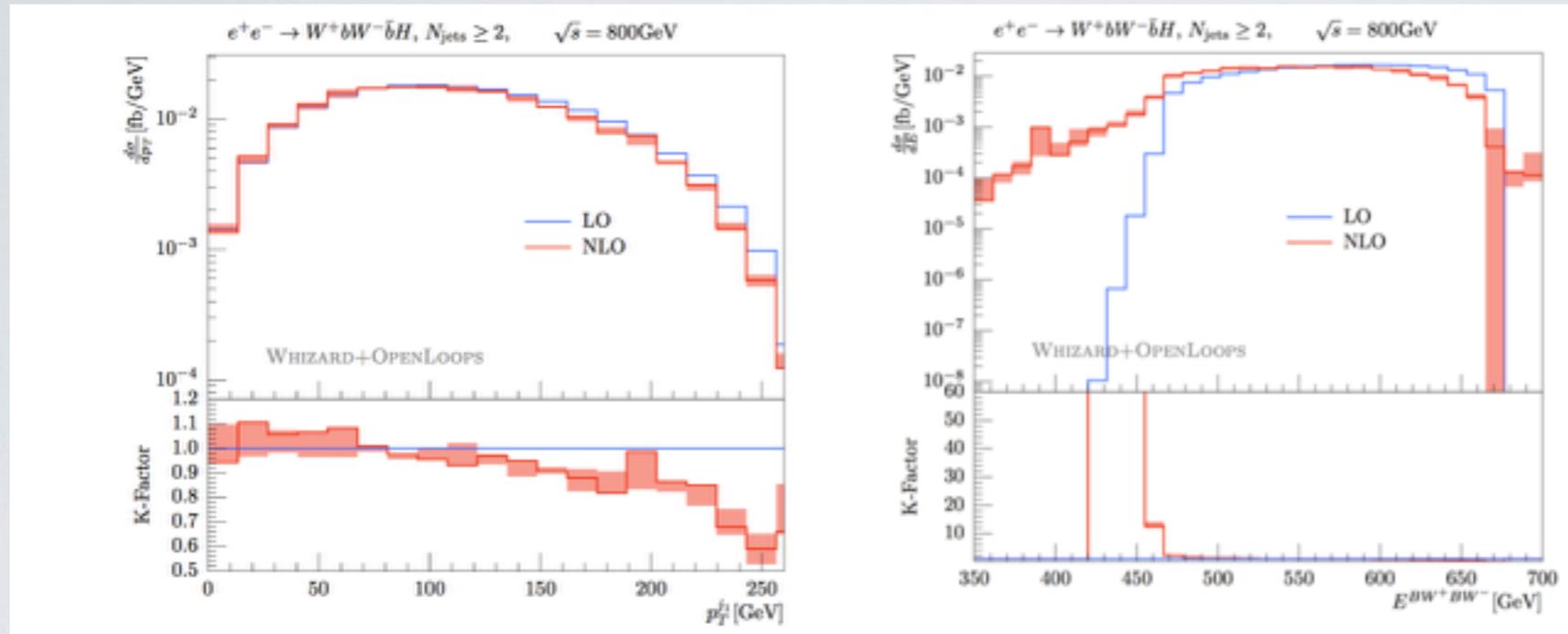
NLO QCD Results for tt and ttH



$$E_h = \frac{1}{2\sqrt{s}} [s + M_h^2 - (k_1 + k_2)^2]$$

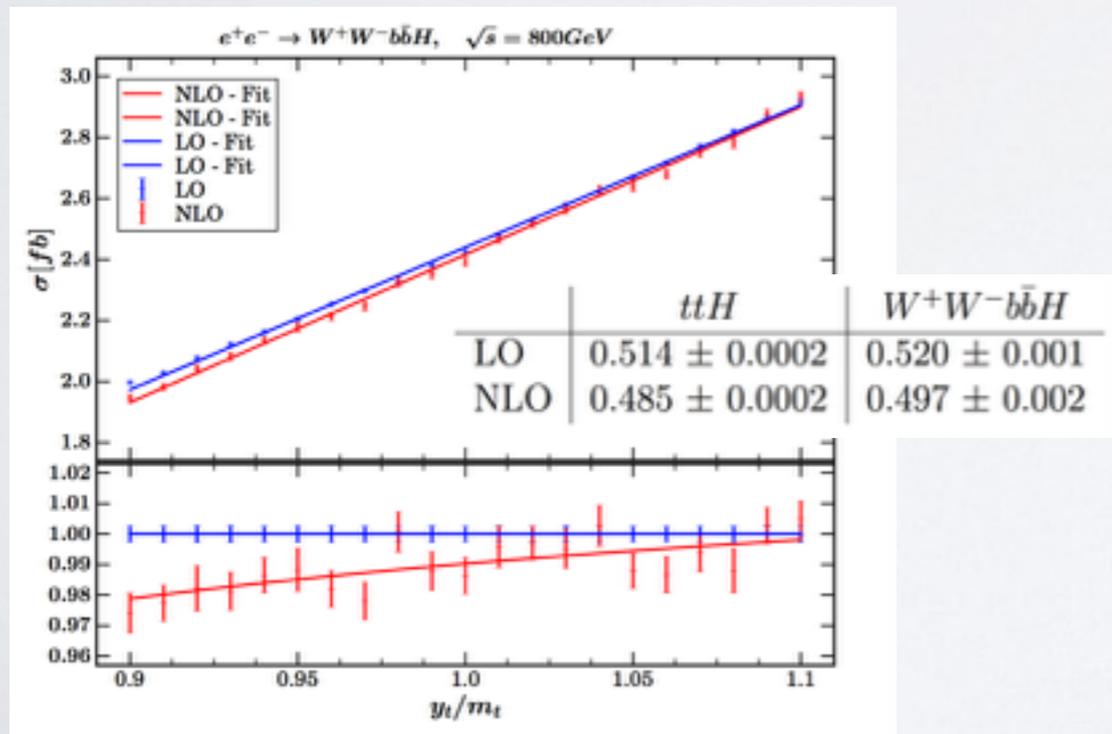


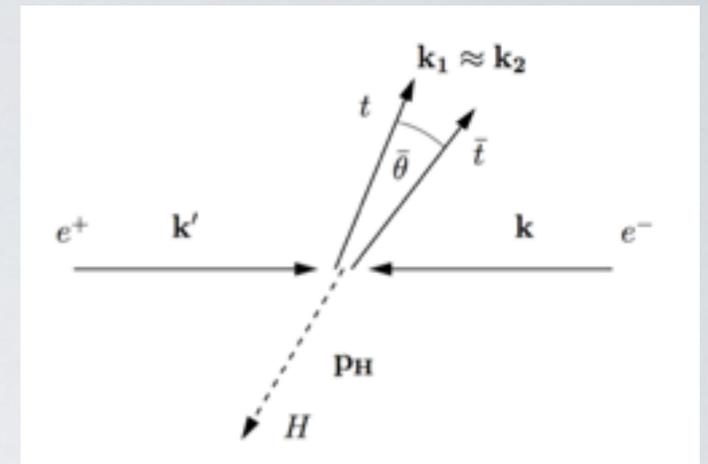
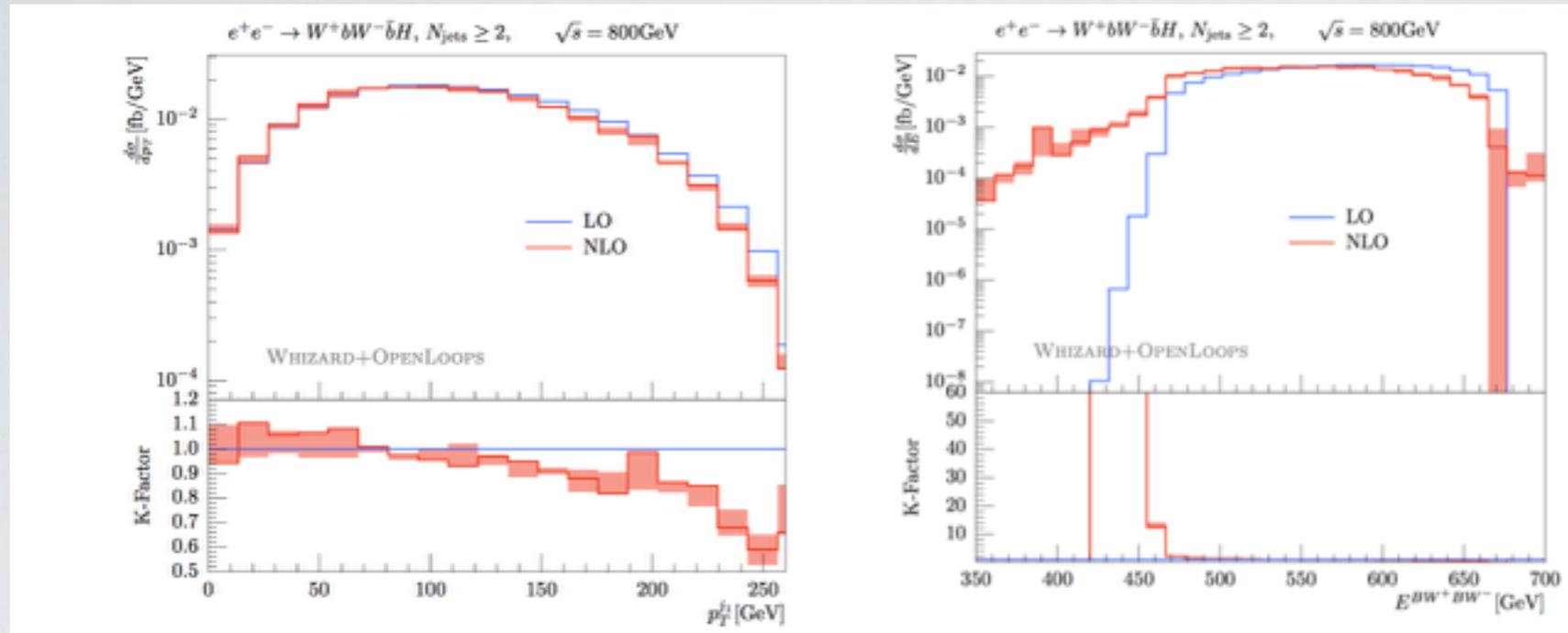
NLO QCD Results for tt and ttH



$$E_h = \frac{1}{2\sqrt{s}} [s + M_h^2 - (k_1 + k_2)^2]$$

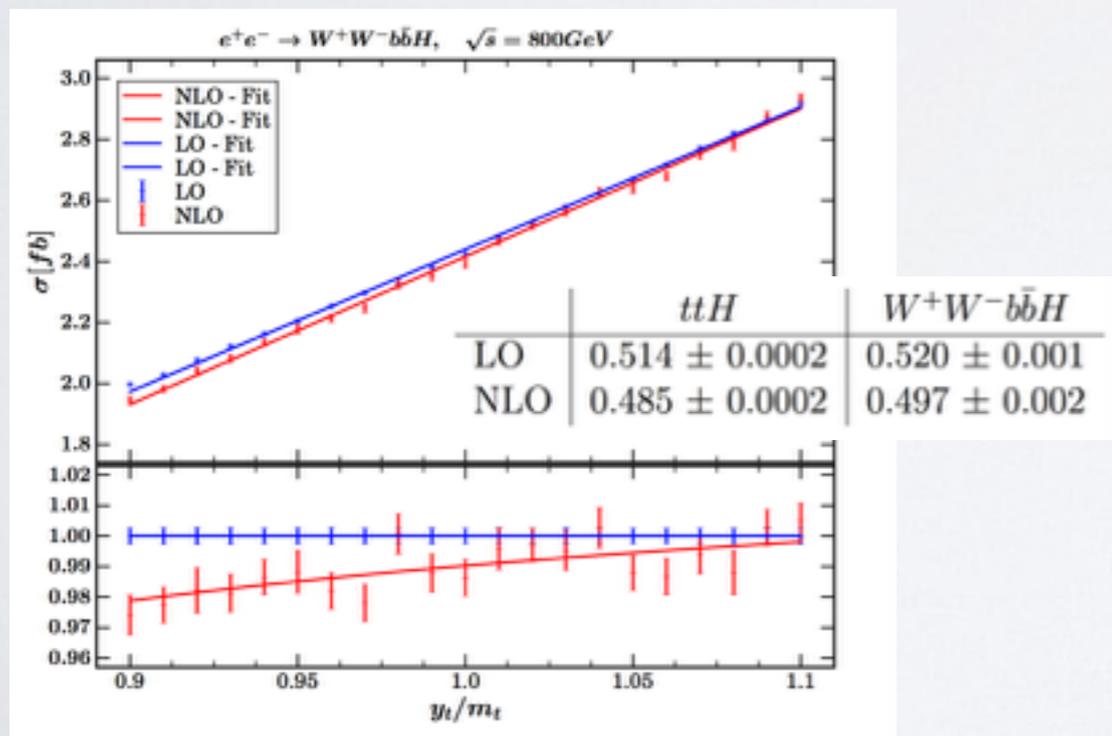
Determination of top Yukawa coupling (ttH)





$$E_h = \frac{1}{2\sqrt{s}} [s + M_h^2 - (k_1 + k_2)^2]$$

Determination of top Yukawa coupling (ttH)



Polarized Results (tt)

- ILC will always run polarized
- Polarized I-loop amplitudes beyond BLHA

$P(e^-)$	$P(e^+)$	$\sqrt{s} = 800 \text{ GeV}$			$\sqrt{s} = 1500 \text{ GeV}$		
		$\sigma^{\text{LO}}[\text{fb}]$	$\sigma^{\text{NLO}}[\text{fb}]$	K-factor	$\sigma^{\text{LO}}[\text{fb}]$	$\sigma^{\text{NLO}}[\text{fb}]$	K-factor
0%	0%	253.7	272.8	1.075	75.8	79.4	1.049
-80%	0%	176.5	190.0	1.077	98.3	103.1	1.049
+80%	0%	176.5	190.0	1.077	53.2	55.9	1.049
-80%	30%	420.8	452.2	1.074	124.9	131.0	1.048
-80%	60%	510.7	548.7	1.074	151.6	158.9	1.048
80%	-30%	208.4	224.5	1.077	63.0	66.1	1.049
80%	-60%	240.3	258.9	1.077	72.7	76.3	1.049



Top-Forward Backward Asymmetry

$$A_{FB} = \frac{\sigma(\cos \theta_t > 0) - \sigma(\cos \theta_t < 0)}{\sigma(\cos \theta_t > 0) + \sigma(\cos \theta_t < 0)}$$

Gluon emission symmetric in $\theta \Rightarrow$
NLO QCD corrections small

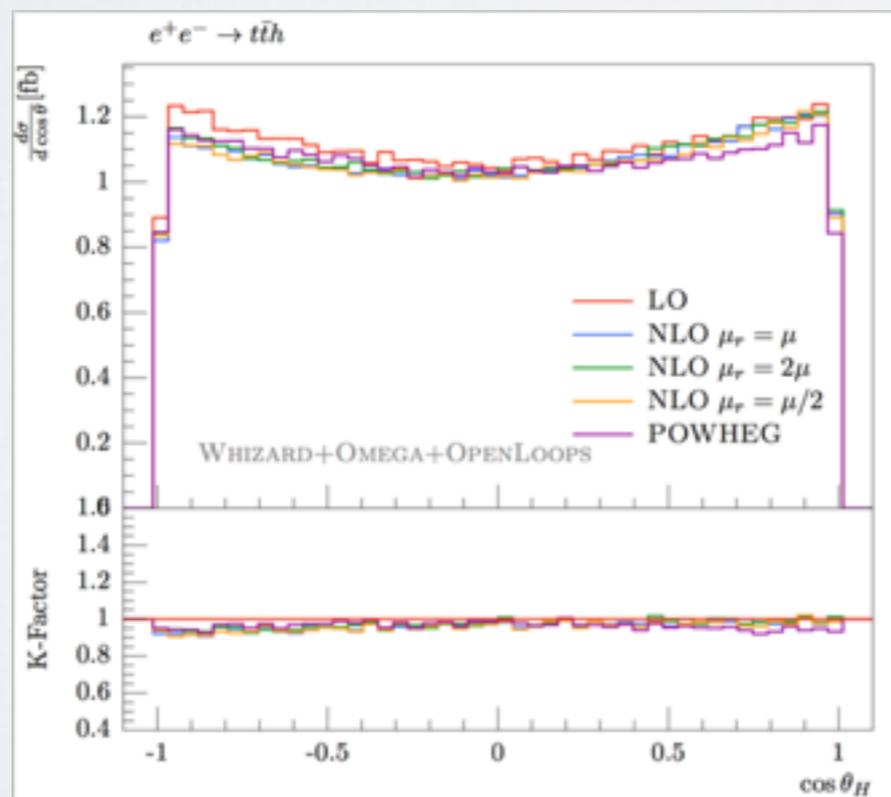
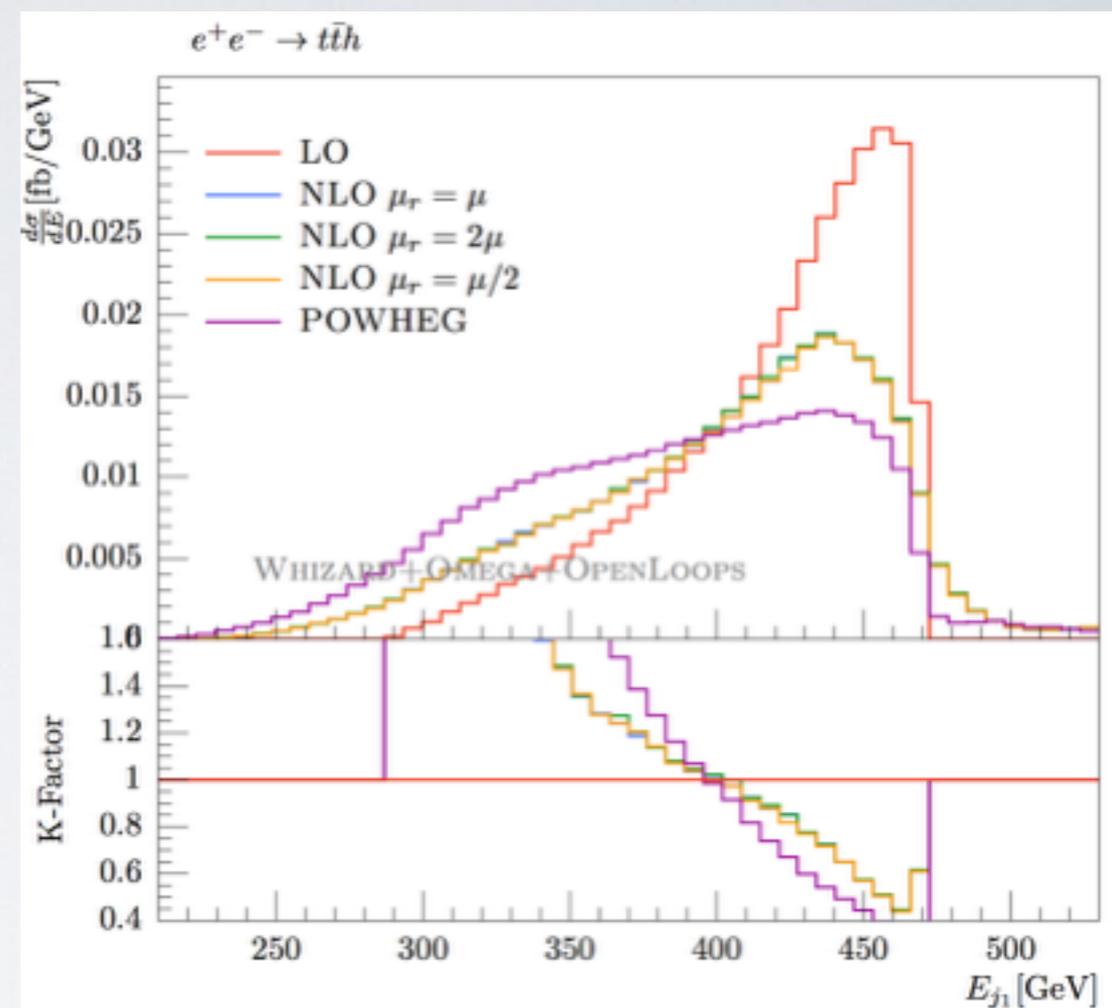
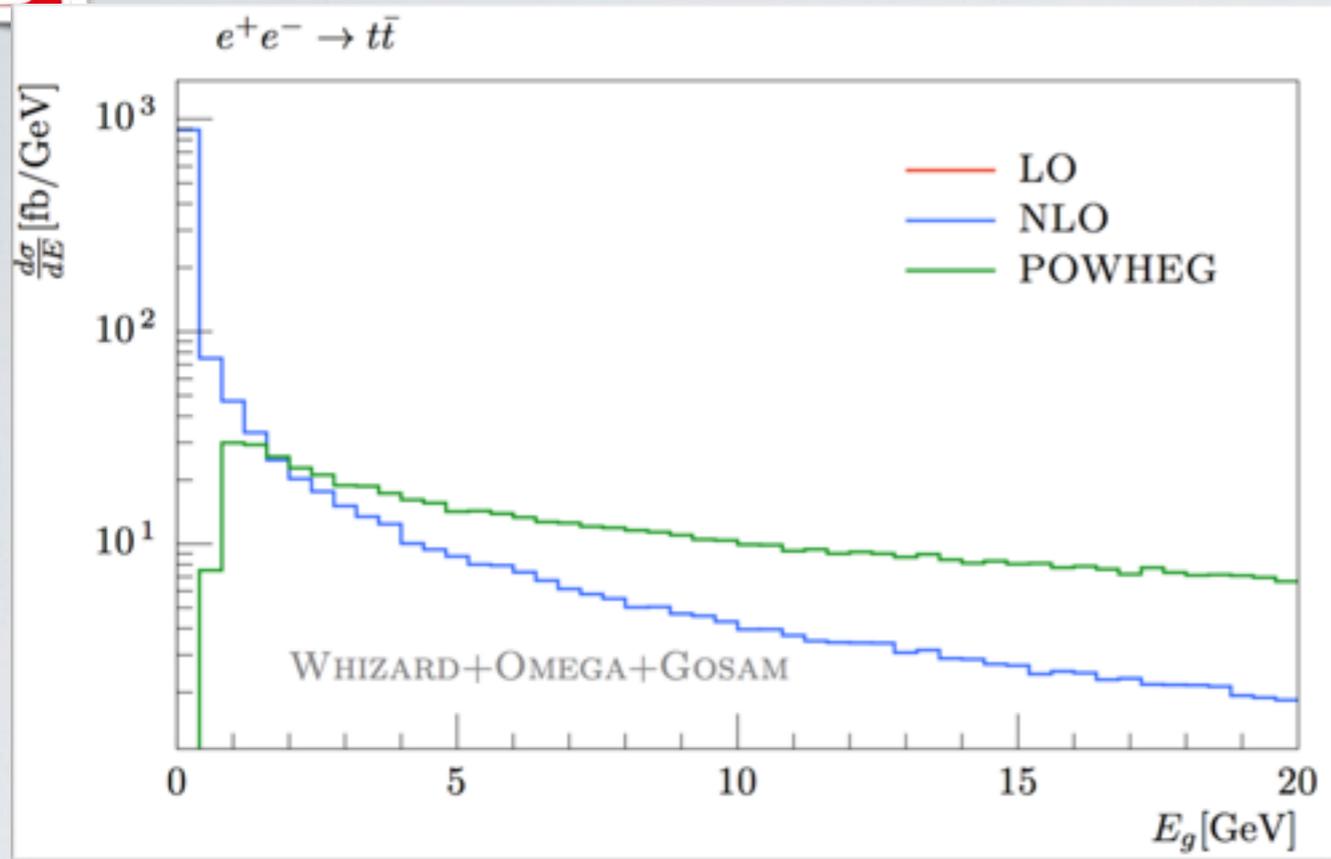
A_{FB} of the top quark

Final state	A_{FB}^{LO}	A_{FB}^{NLO}
$t\bar{t}$	-0.5935 ± 0.0017	-0.5983 ± 0.0048
$W^+W^-b\bar{b}$	-0.4847 ± 0.0017	-0.4778 ± 0.0114
$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.5005 ± 0.0001	-0.4947 ± 0.0088
$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	-0.4854 ± 0.0010	-0.4805 ± 0.0089

A_{FB} of the anti-top quark

Final state	A_{FB}^{LO}	A_{FB}^{NLO}
$t\bar{t}$	0.4764 ± 0.0017	0.4789 ± 0.0047
$W^+W^-b\bar{b}$	0.3674 ± 0.0017	0.3701 ± 0.0104
$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.3267 ± 0.0009	0.3264 ± 0.0084
$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	0.2656 ± 0.0009	0.2603 ± 0.0083

POWHEG-matched results for tt and ttH





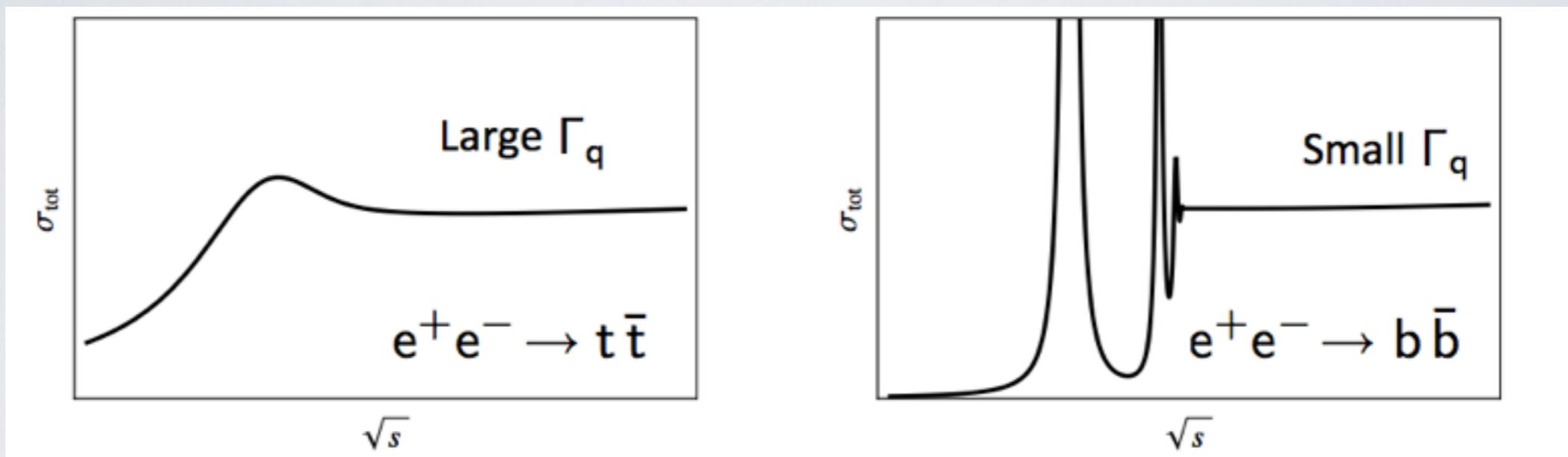
3) Top threshold in (N)LL (p)NRQCD
matched to (N)LO QCD
in WHIZARD



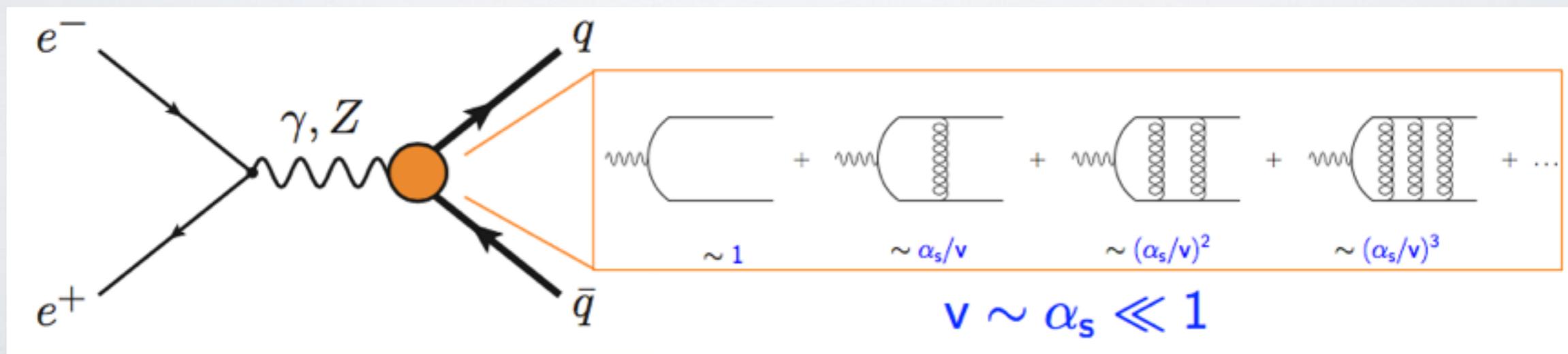
Top Threshold at lepton colliders

ILC top threshold scan best-known method to measure top quark mass, $\Delta M \sim 30\text{-}50$ MeV

Heavy quark production at lepton colliders, qualitatively:



Threshold region: top velocity $v \sim \alpha_s \ll 1$





- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold ($v = 0$) [Hoang/Teubner, 1999](#); [Hoang et al., 2001](#)

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$

$$\times \{1 (\mathbf{LL}); \alpha_s, v (\mathbf{NLL}); \alpha_s^2, \alpha_s v, v^2 (\mathbf{NNLL})\}$$

(p/v)NRQCD EFT w/ RG improvement



- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold ($v = 0$) [Hoang/Teubner, 1999](#); [Hoang et al., 2001](#)

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$

$$\times \{1 (\mathbf{LL}); \alpha_s, v (\mathbf{NLL}); \alpha_s^2, \alpha_s v, v^2 (\mathbf{NNLL})\}$$

(p/v)NRQCD EFT w/ RG improvement

$$R^{\gamma,Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave} \sim v^2: \text{NNLL}}$$

but contributes
at NLL differentially!



- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold ($v = 0$) [Hoang/Teubner, 1999; Hoang et al., 2001](#)

Phase space of two massive particles

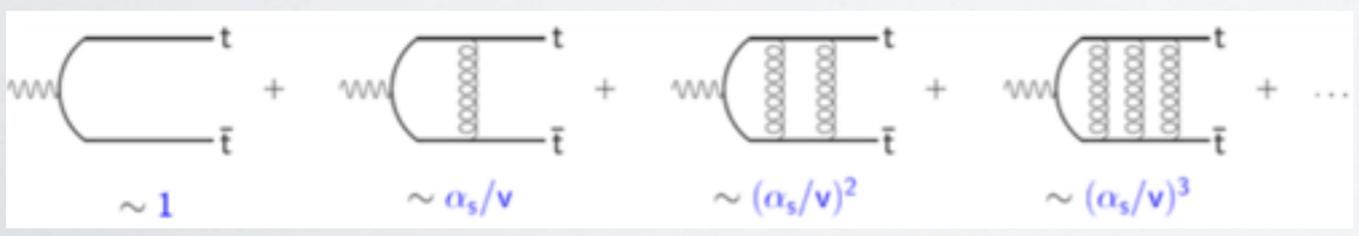
$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \{1 (\mathbf{LL}); \alpha_s, v (\mathbf{NLL}); \alpha_s^2, \alpha_s v, v^2 (\mathbf{NNLL})\}$$

(p/v)NRQCD EFT w/ RG improvement

$$R^{\gamma,Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave} \sim v^2: \text{NNLL}}$$

but contributes at NLL differentially!

Coulomb potential gluon ladder resummation





Top Threshold Resummation in (p)NRQCD

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold ($v = 0$) Hoang/Teubner, 1999; Hoang et al., 2001

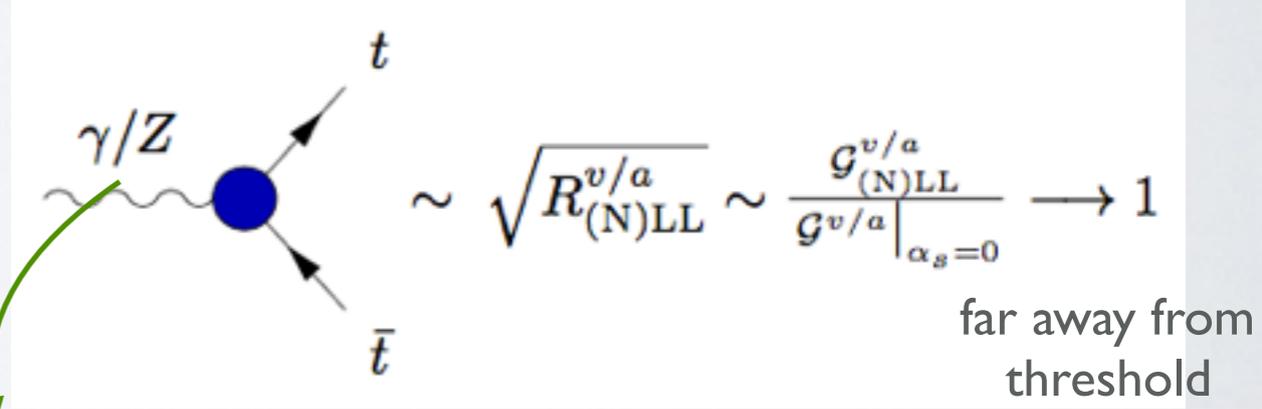
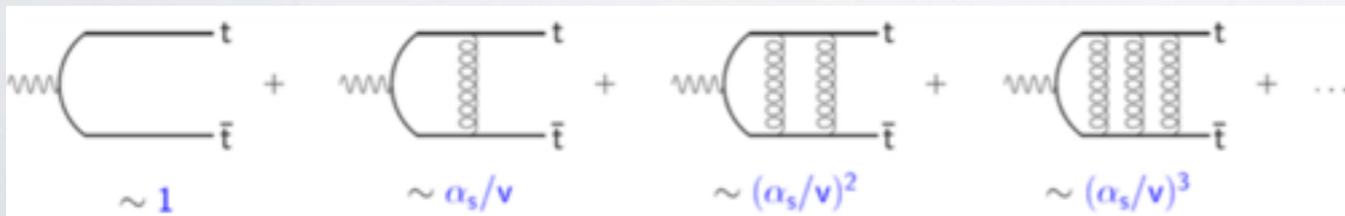
Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \underbrace{\{1 (\mathbf{LL}); \alpha_s, v (\mathbf{NLL}); \alpha_s^2, \alpha_s v, v^2 (\mathbf{NNLL})\}}_{\text{(p/v)NRQCD EFT w/ RG improvement}}$$

$$R^{\gamma,Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave} \sim v^2: \mathbf{NNLL}}$$

but contributes at NLL differentially!

Coulomb potential gluon ladder resummation



$$\mathbb{C} \ni \mathcal{G}_{(N)LL}^{v/a} = \mathcal{G}_{(N)LL}^{v/a}(\alpha_s, M_t^{\text{pole}}, \sqrt{s}, |\vec{p}_t|, \Gamma_t)$$

differential in off-shell tt phase space

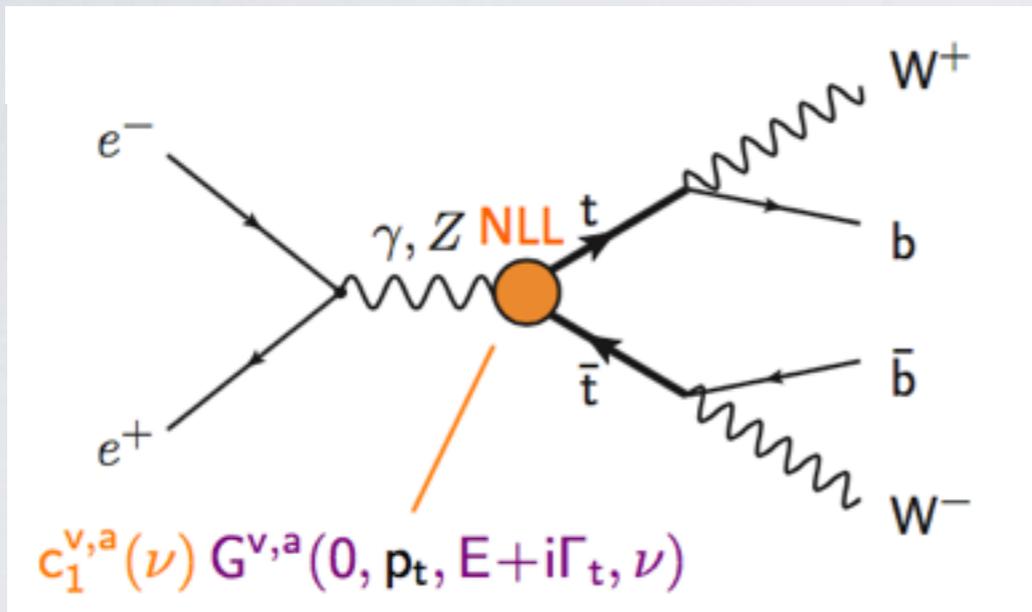




Top Threshold in WHIZARD

with B. Chokouf /A. Hoang/M. Stahlhofen/T. Teubner/C. Weiss

- Implement resummed threshold effects as effective vertex [form factor] in WHIZARD
- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$ from TOPPIK code [Jezabek/Teubner], included in WHIZARD



- Default parameters:

$$M^{1S} = 172 \text{ GeV}, \Gamma_t = 1.54 \text{ GeV},$$

$$\alpha_s(M_Z) = 0.118$$

$$M^{1S} = M_t^{pole} \left(1 - \Delta_{(Coul.)}^{LL/NLL} \right)$$

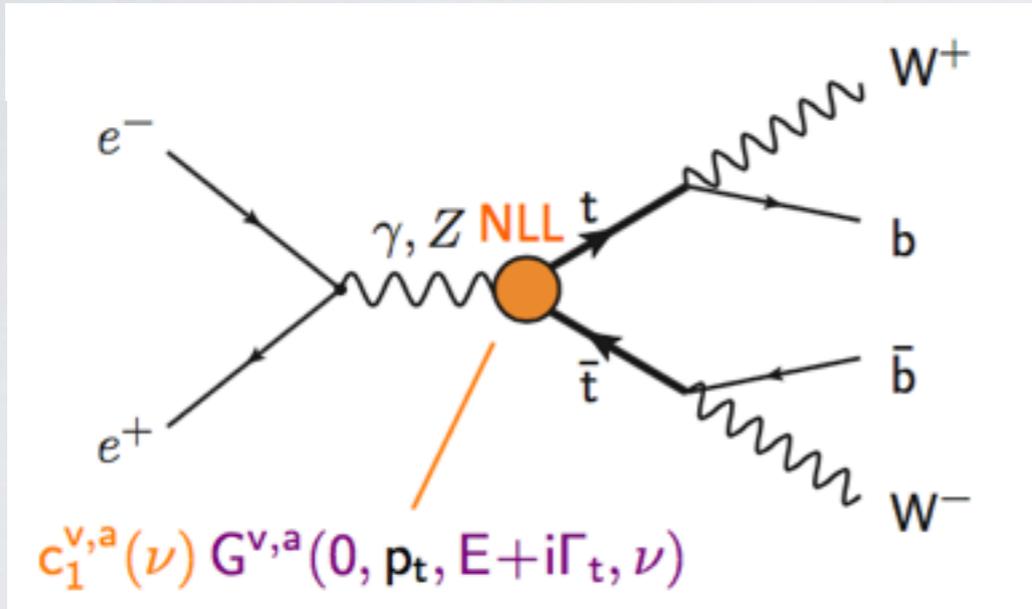
Marquard et al.



Top Threshold in WHIZARD

with B. Chokouf /A. Hoang/M. Stahlhofen/T. Teubner/C. Weiss

- Implement resummed threshold effects as effective vertex [form factor] in WHIZARD
- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$ from TOPPIK code [Jezabek/Teubner], included in WHIZARD

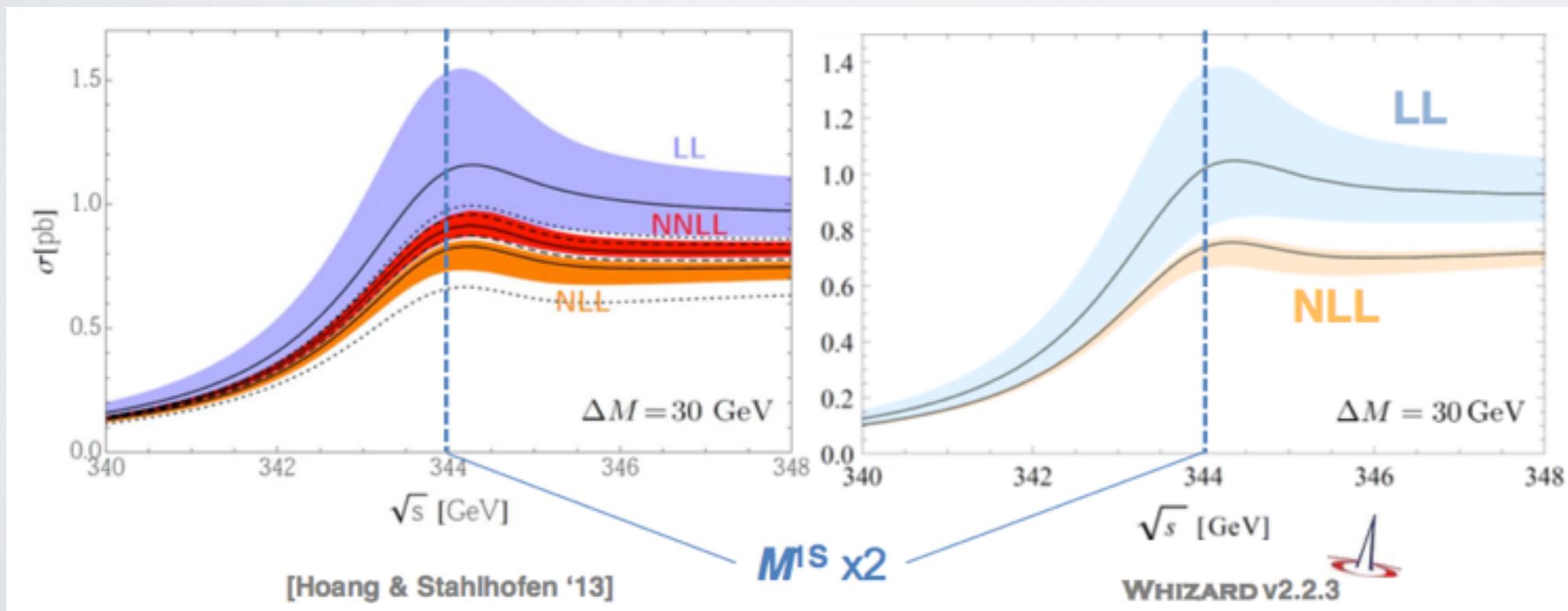


- Default parameters:

$$M^{1S} = 172 \text{ GeV}, \quad \Gamma_t = 1.54 \text{ GeV},$$

$$\alpha_s(M_Z) = 0.118$$

$$M^{1S} = M_t^{pole} \left(1 - \Delta_{(Coul.)}^{LL/NLL} \right) \quad \text{Marquard et al.}$$

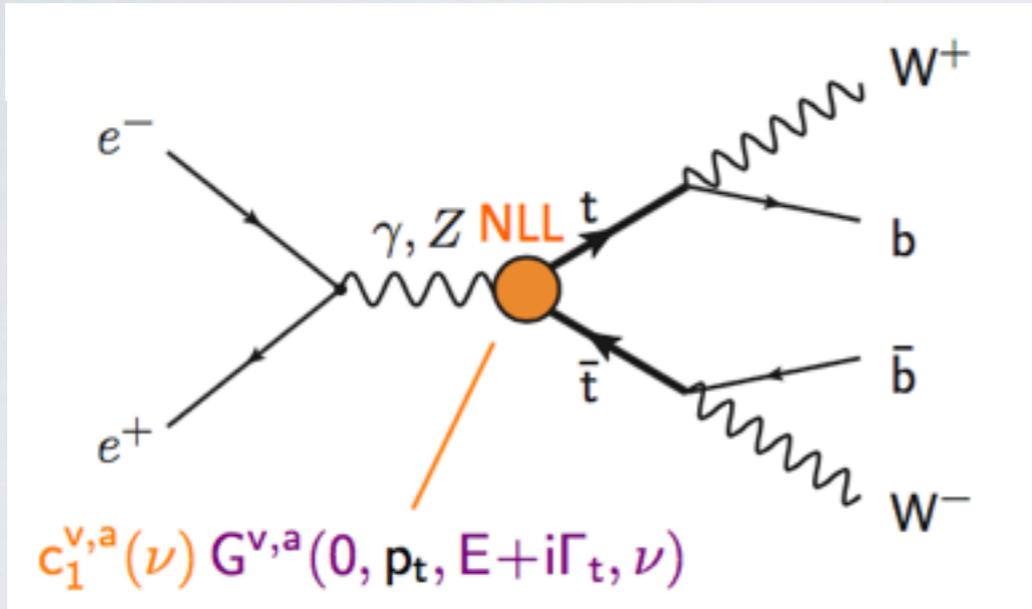




Top Threshold in WHIZARD

with B. Chokouf /A. Hoang/M. Stahlhofen/T. Teubner/C. Weiss

- Implement resummed threshold effects as effective vertex [form factor] in WHIZARD
- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$ from TOPPIK code [Jezabek/Teubner], included in WHIZARD



- Default parameters:

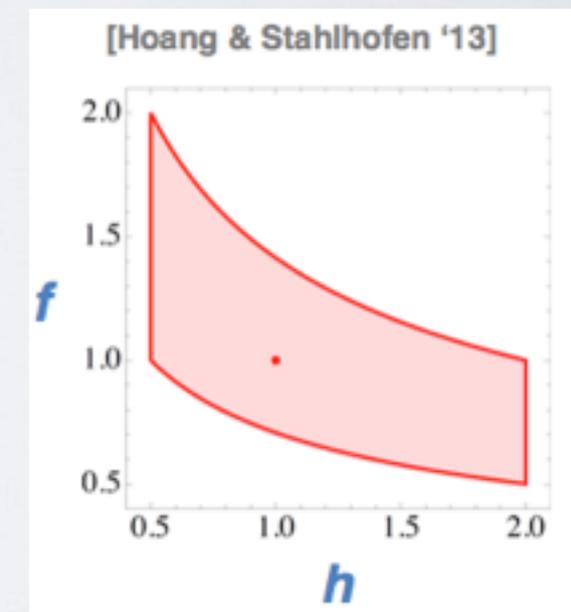
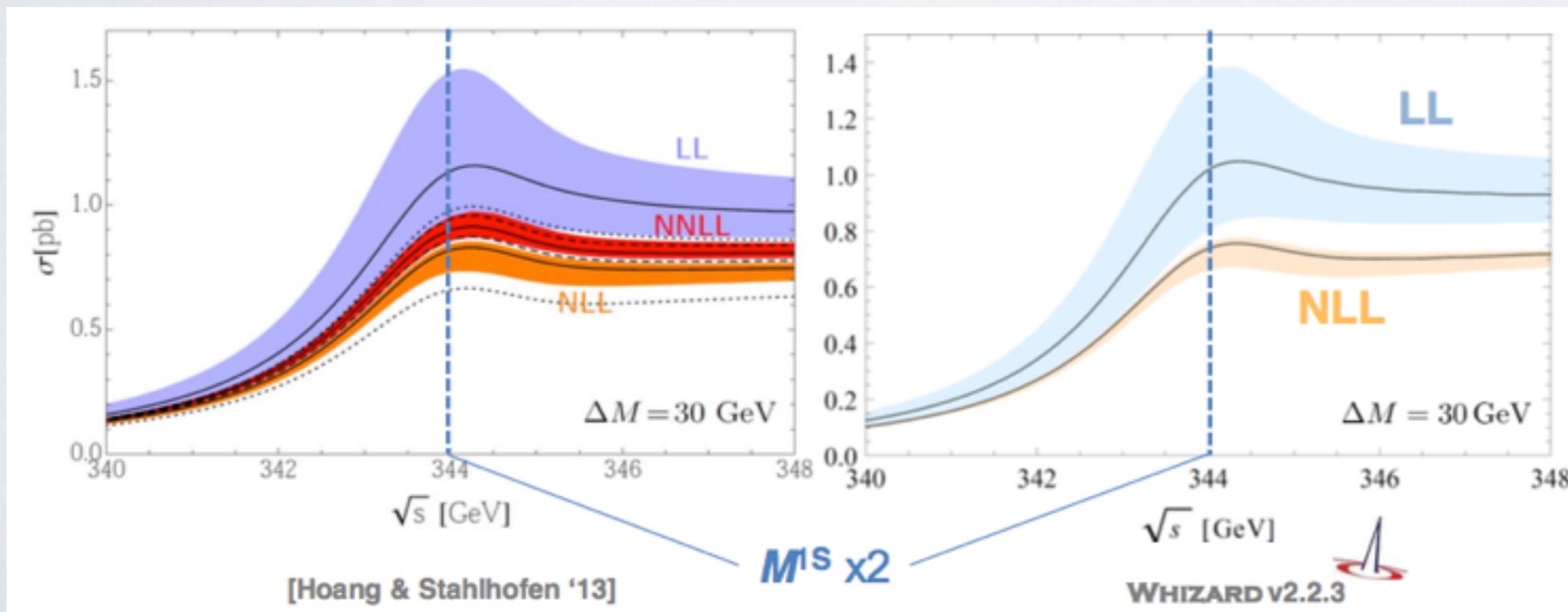
$$M^{1S} = 172 \text{ GeV}, \quad \Gamma_t = 1.54 \text{ GeV},$$

$$\alpha_s(M_Z) = 0.118$$

$$M^{1S} = M_t^{pole} \left(1 - \Delta_{(Coul.)}^{LL/NLL} \right) \quad \text{Marquard et al.}$$

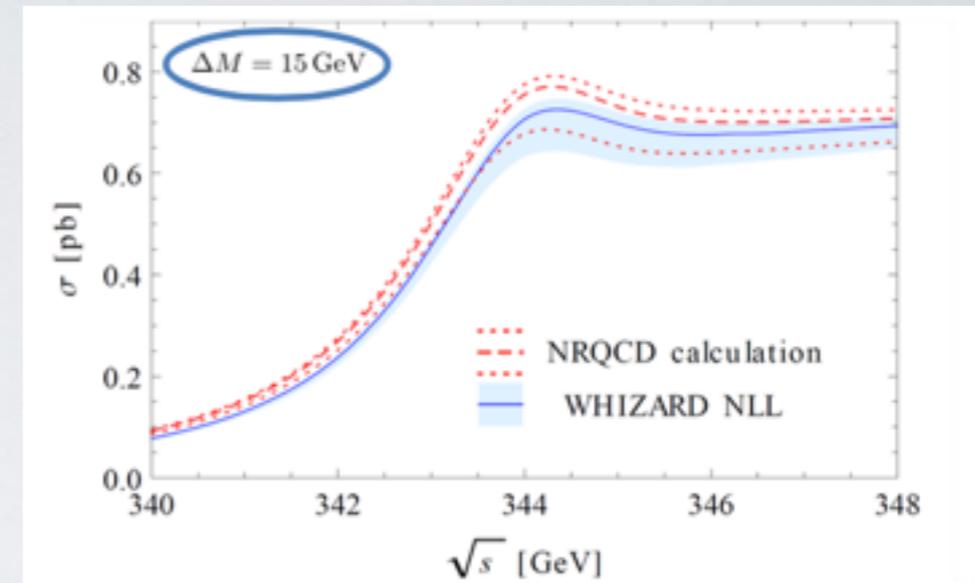
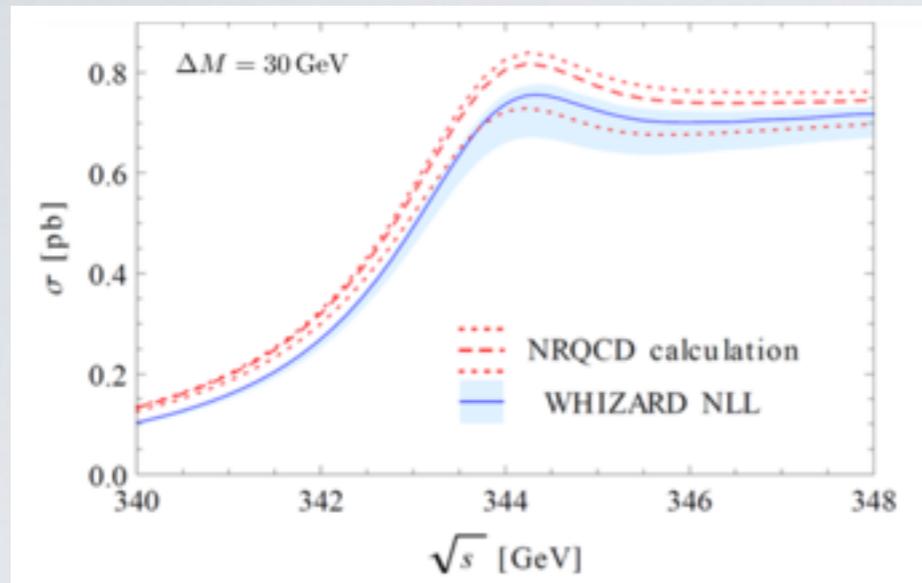
Theory uncertainties from scale variations:
hard and soft scale

$$\mu_h = h \cdot m_t \quad \mu_s = f \cdot m_t v$$





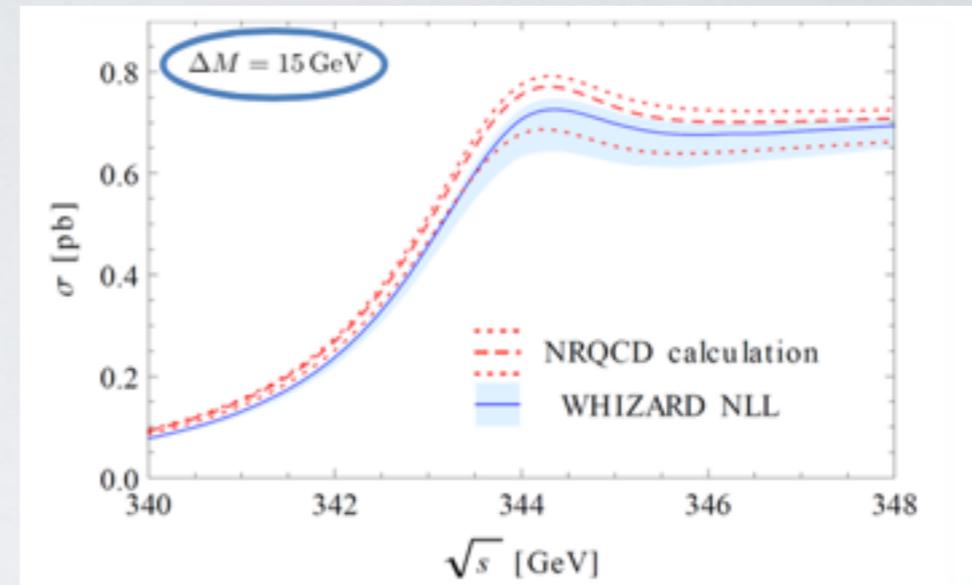
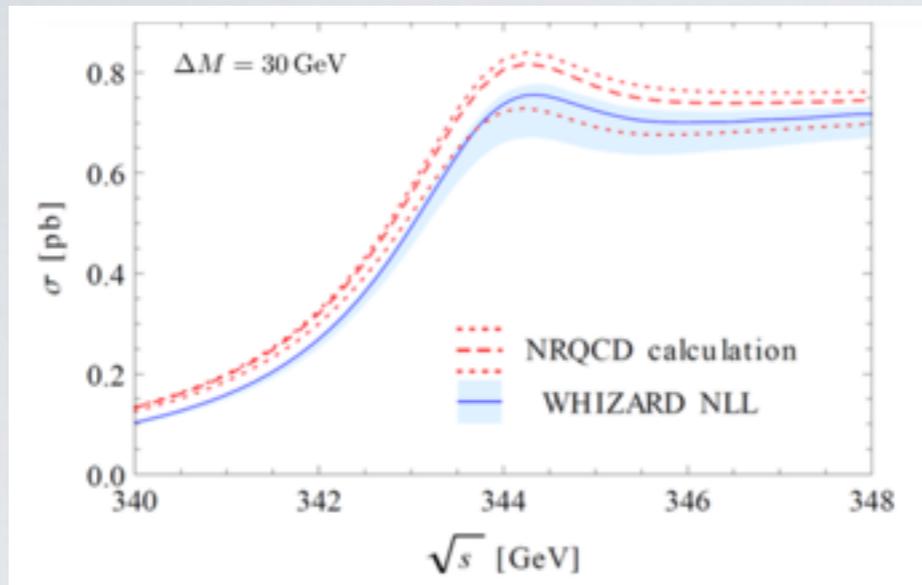
Sanity checks: correct limit for $\alpha_s \rightarrow 0$, stable against variation of cutoff ΔM [15-30 GeV]



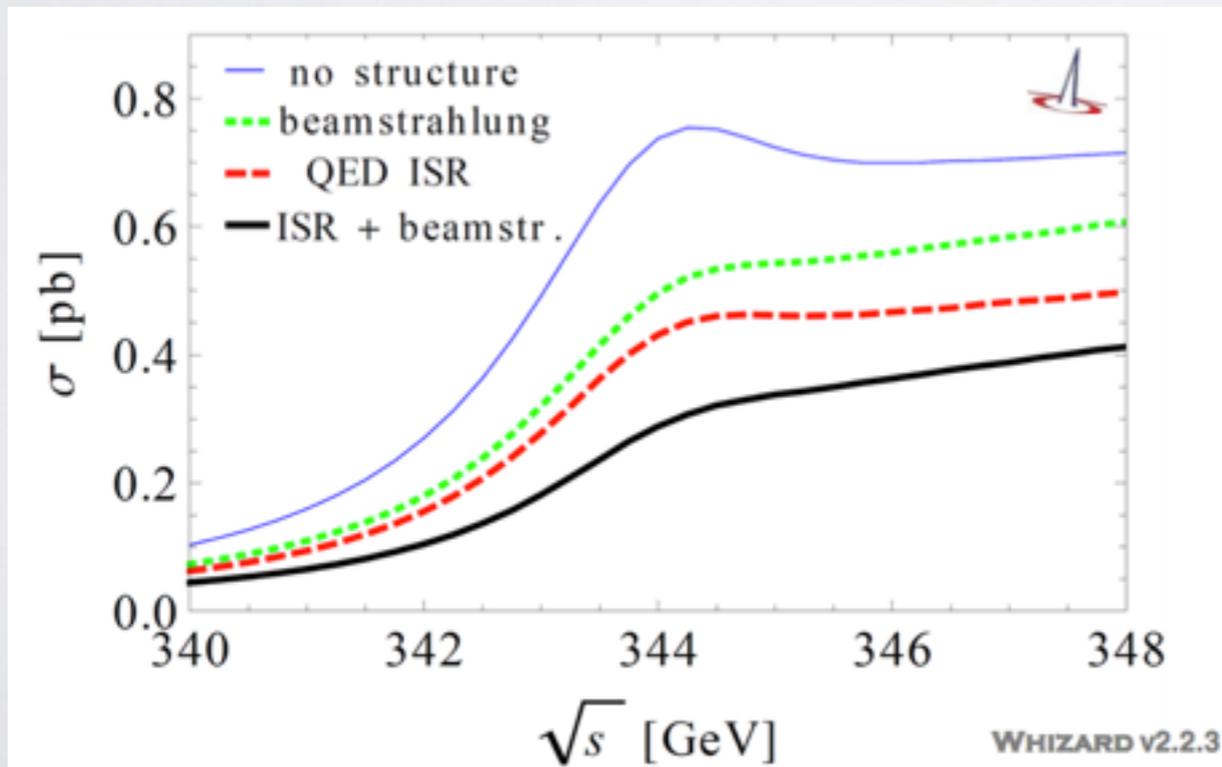
- ▶ Why include LL/NLL in a Monte Carlo event generator?
- ▶ Important effects: beamstrahlung; ISR; LO electroweak terms
- ▶ More exclusive observables accessible



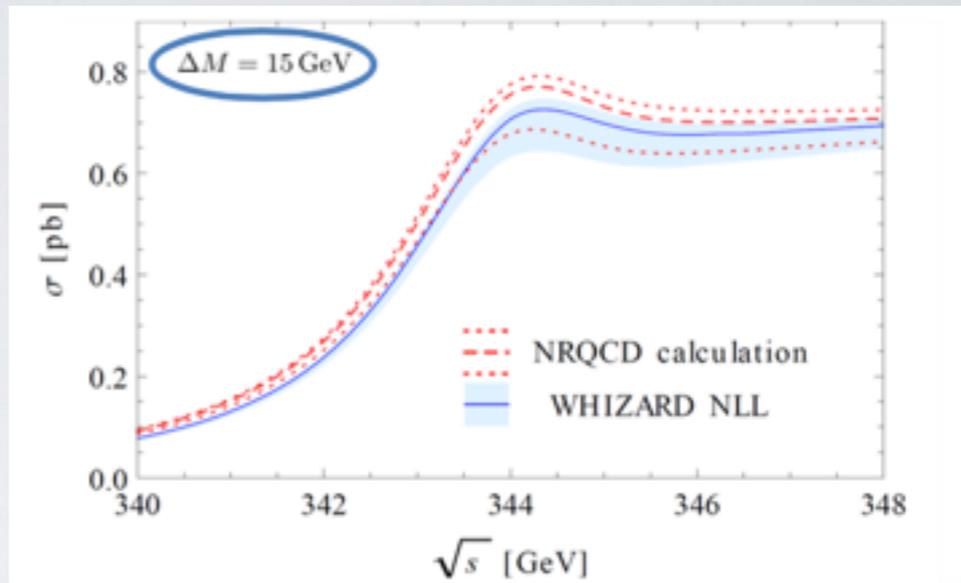
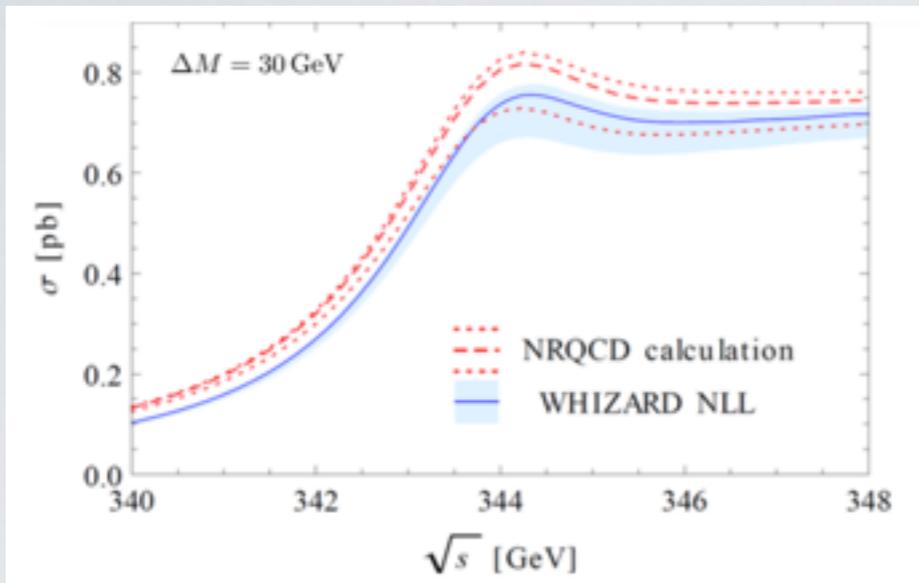
Sanity checks: correct limit for $\alpha_s \rightarrow 0$, stable against variation of cutoff ΔM [15-30 GeV]



- ▶ Why include LL/NLL in a Monte Carlo event generator?
- ▶ Important effects: beamstrahlung; ISR; LO electroweak terms
- ▶ More exclusive observables accessible



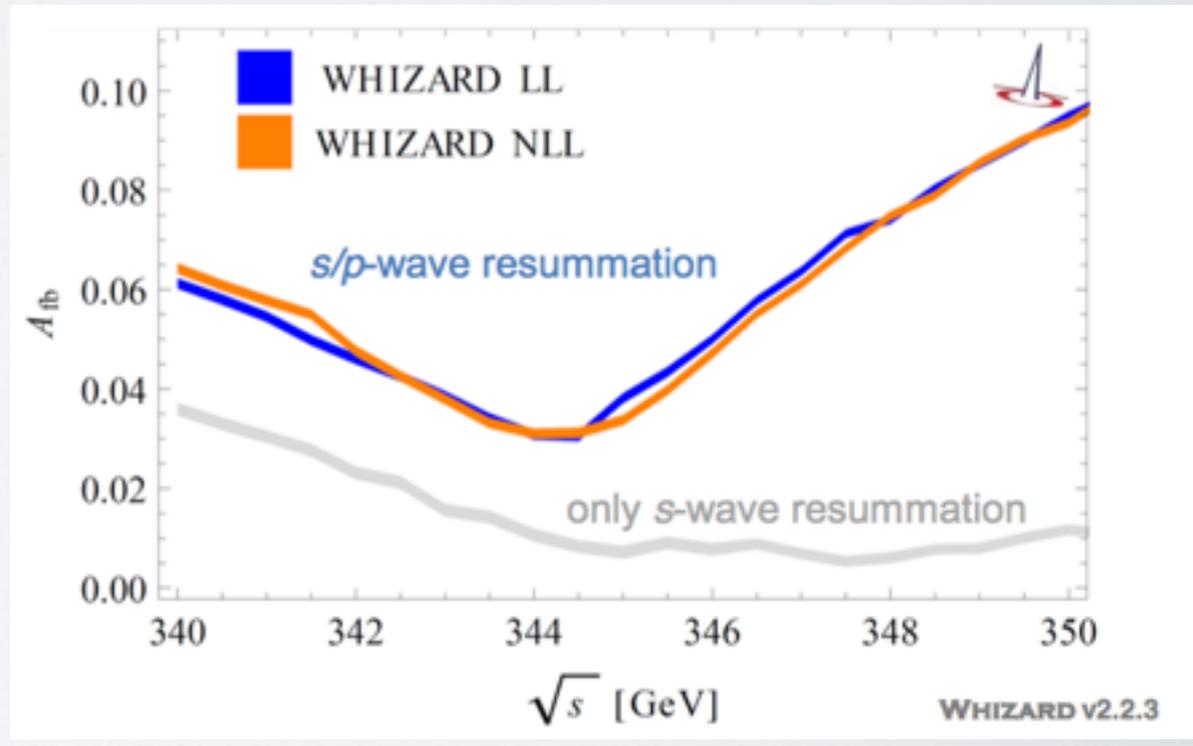
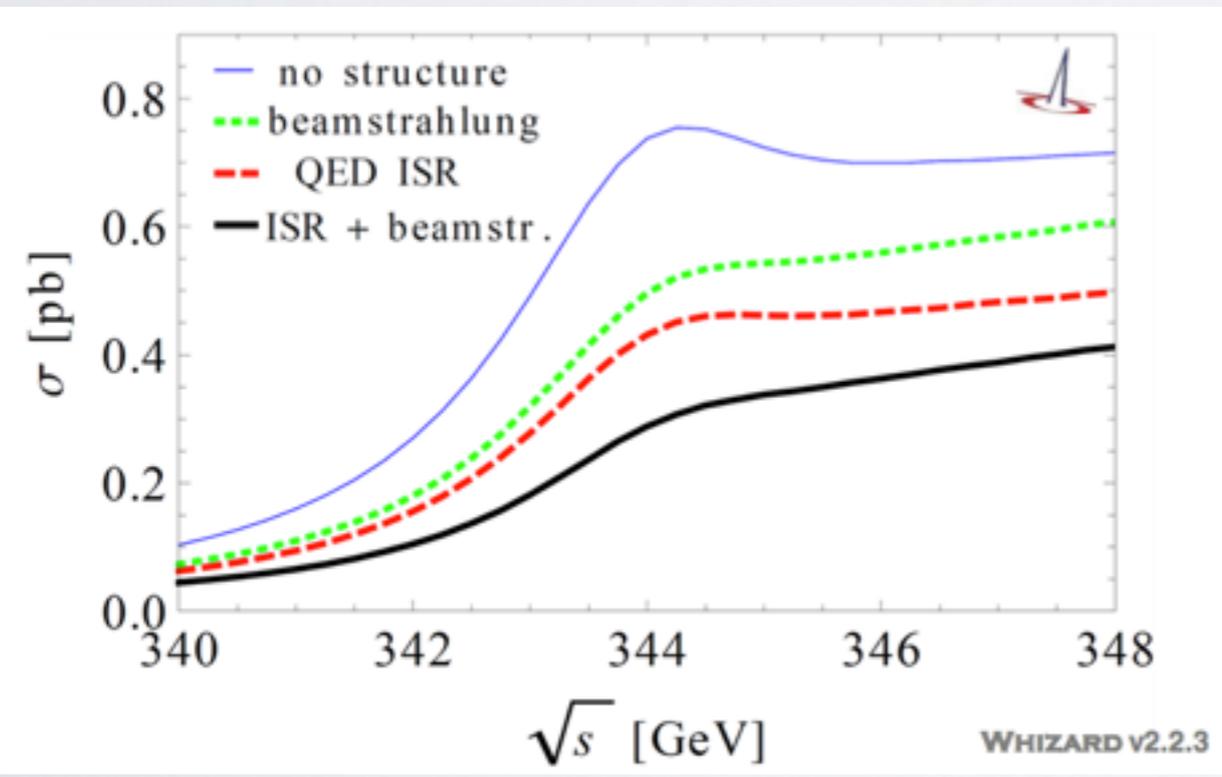
Sanity checks: correct limit for $\alpha_s \rightarrow 0$, stable against variation of cutoff ΔM [15-30 GeV]



- ▶ Why include LL/NLL in a Monte Carlo event generator?
- ▶ Important effects: beamstrahlung; ISR; LO electroweak terms
- ▶ More exclusive observables accessible

Forward-backward asymmetry (norm. \Rightarrow good shape stability)

$$A_{fb} := \frac{\sigma(p_z^t > 0) - \sigma(p_z^t < 0)}{\sigma(p_z^t > 0) + \sigma(p_z^t < 0)}$$

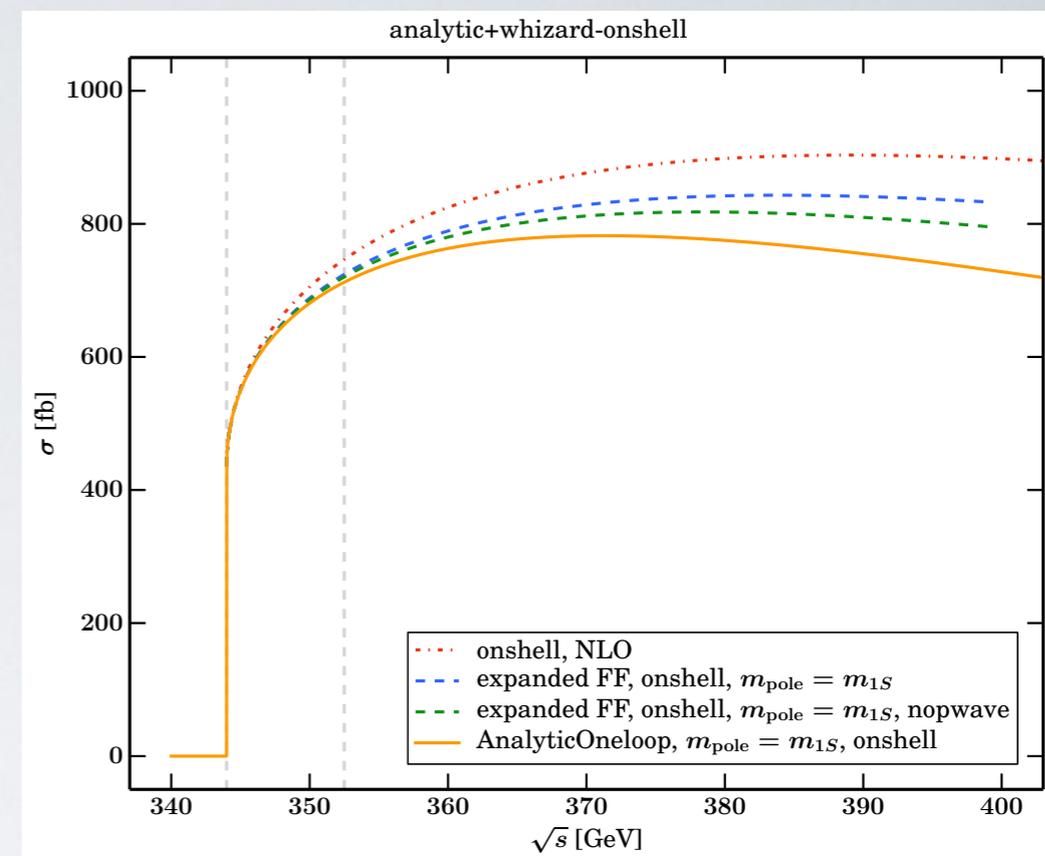




Matching to continuum at (LO and) NLO

28/31

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV
- Remove double-counting NLO / (N)LL





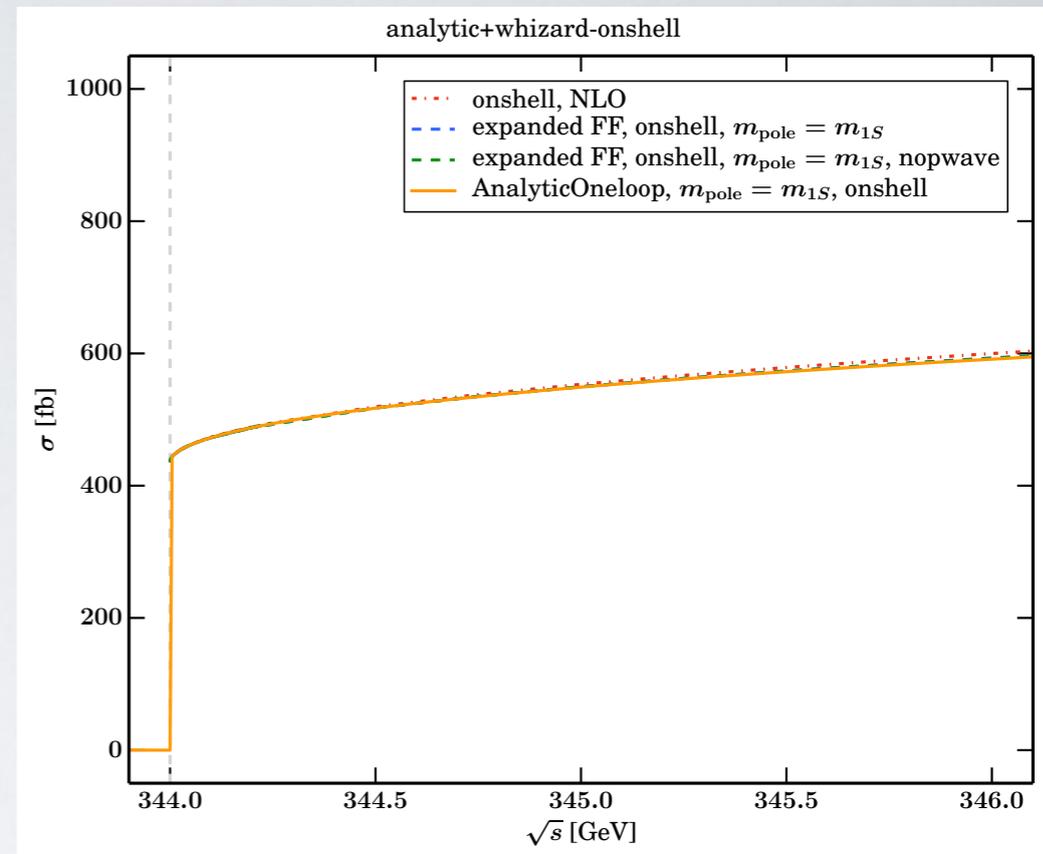
Matching to continuum at (LO and) NLO

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV
- Remove double-counting NLO / (N)LL

Resummed formfactor, expanded to $\mathcal{O}(\alpha_s)$

$$\nu = \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m}} \quad p = |\vec{p}| \quad p_0 = E_t - m_t$$

$$F^{\text{expanded}}[\alpha_H, \alpha_S] = \alpha_H \left(-\frac{2C_F}{\pi} \right) + \alpha_S \left(\frac{iC_F m \log \frac{mv+p}{mv-p}}{2p} \right)$$





Matching to continuum at (LO and) NLO

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV
- Remove double-counting NLO / (N)LL

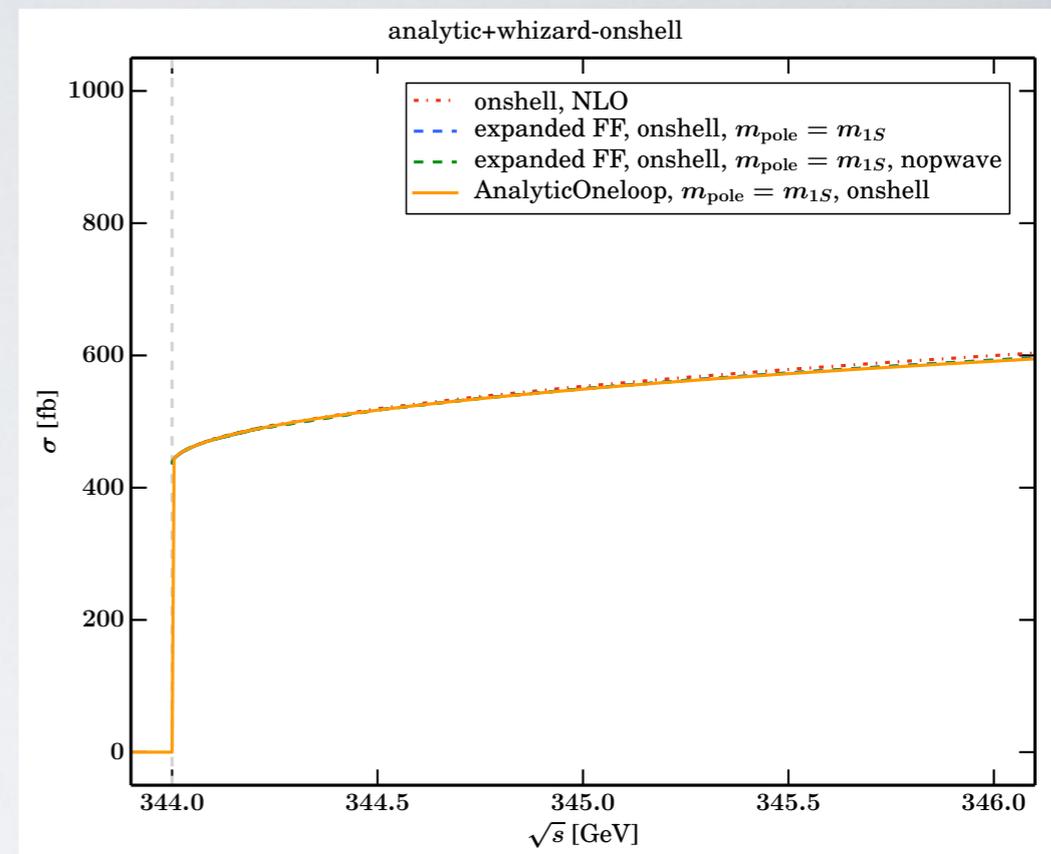
Resummed formfactor, expanded to $\mathcal{O}(\alpha_s)$

$$\nu = \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m}} \quad p = |\vec{p}| \quad p_0 = E_t - m_t$$

$$F^{\text{expanded}}[\alpha_H, \alpha_S] = \alpha_H \left(-\frac{2C_F}{\pi} \right) + \alpha_S \left(\frac{iC_F m \log \frac{mv+p}{mv-p}}{2p} \right)$$

Matching formula

$$\begin{aligned} \sigma_{\text{matched}} &= \sigma_{\text{QCD}}[\alpha_H] - \sigma_{\text{NRQCD}}^{\text{expanded}}[\alpha_H, \alpha_H] \\ &+ \sigma_{\text{NRQCD}}^{\text{expanded}}[\alpha_H, f_s \alpha_S + (1 - f_s) \alpha_H] \\ &+ \sigma_{\text{NRQCD}}^{\text{full}}[f_s \alpha_H, f_s \alpha_S, f_s \alpha_{\text{US}}] - \sigma_{\text{NRQCD}}^{\text{expanded}}[f_s \alpha_H, f_s \alpha_S] \end{aligned}$$



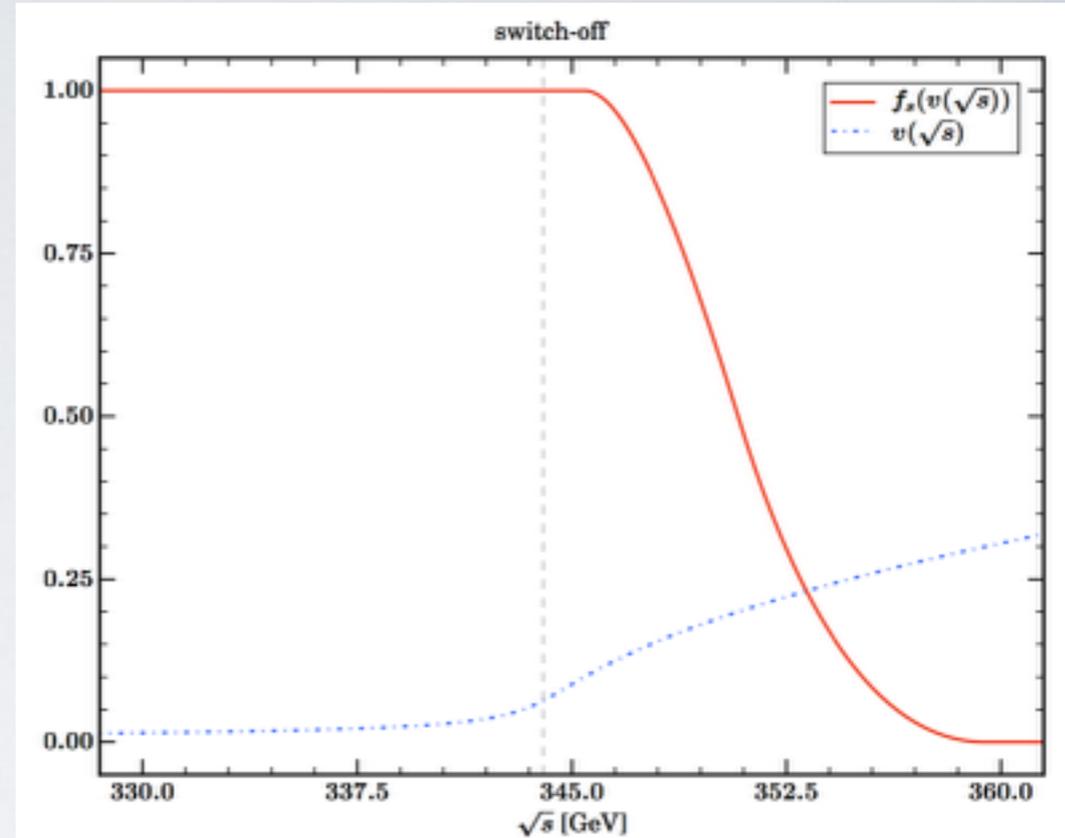


Matching to continuum at (LO and) NLO

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV
- Remove double-counting NLO / (N)LL

Resummed formfactor, expanded to $\mathcal{O}(\alpha_s)$

$$\nu = \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m}} \quad p = |\vec{p}| \quad p_0 = E_t - m_t$$



$$F^{\text{expanded}}[\alpha_H, \alpha_S] = \alpha_H \left(-\frac{2C_F}{\pi} \right) + \alpha_S \left(\frac{iC_F m \log \frac{mv+p}{mv-p}}{2p} \right)$$

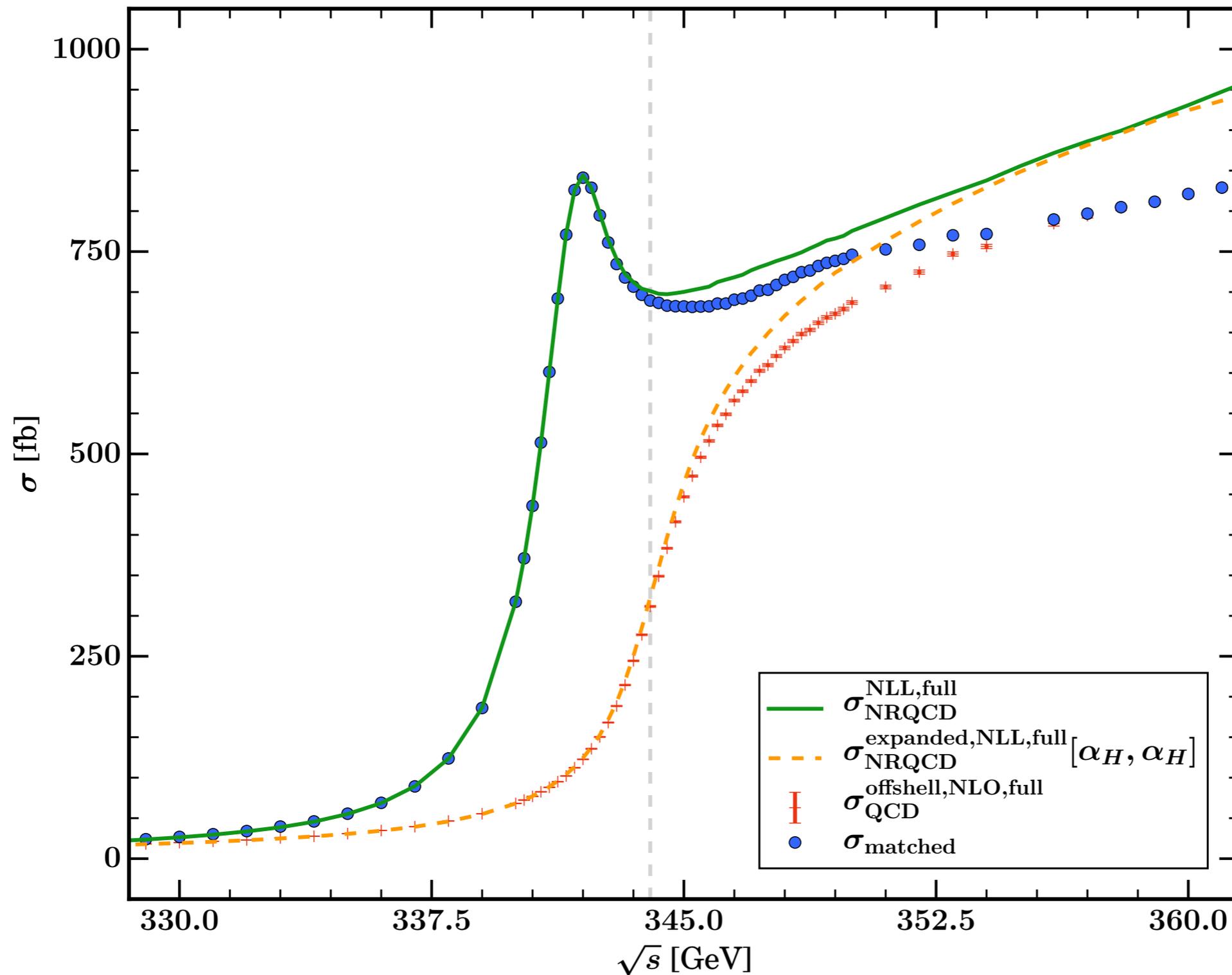
Matching formula

$$\begin{aligned} \sigma_{\text{matched}} = & \sigma_{\text{QCD}}[\alpha_H] - \sigma_{\text{NRQCD}}^{\text{expanded}}[\alpha_H, \alpha_H] \\ & + \sigma_{\text{NRQCD}}^{\text{expanded}}[\alpha_H, f_s \alpha_S + (1 - f_s) \alpha_H] \\ & + \sigma_{\text{NRQCD}}^{\text{full}}[f_s \alpha_H, f_s \alpha_S, f_s \alpha_{\text{US}}] - \sigma_{\text{NRQCD}}^{\text{expanded}}[f_s \alpha_H, f_s \alpha_S] \end{aligned}$$

Switch-off function

$$f_s(v) = \begin{cases} 1 & v < v_1 \\ 1 - 2 \frac{(v-v_1)^2}{(v_2-v_1)^2} & v_1 < v < \frac{v_1+v_2}{2} \\ 2 \frac{(v-v_2)^2}{(v_2-v_1)^2} & \frac{v_1+v_2}{2} < v < v_2 \\ 0 & v > v_2 \end{cases}$$

Threshold-continuum matching



Bach/Chokouf /Hoang/
Kilian/JRR/Stahlhofen/
Teubner/Weiss, 2016 &
work in progress

Conclusions & Outlook

- WHIZARD 2.3 event generator for collider physics (ee, pp, ep)
- Allows to simulate all possible BSM models
- Allows for all SM backgrounds
- NLO automation: reals and subtraction terms (FKS) [+ virtuals externally] → WHIZARD 3.0
- allows to produce NLO fixed-order histograms
- Automated POWHEG matching (other schemes in progress)
- Top threshold in e^+e^- : NLL NRQCD threshold / NLO continuum matching
- Virtual Machine for more efficient matrix elements
- Ongoing projects: showers, merging, EW NLO, ... , ...





New



**Higher Performance
Superior Protection**

▶ Learn More



BACKUP SLIDES



WHIZARD: Manual



WHIZARD is hosted by Hepforge, IPP-Cambridge

- WHIZARD



- HOME

- Main Page

- MANUAL, WIKI, NEWS

- Manual
- Wiki Page
- News
- Tutorials
- ChangeLog

- REPOSITORY, BUG TRACKER

- Subversion Repository
- SVN Browser
- Bug Tracker

- DOWNLOADS

- Download Page
- Patches/Unofficial versions

- CONTACT

- Contact us

- INTERNAL WHIZARD PAGE

- You Shall Not Pass!

WHIZARD 2.2 A generic Monte-Carlo integration and event generation package for multi-particle processes MANUAL ¹

Wolfgang Kilian,² Thorsten Ohl,³ Jürgen Reuter,⁴ with contributions from Fabian Bach,⁵ Bijan Chokoufè Nejad,⁶ Sebastian Schmidt, Christian Speckner⁷, Florian Staub⁸

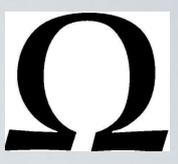
- Contents
- Chapter 1 Introduction
 - 1.1 Disclaimer
 - 1.2 Overview
 - 1.3 Historical remarks
 - 1.4 About examples in this manual
- Chapter 2 Installation
 - 2.1 Package Structure
 - 2.2 Prerequisites
 - 2.3 Installation
 - 2.4 Working With WHIZARD
 - 2.5 Troubleshooting
- Chapter 3 Getting Started
 - 3.1 Hello World
 - 3.2 A Simple Calculation
- Chapter 4 Steering WHIZARD: SINDARIN Overview
 - 4.1 The command language for WHIZARD
 - 4.2 SINDARIN scripts
 - 4.3 Errors

WHIZARD Manual @ HepForge

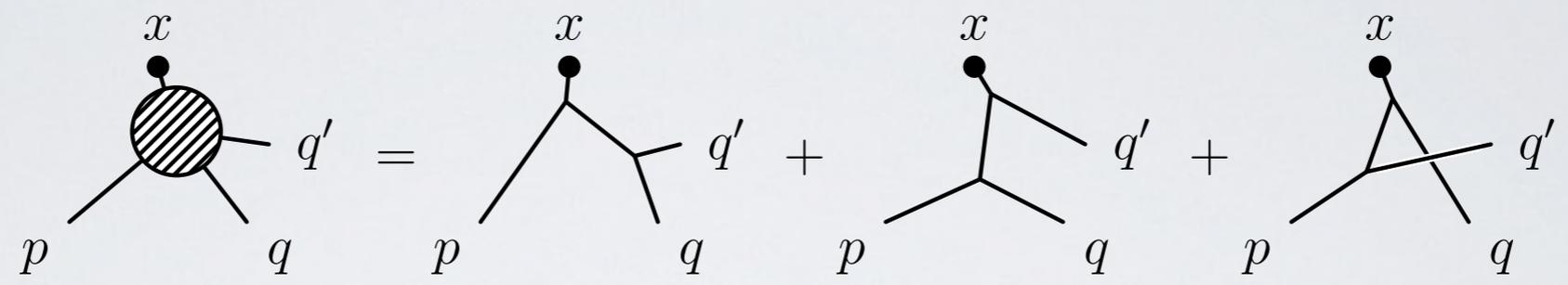




The Optimizing Matrix Element Generator (0' Mega)



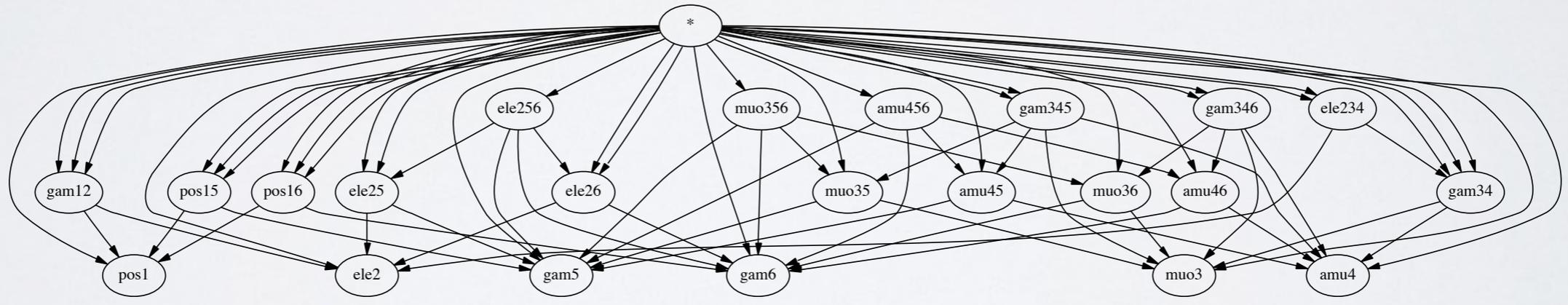
- 0' Mega [Ohl, 2000; Moretti/Ohl/JRR, 2001; JRR, 2002] computes amplitudes with **I-particle off-shell wave functions (IPOWs)**



- Possible to construct set of all currents recursively (tree-/I-loop level)

- Keystones K to replace sum over Feynman diagrams

$$\sum_{i=1}^{F(n)} D_i = \sum_{k,l,m=1}^{P(n)} K_{f_k f_l f_m}^{(3)}(p_k, p_l, p_m) W_{f_k}(p_k) W_{f_l}(p_l) W_{f_m}(p_m)$$



- Calculation forms **Directed Acyclical Graphs (DAGs)**, optimized to consist only of the minimal number of connections by 0' Mega