Charm and strange quark masses and f_{D_s} from LQCD

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χQCD Collaboration:
Y.-B. Yang et al., PRD92, 2015
ZL et al., PRD90, 2014

Quantum ChromoDynamics (QCD)

- QCD is an important part of the Standard Model.
- At low energies, QCD can not be solved by perturbation theory.
- Nonperturbative treatment of QCD is needed to connect hadron properties to those of quarks.
- Lattice QCD are calculating some properties of hadrons precisely and can test SM to a precision of ~1%.

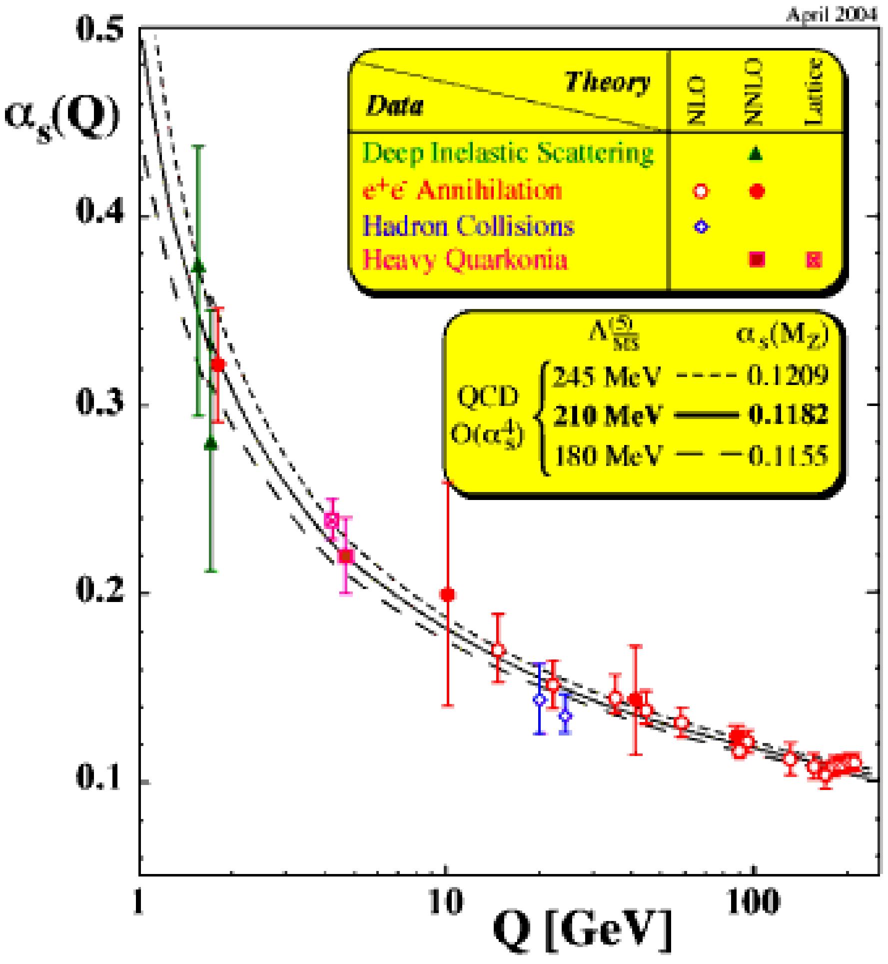


FIG. 1: QCD running coupling constant

Weak decays and LQCD

- **LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons.**
- **Combined with experiments, they can give us CKM matrix elements.**
- **Test the SM (is CKM unitary?).**
- **Or using V_{ab} to test QCD (compare LQCD results with experiments).**

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cd} & V_{cd} \\ V_{td} & V_{td} & V_{td} \end{pmatrix}$$

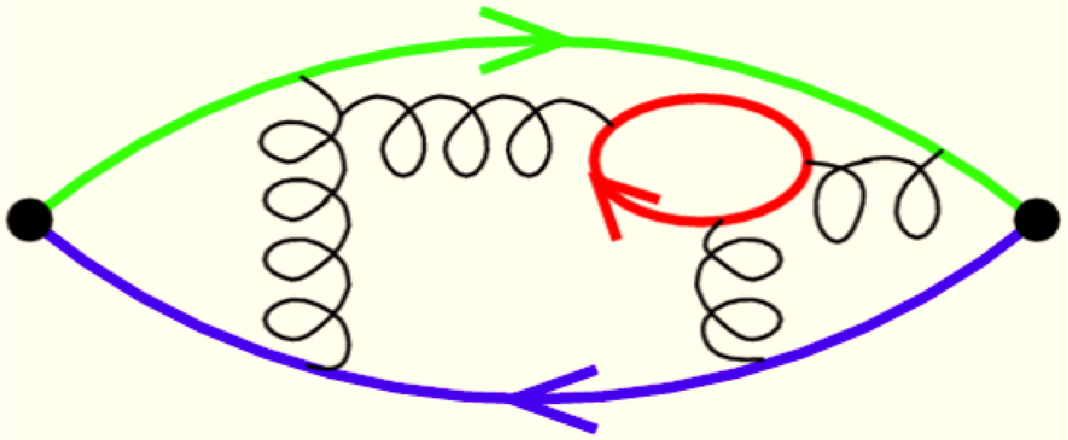
Can be calculated by LQCD

$$\Gamma(P \to \ell\nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

Pseudoscalar decay constants

- $\langle 0|\bar{q}\gamma_\mu\gamma_5 c|P\rangle = f_P p_\mu$
- **Let $O = \bar{q}\gamma_\mu\gamma_5 c$, the above matrix element can be obtained from**

$$C(t) = \langle 0\big|O(t)O^\dagger(0)\big|0\rangle \xrightarrow{t\to\infty} |\langle 0|O|P\rangle|^2 e^{-m_P t}$$



Heavier hadrons with quantum numbers of O are suppressed.

LQCD

- LQCD is based on the path integral formalism in Euclidean space.
- $\langle O \rangle = \dfrac{\int DA_\mu D\bar{\psi} D\psi O[A,\bar{\psi},\psi] e^{-\int \mathcal{L}_{QCD}\, d^4x}}{\int DA_\mu D\bar{\psi} D\psi e^{-\int \mathcal{L}_{QCD}\, d^4x}}$,

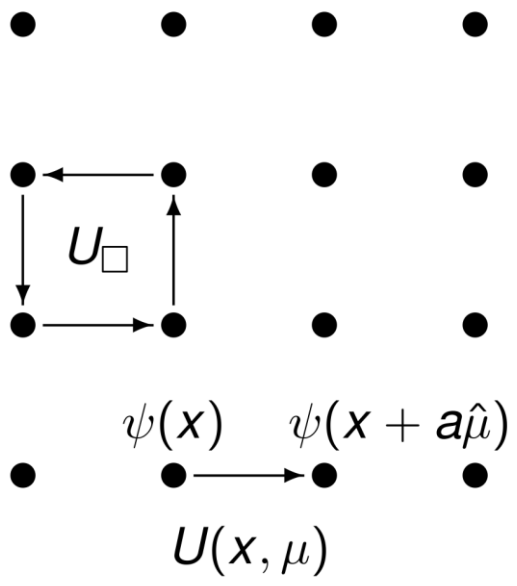
where $\mathcal{L}_{QCD} = \bar{\psi} M[A] \psi + \mathcal{L}_G$, $\quad M = \gamma \cdot D + m_q$

- On the 4D lattice, the A fields are put on the links

$$U(x,\mu) = e^{igaA_\mu}.$$

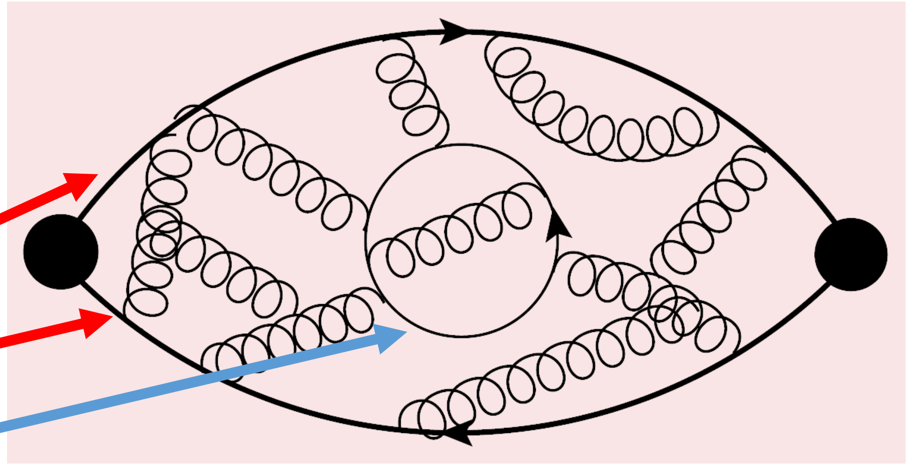
- Integrating over quark fields:

$$\langle O \rangle = \frac{\int DU_\mu O[U, M^{-1}[U]] Det[M[U]] e^{-S_G}}{\int DU_\mu Det[M[U]] e^{-S_G}}$$



LQCD

- $\langle O \rangle = \frac{\int DU_\mu O[U, M^{-1}[U]] Det[M[U]] e^{-S_G}}{\int DU_\mu Det[M[U]] e^{-S_G}}$
- Valence quark propagators $M^{-1}[U]$
- Sea quark effects $\mathrm{Det}[M[U]]$
- Generate gluon fields U (numerically very challenging.)
- Calculate valence quark propagators and combine them to get hadron correlators (numerically costly, data intensive).
- Fit for masses and matrix elements.
- Determine a and fix m_q to get physical results.



Flavor physics and LQCD

- Other examples: $D \to \pi l \nu$, $D \to K l \nu$ can be used to determine $|V_{cd}|$ 和$|V_{cs}|$.

$$\frac{d\Gamma(D \to K\ell\nu)}{dq^2} = (\text{known})\,|\mathbf{p}_K|^3\,|V_{cs}|^2\,|f_+^{D\to K}(q^2)|^2$$

- The form factor $|f_+|^2$ parametrizes the matrix element.

$$\langle K|V^\mu|D\rangle = f_+(q^2)\left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_D^2 - m_K^2}{q^2}q^\mu$$

- There are varies lattice calculations on meson decay constants and form factors. (f_π, f_K, f_D, …, $f_+^{D\to K}$, …)
- The Flavor Lattice Averaging Group (FLAG) was set up to review the current status of lattice results related to flavor physics.

FLAG

- First set up in 2007. http://itpwiki.unibe.ch/flag
- FLAG-2, Eur. J. Phys. C74 (2014) 2890, arXiv:1310.8555
- Continuum extrapolation(# a's, <0.1 fm?)
- Chiral extrapolation of light quark masses ($m_{\pi,min}$<200 MeV, 400 MeV?)
- Finite volume($m_{\pi,\mathrm{min}}L > 4$, or 3?)
- Renormalization(non-perturbative? 1-loop?)
- Heavy quark treatment (O(a) improved?)
- Publication status (published, preprint, conference)

Color-coding of systematic errors:

★ has been estimated in a satisfactory manner.

○ reasonable, could be improved.

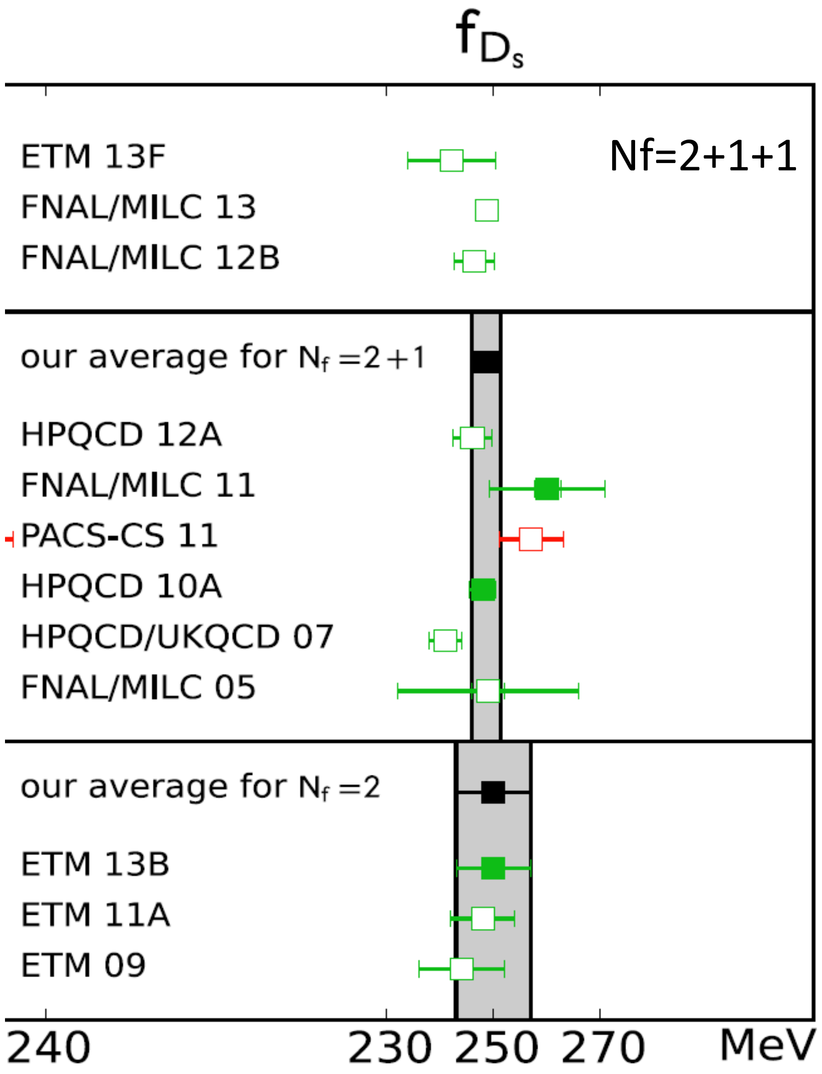
■ no estimation, or unsatisfactory.

■ Included in the average.

□ Not included, but pass quality criteria.

■ Other results.

FLAG2013 for f_{D_s}



Eur. J. Phys. C74 (2014) 2890

- Different LQCD collaborations use different lattice actions.
- Various lattice spacings and volumes.
- Different range of light quark masses.
- Thus different systematic errors.
- The consistency of values from different collaborations is important.

m_c, m_s and f_{D_s} from overlap fermions

- **2+1-flavor domain wall fermion configurations from the RBC-UKQCD Collaborations. Two *a*'s: ~0.08 fm, ~0.11 fm. *O*(100) configurations.**

β	$L^3 \times T$	$m_s^{(s)}a$	$m_l^{(s)}a$		
2.13	$24^3 \times 64$	0.04	0.005	0.01	0.02
2.25	$32^3 \times 64$	0.03	0.004	0.006	0.008

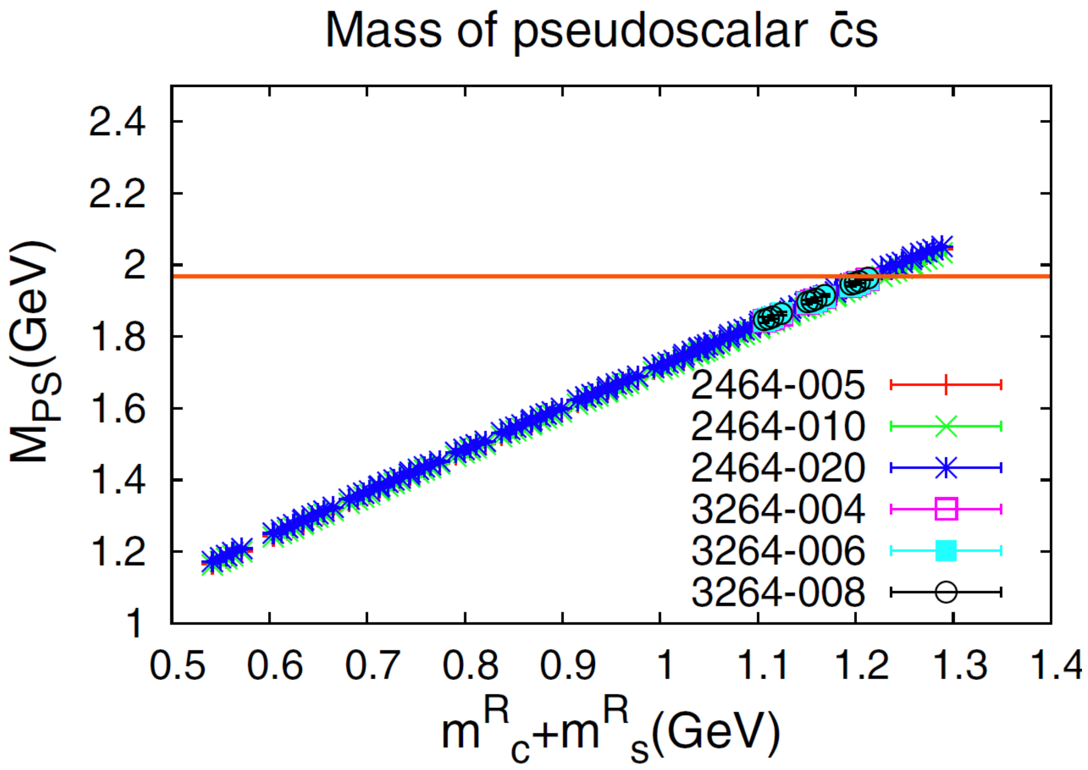
- **Multi-mass algorithm is used to compute valence overlap quark propagators.**

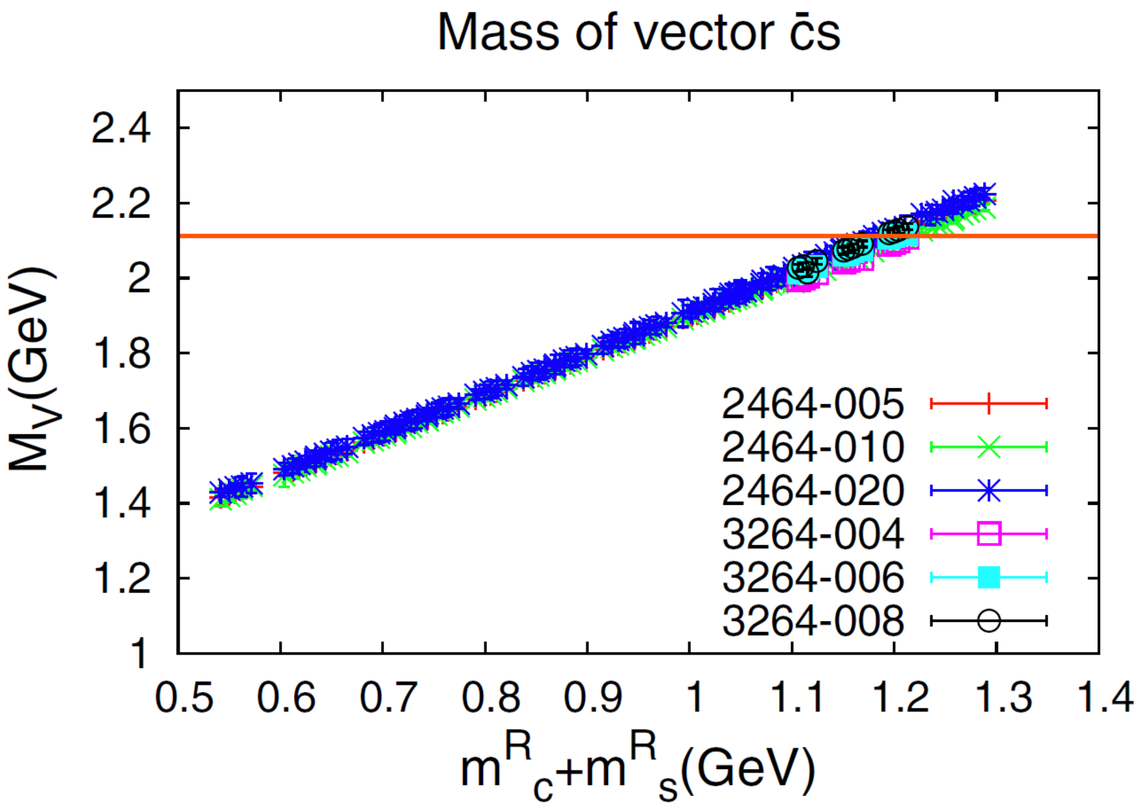
$\beta = 2.13$	$m_s a$	0.0576	0.063	0.067	0.071	0.077		
	$m_c a$	0.29	0.33	0.35	0.38	0.40	0.42	0.45
		0.48	0.50	0.53	0.55	0.58	0.60	0.61
		0.63	0.65	0.67	0.68	0.70	0.73	0.75
$\beta = 2.25$	$m_s a$	0.039	0.041	0.043	0.047			
	$m_c a$	0.38	0.46	0.48	0.50	0.57		

Y.-B. Yang et al. (χQCD Collaboration), PRD92, 2015

Meson masses

- **Masses of pseudo-scalar and vector mesons of $c\bar{s}$ and $c\bar{c}$ are calculated from 2-point functions.**





- **Assume:**

$$M^{(0)}(m_c, m_s, m_l) = A_0 + A_1 m_c + A_2 m_s + A_3 m_l + \cdots$$

Meson masses

- Taking into account discretization effects:

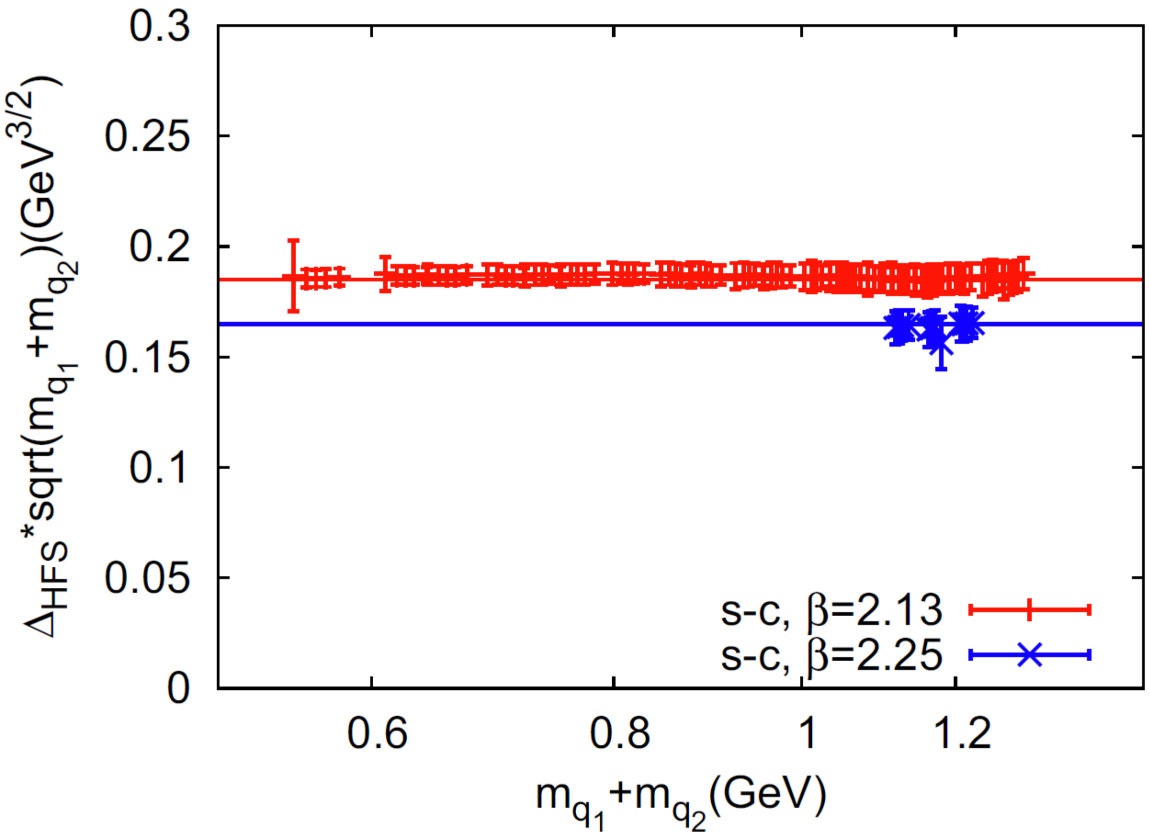
$$\begin{aligned} M(m_c, m_s, m_l, a) \\ &= M(m_c, m_s, m_l) \times (1 + B_1(m_c a)^2 + B_2(m_c a)^4 \\ &\quad + O((m_c a)^6)) + C_1 a^2 + O(a^4). \end{aligned}$$

- **Applying the above to a correlated fit of M_{D_s} gives χ^2/d.o.f = 4.6**
- The above linear behavior expects a linear quark mass dependence in the mass difference of the vector and pseudoscalar mesons.
- **Experiments tell a different story:**

$M_\rho - M_\pi \sim 630$ MeV, $M_{K^*} - M_K \sim 400$ MeV, $M_{D^*} - M_D \sim 140$ MeV, $M_{D_s^*} - M_{D_s} \sim 140$ MeV, $M_{J/\psi} - M_{\eta_c} \sim$ 117 MeV

Hyperfine splitting

We observe the combined quantity $\Delta_{\mathrm{HFS}}\sqrt{m_{q_1}^R + m_{q_2}^R}$ is consistent with a constant for the charm-strange system:



This suggests:

Light sea quark mass dependence

$$\Delta_{\mathrm{HFS}} = \frac{A_4 + A_5 m_l^R}{\sqrt{m_{q_1}^R + m_{q_2}^R + \delta m}}(1 + B_0 a^2)$$

Avoid divergence in the chiral limit

Finite *a*

Correlated Global fit

- **The global fit function for the meson masses is (the quark masses are renormalized):**

$$M(m_c^R, m_s^R, m_l^R, a) = \left[A_0 + A_1 m_c^R + A_2 m_s^R + A_3 m_l^R + (A_4 + A_5 m_l^R)\frac{1}{\sqrt{m_c^R + m_s^R + \delta m}}\right] \times (1 + B_0 a^2 + B_1 (m_c^R a)^2 + B_2 (m_c^R a)^4) + C_1 a^2$$

~450 data points

~20 parameters

χ^2 **/d.o.f = 1.05**

Inputs: $M_{D_s} = 1.9685$ GeV

$M_{D_s^*} - M_{D_s} = 0.1438$ GeV

$M_{J/\psi} = 3.0969$ GeV

Outputs: $r_0 = 0.465(4)(9)$ **fm**

$m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 0.101(3)(6)$ **GeV**

$m_c^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 1.118(6)(24)$ **GeV**

Systematic errors

- Statistical error in Z_m
- Systematic error in Z_m
- Chiral extrapolation function form
- Strange sea quark mass mistuned
- Cutoff for the small eigenvalues of the correlation matrix in the global fit
- Electromagnetic effects
- .....

	χ^2/d.o.f	r_0(fm)	m_c(GeV)	m_s(GeV)
PDG [28]		$\cdots$	1.09(3)	0.095(5)
This work	1.05	0.465	1.118	0.101
σ(stat)		0.004	0.006	0.003
$\sigma(r_0/a)$		0.002	0.001	0.000
$\sigma(\frac{\partial r_0}{\partial a^2})$		0.005	0.007	0.004
σ(MR/stat)		0.001	0.022	0.000
σ(MR/sys)		–	0.003	0.000
σ(SSQMD)		0.006	0.004	0.002
$\sigma(\delta m)$		0.001	0.001	0.000
σ(chiral)		0.004	0.006	0.003
$\sigma(u-d)$		0.001	0.001	0.000
$\sigma(a)$		0.002	0.002	0.001
σ(cut)		0.001	0.001	0.000
σ(EM)		0.002	0.002	0.001
σ(heavy)		0.005	0.007	0.001
σ(all sys)		0.009	0.024	0.006
σ(all)		0.010	0.025	0.007

f_{D_s}

- $\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle = f_{D_s} p_\mu$
- The local current $\bar{s}\gamma_\mu\gamma_5 c$ on the lattice needs renormalization Z_A
- Using the PCAC relation, one could also get f_{D_s} by

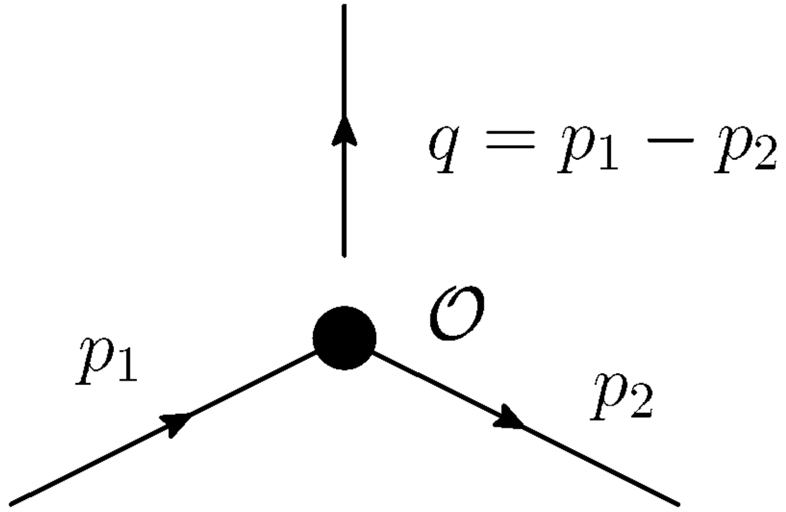
$$(m_s + m_c)\langle 0|\bar{s}\gamma_5 c|D_s\rangle = f_{D_s} M_{D_s}^2$$

No renormalization is needed due to $Z_P Z_m = 1$

- f_{D_s} are obtained from both matrix elements in our calculation.
- We find $f_{D_s} = 254(2)(4)$ MeV

Renormalization: $\boldsymbol{O(\mu) = Z(\mu, a)O(a)}$

- **Quark mass renormalization Z_m and Z_A for the axial vector current are needed.**
- **Lattice perturbation theory is much more complicated than in the continuum. Most calculations only up to one-loop.**
- **We use the so called RI/MOM scheme and then convert to the popular $\overline{\mathrm{MS}}$ scheme.**
- **Work with Green functions, which can be calculated to all loops in LQCD (nonperturbative).**



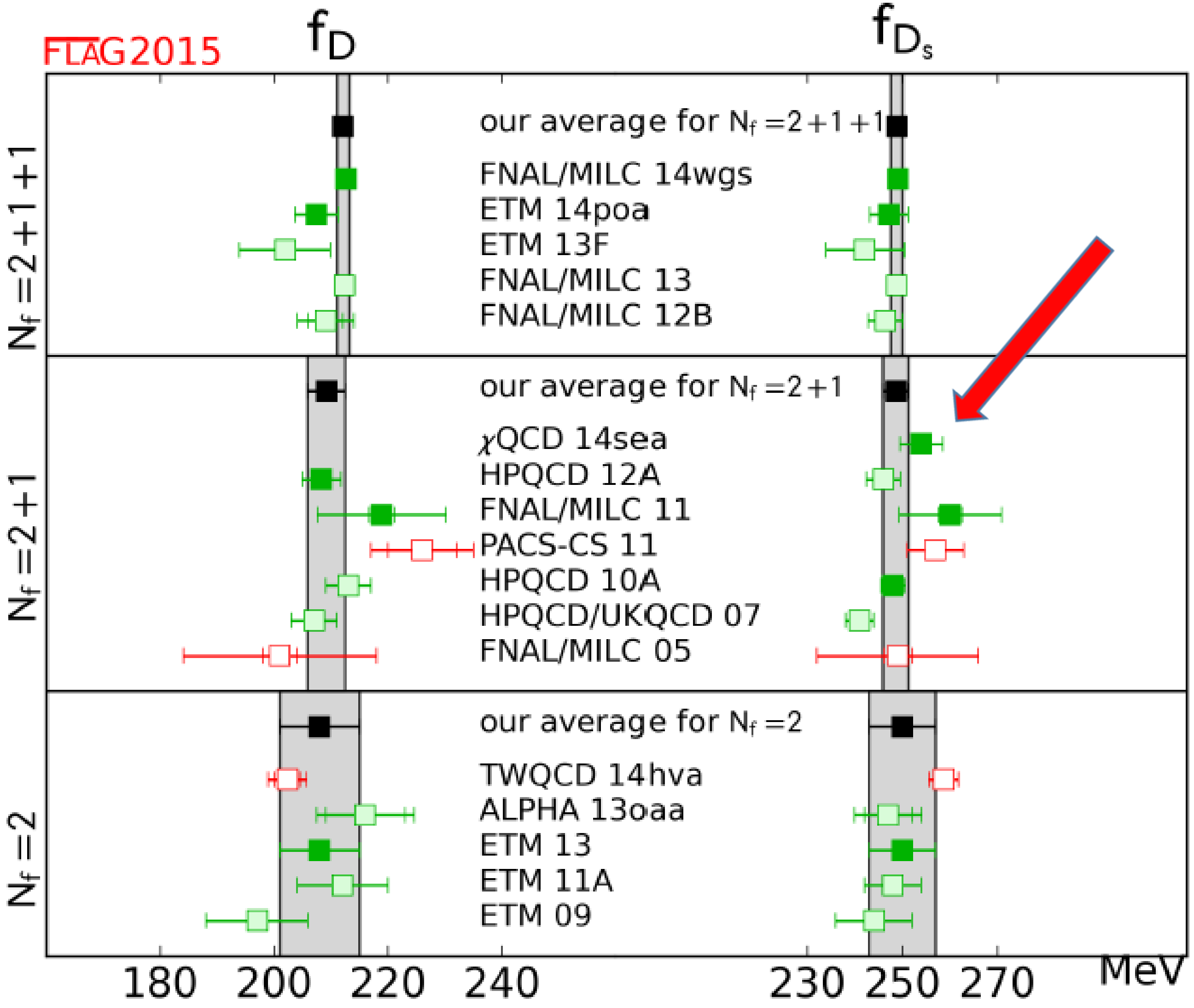
ZL et al., (χQCD Collaboration), PRD90, 2014

Our result of f_{D_s} will be included in the new average FLAG-3.

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_D	f_{D_s}	f_{D_s}/f_D
χQCD 14sea	[10]	2+1	A	○	○	○	★	✓		254(2)(4)	
HPQCD 12A	[11]	2+1	A	○	○	○	★	✓	208.3(1.0)(3.3)	246.0(0.7)(3.5)	1.187(4)(12)
FNAL/MILC 11	[12]	2+1	A	○	○	○	○	✓	218.9(11.3)	260.1(10.8)	1.188(25)
PACS-CS 11	[13]	2+1	A	■	★	■	○	✓	226(6)(1)(5)	257(2)(1)(5)	1.14(3)
HPQCD 10A	[14]	2+1	A	★	○	★	★	✓	213(4)*	248.0(2.5)	
HPQCD/UKQCD 07	[15]	2+1	A	★	○	○	★	✓	207(4)	241 (3)	1.164(11)
FNAL/MILC 05	[16]	2+1	A	○	○	■	○	✓	201(3)(17)	249(3)(16)	1.24(1)(7)

Private Communication

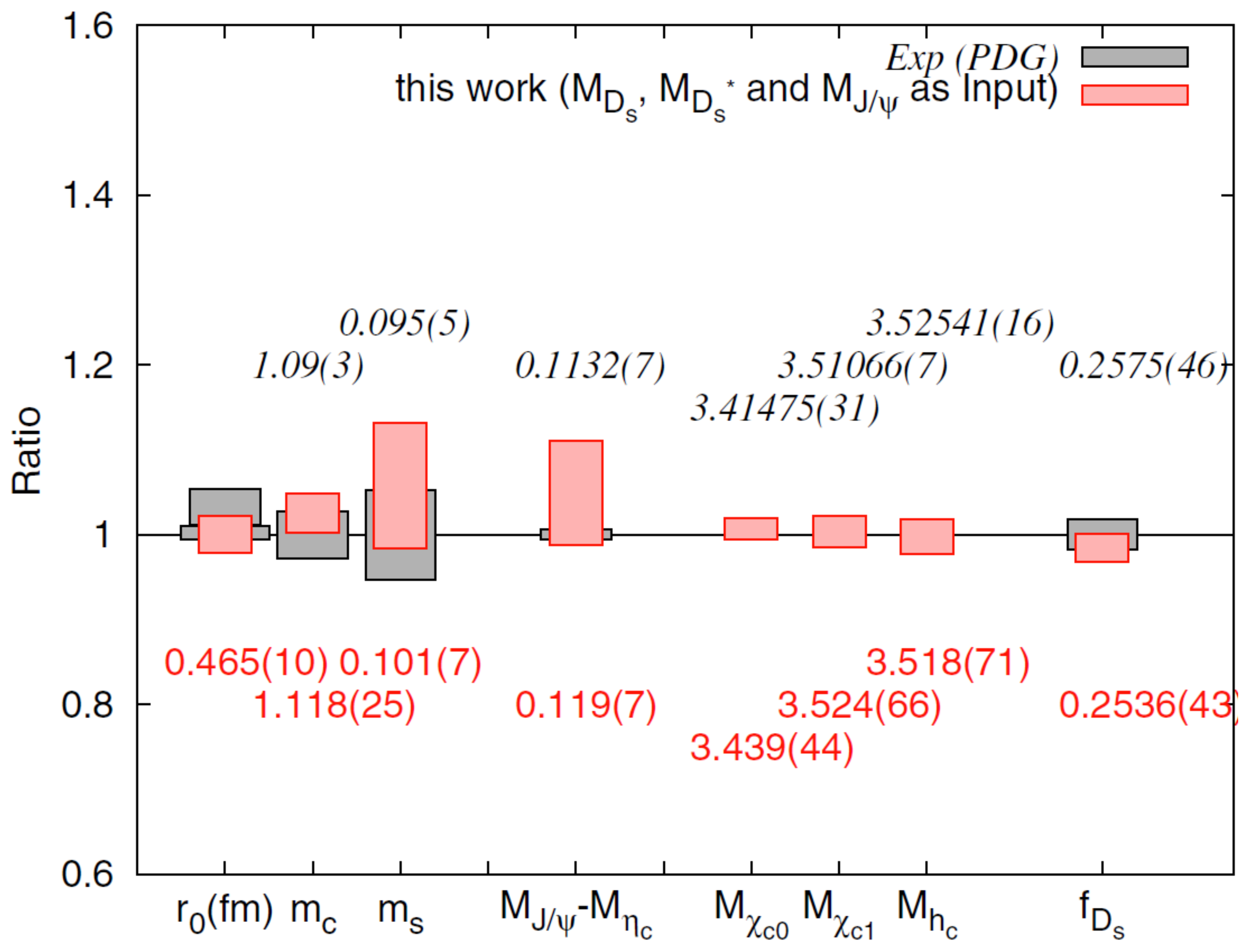
FLAG-3



Private Communication

- $N_f = 2+1$:
  $f_{D_s} = 249.8(2.3)$ MeV
- $N_f = 2+1+1$:
  $f_{D_s} = 248.83(1.27)$ MeV
- **PDG2014:**
  $f_{D_s^+}^{exp} = 257.5(4.6)$ MeV

Summary



- Charm, strange quark masses and f_{D_s} are determined by using overlap fermions on 2+1-flavor configurations.
- We are working on a lattice with almost physical pion mass:

$M_\pi^{(\mathrm{sea})} = 139.2(4)$ MeV

$L^3 \times T = 48^3 \times 96$

$La \sim 5.5$ fm