Two-dimensional Model for mesons in the light-cone and the axial gauge

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The 't Hooft model in the light-cone gauge

1974, 't Hooft. Two-dimensional model for mesons. [1]
QCD_2 Lagrangian in the light-cone gauge:

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}(\partial_- A_+)^2 - \bar{q}\left(\not{\partial} - m + g\gamma^+ A_+\right) q \tag{1}$$

The 't Hooft equation for meson $a\bar{b}$:

$$\mu^2 \Phi^{a\bar{b}}(x) = \left(\frac{m_a^2 - \beta^2}{x} + \frac{m_{\bar{b}}^2 - \beta^2}{1-x}\right)\Phi^{a\bar{b}}(x) - \beta^2 \int dy \mathcal{P}\frac{1}{(x-y)^2}\Phi^{a\bar{b}}(y) \tag{2}$$

where $\beta^2 = \frac{g^2 N_c}{\pi}$ is fixed with $N_c \to \infty$.
Meson mass μ.
Meson wave function $\Phi^{a\bar{b}}(x)$.

The Bathe-Salpeter equation in the axial gauge

1978, Bars and Green. The Hamitonian approach in the axial gauge. [2]
First, we need to get Bogoliubov angle $\theta(p)$:

$$p\cos[\theta(p)] - m\sin[\theta(p)] = \frac{\beta}{8}\int dk \frac{\mathcal{P}}{(k-p)^2}\sin[\theta(p)-\theta(k)] \tag{3}$$

Then, we have the meson energy function E(p):

$$E(p) = m\cos[\theta(p)] + p\sin[\theta(p)] + \frac{\beta}{8}\int dk \frac{\mathcal{P}}{(k-p)^2}\cos[\theta(p)-\theta(k)] \tag{4}$$

The Bathe-Salpeter equation in the axial gauge

Third, solve the coupled equations for the meson wave function :

$$(E(p)+E(P-p)-P_0)\ \psi_+(P,p) = \frac{\beta}{4}\int dk \frac{\mathcal{P}}{(k-p)^2}\Big[C(p,k,P)\,\psi_+(P,k) - S(p,k,P)\,\psi_-(P,k)\Big]$$
$$(E(p)+E(P-p)+P_0)\ \psi_-(P,p) = \frac{\beta}{4}\int dk \frac{\mathcal{P}}{(k-p)^2}\Big[C(p,k,P)\,\psi_-(P,k) - S(p,k,P)\,\psi_+(P,k)\Big] \tag{5}$$

p the quark momentum
P the meson momentum
C and S are functionals of $\theta(p)$,$\theta(k)$,$\theta(P-p)$ and $\theta(P-k)$.

Bars and Green predicted:

$$\begin{aligned} P &\rightarrow \infty \\ \psi_-(P,x) &\rightarrow 0 \\ \psi_+(P,x) &\rightarrow \Psi_{LC}(x) \\ x &= \frac{p}{P} \end{aligned} \tag{6}$$

Numerical solutions

Ming Li et. [3] solved the B-S functions when the meson is at rest.
We construct a basis to solve the coupled eigen-system numerically.
Part of the results:

Table : mass spectra with bare quark mass 2.11

n	0	1	2
't Hooft	4.91409	6.16832	7.05811
p=5.	4.91344	6.16771	7.05838
p=10.	4.91297	6.16738	7.05972
p=20.	4.91239	6.16709	7.06868
p=50.	4.91509	6.16799	7.06866

Numerical solutions

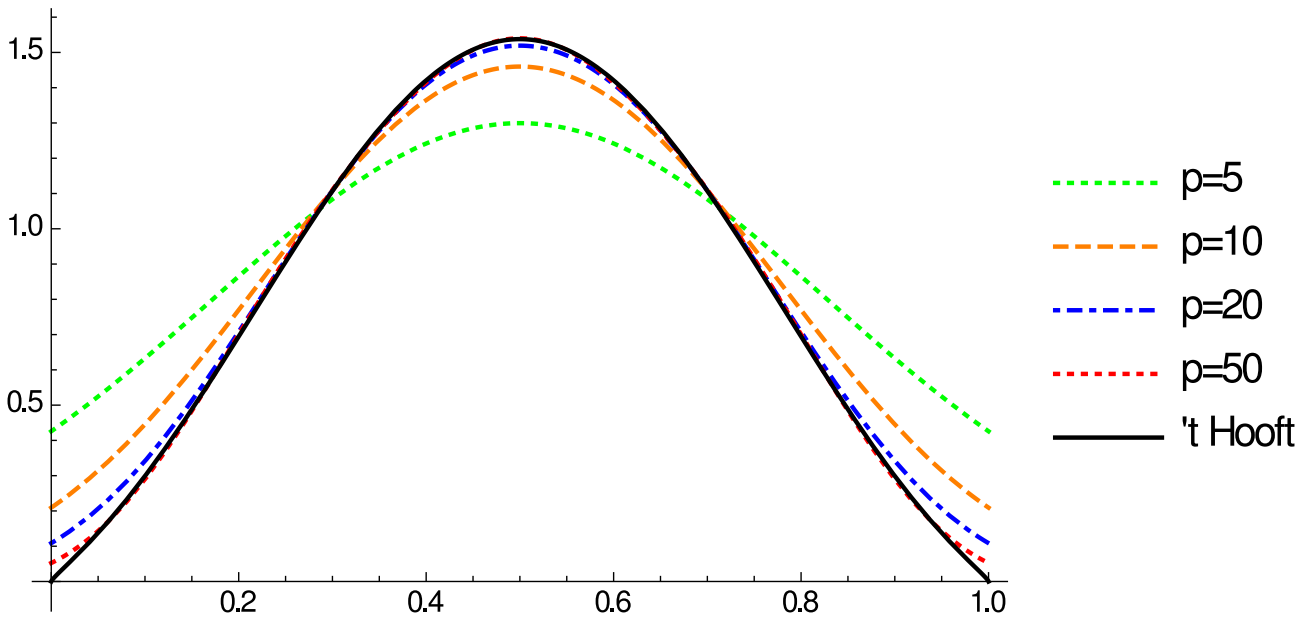

Figure : the ground state wavefunctions for various meson momenta

Numerical solutions

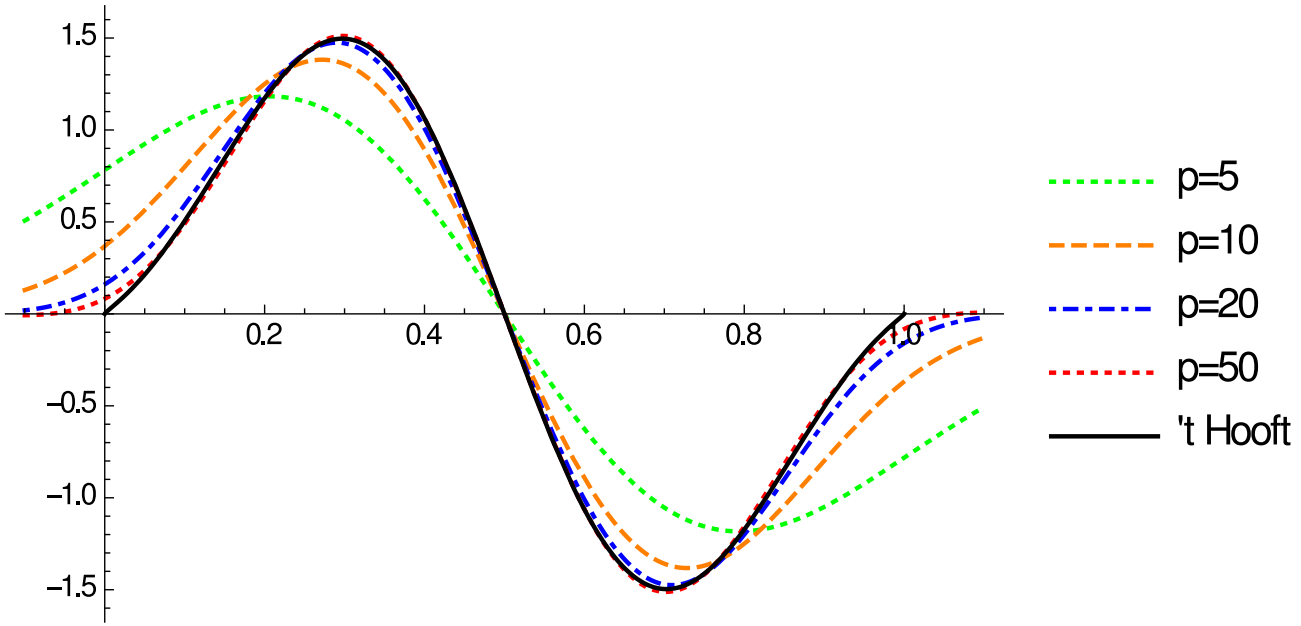

Figure : the first excited state wavefunctions for various meson momenta

Numerical solutions

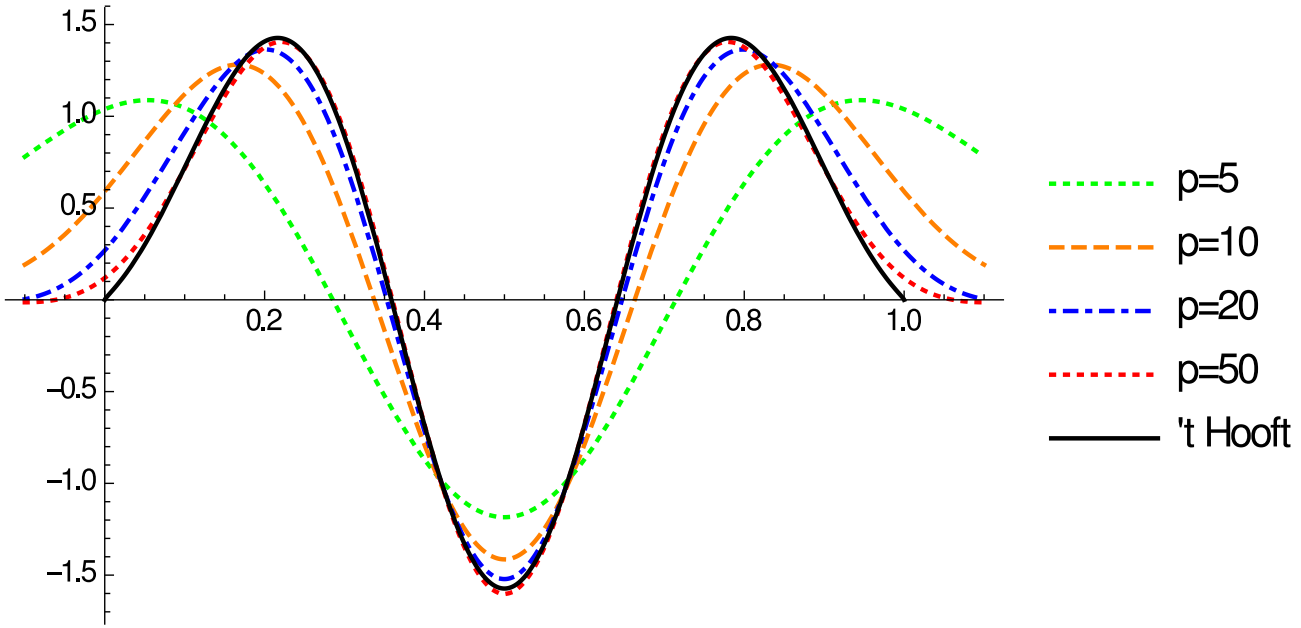

Figure : the second excited state wavefunctions for various meson momenta

Many other papers not included.


G. 't Hooft, Nucl. Phys. B **75**, 461 (1974). doi:10.1016/0550-3213(74)90088-1


M. Li, L. Wilets and M. C. Birse, J. Phys. G **13**, 915 (1987). doi:10.1088/0305-4616/13/7/005


I. Bars and M. B. Green, Phys. Rev. D **17**, 537 (1978). doi:10.1103/PhysRevD.17.537