QCD Factorization in B decays and QCD Corrections

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based on **0810.1230**, **0911.3655**, **1007.3758**, **1507.03700** and **work in progress**

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Outline

Goals of B physics:

- What is B physics: the productions and decays of hadrons containing a b-quark;

$$B_u\ (u\bar{b}),\quad B_d\ (d\bar{b}),\quad B_s\ (s\bar{b}),\quad B_c\ (c\bar{b}),\quad \Lambda_b\ (udb),\quad \cdots$$

- Goals of B physics:
  - to measure the SM parameters, to precisely test the CKM mechanism of CP violation, to search for/constrain on NP signals beyond the SM;

    $\hookrightarrow$ complementary to EWP tests @(LEP, Tevatron) and direct searches @(LHC, ILC)
  - to understand strong-interaction physics related with the confinement of quarks and gluons into hadrons;

    $\hookrightarrow$ Operator product expansion, effective field theories, and factorization theorems
  - to probe the hadronic structure in B hadrons and their decay products;

    $\hookrightarrow$ important theoretical and phenomenological input for other processes

Classification of B decays:

- At the quark level: B-meson weak decays are mediated by flavour-changing charged-current $J^{\mu}_{\rm CC}$ coupled to the W-boson;

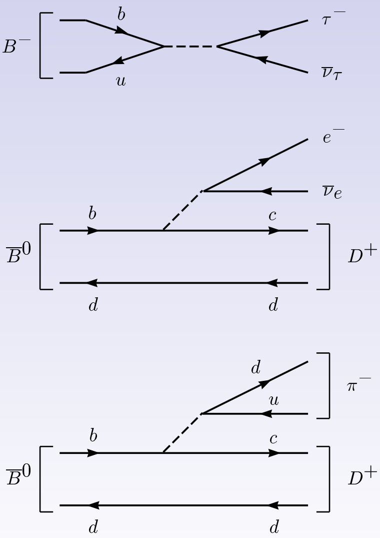
$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^{\mu}_{\rm CC} W^{\dagger}_{\mu} + \text{h.c.}$$

$$J^{\mu}_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_{\mu}, \bar{\nu}_{\tau})\, \gamma^{\mu} \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix}$$

$$+ (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\, \gamma^{\mu}\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

- Three different classes: depending on the different final states, B-meson weak decays can be divided into three classes:

leptonic, semi-leptonic, non-leptonic



Simple quark-line diagrams

Non-leptonic B decays:

- A crucial role in testing the CKM mechanism and in quantifying the CP violation:

  - α: from time-dep. CP asym. in $B \to \pi\pi$, $\pi\rho$ and $\rho\rho$ decays;

    $(90.4^{+2.0}_{-1.0})^\circ$

  - β: from $B \to J/\psi K_S$ and other charmonium modes;

    $(22.62^{+0.44}_{-0.42})^\circ$

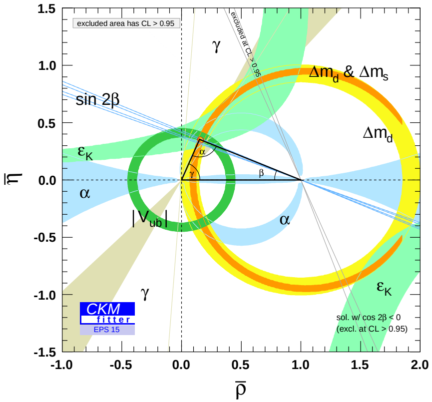
  - γ: from $B \to DK$, $B \to K\pi\pi$, $B \to KKK$ decays;

    $(67.01^{+0.88}_{-1.99})^\circ$

  - β_s: from $B_s \to J/\psi\phi$ and $B_s \to \phi\phi$ decays, $\cdots$;

    $(0.01882^{+0.00036}_{-0.00042})rad$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

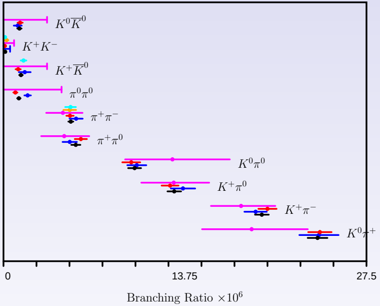


taken from CKMfitter group as of Summer 2015.

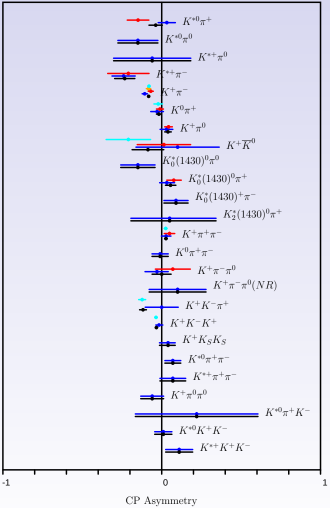
Status of exp. data on hadronic B decays:

- Thanks to BaBar, Belle, Tevatron and LHCb, more and more precise data available now for many hadronic B decays; [*HFAG, 1412.7515*]

$\mathcal{B}(B \to K\pi,\ \pi\pi,\ KK)$



A_{CP}

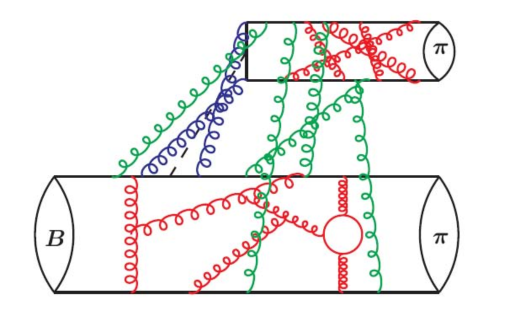


- To catch up with the precise exp. measurements, we need to improve the theor. calculation!

Difficulties in non-leptonic B decays:

- In reality: in the real world, quarks are confined inside hadrons and bound by the exchange of soft gluons;

  ↪the simplicity of weak interactions is overshadowed by the complexity of strong interactions in these decays!



- B-meson decay is a multi-scale problem with highly hierarchical interaction scales:

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV		$m_B \sim 5$ GeV		$\Lambda_{\rm QCD} \sim 1$ GeV

Difficulties in non-leptonic B decays:

- Different faces of strong interactions in B decays:

Different energy scales $\leftrightarrow$ different dynamics $\leftrightarrow$ different methods:

$\mu^2 \sim M_X^2$	$\longrightarrow$	"new physics" ???
$\mu^2 \sim M_W^2$	$\longrightarrow$	standard-model flavour transitions

effective electroweak Hamiltonian including perturbative QCD (QED) corrections

$\mu^2 \sim m_b^2$ $\mu^2 \sim m_b\Lambda$	$\longrightarrow$	short-distance dynamics in hadronic matrix elements

heavy quark expansion $\longrightarrow$ effective theories HQET/SCET

$\mu^2 \sim \Lambda^2$	$\longrightarrow$	long-distance hadronic parameters (form factors, decay constants, parton distributions, . . .)

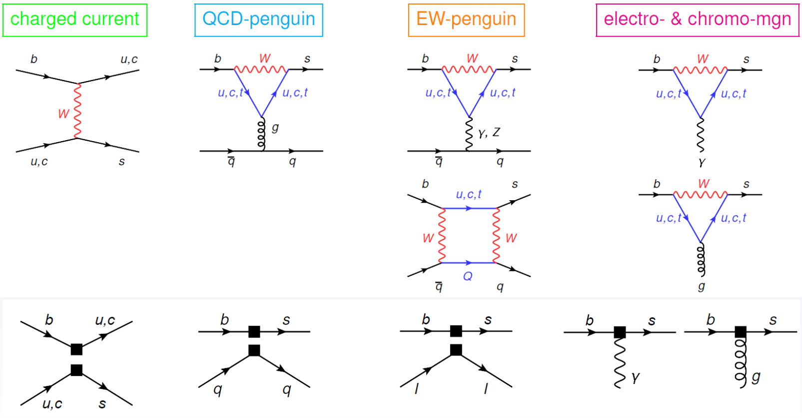
data / non-perturbative methods / approximate symmetries

- Hadronic parameters: Lattice QCD, QCD sum rules, phenomenological models, $\cdots$

Effective weak Hamiltonion for B decays:

- The starting point $H_{\rm eff}$: obtained after integrating out the heavy d.o.f. $(m_W, m_Z, m_t \gg m_b)$, containing physics above $\mu \sim m_b$; [*BBL basis: Buras, Buchalla, Lautenbacher '96*; *CMM basis: Chetyrkin, Misiak, Münz '98*]

$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{p=u,c} \sum_{i=1,2} C_i\, \mathcal{O}_i^p + \sum_{3,\ldots,6} C_i\, \mathcal{O}_i + \sum_{7,\ldots,10} C_i\, \mathcal{O}_i + \sum_{7\gamma,8g} C_i\, \mathcal{O}_i \Big]$$



Weak effective Hamiltonian for non-leptonic B decays:

- Three steps to obtain $\mathcal{H}_{\text{eff}} \propto \sum C_i Q_i$: [*BBL basis: Buras, Buchalla, Lautenbacher '96*;

  - Calculation of matching coefficients c_i in fixed-order perturbation theory:

$$C_i(m_W) = c_i^{(0)} + \frac{\alpha_s}{4\pi} c_i^{(1)} + \ldots$$

⟵ SM! + New Physics?

  - Perturbative calculation of anomalous dimensions γ_{ij} of operators in H_{eff}

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \gamma_{ij}^{(1)} + \ldots$$

⟵ QCD (+QED)

  - Use renormalization group to sum large logarithms $\ln \frac{m_b}{m_W}$:

$$C_i(m_W) \to C_i(m_b) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{-\gamma_{ij}^{(0)}/2\beta_0} C_j(m_W) + \ldots$$

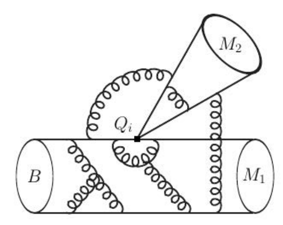
⟵ RGE

- C_i: perturbatively calculable for a given model; the NNLO program now complete;
  - ▷ 2-loop/3-loop matching calculations at the initial scale; [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]
  - ▷ 3-loop/4-loop anomalous dimension matrices for running; [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

Calculation of the hadronic matrix elements of Q_i:

- Dim-6 operators Q_i: their matrix elements $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ depends on spin and parity of $M_{1,2}$; re-scattering determines strong phases, thus direct CP asymmetries;

  a quite difficult, multi-scale, strong-interaction problem!



- Effective theories / Factorization / Approximate symmetries: express exclusive matrix elements in terms of (few) universal hadronic quantities;

- To match the exp. precision, we need to try to improve the calculation of $\langle Q_i \rangle$!
  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, $\cdots$
    [Keum, Li, Sanda, Lü, Yang '00;
    Beneke, Buchalla, Neubert, Sachrajda, '00;
    Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
  - Exploit symmetries of QCD: flavour SU(3) symmetries (Isospin, U-Spin, V-Spin), chiral symmetry, heavy-quark symmetry, $\cdots$ [Zeppenfeld, '81;
    London, Gronau, Rosner, He, Chiang, Cheng]

- *To reach precision predictions, we need to combine all these complementary approaches!*

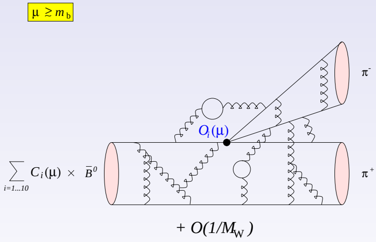
Hadronic matrix elements in QCDF approach:

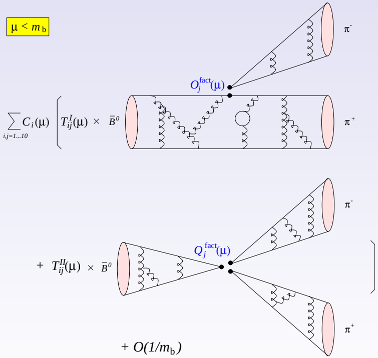
- In the heavy-quark limit, $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ obeys the factorization formula: [*BBNS'99-'04*]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 F_+^{BM_1}(0) f_{M_2} \int du \; T_i^I(u) \; \phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

$$+ f_B f_{M_1} f_{M_2} \int d\omega dv du \; T_i^{II}(\omega, v, u) \; \phi_B(\omega) \; \phi_{M_1}(v) \; \phi_{M_2}(u)$$

$$+ \mathcal{O}(1/m_b)$$





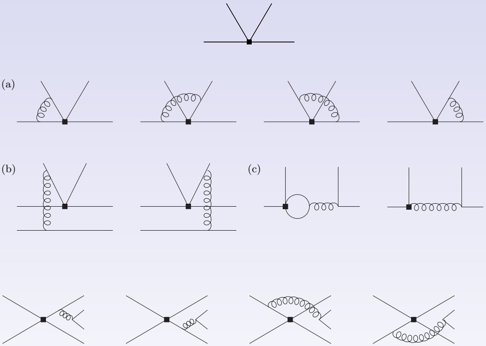
- A systematic framework to all orders in α_s, but limited accuracy by $1/m_b$ corrections.

Perturbative calculation of the hard kernels $T^{I,II}$:

- T^{I} and T^{II}: perturbatively calculable order by order in α_s;

vertex corrections: $T^{I} = 1 + \mathcal{O}(\alpha_s) + \cdots;$ spectator scattering: $T^{II} = \mathcal{O}(\alpha_s) + \cdots.$

- up to NLO in α_s, the relevant Feynman diagrams include:
- hard and IR contributions are separated properly, thus validating the soft-collinear factorization at 1-loop level;
- strong phases from final-state interactions $\sim \mathcal{O}(\alpha_s), \mathcal{O}(1/m_b)$;
- annihilation topologies and higher Fock states give power-suppressed contributions;



- Main result: in the heavy-quark limit, all "non-factorizable" diagrams are dominated by hard gluons/quarks and can be calculated as expansion in $\alpha_s(m_b)$; Soft gluons are suppressed as $\Lambda_{\rm QCD}/m_b$; $\hookrightarrow$ "colour transparency argument" [Bjorken, '89]

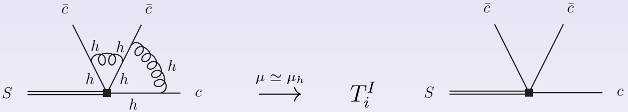
Factorization formulae from the SCET point of view:

- Soft-collinear effective theory (SCET): an EFT designed to describe processes involving energetic hadrons/jets; [Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02; [Becher, Broggio, Ferroglia '14]

- SCET: field-theoretical basis for QCDF, theoretical basis of Feynman diagrammatic QCD factorization; $\hookrightarrow$ SCET factorization is exactly the same as QCDF; [Beneke '15]

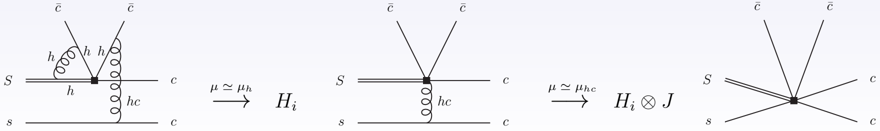
- For the form-factor term T^I: the matching procedure from QCD onto SCET is simpler;

$$\text{QCD} \rightarrow \text{SCET}_{\text{I}}(hc, c, s)$$



- For the spectator-scattering kernel T^{II}: the two-step matching from QCD onto SCET is needed due to μ_{hc};

$$\text{QCD} \rightarrow \text{SCET}_{\text{I}}(hc, c, s) \rightarrow \text{SCET}_{\text{II}}(c, s)$$



QCDF/SCET analyses of $B \to M_1 M_2$ at NLO:

- Analyses of complete sets of final states:
  - $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
  - $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
  - $B \to TP, TV$: [Cheng, Yang, 1010.3309;]
- Well-established successes based on the NLO hard-scattering functions!

- Successes of QCDF/SCET:
  - Colour-allowed tree-dominated and penguin-dominated Brs are usually quantitatively OK;
  - Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes:

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

  - Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation;
  - Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries;

Some issues in QCDF/SCET:

- Some issues in QCDF/SCET:
  - factorization of power corrections is generally broken, due to the appearance of end-point divergence;
  - could not account for some data, such as the large $\mathrm{Br}(B \to \pi^0\pi^0)$, the unmatched CP asymmetries in $B \to \pi K$ decays, $\cdots$;
  - how important is the higher-order perturbative corrections? Factorization theorem is still established?
  - what is the correct theory for power corrections? Can never exclude large sizeable power corrections theoretically!
- Motivation for nontrivial NNLO calculation:
  - conceptual aspect: check if factorization theorem still held at the NNLO?
  - phenomenologically: strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only the NLO to them; quite relevant for precise direct CP prediction;
  - exp. data driven: α_2 seems to be too small, and the $A_{CP}(\pi K)$ puzzle; Does NNLO short-distance prediction tend toward the right direction?

Status of perturbative calculation of the hard kernels $T^{I,II}$:

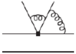
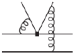
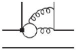
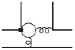
- To ascertain the short-distance contribution: need to have a reliable $\mathcal{O}(\alpha_s^2)$ hard-scattering kernels, at least their imaginary parts;

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1} \, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes

[taken from G. Bell talk at 610TH WE HERAEUS - SEMINAR – BAD HONNEF, '16]

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 [GB, Beneke, Huber, Li 15 + in progress]	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

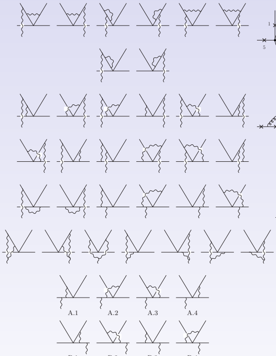
- For the spectator-scattering kernel T_i^{II}: already completed, both for the tree and penguin amplitudes; [*Beneke, Jäger '05; Kivel '06; Pilipp '07; Jain, Rothstein, Stewart '07*]

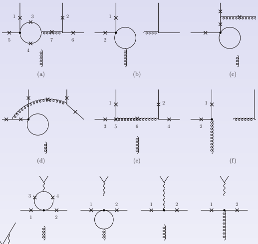
- For the form-factor term T_i^I: known for the tree amplitudes, now only for the penguin amplitudes with $Q_{1,2}^p$; [*Bell '07-'09; Beneke, Huber, Li '09; Bell, Beneke, Huber, Li '15 and work in progress*]

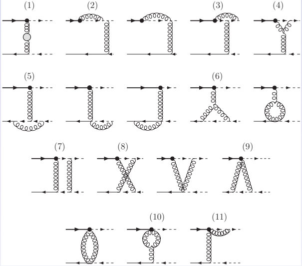
One-loop spectator scattering T^{II}:

- Final results for the hard kernel T_i^{II}: $\qquad T_i^{II}(\omega, v, u) = \int_0^1 dz\, H_i(z, u)\, J(\omega, v, z)$

  - The one-loop calculation of H^{II} using both the diagrammatical approach and the SCET formulation: QCD $\rightarrow$ SCET$_{\mathrm{I}}$ at $\mu_h \sim m_b$; [*Pilipp 07; Beneke, Jäger 05; Kivel 06*]





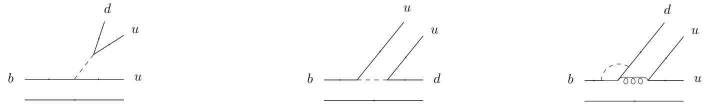


  - The one-loop calculation of the Jet function J: SCET$_{\mathrm{I}}$ $\rightarrow$ SCET$_{\mathrm{II}}$ at $\mu_{hc} \sim \sqrt{m_b \Lambda_{\mathrm{QCD}}}$; [*Beneke, Yang 05; Becher, Hill, Lee, Neubert 04*]

- Factorization theorem does hold at μ_{hc}; perturbative theory is well-behaved!

Typical topological amplitudes for $B \to M_1 M_2$:

- For non-leptonic B decays, three topological amplitudes are mostly relevant:



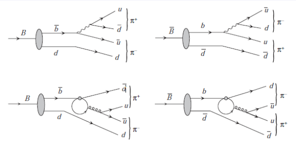
colour-allowed tree α_1 colour-suppressed tree α_2 QCD penguins α_4

- Due to CKM unitarity, the amplitude for a $\bar{B} \to \bar{f}$ decay can always be written as:

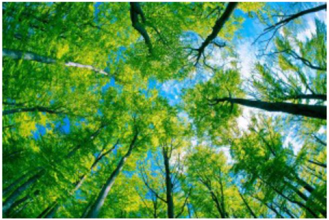
$$\bar{A}(\bar{B} \to \bar{f}) = \lambda_u^{(D)} [T + \ldots] + \lambda_c^{(D)} [P_c + \ldots]$$

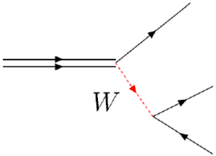
  - $\lambda_u^{(D)}$ part: $b \to u\bar{u}D$ transition, dominated by tree amplitudes $T = \alpha_{1,2}(M_1 M_2)$;
  - $\lambda_c^{(D)}$ part: $b \to Dq\bar{q}$ transition, dominated by penguin amplitudes $P_c = \alpha_4^c(M_1 M_2)$;

- For a specific decay mode, if both T and P_c involved, then direct CP asymmetry:
  - Tree amplitude:
  - Penguin amplitude:



Tree-dominated decay modes:





- Two-loop vertex corrections:

G. Bell, *"NNLO vertex corrections in charmless hadronic B decays: Imaginary part,"* Nucl. Phys. B **795** (2008) 1 [arXiv:0705.3127 [hep-ph]].

G. Bell, *"NNLO vertex corrections in charmless hadronic B decays: Real part,"* Nucl. Phys. B **822** (2009) 172 [arXiv:0902.1915 [hep-ph]].

G. Bell and V. Pilipp, *"$B \to \pi^- \pi^0 / \rho^- \rho^0$ to NNLO in QCD factorization,"* Phys. Rev. D **80** (2009) 054024 [arXiv:0907.1016 [hep-ph]].

M. Beneke, T. Huber and X. Q. Li, *"NNLO vertex corrections to non-leptonic B decays: Tree amplitudes,"* Nucl. Phys. B **832** (2010) 109 [arXiv:0911.3655 [hep-ph]].

The operator basis in QCD and SCET:

- CMM operator basis in full QCD:

$$Q_1^p = \bar{p}\gamma^\mu P_L T^A b \;\; \bar{D}\gamma_\mu P_L T^A p,$$

$$Q_2^p = \bar{p}\gamma^\mu P_L b \;\; \bar{D}\gamma_\mu P_L p,$$

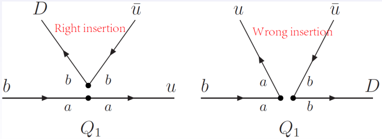
$$Q_3 = \bar{D}\gamma^\mu P_L b \;\; \sum_q \bar{q}\gamma_\mu q,$$

$$Q_4 = \bar{D}\gamma^\mu P_L T^A b \;\; \sum_q \bar{q}\gamma_\mu T^A q,$$

$$Q_5 = \bar{D}\gamma^\mu\gamma^\nu\gamma^\rho P_L b \;\; \sum_q \bar{q}\gamma_\mu\gamma_\nu\gamma_\rho q,$$

$$Q_6 = \bar{D}\gamma^\mu\gamma^\nu\gamma^\rho P_L T^A b \;\; \sum_q \bar{q}\gamma_\mu\gamma_\nu\gamma_\rho T^A q,$$

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, m_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu} b.$$



- Nonlocal SCET operator basis for RI:

$$O_1 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L h_v\right],$$

$$O_2 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \gamma_\perp^\alpha\gamma_\perp^\beta \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L \gamma^\perp_\beta\gamma^\perp_\alpha h_v\right],$$

$$O_3 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L \gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right].$$

- Nonlocal SCET operator basis for WI:

$$\tilde{O}_1 = \left[\bar{\xi}\,\gamma_\perp^\alpha P_L \chi\right]\left[\bar{\chi} P_R \gamma^\perp_\alpha h_v\right],$$

$$\tilde{O}_2 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma P_L \chi\right]\left[\bar{\chi} P_L \gamma^\perp_\alpha\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right],$$

$$\tilde{O}_3 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\gamma_\perp^\epsilon P_L \chi\right]\left[\bar{\chi} P_R \gamma^\perp_\alpha\gamma^\perp_\epsilon\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right].$$

- 8 evanescent operators in QCD: although vanish in 4-dim., but needed to complete the operator basis under renormalization!
  [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05]

Matching calculation from QCD onto SCET$_{\text{I}}$: I

- To extract the hard kernels from matching, construct the factorized QCD operator:

$$O_{\text{QCD}} \equiv \left[\bar{q}\,\frac{\not{n}_-}{2}(1-\gamma_5)q\right]\left[\bar{q}\,\not{n}_+(1-\gamma_5)b\right] = C_{FF}\,C_{\bar{q}q}\,O_1$$

$C_{\bar{q}q}$ and C_{FF}: matching coefficients for QCD currents to SCET currents; [Beneke, Huber, Li, 0810.0987]

$\Longrightarrow \langle O_{\text{QCD}}\rangle$ is the product of a light-meson LCDA and the *full QCD* heavy-to-light transition form factor.

- For "right insertion":

$$\langle Q_i\rangle = T_i\,\langle O_{\text{QCD}}\rangle + \sum_{a>1} H_{ia}\langle O_a\rangle$$

- For "wrong insertion":

$$\langle Q_i\rangle = \widetilde{T}_i\,\langle O_{\text{QCD}}\rangle + \tilde{H}_{i1}\langle\tilde{O}_1 - O_1\rangle + \sum_{a>1}\tilde{H}_{ia}\langle\tilde{O}_a\rangle$$

- On the QCD side, we have:

$$\langle Q_i\rangle = \Big\{A_{ia}^{(0)} + \frac{\alpha_s}{4\pi}\left[A_{ia}^{(1)} + Z_{ext}^{(1)}\,A_{ia}^{(0)} + Z_{ij}^{(1)}A_{ja}^{(0)}\right] + \left(\frac{\alpha_s}{4\pi}\right)^2\left[A_{ia}^{(2)} + Z_{ij}^{(1)}A_{ja}^{(1)} + Z_{ij}^{(2)}A_{ja}^{(0)} + Z_{ext}^{(1)}\,A_{ia}^{(1)} + Z_{ext}^{(2)}\,A_{ia}^{(0)} + Z_{ext}^{(1)}\,Z_{ij}^{(1)}A_{ja}^{(0)} + Z_{\alpha}^{(1)}A_{ia}^{(1)} + (-i)\,\delta m^{(1)}\,A_{ia}^{\prime(1)}\right] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle O_a\rangle^{(0)}$$

- On the SCET side, we have:

$$\langle O_a\rangle = \Big\{\delta_{ab} + \frac{\hat{\alpha}_s}{4\pi}\left[M_{ab}^{(1)} + Y_{ext}^{(1)}\,\delta_{ab} + Y_{ab}^{(1)}\right] + \left(\frac{\hat{\alpha}_s}{4\pi}\right)^2\left[M_{ab}^{(2)} + Y_{ac}^{(1)}M_{cb}^{(1)} + Y_{ab}^{(2)} + Y_{ext}^{(1)}\,M_{ab}^{(1)} + Y_{ext}^{(2)}\,\delta_{ab} + Y_{ext}^{(1)}\,Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)}M_{ab}^{(1)}\right] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle O_b\rangle^{(0)}$$

Matching calculation from QCD to SCET$_{\rm I}$: II

- Final result for RI at 1- and 2-loop level (for $Q_{5,6}$ insertion, more terms on the RHS):

$$
\begin{aligned}
T_i^{(1)} &= A_{i1}^{(1),nf} + Z_{ij}^{(1)}\, A_{j1}^{(0)}\,, \\
T_i^{(2)} &= A_{i1}^{(2),nf} + Z_{ij}^{(1)}\, A_{j1}^{(1)} + Z_{ij}^{(2)}\, A_{j1}^{(0)} + Z_{\alpha}^{(1)}\, A_{i1}^{(1),nf} + (-i)\,\delta_m^{(1)}\, A_{i1}^{\prime(1),nf} \\
&\quad + T_i^{(1)}\big[-C_{FF}^{(1)} - Y_{11} + Z_{ext}^{(1)}\big] - \sum_{b>1} H_{ib}^{(1)}\, Y_{b1}^{(1)}\,.
\end{aligned}
$$

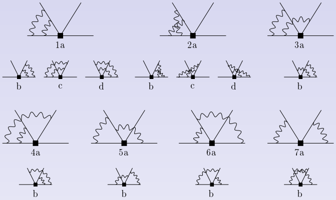
- Final result for WI at 1- and 2-loop level (for $Q_{5,6}$ insertion, more terms on the RHS):

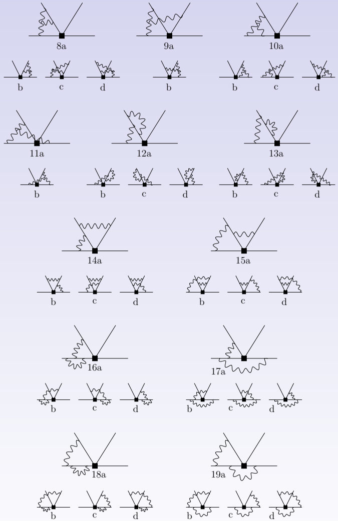
$$
\begin{aligned}
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(0)} + \widetilde{A}_{i1}^{(1),f} - A^{(1),f}\, \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\, \widetilde{A}_{i1}^{(0)} \\
\widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\, \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\, \widetilde{A}_{i1}^{(1),nf} \\
&\quad + (-i)\,\delta_m^{(1)}\, \widetilde{A}_{i1}^{\prime(1),nf} + Z_{ext}^{(1)}\,[\widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(0)}] \\
&\quad - \widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\, \widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2),f} - A^{(2),f}\, \widetilde{A}_{i1}^{(0)}] + (-i)\,\delta_m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1),f} - A^{\prime(1),f}\, \widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)})[\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\, \widetilde{A}_{i1}^{(0)}] \\
&\quad - C_{FF}^{(1)}\, \widetilde{A}_{i1}^{(0)}\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\, \widetilde{A}_{i1}^{(0)}
\end{aligned}
$$

- For $Q_{5,6}$ insertion: evanescent operator contributions are already non-zero at tree-level;

Two-loop Feynman diagrams for tree amplitudes:

- The two-loop non-factorizable diagrams: [*Beneke,Buchalla,Neubert,Sachrajda 00*]





  - totally 62 “non-factorizable” diagrams;
  - vacuum polarization insertions in gluon propagators;
  - the one-loop counter-term insertions;

- For the tree amplitudes α_1 and α_2, now complete and have been cross-checked; [*G.Bell 07, 09; M. Beneke, T. Huber, X. Q. Li 09*]

Multi-loop calculations in a nutshell: I

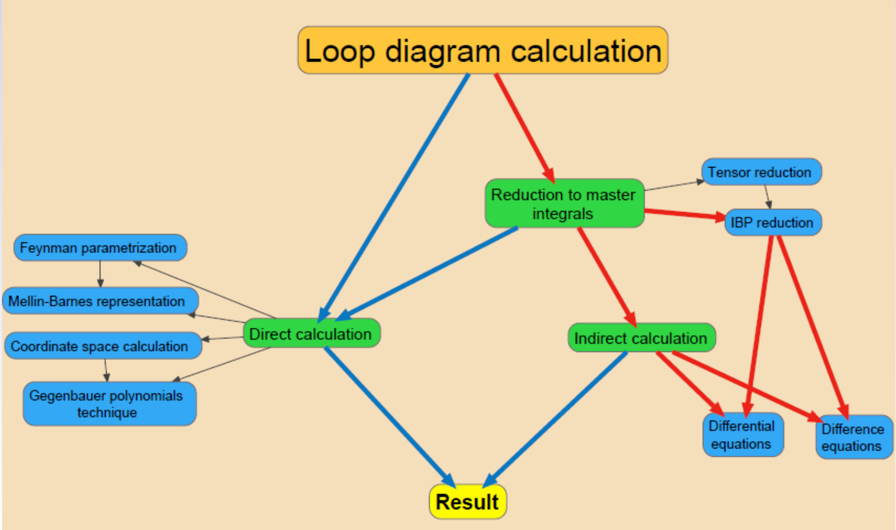
- Adopt the DR scheme with $D = 4 - 2\epsilon$, to regulate both the UV and IR div.; at two-loop order, UV and IR poles appear up to $1/\epsilon^2$ and $1/\epsilon^4$, respectively.

- Basis strategy and procedure:
  - perform the general tensor reduction via Passarino-Veltman ansatz, $\Longrightarrow$thousands of scalar integrals, [*Passarino,Veltman '79*]
  - reduce them to Master Integrals via Laporta algorithm based on IBP identities $\Longrightarrow$totally 42 MIs, [*Tkachov '81; Chetyrkin,Tkachov '81; Laporta '01; Anastasiou,Lazopoulos '04*]
  - calculate these MIs, very challenging as we need analytical results.

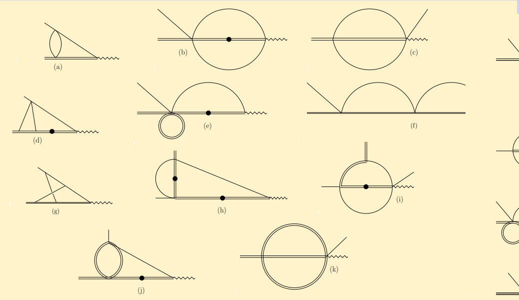
- Techniques used to calculate MIs (developed very rapidly in recent years):
  - standard Feynman/Schwinger parameterisation, only for very simpler MIs;
  - method of differential equations; [*Kotikov '91; Remiddi '97; Henn '13*]
  - Mellin-Barnes techniques; [*Smirnov '99; Tausk '99*]
  - method of sector decomposition, for numerical check! [*Binoth, Heinrich 00*]

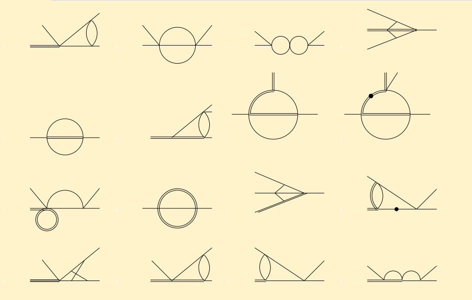
Multi-loop calculations in a nutshell:

- General precedure of the multi-loop calculations: [*R.N. Lee talk at ACAT 2013*]



List of the resulted Master Integrals:





- The double lines are massive, while the single lines massless;
- The dot on lines denotes the squared propagator;
- LHS MIs have been cross-checked in inclusive $B \to X_u l \nu$ calculations;
  [*Bell '08; Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08*]
- RHS MIs are needed for charmless hadronic B decays and have also been cross-checked;
  [*Bell '07, '09; Beneke, Huber, Li '09*]

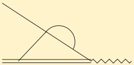
Illustration of the calculation techniques: I

- IBP IDs: for the two-loop case, there are eight IDs per scalar integral;

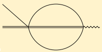
$$\int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0 \; ; \quad a^\mu = k^\mu, \, l^\mu \; ; \quad b^\mu = k^\mu, \, l^\mu, \, p_i^\mu$$

- Solve systems of these equations via Laporta algorithm;

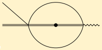
$\Longrightarrow$a scalar integral can be expressed as a linear combination of some MIs:



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$



- Differential equations: [*Kotikov '91; Remiddi '97; Henn '13*]

$$\frac{\partial}{\partial u} \mathrm{MI}_i(u) = f(u,\epsilon), \mathrm{MI}_i(u) + \sum_{j \neq i} g_j(u,\epsilon)\, \mathrm{MI}_j(u)$$

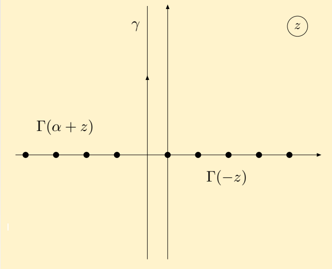
- needs results from Laporta reduction. - $\mathrm{MI}_j(u)$ are known simpler MIs.
- should fix the boundary condition by some other methods.
- choose "optimal" basis of MIs to get simple iterated integrations in each order in ϵ-expansion.

Illustration of the calculation techniques: II

- Mellin-Barnes representation: makes Feynman parameter integrals simpler; [*Smirnov '99; Tausk '99*]

$$\frac{1}{(A_1 + A_2)^{\alpha}} = \oint_{\gamma} \frac{dz}{2\pi i} A_1^z A_2^{-\alpha - z} \frac{\Gamma(-z)\,\Gamma(\alpha + z)}{\Gamma(\alpha)}$$

  - has been partially automated;
  - can be used as a numerical cross-check of our analytic calculation; [*Czakon '05; Gluza, Kajda, Riemann '07*]



- Special functions frequently used:
  - HPL up to weight 4 with argument u or $1 - u$;
  - Generalized polylogarithms Li_2, Li_3, Li_4 with argument u, $1 - u$, $\frac{u}{1-u}$;
  - Hypergeometric function pFq, needs perform ϵ-expansion; [*Maitre,Huber'05,'07*]

Numerical results for α_1 and α_2 at NNLO:

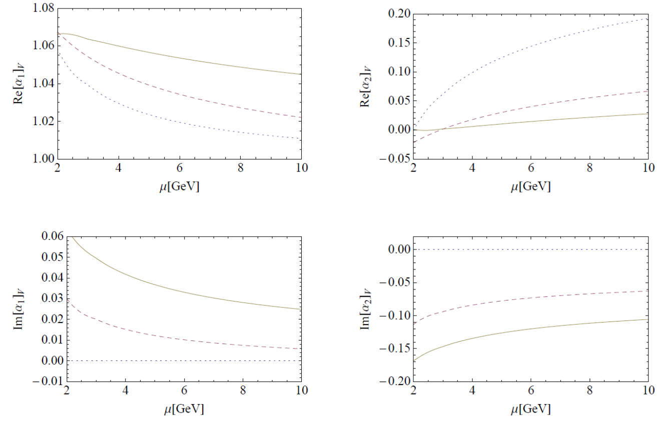
- Numerical results for colour-allowed α_1 and colour-suppress α_2 at NNLO: [*Beneke, Huber, Li, arXiv:0911.3655 [hep-ph]; Bell, arXiv:0902.1915 [hep-ph]; Bell and Pilipp, arXiv:0907.1016 [hep-ph]*]

$$
\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i
\end{aligned}
$$

- individual NNLO corrections significant, but cancelled between vertex and spectator;
- precise prediction for α_1, but larger hadronic uncertainties for α_2 from $r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$;
- *The NNLO contributions have only a marginal effect on tree-dominated B decays*.

Dependence of $\alpha_{1,2}$ on the hard scale μ_h, only vertex part!



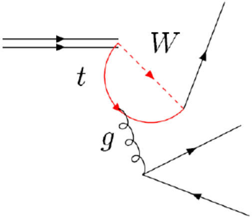
- dotted line: LO result
- dashed line: NLO result
- solid line: NNLO result

- The real parts on the scale dependence substantially reduced!
- The imaginary parts less pronounced, since it is just a first-order effect!
- Sizeable correction to imaginary part (phases), but cancellation between vertex and spectator-scattering!

Summary for tree-dominated B decays:

- NNLO corrections individually sizeable, but ultimately not large due to cancellation between vertex and spectator;
- Colour-allowed modes well described by factorization, less the purely colour-suppressed ones;
- NNLO corrections are end of the road at leading power; No indication of further large radiative corrections;
- Size of the spectator-scattering contributions in QCDF determined by $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega} \phi_B(\omega; \mu)$:
  - the current $B \to \pi\pi$ and $\pi\rho$ data prefers smaller $\lambda_B \sim 200$ MeV, compared to QCD sum rule estimate $\lambda_B(1\text{ GeV}) \sim 350 - 500$ MeV; [*Braun, Ivanov, Korchemsky '03*]
  - λ_B can be measured in $B \to \gamma\ell\nu$ decays: $\Gamma(B \to \gamma\ell\nu) \propto 1/\lambda_B^2$, NLO+$1/m_b$ corrections; [*Beneke, Rohrwild '11; Braun, Khodjamirian '12*]
  - weak constraint from BaBar '09 data: $\lambda_B > 115$ MeV; Belle '15 data: $\lambda_B > 217$ MeV [*Belle, 1504.05831*]
  - much progress in our theoretical understanding of $\phi_B(\omega; \mu)$; [*Bell, Feldmann, Wang, Yip, '13; Braun, Manashov '14; Feldmann, Lange, Wang, '14*]
- Other attempts for enhanced colour-suppressed tree amplitude:
  - $1/m_b$ power correction as a "nuisance parameter": $a_2 \to a_2 \left(1 + \rho_c e^{i\phi_c}\right)$; [*Cheng, Chua, '09*]
  - introduce the Glauber gluon effects in spectator amplitudes; [*Li, Mishima, arXiv:1407.7647 [hep-ph]*]
  - the renormalization scale for spectator interactions is much lower after applying the principle of maximum conformality, $Q_1^H \simeq 0.75 - 0.90$ GeV; [*Qiao, Zhu, Wu, Brodsky, arXiv:1408.1158 [hep-ph]*]

Penguin-dominated decay modes:



- Two-loop vertex corrections:

G. Bell, M. Beneke, T. Huber and X. Q. Li, *"Two-loop current – current operator contribution to the non-leptonic QCD penguin amplitude,"* Phys. Lett. B **750** (2015) 348 [arXiv:1507.03700 [hep-ph]].

C. S. Kim and Y. W. Yoon, *"Order α_s^2 magnetic penguin correction for B decay to light mesons,"* JHEP **1111** (2011) 003 [arXiv:1107.1601 [hep-ph]].

Motivation for NNLO corrections to penguin amplitudes:

- Many decay channels are penguin-dominated, very sensitive to penguin amplitudes α_4^p;

$$\mathcal{A}_{B^- \to \pi^- \bar{K}^0} = \lambda_c^{(s)} [P_c - \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [P_u - \frac{1}{3} P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{B^- \to \pi^0 K^-} = \lambda_c^{(s)} [P_c + P_c^{EW} + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + C + P_u + P_u^{EW} + \frac{2}{3} P_u^{C,EW}]$$

$$\mathcal{A}_{\bar{B}^0 \to \pi^+ K^-} = \lambda_c^{(s)} [P_c + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + P_u + \frac{2}{3} P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{\bar{B}^0 \to \pi^0 \bar{K}^0} = \lambda_c^{(s)} [-P_c + P_c^{EW} + \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [C - P_u + P_u^{EW} + \frac{1}{3} P_u^{C,EW}]$$

Mode	Br $[10^{-6}]$	$A_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+ K^0$	$23.79^{+0.75}_{-0.75}$	-0.015 ± 0.019	
$B^+ \to \pi^0 K^+$	$12.94^{+0.52}_{-0.51}$	0.040 ± 0.021	
$B^0 \to \pi^- K^+$	$19.57^{+0.53}_{-0.52}$	-0.082 ± 0.006	
$B^0 \to \pi^0 K^0$	$9.93^{+0.49}_{-0.49}$	-0.01 ± 0.10	0.57 ± 0.17

- Due to CKM unitarity, the amplitude for a $\bar{B} \to \bar{f}$ decay can always be written as:

$$\bar{A}(\bar{B} \to \bar{f}) = \lambda_u^{(D)} [T + \ldots] + \lambda_c^{(D)} [P_c + \ldots]$$

- *T*o predict direct CP asymmetries, need calculate both T and P_c to a high precision level;

The dominant contribution to a_4^p: I

- The leading penguin amplitudes including the $\mathcal{O}(\alpha_s^2)$ spectator terms: [*Beneke, Jäger '06*]

$$a_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_{\rm V} + [0.003 - 0.013i]_{\rm P} + [??+??i]_{\rm NNLO}$$

$$+ \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{\rm HV} - [0.000 + 0.001i]_{\rm HP} + [0.001]_{\rm tw3}\right\}$$

$$a_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_{\rm V} - [0.001 + 0.007i]_{\rm P} + [??+??i]_{\rm NNLO}$$

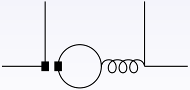
$$+ \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{\rm HV} + [0.000 - 0.000i]_{\rm HP} + [0.001]_{\rm tw3}\right\}$$

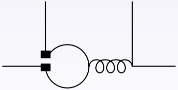
- Although $\mathcal{O}(\alpha_s^2)$ spectator effect on $\alpha_{1,2}$ significant, but small on a_4^p due to numerical cancellation $\Rightarrow$ *how about the 2-loop vertex corrections to a_4^p? significant or marginal?*

- For the penguin topology, need to consider two distinct ways of contraction:

$-$left : $\langle Q_{4,6}\rangle$, $\quad \frac{2}{3}\ln\frac{m_b^2}{\mu^2} - G(s)$

$-$right : $\langle Q_{1-6}\rangle$, $\quad \frac{2}{3}\left(\ln\frac{m_b^2}{\mu^2} + 1\right) - G(s)$





The dominant contribution to a_4^p: II

- The NNLO correction to a_4^p comes mainly from Q_1 and Q_2 insertions:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7^{eff}	C_8^{eff}
LL	-0.536	1.028	-0.006	-0.073	0.0005	0.001	-0.319	-0.151
NLL	-0.320	1.009	-0.005	-0.088	0.0004	0.001	-0.312	-0.171
NNLL	-0.285	1.010	-0.006	-0.087	0.0004	0.001	-0.302	-0.164

- The penguin contractions of $Q_{1,2}^p$ give the largest contribution at any given loop order;

$$a_4^u(\pi\bar{K})_{|\text{NLO}} = (-0.0087 - 0.0172i)_{|Q_{1,2}} + (0.0042 + 0.0041i)_{|Q_{3-6}} + 0.0083_{|Q_{8g}}$$

$$a_4^c(\pi\bar{K})_{|\text{NLO}} = (-0.0131 - 0.0102i)_{|Q_{1,2}} + (0.0042 + 0.0041i)_{|Q_{3-6}} + 0.0083_{|Q_{8g}}$$

- while cancellations exist for the real part, the imaginary part from $Q_{1,2}^p$ is clearly dominant;
- at NLO, the SD direct CP asymmetries mainly determined by the $Q_{1,2}^p$ insertion;
- *reasonable to assume that the $Q_{1,2}^{u,c}$ insertion at two loops also captures the bulk of the yet unknown NNLO form-factor-term contribution to $a_4^{u,c}$.*

Matching from QCD to SCET$_{\rm I}$:

- The CMM operator basis in full QCD:

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\,(\bar{D}_L \gamma_\mu T^A p_L), \qquad Q_2^p = (\bar{p}_L \gamma^\mu b_L)\,(\bar{D}_L \gamma_\mu p_L),$$

$$+\ \text{QCD penguin operators and evanescent operators}$$

- The nonlocal operator basis in SCET:

$$O_1 = \bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\;\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\,,$$

$$\tilde{O}_n = \bar{\xi}\,\gamma_\perp^\alpha \gamma_\perp^{\mu_1}\gamma_\perp^{\mu_2}\ldots\gamma_\perp^{\mu_{2n-2}}\chi\;\bar{\chi}(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}}\ldots\gamma_{\perp\mu_1}h_v,$$

$\tilde{O}_1 - O_1/2$ is another evanescent operator

- The master formulae at LO, NLO, and NNLO read, respectively,

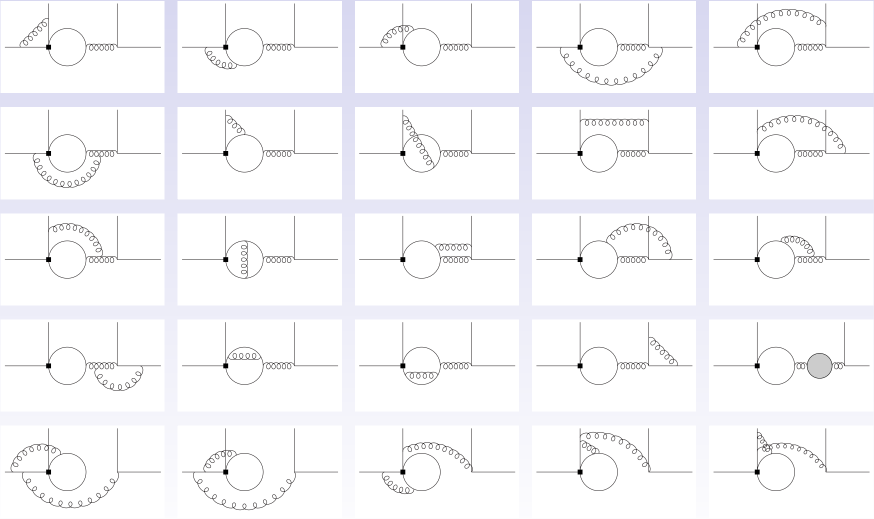
$$\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)}\,, \qquad \widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \ldots\,,$$

$$\widetilde{T}_i^{(2)} = \widetilde{A}_{i1}^{(2)\rm nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_\alpha^{(1)}\,\widetilde{A}_{i1}^{(1)\rm nf} + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\rm nf}$$

$$+ Z_{ext}^{(1)}\left[\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\right] - \widetilde{T}_i^{(1)}\left[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\right] + \ldots$$

NNLO penguin amplitudes with $Q^p_{1,2}$ insertion: I

- On the QCD side, relevant Feynman diagrams with $Q^p_{1,2}$ insertion ($\sim$ 70 diagrams):



NNLO penguin amplitudes with $Q^p_{1,2}$ insertion: II

- The quark in the fermion loop can either be massless (for $p = u$) or massive (for $p = c$);
- In the massive case, a genuine two-loop, two-scale problem involved; also $s_c = m_c^2/m_b^2$;
- Procedure to perform the calculation in QCD:
  - regularize UV and IR divergences dimensionally, with $D = 4 - 2\epsilon$; poles up to $1/\epsilon^3$;
  - perform the Passarino-Veltman decomposition of the tensor integrals;
  - perform the Laporta reduction of the scalar Feynman integrals to the MIs;
  - use modern multi-loop analytic calculation technique to calculate the obtained MIs;
- Procedure to perform the matching calculation:
  - calculate SCET matrix elements, match QCD to SCET, and then extract the matching coefficients;
  - check factorization theorem; compute convolution in u with the light meson LCDAs;
- For the massive charm-type insertions, 29 new MIs found and computed based on the DE approach in a canonical basis; [*Bell, Huber '14; Henn '13*]

Calculate the MIs in a canonical basis:

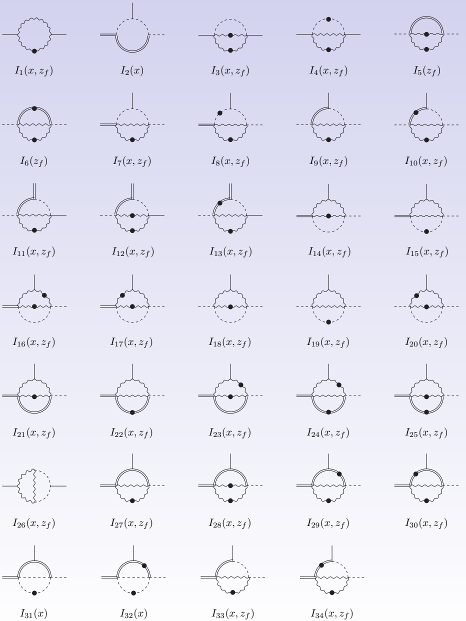
- Choose an "optimal" basis of MIs, so that the DEs decouple order-by-order in ϵ expansion, and the total weight of each MI is zero to all orders in ϵ: [*Henn, 1304.1806*]

$$\frac{\partial}{\partial x_m} \vec{M}(\epsilon, x_n) = \epsilon A_m(x_n) \vec{M}(\epsilon, x_n)$$

- The above simplified form of DEs is trivial to solve in terms of iterated integrals;[*Henn '13; Bell, Huber '14*]

- Together with boundary conditions, analytic results of the MIs are most compactly written in terms of generalised HPLs (or Goncharov polylogarithms); [*Maitre, 0703052*]

- The analytic results make it much easier to handel the threshold at $\bar{u}m_b^2 = 4m_c^2$ and the convolution integral over the product of the kernels and the meson LCDAs; [*Bell, Beneke, Huber, Li '15*]



Numerical result for a_4^p with $Q_{1,2}^p$ insertion: I

- For $Q_{1,2}^u$ insertion: fully analytic;

$$
\begin{aligned}
a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.49-1.32i]_{\mathrm{P}_1} - [0.32+0.71i]_{\mathrm{P}_2} \\
&+ \left[\frac{r_{\rm sp}}{0.434}\right]\left\{[0.13]_{\rm LO} + [0.14+0.12i]_{\rm HV} - [0.01-0.05i]_{\rm HP} + [0.07]_{\rm tw3}\right\} \\
&= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i
\end{aligned}
$$

$\sim 15\%$ correction to real part, $\sim 40\%$ to imaginary part for $a_4^u(\pi\bar{K})$

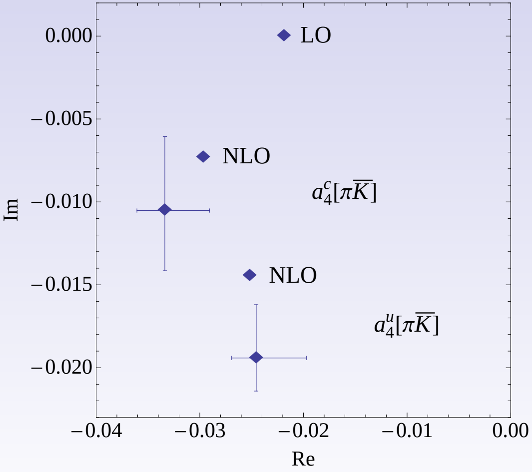
- For $Q_{1,2}^c$ insertion: semi-analytic results;

$$
\begin{aligned}
a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.05-0.62i]_{\mathrm{P}_1} - [0.77+0.50i]_{\mathrm{P}_2} \\
&+ \left[\frac{r_{\rm sp}}{0.434}\right]\left\{[0.13]_{\rm LO} + [0.14+0.12i]_{\rm HV} + [0.01+0.03i]_{\rm HP} + [0.07]_{\rm tw3}\right\} \\
&= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i
\end{aligned}
$$

$\sim 25\%$ correction to real part, $\sim 50\%$ to imaginary part for $a_4^c(\pi\bar{K})$

Numerical result for a_4^p with $Q_{1,2}^p$ insertion: II

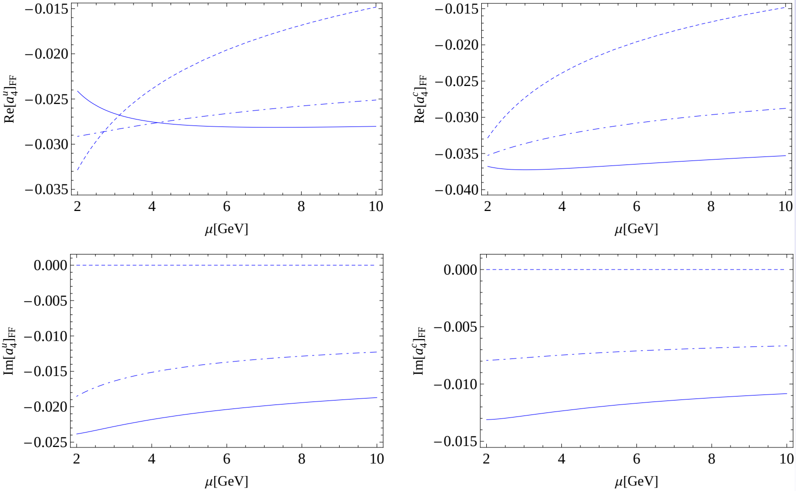
- Graphical representations of $a_4^p(\pi\bar{K})$:



- The larger uncertainty of $\mathrm{Im}(a_4^c(\pi\bar{K}))$ is due to the sensitivity to the charm-quark.

Numerical result for a_4^p with $Q_{1,2}^p$ insertion: III

- With the NNLO corrections included, the scale dependence of a_4^u and a_4^c reduced:



form-factor term only! red-dashed: LO; red-line: NLO; qquad blue-line: NNLO

The full QCD penguin amplitude in QCDF:

- In QCDF, the full QCD penguin amplitude is defined as: [*Beneke, Neubert '03*]

$$\hat{\alpha}_4^p(M_1M_2) = a_4^p(M_1M_2) \pm r_\chi^{M_2} a_6^p(M_1M_2) + \beta_3^p(M_1M_2)$$

  - a_4^p: the only leading-power contribution, with its real part being of order -0.03;
  - β_3^p: $1/m_b$-suppressed annihilation contribution; can only be estimated based on a two-parameter model,

    $\int_0^1 \frac{dx}{x} \to X_A = (1 + \varrho_A\, e^{i\phi_A}) \ln \frac{m_B}{\Lambda_h}$, with $\Lambda_h = 500$ MeV;
  - $r_\chi^{M_2} a_6^p(M_1M_2)$: the power-suppressed scalar penguin amplitude; very small when $M_2 = V$, but larger than $a_4^p(M_1M_2)$ for $M_2 = P$ due to the "chiral enhancement" factor r_χ^P;
  - the interference between $a_4^p(M_1M_2)$ and $a_6^p(M_1M_2)$ is constructive for PP, but destructive for VP;
- The impact of a correction to a_4^p is always diluted by the other power-suppressed terms;
- When $M_2 = V$, the computation of a_4^p ascertains the SD contribution, and hence the direct CP asymmetry, but there is an uncertain annihilation contribution of similar size;
- When $M_2 = P$, there is another NNLO SD contribution from a_6^p, difficult though not impossible to calculate, since it is power-suppressed;

The penguin-to-tree ratio in QCDF: I

- The magnitude of the penguin-to-tree ratio can be extracted from data, and provides a crucial test of the QCDF approach: [*Beneke, Neubert '03*]

$$\left|\frac{\hat{\alpha}_4^c(\pi\bar{K})}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}\right| = \left|\frac{V_{ub}}{V_{cb}}\right| \frac{f_\pi}{f_K} \left[\frac{\Gamma_{\pi^-\bar{K}^0}}{2\Gamma_{\pi^-\pi^0}}\right]^{1/2}$$

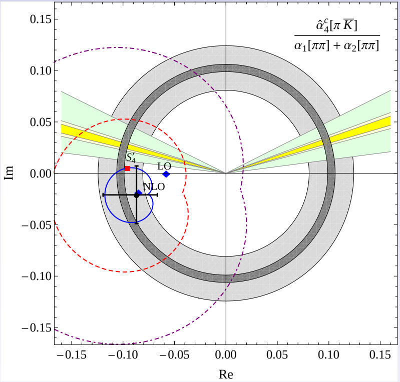
- The relative strong phase of the ratio can be probed by considering: [*Beneke, Neubert '03*]

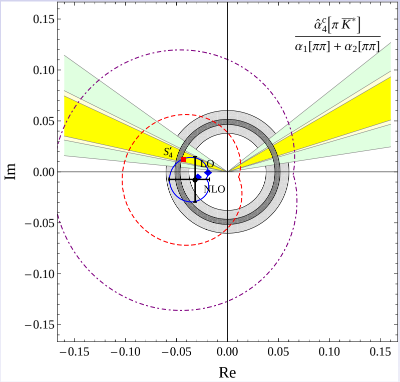
$$-\sin\psi + \frac{\mathrm{Im}\,\mathcal{R}}{\mathrm{Re}\,\mathcal{R}}\cos\psi = \frac{1}{2\sin\gamma\,\mathrm{Re}\,\mathcal{R}} \left|\frac{V_{cs}}{V_{us}}\right| \frac{f_\pi}{f_K} \frac{\Gamma_{\pi^+K^-}}{\sqrt{2\Gamma_{\pi^-\pi^0}\Gamma_{\pi^-\bar{K}^0}}} A_{\mathrm{CP}}(\pi^+K^-)$$

  - ψ: the phase of the amplitude ratio;
  - $\mathcal{R} = \frac{\alpha_1(\pi\bar{K})+\hat{\alpha}_4^u(\pi\bar{K})}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}$

- For $\pi\bar{K}$, $\pi\bar{K}^*$, normalized to the $\pi\pi$ final state, thus free of $f_+^{B\to\pi}$ uncertainty;
- For $\rho\bar{K}$, $\rho_L\bar{K}_L^*$, normalized to the $\rho_L\rho_L$ final state, thus free of $A_0^{B\to\rho}$ uncertainty;
- Together with the exp. data, these two equations provide useful information on the ratio;

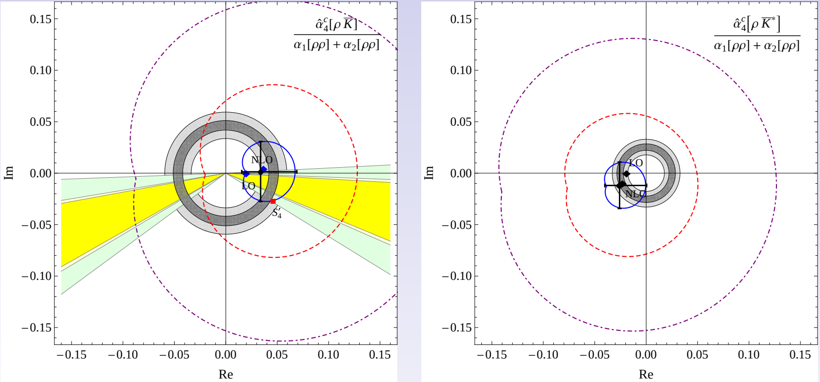
The penguin-to-tree ratio in QCDF: II





- Despite sizable NNLO correction to a_4^c, difference between NNLO and NLO is small due to "dilution" and partial cancellation in the amplitude ratio;
- Two solutions for the wedge, but the one that does not match the theoretical prediction is excluded by $\Gamma_{\pi^+K^-}/\Gamma_{\pi^-\bar{K}^0} < 1$ and similarly for PV, VP modes;
- Only the πK CP asymmetry now requires a value larger than $\varrho_A = 1$ for a perfect fit;

The penguin-to-tree ratio in QCDF: III



- The different magnitude of the *PP* penguin amplitude vs. *PV*, *VP* and *VV* is clearly reflected in the data as predicted;
- An annihilation contribution of 0.02 to 0.03 seems to be required, except for the longitudinal *VV* final states;
- The red square S_4': theoretical prediction with $\varrho_A = 1$ and the phase $\phi_A = -55^\circ$ (PP), $\phi_A = -45^\circ$ (PV), $\phi_A = -50^\circ$ (VP); $\hookrightarrow$ the favoured parameter set;

Conclusion and outlook

- NNLO calculation for hadronic B decays at leading power in QCDF (almostly) complete:
  - two-loop vertex corrections to tree amplitudes $\alpha_{1,2}$ now complete;
  - two-loop corrections with $Q_{1,2}^p$ insertion to α_4^p now complete;
  - two-loop corrections from $Q_{3,4,5,6}$ and Q_{8g} operators in progress;
- The NNLO calculation provides a sizable correction to the SD part of direct CP asymmetry, but its effect tempered by power-suppressed a_6^p and β_3^p terms;
- The NNLO correction does not help resolving the πK CP asymmetry puzzle, nor does it render the poorly known annihilation terms redundant;
- The NNLO SD corrections to the power-suppressed a_6^p possible but more complicated (for spectator term, Beneke, Jaeger, Wang, in progress; for form-factor term, our next project!);
- Given the known NNLO SD contribution and the amount of data, imperative to better determine the annihilation amplitude, presumably through fits to data (to do next!).

Thank you for your attention!

Factorization test with the semi-leptonic data:

$$R_\pi \equiv \frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\, |V_{ud}|^2\, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From exclusive semi-leptonic data (HFAG 2014):

  $[\,|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|\,]_{\text{exp}} = 1.27 \pm 0.04$

- Prediction with $\lambda_B = 0.35$ GeV:

  $|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)| = 1.24^{+0.16}_{-0.10}$

- Good agreements observed, supporting QCD factorization!

- The main uncertainties: λ_B, α_2^π and power corrections.

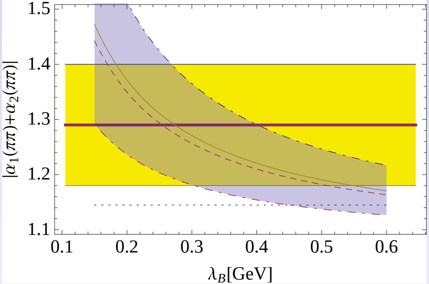


Figure from BHL2009 with obsolete data (yellow band)

$$[\,|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|\,]_{\text{exp}} = 1.29 \pm 0.11$$

- *C*olour-suppressed tree amplitude α_2 can be large only if it also has a large relative phase!

- It is interesting to extend to the other final state, $R_\rho = 1.75^{+0.37}_{-0.24}$ $(2.08^{+0.50}_{-0.46})$;

Branching ratios for tree-dominated decays:

	Theory I		Theory II		Experiment
$B^- \to \pi^- \pi^0$	$5.43^{+0.06}_{-0.06}{}^{+1.45}_{-0.84}$	$(\star)$	$5.82^{+0.07}_{-0.06}{}^{+1.42}_{-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+ \pi^-$	$7.37^{+0.86}_{-0.69}{}^{+1.22}_{-0.97}$	$(\star)$	$5.70^{+0.70}_{-0.55}{}^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0 \pi^0$	$0.33^{+0.11}_{-0.08}{}^{+0.42}_{-0.17}$		$0.63^{+0.12}_{-0.10}{}^{+0.64}_{-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \to \pi^- \rho^0$	$8.68^{+0.42}_{-0.41}{}^{+2.71}_{-1.56}$	$(\star\star)$	$9.84^{+0.41}_{-0.40}{}^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0 \rho^-$	$12.38^{+0.90}_{-0.77}{}^{+2.18}_{-1.41}$	$(\star)$	$12.13^{+0.85}_{-0.73}{}^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+ \rho^-$	$17.80^{+0.62}_{-0.56}{}^{+1.76}_{-2.10}$	$(\star)$	$13.76^{+0.49}_{-0.44}{}^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^- \rho^+$	$10.28^{+0.39}_{-0.39}{}^{+1.37}_{-1.42}$	$(\star\star)$	$8.14^{+0.34}_{-0.33}{}^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm \rho^\mp$	$28.08^{+0.27}_{-0.19}{}^{+3.82}_{-3.50}$	$(\dagger)$	$21.90^{+0.20}_{-0.12}{}^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0 \rho^0$	$0.52^{+0.04}_{-0.03}{}^{+1.11}_{-0.43}$		$1.49^{+0.07}_{-0.07}{}^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho_L^- \rho_L^0$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho_L^+ \rho_L^-$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho_L^0 \rho_L^0$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\text{ GeV}) = 0.35 \pm 0.15$ GeV

Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\text{ GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

- First error from γ and V_{cb}; V_{ub} uncertainty not included;
- Second error from hadronic inputs; form-factor uncertainty not included for marked modes;
- Theory II: small λ_B and form-factor hypothesis;

Direct CP asymmetries in $B \to \pi K^{(*)}$, ρK:

- Besides the direct CP asymmetries themselves, consider the following two observables:
  - the CP asymmetry difference (the so-called "πK" puzzle):

$$\delta(\pi\bar{K}) = A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

  - the CP asymmetry "sum rule": [*Gronau '05*]

$$\Delta(\pi\bar{K}) = A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^-\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^-\bar{K}^0) - \frac{2\Gamma_{\pi^0K^-}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0\bar{K}^0)$$

- $\Delta(\pi\bar{K})$: expected to be small, since the leading CP-violating interference of QCD penguin and tree amplitudes cancels out in the sum;
- For different modes, direct CP asymmetry arises from interference between different terms:

$$\begin{aligned}
\mathcal{A}_{B^-\to\pi^-\bar{K}^0} &= A_{\pi\bar{K}}\,\hat{\alpha}_4^p\,,\\
\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} &= A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right] + A_{\bar{K}\pi}\left[\delta_{pu}\,\alpha_2 + \delta_{pc}\,\tfrac{3}{2}\alpha_{3,\rm EW}^c\right],\\
\mathcal{A}_{\bar{B}^0\to\pi^+K^-} &= A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right],\\
\sqrt{2}\,\mathcal{A}_{\bar{B}^0\to\pi^0\bar{K}^0} &= A_{\pi\bar{K}}\left[-\hat{\alpha}_4^p\right] + A_{\bar{K}\pi}\left[\delta_{pu}\,\alpha_2 + \delta_{pc}\,\tfrac{3}{2}\alpha_{3,\rm EW}^c\right]
\end{aligned}$$

Direct CP asymmetries in $B \to \pi K$ decays:

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^0$	$0.71^{+0.13}_{-0.14}{}^{+0.21}_{-0.19}$	$0.77^{+0.14}_{-0.15}{}^{+0.23}_{-0.22}$	$0.10^{+0.02}_{-0.02}{}^{+1.24}_{-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42^{+1.77}_{-1.76}{}^{+1.87}_{-1.88}$	$10.18^{+1.91}_{-1.90}{}^{+2.03}_{-2.62}$	$-1.17^{+0.22}_{-0.22}{}^{+20.00}_{-6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25^{+1.36}_{-1.36}{}^{+2.13}_{-2.58}$	$8.08^{+1.52}_{-1.51}{}^{+2.52}_{-2.65}$	$-3.23^{+0.61}_{-0.61}{}^{+19.17}_{-3.36}$	-8.2 ± 0.6
$\pi^0 \bar{K}^0$	$-4.27^{+0.83}_{-0.77}{}^{+1.48}_{-2.23}$	$-4.33^{+0.84}_{-0.78}{}^{+3.29}_{-2.32}$	$-1.41^{+0.27}_{-0.25}{}^{+5.54}_{-6.10}$	1 ± 10
$\delta(\pi \bar{K})$	$2.17^{+0.40}_{-0.40}{}^{+1.39}_{-0.74}$	$2.10^{+0.39}_{-0.39}{}^{+1.40}_{-2.86}$	$2.07^{+0.39}_{-0.39}{}^{+2.76}_{-4.55}$	12.2 ± 2.2
$\Delta(\pi \bar{K})$	$-1.15^{+0.21}_{-0.22}{}^{+0.55}_{-0.84}$	$-0.88^{+0.16}_{-0.17}{}^{+1.31}_{-0.91}$	$-0.48^{+0.09}_{-0.09}{}^{+1.09}_{-1.15}$	-14 ± 11

- "NLO" and "NNLO": only perturbatively calculable SD contribution included;
- "NNLO+LD": power-suppressed spectator and annihilation terms included back;
- For LD effect, mainly from β_3^p since penguin-dominated; adopt the S_4' scenario;
- For πK, the NNLO change is minor, since a_4^c only part of the SD penguin amplitude;
- NNLO correction does not help resolving the observed πK CP asymmetry puzzle;

Direct CP asymmetries in $B \to \pi K^*$, ρK decays:

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^{*0}$	$1.36^{+0.25+0.60}_{-0.26-0.47}$	$1.49^{+0.27+0.69}_{-0.29-0.56}$	$0.27^{+0.05+3.18}_{-0.05-0.67}$	-3.8 ± 4.2
π^0K^{*-}	$13.85^{+2.40+5.84}_{-2.70-5.86}$	$18.16^{+3.11+7.79}_{-3.52-10.57}$	$-15.81^{+3.01+69.35}_{-2.83-15.39}$	-6 ± 24
π^+K^{*-}	$11.18^{+2.00+9.75}_{-2.15-10.62}$	$19.70^{+3.37+10.54}_{-3.80-11.42}$	$-23.07^{+4.35+86.20}_{-4.05-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33+7.59}_{-3.00-12.57}$	$-15.11^{+2.93+12.34}_{-2.65-10.64}$	$2.16^{+0.39+17.53}_{-0.42-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72+5.44}_{-0.67-4.30}$	$-1.54^{+0.45+4.60}_{-0.58-9.19}$	$7.26^{+1.21+12.78}_{-1.34-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38+3.38}_{-1.28-5.35}$	$-3.45^{+0.67+9.48}_{-0.59-4.95}$	$-1.02^{+0.19+4.32}_{-0.18-7.86}$	-5 ± 45
$\rho^-\bar{K}^0$	$0.38^{+0.07+0.16}_{-0.07-0.27}$	$0.22^{+0.04+0.19}_{-0.04-0.17}$	$0.30^{+0.06+2.28}_{-0.06-2.39}$	-12 ± 17
ρ^0K^-	$-19.31^{+3.42+13.95}_{-3.61-8.96}$	$-4.17^{+0.75+19.26}_{-0.80-19.52}$	$43.73^{+7.07+44.00}_{-7.62-137.77}$	37 ± 11
ρ^+K^-	$-5.13^{+0.95+6.38}_{-0.97-4.02}$	$1.50^{+0.29+8.69}_{-0.27-10.36}$	$25.93^{+4.43+25.40}_{-4.90-75.63}$	20 ± 11
$\rho^0\bar{K}^0$	$8.63^{+1.59+2.31}_{-1.65-1.69}$	$8.99^{+1.66+3.60}_{-1.71-7.44}$	$-0.42^{+0.08+19.49}_{-0.08-8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17^{+2.80+7.98}_{-2.96-5.39}$	$-5.67^{+0.96+10.86}_{-1.01-9.79}$	$17.80^{+3.15+19.51}_{-3.01-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75^{+1.62+4.78}_{-1.66-6.48}$	$-10.84^{+1.98+11.67}_{-2.09-9.09}$	$-2.43^{+0.46+4.60}_{-0.42-19.43}$	-37 ± 37

- For πK^* a_6^c is small; for ρK cancellation between a_4^c and a_6^c; $\hookrightarrow$ large modification for π^0K^{*-}, π^+K^{*-}, ρ^0K^-, ρ^+K^- due to $\mathrm{Im}(\hat{\alpha}_4^c/\alpha_1)$; Less pronounced for the others;