## Glauber-gluon effects and $B \rightarrow \pi \pi$ , *K* $\pi$ puzzles

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April 2016

**Presented at Shanghai JiaoTong University** 

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## Summary

## **Motivation**

#### > Experimentally, the current data on the BR of $B^0 \rightarrow \pi^0 \pi^0$

 $\begin{cases} (1.83 \pm 0.21 \pm 0.13) \times 10^{-6} & \text{(BaBar),} \\ (0.90 \pm 0.12 \pm 0.10) \times 10^{-6} & \text{(Belle),} \\ (1.17 \pm 0.13) \times 10^{-6} & \text{(HFAG),} \end{cases}$ 

M. Petric@ICHEP2014 arXiv: 1412.7515[hep-ex]

Hierarchy,

$$Br\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right) > (\sim) \qquad Br\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right) > Br\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)$$

and on the  $\triangle A_{K\pi} \equiv A_{CP}^{dir}(K^{\pm}\pi^{0}) - A_{CP}^{dir}(K^{\pm}\pi^{\mp})$   $\Delta A_{K\pi} = 0.119 \pm 0.022$  arXiv: 1412.7515[hep-ex]  $A_{CP}^{dir}(B^{\pm} \to K^{\pm}\pi^{0}) = +0.037 \pm 0.021, \quad A_{CP}^{dir}(B^{0} \to K^{\pm}\pi^{\mp}) = -0.082 \pm 0.006,$  > Theoretically, e.g., in the NLO pQCD formalism,

$$Br (B^{0} \rightarrow \pi^{+}\pi^{-}) > Br (B^{+} \rightarrow \pi^{+}\pi^{0}) >> Br (B^{0} \rightarrow \pi^{0}\pi^{0})$$

Phys.Rev.D 73, 114014(2006), Li and Mishima Phys.Rev.D 90, 014029(2014), Zhang *et al.* 

In particular, in the factorization theorems,

$$Br (B^0 \rightarrow \pi^0 \pi^0) \sim (0.2 - 0.3) \times 10^{-6}$$

For  $B \rightarrow K\pi$  decays,

$$A_{\rm CP}^{\rm dir}(B^0 \to K^{\pm} \pi^{\mp}) \sim A_{\rm CP}^{\rm dir}(B^{\pm} \to K^{\pm} \pi^0)$$

> The above contradictions between theory and experiment for the  $B \rightarrow \pi\pi$  decay rates and the differences of the  $B \rightarrow K\pi$  direct CP asymmetries have been known as long-standing puzzles.



$$\mathcal{B}(B^0 \to \rho^0 \rho^0) = \begin{cases} (0.92 \pm 0.32 \pm 0.14) \times 10^{-6} & \text{(BaBar)}, \\ (1.02 \pm 0.30 \pm 0.15) \times 10^{-6} & \text{(Belle)}, \\ (0.97 \pm 0.24) \times 10^{-6} & \text{(HFAG)}, \end{cases}$$

arXiv: 1412.7515[hep-ex]

Can C be larger? How to reach? Sub-leading corrections or new QCD mechanism?

The  $B \rightarrow \rho \rho$  data seriously constrained the possibility of resolving the  $B \rightarrow \pi \pi$  puzzle. Phys.Rev.D 73, 114014 (2006), Li and Mishima

> Many efforts have been made on this puzzle. But, no satisfactory resolution can be achieved yet. (Naively enhancing the hard spectator amplitudes(HSA) to C!)

> To our best knowledge, all the strategies in the literature adopted to resolve this puzzle either evaded the  $B^0 \rightarrow \rho^0 \rho^0$ constraint or did not survive the constraint in the SM.

A new mechanism is demanded!

Mechanism must differentiate pion from  $\rho$  meson!

# Have checked the k<sub>T</sub> factorization of the spectator nonfactorizable diagrams Phys. Rev. D 83, 034023 (2011); *ibid.* 90, 074018 (2014), Li and Mishima



Considered the factorization of  $M_2$  meson wave function Observed the existence of Glauber gluons

Leading IR regions  $l \sim (l^+, l^-, l_T)$ 

$$l^{+}(Q) \gg l_{T}(\Lambda_{QCD}) \gg l^{-}(\Lambda_{QCD}^{2} / Q) \qquad \text{Collinear}$$

$$l^{+}(\Lambda_{QCD}) \sim l_{T}(\Lambda_{QCD}) \sim l^{-}(\Lambda_{QCD}) \qquad \text{Soft}$$

$$l^{+}(\Lambda_{QCD}^{2} / Q) \sim l^{-}(\Lambda_{QCD}^{2} / Q) \ll l_{T}(\Lambda_{QCD}) \qquad \text{Glauber}$$

> Li and Mishima observed the glauber divergences with the NLO spectator amplitudes in the  $k_{\rm T}$  factorization theorem, then gave universal phase factors by all-order summation.

Phys. Rev. D 83, 034023 (2011); ibid. 90, 074018 (2014), Li and Mishima

 $\succ$  the phase factors associated with the emitted meson will turn the destructive interference between the LO spectator diagrams into a constructive one, then modify *C*;

$$I_a \approx \exp(iS_e)\mathcal{M}_a^{(0)}, \qquad I_b \approx \exp(-iS_e)\mathcal{M}_b^{(0)},$$

• like 
$$1 - 1 \Rightarrow e^{iS_e} - e^{-iS_e}$$
 , large imaginary C

 $\succ$  the phase factors associated with the recoiled meson will rotate the enhanced *C* then modify the interference between *C* and *T*. ► After treating the glauber phases as free real parameters  $-\frac{\pi}{2}$ in the  $B \rightarrow \pi\pi$  decays, the BR was  $Br (B^0 \rightarrow \pi^0 \pi^0) \sim 1.2 \times 10^{-6}$ 

▷ the postulation on vanishing of glauber phases in the  $B \rightarrow \rho \rho$  decays was made.

But, the question should be answered:

Why the color-suppressed tree amplitudes are so different in the  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  decays?

→ We attempt to answer this question by quantitatively estimating different glauber effects through convolution in the  $B \rightarrow \pi \pi$  and  $B \rightarrow \rho \rho$  decays.

## **Factorization formulas**

➤ Glauber gluons have been identified from the higher order corrections to the spectator diagrams in  $B \rightarrow M_1 M_2$  decays. Phys. Rev. D 83, 034023 (2011); *ibid.* 90, 074018 (2014), Li and Mishima



NLO spectator diagrams contain the glauber divergences associated with the  $M_2$  meson for  $B \rightarrow M_1 M_2$  decay. Other NLO diagrams with the glauber divergences are referred to Phys.Rev.D 83, 034023(2011).



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$$\int d^{2}\mathbf{b}_{1}d^{2}\mathbf{b}_{2} \int d^{2}\mathbf{b}_{s1}d^{2}\mathbf{b}_{s2}\bar{\phi}_{B}(\mathbf{b}_{1})\bar{\phi}_{1}(\mathbf{b}_{s1}+\mathbf{b}_{1},\mathbf{b}_{s1}) \\ \times \bar{\phi}_{2}(\mathbf{b}_{s2},\mathbf{b}_{s2}+\mathbf{b}_{2}) \exp\left[-iS(\mathbf{b}_{s1}-\mathbf{b}_{2})+iS(\mathbf{b}_{s2}-\mathbf{b}_{1})\right]H_{a}(\mathbf{b}_{1},\mathbf{b}_{2})$$

$$\int d^{2}\mathbf{b}_{1}d^{2}\mathbf{b}_{2} \int d^{2}\mathbf{b}_{s1}d^{2}\mathbf{b}_{s2}\bar{\phi}_{B}(\mathbf{b}_{1})\bar{\phi}_{1}(\mathbf{b}_{s1}+\mathbf{b}_{1},\mathbf{b}_{s1})$$

$$\times \bar{\phi}_{2}(\mathbf{b}_{s2}+\mathbf{b}_{2},\mathbf{b}_{s2})\exp\left[-iS(\mathbf{b}_{s1}-\mathbf{b}_{2})-iS(\mathbf{b}_{s2}-\mathbf{b}_{1})\right]H_{b}(\mathbf{b}_{1},\mathbf{b}_{2})$$
Comments:

 $\succ$  the universal glauber factor associated with  $M_1$ , same for both amplitudes

> the universal glauber factor associated with  $M_2$ , "(-)+" denotes the glauber gluons radiated from valence (anti-)quark

> Though the glauber factor is universal, the glauber effect appears different through the convolution with the TMD wave functions  $\bar{\phi}_1(\mathbf{b}_{s1} + \mathbf{b}_1, \mathbf{b}_{s1})$  and  $\bar{\phi}_2(\mathbf{b}_{s2} + \mathbf{b}_2, \mathbf{b}_{s2})$  which contains different intrinsic *b* dependences for pion, kaon, and  $\rho$  meson.

## Numerical analysis and discussions

- Two important elements:
  - (1) TMD meson wave function(different intrinsic  $k_{\rm T}$ )
  - ② parameterization of glauber phase factor
- $\succ$  TMD meson wave function with the intrinsic  $k_{\rm T}$  part in the<br/>Gaussian formSLAC-PUB-2540, Brodsky et al., AIP Conf. Proc. 68,1000(1981), Huang

J.Phys.G 34, 1845 (2007), Yu et al.

$$\phi_M(x, \mathbf{k}_T) = \frac{\pi}{2\beta_M^2} \exp\left(-\frac{\mathcal{M}^2}{8\beta_M^2}\right) \frac{\phi_M(x)}{x(1-x)},$$
  
For pion:  $\mathcal{M}^2 = \frac{k_T^2 + m^2}{x} + \frac{k_T^2 + m^2}{1-x},$   
For kaon:  $\mathcal{M}^2 = \frac{k_T^2 + m_q^2}{x} + \frac{k_T^2 + m_s^2}{1-x}$ 

#### Then the modified wave function is expressed as,

$$\bar{\phi}_{M}(x, \mathbf{b}', \mathbf{b}) \equiv \bar{\phi}_{M}(\mathbf{b}', \mathbf{b}) \phi_{M}(x) = \frac{2\beta_{M}^{2}}{\pi} \exp\left[-2\beta_{M}^{2}xb'^{2} - 2\beta_{M}^{2}(1-x)b^{2}\right] \phi_{M}(x)$$
$$\bar{\phi}_{K}(x, \mathbf{b}', \mathbf{b}) = \frac{2\beta_{K}^{2}}{\pi} \exp\left[-\frac{1}{8\beta_{K}^{2}}\left(\frac{m_{q}^{2}}{x} + \frac{m_{s}^{2}}{1-x}\right)\right] \times \exp\left[-2\beta_{K}^{2}xb'^{2} - 2\beta_{K}^{2}(1-x)b^{2}\right] \phi_{K}(x)$$

Solution We simply parameterize the glauber phase S(b) by a sinusoidal function,

$$S(\boldsymbol{b}) = r \pi \sin(p \boldsymbol{b})$$

Image: series of pion, kaon and  $\rho$  mesonI.Phys.G 34, 1845 (2007), Yu et al.

(1) widely adopted  $\beta_{\pi} \sim 0.4$  GeV for pion in the literature (2)  $\beta_{\rho} \sim 1/3 \beta_{\pi}$  fixed through the  $B \rightarrow \rho$  form factor (3)  $\beta_{K} \sim 0.25$  GeV fixed through the  $B \rightarrow K$  form factor

#### Important implication:

▶ pion( $\rho$ -meson) WF with a weak(strong) falloff in parton TM  $k_{\rm T}$ , and kaon WF reveals a stronger(weaker) falloff in  $k_{\rm T}$ compared to the pion( $\rho$ -meson) one, which means pion requires a tighter spatial distribution of its leading Fock state relative to higher Fock states.

> consistent with the dual role of pion as a massless NGB and as a qq bound state simultaneously.

Two sets of parameters *r* and *p* corresponding to largest BR of  $B^0 \rightarrow \pi^0 \pi^0$  decay 15

<b>×</b> <i>r</i> ~ 0.47	$p \sim -0.632$	GeV	$\checkmark$ $r \sim 0.60$	), $p \sim 0.544 {\rm GeV}$
$Br(B^0 \rightarrow \pi^-)$	$(\pi^{-}) = 5.902$	×10 <sup>-6</sup>	$Br(B^0 \rightarrow \pi^+)$	$(\pi^{-}) = 5.39 \times 10^{-6}$
$Br(B^+ \rightarrow \pi$	$(\pi^{+}\pi^{0}) = 3.88$	×10 <sup>-6</sup>	$Br(B^+ \rightarrow \pi$	$(\pi^{+}\pi^{0}) = 4.45 \times 10^{-6}$
$Br(B^0 \to \pi^0 \pi^0) = 0.62 \times 10^{-6}$			$Br(B^0 \to \pi^0 \pi^0) = 0.61 \times 10^{-6}$	
$Br(B^0 \rightarrow \rho$	$(^{0}\rho^{0}) = 1.07 \times$	<b>×</b> 10 <sup>-6</sup>	$Br(B^0 \rightarrow \rho$	$^{0}\rho^{0}) = 0.89 \times 10^{-6}$
Modes	Data [1, 2]	Ν	ILO	NLOG
$B^0 \to \pi^+\pi^-$	$5.10\pm0.19$	$6.19^{+2.09}_{-1.48}$	$(\omega_B)^{+0.38}_{-0.34}(a_2^{\pi})$	$5.39^{+1.86}_{-1.31}(\omega_B)^{+0.28}_{-0.25}(a_2^{\pi}$
$B^+ \to \pi^+ \pi^0$	$5.48^{+0.35}_{-0.34}$	$3.35^{+1.08}_{-0.77}$	$(\omega_B)^{+0.23}_{-0.22}(a_2^{\pi})$	$4.45^{+1.38}_{-0.99}(\omega_B)^{+0.39}_{-0.36}(a_2^{\pi}$
$B^0 \to \pi^0 \pi^0$	$0.90\pm0.16$	$0.29^{+0.11}_{-0.07}$	$(\omega_B)^{+0.03}_{-0.02}(a_2^{\pi})$	$0.61^{+0.16}_{-0.12}(\omega_B)^{+0.14}_{-0.12}(a_2^{\pi}$
$B^0  o  ho^0  ho^0$	$0.97 \pm 0.24$	$1.06^{+0.29}_{-0.21}$	$(\omega_B)^{+0.19}_{-0.16}(a_2^{ ho})$	$0.89^{+0.26}_{-0.18}(\omega_B)^{+0.13}_{-0.10}(a_2^{\rho}$

Phys.Rev.D 91, 114019(2015), Liu, Li, and Xiao

ℜ The results obtained with second set of parameters show the preferred pattern:  $B^0 \rightarrow \pi^+\pi^-$  and  $B^0 \rightarrow \rho^0\rho^0$  BRs decrease 13% and 16%, respectively, while  $B^+ \rightarrow \pi^+\pi^0$  and  $B^0 \rightarrow \pi^0\pi^0$  ones increase by 33% and a factor of 2.1, respectively.

% The agreement between the theoretical predictions and the data for all the  $B \rightarrow \pi\pi$  and  $B^0 \rightarrow \rho^0 \rho^0$  BRs is improved simultaneously.

% The ratio of enhancement factor of  $B^0 \rightarrow \pi^0 \pi^0$  over reduction factor of  $B^0 \rightarrow \rho^0 \rho^0$  is about 2.5, close to 3 observed in Phys.Rev.D 90, 074018 (2014). % To clarify the glauber effect, the results(in units of 10<sup>-2</sup> GeV<sup>3</sup>) of the nonfactorizable spectator amplitudes from operator O<sub>2</sub> without and with glauber phase are

$$\mathcal{A}_{a,b}(B^0 \to \pi^0 \pi^0) = \begin{cases} 11.86 - i9.04, \\ 10.80 - i7.25, \end{cases} \begin{vmatrix} -7.13 + i6.18, \\ 7.67 - i3.42, \end{vmatrix}$$
(NLO), (NLOG),

$$5.53e^{-i0.54} \longrightarrow 21.33e^{-i0.52}$$

$$\mathcal{A}_{a,b}(B^0 \to \rho^0 \rho^0) = \begin{cases} (-42.44 + i24.42, \\ -5.78 + i4.32, \end{cases} \begin{vmatrix} 28.88 - i18.07, \\ -3.61 - i3.23, \end{vmatrix}$$
(NLO), (NLOG),

It is possible to resolve the  $B \rightarrow \pi \pi$  puzzle!

> CP-averaged branching ratios of  $B \rightarrow K\pi$  decays,

Modes	Data [13,14]	NLO	NLOG
$B^0  o K^{\pm} \pi^{\mp}$	$1.96\pm0.05$	$2.33^{+0.74}_{-0.52}(\omega_B)^{+0.12}_{-0.11}(a^K)^{+0.21}_{-0.20}(a^{\pi})$	$2.17^{+0.71}_{-0.49}(\omega_B)^{+0.11}_{-0.10}(a^K)^{+0.17}_{-0.16}(a^\pi)$
$B^{\pm}  ightarrow K^{\pm} \pi^0$	$1.29\pm0.05$	$1.53^{+0.50}_{-0.35}(\omega_B)^{+0.08}_{-0.07}(a^K)^{+0.12}_{-0.12}(a^\pi)$	$1.40^{+0.46}_{-0.33}(\omega_B)^{+0.06}_{-0.06}(a^K)^{+0.10}_{-0.10}(a^\pi)$
$B^{\pm}  o \pi^{\pm} K^0$	$2.37\pm0.08$	$2.72^{+0.88}_{-0.61}(\omega_B)^{+0.15}_{-0.13}(a^K)^{+0.25}_{-0.24}(a^\pi)$	$2.41^{+0.80}_{-0.56}(\omega_B)^{+0.11}_{-0.11}(a^K)^{+0.17}_{-0.17}(a^\pi)$
$B^0 \to K^0 \pi^0$	$0.99\pm0.05$	$1.02^{+0.32}_{-0.22}(\omega_B)^{+0.05}_{-0.05}(a^K)^{+0.11}_{-0.10}(a^\pi)$	$0.93^{+0.30}_{-0.21}(\omega_B)^{+0.06}_{-0.05}(a^K)^{+0.08}_{-0.07}(a^\pi)$

Glauber-gluon effects modify the branching ratios moderately with around 10% reduction;

Dominance of the penguin contributions make the values be insensitive to amplitude C. Phys.Rev.D 93, 014024(2016), Liu, Li, and Xiao

 $\succ$  Direct CP asymmetries of  $B \rightarrow K\pi$  decays,

Modes	Data [13,14]	NLO	NLOG
$B^0  o K^{\pm} \pi^{\mp}$	$-0.082 \pm 0.006$	$-0.076^{+0.008}_{-0.009}(\omega_B)^{+0.013}_{-0.013}(a^K)^{+0.007}_{-0.007}(a^\pi)$	$-0.081^{+0.009}_{-0.009}(\omega_B)^{+0.011}_{-0.011}(a^K)^{+0.010}_{-0.009}(a^\pi)$
$B^{\pm}  ightarrow K^{\pm} \pi^0$	$+0.037 \pm 0.021$	$-0.008^{+0.008}_{-0.009}(\omega_B)^{+0.009}_{-0.009}(a^K)^{+0.006}_{-0.006}(a^\pi)$	$+0.021^{+0.008}_{-0.008}(\omega_B)^{+0.003}_{-0.004}(a^K)^{+0.014}_{-0.013}(a^\pi)$
$B^{\pm}  ightarrow \pi^{\pm} K^0_S$	$-0.017 \pm 0.016$	$+0.003^{+0.001}_{-0.000}(\omega_B)^{+0.001}_{-0.001}(a^K)^{+0.000}_{-0.000}(a^\pi)$	$+0.004^{+0.000}_{-0.001}(\omega_B)^{+0.001}_{-0.002}(a^K)^{+0.000}_{-0.000}(a^\pi)$
$B^0 \to K^0_S \pi^0$	$0.00\pm0.13$	$-0.056^{+0.001}_{-0.001}(\omega_B)^{+0.004}_{-0.004}(a^K)^{+0.000}_{-0.001}(a^\pi)$	$-0.089^{+0.001}_{-0.000}(\omega_B)^{+0.013}_{-0.009}(a^K)^{+0.005}_{-0.005}(a^\pi)$

Glauber-gluon effects modified the DCP of  $B^+ \rightarrow K^+ \pi$  mode significantly with sign flipping attributed to its sensitivity to amplitude *C* 19

#### > CP-averaged branching ratios of $B \rightarrow K\underline{K}$ decays,

Modes	Data [13,14,23]	NLO	NLOG
$egin{array}{lll} B^{\pm} & ightarrow K^{\pm} ar{K}^0 \ B^0 & ightarrow K^0 ar{K}^0 \end{array}$	$1.52 \pm 0.22^{\mathrm{a}}$ $1.21 \pm 0.16$	$2.45^{+0.83}_{-0.58}(\omega_B)^{+0.17}_{-0.17}(a^K) \ 2.19^{+0.77}_{-0.54}(\omega_B)^{+0.09}_{-0.09}(a^K)$	$2.27^{+0.79}_{-0.54}(\omega_B)^{+0.17}_{-0.14}(a^K) \ 2.02^{+0.72}_{-0.50}(\omega_B)^{+0.08}_{-0.08}(a^K)$

<sup>a</sup>This is the very recent measurement reported by the LHCb Collaboration [23], which is comparable with  $1.64 \pm 0.45$  by the *BABAR* Collaboration [24] and a bit larger than  $1.11 \pm 0.20$  by the Belle Collaboration [25].

Phys.Rev.D 93, 014024(2016), Liu, Li, and Xiao

#### > Direct CP asymmetries of $B \rightarrow K\underline{K}$ decays,

Modes	Data [13,14,23]	NLO	NLOG
$egin{array}{lll} B^{\pm} & ightarrow K^{\pm} ar{K}^0_S \ B^0 & ightarrow K^0_S ar{K}^0_S \end{array}$	$-0.21 \pm 0.14$ $0.0 \pm 0.4$	$\begin{array}{l}-0.03^{+0.01}_{-0.01}(\omega_B)^{+0.02}_{-0.02}(a^K)\\-0.09^{+0.00}_{-0.00}(\omega_B)^{+0.01}_{-0.01}(a^K)\end{array}$	$\begin{array}{c} -0.03^{+0.01}_{-0.01}(\omega_B)^{+0.02}_{-0.02}(a^K) \\ -0.09^{+0.00}_{-0.00}(\omega_B)^{+0.00}_{-0.00}(a^K) \end{array}$

No amplitude C; only the spectator amplitudes induced by penguin operators;

Glauber-gluon effects decrease the branching ratios by only a few percent; the DCP remains unchanged.

 $\succ$  for pion emission from the weak vertex with operator O<sub>2</sub>,

$$\begin{aligned} \mathcal{A}_{a,b}(B^{\pm} \to \pi^0 K^{\pm}) \\ &= \begin{cases} -16.71 + i13.71, & 10.85 - i9.96, & (\text{NLO}), \\ -12.57 + i10.80, & -9.96 + i5.85, & (\text{NLOG}), \end{cases} \end{aligned}$$

 $\succ$  for kaon emission from the weak vertex with operator O<sub>1</sub>,

$$\mathcal{A}_{a,b}(B^{\pm} \to K^{\pm} \pi^{0}) = \begin{cases} 4.55 - i3.98, & -3.37 + i2.25, & (\text{NLO}), \\ 3.59 - i3.23, & 3.14 - i0.90, & (\text{NLOG}) \end{cases}$$

$$6.96e^{i2.57} \quad (2.09e^{-i0.97}) \times 10^{-2} \text{ GeV}^{3}$$

$$28.01e^{i2.51} \quad (7.90e^{-i0.55}) \times 10^{-2} \text{ GeV}^{3}$$

➤ to examine the similarity between pion and kaon from Glauber gluons

$$R_{\pi} \equiv \frac{|\mathcal{A}_{a}(\pi^{0}K^{\pm}) + \mathcal{A}_{b}(\pi^{0}K^{\pm})|_{\text{NLOG}-S_{e^{2}}}}{|\mathcal{A}_{a}(\pi^{0}K^{\pm}) + \mathcal{A}_{b}(\pi^{0}K^{\pm})|_{\text{NLO}}} \approx 4.69,$$

$$R_{K} \equiv \frac{|\mathcal{A}_{a}(K^{\pm}\pi^{0}) + \mathcal{A}_{b}(K^{\pm}\pi^{0})|_{\text{NLOG}-S_{e2}}}{|\mathcal{A}_{a}(K^{\pm}\pi^{0}) + \mathcal{A}_{b}(K^{\pm}\pi^{0})|_{\text{NLO}}} \approx 3.90,$$

➢ both pion and kaon are pseudo-NGBs, but the latter one with non-negligible SU(3) symmetry breaking effect, which resulting in the weaker Glauber-gluon effects than the pion through the convolution with the TMD WF. % A plausible explanation to the dynamical origin of glauber phase:

the overlap between the  $k_{\rm T}$  distributions of the leading Fock state in a meson and the glauber gluons in a decay process

 $\Re$  It is stressed that the  $B \rightarrow \pi\pi$ ,  $K\pi$  puzzles must be resolved by resorting to a mechanism and that the glauber gluons should be one of the most crucial mechanisms.

## **Conclusions and Summary**

% The model estimate of the glauber effect has been performed and the convoluted factorization formulas have been obtained.

 $\Re$  A weak falloff in  $k_{\rm T}$  of pion WF is consistent with the dual role of the pion as a massless NGB and as a qq bound state.

 $\Re$  The universal glauber factors make distinct impacts on the  $B^0 \rightarrow \pi^0 \pi^0$  and  $B^0 \rightarrow \rho^0 \rho^0$  BRs through convolution with TMD WF and reasonable parameterization of glauber phase.

% The more significant glauber effect from pion was observed and the consistency between th. and ex. for all the modes was improved simultaneously.

 $\Re$  The glauber gluons should be one of the most crucial mechanisms to resolve the long-standing  $B \rightarrow \pi\pi$ ,  $K\pi$  puzzles.



# **BACKUP SLIDES**

#### Contributing to glauber phase S<sub>2</sub>



Phys. Rev. D 90, 074018 (2014), Li and Mishima

## Contributing to glauber phase S<sub>1</sub>



boson [30]: the valence quark and antiquark of the pion are separated by a short distance, like those of the  $\rho$  meson, in order to reduce the confinement potential energy. The multiparton states of the pion spread over a huge spacetime in order to meet the role of a massless NG boson, which result in a strong Glauber effect.

## Nambu-Goldstone boson

- Pion as a qq bound state and as a massless
   Nambu-Goldstone boson?
- Massless boson => huge spacetime => large separation between qq => high mass under confinement => contradiction!
- Reconciliation: leading qq state is tight, higher Fock state gives soft cloud (Lepage, Brodsky 79; Nussinov, Shrock 08; Duraisamy, Kagan 08)
   Pion is unique.
- Glauber factor for pion corresponds to this soft cloud: 3 partons in k<sub>T</sub> space

 $\int_0^{\Lambda} d^2 l_T \exp(-i\mathbf{l}_T \cdot \mathbf{b})/(l_T^2 + m_q^2)$ 

Loop momentum cutoff:  $\Lambda \sim 0.5$  GeV, collecting soft contributions, roughly yielding the period *p* of the oscillatory parameterization;

Gluon mass:  $m_g$ , together with the coefficient in the associated loop corrections such as strong coupling, controlling the magnitude r of the oscillation;

 $b \rightarrow 0$  limit, corresponding to the integration over the transverse momentum in the collinear factorization theorem;