Emergent SUSY in deformed $\mathcal{N}$=4 SYM

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April 4, 2016

arxiv 1601.01943

$\mathcal{N}$=4 SYM and Deformations

- $\mathcal{N}$=4 SYM: dimensional reduction of $\mathcal{N}$=1 SYM in 10-d.

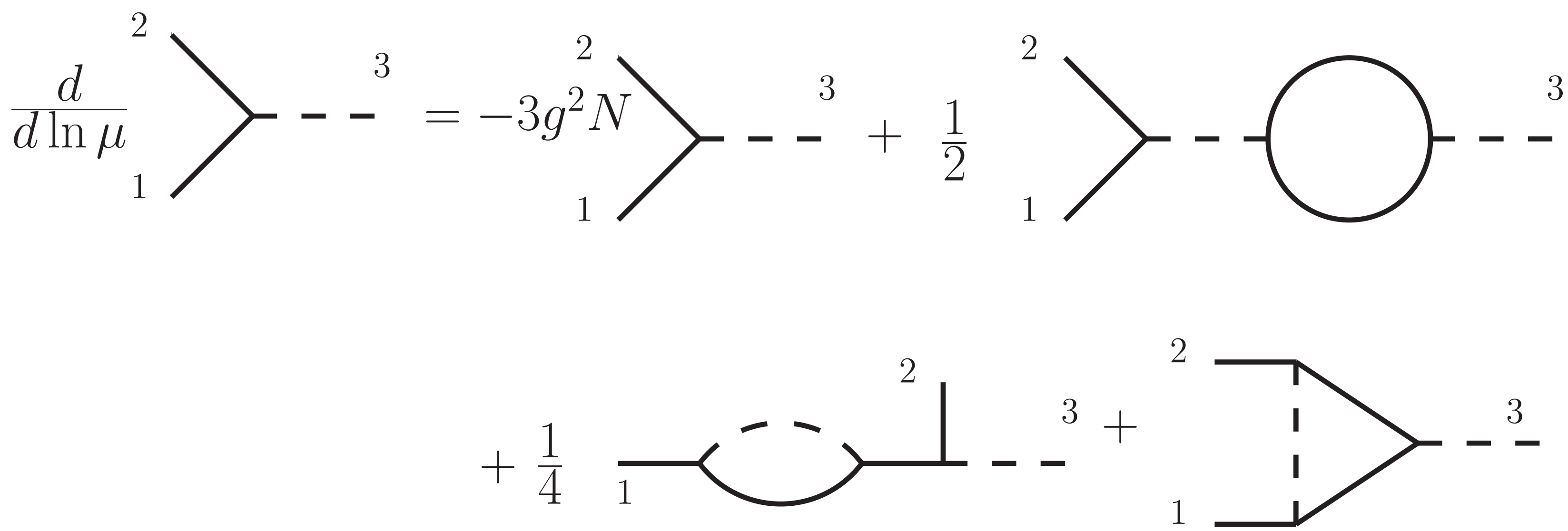
  $$A_{(10)} \to (A_\mu, \phi^I),\ \psi_{(10)} \to \psi^A$$

- One gluon, 6 real scalars, 4 chiral fermions.
- Deformation: Same fields, different couplings(parameters).

Lagrangian of deformed theory

$$
\begin{aligned}
L =& Tr\left(L_0 - \frac{i}{\sqrt{2}}(L_Y - \bar{L}_Y) + L_\phi\right) \\
L_0 =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi_I D^\mu\phi^I + i\bar{\psi}_A\bar{\sigma}^\mu D_\mu\psi^A \\
L_Y =& \lambda_{IAB}\phi^I\psi^A\psi^B,\ \bar{L}_Y = \bar{\lambda}_I^{BA}\phi^I\bar{\psi}_A\bar{\psi}_B \\
L_\phi =& \frac{1}{4}a_{IJKL}\phi^I\phi^J\phi^K\phi^L
\end{aligned}
$$

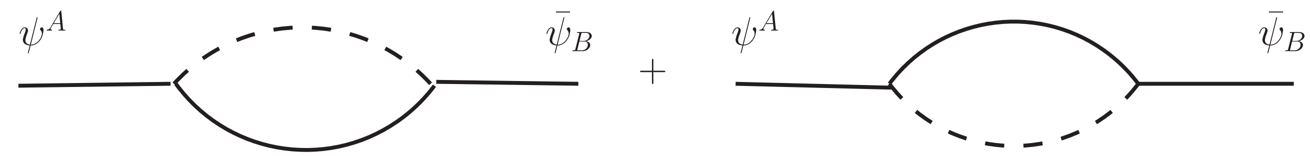
β-function of Yukawa Coupling



- Dashed line: scalar   Solid line: fermion

$$\frac{1}{N}\frac{d\lambda_{IAB}}{d\ln\mu} = -\,3g^2\lambda_{IAB} + \frac{1}{2}\lambda_{JAB}R_{IJ} + \frac{1}{4}f_A^C\lambda_{ICB} + \frac{1}{4}f_B^C\lambda_{IAC} + \bar{\lambda}_I^{CD}\lambda_{JDA}\lambda_{JBC}$$

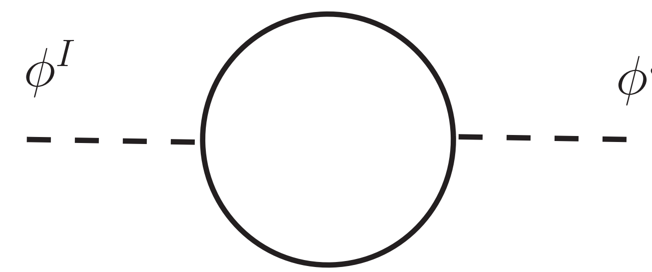
$$f_A^B = \lambda_{IAC}\bar{\lambda}_I^{BC} + \lambda_{ICA}\bar{\lambda}_I^{CB}$$



$$d_{IJ} = \frac{1}{2}(R_{IJ} + iI_{IJ}) = \lambda_{JCD}\bar{\lambda}_I^{CD}$$

$$R_{IJ} = \lambda_{JCD}\bar{\lambda}_I^{CD} + \lambda_{ICD}\bar{\lambda}_J^{CD}$$

$$I_{IJ} = \frac{1}{i}(\lambda_{JCD}\bar{\lambda}_I^{CD} - \lambda_{ICD}\bar{\lambda}_J^{CD})$$



Nonlinear differential equations with 96 complex variables!

$\mathcal{N}=1$ deformations

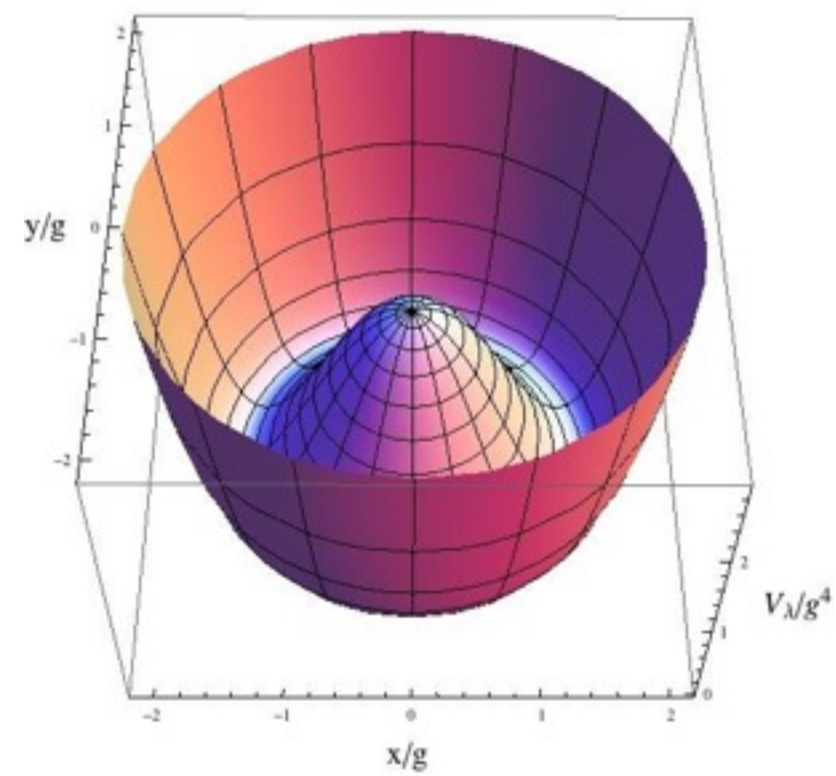
- SYM with chiral supermultiplets
- Superpotential $W = \lambda_{ijk} Tr(\Phi^i \Phi^j \Phi^k)$
- Beta functions

$$\frac{1}{N}\frac{d\lambda_{ijk}}{d\ln\mu} = \frac{1}{2}\left(D_i^l \lambda_{ljk} + D_j^l \lambda_{ilk} + D_k^l \lambda_{ijl}\right)$$

$$D_i^j \equiv -2g^2 \delta_i^j + \lambda_{ikl} \bar{\lambda}^{jkl}$$

Renormalization Potential

$$\frac{1}{N}\frac{d\lambda_{ijk}}{d\ln\mu} = \frac{\partial V_\lambda}{\partial\bar{\lambda}^{ijk}},\ \ V_\lambda \equiv \frac{1}{4}D_i^j D_j^i$$



The potential for

$$W = Tr(x\Phi^1\Phi^2\Phi^3 - y\Phi^1\Phi^3\Phi^2)$$

$$V(x,y) = (x^2 + y^2 - 2g^2)^2$$

Properties of Renormalization Potential

- Covariant under re-parameterization.

$$\frac{dx^i}{d\ln\mu} = g^{ij}\frac{\partial V}{\partial x^j}$$

- Assuming the metric g is positive definite, V increases monotonically with energy.

$$\frac{dV}{d\ln\mu} = \frac{\partial V}{\partial x^i}\frac{dx^i}{d\ln\mu} = g^{ij}\frac{\partial V}{\partial x^i}\frac{\partial V}{\partial x^j} \geq 0$$

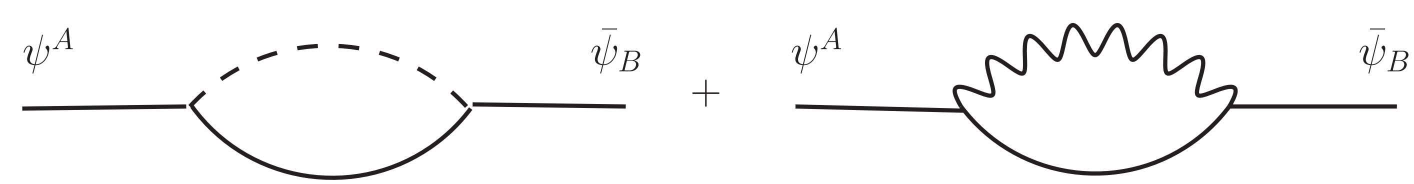
- If V is bounded below, the flow always have a infrared fixed point.

Renormalization Potential of Yukawa Coupling

$$\frac{1}{N}\frac{d\lambda_{IAB}}{d\ln\mu} = \frac{\partial\rho}{\partial\bar{\lambda}_I^{AB}}$$

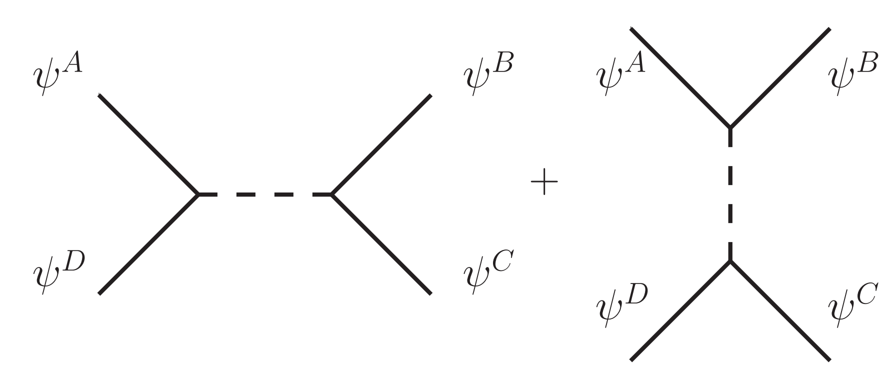
$$\rho = \frac{1}{8}F_A^B F_B^A + \frac{1}{8}I_{IJ}I_{IJ} + \frac{1}{4}T_{ABCD}\bar{T}^{ABCD}$$

$$F_A^B = f_A^B - 6g^2\delta_A^B$$

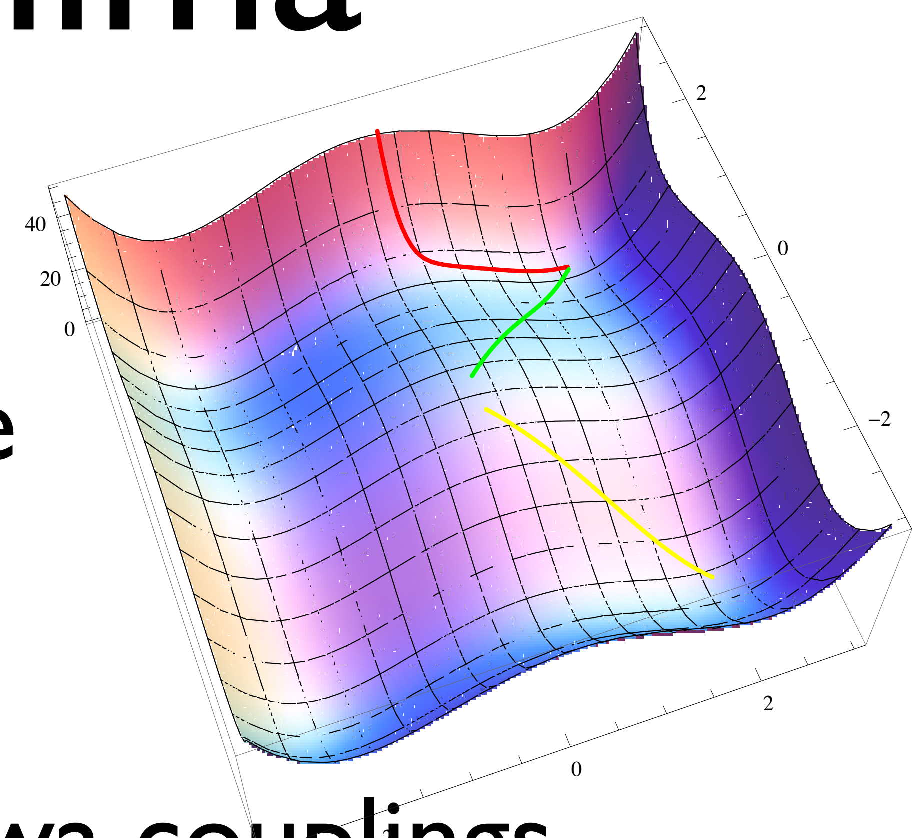


$$I_{IJ} = \frac{1}{i}(\lambda_{JCD}\bar{\lambda}_I^{CD} - \lambda_{ICD}\bar{\lambda}_J^{CD})$$

$$T_{ABCD} = \lambda_{IDA}\lambda_{IBC} + \lambda_{IAB}\lambda_{ICD}$$



Global minima



- $\boldsymbol{\rho}$ has global minima where

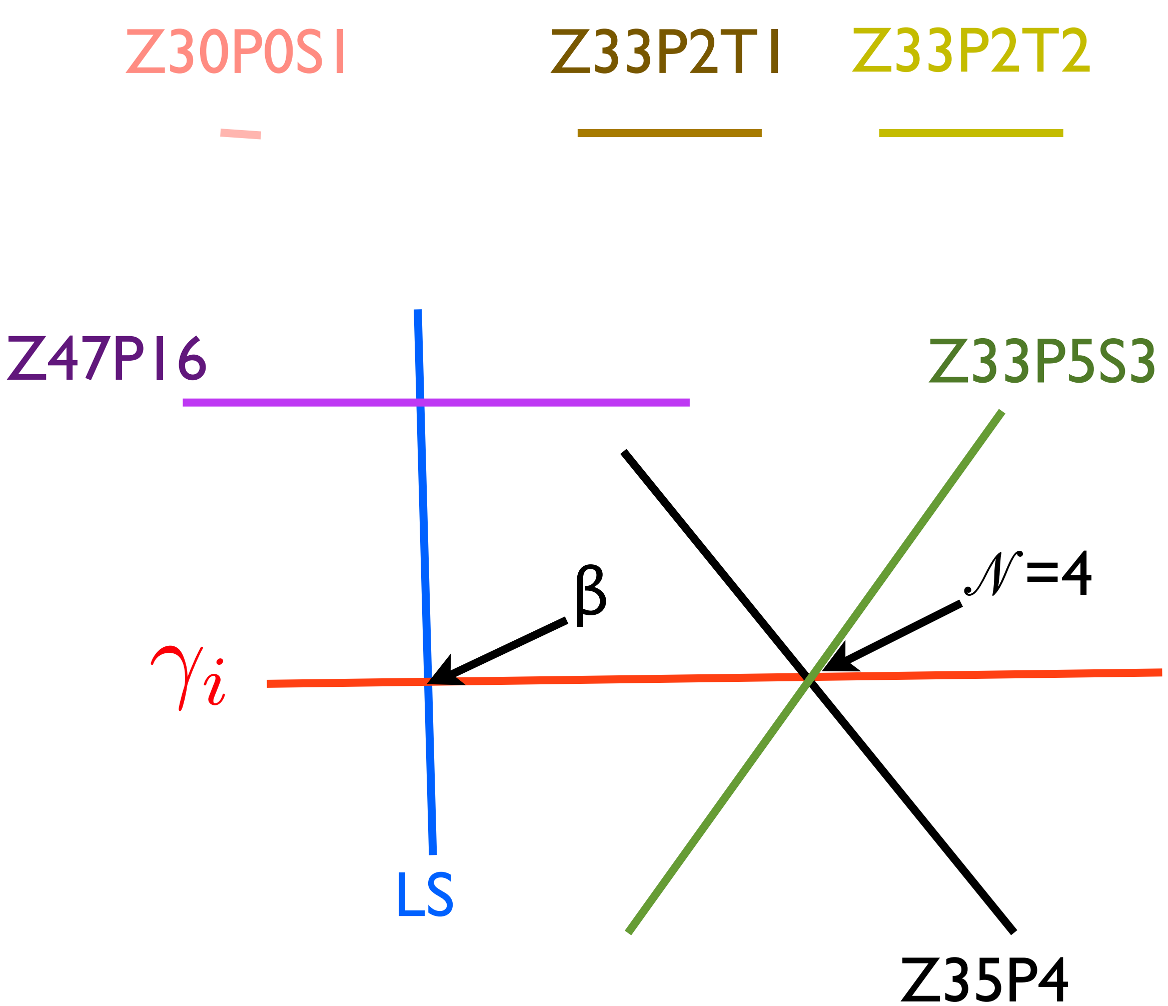
$$F_A^B = I_{IJ} = T_{ABCD} = 0$$

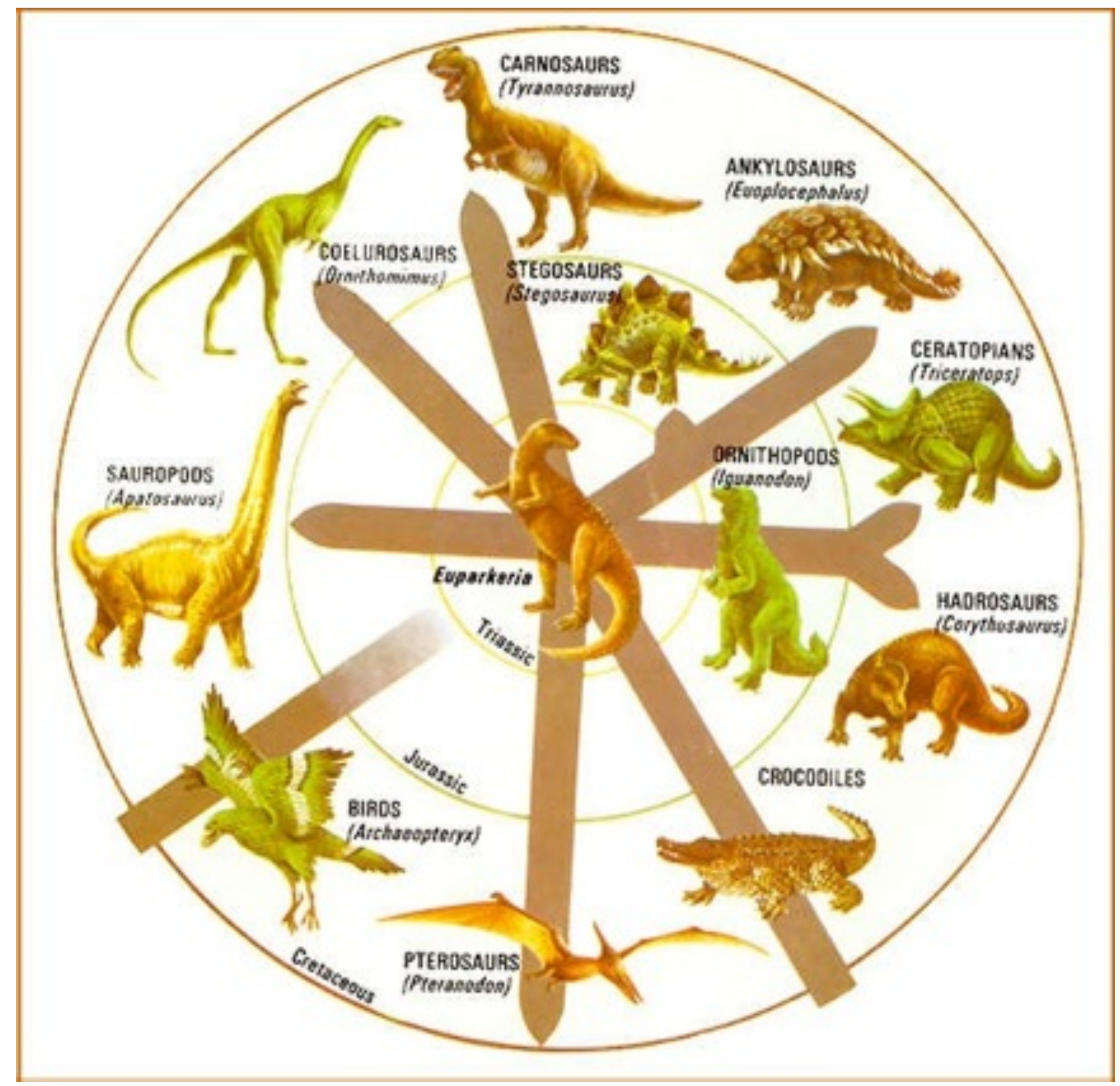
- As energy decreases, Yukawa couplings always flow to a fixed point!
- Start with random values for couplings, a fixed point is found at the end of Flow.

New fixed points vs new dinosaurs

- Find a numerical solution(dinosaur fossil).
- Find characteristics of the solution(describe the fossil).
- Compare with known theories(species).
- Derive the analytical Lagrangian(Name and study a new species).

The Jurassic Park of CFT?





Anomalous Dimension Matrix

- Anomalous dimension matrix is defined by:

$$\Delta_{ij} = \frac{\partial \beta_i}{\partial g_j} = N \frac{\partial^2 \rho}{\partial g_i \partial g_j}$$

- Expand couplings around a minima of ***ρ***:

$$g_i = g_i^0 + \epsilon \kappa_i$$

$$\rho = \rho_0 + \epsilon \frac{\partial \rho(g^0)}{\partial g_i} \kappa_i + \frac{\epsilon^2}{2} \frac{\partial^2 \rho(g^0)}{\partial g_i \partial g_j} \kappa_i \kappa_j + O(\epsilon^3)$$

- The matrix is Positive-semidefinite. (NOT true the other way around.)

Number of parameters

- Each zero eigenvalue of the anomalous dimension matrix corresponds to a ‘protected operator’.

$$L_\kappa = L + \kappa \mathcal{O}, \ \frac{d\kappa}{d\ln\mu} = \mathcal{O}(\kappa^2)$$

$$L_{cft} = L_{cft}(g_a, A_\mu, U^A_B(\omega_i)\psi^A, O^{IJ}(\omega_i)\phi^J)$$

$$\mathcal{O}_a = \frac{\partial L_{cft}}{\partial g_a}, \ \mathcal{O}_i = \frac{\partial L_{cft}}{\partial \omega_i}$$

Number of parameters

- Number of $\mathcal{O}_a$: number of parameters N_p .
- Number of $\mathcal{O}_i$: $31 - Dim(G_{sym})$
- 'Accidentally' protected operators.

$$N_p = Dim(Ker(\Delta_{ij})) + Dim(G_{sym}) - N_{acci} - 31$$

- Leigh Strassler: $N_p = 4$, γ_i deformed, $N_p = 6$

Leigh-Strassler Theory

$$L_Y = \kappa \epsilon^+_{ijk} \phi^i \left(q\psi^j \psi^k - \frac{1}{q} \psi^k \psi^j \right) + h \sum_i \phi^i \psi^i \psi^i + g\phi_i [\psi^i, \psi^4]$$

$$\epsilon^+_{ijk} = \frac{1}{2}(\epsilon_{ijk} + |\epsilon_{ijk}|)$$

- 4 parameters, a U(1) symmetry, 12 accidentally protected operators.
- #zeros of anomalous dimension matrix: 31-1+4+12=46

Renormalization potential of the scalar couplings

$$V_S = \frac{65}{2} g^6 - \frac{3}{8} g^4 a_{IIJJ} - \frac{3}{4} g^2 a_{IJKL} a_{LKJI} + \frac{1}{4} d_{MN} a_{MIJK} a_{KJIN},$$
$$+ B_{IJKL} a_{LKJI} - \frac{1}{6} a_{IJKL} a_{LKMN} a_{NMJI},$$

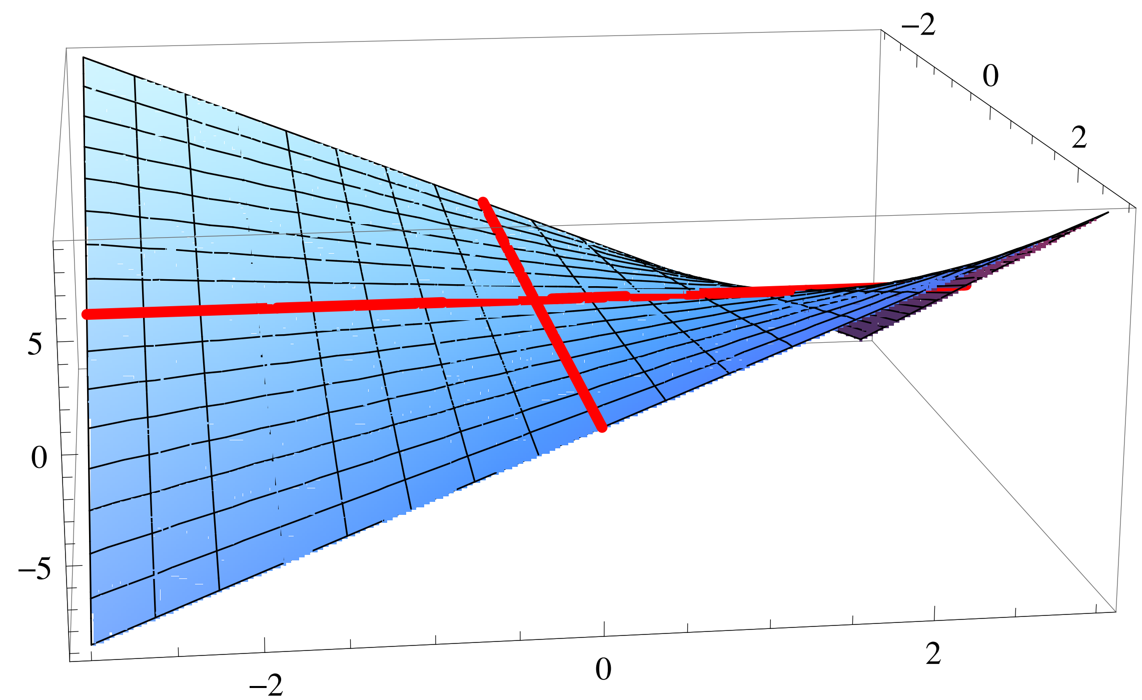
- The potential is not bounded below.
- anomalous dimension matrix: positive semi-definite.
- 6 protected operators: (anti-)holomorphic.

Leigh-Strassler theory is a saddle point

$$L = L_{LS} + z_i \mathcal{O}_i + \bar{z}_i \bar{\mathcal{O}}_i.$$

$$V_S(z_i, \bar{z}_i) = V_S^{LS} - \frac{1}{6} \delta a_{IJKL} \delta a_{LKMN} \delta a_{NMJI}$$

$$V_S(z_i, \bar{z}_i) = V_S^{LS}, \ \beta_i \neq 0$$



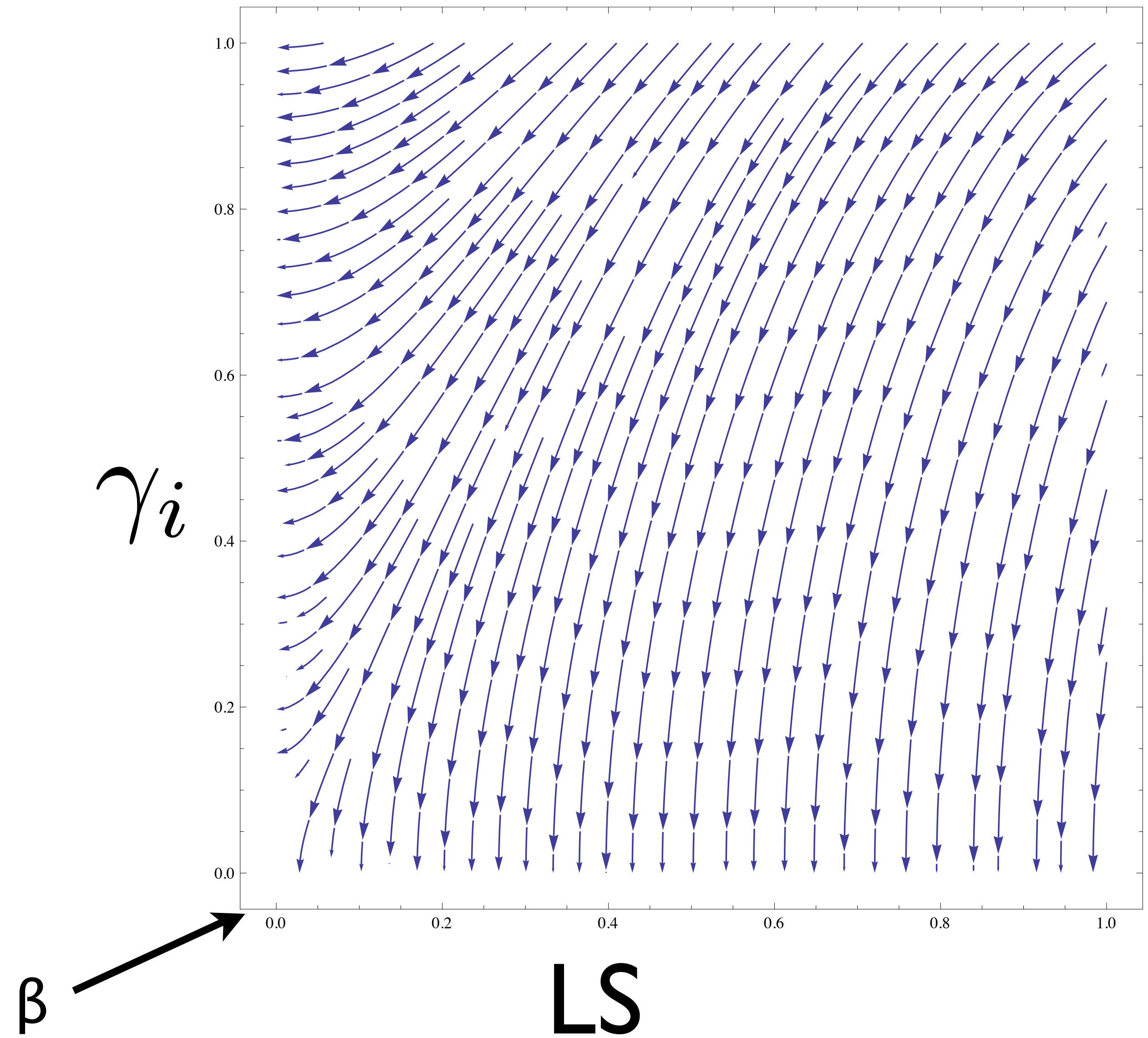
A subsystem in which LS is stable:

$$L_\phi = a_{ij\bar{k}\bar{l}} \phi^i \phi^j \bar{\phi}_k \bar{\phi}_l + \frac{1}{2} a_{i\bar{k}j\bar{l}} \phi^i \bar{\phi}_k \phi^j \bar{\phi}_l$$

Emergence of other symmetries

LS: U(1)×SUSY

$\gamma_i : U(1)^3$



Discussion

- Classification of the fixed points.
- The gravity dual of the fixed points. Strong evidence for a conformal field theory since it has both weak and strong description.
- Behavior of accidentally protected operators should be sensitive to higher loop corrections.