Evolution equation for quasi PDF

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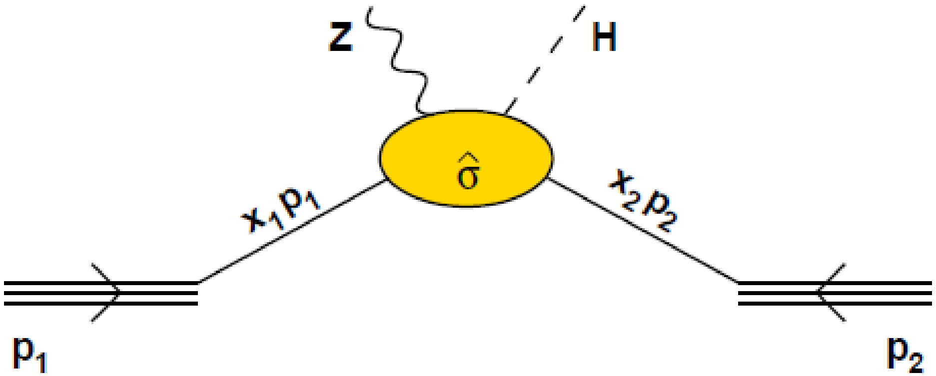
1. Motivation

Collinear factorization

Cross section for some hard process in hadron-hadron collisions



$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)\,, \quad \hat{s} = x_1 x_2 s$$

- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).

1. Motivation

❖ Parton distribution function (PDF)

- **Defined on light cone, i.e. in the infinite momentum frame**

$$q(x) \sim \lim_{P_z \to \infty} \int \mathrm{d}^2 k_\perp \mathrm{d}k^z n(\vec{k}, P^z) \delta(x - k^z / P^z) \ .$$

- **Fundamental field operators description**

$$q(x,\mu^2) \;=\; \int \frac{d\xi^-}{4\pi} e^{-i\xi^- x P^+} \langle P | \overline{\psi}(\xi^-) \gamma^+ \mathcal{P} \exp\left(-ig \int_0^{\xi^-} d\eta A^+(\eta) \right) \psi(0) | P \rangle \ ,$$

$$x g(x,\mu^2) \;=\; \int \frac{d\xi^-}{2\pi P^z} e^{-i\xi^- x P^+} \langle P | G^+_{\ \mu}(\xi^-) \mathcal{P} \exp\left(-ig \int_0^{\xi^-} d\eta A^+(\eta) \right) G^{\mu +}(0) | P \rangle \ .$$

Non local, time dependent , on light cone

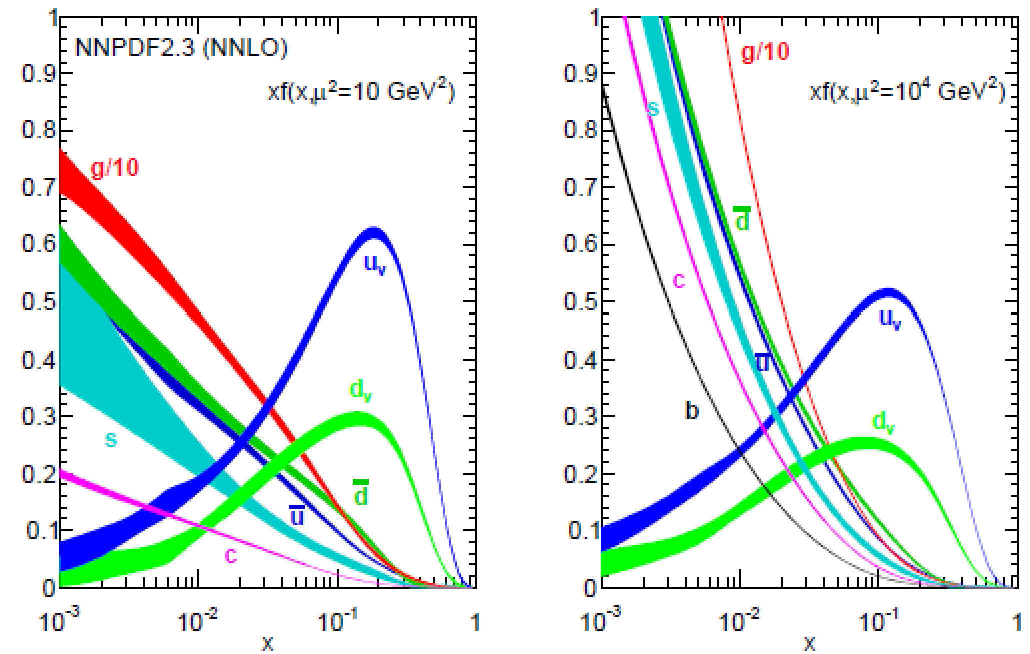
1. Motivation

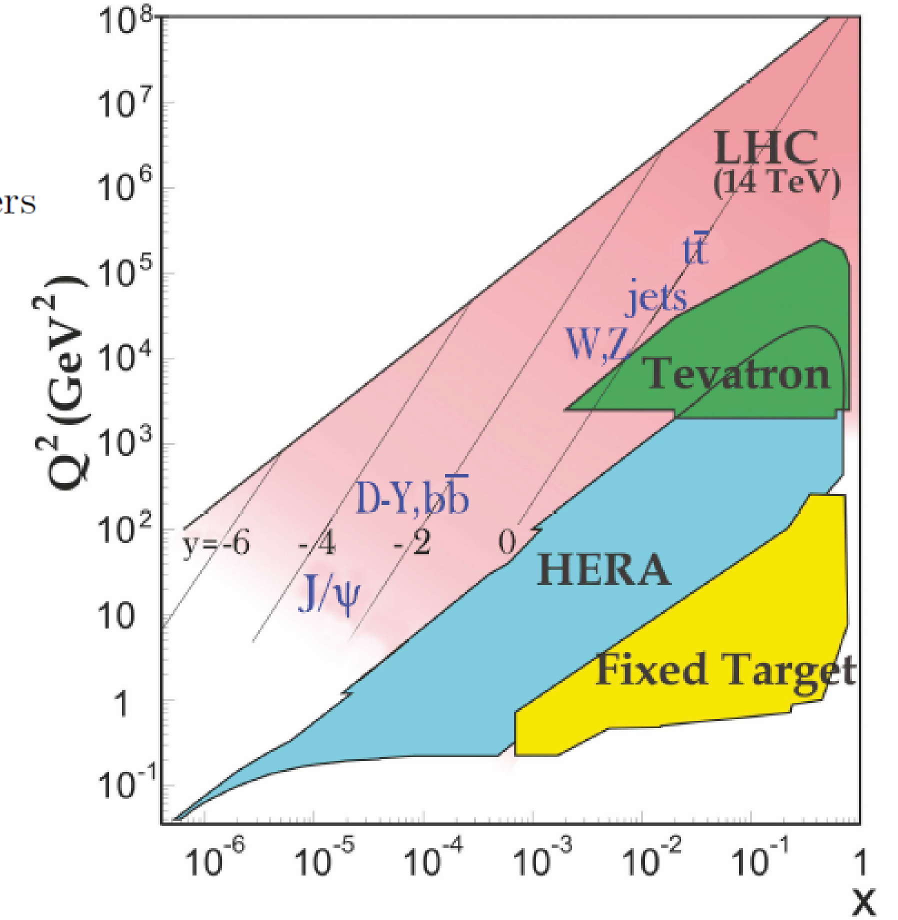
❖ Methods to extract PDF

- **Fitting data**

  Groups: CTEQ, MSTW, NNPDF HERAPDF, AB(K)M, GJR

the form $xf = x^a(\ldots)(1-x)^b$ with 10-25 free parameters





PDG2014

1. Motivation

❖ Other Methods to extract PDF

- **Nucleon's wave function (wave function is model-dependent)**

$$|P\rangle = \sum_{n\alpha,\lambda_i} \int \Pi_i \frac{\mathrm{d}x_i \mathrm{d}^2 k_{\perp i}}{\sqrt{2x_i}(2\pi)^3} \psi_{n\alpha}(x_i, k_{\perp i}, \lambda_i) \left| n\alpha : x_i P^+, k_{\perp,i}, \lambda_i \right\rangle$$

- **Operator product expansion (only limited to the first few moments)**

- **Lattice calculation (can not be performed on light cone)**

- **Large momentum effective theory (LaMET)**

quasi PDFs related to light cone PDF s can be measured on lattice

X.D Ji,PRL110,262002(2013);

1. Motivation

- **Definition (nonlocal, time independent, related to Lattice measurements)**

$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izxP^z} \langle P|\overline{\psi}(z)\gamma^z \mathcal{P}\exp\left(-ig\int_0^z dz'_z A^z(z')\right)\psi(0)|P\rangle \ ,$$

$$x\tilde{g}(x,\mu^2,P^z) = \int \frac{dz}{2\pi} e^{izxP^z} \langle P|G^z{}_\mu(z)\mathcal{P}\exp\left(-ig\int_0^z dz'_z A^z(z')\right)G^{\mu z}(0)|P\rangle \ ,$$

X.D Ji,PRL110,262002(2013);Xiong-Ji-Zhang-Zhao,PRD90,014051(2014)

- **Finite momentum**
- **Large scale : Pz,** Λ
- **Matching condition exists between quasi PDF and light cone PDF.**

1. Motivation

Current progresses in Quasi PDF

- **Matching between quasi PDF and light cone PDF (Non-singlet)**

$$\tilde{q}(x,\Lambda,P^z)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right)q(y,\mu)$$

X.N.Xiong, X.D.Ji, J.H.Zhang, Y.Zhao,PRD90,014051(2014)

- **Quasi distribution factorization, Y.Q. Ma, J.W. Qiu, arXiv:1404.6860**
- **General quasi PDF: Ji –Schafer-Xiong –Zhang,PRD92,014039(2015)**
- **Renormalization of quasi PDF: Ji-Zhang, PRD92,034006(2015)**
- **Quasi TMD PDF:Ji-Sun-Xiong-Yuan,PRD91,074009(2015)**
- **Quasi DA of Heavy Quarkonia, Y. Jia, X.N. Xiong,arXiv:1511.04430**
- **Non-dipolar Wilson links for Quasi PDF: H.N. Li, arXiv:1602.075075**
- **Lattice QCD: Chen-Cohen-Ji-Lin-Zhang, arXiv:1603.06664; PRD92,014502(2015);K.F. Liu, arXiv:1603.07352**
- **And so on: ……**
- **Many challenging problems: renormalization, light-cone divergences (see talks of Prof. X.N. Li and Dr. J.H. Zhang)**

2. Evolution equation for Quasi PDFs

❖ Quasi PDF to light cone PDF

- **quark quasi distribution matching equation (Xiong-Ji-Zhang-Zhao2015)**

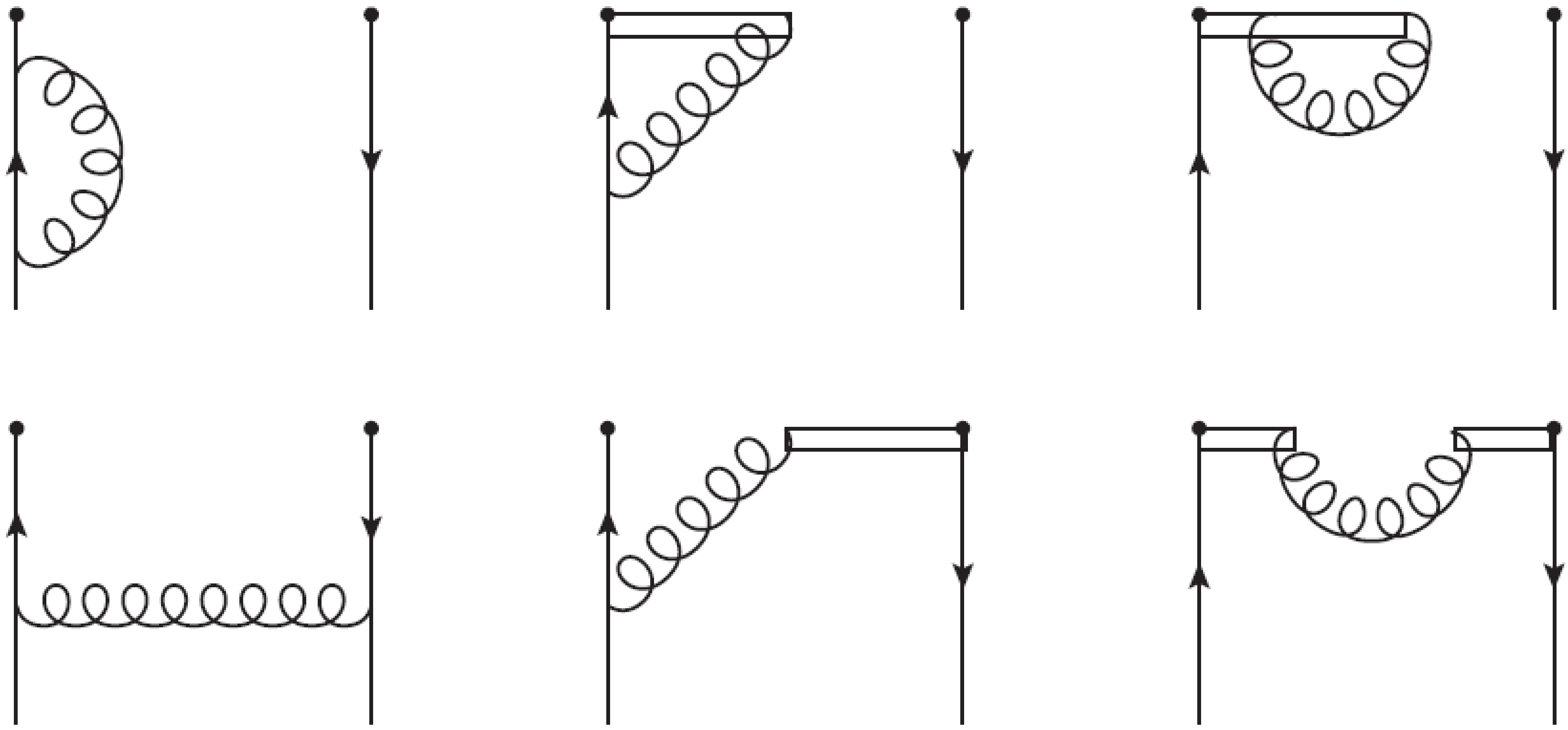
$$\tilde{q}(x, \mu^2, P^z) = \int_0^1 \frac{dy}{|y|} Z_{QQ}\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2)$$

- **gluon quasi distribution matching equation**

$$\tilde{g}(x, \mu^2, P^z) = \int_0^1 \frac{dy}{|y|} Z_{GG}\left(\frac{x}{y}, \frac{\mu}{P^z}\right) g(x, \mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) ,$$

2. Evolution equation for Quasi PDFs

- Quark to quark (Xiong-Ji-Zhang-Zhao2015)



2. Evolution equation for Quasi PDFs

- Quark to quark (Xiong-Ji-Zhang-Zhao2015)

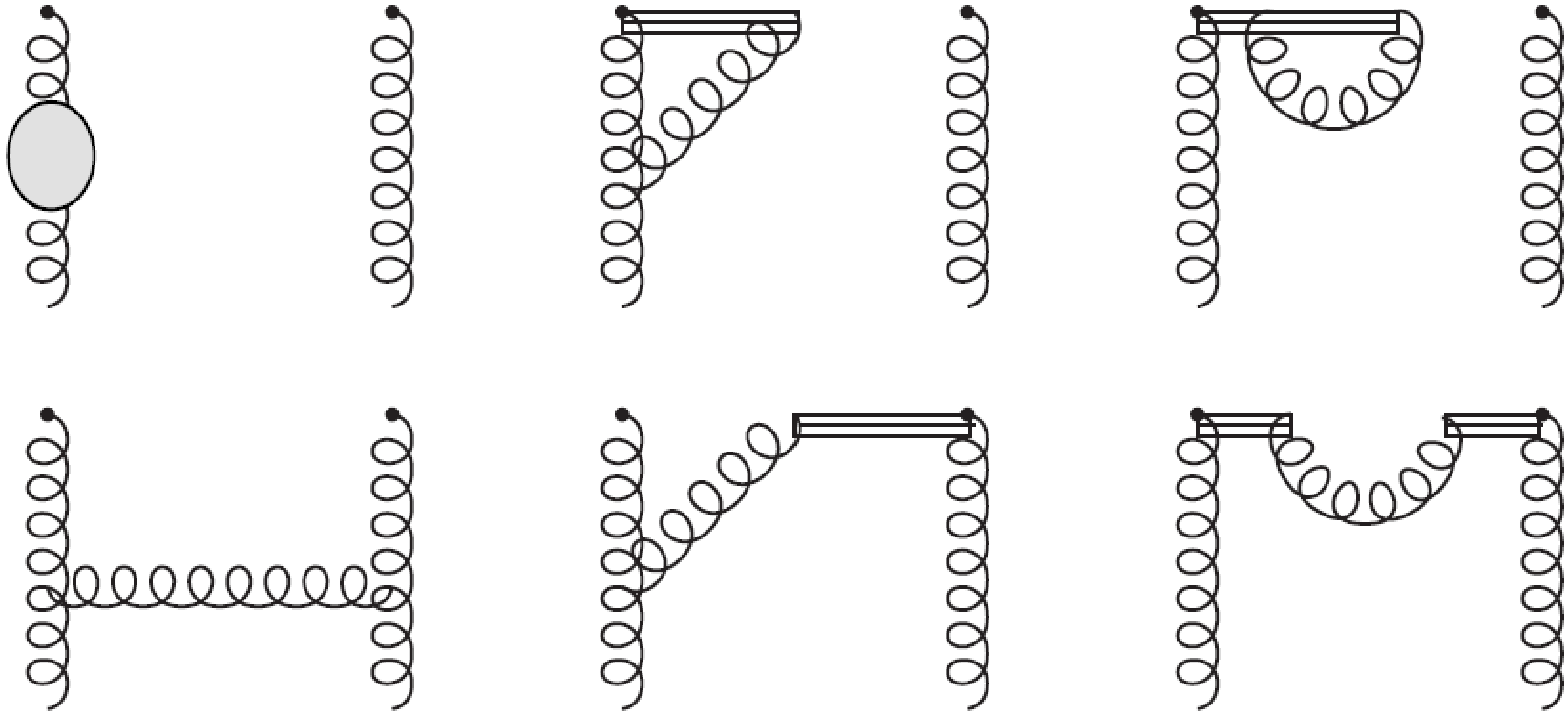
$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \qquad \text{For } \xi > 1,$$

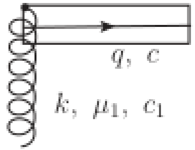
$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln[4\xi(1-\xi)] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2 P^z}, \qquad 0 < \xi < 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1 + \frac{\Lambda}{(1-\xi)^2 P^z}. \qquad \text{for } \xi < 0$$

2. Evolution equation for Quasi PDFs

❖ Gluon to Gluon





including both gauge links and non-abelian contributions

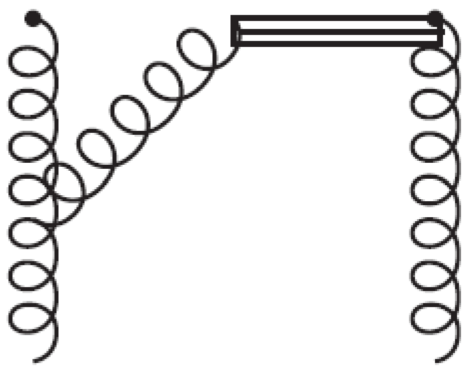
$$G_{\mu\nu} = T^a G^a_{\mu\nu} = -\frac{i}{g}[D_\mu, D_\nu] = T^a(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu).$$

2.Evolution equation for Quasi PDFs

❖ Quark to Gluon, Gluon to Quark

2. Evolution equation for Quasi PDFs

❖ Take an example



$$g_2 = \frac{\alpha_S C_F}{2x\pi} \begin{cases} 0\ , & x > 1 \text{ or } x < 0\ , \\ \frac{x^2(x+1)}{x-1} \log\left(\frac{m^2(x^2-x+1)}{\Lambda^2}\right)\ , & 0 < x < 1\ , \end{cases}$$

$$\tilde{g}_2 = \frac{\alpha_S C_F}{2x\pi} \begin{cases} \frac{x((x+1)\log(\frac{x-1}{x})x+2x-1)}{x-1} - \frac{x\Lambda}{(x-1)p^z}\ , & x > 1\ , \\ \frac{x^2(x+1)}{x-1}\log(\frac{m^2(x^2+1-x)}{4x(1-x)(p^z)^2}) + \frac{x(2x^2-2x+1)}{x-1} - \frac{x\Lambda}{(x-1)p^z}\ , & 0 < x < 1\ , \\ -\frac{x(x(x+1)\log(\frac{x-1}{x})+2x-1)}{x-1} - \frac{x\Lambda}{(x-1)p^z}\ , & x < 0\ , \end{cases}$$

2. Evolution equation for Quasi PDFs

Quasi PDF evolution equation

$$\frac{d}{d\log p^z}\tilde{q}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left[\tilde{P}_{q\leftarrow q}(y)\tilde{q}\left(\frac{x}{y},\frac{\mu}{P^z}\right)+\tilde{P}_{q\leftarrow g}(y)\tilde{g}\left(\frac{x}{y},\frac{\mu}{P^z}\right)\right],$$

$$\frac{d}{d\log p^z}\tilde{\bar{q}}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left[\tilde{P}_{q\leftarrow q}(y)\tilde{\bar{q}}\left(\frac{x}{y},\frac{\mu}{P^z}\right)+\tilde{P}_{q\leftarrow g}(y)\tilde{g}\left(\frac{x}{y},\frac{\mu}{P^z}\right)\right],$$

$$\frac{d}{d\log p^z}\tilde{g}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left[\tilde{P}_{g\leftarrow q}(y)\sum_f\left(\tilde{q}_f\left(\frac{x}{y},\frac{\mu}{P^z}\right)+\tilde{\bar{q}}_f\left(\frac{x}{y},\frac{\mu}{P^z}\right)\right)+\tilde{P}_{g\leftarrow g}(y)\tilde{g}\left(\frac{x}{y},\frac{\mu}{P^z}\right)\right.$$

We found that:
Pz evolution kernels
are identical to
Splitting func. of PDF

$$\tilde{P}_{q\leftarrow q}(y) = C_F\begin{cases}0\,, & y>1 \text{ or } y<0\,,\\ \frac{1+y^2}{(1-y)^+}+\frac{3}{2}\delta(1-y)\,, & 0<y\leq 1\,.\end{cases}$$

$$\tilde{P}_{q\leftarrow g}(y) = T_F\begin{cases}0\,, & y>1 \text{ or } y<0\,,\\ y^2+(1-y)^2\,, & 0<y\leq 1\,.\end{cases}$$

$$\tilde{P}_{g\leftarrow q}(y) = C_F\begin{cases}0\,, & y>1 \text{ or } y<0\,,\\ \frac{1+(1-y)^2}{y}\,, & 0<y\leq 1\,.\end{cases}$$

One loop results

$$\tilde{P}_{g\leftarrow g}(y) = \begin{cases}0\,, & y>1 \text{ or } y<0\,,\\ \frac{2C_A(1-y+y^2)^2}{y(1-y)^+}+\frac{\beta_0}{2}\delta(1-y)\,, & 0<y\leq 1\,.\end{cases}$$

Summary

- Quasi PDFs link light cone PDFs and Lattice measurements

- Many progresses are made in the last three years

- The Pz evolution equation of Quasi PDFs is obtained.

Thank you for your attention!