QCD Corrections to B→ π Form Factors From Light-Cone Sum Rules

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Importance of B to Pion form factors

- Determination of $|V_{ub}|$

- Exploring QCD dynamics

- New physics search: FCNC mode

B meson LCSR at leading order

- The correlation function(Khodjamirian, Mannel ,Offen,2005; De Fazio, Feldman, Hurth 2005)

$$\Pi^\mu(p,q) = i\int d^4x\, e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\not{n}\gamma_5 u(x), \bar{u}(0)\Gamma^\mu b(0)\right\}|\bar{B}(P_B)\rangle$$

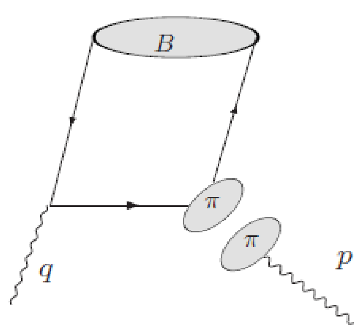
$$\Pi^\mu(p,q) = \Pi(n\cdot p, \bar{n}\cdot p)n^\mu + \tilde{\Pi}(n\cdot p, \bar{n}\cdot p)\bar{n}^\mu \qquad \Gamma^\mu = \gamma^\mu(\sigma^{\mu\nu}q_\nu)$$

$$\Pi_\mu(p,q) = \Pi_{\mathrm{T}}(n\cdot p, \bar{n}\cdot p)\epsilon^{\mu\nu}q_\nu \quad \epsilon^{\mu\nu} = (n^\mu\bar{n}^\nu - \bar{n}^\nu n^\mu)/2.$$

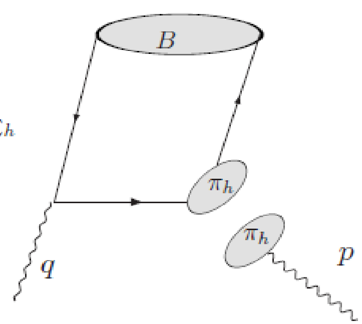
$$n\cdot p \simeq \frac{m_B^2 + m_\pi^2 - q^2}{m_B} = 2E_\pi\,, \qquad \bar{n}\cdot p \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

- Inserting complete set of Pion states

$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) =$

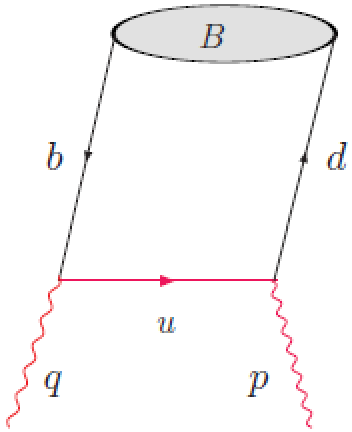


$+$ Σ_h



$$\underbrace{\frac{f_\pi(n\cdot p)m_B}{2(m_\pi^2 - p^2)}\left[\frac{n\cdot p}{m_B}f_{B\pi}^{+}(n\cdot p) + f_{B\pi}^{0}(n\cdot p)\right]}_{\text{relative sign changes for } \Pi(n\cdot p,\bar{n}\cdot p)} \qquad \int_{\omega_s}^{+\infty} d\omega' \frac{\widetilde{\rho}^h(n\cdot p, \omega')}{\omega' - \bar{n}\cdot p}$$

- OPE calculation of the correlation function



$$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) \;=\; \tilde{f}_B(\mu)\, m_B \int_0^\infty d\omega' \, \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p - i\,0} + \mathcal{O}(\alpha_s)$$

$$\Pi(n\cdot p, \bar{n}\cdot p) \;=\; \mathcal{O}(\alpha_s)\,.$$

$$\Pi_{\mathrm{T}}(n\cdot p, \bar{n}\cdot p) = \tilde{f}_B(\mu)\, m_B \int_0^\infty d\omega' \, \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p - i\,0} + \mathcal{O}(\alpha_s)$$

symmetry breaking

- Borel improved LCSRs

$$f_{B\pi}^T(q^2) = \frac{f_B(m_B + m_\pi)}{n\cdot p f_\pi} e^{m_\pi^2/(n\cdot p\omega_M)} \int_0^{\omega_s} d\omega e^{-\omega/\omega_M} \phi_-^B(\omega),$$

$$f_{B\pi}^+(q^2) = \frac{m_B}{n\cdot p} f_{B\pi}^0(q^2) = \frac{m_B}{m_B + m_\pi} f_{B\pi}^T(q^2)$$

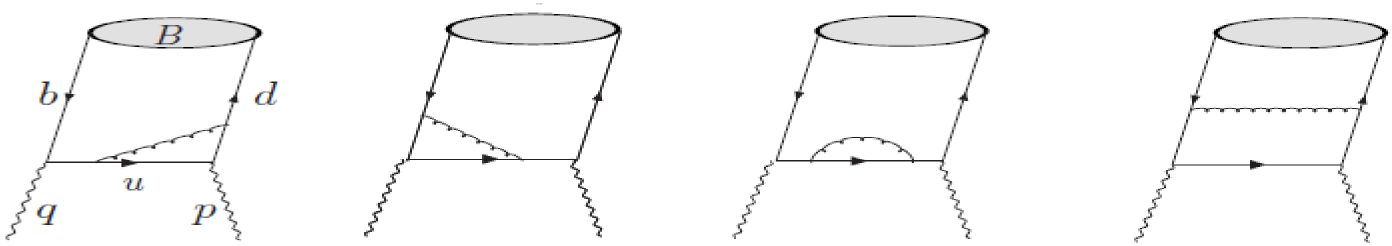
QCD corrections to the correlation functions

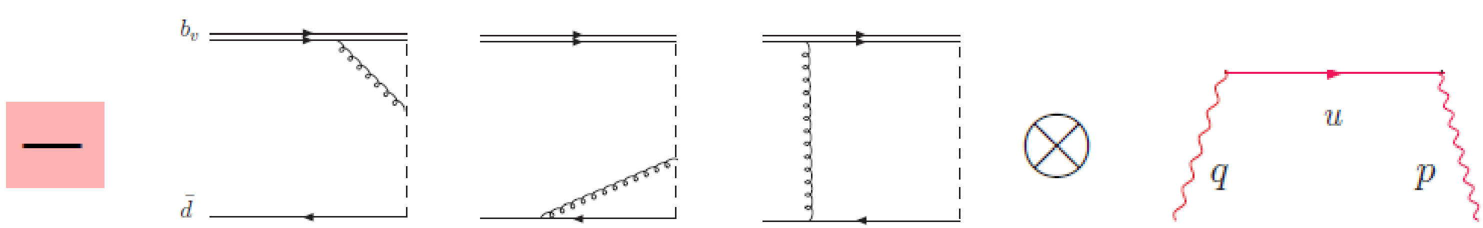
$$\begin{aligned}\Pi_\mu &= \Pi_\mu^{(0)} + \Pi_\mu^{(1)} + \ldots = \Phi_B \otimes T \\ &= \Phi_B^{(0)} \otimes T^{(0)} + \left[\Phi_B^{(0)} \otimes T^{(1)} + \Phi_B^{(1)} \otimes T^{(0)}\right] + \ldots .\end{aligned}$$

$$\Downarrow$$

$$\boxed{\Phi_B^{(0)} \otimes T^{(1)} = \Pi_\mu^{(1)} - \Phi_B^{(1)} \otimes T^{(0)} .}$$

diagrammatical factorization





Calculation of one-loop QCD corrections

- Method of region: hard, hard-collinear and soft
- Separation of energy scales
- Soft cancellation

The NLO order correlation function

$$
\begin{aligned}
\Phi^{(0)}_{b\bar{d}} \otimes T^{(1)} &= \left[\Pi^{(1)}_{\mu,\,weak} + \Pi^{(1)}_{\mu,\,pion} + \Pi^{(1)}_{\mu,\,wfc} + \Pi^{(1)}_{\mu,\,box} + \Pi^{(1)}_{\mu,\,bwf} + \Pi^{(1)}_{\mu,\,dwf}\right] \\
&\quad - \boxed{\left[\Phi^{(1)}_{b\bar{d},\,a} + \Phi^{(1)}_{b\bar{d},\,b} + \Phi^{(1)}_{b\bar{d},\,c} + \Phi^{(1)}_{b\bar{d},\,bwf} + \Phi^{(1)}_{b\bar{d},\,dwf}\right] \otimes T^{(0)}} \\
&= \boxed{\left[\Pi^{(1),\,h}_{\mu,\,weak} + \left(\Pi^{(1)}_{\mu,\,bwf} - \Phi^{(1)}_{b\bar{d},\,bwf}\right)\right]} \\
&\quad + \boxed{\left[\Pi^{(1),\,hc}_{\mu,\,weak} + \Pi^{(1),\,hc}_{\mu,\,pion} + \Pi^{(1),\,hc}_{\mu,\,wfc} + \Pi^{(1),\,hc}_{\mu,\,box}\right]} \,,
\end{aligned}
$$

Hard function

Jet function

$$
\Pi = \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} C^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)
$$

$$
\widetilde{\Pi} = \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)
$$

Hard function:

In agreement with the matching coefficient of QCD weak current to SCET I operator(Bauer et al, 2001;Beneke, Yang,2003)

$$
\begin{aligned}
C^{(+)} &= \tilde{C}^{(+)} = 1, \\
C^{(-)} &= \frac{\alpha_s C_F}{4\pi} \frac{1}{\bar{r}} \left[\frac{r}{\bar{r}} \ln r + 1\right], \\
\tilde{C}^{(-)} &= 1 - \frac{\alpha_s C_F}{4\pi} \left[2 \ln^2 \frac{\mu}{n \cdot p} + 5 \ln \frac{\mu}{m_b} - \ln^2 r - 2\,\mathrm{Li}_2\left(-\frac{\bar{r}}{r}\right) \right. \\
&\quad \left. + \frac{2-r}{r-1} \ln r + \frac{\pi^2}{12} + 5\right],
\end{aligned}
$$

Jet function:

In agreement with that computed in SCET sum rules, De Fazio et al 2005)

$$
\begin{aligned}
J^{(+)} &= \frac{1}{r} \tilde{J}^{(+)} = \frac{\alpha_s C_F}{4\pi} \left(1 - \frac{\bar{n} \cdot p}{\omega}\right) \ln\left(1 - \frac{\omega}{\bar{n} \cdot p}\right), \\
J^{(-)} &= 1, \\
\tilde{J}^{(-)} &= 1 + \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{\mu^2}{n \cdot p(\omega - \bar{n} \cdot p)} - 2 \ln \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} \ln \frac{\mu^2}{n \cdot p(\omega - \bar{n} \cdot p)} \right. \\
&\quad \left. - \ln^2 \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} - \left(1 + \frac{2\bar{n} \cdot p}{\omega}\right) \ln \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} - \frac{\pi^2}{6} - 1\right].
\end{aligned}
$$

Cancellation of scale dependence

$$\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{n\cdot p}+5\right]\tilde{C}^{(-)}(n\cdot p,\mu),$$

$$\frac{d}{d\ln\mu}\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu^2}{n\cdot p\,\omega}\right]\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) + \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\,\tilde{J}^{(-)}(\bar{n}\cdot p,\omega',\mu),$$

$$\frac{d}{d\ln\mu}\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{\omega}-5\right]\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] - \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\,\left[\tilde{f}_B\,\phi_B^-(\omega',\mu)\right],$$

Running hard function and inverse moment of B meson

$$\frac{d}{d\ln\mu} C^{(-)}(n\cdot p,\mu,\nu) = \Gamma_c(\mu) C^{(-)}(n\cdot p,\mu) \quad \Gamma_c(\mu) = -\left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{n\cdot p} + \gamma_h(\alpha_s)\right]$$

$$\Gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s C_F}{4\pi}\left[\Gamma^{(0)}_{\rm cusp} + \left(\frac{\alpha_s}{4\pi}\right)\Gamma^{(1)}_{\rm cusp} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma^{(2)}_{\rm cusp} + \ldots\right]$$

3-loop level

$$C^{(-)}(n\cdot p,\mu) = U_1(n\cdot p,\mu_{h1},\mu)\, C^{(-)}(n\cdot p,\mu_{h1})$$

Beneke, Rohrwild, 2011

$$\frac{\lambda_B(\mu_0)}{\lambda_B(\mu)} = 1 + \frac{\alpha_s(\mu_0) C_F}{4\pi}\ln\frac{\mu}{\mu_0}\left[2 - 2\ln\frac{\mu}{\mu_0} - 4\,\sigma_B^{(1)}(\mu_0)\right] + \mathcal{O}(\alpha_s^2)\,;$$

Factorziation scale fixed at hard collinear scale

Wang, Shen 2015

B→Pion form factors at NLO

$$f_\pi \; e^{-m_\pi^2/(n\cdot p\,\omega_M)} \left\{ \frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p)\,,\; f_{B\pi}^{0}(n\cdot p) \right\}$$

$$= \tilde{f}_B(\mu) \int_0^{\omega_s} d\omega' \; e^{-\omega'/\omega_M} \Bigg[r\, \widetilde{C}^{(+)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{+}(\omega',\mu) + \widetilde{C}^{(-)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{-}(\omega',\mu)$$

$$\pm \frac{n\cdot p - m_B}{m_B} \left(C^{(+)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{+}(\omega',\mu) + C^{(-)}(n\cdot p,\mu)\, \phi_B^{-}(\omega',\mu) \right) \Bigg] .$$

Symmetry breaking effect

The effective wave functions

$$\phi_{B,\mathrm{eff}}^{+}(\omega',\mu) = 0 + \frac{\alpha_s C_F}{4\pi} \int_{\omega'}^{\infty} \frac{d\omega}{\omega}\, \phi_B^{+}(\omega,\mu)\,,$$

$$\phi_{B,\mathrm{eff}}^{-}(\omega',\mu) = \phi_B^{-}(\omega',\mu) + \frac{\alpha_s C_F}{4\pi} \Bigg\{ \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} - 4\ln\frac{\omega'-\omega}{\omega'} \right) \right]_+ \phi_B^{-}(\omega,\mu)$$

$$- \int_{\omega'}^{\infty} d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega} - \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} + 3 \right) \ln\frac{\omega-\omega'}{\omega'} + 2\ln\frac{\omega}{\omega'} + \frac{\pi^2}{6} - 1 \right] \frac{d\phi_B^{-}(\omega,\mu)}{d\omega} \Bigg\} .$$

End point singularity

The B meson LCDA

$$\phi_{B,\mathrm{I}}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,,$$ [Grozin and Neubert, 1997]

$$\phi_{B,\mathrm{II}}^{+}(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\ln k\right],\; k = \frac{\omega}{1\ \mathrm{GeV}}\,,$$ [Braun et al, 2004]

$$\phi_{B,\mathrm{III}}^{+}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2}\, e^{-(\omega/\omega_1)^2},\; \omega_1 = \frac{2\omega_0}{2\sqrt{\pi}}\,,$$ [De Fazio, Feldmann, Hurth, 2008]

$$\phi_{B,\mathrm{IV}}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0\omega_2}\,\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}},\; \omega_2 = \frac{4\omega_0}{4-\pi}\,,$$ [De Fazio, Feldmann, Hurth, 2008]

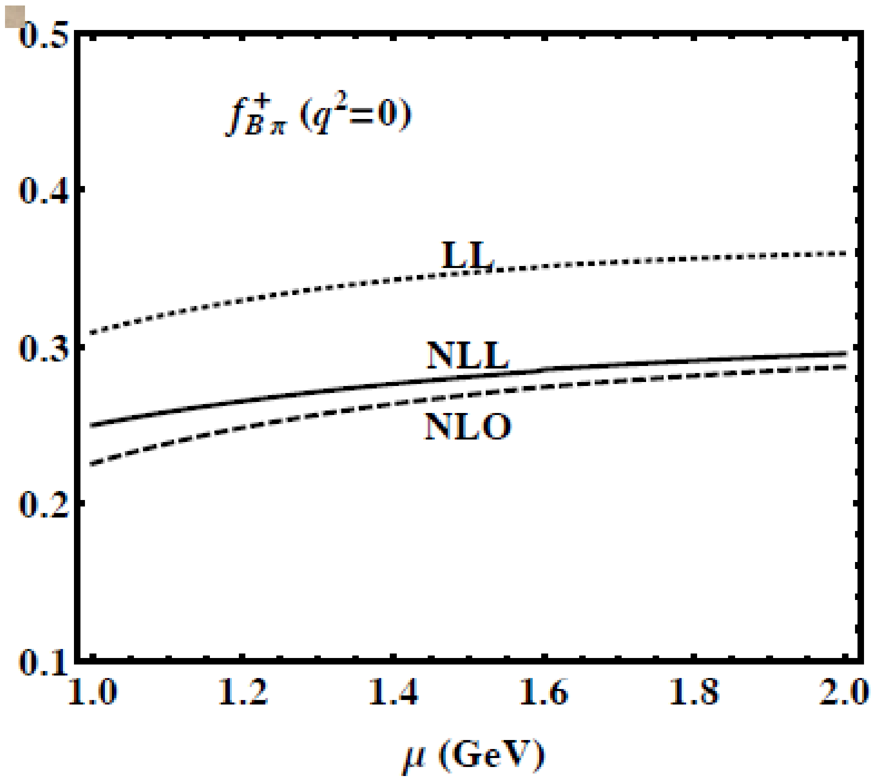
fitting $f_{B\pi}^{+}(q^2=0) = 0.28 \pm 0.03$ from pion LCSR $\Rightarrow$

Model-I: $\omega_0 = 360^{+40}_{-30}\,\mathrm{MeV}$,

Model-II: $\omega_0 = 375^{+40}_{-35}\,\mathrm{MeV}$,

Model-III: $\omega_0 = 395^{+35}_{-30}\,\mathrm{MeV}$,

Model-IV: $\omega_0 = 310^{+40}_{-30}\,\mathrm{MeV}$.



Operator Renormalization of tensor form factor

- Hard function of tensor form factor

$$C^{(-)}(n\cdot p,\mu,\nu) = 1 - \frac{\alpha_s C_F}{4\pi}\left[2\ln\frac{\nu}{m_B} + 2\ln^2\frac{\mu}{m_B} - (4\ln r - 5)\ln\frac{\mu}{m_B}\right.$$
$$\left. + 2\ln^2 r + 2\mathrm{Li}_2(\bar{r}) - \frac{4r-2}{r-1}\ln r + \frac{\pi^2}{12} + 6\right].$$

- RGE

$$\frac{d}{d\ln\nu}C^{(-)}(n\cdot p,\mu,\nu) = \gamma_T(\alpha_s)C^{(-)}(n\cdot p,\mu,\nu)$$

$$\gamma_T(\alpha_s) = \frac{\alpha_s C_F}{4\pi}\left[-2 + \frac{\alpha_s}{4\pi}\left(19C_F - \frac{257}{9}C_A + \frac{52}{9}(n_l+1)T_F\right) + ...\right]$$

- Solution

$$C^{(-)}(n\cdot p,\mu,\nu) = e^{\int_{m_b}^{\nu}\frac{d\nu'}{\nu'}\gamma_T(\alpha_s)}C^{(-)}(n\cdot p,\mu,m_b)$$

Vanishes at mb scale

RGE of jet function and wave function

$$\frac{d}{d\ln\mu} J^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) = \left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu^2}{n\cdot p\,\omega}\right] J^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right)$$
$$+\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\; J^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega'}, \frac{\omega'}{\bar{n}\cdot p}\right)\,,$$

$$\frac{d}{d\ln\mu}\phi_{\bar{B}}^-(\omega,\mu) = -\left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{\omega} + \gamma_+(\alpha_s)\right]\phi_{\bar{B}}^-(\omega,\mu)$$
$$-\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\;\phi_{\bar{B}}^-(\omega',\mu)\,,$$

Lange-Neubert function(2003)

$$\Gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s C_F}{4\pi}\left[\Gamma^{(0)}_{\rm cusp} + \left(\frac{\alpha_s}{4\pi}\right)\Gamma^{(1)}_{\rm cusp} + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma^{(2)}_{\rm cusp} + \ldots\right]$$

Diagonal RGE

- Integral transformation (Bell, Feldmann, Wang and Yip, 2013)

$$\rho_B^-(\omega',\mu) = \int_0^\infty \frac{d\omega}{\omega'} J_0(2\sqrt{\frac{\omega}{\omega'}})\phi_B^-(\omega,\mu)$$

$$\frac{d}{d\ln\mu}\rho_B^-(\omega',\mu) = \Gamma_\rho(\mu)\rho_B^-(\omega',\mu) \qquad \Gamma_\rho(\mu) = -\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{\hat\omega'} - \gamma_+(\alpha_s)$$

$$j^{(-)}\left(\frac{\mu_{hc}^2}{n\cdot p\hat\omega'},\frac{\hat\omega'}{\bar n\cdot p}\right) = \int_0^\infty \frac{d\omega}{\omega-\bar n\cdot p} J_0(2\sqrt{\frac{\omega}{\omega'}}) J^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar n\cdot p}\right)$$

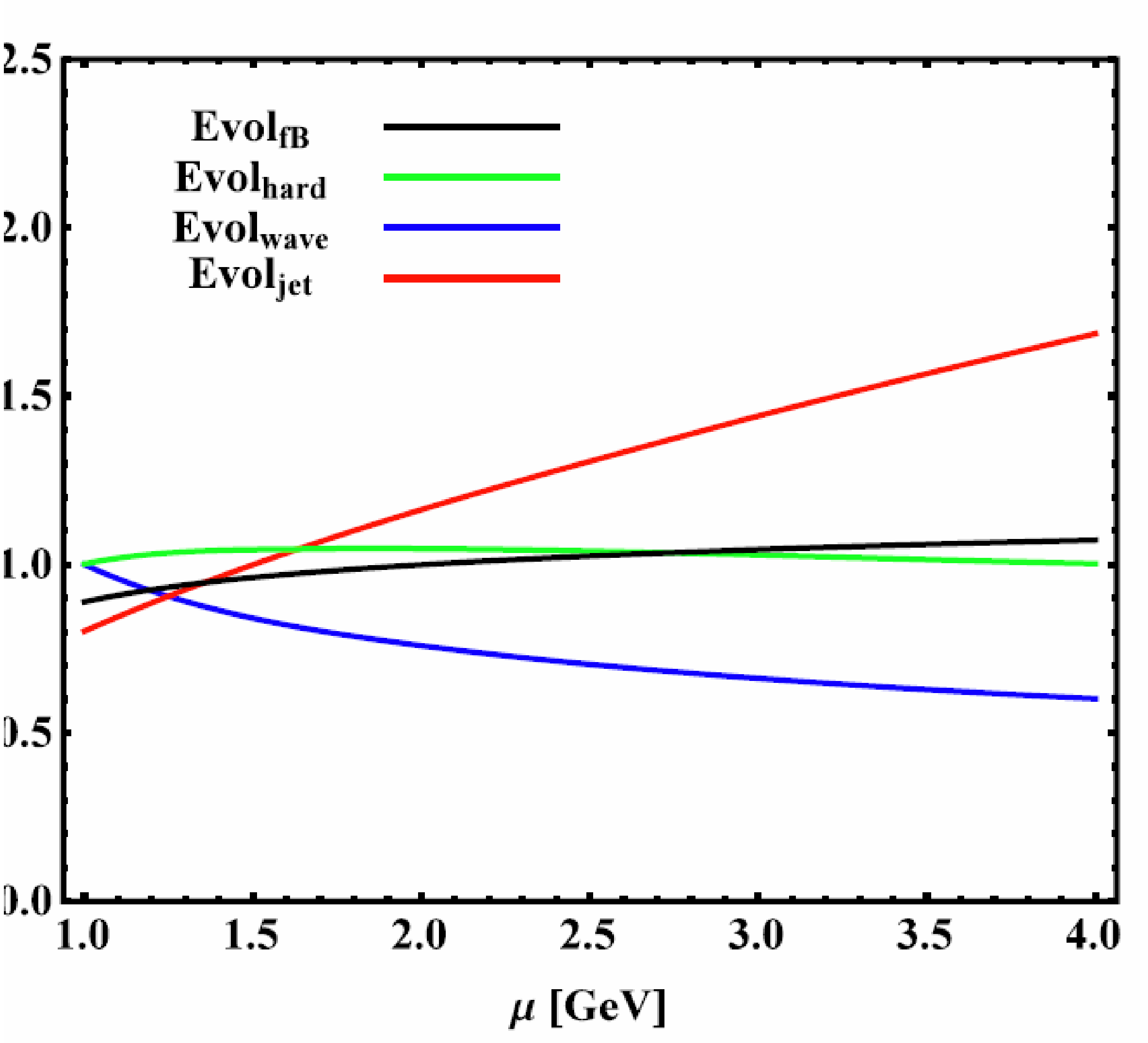
$$\frac{d}{d\ln\mu} j(\hat\omega',\mu) = \Gamma_j j(\hat\omega',\mu) \qquad \Gamma_j = -\Gamma_C - \Gamma_\rho - \tilde\gamma(\alpha_s)$$

- Formal solution

$$\rho_B^-(\omega',\mu) = e^{V(\mu,\mu_0)}\left(\frac{\mu_0}{\hat\omega'}\right)^{-g(\mu,\mu_0)}\rho_B^-(\omega',\mu_0)$$

$$j(\hat\omega',\mu) = e^{-2V_{hc}(\mu,\mu_{hc})}\left(\frac{\mu_{hc}^2}{\hat\omega'\bar n\cdot p}\right)^{g(\mu,\mu_{hc})} j(\hat\omega',\mu_{hc})$$

Behavior of the evolution factors



Shen ,Wang, Wei, Lu,

in preparation

Jet function and wave function in dual space

$$j(\hat{\omega}', \mu_{hc}) = 2K_0\left(2\sqrt{\frac{1}{\eta'}}\right)\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{-p^2} - \frac{\pi^2}{6} - 1 - \frac{1}{2}\ln\hat{\eta}'(4\ln\frac{\mu^2}{-p^2} + 3) + \frac{1}{4}\ln^2\hat{\eta}' - \frac{1}{6}\pi^2\right]\right\} + \frac{\alpha_s C_F}{2\pi}K_0^{(2,0)}\left(2\sqrt{\frac{1}{\eta'}}\right) + \frac{\alpha_s C_F}{2\pi}G\left(2\sqrt{\frac{1}{\eta'}}\right) \quad (2.42)$$

$$\rho_B^-(\omega', \mu) = \rho_B^+(\omega', \mu) = \frac{1}{\omega'}e^{-\omega_0/\omega'}$$

$$\rho_{B,\mathrm{III}}^-(\omega', \mu) = \frac{\omega_1[\omega'\sqrt{\pi}{}_0F_2(;\frac{1}{2},1;\frac{\omega_1^2}{16\omega'^2}) - \omega_{1\,0}F_2(;\frac{3}{2},\frac{3}{2};\frac{\omega_1^2}{16\omega'^2})]}{2\omega_0\omega'}$$

B→Pion form factors at NLO (running jet and wave function)

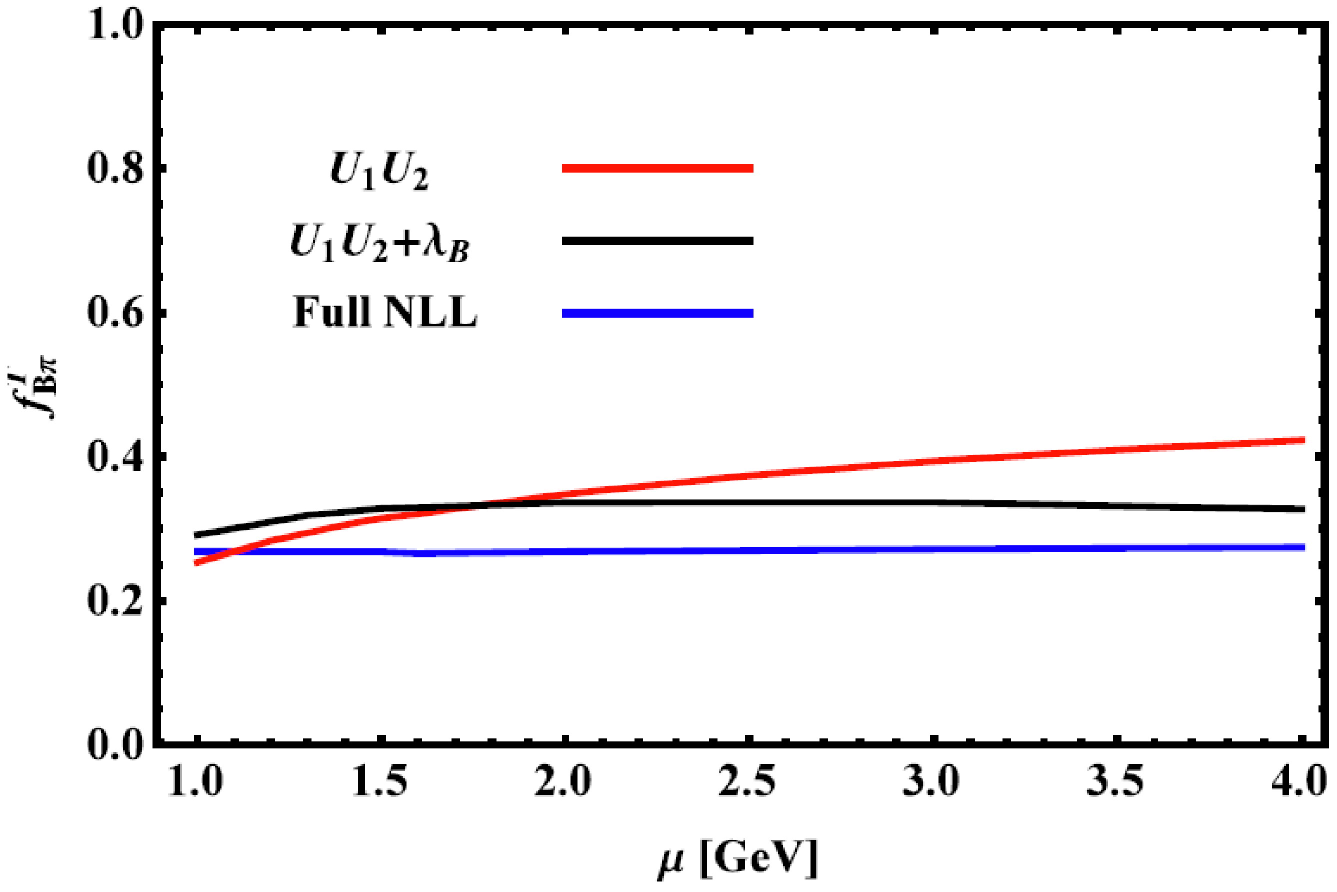
$$f_\pi e^{-m_\pi^2 n\cdot p/\omega_M^2} f_{B\pi}^T(q^2) = U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2}) \int_0^{\omega_s} d\omega\; e^{-\omega/\omega_M} \Big[\phi_{eff}^+(\omega) + U_1(n\cdot p,\mu_{h1},\mu) C^{(-)}(n\cdot p,\mu_{h1}) \rho_{eff}^-(\omega)\Big]$$

$$\begin{aligned}\rho_{eff}^-(\Omega,\mu) = &\int_0^\infty \frac{d\omega'}{\omega'} \bigg\{\bigg[1+\frac{\alpha_s C_F}{4\pi}\bigg(\ln^2\frac{\mu^2}{n\cdot p\Omega} - 2\ln\frac{\mu^2}{n\cdot p\Omega}\ln\frac{\hat{\omega}'}{\Omega} \\ &+\frac{1}{2}\ln^2\frac{\hat{\omega}'}{\Omega} - \frac{3}{2}\ln\frac{\hat{\omega}'}{\Omega} + \frac{\pi^2}{2} - 1\bigg)\bigg] J_0\left(2\sqrt{\frac{\Omega}{\omega'}}\right) \\ &+\frac{\alpha_s C_F}{4\pi}\left(\ln\frac{\hat{\omega}'}{\Omega}+\frac{3}{2}\right)\pi N_0\left(2\sqrt{\frac{\Omega}{\omega'}}\right) \\ &+\frac{\alpha_s C_F}{2\pi}\bigg[J_0^{(2,0)}\left(2\sqrt{\frac{\Omega}{\omega'}}\right) + \frac{\Omega}{\omega'}\,{}_2F_3(1,1;2,2,2;-\frac{\Omega}{\omega'}) - \ln\frac{\Omega}{\hat{\omega}'}\bigg]\bigg\} \\ &\times U_j(\mu_{hc},\mu) U_\rho(\mu_0,\mu) \rho_B^{(-)}(\omega',\mu_0)\end{aligned}$$

The effective wave functions

$$\phi_{B,\mathrm{eff}}^+(\Omega,\mu) = \frac{\alpha_s C_F}{4\pi} \int_\Omega^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega,\mu)$$

Scale dependence of the tensor form factor



Summary and outlook

- We establish a framework of the NLO corrections to the B→Pion form factors from B meson LCSRs
- We perform a “complete” RGE evolution at one-loop level ;
- Understanding the subleading correction is important, such as 3-particle DAs;
- This framework can be extended to various heavy-to-light transition processes .

Form factors

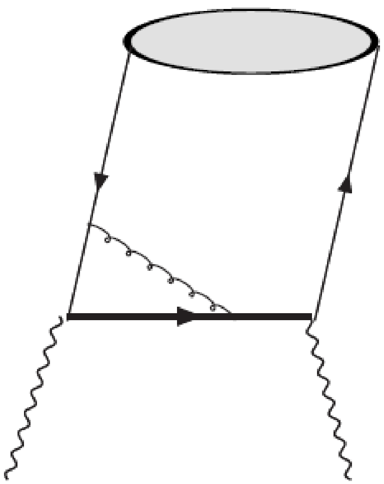
$$\langle \pi(p)|\bar{u}\gamma_\mu b|\bar{B}(p_B)\rangle = f^+_{B\pi}(q^2)\left[p_B + p - \frac{m_B^2 - m_\pi^2}{q^2} q\right]_\mu + f^0_{B\pi}(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2} q_\mu ,$$

$$\langle \pi(p)|\bar{q}\sigma^{\mu\nu} q_\nu b|B(p_B)\rangle = \frac{i f^T_{B\pi}(q^2)}{m_B + m_\pi}[(p_B + p)_\mu q^2 - (m_B^2 - m_\pi^2) q^\mu] ,$$

B meson DA

$$\langle 0|\bar{d}_\beta(\tau\,\bar{n})\,[\tau\bar{n},0]\,b_\alpha(0)|\bar{B}(p+q)\rangle$$
$$= -\frac{i\tilde{f}_B(\mu)\,m_B}{4}\left\{\frac{1+\not{v}}{2}\left[2\,\tilde{\phi}_B^+(\tau) + \left(\tilde{\phi}_B^-(\tau) - \tilde{\phi}_B^+(\tau)\right)\not{n}\right]\gamma_5\right\}_{\alpha\beta}$$

Strategy of calculation: method of regions



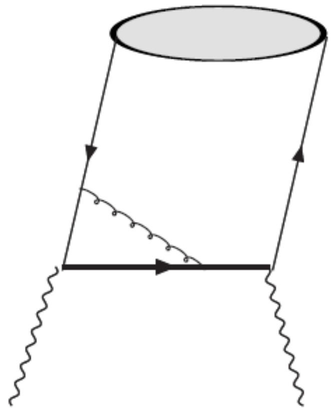
Leading region

- Hard $\quad l \sim m_b(\lambda^0, \lambda^0, \lambda^0)$
- Hard-collinear $\quad l \sim m_b(\lambda^0, \lambda^{\frac{1}{2}}, \lambda)$
- soft $\quad l \sim m_b(\lambda, \lambda, \lambda)$

$$\Pi^{(1),\,h}_{\mu,\,weak} = i\, g_s^2\, C_F\, \tilde{f}_B(\mu)\, m_B\, \frac{\phi^-_{b\bar{d}}(\omega)}{\bar{n}\cdot p - \omega} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2 + n\cdot p\ \bar{n}\cdot l + i0][l^2 + 2\, m_b\, v\cdot l + i0][l^2 + i0]} \times \left\{ \bar{n}_\mu \left[2\, m_b\, n\cdot(p+l) + (D-2)\, l_\perp^2\right] - n_\mu\, (D-2)\, (\bar{n}\cdot l)^2 \right\}$$

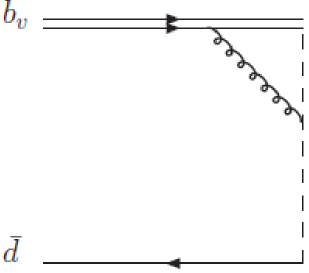
$$\Pi^{(1),\,h}_{\mu,\,weak} = \frac{\alpha_s\, C_F}{4\,\pi}\, \tilde{f}_B(\mu)\, m_B\, \frac{\phi^-_{b\bar{d}}(\omega)}{\bar{n}\cdot p - \omega} \left\{ \bar{n}_\mu \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(2\,\ln\frac{\mu}{n\cdot p} + 1\right) + 2\,\ln^2\frac{\mu}{n\cdot p} + 2\,\ln\frac{\mu}{m_b} - \ln^2 r - 2\,\mathrm{Li}_2\left(-\frac{\bar{r}}{r}\right) + \frac{2-r}{r-1}\ln r + \frac{\pi^2}{12} + 3 \right] \boxed{+ n_\mu \left[\frac{1}{r-1}\left(1 + \frac{r}{\bar{r}}\ln r\right)\right]} \right\}$$

Infrared safe



$$\Pi^{(1),s}_{\mu,weak} = \frac{g_s^2 C_F}{2(\bar n\cdot p-\omega)}\int\frac{d^D l}{(2\pi)^D}\frac{1}{[\bar n\cdot(p-k+l)+i0][v\cdot l+i0][l^2+i0]}$$
$$\bar d(k)\ \not{\bar n}\,\gamma_5\ \not{n}\,\gamma_\mu\ b(p_b)$$
$$= \frac{\alpha_s C_F}{4\pi}\tilde f_B(\mu)\,m_B\frac{\phi^-_{b\bar d}(\omega)}{\bar n\cdot p-\omega}\,\bar n_\mu$$
$$\times\left[\frac{1}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{\mu}{\omega-\bar n\cdot p}+2\ln^2\frac{\mu}{\omega-\bar n\cdot p}+\frac{3\pi^2}{4}\right],$$

$$\Phi^{\alpha\beta,(1)}_{b\bar d,a}(\omega,\omega') = i g_s^2 C_F\int\frac{d^D l}{(2\pi)^D}\frac{1}{[\bar n\cdot l+i0][v\cdot l+i0][l^2+i0]}$$
$$\times[\delta(\omega'-\omega-\bar n\cdot l)-\delta(\omega'-\omega)]\,[\bar d(k)]_\alpha\,[b(v)]_\beta$$

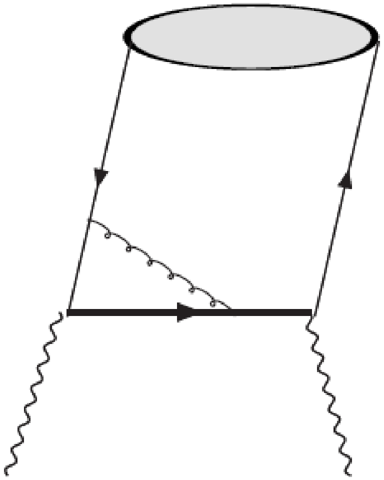


Soft cancellation

$$\boxed{\Pi^{(1),s}_{\mu,weak} = \Phi^{(1)}_{b\bar d,a}\otimes T^{(0)}}$$

- The convolution integrals involves $\phi^+_{b\bar d}(\omega)$ must be finite
- The symmetry breaking contribution must be infrared finite

Strategy of calculation: method of regions



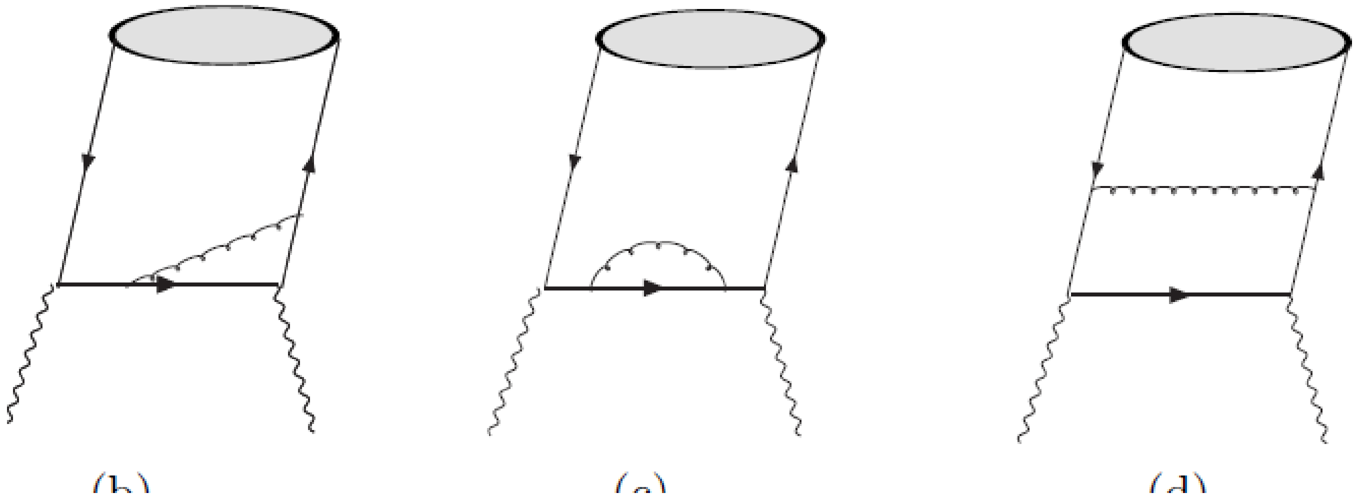
Leading region

- Hard $\quad l \sim m_b(\lambda^0, \lambda^0, \lambda^0)$
- Hard-collinear $\quad l \sim m_b(\lambda^0, \lambda^{\frac{1}{2}}, \lambda)$
- soft $\quad l \sim m_b(\lambda, \lambda, \lambda)$

$$\Pi^{(1),\,hc}_{\mu,\,weak} = i\, g_s^2\, C_F\, \tilde{f}_B(\mu)\, m_B\, \frac{\phi^-_{b\bar{d}}(\omega)}{\bar{n}\cdot p - \omega} \int \frac{d^D l}{(2\pi)^D}$$

$$\frac{2\, m_b\, n\cdot(p+l)}{[n\cdot(p+l)\,\bar{n}\cdot(p-k+l) + l_\perp^2 + i0][m_b\, n\cdot l + i0][l^2 + i0]}$$

$$\Pi^{(1),\,hc}_{\mu,\,weak} = \frac{\alpha_s\, C_F}{4\,\pi}\, \tilde{f}_B(\mu)\, m_B\, \frac{\phi^-_{b\bar{d}}(\omega)}{\omega - \bar{n}\cdot p}\, \bar{n}_\mu \left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\left(\ln \frac{\mu^2}{n\cdot p\,(\omega - \bar{n}\cdot p)} + 1 \right) + \ln^2 \frac{\mu^2}{n\cdot p\,(\omega - \bar{n}\cdot p)} + 2\, \ln \frac{\mu^2}{n\cdot p\,(\omega - \bar{n}\cdot p)} - \frac{\pi^2}{6} + 4 \right].$$



- Only hard collinear region is relevant in Pion vertex and box diagram
- In the pion vertex diagram, the transverse momentum part in the B wave function is important

$$
\begin{aligned}
M_{\beta\alpha} \;=\; & -\frac{i\tilde{f}_B(\mu)\, m_B}{4} \\
& \times \left\{ \frac{1+\not{v}}{2} \left[\phi_B^+(\omega')\;\not{n} + \phi_B^-(\omega')\;\bar{\not{n}} - \frac{2\,\omega'}{D-2}\,\phi_B^-(\omega')\,\gamma_\perp^\rho\,\frac{\partial}{\partial k'_{\perp\rho}} \right] \gamma_5 \right\}_{\alpha\beta}
\end{aligned}
$$

Evolution function

$$V := V(\mu, \mu_0) = -\int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \left[\Gamma_{\text{cusp}}(\alpha) \int\limits_{\alpha_s(\mu_0)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} + \gamma_+(\alpha) \right]$$

$$g := g(\mu, \mu_0) = \int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d\alpha \, \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \, .$$