Three dimensional fragmentation functions from parton correlators

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Introduction

- **Parton distribution functions (PDFs):**

  parton number density in hadron   (hadron structure)

- **Fragmentation functions (FFs):**

  hadron number density in jet   (hadronization)

- **PDFs & FFs are important inputs for high energy reactions.**

Introduction

Intuitive definition of FFs

One dimensional:

$$D(z;k) = D_1(z) \qquad \sum_h \int dz\, z D_1^h(z) = 1$$

$$D\big(z,S;k,S_q\big) = D_1(z) + \lambda_q \lambda G_{1L}(z) + \big(\vec{S}_{\perp q}\cdot\vec{S}_T\big)H_{1T}(z)$$

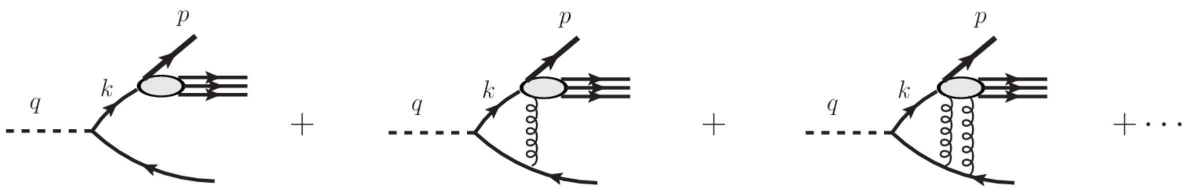
Three dimensional:

$$\begin{aligned} D\big(z,p_T,S;k,S_q\big) = {} & D_1(z,p_T) + \lambda_q \lambda G_{1L}(z,p_T) + \big(\vec{S}_{\perp q}\cdot\vec{S}_T\big)H_{1T}(z,p_T) \\ & + \frac{1}{M}\vec{S}_T\cdot\big(\hat{k}\times\vec{p}_T\big)D_{1T}^{\perp}(z,p_T) + \frac{1}{M}\vec{S}_{\perp q}\cdot\big(\hat{k}\times\vec{p}_T\big)H_1^{\perp}(z,p_T) \\ & + \frac{1}{M^2}\big(\vec{S}_{\perp q}\cdot\vec{p}_T\big)\big(\vec{S}_T\cdot\vec{p}_T\big)H_{1T}^{\perp}(z,p_T) + \frac{1}{M}\lambda\big(\vec{S}_{\perp q}\cdot\vec{p}_T\big)H_{1L}^{\perp}(z,p_T) \\ & + \frac{1}{M}\lambda_q\big(\vec{S}_T\cdot\vec{p}_T\big)G_{1T}^{\perp}(z,p_T) \end{aligned}$$

Introduction

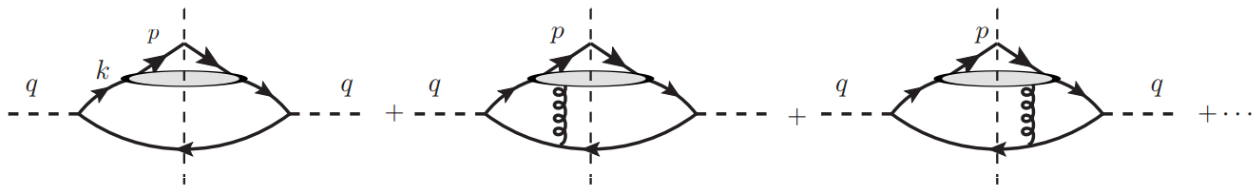
In the language of quantum field theory

$\mathcal{M}$



interference effect

$|\mathcal{M}|^2$



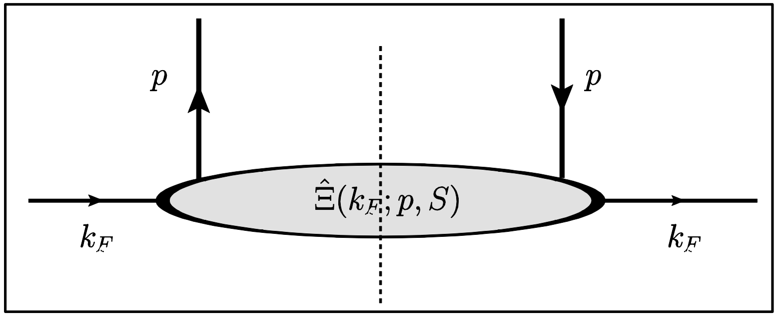
Gauge invariant definition of FFs using QFT operators

Procedure of FFs decomposition

Quark-quark correlator for fragmentation process

$$\hat{\Xi}^{(0)}_{ij}(k_F;p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi\, e^{-ik_F\xi}\langle 0|\mathcal{L}^\dagger(0;\infty)\psi_i(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

$$\mathcal{L}(\xi,\infty) = \mathcal{P}e^{ig\int_{\xi^-}^{\infty} d\eta^- A^+(\eta^-;\xi^+,\vec{\xi}_\perp)}$$



Constraints

- $\hat{\Xi}^{\dagger 0}(k_F;p,S) = \gamma^0\hat{\Xi}^0(k_F;p,S)\gamma^0$ (Hermiticity)
- $\hat{\Xi}^0(k_F;p,S) = \gamma^0\hat{\Xi}^0(\tilde{k}_F;\tilde{p},S^{\mathcal{P}})\gamma^0$ (Parity) $\tilde{A}^\mu = A_\mu$

Procedure of FFs decomposition

Decompose quark-quark correlator under gamma matrices

$$\hat{\Xi}_{ij}^{(0)}(z,k_{F\perp};p,S) = \int \frac{p^+ dk_F^+ dk_F^-}{(2\pi)^2} 2\pi\delta(k_F^+ - \frac{p^+}{z})\hat{\Xi}^{(0)}(k_F;p,S)$$

4 × 4 matrix

$$= \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-i(p^+\xi^-/z + k_{F\perp}\cdot\xi_\perp)} \langle 0|\mathcal{L}^\dagger(0;\infty)\psi_j(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}_i(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

Decompose in terms of Γ matrices

$$\Gamma = \{\, \mathbf{I}, \quad i\gamma_5, \quad \gamma^\alpha, \quad \gamma_5\gamma^\alpha, \quad i\sigma^{\alpha\beta}\gamma_5 \,\}$$

$$\hat{\Xi}^{(0)}(z,k_{F\perp};p,S) = \frac{1}{2}[\Xi^{(0)}(z,k_{F\perp};p,S) + i\gamma_5\tilde{\Xi}^{(0)}(z,k_{F\perp};p,S) + \gamma^\alpha\Xi_\alpha^{(0)}(z,k_{F\perp};p,S) + \gamma_5\gamma^\alpha\tilde{\Xi}_\alpha^{(0)}(z,k_F;p,S) + i\sigma^{\alpha\beta}\gamma_5\Xi_{\alpha\beta}^{(0)}(z,k_{F\perp};p,S)]$$

$$\Xi^{(0)[\Gamma]}(z,k_{F\perp};p,S) = \frac{1}{2}\mathrm{Tr}\left[\Gamma\hat{\Xi}^{(0)}(z,k_{F\perp};p,S)\right]$$

Procedure of FFs decomposition

Spin dependence

Description of spin states: $$\rho_{(S=1/2)} = \frac{1}{2}(1 + \vec{S} \cdot \vec{\sigma}),$$
$$\rho_{(S=1)} = \frac{1}{3}(1 + \frac{3}{2}\vec{S} \cdot \vec{\Sigma} + 3T^{ij}\Sigma^{ij}).$$

Polarization vector: $S^{\mu} = (0, \vec{S}_T, \lambda),$

polarization tensor: $$T^{ij} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}.$$

Available variables: $p, \quad k_{F\perp}, \quad n, \quad \lambda, \quad S_T, \quad S_{LL}, \quad S_{LT}, \quad S_{TT}.$

Results of TMD FFs from parton correlators

TMD FFs for unpolarized hadron:

scalar	pseudo-scalar	vector	axial-vector	pseudo-tensor
M	$\times$	p_α	$\varepsilon_{\perp\alpha\beta}k_{F\perp}^{\beta}$	$p_{[\rho}\varepsilon_{\perp\alpha]\beta}k_{F\perp}^{\beta}$
		$k_{F\perp\alpha}$		$\varepsilon_{\perp\rho\alpha}$
		n_α		$n_{[\rho}\varepsilon_{\perp\alpha]\beta}k_{F\perp}^{\beta}$

$$z\Xi^{U(0)}(z,k_{F\perp};p) = ME(z,k_{F\perp}),$$

$$z\tilde{\Xi}^{U(0)}(z,k_{F\perp};p) = 0,$$

$$z\Xi_{\alpha}^{U(0)}(z,k_{F\perp};p) = p^+\bar{n}_\alpha D_1(z,k_{F\perp}) + k_{F\perp\alpha}D^{\perp}(z,k_{F\perp}) + \frac{M^2}{p^+}n_\alpha D_3(z,k_{F\perp}),$$

$$z\tilde{\Xi}_{\alpha}^{U(0)}(z,k_{F\perp};p) = \varepsilon_{\perp\alpha\beta}k_{F\perp}^{\beta}G^{\perp}(z,k_{F\perp}),$$

$$z\Xi_{\rho\alpha}^{U(0)}(z,k_{F\perp};p) = \frac{p^+\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\beta}k_{F\perp}^{\beta}}{M}H_1^{\perp}(z,k_{F\perp}) + M\varepsilon_{\perp\rho\alpha}H(z,k_{F\perp}) + \frac{Mn_{[\rho}\varepsilon_{\perp\alpha]\beta}k_{F\perp}^{\beta}}{p^+}H_3^{\perp}(z,k_{F\perp}).$$

$$D_1(z) = \frac{z}{2}\sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle\gamma^+\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle$$

$$\xRightarrow{(\mathcal{L}=1)} \frac{z}{4p^{+2}}\delta(1/z - k_F^+/p^+)\langle hX|a_k^\dagger|0\rangle\langle 0|a_k|hX\rangle$$

Number density!

Results of TMD FFs from parton correlators

Vector polarization dependent TMD FFs:

$$z\Xi^{V(0)}(z,k_{F\perp};p,S) = \varepsilon_\perp^{k_{F\perp}S_T} E_T^\perp(z,k_{F\perp}),$$

$$z\tilde{\Xi}^{V(0)}(z,k_{F\perp};p,S) = M[\lambda E_L(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} E_T'^\perp(z,k_{F\perp})],$$

$$z\Xi_\alpha^{V(0)}(z,k_{F\perp};p,S) = \frac{p^+\bar{n}_\alpha\varepsilon_\perp^{k_{F\perp}S_T}}{M} D_{1T}^\perp(z,k_{F\perp}) + M\varepsilon_{\perp\alpha\rho}S_T^\rho D_T(z,k_{F\perp})$$
$$+\varepsilon_{\perp\alpha\rho}k_{F\perp}^\rho[\lambda D_L^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} D_T^\perp(z,k_{F\perp})] + \frac{Mn_\alpha\varepsilon_\perp^{k_{F\perp}S_T}}{p^+} D_{3T}^\perp(z,k_{F\perp}),$$

$$z\tilde{\Xi}_\alpha^{V(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_\alpha[\lambda G_{1L}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_{1T}^\perp(z,k_{F\perp})]$$
$$-k_{F\perp\alpha}[\lambda G_L^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_T^\perp(z,k_{F\perp})] - MS_{T\alpha}\, G_T(z,k_{F\perp})$$
$$+\frac{M^2 n_\alpha}{p^+}[\lambda G_{3L}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G_{3T}^\perp(z,k_{F\perp})].$$

$$z\Xi_{\rho\alpha}^{V(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z,k_{F\perp}) + \frac{p^+\bar{n}_{[\rho}k_{F\perp\alpha]}}{M}[\lambda H_{1L}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_{1T}^\perp(z,k_{F\perp})]$$
$$+k_{F\perp[\rho}S_{T\alpha]}H_T^\perp(z,k_{F\perp}) + M\bar{n}_{[\rho}n_{\alpha]}[\lambda H_L^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_T'^\perp(z,k_{F\perp})]$$
$$+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}H_{3T}(z,k_{F\perp}) + \frac{Mn_{[\rho}k_{F\perp\alpha]}}{p^+}[\lambda H_{3L}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} H_{3T}^\perp(z,k_{F\perp})].$$

Results of TMD FFs from parton correlators

Tensor polarization dependent TMD FFs:

$$
\begin{aligned}
z\Xi_\alpha^{T(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_\alpha\Big[& S_{LL}D_{1LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{1LT}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}D_{1TT}^\perp(z,k_{F\perp})\Big] \\
& + MS_{LT\alpha}D_{LT}(z,k_{F\perp}) + k_{F\perp}^\sigma S_{TT\sigma\alpha}D_{TT}'^\perp(z,k_{F\perp}) \\
& + k_{F\perp\alpha}\Big[S_{LL}D_{LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{LT}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}D_{TT}^\perp(z,k_{F\perp})\Big] \\
& + \frac{M^2}{p^+}n_\alpha\Big[S_{LL}D_{3LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}D_{3LT}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}D_{3TT}^\perp(z,k_{F\perp})\Big], \\
z\tilde{\Xi}_\alpha^{T(0)}(z,k_{F\perp};p,S) = p^+\bar{n}_\alpha\Big[& \frac{\varepsilon_\perp^{k_{F\perp}S_{LT}}}{M}G_{1LT}^\perp(z,k_{F\perp}) + \frac{\varepsilon_{\perp\beta\rho}k_{F\perp}^\beta k_{F\perp\sigma}S_{TT}^{\rho\sigma}}{M^2}G_{1TT}^\perp(z,k_{F\perp})\Big] \\
& + M\varepsilon_{\perp\alpha\rho}S_{LT}^\rho G_{LT}(z,k_{F\perp}) + \varepsilon_{\perp\alpha\rho}k_{F\perp\sigma}S_{TT}^{\rho\sigma}G_{TT}'^\perp(z,k_{F\perp}) \\
& + \varepsilon_{\perp\alpha\rho}k_{F\perp}^\rho\Big[S_{LL}G_{LL}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}G_{LT}^\perp(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}G_{TT}^\perp(z,k_{F\perp})\Big] \\
& + \frac{M^2}{p^+}n_\alpha\Big[\frac{\varepsilon_\perp^{k_{F\perp}S_{LT}}}{M}G_{3LT}^\perp(z,k_{F\perp}) + \frac{\varepsilon_{\perp\beta\rho}k_{F\perp}^\beta k_{F\perp\sigma}S_{TT}^{\rho\sigma}}{M^2}G_{3TT}^\perp(z,k_{F\perp})\Big].
\end{aligned}
$$

Results of TMD FFs from parton correlators

Tensor polarization dependent TMD FFs:

$$z\Xi^{T(0)}(z,k_{F\perp};p,S) = M\Big[S_{LL}E_{LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}E_{LT}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}E_{TT}^{\perp}(z,k_{F\perp})\Big],$$

$$z\tilde{\Xi}^{T(0)}(z,k_{F\perp};p,S) = M\Big[\frac{\varepsilon_{\perp}^{k_{F\perp}S_{LT}}}{M}E_{LT}'^{\perp}(z,k_{F\perp}) + \frac{\varepsilon_{\perp\beta\rho}k_{F\perp}^{\beta}k_{F\perp\sigma}S_{TT}^{\rho\sigma}}{M^2}E_{TT}'^{\perp}(z,k_{F\perp})\Big],$$

$$\begin{aligned}
z\Xi_{\rho\alpha}^{T(0)}(z,k_{F\perp};p,S) = {}& p^{+}\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\sigma}S_{LT}^{\sigma}H_{1LT}(z,k_{F\perp}) + \frac{p^{+}}{M}\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\sigma}k_{F\perp\delta}S_{TT}^{\sigma\delta}H_{1TT}'^{\perp}(z,k_{F\perp})\\
&+ \frac{p^{+}\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\sigma}k_{F\perp}^{\sigma}}{M}\Big[S_{LL}H_{1LL}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{1LT}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}H_{1TT}^{\perp}(z,k_{F\perp})\Big]\\
&+ M\varepsilon_{\perp\rho\alpha}\Big[S_{LL}H_{LL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{LT}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}H_{TT}^{\perp}(z,k_{F\perp})\Big]\\
&+ \bar{n}_{[\rho}n_{\alpha]}\Big[\varepsilon_{\perp}^{k_{F\perp}S_{LT}}H_{LT}'^{\perp}(z,k_{F\perp}) + \frac{\varepsilon_{\perp\beta\sigma}k_{F\perp}^{\beta}k_{F\perp\delta}S_{TT}^{\sigma\delta}}{M}H_{TT}'^{\perp}(z,k_{F\perp})\Big]\\
&+ \frac{M}{p^{+}}n_{[\rho}\varepsilon_{\perp\alpha]\sigma}k_{F\perp}^{\sigma}\Big[S_{LL}H_{3LL}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{LT}}{M}H_{3LT}^{\perp}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}H_{3TT}^{\perp}(z,k_{F\perp})\Big]\\
&+ \frac{M}{p^{+}}n_{[\rho}\varepsilon_{\perp\alpha]\sigma}\Big[MS_{LT}^{\sigma}H_{3LT}(z,k_{F\perp}) + k_{F\perp\delta}S_{TT}^{\sigma\delta}H_{3TT}'^{\perp}(z,k_{F\perp})\Big].
\end{aligned}$$

Results of TMD FFs from parton correlators

Twist-3 FFs from quark-gluon-quark correlator

$$\hat{\Xi}^{(1)}_{\rho,ij}(k_F;p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi \; e^{-ik_F\xi} \langle 0 \big| \mathcal{L}^\dagger(0;\infty) D_\rho(0)\psi_i(0) \big| p,S;X\rangle \langle p,S;X \big| \bar{\psi}_j(\xi)\mathcal{L}(\xi;\infty) \big| 0\rangle$$

$$\hat{\Xi}^{(1)}_{\rho} = \frac{1}{2}[\gamma^\alpha \Xi^{(1)}_{\rho\alpha} + \gamma_5\gamma^\alpha \tilde{\Xi}^{(1)}_{\rho\alpha} + i\sigma^{\alpha\beta}\gamma_5 \Xi^{(1)}_{\rho\alpha\beta} + \cdots]$$

$$z\Xi^{U(1)}_{\rho\alpha}(z,k_{F\perp};p) = -p^+\bar{n}_\alpha k_{F\perp\rho} D^\perp_d(z,k_{F\perp}) + \cdots,$$
$$z\tilde{\Xi}^{U(1)}_{\rho\alpha}(z,k_{F\perp};p) = ip^+\bar{n}_\alpha \varepsilon_{\perp\rho\sigma} k^\sigma_{F\perp} G^\perp_d(z,k_{F\perp}) + \cdots,$$
$$z\Xi^{U(1)}_{\rho\alpha\beta}(z,k_{F\perp};p) = -p^+[M\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]} H_d(z,k_{F\perp}) + \frac{1}{M}\varepsilon_{\perp\rho\sigma} k^\sigma_{F\perp} k_{F\perp[\alpha}\bar{n}_{\beta]} H^\perp_d(z,k_{F\perp})] + \cdots.$$

$$z\Xi^{V(1)}_{\rho\alpha}(z,k_{F\perp};p,S) = -p^+\bar{n}_\alpha\Big\{M\varepsilon_{\perp\rho\sigma}S^\sigma_T D_{dT}(z,k_{F\perp}) + \varepsilon_{\perp\rho\sigma}k^\sigma_{F\perp}\Big[\lambda D^\perp_{dL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} D^\perp_{dT}(z,k_{F\perp})\Big]\Big\} + \ldots,$$

$$z\tilde{\Xi}^{V(1)}_{\rho\alpha}(z,k_{F\perp};p,S) = -ip^+\bar{n}_\alpha\Big\{MS_{T\rho}G_{dT}(z,k_{F\perp}) + k_{\perp\rho}\Big[\lambda G^\perp_{dL}(z,k_{F\perp}) + \frac{k_{F\perp}\cdot S_T}{M} G^\perp_{dT}(z,k_{F\perp})\Big]\Big\} + \ldots,$$

$$\begin{aligned} z\Xi^{V(1)}_{\rho\alpha\beta}(z,k_{F\perp};p,S) =& p^+\Big\{\lambda\Big[Mg_{\perp\rho[\alpha}\bar{n}_{\beta]}H_{dL}(z,k_{F\perp}) + \frac{1}{M}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^\perp_{dL}(z,k_{F\perp})\Big] \\ & - \varepsilon^{k_{F\perp}S_T}_\perp\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^\perp_{dT}(z,k_{F\perp}) + \frac{1}{M^2}\varepsilon_{\perp\rho\sigma}k^\sigma_{F\perp}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dT}(z,k_{F\perp})\Big] \\ & + (k_{F\perp}\cdot S_T)\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\prime\perp}_{dT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\prime\perp\prime}_{dT}(z,k_{F\perp})\Big]\Big\} + \ldots. \end{aligned}$$

Results of TMD FFs from parton correlators

Twist-3 FFs from Quark-gluon-quark correlator

$$
\begin{aligned}
z\Xi^{T(1)}_{\rho\alpha}(z,k_{F\perp};p,S) =& -p^+\bar{n}_\alpha\Big[k_{F\perp\rho}S_{LL}D^{\perp}_{dLL}(z,k_{F\perp}) + MS_{LT\rho}D_{dLT}(z,k_{F\perp}) + k_{F\perp\rho}\frac{k_{F\perp}\cdot S_{LT}}{M}D^{\perp}_{dLT}(z,k_{F\perp}) \\
&+ S_{TT\rho\beta}k^{\beta}_{F\perp}D'^{\perp}_{dTT}(z,k_{F\perp}) + k_{F\perp\rho}\frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}D^{\perp}_{dTT}(z,k_{F\perp})\Big] + \dots, \\
z\tilde{\Xi}^{(1)}_{\rho\alpha}(z,k_{F\perp};p,S) =& ip^+\bar{n}_\alpha\Big[\varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}S_{LL}G^{\perp}_{dLL}(z,k_{F\perp}) + M\varepsilon_{\perp\rho\sigma}S^{\sigma}_{LT}G_{dLT}(z,k_{F\perp}) + \frac{1}{M}\varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}k_{F\perp}\cdot S_{LT}G^{\perp}_{dLT}(z,k_{F\perp}) \\
&+ \varepsilon_{\perp\rho\beta}S^{\beta\gamma}_{TT}k_{F\perp\gamma}G'^{\perp}_{dTT}(z,k_{F\perp}) + \varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}\frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M^2}G^{\perp}_{dTT}(z,k_{F\perp})\Big] + \dots, \\
z\Xi^{T(1)}_{\rho\alpha\beta}(z,k_{F\perp};p,S) =& p^+\Big\{S_{LL}\Big[M\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H_{dLL}(z,k_{F\perp}) + \frac{1}{M}\varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp}_{dLL}(z,k_{F\perp})\Big] \\
&+ (k_{F\perp}\cdot S_{LT})\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\perp}_{dLT}(z,k_{F\perp}) + \frac{1}{M^2}\varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dLT}(z,k_{F\perp})\Big] \\
&- \varepsilon^{k_{F\perp}S_{LT}}_{\perp}\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H'^{\perp}_{dLT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H'^{\perp\prime}_{dLT}(z,k_{F\perp})\Big] \\
&+ \frac{k_{F\perp}\cdot S_{TT}\cdot k_{F\perp}}{M}\Big[\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H^{\perp}_{dTT}(z,k_{F\perp}) + \frac{1}{M^2}\varepsilon_{\perp\rho\sigma}k^{\sigma}_{F\perp}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp\prime}_{dTT}(z,k_{F\perp})\Big] \\
&- \frac{\varepsilon_{\perp\gamma\delta}k^{\delta}_{F\perp}S^{\gamma\sigma}_{TT}k_{F\perp\sigma}}{M}\Big[g_{\perp\rho[\alpha}\bar{n}_{\beta]}H'^{\perp}_{dTT}(z,k_{F\perp}) + \frac{1}{M^2}k_{F\perp\rho}k_{F\perp[\alpha}\bar{n}_{\beta]}H'^{\perp\prime}_{dTT}(z,k_{F\perp})\Big]\Big\} + \dots.
\end{aligned}
$$

Results of TMD FFs from parton correlators

QCD equation of motion: relations among FFs

$$\gamma^\mu D_\mu(x)\psi(x) = 0 \quad \Rightarrow$$

$$p^+\Xi^{(0)\rho} = -zn_\alpha\left(\mathrm{Re}\Xi^{(1)\rho\alpha} + \varepsilon_{\perp\sigma}^{\rho}\mathrm{Im}\tilde{\Xi}^{(1)\sigma\alpha}\right),$$

$$p^+\tilde{\Xi}^{(0)\rho} = -zn_\alpha\left(\mathrm{Re}\tilde{\Xi}^{(1)\rho\alpha} + \varepsilon_{\perp\sigma}^{\rho}\mathrm{Im}\Xi^{(1)\sigma\alpha}\right),$$

$$p^+\left(\Xi^{(0)\rho\sigma} - i\varepsilon_\perp^{\rho\sigma}\Xi^{(0)}\right) = -zn_\gamma\varepsilon_\perp^{\rho\sigma}\varepsilon_{\perp\alpha\beta}\Xi^{(1)\alpha\gamma\beta},$$

$$p^+\left(\Xi^{0+-} - i\tilde{\Xi}^{(0)}\right) = zn_\gamma g_{\perp\alpha\beta}\Xi^{(1)\alpha\gamma\beta}.$$

$$D_{dS}^K(z,k_{F\perp}) + G_{dS}^K(z,k_{F\perp}) = \frac{1}{z}\left[D_S^K(z,k_{F\perp}) + iG_S^K(z,k_{F\perp})\right]$$

K	***S***
null	*T LT*
⊥	*null L T LL LT TT*
′⊥	*TT*

$$H_{dS}^K(z,k_{F\perp}) + \frac{k_{F\perp}^2}{2M^2}H_{dS}^{K'}(z,k_{F\perp}) = \frac{1}{2z}\left[H_S^K(z,k_{F\perp}) + \frac{i}{2}E_S^K(z,k_{F\perp})\right]$$

K	***K′***	***S***
null	⊥	*null L LL*
⊥	⊥′	*T LT TT*
′⊥	′⊥′	*T LT TT*

Summary & outlook

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- Most general decomposition procedure of TMD FFs for spin-1 hadron is given in the framework of parton correlation matrix.
- Totally 72 TMD FFs are defined via quark-quark correlator. 8 spin independent, 24 vector polarization dependent, 40 tensor polarization dependent. The number of T-odd and T-even (χ-odd and χ-even)TMD FFs are equal.
- Twist-3 FFs from quark-gluon-quark correlator are given, and they are related to the FFs from quark-quark correlator by QCD EOM in a unified form.

Outlook

Study tensor polarization dependent TMD FFs in high energy reactions

$e^+e^- \to \pi + V + X$, ……

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Thank you!

☐ Parity constraints

$\hat{\mathcal{P}}$	$\bar{\psi}\psi$	$\bar{\psi}i\gamma_5\psi$	$\bar{\psi}\gamma_\alpha\psi$	$\bar{\psi}\gamma_5\gamma_\alpha\psi$	$\bar{\psi}i\sigma_{\alpha\beta}\gamma_5\psi$
	$\bar{\psi}\psi$	$-\bar{\psi}i\gamma_5\psi$	$\bar{\psi}\gamma^\alpha\psi$	$-\bar{\psi}\gamma_5\gamma^\alpha\psi$	$-\bar{\psi}i\sigma^{\alpha\beta}\gamma_5\psi$

$$\hat{\Xi}^{(0)}(z,k_{F\perp};p,S) = \frac{1}{2}[\underline{\Xi^{(0)}(z,k_{F\perp};p,S)} + i\gamma_5\underline{\tilde{\Xi}^{(0)}(z,k_{F\perp};p,S)}$$
$$+\gamma^\alpha\underline{\Xi_\alpha^{(0)}(z,k_{F\perp};p,S)} + \gamma_5\gamma^\alpha\underline{\tilde{\Xi}_\alpha^{(0)}(z,k_F;p,S)} + i\sigma^{\alpha\beta}\gamma_5\underline{\Xi_{\alpha\beta}^{(0)}(z,k_{F\perp};p,S)}]$$

scalar; pseudo-scalar; vector; axial-vector; pseudo-tensor

$\hat{\mathcal{P}}$	p_α	$k_{F\perp\alpha}$	n_α	λ	$S_{T\alpha}$	S_{LL}	$S_{LT\alpha}$	$S_{TT\alpha\beta}$
	p^α	$k_{F\perp}^\alpha$	n^α	$-\lambda$	$-S_T^\alpha$	S_{LL}	S_{LT}^α	$S_{TT}^{\alpha\beta}$

Spin-0

Chirality

$$\psi_{R/L} = \frac{1 \pm \gamma^5}{2}\psi, \quad \bar{\psi}_{R/L} = \bar{\psi}\frac{1 \mp \gamma^5}{2}$$

$$\bar{\psi}\Gamma\psi \sim \begin{cases} \bar{\psi}_R\Gamma\psi_R + \bar{\psi}_L\Gamma\psi_L & (\Gamma = \gamma^\mu,\ \gamma^5\gamma^\mu) \quad \chi\text{-even} \\ \bar{\psi}_R\Gamma\psi_L + \bar{\psi}_L\Gamma\psi_R & (\Gamma = \mathrm{I},\ i\gamma^5, i\sigma^{\mu\nu}\gamma^5) \quad \chi\text{-odd} \end{cases}$$

χ-even: $(D_1,\quad D^\perp,\quad D_3,\quad G^\perp)$ χ-odd: $(E,\quad H_1^\perp,\quad H,\quad H_3^\perp)$

Time reversal

$\hat{\mathcal{T}}$	$\bar{\psi}\psi$	$\bar{\psi}i\gamma_5\psi$	$\bar{\psi}\gamma_\alpha\psi$	$\bar{\psi}\gamma_5\gamma_\alpha\psi$	$\bar{\psi}i\sigma_{\alpha\beta}\gamma_5\psi$
	$\bar{\psi}\psi$	$-\bar{\psi}i\gamma_5\psi$	$\bar{\psi}\gamma^\alpha\psi$	$\bar{\psi}\gamma_5\gamma^\alpha\psi$	$\bar{\psi}i\sigma^{\alpha\beta}\gamma_5\psi$

$\hat{\mathcal{T}}$	M	p_α	$k_{F\perp\alpha}$	n_α	$\varepsilon_{\perp\alpha k_{F\perp}}$	$\varepsilon_{\perp\alpha\beta}$
	M	p^α	$k_{F\perp}^\alpha$	n^α	$-\varepsilon_\perp^{\alpha k_{F\perp}}$	$-\varepsilon_\perp^{\alpha\beta}$

T-even: $(E,\quad D^\perp,\quad D_3,\quad G^\perp)$ T-odd: $(G^\perp,\quad H_1^\perp,\quad H,\quad H_3^\perp)$

Quark polarization	Hadron polarization	Chiral-even		Chiral-odd	
		T-even	T-odd	T-even	T-odd
U	**U**	D_1 $D^\perp$ D_3		E	
	L		$D_L^\perp$		
	T		$D_{1T}^\perp$ D_T $D_T^\perp$ $D_{3T}^\perp$		$E_T^\perp$
	LL	D_{1LL} $D_{LL}^\perp$ D_{3LL}		E_{LL}	
	LT	$D_{1LT}^\perp$ D_{LT} $D_{LT}^\perp$ $D_{3LT}^\perp$		$E_{LT}^\perp$	
	TT	$D_{1TT}^\perp$ $D_{TT}^\perp$ $D_{TT}'^\perp$ $D_{3TT}^\perp$		$E_{TT}^\perp$	
L	**U**		$G^\perp$		
	L	G_{1L} $G_L^\perp$ G_{3L}			E_L
	T	$G_{1T}^\perp$ G_T $G_T^\perp$ $G_{3T}^\perp$			$E_T'^\perp$
	LL		$G_{LL}^\perp$		
	LT		$G_{1LT}^\perp$ G_{LT} $G_{LT}^\perp$ $G_{3LT}^\perp$	$E_{LT}'^\perp$	
	TT		$G_{1TT}^\perp$ $G_{TT}^\perp$ $G_{TT}'^\perp$ $G_{3TT}^\perp$	$E_{TT}'^\perp$	
T	**U**				$H_1^\perp$ H $H_3^\perp$
	L			$H_{1L}^\perp$ H_L $H_{3L}^\perp$	
	T(∥)			H_{1T} $H_T^\perp$ H_{3T}	
	T(⊥)			$H_{1T}^\perp$ $H_T'^\perp$ $H_{3T}^\perp$	
	LL				
	LT				H_{1LT} $H_{1LT}^\perp$ $H_{LT}^\perp$ $H_{LT}'^\perp$ H_{3LT} $H_{3LT}^\perp$
	TT				$H_{1TT}^\perp$ $H_{1TT}'^\perp$ $H_{TT}^\perp$ $H_{TT}'^\perp$ $H_{3TT}^\perp$ $H_{3TT}'^\perp$