在 i 和 j 处各有一块 sextupole s_i s_j.

$$\mathcal{M} = \mathcal{A}_0^{-1} \mathcal{R}_{0\to i} \mathcal{A}_i e^{:f_i:} \mathcal{A}_i^{-1} \mathcal{R}_{i\to j} \mathcal{A}_j e^{:f_j:} \mathcal{A}_j^{-1} \mathcal{R}_{j\to 0} \mathcal{A}_0$$

$$= \mathcal{A}_0^{-1} \mathcal{R}_{0\to i} e^{:\mathcal{A}_i f_i:} \mathcal{R}_{i\to j} e^{:\mathcal{A}_j f_i:} \mathcal{R}_{j\to 0} \mathcal{A}_0$$

$$\doteq \mathcal{A}_0^{-1} \mathcal{R}_{0\to i} e^{:\mathcal{A}_i f_i:} \mathcal{R}_{i\to j} \mathcal{R}_{j\to 0} \mathcal{R}_{j\to 0}^{-1} e^{:\mathcal{A}_j f_j:} \mathcal{R}_{j\to 0} \mathcal{A}_0$$

$$= \mathcal{A}_0^{-1} \mathcal{R}_{0\to i} e^{:\mathcal{A}_i f_i:} \mathcal{R}_{i\to j\to 0} e^{:\mathcal{R}_{j\to 0}^{-1} \mathcal{A}_j f_j:} \mathcal{A}_0$$

$$= \mathcal{A}_0^{-1} \mathcal{R}_{0\to i} \mathcal{R}_{i\to j\to 0} e^{:\mathcal{R}_{i\to j\to 0}^{-1} \mathcal{A}_i f_i:} e^{:\mathcal{R}_{j\to 0} \mathcal{A}_j f_j:} \mathcal{A}_0$$

$$= \mathcal{A}_0^{-1} \mathcal{R}_{2\pi\nu} e^{:g_i:} e^{:g_j:} \mathcal{A}_0.$$

$$= \mathcal{A}_0^{-1} \mathcal{R}_{2\pi\nu} e^{:g_i + g_j + \frac{[g_i, g_j]}{2} \cdots :} \mathcal{A}_0.$$

$\cancel{\mathcal{A}_i g}$, $\mathcal{A}_i f_i = -\frac{1}{3} s_i \left(\left(\sqrt{\beta_{xi}}\, x + \eta_{xi} \delta\right)^3 - 3 \cdot \left(\sqrt{\beta_{xi}}\, x + \eta_{xi} \delta\right) \left(\sqrt{\beta_{yi}}\, y + \eta_{yi} \delta\right)^2 \right)$

$$\underset{i\to j}{R} = \begin{pmatrix} \cos\mu_{ij} & \sin\mu_{ij} & & \\ -\sin\mu_{ij} & \cos\mu_{ij} & & \\ & & \cos\mu_{ij} & \sin\mu_{ij} \\ & & -\sin\mu_{ij} & \cos\mu_{ij} \end{pmatrix}$$

$$\mathcal{M}_f = \mathcal{R}_{2\pi\nu}\, e^{:g_i:} e^{:g_j:} = \mathcal{R}_{2\pi\nu}\, e^{:g_i + g_j + \frac{[g_i, g_j]}{2}:} \Omega$$

$$= \mathcal{R}_{2\pi\nu}\, e^{:g_i+g_j:} e^{:\frac{[g_i,g_j]}{2}:} \Omega'$$

$$\mathcal{R}_{2\pi\nu}\, e^{:h_4(\vec{J}):} = e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu}\, e^{:\overset{g_i+g_j}{O_3}:} e^{:\overset{\frac{1}{2}[g_i,g_j]}{O_4}:} e^{-:F_3:} e^{-:F_4:}$$

$$= e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu}\, e^{:O_3:} e^{:-F_3:} e^{:F_3:} e^{:O_4:} e^{-:F_3:} e^{-:F_4:}$$

$$= e^{:F_4:} e^{:F_3:} \mathcal{R}_{2\pi\nu}\, e^{:O_3:} e^{:-F_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$= e^{:F_4:} \mathcal{R}_{2\pi\nu}\, e^{:\mathcal{R}_{2\pi\nu}^{-1} F_3:} e^{:O_3:} e^{:-F_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$= e^{:F_4:} \mathcal{R}_{2\pi\nu}\, e^{:(\mathcal{R}_{2\pi\nu}^{-1} - I)F_3 + O_3:} e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$\boxed{\text{其中 } (\mathcal{R}_{2\pi\nu}^{-1} - I)F_3 + O_3 = h_3(\vec{J}) = 0}$$

$$\therefore = e^{:F_4:} \mathcal{R}_{2\pi\nu}\, e^{:e^{:F_3:}O_4:} e^{-:F_4:}$$

$$\boxed{\text{其中 } F_4 = \frac{\left(e^{:F_3:} O_4 - h_4(\vec{J})\right)}{I - \mathcal{R}_{2\pi\nu}^{-1}}}$$

tune shift with amplitude:

$$\vec{\mu}(\vec{J}) = \vec{\mu} + \frac{\partial h_4(\vec{J})}{\partial \vec{J}} \cdots\cdots$$

interlived sextupole terms.

$$O_4 = \frac{1}{2}\left[g_i,\, g_j\right]$$

resonance basis:

$$h_x^+ = \sqrt{2J_x}\, e^{+i\phi_x} \qquad\qquad h_x^- = \sqrt{2J_x}\, e^{-i\phi_x}.$$

$$\shortparallel \qquad\qquad\qquad\qquad \shortparallel$$

$$x - ip_x \qquad\qquad\qquad\qquad x + ip_x$$

$$\therefore\; x = \sqrt{2J_x}\cos\phi_x = \frac{1}{2}\left(h_x^+ + h_x^-\right)$$

$$p_x = -\frac{1}{2i}\left(h_x^+ - h_x^-\right) = -\sqrt{2J_x}\sin\phi_x$$

$$\mathcal{R}x = \frac{1}{2}\mathcal{R}\left(h_x^+ + h_x^-\right) = \frac{1}{2}\mathcal{R}\left(\sqrt{2J_x}\, e^{+i\phi_x} + \sqrt{2J_x}\, e^{-i\phi_x}\right).$$

$$= \frac{\sqrt{2J_x}}{2}\left(e^{i(\phi_x+\mu)} + e^{-i(\phi_x+\mu)}\right)$$

Only pick up ~~JQ~~ coefficients of J_x^2 : $\left(\frac{\partial h_4(\vec{J})}{\partial J_x}\right)$.

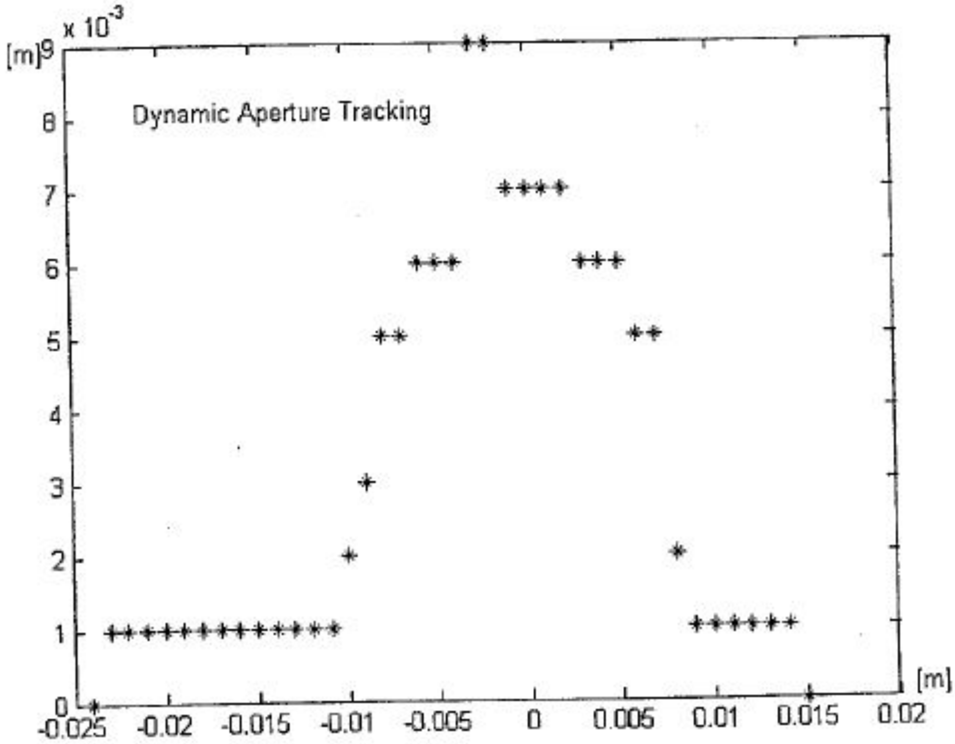
$$\boxed{\frac{\partial \nu_x}{\partial J_x} = \frac{1}{2\pi}\cdot\frac{1}{8}\cdot S_i S_j\, \beta_{xi}^{3/2}\, \beta_{xj}^{3/2}\left(3\cdot\sin(\mu_{ij}) + \sin(3\mu_{ij})\right).}$$

$$\Uparrow$$

Goal of Optimization

Maybe something wrong.

$$
\begin{aligned}
\frac{\partial \nu_x}{\partial J_x} &= -\frac{1}{16\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\beta_{xj}^{3/2}\beta_{xk}^{3/2} \\
&\quad\times\left[\frac{3\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}+\frac{\cos\left(\left|3\mu_{j\to k,x}\right|-3\pi\nu_x\right)}{\sin\left(3\pi\nu_x\right)}\right], \\
\frac{\partial \nu_x}{\partial J_y} &= \frac{\partial \nu_y}{\partial J_x} \\
&= \frac{1}{8\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\sqrt{\beta_{xj}\beta_{xk}}\beta_{yj}\left[\frac{2\beta_{xk}\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}\right. \\
&\quad-\frac{\beta_{yk}\cos\left[|\mu_{j\to k,x}+2\mu_{j\to k,y}|-\pi\left(\nu_x+2\nu_y\right)\right]}{\sin\pi\left(\nu_x+2\nu_y\right)} \\
&\quad\left.+\frac{\beta_{yk}\cos\left[|\mu_{j\to k,x}-2\mu_{j\to k,y}|-\pi\left(\nu_x-2\nu_y\right)\right]}{\sin\pi\left(\nu_x-2\nu_y\right)}\right], \\
\frac{\partial \nu_y}{\partial J_y} &= -\frac{1}{16\pi}\sum_{j=1}^{N}\sum_{k=1}^{N}(b_3L)_j(b_3L)_k\sqrt{\beta_{xj}\beta_{xk}}\beta_{yj}\beta_{yk}\left[\frac{4\cos\left(|\mu_{j\to k,x}|-\pi\nu_x\right)}{\sin\left(\pi\nu_x\right)}\right. \\
&\quad+\frac{\cos\left[|\mu_{j\to k,x}+2\mu_{j\to k,y}|-\pi\left(\nu_x+2\nu_y\right)\right]}{\sin\pi\left(\nu_x+2\nu_y\right)} \\
&\quad\left.+\frac{\cos\left[|\mu_{j\to k,x}-2\mu_{j\to k,y}|-\pi\left(\nu_x-2\nu_y\right)\right]}{\sin\pi\left(\nu_x-2\nu_y\right)}\right] \qquad (119)
\end{aligned}
$$



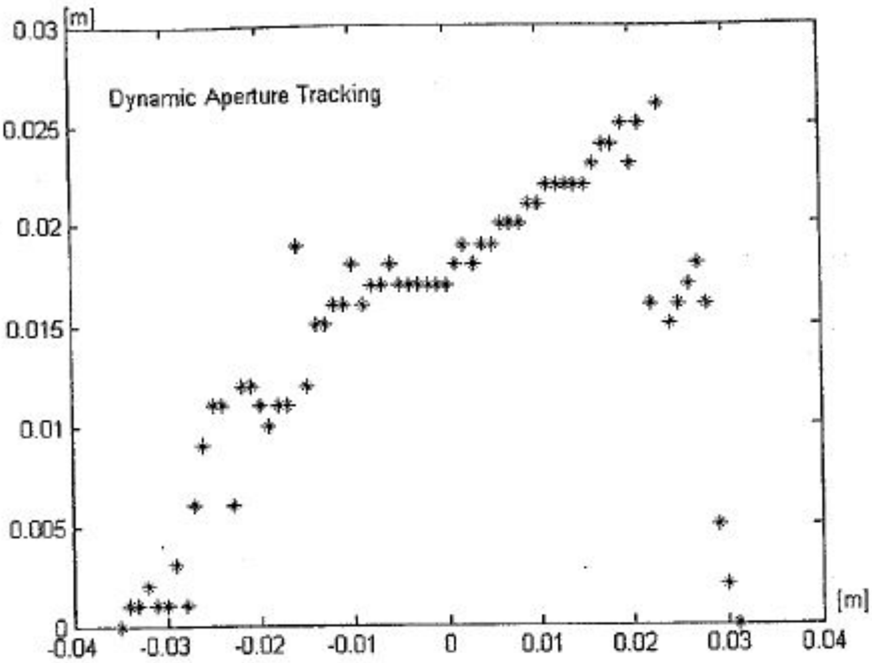


图 3-13 进一步优化后动力学孔径跟踪的结果 4（对应各磁铁族的强度为：

本周工作情况：

1. 选用 Matlab 用于 Sextupole 分组~~合~~优化

2. 跑通 Matlab 的 AT 工具~~箱~~，
学习 AT 中 tune、chrom 的计算方法。

$\Downarrow$

重温 dispersion closed orbit.

3. $\frac{\partial \nu_x}{\partial J_x}$、$\frac{\partial \nu_x}{\partial J_y}$、$\frac{\partial \nu_y}{\partial J_y}$ 均在 10^5 量级.

4. 程序编写接近完成：

$\sum_{i}^{N}\sum_{j}^{N} \Rightarrow$ 双重 for 循环 $\Rightarrow \times$

N块六极铁 1 …… N.

$$\text{N行}\downarrow\begin{pmatrix} 1 & 2 & 3 & \cdots & N \\ \vdots & & & & \vdots \\ 1 & 2 & 3 & \cdots & N \end{pmatrix} \xrightarrow[\text{对角}]{\text{去掉}} \begin{pmatrix} 0 & 2 & 3 & \cdots & N \\ 1 & 0 & & & \\ \vdots & & 0 & & \\ & & & \ddots & \\ 1 & 2 & 3 & \cdots & 0 \end{pmatrix}$$

$$\text{上三角}\times\text{下三角}: \begin{pmatrix} 0 & 2 & 3 & \cdots & N \\ 1 & 0 & & & \\ \vdots & & \ddots & & \\ 1 & 2 & 3 & \cdots & 0 \end{pmatrix}$$

上三角 × 下三角:

$$\begin{pmatrix} 0 & 2 & 3 & 4 & \cdots & N \\ 0 & 0 & 3 & 4 & \cdots & N \\ 0 & 0 & \cdots & & \ddots & \vdots \\ 0 & \cdots & & & & 0 \end{pmatrix} \begin{pmatrix} 0 & & & & \\ 1 & 0 & & & \\ 1 & 2 & 0 & \ddots & \\ \vdots & \vdots & & & \\ 1 & 2 & \cdots & & 0 \end{pmatrix}$$

举例:

$$\begin{pmatrix} 0 & 2 & 3 & 4 \\ & 0 & 3 & 4 \\ & & 0 & 4 \\ & & & 0 \end{pmatrix} \begin{pmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 2 & 0 & \\ 1 & 2 & 3 & 0 \end{pmatrix} \Rightarrow$$ 只需求对角线元素即可

5.

Extro

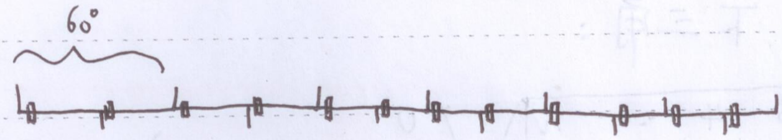
$$\frac{\partial \nu_x}{\partial J_x} = -\frac{1}{16\pi} \sum_{j=1}^{N} \sum_{k=1}^{N} (b_3L)_j (b_3L)_k \beta_{xj}^{3/2} \beta_{xk}^{3/2} \left[\frac{3\cos(|\mu_{j\to k,x}| - \pi\nu_x)}{\sin(\pi\nu_x)} + \frac{\cos(|3\mu_{j\to k,x}| - 3\pi\nu_x)}{\sin 3\pi\nu_x} \right].$$

$$\Downarrow$$

$$(b_3L)_j \beta_{xj}^{3/2} \quad \begin{aligned} &\cos(\mu_i)\cos(\mu_j)\cot(\pi\nu) \\ &+\sin(\mu_i)\sin(\mu_j)\cot(\pi\nu) \\ &-\cos(\mu_j)\sin(\mu_i) \\ &+\sin(\mu_j)\cos(\mu_i) \end{aligned}$$

$$\left[(b_3L)_j \beta_{xj}^{3/2} \cos\mu_j\right] \cdot \left[(b_3L)_i \beta_{xi}^{3/2} \cos(\mu_i)\right] \Rightarrow$$ 上三角 × 下三角

6.



J_x^2 的系数($h_4(J_x^2)$):

$$2\times\left(\frac{9}{64}\sqrt{3}\, Sf^2 \beta_x^2 + \frac{21}{32} sf\cdot sd\, \beta^{3/2} \beta_{xm}^{3/2} + \frac{9}{64}\sqrt{3}\, sd^2 \beta_{xm}^3\right)$$

∴每6个连续FODO不能成一组。⇒破坏掉此结构。

目前 sextupole scheme 为:

全环 8个 Arc, 1920块 sextupole

每在一个Arc中, 有 1920/8 = 240 块 sextupole

每隔 6块 sextupole, 为一组。

即:

$$\left.\begin{array}{lllll} 1 & 7 & 13 & 19 & \cdots \\ 2 & 8 & 14 & 20 & \cdots \\ 3 & 9 & 15 & 21 & \cdots \\ \vdots & & & & \\ 6 & 12 & 18 & 24 & \cdots \end{array}\right\}\ \text{共 } 6\times 8 = 48 \text{ 组}$$